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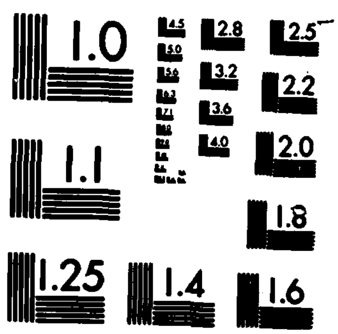

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The class of nonnegative random vectors $I = (T_1,\dots,T_n)$ for which $\min_{1\le i\le n} a_i T_i$ is IFRA for all $0 < a_i \le \infty$, $i = 1,\dots,n$, is closed under convolution.

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CONVOLUTION OF THE IFRA SCALED-MINS CLASS

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Convolution of the IFRA Scaled-mins Class

By

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Abstract

The class of nonnegative random vectors $\underline{T} = (T_1,\ldots,T_n)$ for which $\min_{1 \le i \le n} a_i T_i$ is IFRA for all $0 < a_i \le \infty$, $i = 1,\ldots,n$, is closed under convolution.

Key Words and Phrases: Increasing failure rate average, characterizations, convolution.

AMS 1980 subject classifications: Primary 62H05, Secondary 62N05.

1. Introduction and statement of main result.

Start In recent years various multivariate extensions of the univariate classes of life distributions that are important in reliability theory have been proposed. A survey of many of these classes may be found in Block and Savits (1981). In this paper we focus on one particular extension of the IFRA (increasing failure rate average) class due to Esary and Marshall (1979). ~~A~~ Key words include: nonnegative random vector $\underline{T} = (T_1,\ldots,T_n)$ is said to satisfy condition (F) if $\min_{1 \leq i \leq n} a_i T_i$ is IFRA for all choices $0 < a_i \leq \infty$, $i = 1,\ldots,n$. (Recall that a nonnegative random variable T is IFRA if $\bar{F}(\alpha t) \geq \bar{F}^{\alpha}(t)$ for all $t \geq 0$, $0 < \alpha < 1$, where $\bar{F}(t) = P(T > t)$ is the survival probability). Here we interpret $\infty \cdot 0 = \infty$.

Although Esary and Marshall (1979) considered some closure properties of this class, they did not study closure with respect to the operation of convolution. Recently, El-Neweihi (1984) showed that the class is closed under convolution provided one of the two vectors has independent components; however, the general problem was not resolved. The purpose of this paper is to prove the general result as stated below.

Let $\mathcal{I}$ denote the class of all nonnegative random vectors $\underline{T} = (T_1,\ldots,T_n)$ satisfying condition (F).

Theorem 1.1. The class $\mathcal{I}$ is closed under convolution.

The proof of this result is contained in section 2. In section 3 we consider a further characterization of the class $\mathcal{I}$. As usual R_+^n denotes the nonnegative upper orthant.

2. Proof of the main result.

We shall make use of the following characterization of the class $\mathcal{I}$ due to El-Neweihi (1984).

Theorem 2.1 (El-Neweihi). A nonnegative random vector $\underline{T} = (T_1,\ldots,T_n)$ belongs to I if and only if

$$E\left[\prod_{i=1}^{n} h_i(T_i)\right] \le E^{1/\alpha}\left[\prod_{i=1}^{n} h_i^{\alpha}(T_i/\alpha)\right] \tag{2.1}$$

for all $0<\alpha<1$ and all nonnegative nondecreasing functions h_i defined on $[0,\infty)$, $i=1,\ldots,n$.

Thus to show that I is closed under convolution, we need only show that if $\underline{S} = (S_1,\ldots,S_n)$ and $\underline{T} = (T_1,\ldots,T_n)$ are independent vectors in I, then

$$E\left[\prod_{i=1}^{n} h_i(S_i+T_i)\right] \le E^{1/\alpha}\left[\prod_{i=1}^{n} h_i^{\alpha}((S_i+T_i)/\alpha)\right]$$

for all $0<\alpha<1$ and all nonnegative nondecreasing h_i on $[0,\infty)$, $i=1,\ldots,n$. As usual we may assume without loss of generality that each h_i is continuous and bounded (see, e.g., Block and Savits (1980)).

First we prove a lemma.

Lemma 2.2. Let $H(\underline{s},\underline{t})$ be bounded, nonnegative and continuous on $R_+^n \times R_+^n$. Let μ and ν be two finite measures on R_+^n. For $0<\alpha<1$, define $\|H(\cdot,\underline{t})\|_\alpha = \{\int H^\alpha(\underline{s},\underline{t})d\nu(\underline{s})\}^{1/\alpha}$. Then

$$\int \|H(\cdot,\underline{t})\|_\alpha d\mu(\underline{t}) \le \{\int[\int H(\underline{s},\underline{t})d\mu(\underline{t})]^\alpha d\nu(\underline{s})\}^{1/\alpha} \tag{2.2}$$

$$= \left\|\int H(\cdot,\underline{t})d\mu(\underline{t})\right\|_\alpha .$$

Proof. If $m>0$ and $\underline{i} = (i_1,\ldots,i_n)$, let $A_{\underline{i}}^m = [\frac{i_1-1}{2^m}, \frac{i_1}{2^m})\times\ldots\times[\frac{i_n-1}{2^m}, \frac{i_n}{2^m})$ for $1\le i_j \le m\cdot 2^m$, $j=1,\ldots,n$, $m=1,2,\ldots$. Set $H_m(\underline{s},\underline{t}) = H(\underline{s},2^{-m}\underline{i})$ for $\underline{t}\in A_{\underline{i}}^m$ and zero otherwise. Since H is bounded and continuous, $H_m(\underline{s},\underline{t}) \to H(\underline{s},\underline{t})$ boundedly as $m\to\infty$. Hence $\|H_m(\cdot,\underline{t})\|_\alpha \to \|H(\cdot,\underline{t})\|_\alpha$ boundedly and

$\int \| H_m(\cdot,\underline{t}) \|_\alpha d\mu(\underline{t}) \to \int \| H(\cdot,\underline{t}) \|_\alpha d\mu(t)$ as $m \to \infty$ by the bounded convergence theorem. But

$$\int \| H_m(\cdot,\underline{t}) \|_\alpha d\mu(\underline{t}) = \sum_{\underline{i}} \| H_m(\cdot, 2^{-m}\underline{i}) \|_\alpha \mu(A_{\underline{i}}^m)$$

$$= \sum_{\underline{i}} \| \mu(A_{\underline{i}}^m) H_m(\cdot, 2^{-m}\underline{i}) \|_\alpha$$

$$\leq \| \sum_{\underline{i}} \mu(A_{\underline{i}}^m) H_m(\cdot, 2^{-m}\underline{i}) \|_\alpha$$

$$= \| \int H_m(\cdot,\underline{t}) d\mu(\underline{t}) \|_\alpha .$$

The inequality follows from Minkowski's inequality for $0 < \alpha < 1$. The desired result is obtained by passing to the limit as $m \to \infty$.

(2.3) Remark. The above lemma remains valid if we weaken the continuity assumption on H. It suffices that $H(\underline{s},\underline{t})$ be measureable and right-continuous in $\underline{t}$ for each fixed $\underline{s}$. We can also replace right-continuity with left-continuity if we redefine $H_m(\underline{s},\underline{t})$ as $H(\underline{s}, 2^{-m}(\underline{i}-\underline{1}))$ on $A_{\underline{i}}^m$ where $\underline{1} = (1,\ldots,1)$.

We are now ready to prove the main result. Let $\underline{S} = (S_1,\ldots,S_n)$ and $\underline{T} = (T_1,\ldots,T_n)$ be independent vectors in I with corresponding distribution functions F and G respectively. Fix $0 < \alpha < 1$ and let h_i be nonnegative, nondecreasing, continuous bounded functions on $[0,\infty)$. Then

$$E[\prod_{i=1}^n h_i(S_i+T_i)] = \iint \prod_{i=1}^n h_i(s_i+t_i) dF(\underline{s}) dG(\underline{t})$$

$$\leq \int [\int \prod_{i=1}^n h_i^\alpha(\frac{s_i}{\alpha} + t_i) dF(\underline{s})]^{1/\alpha} dG(\underline{t}) \qquad \text{(since } \underline{S} \in I)$$

$$\leq \{ \int [\int \prod_{i=1}^n h_i(\frac{s_i}{\alpha} + t_i) dG(\underline{t})]^\alpha dF(\underline{s}) \}^{1/\alpha} \qquad \text{(By Lemma 2.2).}$$

$$\leq \{\int [(\int \prod_{i=1}^{n} h_i^{\alpha}(\frac{s_i}{\alpha} + \frac{t_i}{\alpha})dG(\underline{t}))^{1/\alpha}]^{\alpha} dF(\underline{s})\}^{1/\alpha} \qquad \text{(since } \underline{T} \in I)$$

$$= E^{1/\alpha}[\prod_{i=1}^{n} h_i^{\alpha}(\frac{S_i+T_i}{\alpha})].$$

(2.4) Remark. Suppose H is any class of nonnegative functions and we define $\underline{T}$ to be H-IFRA if $E[h(\underline{T})] \leq E^{1/\alpha}[h^{\alpha}(\underline{T}/\alpha)]$ for all $h \in H$, $0 < \alpha < 1$. Then this same argument shows that such a class is closed under convolution provided whenever $h \in H$, it follows that $h(\underline{s} + \underline{t})$ belongs to H for fixed $\underline{s}$ and for fixed $\underline{t}$.

3. Another characterization of I

As was mentioned in Section 2, El-Neweihi characterized the class I by the requirement that

$$E[H(\underline{T})] \leq E^{1/\alpha}[H^{\alpha}(\underline{T}/\alpha)] \qquad (3.1)$$

for all $0 < \alpha < 1$ and $H(\underline{t})$ of the form $\prod_{i=1}^{n} h_i(t_i)$, where each h_i is nonnegative and nondecreasing on $[0,\infty)$. With the help of Lemma 2.2 we can extend the inequality (3.1) to a larger class.

Let H denote the class of all nonnegative distribution functions on R_+^n; i.e., $H \in H$ if and only if there exists a measure μ on R_+^n such that $H(\underline{t}) = \mu([\underline{0},\underline{t}])$. We denote this unique measure μ by dH.

Theorem 3.2. $\underline{T} \in I$ if and only if

$$E[H(\underline{T})] \leq E^{1/\alpha}[H^{\alpha}(\underline{T}/\alpha)]$$

for all $0 < \alpha < 1$ and all $H \in H$.

Proof. The sufficiency is clear since $h(\underline{t}) = \prod_{i=1}^{n} h_i(t_i) \in H$ whenever each h_i is nonnegative, nondecreasing and right-continuous. Now suppose $\underline{T} \in I$ and $H \in H$. Let F be the distribution of $\underline{T}$. Then

$$
\begin{aligned}
E[H(\underline{T})] &= \int H(\underline{t})dF(\underline{t}) = \iint I_{[\underline{0},\underline{t}]}(\underline{s})dH(\underline{s})dF(\underline{t}) \\
&= \int [\int I_{[\underline{s},\infty)}(\underline{t})dF(\underline{t})]dH(s) \\
&\le \int [\int I_{[\underline{s},\infty)}(\underline{t}/\alpha)\, dF(\underline{t})]^{1/\alpha} dH(\underline{s}) && (\text{since } \underline{T} \in \mathcal{I}) \\
&\le \{\int [\int I_{[\underline{0},\underline{t}/\alpha]}(\underline{s})dH(\underline{s})]^{\alpha} dF(\underline{t})\}^{1/\alpha} && (\text{Remark } (2.3)) \\
&= E^{1/\alpha}[H^{\alpha}(\underline{T}/\alpha)].
\end{aligned}
$$

(3.3) Remark. The characterization of the NBU class considered by El-Neweihi (1984) also extends to this class $\mathcal{H}$.

[1] Block, H.W. and Savits, T.H. (1981). Multivariate classes in reliability theory. Math. of O.R., 6, 453-461.

[2] Block, H.W. and Savits, T.H. (1980). Multivariate increasing failure rate average distributions. Ann. Prob., 8, 793-801.

[3] El-Neweihi, E. (1984). Characterizations and closure under convolution of two classes of multivariate life distributions. Statistics and Probability Letters. To appear.

[4] Esary, J.D. and Marshall, A.W. (1979). Multivariate distributions with increasing hazard rate averages. Ann. Prob., 7, 359-370.

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