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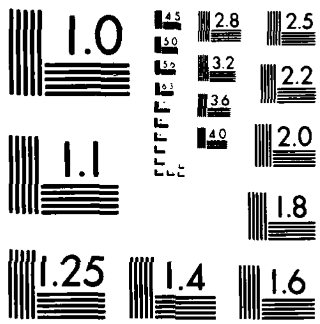
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TESTS OF FIT BASED ON NORMALIZED SPACINGS

by

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1. INTRODUCTION.

Let $x_{(1)} < x_{(2)} < \ldots < x_{(n)}$ be the order statistics of a random sample from a continuous distribution $G(x)$ with location and scale parameters α and β ; thus $G(x) = F(w)$ where $F(w)$ is a completely specified distribution, $x = \alpha + \beta w$, and $x_{(i)} = \alpha + \beta w_{(i)}$, $i = 1,\ldots,n$. Let $m_i = E\{w_{(i)}\}$ where E denotes expectation. Suppose, to fix ideas, that $G(x)$ is the exponential distribution $G(x) = 1 - \exp(-x/\beta)$, so that α is zero, and suppose β is unknown. A well-known technique exists to transform the $x_{(i)}$ to a set $z_{(i)}$, which will be distributed as a set of uniform order statistics.

The transformation, first introduced by Sukhatme (1937) depends on normalized spacings y_i , defined as follows:

$$y_i = \{x_{(i)} - x_{(i-1)}\}/(m_i - m_{i-1}), \quad i = 1,\ldots,n \tag{1}$$

where $x_{(0)} = m_0 \equiv 0$. For an exponential set x , from $G(x)$, the values y_i will be a random sample from the same distribution. A further transformation J gives values $z_{(i)}$, as follows:

$$z_{(i)} = \Sigma_{j=1}^{i} y_j / \Sigma_{j=1}^{n} y_j \ , \ i = 1,\ldots,n \ . \tag{2}$$

It is well known that the values $z_{(i)}$, $i = 1,\ldots,n-1$, are distributed as the order statistics of a sample of size $n - 1$ from the uniform distribution, with limits 0 and 1 , written $U(0,1)$. A test of the null hypothesis that the x_i are from $G(x)$ can then be made by testing that the $z_{(i)}$ are (ordered) uniform random variables. Seshadri, Csörgo and Stephens (1969) defined the K transformation to be the combination of the Sukhatme transformation, say N, and the J transformation above, to produce $z_{(i)}$ from the original x-set; symbolically, $\underset{\sim}{z} = K\underset{\sim}{x} = JN\underset{\sim}{x}$, where $\underset{\sim}{z}$, $\underset{\sim}{x}$ are the vectors of the $z_{(i)}$ and $x_{(i)}$. These authors found that the Anderson-Darling statistic A^2, applied to the $z_{(i)}$, provided a powerful test for exponentiality of the original sample $\underset{\sim}{x}$. O'Reilly and Stephens (1982) gave characterization properties in support of transformation K, as well as further power studies.

An important property of normalized spacings is that, in the limit, for any regular parent population for x, and for "sufficiently separate" indices k and ℓ, y_k and y_ℓ converge to independent exponentials, as $k, \ell, n \to \infty$, and $k/n \to p_i$ and $\ell/n \to p_j$, with both p_i and p_j in $(0,1)$ and different; see Pyke (1965, p. 407) for more rigour and details. Thus it is attractive to devise tests for various distributions, based on their normalized spacings and on the values $z_{(i)}$. A good feature of the tests is that, although it is necessary to know the values m_i, it is not necessary to estimate unknown parameters: furthermore, the tests may be easily applied to both left- and right-censored data. On the other hand, it is not easy to find distribution theory for test statistics; in particular, despite the above result, it is not correct to regard the $z_{(i)}$ as ordered iid uniforms, even asymptotically. In this article we discuss asymptotic theory for the mean, the median, and the Anderson-Darling statistic, in particular for tests for the normal, logistic, and extreme-value (or Weibull) distributions, and follow up the normal tests with a power study.

Censored data. Normalized spacings can be used for the more general problems where the data is singly- or doubly-censored. For doubly-censored samples, or for samples with no endpoints, the normalized spacings are defined as follows. Suppose there are $r + 2$ ordered observations available, namely $x_{(k)}, x_{(k+1)}, \ldots, x_{(k+r+1)}$; then define

$$y_i = \{x_{(k+i)} - x_{(k+i-1)}\}/(m_{k+i} - m_{k+i-1}),\ i = 1,\ldots,r+1 . \tag{3}$$

For singly-censored samples, and with a known endpoint, an extra spacing can arise. If the distribution has a known lower endpoint A, and if the x-sample is right-censored only, the first spacing is $y_1 = \{x_{(1)} - A\}/\{m_1 - A\}$, and y_{i+1}, $i = 1,\ldots,r+1$, will now be given by the right-hand side of (3). Similarly, if the distribution of x has a known upper end-point B, and if the sample is left-censored only, y_i will be given by (3), for $i = 1,\ldots,r+1$, and the added last spacing is $y_{r+2} = \{B - x_{(k+r+1)}\}/\{B - m_{(k+r+1)}\}$. For either $r + 2$ or $r + 1$ values of y_i, the J-transformation can be applied to give $r + 1$ or r values $z_{(i)}$; for the rest of this article we shall assume there are $r + 1$ values of y_i. Then transformation J gives r values $z_{(i)}$:

$$z_{(i)} = \sum_{j=1}^{i} y_i \Big/ \sum_{j=1}^{r+1} y_i ,\ i = 1,\ldots,r . \tag{4}$$

If the original x were exponentials, the y are exponential, and the $z_{(i)}$ are again ordered uniforms. For other parent populations, the $z_{(i)}$ are not ordered uniforms, but statistics used for tests of uniformity can still be used. We investigate tests based on the Anderson-Darling statistic A^2, calculated from

$$A^2 = -r - (1/r)\left(\sum_{i=1}^{r}(2i-1)[\log z_{(i)} + \log\{1 - z_{(r+1-i)}\}]\right) \tag{5}$$

where log x refers to natural logarithm; also two tests based on the median and the mean of the $z_{(i)}$. These test statistics are M and T given by

$$M = r^{\frac{1}{2}}[z_{(\{r+1\}/2)} - \tfrac{1}{2}], \quad r \text{ odd}$$

$$= r^{\frac{1}{2}}[z_{(\{r+2\}/2)} - (r+2)/\{2(r+1)\}] \quad r \text{ even}$$

$$T = r^{\frac{1}{2}}(\bar{z} - \tfrac{1}{2}) \quad \text{where} \quad \bar{z} = \sum_{j=1}^{r} z_{(j)}/r .$$

These statistics are investigated because they are closely related to statistic S, introduced by Mann, Scheuer and Fertig (1973) and statistic S^*, introduced by Tiku and Singh (1981), for tests for the extreme-value and Weibull distributions. When the x set is from an extreme-value distribution the statistic S is the same as $1 - z_{(t)}$, where $t = (r+1)/2$ when r is odd and $t = (r+2)/2$ when r is even; hence $M = r^{\frac{1}{2}}(0.5 - S)$ for r odd and $M = r^{\frac{1}{2}}[r/\{2(r+1)\} - S]$ for r even. Statistic S^* is $2\bar{z}$. These statistics were found, by the authors above, to give powerful tests for the extreme-value distribution (or for the 2-parameter Weibull distribution by taking logarithms of the data) against certain types of alternative.

In order to calculate the statistics, values of m_i (or, more precisely, values of the difference $k_i = m_i - m_{i-1}$) are needed. For the normal distribution extensive tables can be found in Harter (1961), and are reproduced in Biometrika Tables for Statisticians, Vol. 2; also computer routines exist to calculate the m_i very accurately. For the extreme-value distribution tables of k_i are given for $3 \le n \le 25$ by Mann, Scheuer and Fertig (1973). For the logistic distribution $k_i = n/\{(i-1)(n-i+1)\}$, $i = 2,\ldots,n$.

In this article we give general asymptotic theory for the above test statistics, and apply the theory to tests for the normal, logistic and extreme-value distributions. Significance points are given for the normal and logistic tests, and the power of the normal tests is investigated in detail. Percentage points and power results for tests for the extreme-value distribution are recorded elsewhere (Lockhart, O'Reilly and Stephens, 1984).

2. ASYMPTOTIC THEORY OF THE TESTS.

The statistics A^2, M and T are functionals of the quantile process $Q_n(t)$ of the z_i, where

$$Q_n(t) = r^{\frac{1}{2}}(z_{(v)}-t), \quad 0 \le t \le 1 ,$$

(here v is the greatest integer in $(r+1)t$, and $z_{(0)} \equiv 0$ and $z_{(r+1)} \equiv 1$ by definition), and of the empirical process

$$R_n(t) = r^{\frac{1}{2}}[(r^{-1} \Sigma_1^r I(z_i \le t) - t] \quad 0 \le t \le 1 .$$

Here $I(B)$ is the indicator function; $I(B) = 1$ if event B occurs, and $I(B) = 0$ otherwise. Specifically, it may be shown that

$$M = Q_n(\tfrac{1}{2}) + o_p(1) ;$$

$$T = \int_0^1 Q_n(t)\, dt + o_p(1) ; \quad \text{and}$$

$$A^2 = \int_0^1 R_n^2(s)ds/\{s(1-s)\}.$$

Suppose distribution F has density f with derivative $\dot{f}$. Define

$$c(x) = -(1 + (1-x)\dot{f}(F^{-1}(x))/f^2(F^{-1}(x))) ,$$

and set

$$I_1(s) = \int_0^s (1 + uc(u))/(1-u)du ,$$

$$I_2(s) = \int c(u)I_1(u)du .$$

and $$I_3(s,t) = \int_s^t c(x)dx .$$

Set $\rho_0(t,s) = \rho_0(s,t) = s + 2\, I_2(s) + I_1(s)I_3(s,t) \quad 0 \le s \le t \le 1 .$ (6)

Finally if $0 \le p < q \le 1$ and $0 \le s,t \le 1$ set $t^* = p + t(q-p)$ and $s^* = p + s(q-p)$ and let

$$\rho(s,t) = (q-p)^{-1} \{\rho_0(t^*,s^*) - s\rho_0(t^*,q) - (1-s)\rho_0(t^*,p)$$

$$- t\rho_0(s^*,q) - (1-t)\rho_0(s^*,p) + st\rho_0(q,q) \tag{7}$$

$$- (1-s)(1-t)\rho_0(p,p) + (s+t-2st)\rho_0(p,q)\}.$$

To simplify notation we shall sometimes omit the arguments of, for example, $Q_n(t)$, and of $\rho(s,t)$. The asymptotic theory of the test statistics is based on the following conjecture:

Under H_0 and some regularity conditions, as $n \to \infty$, $k/n \to p$ and $(k+r+1)/n \to q$,

(i) Q_n converges weakly in $D[0,1]$ to a Gaussian process Q with mean zero and covariance ρ ,

(ii) R_n converges weakly to $R = - Q$, and

(iii) A^2 converges in distribution to $\int_0^1 R^2(s)/\{s(1-s)\}ds$.

If the conjecture is correct then A^2 is distributed asymptotically as $\Sigma \lambda_i \omega_i$ where $\lambda_1 \geq \lambda_2 \geq \dots$ are the eigenvalues of

$$\lambda f(x) = \int f(t)\rho^*(s,t)dt \tag{8}$$

with $\rho^*(s,t) = \rho(s,t)/\{(s(1-s)t(1-t)\}^{\frac{1}{2}}$, and ω_i are independent χ_1^2 variates. Moreover, $T \xrightarrow{\mathcal{D}} N(0,\sigma_T^2)$ and $M \xrightarrow{\mathcal{D}} N(0,\sigma_M^2)$.

Where

$$\sigma_T^2 = \int_0^1\int_0^1 \rho(s,t)dsdt \quad \text{and} \quad \sigma_M^2 = \rho(\tfrac{1}{2},\tfrac{1}{2}) \ . \tag{9}$$

The argument leading to the conjecture is as follows. First the weak convergence of Q_n implies that of R_n by a standard Skorohod construction argument. Similarly weak convergence of Q_n follows immediately from weak convergence (under $\alpha = 0$, $\beta = 1$) of $\eta_n(t) = n^{\frac{1}{2}}\{n^{-1}\sum_{j=1}^{[nt]} Y_j - t\}$ to a Gaussian process η with mean zero and covariance ρ_0 . The process Q is then $Q(t) = (q-p)^{-\frac{1}{2}}[\eta\{p+t(q-p)\} - t\eta(q) - (1-t)\eta(p)]$.

To deal with η_n , set $v_i = -\log\{1 - F(x_{(i)})\}$ and $d_i = (n-i+1)(v_i - v_{i-1})$. Then the v_i are ordered standard exponential varieties and the d_i are independent standard exponential variates. We have $E(v_i) \equiv u_i = \Sigma_{j=1}^{i} (n-j+1)^{-1} = -\log(1 - \frac{i}{n+1}) + O(n^{-2})$ for i/n bounded away from 1. Set $H(y) = F^{-1}(1-e^{-y})$ and expand the relation

$x_{(i)} - x_{(i-1)} = H(v_i) - H(v_{i-1})$ as a Taylor series about u_{i-1} to get

$$x_{(i)} - x_{(i+1)} = d_i\{(n-i+1)^{-1} H'(u_{i-1}) + (n-i+1)^{-1} H''(u_{i-1})(v_{i-1} - u_{i-1})\}$$
$$+ O_p(n^{-2}).$$

Take expectations to get

$$E\{x_{(i)} - x_{(i-1)}\} = (n-i+1)^{-1} H'(u_{i-1}) + O(n^{-2}) \ ;$$

then

$$y_i = d_i\{1 + (v_{i-1} - u_{i-1}) H''(u_{i-1})/H'(u_{i-1})\} + O_p(n^{-1}) \ .$$

Thus

$$\eta_n(t) = n^{-\frac{1}{2}}[\sum_{i=1}^{[nt]} d_i\{1 + (v_{i-1} - u_{i-1}) H''(u_{i-1})/H'(u_{i-1})\} - nt\} + O_p(n^{-\frac{1}{2}})$$
$$\equiv \eta_n^*(t) + O_p(n^{-\frac{1}{2}}) \ .$$

Under mild conditions η_n^* has $E\{\eta_n^*(t)\} \to 0$ and $\mathrm{Cov}\{\eta_n^*(t),\eta_n^*(s)\} \to \rho_0(s,t)$. A martingale central limit theorem can be applied to prove (under somewhat more stringent conditions) that η_n^* converges weakly to η .

This falls short of a rigorous proof because the remainder terms are not small uniformly in t . Moreover weak convergence of R_n does not automatically imply conjecture (iii). Nevertheless Monte Carlo results indicate clearly that the conjecture is true for the normal, logistic and extreme value distributions.

Furthermore, in the case $p > 0$, $q < 1$ the arguments for (i) and (ii) can be made rigorous for these distributions.

3. COVARIANCE FUNCTIONS FOR SPECIFIC DISTRIBUTIONS.

3.1 In the case where $F(x) = 1 - e^{-x}$, the standard exponential distribution, the values $z_{(1)},\ldots,z_{(r)}$ are distributed exactly as the order statistics of sample of size r from a uniform distribution on $(0,1)$. The asymptotic distribution of η_n is then that of Brownian motion and Q, R are Brownian bridges. See Seshadri, Csörgo and Stephens (1969) for details.

3.2 When F is uniform we have $m_i - m_{i-1} = (n+1)^{-1}$, a constant. Then $z_{(i)} = (x_{(k+i)} - x_{(k)})/(x_{(k+r+1)} - x_{(k)})$ so that $z_{(1)} \ldots z_{(r)}$ are again distributed exactly as the order statistics in a sample of r uniform variates on $(0,1)$. Again Q and R are Brownian bridges.

3.3 When $F = \Phi$, the standard normal distribution with density ϕ, we find

$$c(x) = (1-x)\Phi^{-1}(x)/\phi(\Phi^{-1}(x))$$

$$I_1(s) = [s + s(\Phi^{-1}(s))^2 + \Phi^{-1}(s)\phi(\Phi^{-1}(s))]/2$$

$$I_2(s) = -[(s^2+s)/4 + (s^2-s)(\Phi^{-1}(s))^2/s + (2s-1)\Phi^{-1}(s)\phi(\Phi^{-1}(s))/4$$

$$+ (s^2-s)\Phi^{-1}(s))^4/f + (2s-1)(\Phi^{-1}(s))^3\phi(\Phi^{-1}(s))/4$$

$$+ (\Phi^{-1}(s))^2\phi^2(\Phi^{-1}(s))/4]/2$$

$$I_3(s,t) = J(t) - J(s) \quad \text{for} \quad 0 < s \le t < 1$$

where

$$J(t) = [(\Phi^{-1}(t))^2(1-t) - \Phi^{-1}(t)\phi(\Phi^{-1}(t)) - t]/2$$

These are used in (6) and (7) to give $\rho(s,t)$.

3.4 For $F(x) = \exp\{-\exp(-x)\}$, the extreme value distribution, we find

$$c(x) = (\log x)^{-1} - x^{-1} - (x \log x)^{-1}$$

$$I_1(s) = E_1(-\log s)$$

$$I_2(s) = -E_1^2(-\log s)/2 + (\log s - \log(-\log s))\,E_1(-\log s) - s$$

$$+ \int_{-\log s}^{\infty} y^{-1} \log(y) e^{-y} dy$$

$$I_3(s,t) = K(s) - K(t) \quad 0 < s \le t < 1$$

where

$$K(s) = E_1(-\log s) + \log(-\log(s)) + \log s$$

and

$$E_1(y) = \int_y^{\infty} x^{-1} e^{-x} dx .$$

These expressions are used in (6) and (7) to give $\rho(s,t)$. The extreme value distribution is sometimes written in the form $F^*(x) = 1 - \exp\{-\exp(x)\}$, $-\infty < x < \infty$. $F^*(x)$ is the distribution of $-x'$, where x' has the distribution $F(\cdot)$ at the beginning of this subsection.

For $F^*(x)$, the covariance $\rho^*(s,t)$ is found from $\rho(s,t)$ for $F(x)$, by the relation $\rho^*(s,t) = \rho(1-s,1-t)$.

3.5 For the logistic distribution, $F(x) = (1 + e^{-x})^{-1}$, we have

$$c(x) = (x-1)/x$$

$$I_1(s) = -s - \log(1-s)$$

$$I_2(s) = s - s^2/s + \int_0^s u^{-1}(1-u)\log(1-u)\,du$$

$$I_3(s,t) = t-s + \log s - \log t \ , \quad 0 < s \le t < 1 \ ;$$

$\rho(s,t)$ is again calculated from (6) and (7).

4. DISTRIBUTIONS AND PERCENTAGE POINTS.

Statistic A^2. Using the covariance functions calculated above we have found the eigenvalues λ_i of ρ^* by discretization of the integral involved in (5). That is, we found the eigenvalues $\lambda_1,\ldots,\lambda_k$ of the matrix system

$$\lambda f\{(i-\tfrac{1}{2})/k\} = [\sum_{j=1}^{k} f\{(j-\tfrac{1}{2})/k\}\rho\{(i-\tfrac{1}{2})/k,(j-\tfrac{1}{2})/k\}]/k .$$

In these calculations $k = 100$ is adequate (in the sense that further increases of k do not significantly change the critical points). Having found the λ_i we evaluated the critical points of $\Sigma_1^{100} \lambda_i \omega_i$ by Imhof's method (see Durbin and Knott, 1972). The resulting critical points are in Table 1, for the normal and logistic distributions, for various $0 \le p < q \le 1$. (Recall that $p = k/n$ and $q = (k + r + 1)/n$, as $k, r, n \to \infty$). In the case of symmetric distributions such as these, censoring at p, q leads to the same points as censoring at $1-q$, $1-p$ so that the tables are quite compact.

An interesting fact is that the asymptotic points change fairly slowly with the censoring pattern. It seems possible that more detailed study would suggest a useful approximate correction factor connecting the p, q censored case to the uncensored case; in any case interpolation in the tables works well.

For finite samples, Monte Carlo points have been found for the normal test, and for uncensored samples of sizes $n = 20$ and for $n = 40$. These are given in Table 2. For A^2, the points for finite n converge fairly quickly to the asymptotic points; we observed that use of the exact values for the m_i (rather than, say, Blom's approximation $m_i \simeq \Phi^{-1}\{(i-3/8)(n+1/4)\}$) makes the convergence faster.

Statistics T and M. Statistics T and M are asymptotically normally distributed with mean 0, and variance given by (9). For the uncensored case, these have been worked out analytically. For the normal distribution, $\sigma_T^2 = (1 - 3^{\frac{1}{2}}/\pi)/8 = 0.056084$ and $\sigma_M^2 = 3/16 = 0.1875$. For the logistic distribution the values are $\sigma_T^2 = (\pi^2 - 9)/12 = 0.07247$ and $\sigma_M^2 = 1 - \pi^2/12 + (0.5 - \log 2)^2 = 0.21484$. These statistics also converge quickly to their asymptotic distributions, as one would expect.

Thus, to make a test for normality based on the median, we calculate $M^* = \{r/0.1875\}^{\frac{1}{2}} (z_{(\{r+1\}/2} - 0.5)$ if r is odd, and refer M^* to a standard normal distribution; if r is even, the bracket including z is replaced by $[z_{(\{r+1\}/2)} - (r+2)/\{2(r+1)\}]$. For the test based on the mean, $T^* = \{r/0.0561\}^{\frac{1}{2}}(\bar{z} - 0.5)$ is refered to the standard normal distribution.

Note that if the z_i were ordered uniforms, the variance σ_T^2 would be $1/12 = 0.0833$, and σ_M^2 would be 0.25. The true variances are much smaller especially in the normal case.

Some calculations have also been made when the test is for the normal or the logistic distribution, but the sample tested is actually uniform. For the logistic test, statistics T and M are again asymptotically normal with mean 0, and the variances are $\sigma_T^2 = 3/70 = 0.04286$ and $\sigma_M^2 = 0.3$. The algebra involved in the calculations is extensive and will be published elsewhere. Straightforward calculations then show that the asymptotic power of T, for a 5% test against a uniform alternative, is 0.011, that is statistic T is both inconsistent and biased. For M the asymptotic power is 0.097, very low, and showing that M is not consistent. Similar results hold for T and M in the test for normality against the uniform alternative; the asymptotic power of M is 0.11, so that M is not consistent, and that of T is 0.031, so that T is biased and inconsistent. The Monte Carlo studies in Section 6 below verify these results.

5. EXAMPLE.

Example. Table 3, part (a), give 15 values of X, a measure of endurance of industrial specimens, taken from Section 6.2 of Biometrika Tables for Statisticians, Vol. 2. Graphical plots are given there and suggest that the logarithms might be normally distributed. Also given in Table 3 are the values $x_{(i)}$, the logarithms of $X_{(i)}$, values of m_i, the normalized spacings y_i, and the values $z_{(i)}$, together with the values of the test statistics. Reference to Table 1 shows that A^2 is not nearly significant, so that lognormality of the original values is acceptable. The values of M^* and T^* (Section 4 above) are -0.958 and -0.594 and these too are not significant.

In part (b) the calculations are shown for a censored sample consisting of the first 11 of the ordered X set; again normality can be accepted. If the original X are used without taking logarithms, values of A^2 are 7.424 for the whole set, and 3.262 for the censored set. Reference to Table 1, with $p = 0$ and $q = 11/15$, shows both of these to be significant at the 1% level. These results agree with results of other tests described in Biometrika Tables for Statisticians.

6. POWER COMPARISONS.

In this section, we examine the power of the tests for normality. Table 4 gives the results of Monte Carlo power studies, for tests with uncensored samples. The tests are for sample sizes $n = 20$ and $n = 40$, and the test level is 5%. The test statistics compared are A^2, M and T, against the well-known Anderson-Darling statistic A^2 (Case 3) and the Shapiro-Wilk (1965) statistic W. In A^2 (Case 3), the Anderson-Darling statistic is calculated using values $z_{(i)} = G\{x_{(i)}\}$, with estimators $\bar{x}$ and s^2 for the normal distribution parameters μ and σ^2. Critical points are given by Stephens (1974).

The power studies show A^2, A^2 (Case 3) and W to have much the same power overall. A^2 detects skew alternatives better, and W and A^2 (Case 3) are better against symmetric alternatives. M and T are poor in power against symmetric alternatives; the results for the uniform and logistic alternatives, for example, verify the asymptotic results of Section 3, that T and M can be not consistent or even biased. M was originally introduced in connection with tests for the 2-parameter Weibull distribution against a special class of alternatives, and was suggested as a one-tailed test. Here we have a wide range of alternatives and M and T have both been used as two-tailed tests. Further examination of M, T and A^2 in connection with tests for the Weibull distribution is in Lockhart, O'Reilly and Stephens (1984); again A^2 has good power.

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Table 1

Asymptotic percentage points for A^2, for samples from normal or logistic populations.

Left Censoring point, p	Right censoring point, q	0.25	0.20	0.15	0.10	0.05	0.025	0.01
					Significance level α			
Normal Distribution								
0	1	0.955	1.066	1.211	1.422	1.798	2.191	2.728
0	0.75	1.056	1.183	1.350	1.592	2.026	2.479	3.100
0	0.50	1.098	1.232	1.409	1.667	2.129	2.612	3.273
0	0.25	1.133	1.273	1.459	1.730	2.215	2.722	3.416
0.25	0.75	1.178	1.324	1.518	1.800	2.306	2.835	3.559
0.25	0.50	1.225	1.381	1.587	1.889	2.430	2.996	3.770
Logistic Distribution								
0	1	1.123	1.263	1.448	1.720	2.206	2.716	3.413
0	0.75	1.141	1.281	1.468	1.741	2.230	2.741	3.441
0	0.50	1.178	1.325	1.521	1.806	2.318	2.852	3.584
0	0.25	1.215	1.369	1.574	1.873	2.409	2.969	3.736
0.25	0.75	1.177	1.323	1.517	1.801	2.308	2.838	3.564
0.25	0.50	1.223	1.378	1.584	1.885	2.424	2.989	3.761

Table 2

Percentage points for A^2 for complete samples of size n from a normal distribution.

A^2	Upper tail significance level				
n	0.25	0.10	0.05	0.025	.01
20	1.016	1.521	1.946	2.345	2.952
40	0.980	1.487	1.887	2.313	2.832
∞	0.955	1.422	1.798	2.191	2.728

Table 3

Values X of endurance measurements and calculations for A^2, T^* and M^*.

Part (a)

Values	$X_{(i)}$	0.20	0.33	0.45	0.49	0.78
		0.92	0.95	0.97	1.04	1.71
		2.22	2.275	3.65	7.00	8.80
Values	$x_{(i)}$:	-1.609	-1.109	-0.799	-0.713	-0.248
		-0.084	-0.051	-0.030	0.039	0.536
		0.798	0.822	1.295	1.946	2.175
	m_i	-0.335	-1.248	-0.948	-0.715	-0.516
		-0.335	-0.165	0.000	0.335	...
	y_i	1.026	1.033	0.366	2.334	0.915
		0.189	0.126	0.422	2.925	1.447
		0.123	2.031	2.169	0.469	
	$z_{(i)}$	0.066	0.132	0.156	0.306	0.364
		0.376	0.385	0.412	0.598	0.692
		0.700	0.831	0.970		

$A^2 = 0.375$

Median $z_{(7)} = 0.385$ $M^* = (13/0.1875)^{\frac{1}{2}}(0.385 - 0.5) = -0.958$

Mean $\bar{z} = 0.461$ $T^* = (13/0.0561)^{\frac{1}{2}}(0.461 - 0.5) = -0.594$

Part (b) z_i =

0.095	0.191	0.225	0.441	0.526
0.544	0.555	0.595	0.866	

$A^2 = 0.6067$

Median $z_{(5)} = 0.526$ $M^* = (9/0.1875)^{\frac{1}{2}}(0.526 - 0.5) = 0.104$

Mean $\bar{z} = 0.449$ $T^* = (9/0.0561)^{\frac{1}{2}}(0.449 - 0.5) = -0.646$

Table 4

Power comparisons: Tests of normality. Test Level = 5%. The table gives the percentage of 5000 Monte Carlo samples declared significant by the appropriate statistics.

n = 20

Alternative	A^2 (Case 3)	Shapiro Wilk	A^2	M	T
χ^2 1 d.f.	98	99	99	96	98
χ^2 2 d.f.	78	83	87	69	83
χ^2 3 d.f.	59	63	69	52	68
χ^2 4 d.f.	50	54	58	42	58
χ^2 10 d.f.	24	24	26	20	29
Exponential	79	83	87	72	82
Log Normal	93	94	96	90	95
Uniform	20	22	14	10	4
Logistic	11	10	9	6	10
Laplace	30	25	20	10	20
t_1	90	88	84	50	67
t_2	53	51	47	24	40
t_3	34	34	28	6	25
t_4	26	26	21	7	21

n = 40

Alternative	A^2 (Case 3)	Shapiro Wilk	A^2	M	T
χ^2 1 d.f.	100	100	100	100	100
χ^2 2 d.f.	98	100	100	96	99
χ^2 3 d.f.	97	98	97	88	96
χ^2 4 d.f.	82	89	92	76	90
χ^2 10 d.f.	39	44	50	38	53
Exponential	99	99	100	96	99
Log Normal	100	100	100	99	100
Uniform	46	62	43	11	4
Logistic	14	12	12	6	13
Laplace	50	42	36	10	21
t_1	100	99	98	62	76
t_2	79	75	71	30	49
t_3	51	50	45	18	31
t_4	35	36	32	12	26

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20. ABSTRACT

Normalized spacings provide useful tests of fit for many suitably regular continuous distributions; attractive features of the tests are that they can be used with unknown parameters and also with samples which are censored (Type 2) on the left and/or right. A transformation of the spacings leads, under the null hypothesis, to a set of z-values in [0,1]; these are not however uniformly distributed except for spacings from the exponential or uniform distributions. Statistics based on the mean or the median of the z-values have already been suggested for tests for the Weibull (or equivalently the extreme-value) distribution; we now add the Anderson-Darling statistic. Asymptotic theory of the test statistics is given in general, and specialized to the normal, logistic and extreme-value distributions. Monte Carlo results show the asymptotic points can be used for relatively small samples. Also, a Monte Carlo study on power of the normal tests is given, which shows the Anderson-Darling statistic to be powerful against a wide range of alternatives; it is possible for the mean and median to be not consistent and even biased.

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