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MODAL CONTROL OF STRUCTURAL SYSTEMS

D. F. Miller
W. R. Wells

Wright State University
Dayton, Ohio 45435

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NOTICE

This technical report has been reviewed and is approved for publication.

VIPPERLA B. VENKAYYA
Project Engineer

FREDERICK A. PICCHIONI, Lt Col, USAF
Chief, Analysis & Optimization Branch

FOR THE COMMANDER

ROGER J. HEGSTROM, Col, USAF
Chief, Structures & Dynamics Division

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This paper investigates the control of flexible systems described by a generalized one-dimensional wave equation which relates the structure displacement to the force distribution acting on the structure. The approach taken is that of modal control which bypasses some of the problems, both theoretical and practical, associated with the control of distributed parameter systems. As an example of the control design, the suppression of a vibrating cantilevered beam is considered in detail.

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FOREWORD

This work was performed by Dr. David F. Miller and Dr. William R. Wells of Wright State University in the role of Visiting Scientists to the Design Analysis Methods Group, Analysis and Optimization Branch, Structures and Dynamics Division, Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories (AFSC), Wright-Patterson Air Force Base, Ohio. The study reported in this technical report was conducted in support of Government Contract number F33615-83-C-3000, Project number FY1456-82-00021, Task number 83-9.

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TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION

At present a great deal of interest is being given to the practical control of large space structures such as space transportation systems and large communication satellites [1]. The control task is normally thought of in terms of maintaining specified shape configurations, orientation and alignment, vibration suppression and pointing accuracy, to name a few. Because of the inherent flexibility associated with these systems they are generally analyzed as distributed parameter systems which creates difficulties in the design and analysis of controllers for them. Further, control laws derived using distributed parameter modeling cannot be implemented efficiently with current sensor and actuator technology. Modal control techniques have been developed specifically to bypass the problems associated with distributed parameter theory [2,3]. The concept of modal control is built upon the notion that certain specified system modes can be controlled by appropriate design of the associated closed-loop eigenvalues. This reduces the number of sensors and actuators needed to effect the control of the structure. However, an undesirable phenomenon referred to as observation and control spillover can occur if the number of sensors and actuators used is small. Spillover refers to the phenomenon in which energy intended to go solely into the controlled modes also leaks into the uncontrolled modes.

This report discusses the control of flexible systems described by a generalized one-dimensional wave equation which relates the structure displacement to the force distribution acting on the structure. Optimal control involving the minimization of a quadratic performance index representing control and modal energy content is considered. Typically this control formulation leads to a state feedback algorithm. Since the state components are the modes of vibration which are not directly measurable, a means of state reconstruction must be considered. The approach taken in this report is a deterministic one for which the Luenberger state observer is sufficient for state estimation.

II. PROBLEM FORMULATION

The structural system considered in this analysis is expressed in the following distributed parameter form

$$\frac{\partial^2 u}{\partial t^2}(x,t) + A\, u(x,t) = F(x,t) \tag{1}$$

where $u(x,t)$ is the displacement at point x and time t, A is a linear self-adjoint differential operator and $F(x,t)$ is an externally applied force on the system which can represent the control force or an external disturbance or both. If, for instance, the control is provided by M point actuators located at x_i, $i=1,2,\cdots,M$ and there is no other external disturbance, then

$$F(x,t) = \sum_{i=1}^{M} \delta(x-x_i)q_i(t) \tag{2}$$

where δ is the Dirac delta function and $q_i(t)$ is the force applied at x_i.

System Modal Equations

The system equations of motion expressed by equation (1) and accompanying boundary conditions can be transformed to the modal representation by use of the expansion theorem [4]

$$u(x,t) = \sum_{r=1}^{\infty} \phi_r(x)u_r(t) \tag{3}$$

$$u_r(t) = \int u(x,t)\phi_r(x)dx, \quad r = 1,2,\cdots \tag{4}$$

where $u_r(t)$ are modal amplitudes and $\phi_r(x)$ are the eigenfunctions of the operator A appearing in equation (1). The eigenfunctions are determined from the equation

$$A\phi_r(x) = \lambda_r\phi_r(x) \quad , \; r = 1,2,\cdots \tag{5}$$

where λ_r are the eigenvalues of the operator A. Similarly, the control force $F(x,t)$ can be represented in modal form as

$$F(x,t) = \sum_{r=1}^{\infty} \phi_r(x)f_r(t) \tag{6}$$

$$f_r(t) = \int F(x,t)\phi_r(x)dx \tag{7}$$

where $f_r(t)$ are the modal control forces.

The modal amplitudes are computed from the following equations obtained by substitution of the results of equations (3) and (6) into equation (1) while imposing the orthogonality properties of the eigenfunctions

$$\ddot{u}_r(t) + \lambda_r u_r(t) = f_r(t) \quad , \; r = 1,2,\cdots \tag{8}$$

Initial values for $u_r(t)$ are obtained from

$$u_r(0) = \int u(x,0)\phi_r(x)dx$$

$$\dot{u}_r(0) = \int \frac{\partial u}{\partial t}(x,0)\phi_r(x)dx \quad .$$

Modal State Equation

A larger number of elastic modes may be needed in equation (3) for accurate representation of the displacement of a large flexible structure. However, the concept of modal control is to restrict active control to a few critical modes referred to as controlled modes. The remaining modes which are needed for the accurate representation of the structure displacement are referred to as residual modes. If we choose L modes for descriptive purposes but only control N we have

$$u(x,t) = u_C(x,t) + u_R(x,t) \tag{9}$$

where

$$u_C(x,t) = \sum_{r=1}^{N} \phi_r(x) u_r(t) \tag{10}$$

$$u_R(x,t) = \sum_{r=N+1}^{L} \phi_r(x) u_r(t) \tag{11}$$

The subscripts "C" and "R" refer to controlled and residual respectively. The control design for motion suppression based on only N modes, when actually L modes are represented in the physical observations and are also effected when the control is activated, can lead to stability problems referred to as spillover. This phenomenon is discussed in some detail in references [1,3,5].

Equation (8) for the dynamics of the controlled modal amplitudes can be expressed in state space form as

$$\dot{v}(t) = Av(t) + \bar{B}f(t) \tag{12}$$

where

$$f(t) = [f_1(t)\ f_2(t) \cdots f_N(t)]^T \tag{13}$$

$$v(t) = [u_1(t)\cdots u_N(t)\dot{u}_1(t)\cdots\dot{u}_N(t)]^T \tag{14}$$

$$\bar{B} = \left[\begin{array}{c} 0 \\ \hline I_N \end{array}\right] \tag{15}$$

$$A = \left[\begin{array}{c:c} 0 & I_N \\ \hline -\Lambda & 0 \end{array}\right] \tag{16}$$

where

$$I_N = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 1 \end{bmatrix}, \text{ (NxN)} \tag{17}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \lambda_2 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \lambda_N \end{bmatrix} \tag{18}$$

Control Force Description

In this analysis it was assumed that the structure control could be effected by M point actuators placed at discrete points x_i along the structure. An expression for this force was given in equation (2) and is repeated here:

$$F(x,t) = \sum_{i=1}^{M} \delta(x-x_i)q_i(t) \tag{19}$$

The modal force is computed from equation (7) as

$$f_r(t) = \int F(x,t)\phi_r(x)dx$$

$$= \sum_{i=1}^{M} \int q_i(t)\phi_r(t)\delta(x-x_i)dx$$

$$= \sum_{i=1}^{M} \phi_r(x_i)q_i(t) \tag{20}$$

Substitution of $f_r(t)$ as given by equation (20) into equation (12) results in the state equation

$$\dot{v}(t) = A\ v(t) + B\ q(t) \tag{21}$$

where

$$q(t) = [q_1(t)\ q_2(t)\ \cdots\ q_M(t)]^T \tag{22}$$

and

$$B = \begin{bmatrix} 0 \cdots\cdots 0 \\ \cdot \qquad \cdot \\ \cdot \qquad \cdot \\ 0 \cdots\cdots 0 \\ \phi_1(x_1)\cdots\phi_1(x_M) \\ \cdot \\ \cdot \\ \phi_N(x_1)\cdots\phi_N(x_M) \end{bmatrix} \tag{23}$$

Measurements

For purposes of feedback control it is asssumed that displacements

$$y_j(t) = u(z_j,t) \quad , \quad j = 1,2,\cdots,P \tag{24}$$

are measured at P points z_j along the structure. The modal representation for equation (24) is

$$y_j(t) = \sum_{k=1}^{N} u_r(t)\phi_r(z_j) + r_j(t) \tag{25}$$

where $r_j(t)$ is the residual spillover. If we define the displacement vector

$$y(t) = [y_1(t)\ y_2(t) \cdots y_p(t)]^T \tag{26}$$

then we have

$$y(t) = C\ v(t) + R(t) \tag{27}$$

where

$$C = \begin{bmatrix} \phi_1(z_1) \cdots \phi_N(z_1) & 0 \cdots 0 \\ \cdot \qquad\qquad\quad \cdot & \cdot \quad \cdot \\ \cdot \qquad\qquad\quad \cdot & \cdot \quad \cdot \\ \cdot \qquad\qquad\quad \cdot & \cdot \quad \cdot \\ \phi_1(z_p) \cdots \phi_N(z_p) & 0 \cdots 0 \end{bmatrix} \tag{28}$$

and

$$R(t) = [r_1(t)\ r_2(t)\ \cdots\ r_p(t)]^T \tag{29}$$

III. THE CONTROL PROBLEM

In this study, we address the problem of vibration suppression in distributed systems governed approximately by the modal equations (21) and (27). We assume that spillover effects are negligible and set $R(t)=0$ in (27). The active control of large space structures serves as our primary motivation. In these structures, typically large essential masses dictated by mission requirements are connected by low mass, flexible trusses. Normal operations aboard these structures naturally introduce vibrations in the supporting trusses which must be damped effectively. Mathematically, two primary sources of vibrations can be identified:

(i) Changes in initial conditions in the dynamics (21) (because of slewing or fine pointing effects, sudden disturbances, etc.);

(ii) Fixed, known disturbances (because of onboard manufacturing machinery, pumps and motors, etc.).

Each of these sources of vibration will be discussed in detail below.

An initial configuration for the structure governed by (1) is determined once the functions

$$u(0,x) \text{ and } \frac{\partial u}{\partial t}(0,x) \tag{30}$$

are specified, since these determine initial conditions

$$u_r(0),\ \dot{u}_r(0),\ r = 1,2,\cdots \tag{31}$$

for the modal equations (8). Conversely, initial conditions (31) determine functions (30). Retaining N modes to approximate (1), as in (21), appropriate system initial conditions are given by specifying a vector

$$v_0 = [v_1(0)\ v_2(0)\ \cdots\ v_{2N}(0)]^T. \tag{32}$$

The suppression of vibrations in system (21) due to changes in initial conditions (32), as described in (i) above, may be stated precisely as follows:

$$\text{Given } v(0) = v_0\text{, find control } q(t) \text{ such that } v(t)\to 0. \tag{33}$$

Since the solution $u(x,t)$ of (1) is given approximately by $u_C(x,t)$ of equation (10), if $v(t)\to 0$, $u_C(x,t)\to 0$ also for each x and vibrations due to the initial conditions

$$u_c(x,0) = \sum_{r=1}^{N} \phi_r(x)v_r(0) \tag{34}$$

and

$$\frac{\partial u}{\partial t}(x,0) = \sum_{r=1}^{N} \phi_r(x)v_{r+N}(0) \tag{35}$$

are suppressed.

The problem in (33) above is customarily viewed as a linear regulator problem in optimal control theory. The criterion for the design of the control algorithm is taken to be the minimization of a quadratic performance index which is a function of energy associated with the N controlled modes and the control effort q(t). More precisely, we minimize over all controls q = q(t)

$$J(q) = \frac{1}{2}\int_0^\infty [v^T Q v + q^T R q]dt \tag{36}$$

subject to

$$\dot{v}(t) = Av(t) + Bq(t) \tag{37}$$

$$y(t) = Cv(t) \tag{38}$$

where

$$Q = \left[\begin{array}{c|c} \Lambda & 0 \\ \hline 0 & I_N \end{array}\right], \tag{39}$$

Λ as in (18), and R is a positive definite weighting matrix. It is well-known that the function $q^*(t)$ minimizing (36) is

$$q^*(t) = -R^{-1}B^T Sv(t) \tag{40}$$

where S is an NxN symmetric, non-negative definite solution of the algebraic Riccati equation

$$SA + A^T S - SBR^{-1}B^T S + Q = 0 \tag{41}$$

It should be noted that in practice, in order to implement the control design (40), a state observer $\hat{v}(t)$ must be constructed satisfying

$$\dot{\hat{v}}(t) = A\hat{v}(t) + Bq(t) + G_0[\hat{y}(t)-y(t)] \tag{42}$$

$$\hat{y}(t) = C\hat{v}(t) \tag{43}$$

The suboptimal control

$$\tilde{q}^*(t) = -R^{-1}B^T S\hat{v}(t) \tag{44}$$

is then used to actually control the system (37).

We now consider the suppression of vibrations due to fixed, known periodic disturbances, as described in (ii) above. If M' point disturbances $d_1(t),\cdots,d_{M'}(t)$ are located at positions $x = x_i'$, $i = 1,2,\cdots,M'$ along the distributed system (1), the total force $F(x,t)$ acting on the controlled system is

$$F(x,t) = F_C(x,t) + F_D(x,t) \tag{45}$$

where $F_C(x,t)$ and $F_D(x,t)$ are the control and disturbance forces respectively. As in (19), we may write

$$F(x,t) = \sum_{i=1}^{M} \delta(x-x_i)q_i(t) + \sum_{i=1}^{M'} \delta(x-x_i')d_i(t) \tag{46}$$

The approximate system dynamics (21) then becomes

$$\dot{v}(t) = Av(t) + Bq(t) + B'd(t) \tag{47}$$

where

$$d(t) = [d_1(t)\ d_2(t)\cdots d_{M'}(t)]^T \tag{48}$$

and

$$B' = \begin{bmatrix} 0 & \cdots\cdots\cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots\cdots\cdots & 0 \\ \phi_1(x_1') & \cdots & \phi_1(x_{M'}') \\ \vdots & & \\ \phi_N(x_1') & \cdots & \phi_N(x_{M'}') \end{bmatrix} \tag{49}$$

To control (47), we make use of a very simple idea. Namely, determine a control gain matrix

$$K = \begin{bmatrix} k_{11} & \cdots\cdots & k_{1M'} \\ \vdots & & \vdots \\ k_{M1} & \cdots\cdots & k_{MM'} \end{bmatrix} \tag{50}$$

such that

$$q(t) = [q_1(t)\ q_2(t)\ \cdots\ q_M(t)]^T, \tag{51}$$

with

$$q_i(t) = K_i d(t) \quad i = 1,\cdots,M \tag{52}$$

suppresses the effects of vibrations in (47) caused by $d(t)$. K_i is the i^{th} row of K. (In application, $d(t)$ would have to be fed back with a slight time delay.) More precisely, the vibration suppression problem of (ii) above may be stated as follows:

Given known disturbances $d(t) = [d_1(t)\cdots d_{M'}(t)]^T$, find K such that when (47) is controlled by $q(t)$ as constructed in (51) and (52), the steady state magnitudes $|v_i(t)|$ are as small as possible. (53)

To minimize the effects of $d(t)$, one should attempt to minimize the effects of the vector of generalized forces

$$BKd(t) + B'd(t) = [BK+B']d(t) \tag{54}$$

in (47). To do this, we attempt to minimize the magnitudes of the elements in the matrix

$$BK + B' \tag{55}$$

by minimizing the quadratic function

$$F(K) = \frac{1}{2}\sum_{i=N+1}^{2N} w_i (B_iK+B_i')\,(B_iK+B_i')^T. \tag{56}$$

The minimization in (56) is over the elements k_{ij} of K and the weights w_i are to be suitably chosen. F(K) may be minimized numerically, or, the linear equations that result when its gradient is equated to zero may be solved.

As a special case, to reduce computations in the minimization of $F(K)$, one could seek a vector

$$K = [k_1 k_2 \cdots k_{M'}] \tag{57}$$

that minimizes the effects of $d(t)$ through the control $q(t)$ with

$$q_i(t) = Kd(t) \quad i = 1,2,\cdots,M. \tag{58}$$

In this simpler case, we need only find M' constants k_1, k_2, $\cdots$ $k_{M'}$ which minimize (56), rather than the MM' constants for K given by (50).

IV. A NUMERICAL EXAMPLE

As a simple illustrative example of vibration suppression in distributed parameter systems, we consider the vibrational control of the cantilevered beam pictured in Fig. 1. The partial differential equation governing undamped displacements $u(x,t)$ of points x (along the axis of the beam) at times t is

$$\frac{\partial^2 u}{\partial t^2}(x,t) + \frac{EI}{m} \frac{\partial^4 u}{\partial x^4}(x,t) = \frac{1}{m}F(x,t) \tag{59}$$

with boundary conditions

$$u(0,t) = \frac{\partial u}{\partial x}(0,t) = \frac{\partial^2 u}{\partial x^2}(L,t) = \frac{\partial^3 u}{\partial x^3}(L,t) = 0 \tag{60}$$

Here m = mass, E = modulus of elasticity, I = cross-sectional area, and L = length. For simplicity, we will assume that $m = E = I = L = 1$, a single point accuator is located at $x = \frac{1}{3}$, and (possibly) a single disturbance acts at $x = \frac{2}{3}$.

Simulations of the suppression of vibrations in the beam due to changes in initial conditions and known external disturbances (as discussed in the preceding section) were performed. Programs were coded in FORTRAN 77 and run on a VAX 11-780 computer. Time responses were obtained using the IBM 360 SSP routine DRKGS, a fourth order, variable step size Runge-Kutta integration technique. Algebraic Riccati equation solutions were obtained using a collection of Kleinman algorithms. The results of these simulations will now be discussed.

Transforming (59) and (60) into the state space, retaining two modes for analysis (N=2), and introducing internal damping via the damping ratio ζ, equations (21) for the cantilevered beam become

$$\dot{v}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2 & 0 & -2\zeta\omega_1 & 0 \\ 0 & -\omega_2^2 & 0 & -2\zeta\omega_2 \end{bmatrix} v(t) + \begin{bmatrix} 0 \\ 0 \\ \phi_2(\frac{1}{3}) \\ \phi_2(\frac{1}{3}) \end{bmatrix} q(t) \tag{61}$$

Ignoring spillover effects, we have the approximate equality

$$u(x,t) \simeq \phi_1(x)v_1(t) + \phi_2(x)v_2(t) \tag{62}$$

where in general, the mode shapes $\phi_r(x)$ for (59) are given explicitly as

$$\phi_r(x) = A_r(\sin \alpha_r - \sinh \alpha_r)(\sin \frac{\alpha_r}{L} x - \sinh \frac{\alpha_r}{L} x)$$

$$+ (\cos \alpha_r + \cosh \alpha_r)(\cos \frac{\alpha_r}{L} x - \cosh \frac{\alpha_r}{L} x) \tag{63}$$

for explicitly computable constants α_2 (see [4]). The A_r are normalizing constants. The associated natural vibrational frequencies ω_r satisfy

$$\omega_r = \sqrt{\lambda_r} = \frac{\alpha_r^2}{L} \sqrt{\frac{EI}{m}} \, . \tag{64}$$

(Note that $A = \frac{EI}{m} \frac{\partial^4}{\partial x^4}$ in equation (5).)

The mode shapes $\phi_1(x)$ and $\phi_2(x)$ are represented schematically in Fig. 2. For all simulations, the beam is assumed configured as in Fig. 1 with, as previously stated, m = E = I = L = 1. We also set ζ = .05.

We first consider problem (33), the suppression of vibrations due to changes in initial conditions (Recall, $d(t)=0$). An effective manner in which to simulate abrupt changes in a system's initial conditions is to excite it with an impulse disturbance. For the beam of Fig. 1, we consider excitation forces of the form

$$F(x,t) = \alpha\ \delta(x-1)\delta(t). \tag{65}$$

Such a force corresponds to an impulse of magnitude α applied to the free end of the beam at $t = 0$. With respect to the state space coordinates v, it is easy to check that excitation with (65) is equivalent to specifying initial conditions

$$v_1(0) = v_2(0) = 0 \quad , \dot{v}_1(0) = \alpha\phi_1(1),\ v_2 = \alpha\phi_2(1) \quad . \tag{66}$$

The uncontrolled time response of (61) excited by (65) with $\alpha = .002$ is given in Fig. 3. Here tip displacements are plotted against elapsed time. When the optimal control (40) is applied to suppress the vibrations (here $R = [1]$) assuming that the entire state $v(t)$ is available to the controller, the vibrations are quickly arrested as revealed by Fig. 4. Assuming that only observations of $v_1(t)$ and $v_2(t)$ are available ($C = [1\ 1\ 0\ 0]$ in (38)), a state observer $\hat{v}(t)$ as in (42) and (43) was constructed. Letting $K^* = R^{-1}B^TS$, G_o in (42) was determined so that the eigenvalues of $A - G_oC$ were positioned at fifty times the real part of that eigenvalue of $A - BK^*$ with smallest real part (in absolute value). The response of (61) when excited by (65) and controlled through (44) is given in Fig. 5. It is assumed that the observer initial conditions are

$$\hat{v}_1(0) = \hat{v}_2(0) = \hat{v}_3(0) = \hat{v}_4(0) = 0$$

so that incomplete knowledge of $v(t)$ is present at $t = 0$. Some degradation in performance is obvious, but $\hat{v}(t)$ effectively converges to $v(t)$ after $t = 1.5$.

Consider now problem (53) in which an external disturbance d(t) is present (refer to Fig. 1). Let

$$d(t) = \beta \sin 9t \tag{68}$$

where $\omega_1 = 3.516 < 9 < 22.034 = \omega_2$. Tip displacements when system (61) is driven by (68) with $\beta = .03$ from zero initial conditions ($v(0) = 0$) are pictured in Fig. 6.

Beam control is provided by q(t) as in (58) ($M = 1$). In this example, K in (57) is simply a scalar which is found by minimizing (56). The minimizing K is easily found to be

$$K = -.717. \tag{69}$$

Fig. 6 displays the tip time response when (61) is controlled through (58) and (69). Obviously, this simple approach to vibration suppression is very effective.

Figures 7 and 8 show beam configurations at various instants (time slices). It is interesting to note that when (68) is applied at $x = \frac{2}{3}$, even though this force is directed in an upward direction initially, the tip is thrust downward. This clearly displays the true flexibility of the simple model chosen for analysis.

V. EXTENSIONS AND FUTURE WORK

The analyses presented in the preceding sections can be extended naturally in numerous ways, and the authors intend to pursue this work in the future. In particular, attention should be given to the following further investigations:

(1) Increase the number of actuators, sensors, and external disturbances and evaluate the consequent effects upon system performance;

(2) Compute actuator power requirements and perform total control energy sensitivity analyses with respect to the placement of the components in (1);

(3) Consider more general disturbances d(t), which are not clearly identified, via spectral analysis;

(4) Consider the effects of spillover on performance;

(5) Couple the ideas of (1)-(4) with FDL finite element analysis software to study realistic, possibly high dimensional models of large space structures.

Finally, there has been considerable interest recently (e.g., see [6]) in combined structural and control analysis. Traditionally, in aerospace engineering, control system design has been completed only after the controlled vehicle has been fully designed. Since large space structures are so costly to deploy and since in practice only limited control energy is available for maneuvering and vibration suppression, the idea arises that perhaps a simultaneous consideration of structural and control design can result in substantial savings in cost, complexity, and mission effectiveness. With respect to the cantilevered beam, optimal controls depend directly upon the structural parameters m, E, L, and I. The variation of these parameters eventually also should be included in the analyses of (1)-(5) above, and ultimately, again using finite element methods, the authors hope to extend their study to the difficult questions of combined structural and control design for realistic large space structures.

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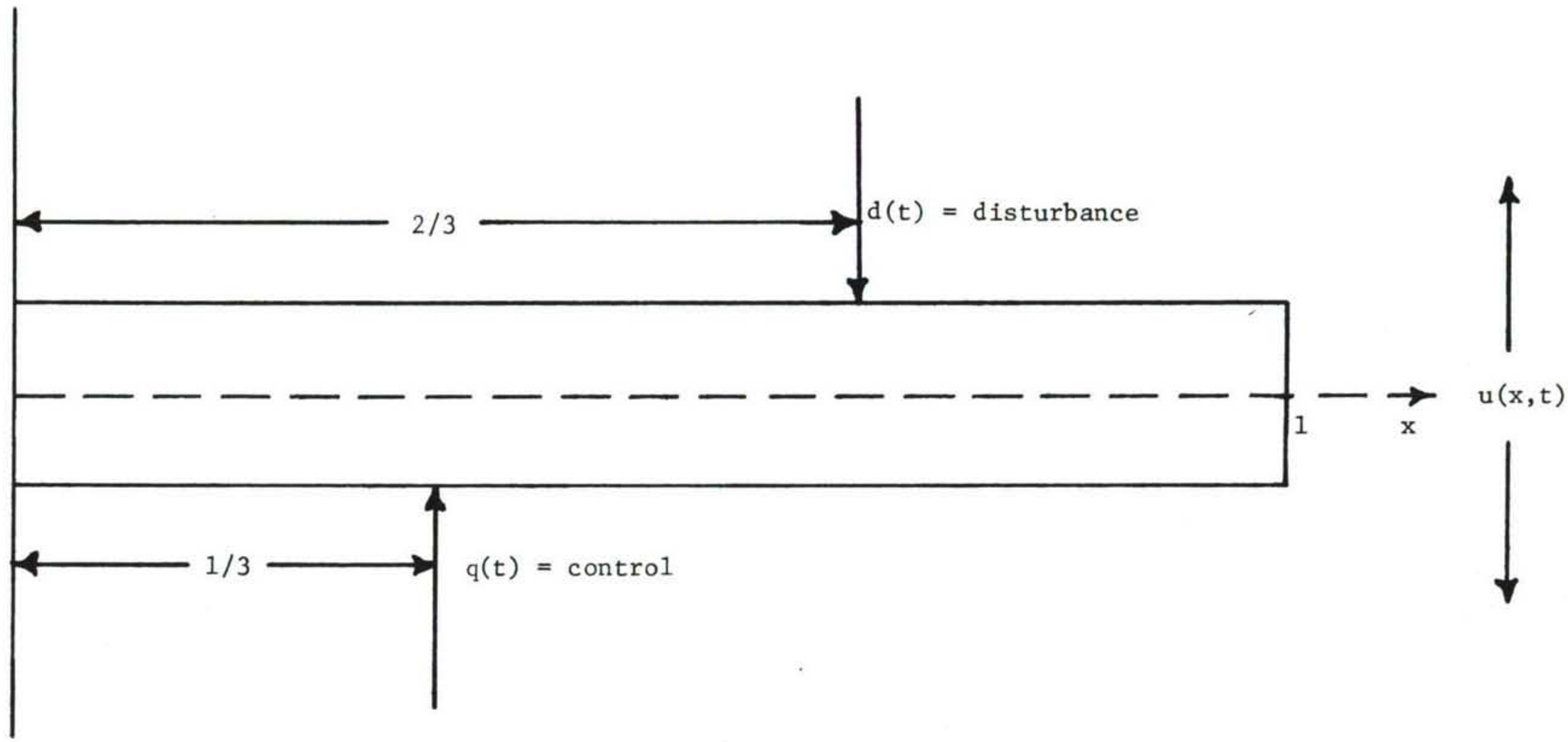

Figure 1 - The Simple Cantilevered Beam - Unit Parameters

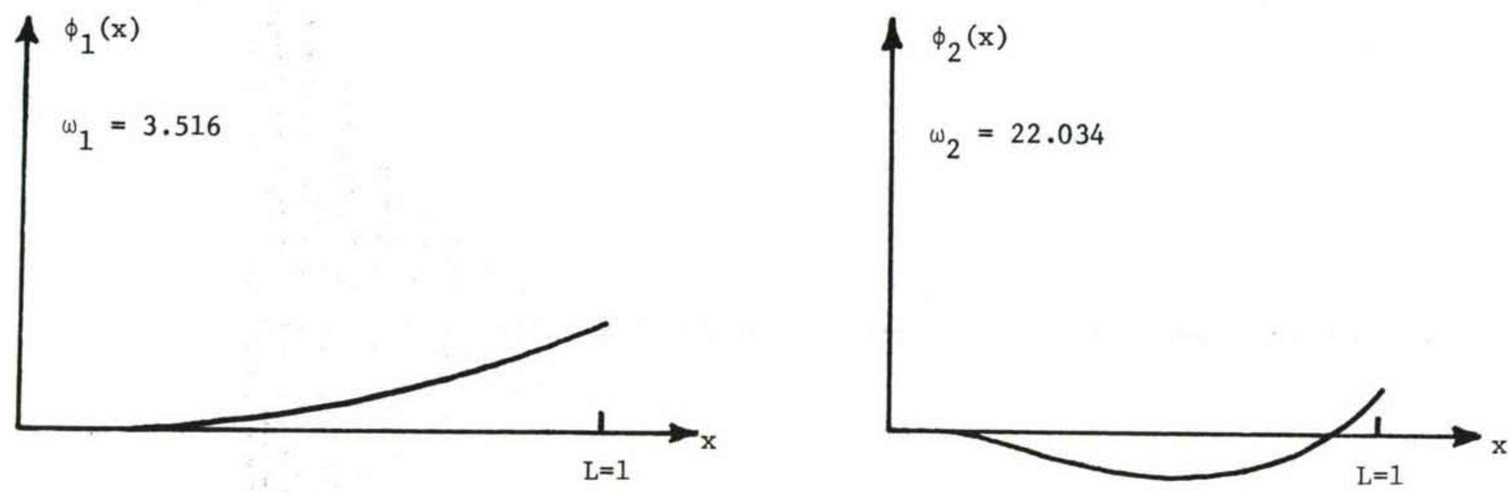

Figure 2 - $\phi_1(x)$ and $\phi_2(x)$ for the Cantilevered Beam

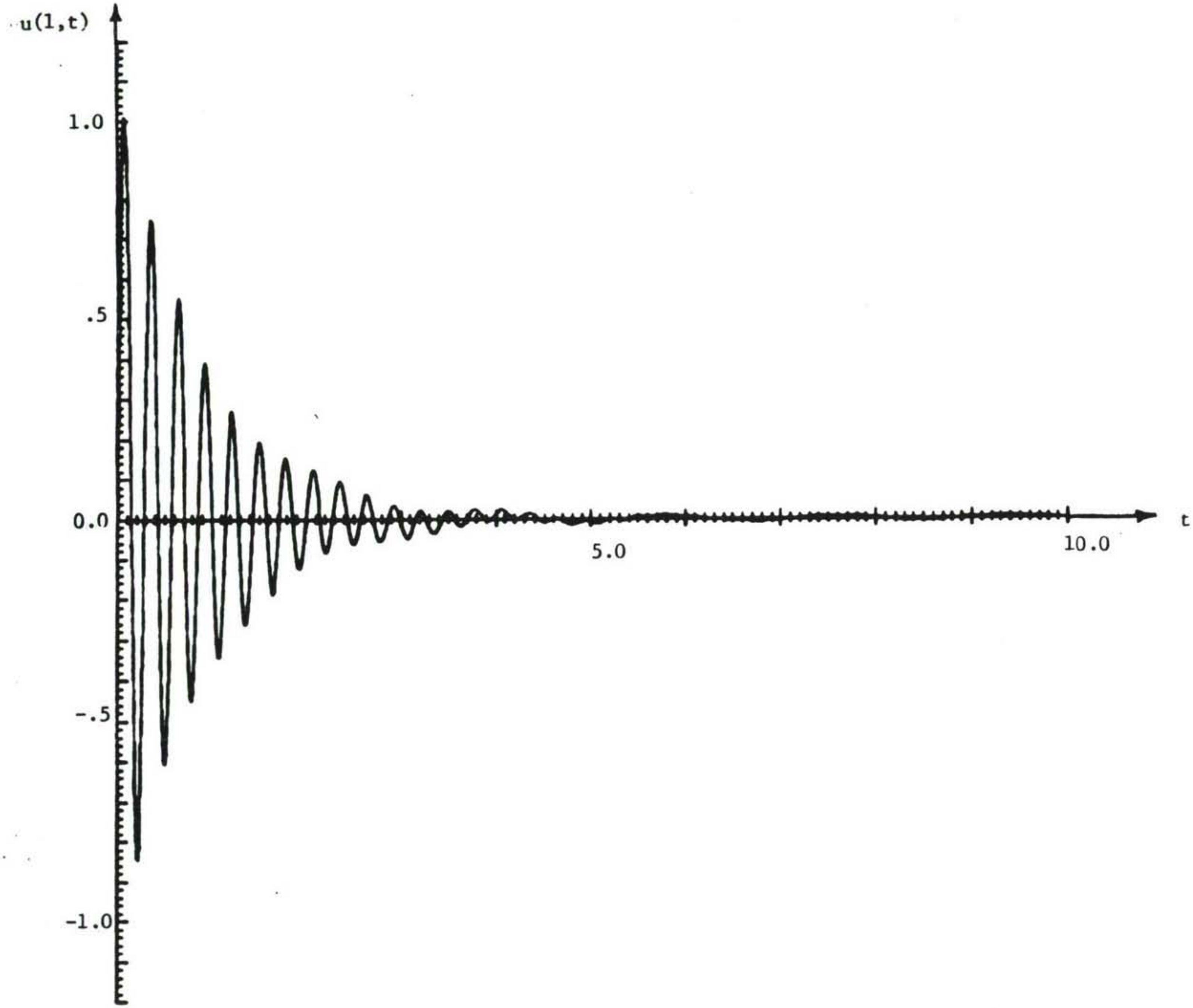

Figure 3 - Uncontrolled Tip Displacement - Impulse Excitation

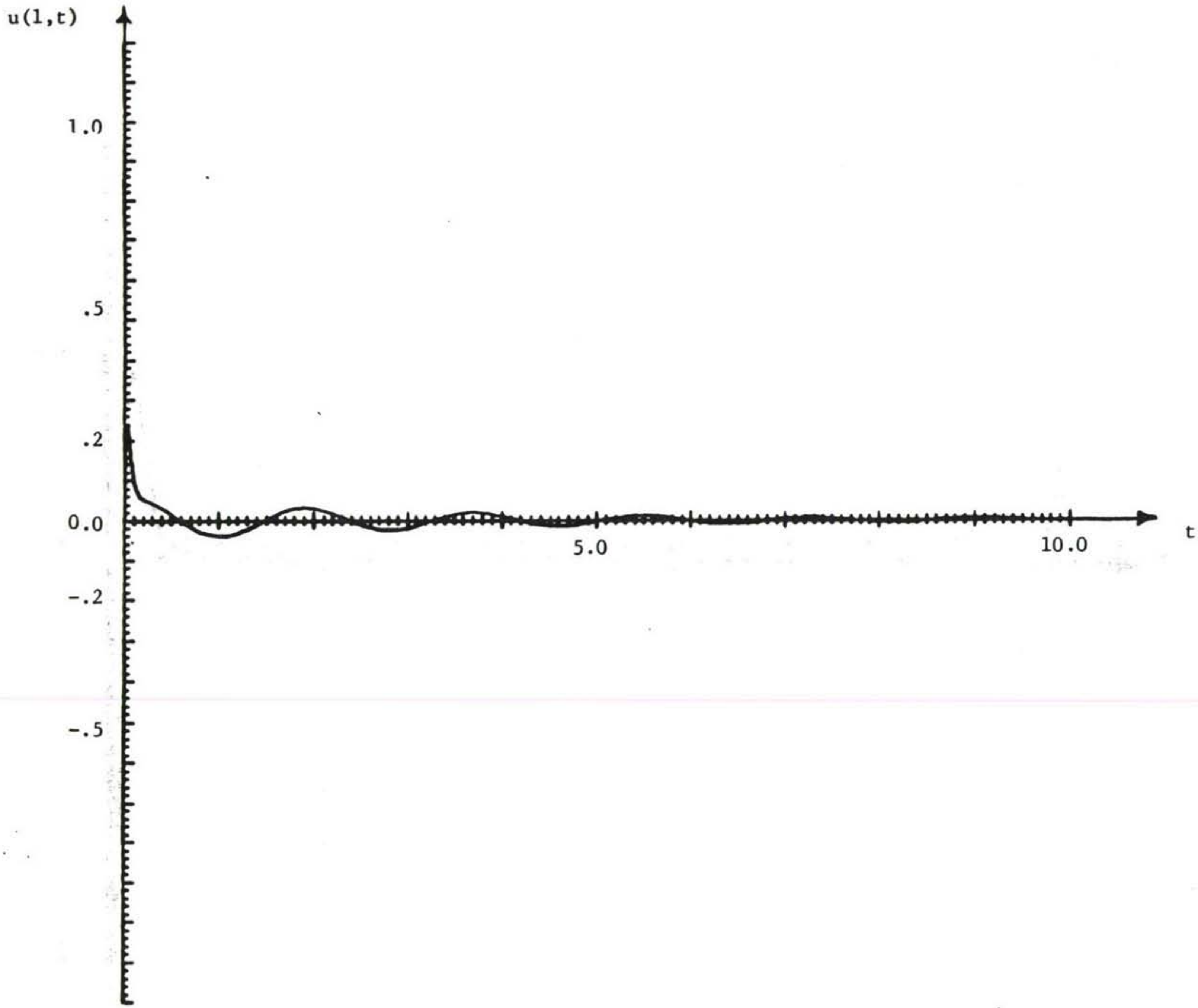

Figure 4 - Controlled Tip Displacement - Impulse Excitation and Complete Observations

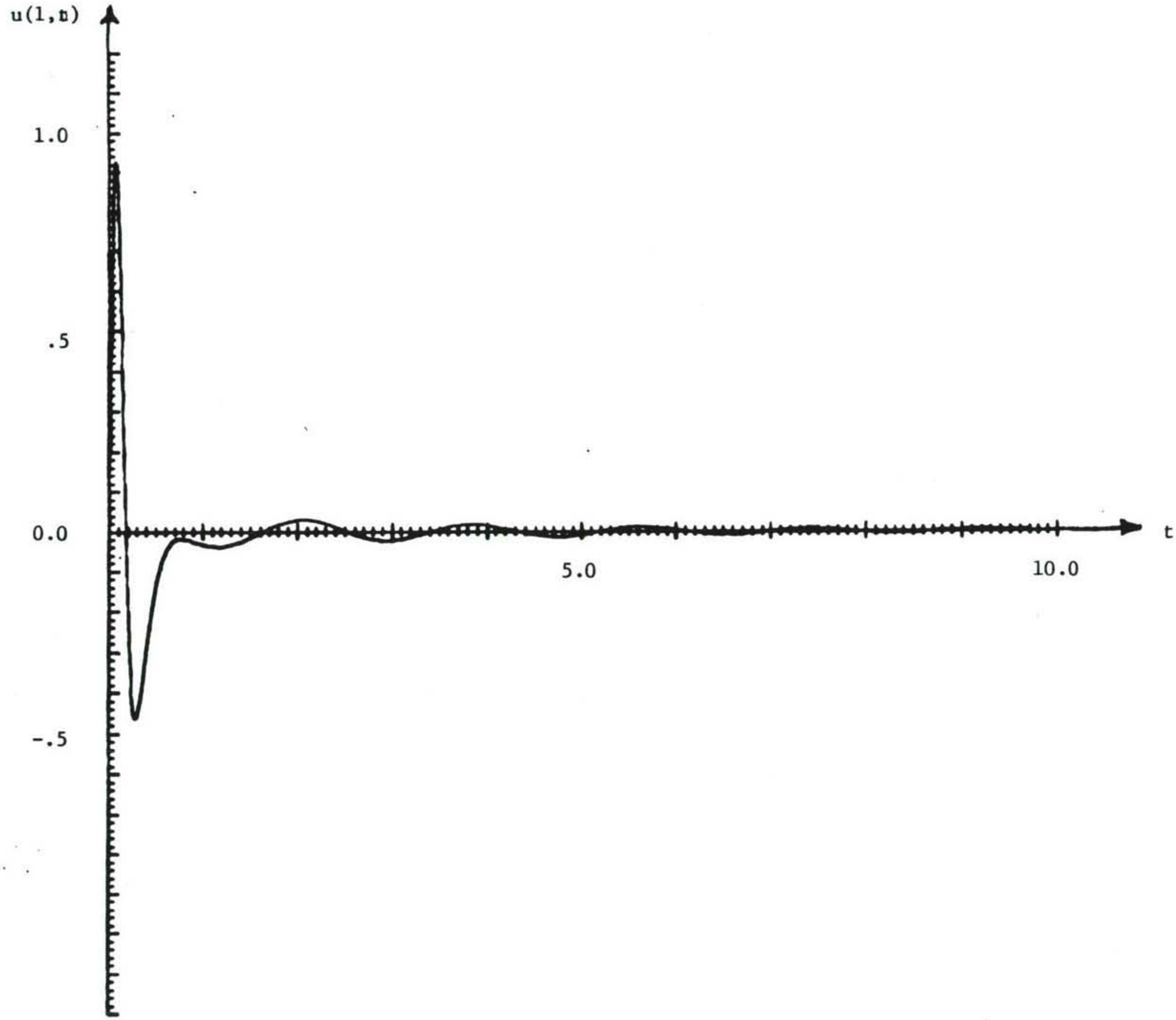

Figure 5 - Controlled Tip Displacement - Impulse Excitation and Incomplete Observations

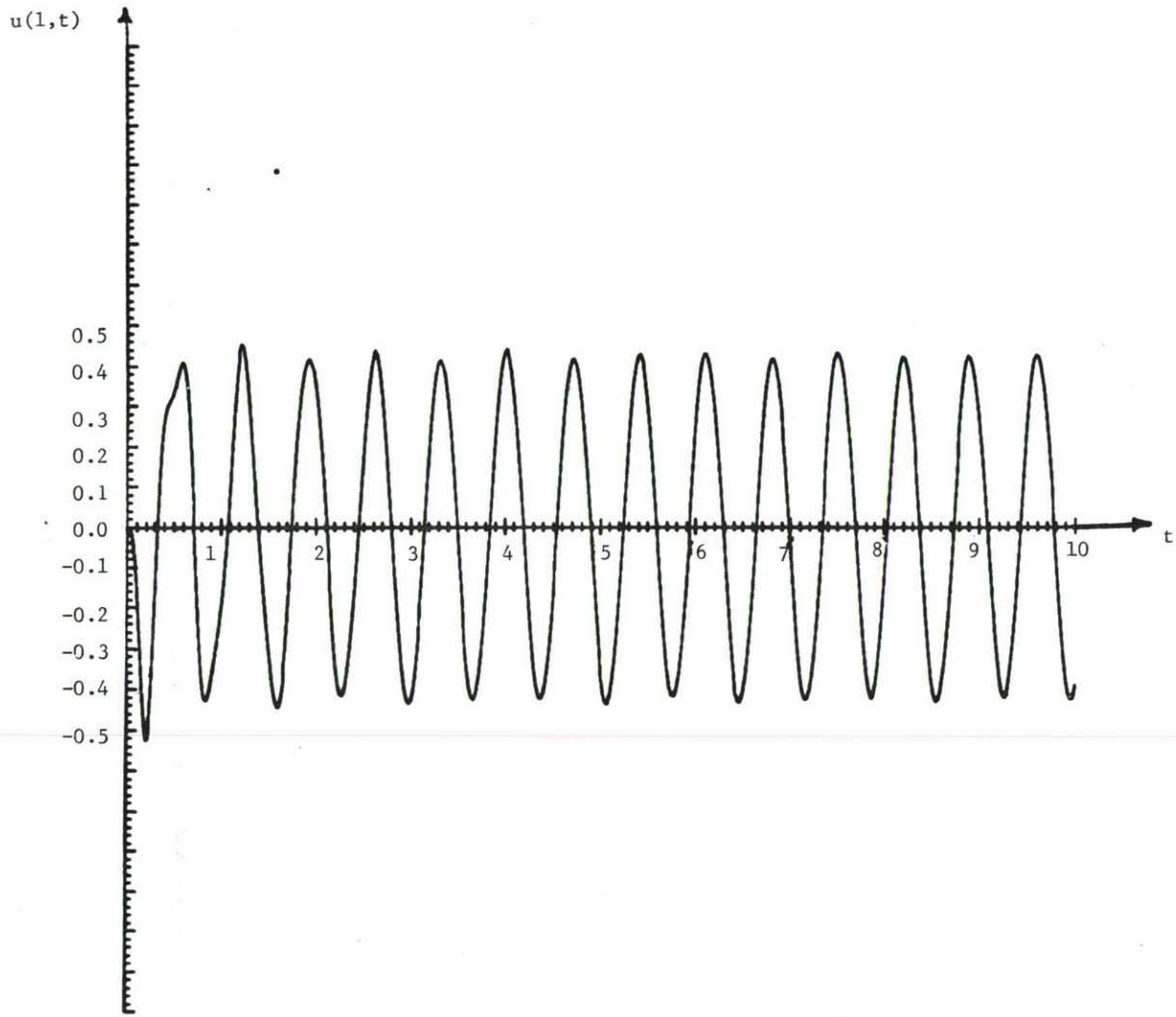

Figure 6 - Uncontrolled Tip Displacement - Known Disturbance $d(t) = .03 \sin 9t$

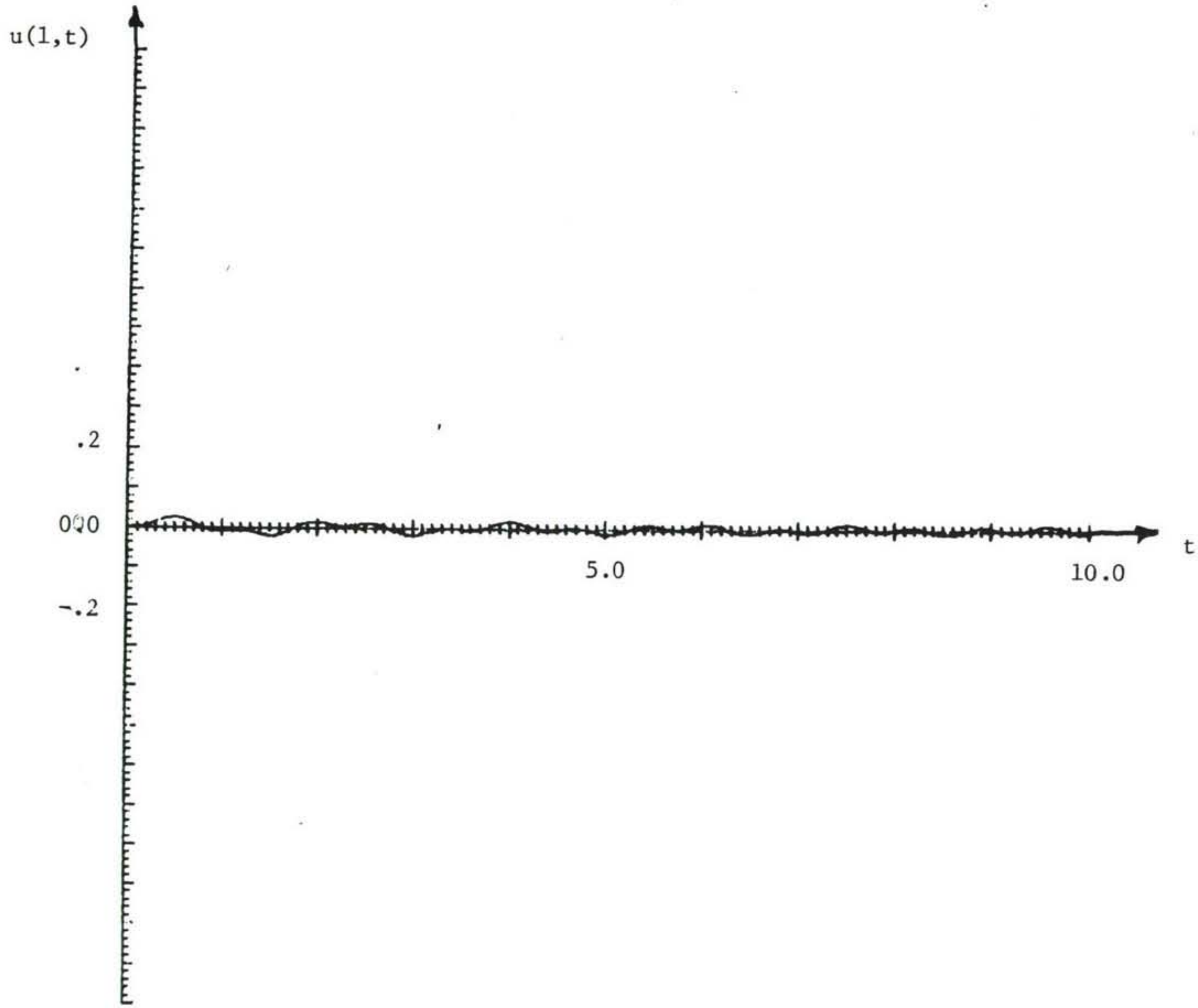

Figure 7 - Controlled Tip Displacement - Known Disturbance $d(t) = .03 \sin 9t$ $(K = -.717)$

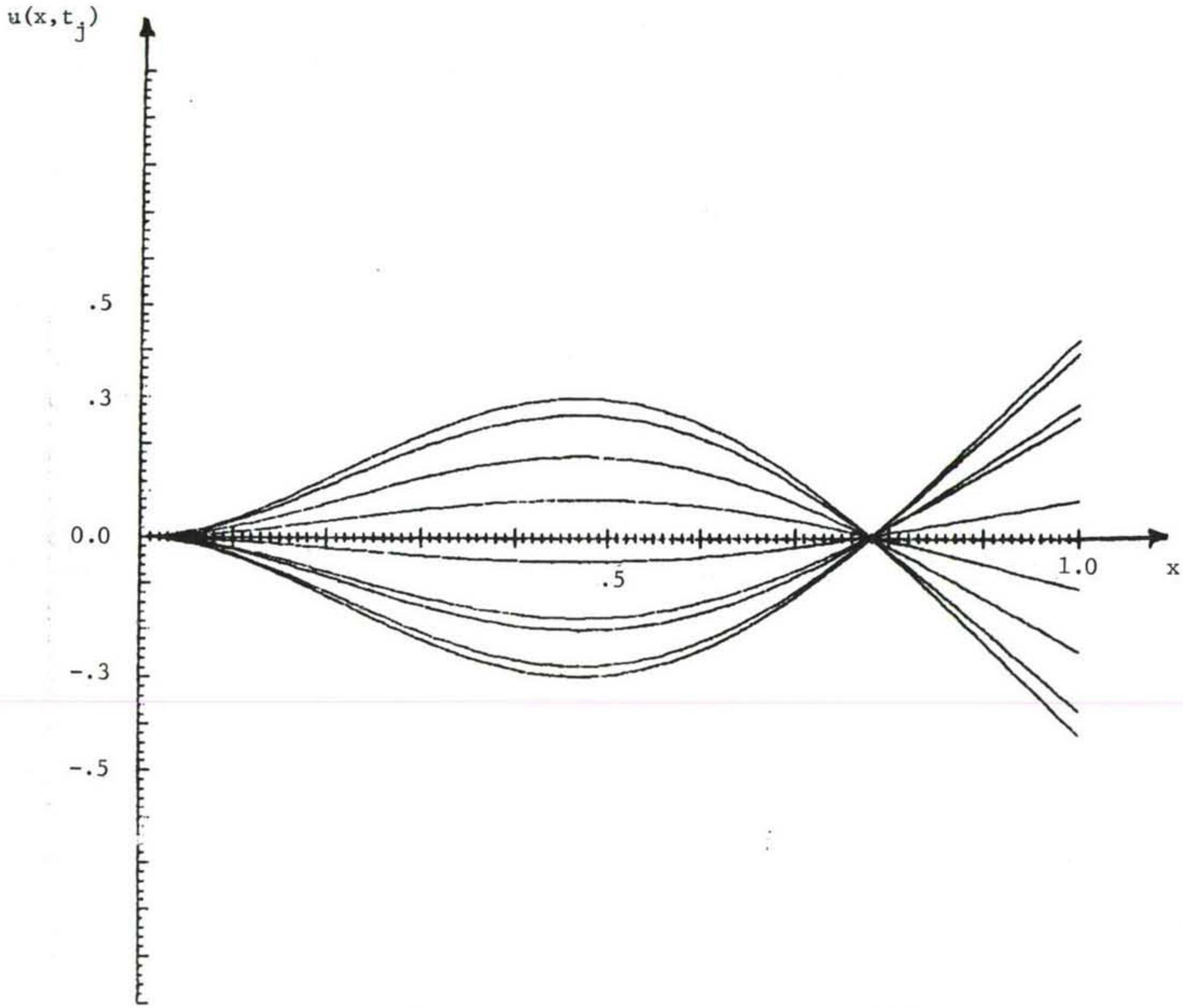

Figure 8 - Uncontrolled Beam Configurations at Selected Times t_j

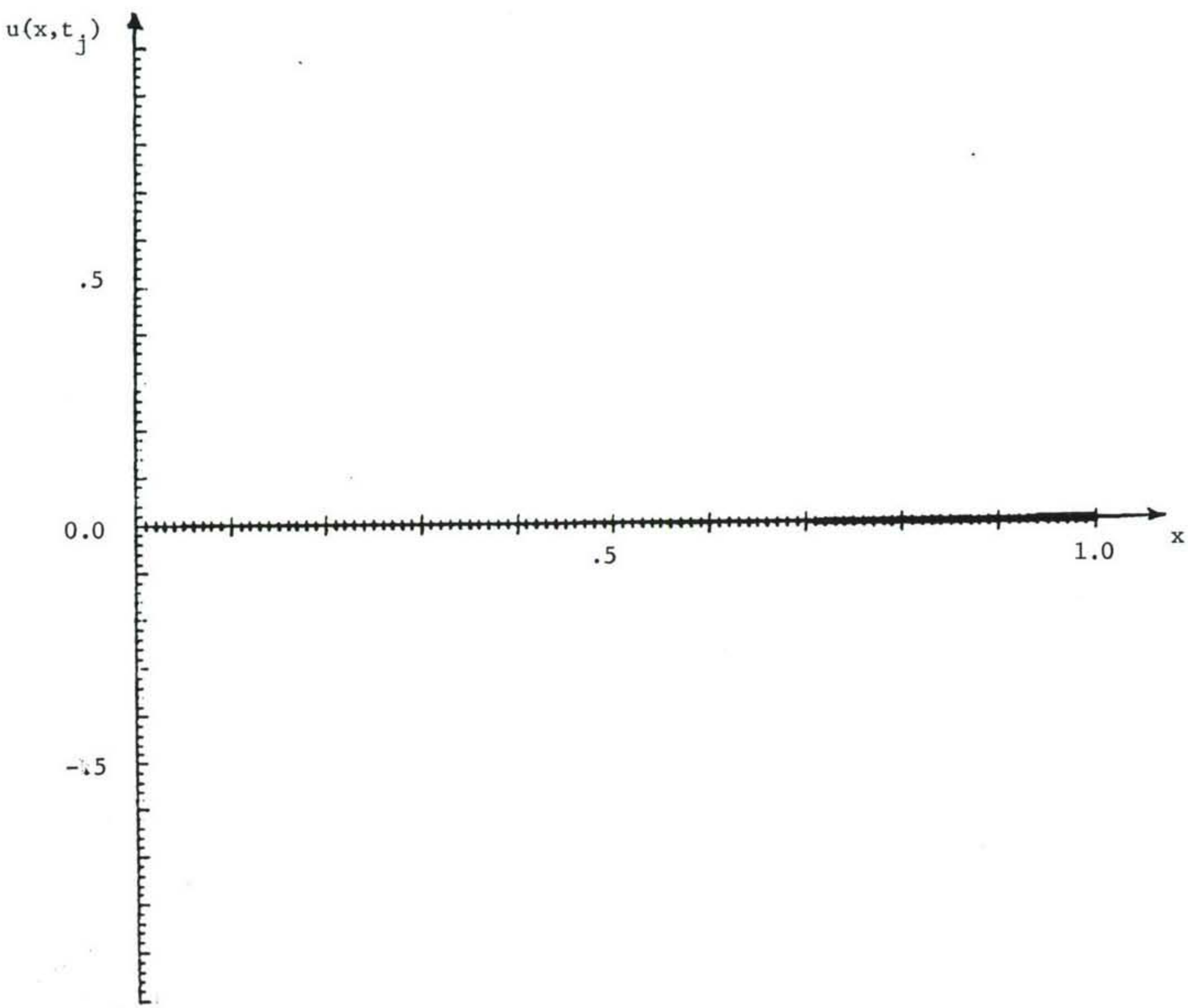

Figure 9 - Controlled Beam Configurations at Selected Times t_j ($K = -.717$)

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