AFOSR-TR- 84-1124

(12)

Materials Sciences Corporation

AD-A149 275

THERMOVISCOELASTIC ANALYSIS OF DIMENSIONALLY STABLE FIBER COMPOSITE SPACE STRUCTURES

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JONATHAN GOERING

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MSC/TFR/1504/1504
August, 1984

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ~~MSC TFR 1504/1204~~ AFOSR-TR- 84 - 1124	2. GOVT ACCESSION NO. A149 273	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THERMOVISCOELASTIC ANALYSIS OF DIMENSIONALLY STABLE FIBER COMPOSITE SPACE STRUCTURES		5. TYPE OF REPORT & PERIOD COVERED Final Report 4/15/83 - 9/30/84
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Zvi Hashin, E. A. Humphreys and Jonathan Goering		8. CONTRACT OR GRANT NUMBER(s) F49620-83-C-0103
9. PERFORMING ORGANIZATION NAME AND ADDRESS Materials Sciences Corporation Gwynedd Plaza II, Bethlehem Pike Spring House, PA 19477		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2307/B1
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NA Building 410 Bolling AFB, DC 20332		12. REPORT DATE August 1984
		13. NUMBER OF PAGES 57
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Carbon fiber composites
Viscoelasticity
Thermo-viscoelasticity
Micro-mechanics analyses
Dimensional stability

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An analytical program was performed in which the thermo-viscoelastic response of a polymeric matrix on unidirectional composite properties were investigated. The fibers were assumed to be linear, elastic, temperature independent transversely isotropic materials while the matrix was modelled as a linear thermo-viscoelastic, isotropic material. The matrix was characterized as a thermorheologically complex material. Micromechanical analyses using the Composite Cylinder Assemblage and periodic hexagonal array were util-

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20. (Continued)

~~ized~~ to model the unidirectional composite. Predicted composite response under combined thermal cycling and mechanical loadings demonstrated the significance of the temperature variation of the matrix material properties. Originator-supplied Keywords include: Carbon fiber composites, Viscoelasticity, Micro-mechanics analysis, and Dimensional stability.

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THERMOVISCOELASTIC PROPERTIES OF UNIDIRECTIONAL FIBER COMPOSITES

1. INTRODUCTION

The present work is concerned with evaluation of the time dependent thermo-mechanical response of a unidirectional fiber composite consisting of transversely isotropic elastic fibers and of linear viscoelastic isotropic matrix. The problem is of fundamental importance for fiber composite structures which undergo severe temperature variation. A pertinent example is a graphite/polymer space structure revolving around the earth and thus, subjected to a temperature cycle whose minimum is 116°K in the earths shadow and maximum 589°K in the sunlight. Under such conditions there are significant variations and time dependences of stiffnesses, compliances and thermal expansions and contractions. The knowledge of such properties for the undirectional material is essential for analysis of laminates made of unidirectionally reinforced laminae.

Most of the methods of analysis of viscoelastic properties of composite materials have been concerned with the isothermal case. The fundamental method in this case has been developed in [1, 2]. It was shown that elastic and viscoelastic properties are related by the usual correspondence principle of quasi-static linear viscoelasticity. Thus, an expression for an elastic property can be converted into the Laplace transform of the corresponding viscoelastic property which can then be obtained by Laplace transform inversion. Detailed analyses have been given in [3].

When the temperature varies with time, thermoviscoelasticity of the matrix must be taken into account. This case has been discussed in [4]. A common idealization of linear thermoviscoelastic behavior is the so-called thermorheologically simple material. This implies that viscoelastic responses at any temper-

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ature can be obtained from a master response at reference temperature by a constant horizontal shift along the log t axis. In such a case the previously mentioned correspondence principle can still be retained in modified form with respect to a reduced time variable.

However, the thermorheologically simple material is of limited usefulness. The main problem with this idealization is that it implies that initial (elastic) response of the material is not temperature dependent while the developing time dependent response does depend on temperature. Such an idealization does not appear justified for polymers with large temperature variations. Consequently, the present study is concerned with the case of thermorheologically complex viscoelastic matrix.

In the following, methods of analysis will be developed to evaluate the thermoviscoelastic response of such a unidirectional composite on the basis of the composite cylinder assemblage model (CCA) and the hexagonal array model. Experience with elastic analyses shows that the predictions of both models are numerically extremely close and agree well with experiments. The preference of one model over the other is a matter of computational convenience. The CCA analysis is very simple and in the present case requires only the solution of simple one dimensional integral equations. The hexagonal array model requires finite element analysis and is thus used only in those cases where the CCA model is not applicable.

2. THERMOVISCOELASTIC STRESS STRAIN RELATIONS OF MATRIX

The stress strain relations of a linear viscoelastic material at constant temperature may be written in the general form

$$\sigma_{ij}(t) = C_{ijk\ell}(t)\varepsilon_{k\ell}(o) + \int_{o+}^{t} C_{ijk\ell}(t-t')\frac{\partial\varepsilon_{k\ell}}{\partial t'}\,dt'$$

$$\varepsilon_{ij}(t) = S_{ijk\ell}(t)\sigma_{k\ell}(o) + \int_{o+}^{t} S_{ijk\ell}(t-t')\frac{\partial\sigma_{k\ell}}{\partial t'}\,dt' \tag{2.1}$$

where $C_{ijk\ell}(t)$ is the relaxation moduli tensor and $S_{ijk\ell}(t)$ is the creep compliance tensor.

For isotropic materials it may be usually assumed that the viscoelastic effect under isotropic stress and strain is much smaller than the viscoelastic effect in shear. Separating stress and strain into isotropic parts σ and ε and into deviatoric parts s_{ij} and e_{ij} according to the scheme

$$\sigma_{ij} = \sigma\delta_{ij} + s_{ij} \qquad \sigma = 1/3\ \sigma_{kk}$$
$$\varepsilon_{ij} = \varepsilon\delta_{ij} + e_{ij} \qquad \varepsilon = 1/3\ \varepsilon_{kk} \tag{2.2}$$

the stress-strain relations become

$$\sigma(t) = 3K\varepsilon(t)$$
$$s_{ij}(t) = 2G(t)\, e_{ij}(o) + 2\int_o^t G(t-t')\frac{\partial e_{ij}}{\partial t'}\,dt'$$
$$e_{ij}(t) = 1/2g(t)s_{ij}(o) + 1/2\int_o^t g(t-t')\frac{\partial s_{ij}}{\partial t'}\,dt' \tag{2.3}$$

where K is the elastic bulk modulus, G(t) is the shear relaxation modulus and g(t) is the shear creep compliance. It is understood here and from now on that the hereditary integrations commence at 0+.

In view of future developments it is noted that a relaxation modulus or creep compliance can be written in the form

$$G(t) = G(o)\ H(t) + \Delta G(t)$$
$$g(t) = g(o)\ H(t) + \Delta g(t) \tag{2.4}$$
$$g(o) = 1/G(o)$$

where $H(t)$ is the Heaviside unit step function. Inserting (2.4) into (2.3) we have

$$s_{ij}(t) = 2G(o)e_{ij}(t) + 2\int_o^t \Delta G(t-t')\frac{\partial e_{ij}}{\partial t'}\,dt'$$
$$e_{ij}(t) = 1/2g(o)\; s_{ij}(t) + 1/2\int_o^t \Delta g(t-t')\frac{\partial s_{ij}}{\partial t'}\,dt' \qquad (2.5)$$

The material properties in (2.2-5) are temperature dependent and the question which arises is how to represent such stress-strain relations when the temperature varies with time. This question has been the subject of much work and discussion and references may be found in Christensen [4] and Schapery [5].

The most well known and common representation is the so called thermorheologically simple material (TSM). By this it is implied that if a relaxation shear modulus, say, at constant reference temperature ϕ_o is expressed in terms of logt, thus

$$G(t,\phi_o) = F(\log t) \qquad (2.6)$$

then at some other temperature ϕ

$$G(t,\ \phi) = F[\log t + \psi(\phi)] \qquad (2.7)$$

which implies that the function F is shifted by an amount ψ along the logt axis. With the definitions

$$\log h(\phi) = \psi(\phi)$$
$$\xi(t) = \int_o^t h[\phi(u)]\,du \qquad (2.8)$$

the TSM equivalent of (2.3) and (2.5) become

$$s_{ij}(t) = 2G(\xi)e_{ij}(o) + 2\int_o^t G(\xi-\xi')\frac{\partial e_{ij}}{\partial t'}dt'$$

$$= 2G(o)e_{ij}(\xi) + 2\int_o^t \Delta G(\xi-\xi')\frac{\partial e_{ij}}{\partial t'}dt' \tag{2.9}$$

$$e_{ij}(t) = 1/2g(\xi)s_{ij}(o) + 1/2\int_o^t g\,(\xi-\xi')\frac{\partial s_{ij}}{\partial t'}dt'$$

$$= 1/2g(o)s_{ij}(\xi) + 1/2\int_o^t \Delta g(\xi-\xi')\frac{\partial s_{ij}}{\partial t'}dt' \tag{2.10}$$

According to (2.6-7) the relaxation modulus (or creep compliance) at any temperature can be found by shifting the corresponding property at reference temperature horizontally along the logt axis by amount $h(\phi)$ which is also often denoted by $1/a_T(\phi)$. The main problem with the TSM representation is that it does not admit temperature dependence of the initial (elastic) modulus or compliance. Indeed it follows from (2.9) and (2.10) that

$$s_{ij}(o) = 2G(o)e_{ij}(o) \tag{2.11}$$

Thus the TSM concept is not suitable for materials whose elastic and viscoelastic properties change with temperature by similar orders of magnitude. This is in particular the case for polymers at significant temperature variations. More complicated linear viscoelastic materials are called thermorheologically complex materials (TCM). The most natural generalization of the TSM representation is to add a vertical shift in log property-log t space, thus conserving the concept of a master viscoelastic time response which is shifted to obtain properties. For discussion see Schapery [5]. More general creep type stress-strain relations have been discussed by Harper and Weitsman [6] and these will here be adapted to the case of shear. Let the shear creep function at some reference temperature ϕ_o be written as

$$g(t, \phi_o) = g_o H(t) + \Delta g(t) \tag{2.12}$$

Then the strain-stress relation for arbitrary temperature variation $\phi(t)$ is written as

$$2e_{ij}(t) = g_o V_o(\phi) s_{ij}(o) + V_1(\phi) \int_o^t \Delta g(\xi - \xi') \frac{\partial}{\partial t'} [V_2(\phi') s_{ij}(t')] dt' \tag{2.13}$$

where $\phi = \phi(t)$ and $\phi' = \phi(t')$ and the strain-stress relations involves the four functions V_o, V_1, V_2 and h, the last through (2.8).

The creep function at some temperature $\phi \neq \phi_o$ is found from (2.13) by introducing $s_{ij}(t) = s_{ij}(o)H(t)$ and assuming constant temperature. This yields

$$g(t, \phi) = V(\phi)[g_o H(t) + \Delta g(t)] = V(\phi) g(\phi_o, t) \tag{2.15}$$

All of the stress-strain relations discussed are representations of varying complexity. The various shift functions are introduced for reasons of convenient mathematical approximation and not on the basis of physical arguments. Relations of type (2.13) can be written in analogous fashion for relaxation stress-strain relations but the connections between relaxation and creep shift functions do not appear to be known.

In the sequel (2.13) and it's inverse will be written symbolically

$$\begin{aligned} e_{ij}(t) &= 1/2\underline{\gamma}\, s_{ij}(t) \\ s_{ij}(t) &= 2\underline{\Gamma}\, e_{ij}(t) \end{aligned} \tag{2.16}$$

where $\underset{\sim}{\gamma}$ and $\underset{\sim}{\Gamma}$ are creep and relaxation operators, respectively.

The stress-strain relation connecting isotropic part of stress with isotropic part of strain is assumed elastic and therefore has the form

$$\sigma(\phi,\ t) = 3K[\phi(t)]\left\{\varepsilon(t) - \alpha[\phi(t)]\ \phi(t)\right\}$$

where α is secant thermal expansion coefficient in reference to a free thermal strain-temperature diagram.

3. THERMOVISCOELASTIC STRESS STRAIN RELATIONS OF UNIDIRECTIONAL FIBER COMPOSITE

3.1 General

It is assumed that the Unidirectional Fiber Composite (UFC) consists of thermoviscoelastic matrix, characterized by the stress-strain relations described above, and of transversely isotropic elastic fibers, thus carbon or graphite fibers. This includes as a special case glass fibers which are isotropic. The fibers are randomly placed and therefore the UFC is transversely isotropic in the macroscopic sense. It follows, just as for elasticity, that the UFC has five independent relaxation moduli and five independent creep compliances. The former characterize the response of the UFC to average strains which are constant in time and the latter - the response to average stresses constant in time. For details see [3]. However, unlike the isothermal linear viscoelastic case (2.1), it is not possible in the case of present complex matrix stress-strain relations to write composite stress-strain relations of the composite in terms of relaxation moduli and creep compliances. Nor is it possible to use the powerful Laplace Transform Method to achieve such a goal. The specific form of such stress-strain relations must in the present case be uncovered by analysis of a model of the UFC.

For reasons of simplicity and practical significance we shall confine the analysis to those properties which enter into the stress-strain behavior of a lamina in a laminate under conditions of plane stress. In the thermoelastic case such stress-strain relations are

$$\bar{\varepsilon}_{11} = S^*_{11}\,\bar{\sigma}_{11} + S^*_{12}\,\bar{\sigma}_{22} + \alpha^*_1\phi \qquad \text{(a)}$$

$$\bar{\varepsilon}_{22} = S^*_{12}\,\bar{\sigma}_{11} + S^*_{22}\,\bar{\sigma}_{22} + \alpha^*_2\phi \qquad \text{(b)}$$

$$\bar{\varepsilon}_{12} = \bar{\sigma}_{12}/2C^*_{66} \qquad \text{(c)} \qquad (3.1.1)$$

$$\bar{\sigma}_{11} = C^*_{11}\,\bar{\varepsilon}_{11} + C^*_{12}\,\bar{\varepsilon}_{22} + D^*_1\phi \qquad \text{(d)}$$

$$\bar{\sigma}_{22} = C^*_{12}\,\bar{\varepsilon}_{11} + C^*_{22}\,\bar{\varepsilon}_{22} + D^*_2\phi \qquad \text{(e)}$$

where x_1 is in fiber direction, x_2 is transverse to fibers, figure 1, overbar denotes average, * denotes effective property and ϕ denotes temperature rise. The compliances and stiffnesses are related to UFC properties in following fashion

$$S^*_{11} = 1/E^*_L \qquad S^*_{12} = -\,\nu^*_L/E^*_L$$

$$S^*_{22} = 1/E^*_T \qquad C^*_{66} = G^*_L$$

$$C^*_{11} = E^*_L/(1 - \nu^{*2}_L\, E^*_T/E^*_L) \cong E^*_L$$

$$C^*_{12} = \nu^*_L\, E^*_L/(1 - \nu^{*2}_L\, E^*_T/E^*_L) \cong \nu^*_L E^*_T$$

$$C^*_{22} = E^*_T/(1 - \nu^{*2}_L E^*_T/E^*_L) \cong E^*_T \qquad (3.1.2)$$

$\alpha_1^* = \alpha_L^*$ $\qquad\qquad\qquad$ $\alpha_2^* = \alpha_T^*$

$$D_1^* = -\ (n^*\alpha_L^* + 2\ell^*\alpha_T^*)$$

$$D_2^* = -\ (\ell^*\alpha_L^* + 2k^*\alpha_T^*)$$

where

E_L^* - Longitudinal Young's modulus

E_T^* - Transverse Young's modulus

ν_L^* - Longitudinal Poisson's ratio

G_L^* - Longitudinal shear modulus

n^*, ℓ^* - Certain effective moduli of UFC, see [3]

k^* - Transverse bulk modulus of UFC, [3]

α_L^* - Longitudinal thermal expansion coefficient of UFC

α_T^* - Transverse thermal expansion coefficient of UFC

Suppose the matrix is thermoviscoelastic and that the UFC specimen is subjected to constant average stress and temperature at time t=0, thus these quantities are given by $\bar{\sigma}_{ij}H(t)$ and $\phi H(t)$ where $H(t)$ is the Heaviside step function; then the strains are given by (3.1.1a-c) with S_{ij}^* replaced by time and temperature dependent creep compliances and α_i^* replaced by time and temperature dependent expansion coefficients. Similarly, for application of Heaviside strains $e_{ij}H(t)$ and temperature $\phi H(t)$ the time dependent average stresses are now given in terms of time and temperature dependent relaxation moduli C^*_{ij} and D^*_i. The general case of average strain, stress or temperature inputs which are arbitrary functions of time is much more complex. For constant temperature the stress-strain relations of the composite will be of form (2.1) but for variable temperature this is no longer the case and only analysis of a model can uncover the nature of the stress-strain relations.

In the following we shall use two kinds of geometrical models to represent the UFC. The first one is the composite cylinder assemblage which was first introduced in [8] and which has been discussed in greater detail in [3]. This model consists of a collection of contiguous composite cylinders in each of which the central cylindrical core represents a fiber which is surrounded by a concentric matrix shell. All space is filled out by composite cylinders whose diameters diminish from finite to infinitesimal sizes, figure 2. In all composite cylinders fiber to matrix shell radius ratios are the same.

The second model is a periodic hexagonal array of identical circular cylinders, figure 3. Both models are macroscopically transversely isotropic. Elastic analyses have shown that the effective elastic moduli of these two models are extremely close and are in good agreement with experimental results. The first model is employed in those cases when it admits an analytical solution. The second model is used in the remaining cases and must be analyzed numerically.

3.2 Axisymmetric Deformation

We consider a homogeneous cylindrical specimen of some transversely isotropic material, without as yet specifying its mechanical nature, which is subjected to the displacement field

$$u_1 = \varepsilon_L(t)x_1$$

$$u_2 = \varepsilon_T(t)x_2 \qquad (3.2.1)$$

$$u_3 = \varepsilon_T(t)x_3$$

Then the only nonvanishing strains are

$$\varepsilon_{11} = \varepsilon_L \qquad \varepsilon_{22} = \varepsilon_{33} = \varepsilon_T \qquad (3.2.2)$$

and it follows from the transverse isotropy that the only surviving stresses are

$$\sigma_{11} = \sigma_L(t) \qquad \sigma_{22} = \sigma_{33} = \sigma_T(t) \qquad (3.2.3)$$

which are related to the strains by the stress-strain law of the material. Next we consider any circular cylinder of radius b with generators in x_1 direction within the specimen, extending throughout its length. The center of this cylinder is located at point x_2^o, x_3^o, figure 4. Introducing local coordinates x_2', x_3' located at the center and defined by

$$x_2 = x_2^o + x_2' \qquad x_3 = x_3^o + x_3' \qquad (3.2.4)$$

The displacements (3.2.1) become

$$u_1 = \varepsilon_L x_1$$

$$u_2 = \varepsilon_T\, x_2^o + \varepsilon_T\, x_2' \qquad (3.2.5)$$

$$u_3 = \varepsilon_T\, x_3^o + \varepsilon_T\, x_3'$$

The first parts of the right sides of (3.2.5 b,c) are rigid body motions of the cylinder while the second parts when converted to cylindrical coordinates become

$$u_r = \varepsilon_T r \qquad u_\theta = 0 \tag{3.2.6}$$

The stresses (3.2.3) converted to cylindrical coordinates are

$$\sigma_{11} = \sigma_{zz} = \sigma_L \qquad \sigma_{rr} = \sigma_T \tag{3.2.7}$$

and the rest of cylindrical stress components vanish. Thus the cylinder is in a state of axisymmetric deformation and stress.

Next we consider a composite cylinder of inner radius a and outer radius b. The inner core is transversely isotropic fiber material and the concentric shell is isotropic (or transversely isotropic) matrix material. The cylinder surface is subjected to the displacements (3.2.5-6). Thus

$$u_1(s) = \varepsilon_L x_1, \qquad u_r(s) = \varepsilon_T b \qquad u_\theta(s) = 0 \tag{3.2.8}$$

Solving this axisymmetric problem subject to the proper continuity conditions at interface r=a we obtain on r=b a stress $\sigma_{rr}(b)=\sigma_T$ and on the end sections an average stress $\bar{\sigma}_{11}=\sigma_L$. Thus, to an external observer the composite cylinder appears as some homogeneous transversely isotropic cylinder whose effective stress strain law is defined by the relations between ε_L, ε_T and σ_L, σ_T. This stress-strain law is now assigned to the homogeneous cylindrical specimen which was our starting point. It then follows that the surface displacements and tractions on the homogeneous circular cylinder of radius b are the same as those on the composite cylinder of radius b and therefore replacement of the first cylinder by the second does not perturb the state of stress and strain in the homogeneous specimen.

Obviously, the effective stress-strain law of the composite cylinder depends geometrically only on the ratio a/b. Therefore, any part of the volume of the cylindrical specimen can be replaced by non-overlapping composite cylinders with same a/b without perturbing the homogeneous state of stress and strain in the remaining volume. This remaining volume is made indefinitely small by adding more and more composite cylinders of diminishing sizes. Thus in the limit there is obtained a composite cylinder assemblage, figure 2, whose symmetric stress-strain relation is that of a single composite cylinder. Accordingly, we now consider a composite cylinder composed of transversely isotropic fiber and thermoviscoelastic transversely isotropic matrix which is in an axisymmetric state. The nonzero displacements and stresses are u_r, u_z, σ_{rr}, $\sigma_{\theta\theta}$ and σ_{zz}. They may be functions of r, $z=x_1$ and t and will be assigned superscript 1 for matrix and superscript 2 for fiber. The cylinder is subjected to some uniform temperature variation $\phi(t)$ and to mechanical boundary conditions

$$u_z(r,L,t) = \varepsilon_L(t)L \qquad u_z(r,o,t) = 0$$

$$\sigma_{rr}(b,z,t) = \sigma_T(t) \tag{3.2.9}$$

The interface conditions at r=a are

$$u_r^{(1)}(a,z,t) = u_r^{(2)}(a,z,t)$$

$$\sigma_{rr}^{(1)}(a,z,t) = \sigma_{rr}^{(2)}(a,z,t) \tag{3.2.10}$$

$$u_z^{(1)}(a,z,t) = u_z^{(2)}(a,z,t)$$

Guided by the elastic axisymmetric generalized plane strain solution the displacements will be assumed to have the form

$$u_r^{(1)} = A_1(t)r + B_1(t)/r$$

$$u_r^{(2)} = A_2(t)r \qquad (3.2.11)$$

$$u_z^{(1)} = u_z^{(2)} = \varepsilon_L(t)z$$

where A_1, B_1 and A_2 are unknown functions of time. The associated strains are

$$\varepsilon_{rr}^{(1)} = A_1 - B_1/r^2 \qquad \varepsilon_{rr}^{(2)} = A_2$$

$$\varepsilon_{\theta\theta}^{(1)} = A_1 + B_1/r^2 \qquad \varepsilon_{\theta\theta}^{(2)} = A_2 \qquad (3.2.12)$$

$$\varepsilon_{zz}^{(1)} = \varepsilon_{zz}^{(2)} = \varepsilon_L$$

Using the transversely isotropic thermoelastic stress-strain relations of the fiber material the fiber stresses are given by

$$\sigma_{zz}^{(2)} = n\varepsilon_{zz}^{(2)} + \ell\varepsilon_{rr}^{(2)} + \ell\,\varepsilon_{\theta\theta}^{(2)} + D_L\phi$$

$$\sigma_{rr}^{(2)} = \ell\varepsilon_{zz}^{(2)} + (k + G_T)\varepsilon_{rr}^{(2)} + (k-G_T)\varepsilon_{\theta\theta}^{(2)} + D_T\phi \qquad (3.2.13)$$

$$\sigma_{\theta\theta}^{(2)} = \ell\varepsilon_{zz}^{(2)} + (k-G_T)\varepsilon_{rr}^{(2)} + (k+G_T)\varepsilon_{\theta\theta}^{(2)} + D_T\phi$$

where

$$D_L = -(n\alpha_L + 2\ell\alpha_T) \qquad D_T = -(\ell\alpha_L + 2k\alpha_T) \qquad (3.2.14)$$

k is transverse bulk modulus, G_T is transverse shear modulus, n and ℓ are additional moduli and α_L, α_T are longitudinal and transverse expansion coefficients, respectively. All of these quantities may be temperature dependent. Separating the cylindrical stresses and strains according to the scheme (2.2) and using (2.17) we have

$$\sigma_{rr}^{(1)} = K_1(2A_1 + \varepsilon_o - 3\alpha_1\phi) + 2\underline{\Gamma}(A_1/3 - B_1/r^2-\varepsilon_o/3)$$

$$\sigma_{\theta\theta}^{(1)} = K_1(2A_1+\varepsilon_o-3\alpha_1\phi) + 2\underline{\Gamma}(A_1/3+B_1/r^2-\varepsilon_o/3) \qquad (3.2.15)$$

$$\sigma_{zz}^{(1)} = K_1(2A_1+\varepsilon_o-3\alpha_1\phi) + 2\underline{\Gamma}(-2A_1/3 + 2\varepsilon_o/3)$$

Fiber and matrix stresses must satisfy the equilibrium equation

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr}-\sigma_{\theta\theta}}{r} = 0 \qquad (3.2.16)$$

This is obviously true for fiber stresses and is easily verified for the stresses (3.2.15).

Introduction of the relevant results into (3.2.9-10) yields equations for the unknowns A_1, B_1 and A_2. After some rearrangement these may be put into the form

$$2K_1A_1(t) + 2(1/3+v_2)\underline{\Gamma}A_1(t)-2v_2\underline{\Gamma}A_2(t) =$$

$$= \sigma_T(t)+3K_1\alpha_1\phi(t) - K_1\varepsilon_L(t) + 2/3\underline{\Gamma}\varepsilon_L(t)$$

$$(3.2.17)$$

$$2K_1A_1(t) + 8/3\underline{\Gamma}A_1(t) - 2k_2A_2(t) - 2\underline{\Gamma}A_2(t) =$$

$$= (3K_1\alpha_1+D_T)\phi(t) + (\ell_2-K_1)\varepsilon_L(t)+2/3\underline{\Gamma}\varepsilon_L(t)$$

where

$$v_2 = (a/b)^2 \tag{3.2.18}$$

Another important case is uniform stressing on the terminal sections of the cylinder. Thus

$$\sigma_{zz}(r,o,t) = \sigma_{zz}(r,L,t) = \sigma_L(t) \tag{3.2.19}$$

The solution form (3.2.11) remains valid but $\varepsilon_L(t)$ is an additional unknown. An additional equation is obtained by averaging the stress σ_{zz} from (3.2.13) and (3.2.15). The resulting equation is after rearrangement

$$2v_1K_1A_1(t) - 4/3v_1\underline{\Gamma}A_1(t) + 2\ell_2v_2A_2(t)$$

$$+(v_1K_1+\nu_2n_2)\varepsilon_L(t) + 4/3v_1\underline{\Gamma}\varepsilon_L(t) =$$

$$= \sigma_L(t) + (3v_1K_1\alpha_1 - v_2D_L)\phi(t) \tag{3.2.20}$$

where

$$v_1 = 1 - v_2 \tag{3.2.21}$$

It should be noted that σ_{zz} in fibers and matrix are very different and therefore satisfaction of end conditions in average fashion produces Saint Venant type local perturbations near the terminal sections. Equations (3.2.17), (3.2.20) now determine the unknowns A_1, A_2 and ε_L. In both cases B_1 is related to A_1 and A_2 by

$$A_1 + B_1/a^2 = A_2 \tag{3.2.22}$$

which is a consequence of (3.2.10a) and (3.2.11)

The important case of free thermal expansion is obtained by subjecting the composite cylinder without load to a temperature step function rise $\Delta\phi$, relative to reference temperature ϕ_o. Then

$$\sigma_L(t) = \sigma_T(t) = 0 \tag{3.2.23}$$

Solving equations (3.2.18), (3.2.20) for this case the effective longitudinal secant expansion coefficient α_L^* is defined by ε_L while the transverse expansion coefficient is defined by the radial surface displacement per unit radius. Thus

$$\begin{aligned} \alpha_L^*(t,\phi_o,\Delta\phi) &= \varepsilon_L(t)/\Delta\phi \\ \alpha_T^*(t,\ \phi_o,\ \Delta\phi) &= [A_1(t) + B_1(t)/b^2]\ /\Delta\phi \end{aligned} \tag{3.2.24}$$

Another important case defined by the present formulation is axial stressing $\sigma_L(t)$ or axial straining $\varepsilon_L(t)$ in isothermal conditions or together with temperature changes.

3.3 Longitudinal Shear

We consider a unidirectionally reinforced specimen which is subjected to the boundary conditions

$$u_1(S) = \beta(t)x_2 \qquad u_2(S) = \beta(t)x_1 \qquad u_3(S) = 0 \tag{3.3.1}$$

which represents a pure shear displacement. Then the only surviving average strain is $\bar{\varepsilon}_{12}(t) = \beta(t)$ and if the specimen is transversely isotropic the only surviving average stress is $\bar{\sigma}_{12}(t)$.

Dually, the specimen is subjected to the shearing boundary conditions

$$T_1(S) = \tau(t)n_2 \qquad T_2(S) = \tau(t)n_1 \qquad T_3(S) = 0 \tag{3.3.2}$$

in which case the only surviving average stress is $\bar{\sigma}_{12}(t) = \tau(t)$ and the only surviving average strain is $\bar{\varepsilon}_{12}(t)$. It has been shown [3] that for an elastic fiber composite such a problem is solved by antiplane strain formulation. The same formulation is valid in the present case. The displacements in fiber region R_2 and in matrix region R_1 have the form

$$\begin{aligned} &u_1(\underline{x}) = \phi - \beta x_2 && \text{(a)} \\ &u_2(\underline{x}) = \beta x_2 && \text{(b)} \\ &u_3(\underline{x}) = 0 && \text{(c)} \end{aligned} \tag{3.3.3}$$

where

$$\phi = \begin{cases} \phi^{(1)}(x_2,x_3,t) & \text{in } R_1 \\ \phi^{(2)}(x_2,x_3,t) & \text{in } R_2 \end{cases} \tag{3.3.4}$$

Then the only surviving strains and stresses are

$$\begin{aligned} &2\varepsilon_{12} = \phi_{,2} && 2\varepsilon_{13} = \phi_{,3} && && \text{(a)} \\ &\sigma_{12} = \underline{\Gamma}\,\phi^{(1)}_{,2} && \sigma_{13} = \underline{\Gamma}\,\phi^{(1)}_{,3} && \text{in } R_1 && \text{(b)} \\ &\sigma_{12} = G_L\,\phi^{(2)}_{,2} && \sigma_{13} = G_L\,\phi^{(2)}_{,3} && \text{in } R_2 && \text{(c)} \end{aligned} \tag{3.3.5}$$

where $\underline{\Gamma}$ is defined by (2.17), G_L is the fiber longitudinal shear modulus and a comma denotes partial differentiation. The only surviving equilibrium equation is

$$\sigma_{12,2} + \sigma_{13,3} = 0 \tag{3.3.6}$$

Introducing (3.3.5 b,c) into (3.3.6) we have

$$\begin{aligned} \nabla^2 \phi^{(1)} &= 0 \qquad \text{in } R_1 \\ \nabla^2 \phi^{(2)} &= 0 \qquad \text{in } R_2 \end{aligned} \tag{3.3.7}$$

It follows from (3.3.3) and (3.3.1) that Φ must satisfy the boundary condition

$$\Phi \ (C) = 2\beta x_2 \tag{3.3.8}$$

where C is the contour of the specimen section. At fiber/matrix interface C_{12} the displacements and tractions must be continuous. Since the only surviving stresses are σ_{12} and σ_{13} the only surviving traction on C_{12} is

$$T_1 = \sigma_{12}n_2 + \sigma_{13}n_3 \tag{3.3.9}$$

Introducing (3.3.5 b,c) into (3.3.9) and equating fiber T_1 to matrix T_1 we have

$$\underline{\Gamma} \frac{\partial\phi^{(1)}}{\partial n} = G_L \frac{\partial\phi^{(2)}}{\partial n} \qquad \text{on } C_{12} \tag{3.3.10}$$

and displacement continuity in terms of (3.3.3) is satisfied if

$$\phi^{(1)} = \phi^{(2)} \qquad \text{on } C_{12} \tag{3.3.11}$$

If the boundary conditions are (3.3.2), and assuming that the entire boundary C is in the matrix, then (3.3.8) is replaced by

$$\underline{\Gamma}\,\frac{\partial\phi^{(1)}}{\partial n} = \tau(t)\,n_2 \tag{3.3.12}$$

This completes the mathematical formulation for analysis of any unidirectional specimen under longitudinal shear.

Our purpose is to analyze the composite cylinder assemblage model in terms of the above given formulation. A detailed analysis of elastic longitudinal shear has been given in [3] and the present situation is very similar. A composite cylinder subjected to (3.3.1) will have on its boundary tractions of form (3.3.2) and vice versa. Thus a composite cylinder behaves to an external observer as a homogeneous circular cylinder with some viscoelastic stress-strain relation. If this stress strain relation is assigned to a homogeneous cylindrical specimen under longitudinal shear, circular cylindrical portions of the specimen can be replaced by composite cylinders without perturbing the states of stress and strain in the remainder. Thus just as in the axisymmetric case, the stress-strain law of the assemblage becomes that of one composite cylinder.

Consequently, we consider one composite cylinder subjected to (3.3.1). The boundary and interface conditions (3.3.8), (3.3.10-11) for the two harmonic functions $\phi^{(1)}(r,\theta,t)$, $\phi^{(2)}(r,\theta,t)$ then become

$$\begin{aligned}
&\phi^{(1)}(b,\theta,t) = 2\beta(t)b \text{ as } \theta \\
&\phi^{(1)}(a,\theta,t) = \phi^{(2)}(a,\theta,t) \\
&\underline{\Gamma}\,\frac{\partial\phi^{(1)}}{\partial r} = G_L\,\frac{\partial\phi^{(2)}}{\partial r}
\end{aligned} \tag{3.3.13}$$

The functions Φ which solve this problem are

$$\phi^{(1)}(r,\theta,t) = \left[A_1(t) + B_1(t)/r\right] \cos\theta$$

$$\phi^{(2)}(r,\theta,t) = A_2(t) \cos\theta \qquad (3.3.14)$$

where A_1, A_2 and B_2 are defined through (3.3.13) by the equations

$$A_1(t) = \left[2\beta(t) - v_2 A_2(t)\right]/v_1 \qquad (a)$$

$$B_1(t) = a^2\left[A_2(t) - 2\beta(t)\right]/v_1 \qquad (b) \qquad (3.3.15)$$

$$(1+v_2)\ \underline{\Gamma}\ A_2(t) + v_1 G_L A_2(t) = 4\ \underline{\Gamma}\beta(t) \qquad (c)$$

The last equation is an integral equation which can be simplified by operating on both sides with $\underline{\gamma}$ which is the inverse of $\underline{\Gamma}$, see (2.16). Remembering that $B(t)=\bar{\varepsilon}_{12}(t)$ we have

$$(1+v_2)\ A_2(t) + v_1 G_L \underline{\gamma}\ A_2(t) = 4\bar{\varepsilon}_{12}(t) \qquad (3.3.16)$$

The boundary traction $T_1(b,\theta,t)$ is the left side of (3.3.13c) evaluated at $r=b$. It follows from (3.3.14) that it is given by

$$T_1(b,\theta,t) = \underline{\Gamma}\ (A_1 - B_1/b^2) \cos\theta \qquad (3.3.17)$$

If the cylinder were homogeneous the boundary tractions would have the form (3.3.2). Since $\cos\theta=n_2$ on $r=b$ it is seen that (3.3.17) is of that form (T_2 vanishes on $r=b$ since $n_1=0$ but does exist on the end sections where $n_2=0$, $n_1=1$). Therefore the average stress response $\bar{\sigma}_{12}(t)$ is identified by the multiplier of $\cos\theta$ in (3.3.18). Using (3.3.16) we have after rearrangement

$$\bar{\gamma}\ \bar{\sigma}_{12}(t) = \frac{2}{v_1}\ (1+v_2)\bar{\varepsilon}_{12}(t) - v_2 A_2(t) \qquad (3.3.18)$$

where A_2 is defined by (3.3.16). If desired, A_2 can be eliminated by use of (3.3.16) and then (3.3.18) assumes the operational form

$$(1+v_2+v_1 G_L \underline{\gamma})\bar{\sigma}_{12}(t) = 2\left[v_1 \underline{\Gamma} + (1+v_2) G_L\right]\bar{\varepsilon}_{12}(t) \tag{3.3.19}$$

Equation (3.3.19) defines the stress-strain relation of the fiber composite as modelled by the CCA. The relaxation modulus is given from (3.3.19) as $\bar{\sigma}_{12}(t)$ when $\bar{\varepsilon}_{12}(t) = H(t)$ and dually the creep compliance is given as $\bar{\varepsilon}_{12}(t)$ when $\bar{\sigma}_{12}(t) = H(t)$. When the matrix is elastic $\underline{\Gamma} = G_1$ and $\underline{\gamma} = 1/G_1$ and the known results for the elastic longitudinal effective shear modulus [8,3] are obtained.

3.4 Transverse Shear and Transverse Uniaxial Stress

Let the composite be subjected to the average stress state

$$\left[\bar{\sigma}_{ij}\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma^0_{22}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.4.1}$$

The loading which produces (3.4.1) is shown in figure 5. The stress (3.4.1) may be split in the following fashion

$$\left[\bar{\sigma}_{ij}\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2}\sigma^0_{22}(t) & 0 \\ 0 & 0 & \frac{1}{2}\sigma^0_{22}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2}\sigma^0_{22}(t) & 0 \\ 0 & 0 & -\frac{1}{2}\sigma^0_{22}(t) \end{bmatrix} \tag{3.4.2}$$

The first part is an isotropic transverse stress which has been considered in section 3.2 while the second is a pure transverse shear of magnitude $\tau(t)=\frac{1}{2}\sigma_{22}^{o}(t)$ oriented at 45° with respect to the material axes x_1, x_2, x_3. Since the material is linear the response to transverse uniaxial strain is obtained by superposition of the isotropic stress response and the transverse shear response.

Unfortunately, however, the CCA model cannot be directly analyzed for transverse shear. This is a well known difficulty which arises also in the elastic context. Therefore, a different model is needed for this case. A suitable model is hexagonal array of identical circular fibers. Such a model has the required transverse material isotropy. It is well known that in all cases where CCA analyses can be performed hexagonal array and CCA results are numerically extremely close (see i.e. [3]), therefore no practical inconsistency is expected by reverting more to the hexagonal array model.

Analysis of the periodic hexagonal array is performed using finite element analysis techniques. The finite element developed for this analysis is a four noded linear rectangle of the serendipity type. The inclusion of a linear displacement function in the x_1 direction (independent of x_2,x_3) yields the capability for generalized plane strain analysis. Viscoelastic material properties are incorporated into a pre-stress as described in the following section.

The periodic hexagonal array (figure 3) is such that only a single repeating element need be modelled. The various symmetrices of the model and loading allow for the prescription of the necessary boundary conditions on the repeating element. The appropriate boundary conditions for the repeating element can be determined by noting that each of the hexagonal cylinders are identical and that under any loading state, the array of hexagons must be contiguous.

In the case of transverse (x_2) extensional or compressional loading, the loading and model symmetrices require that sides 1, 3, and 4 of the repeating element (figure 3) remain straight, and parallel to their original positions. Contiguity between hexagonal cylinders prescribes that along side 2, the line connecting the fiber centers define a point of cylic symmetry. Therefore, points on side 2 above the intersection of the fiber center line deflect exactly opposite of equidistant points on the other sides. Stress boundary conditions are incorporated in the usual fashion with the stresses being integrated to determine applied nodal forces.

4. NUMERICAL PROCEDURES AND IMPLEMENTATION

The analytical models developed in the previous section involve the integral operator representation of the matrix shear response. The presence of the integral operator necessitates that the models be evaluated incrementally in time. The procedure utilized for both the CCA and hexagonal array models involves specifying the time history of applied load and temperature and determining the response of the composite at times 0, Δt, $2\Delta t$, $3\Delta t$, etc. where Δt is an increment of time. At each time step, the hereditary integrals are evaluated numerically using simple trapezoidal integration.

4.1 Composite Cylinder Assemblage

Literature review has yielded matrix thermoviscoelastic material properties in the form of compliance data only. The equations 3.2.17, 18 and 3.2.20 have therefore been converted from relaxation to compliance form utilizing the relation $\underset{\sim}{\gamma}\,\underset{\sim}{\Gamma}x=x$. This operation yields the equations

$$2\underset{\sim}{\gamma}\left[k_1A_1(t)\right] + 2(1/3 + v_2)A_1(t)-2v_2A_2(t)=$$

$$\underset{\sim}{\gamma}\sigma_1(t)+3\underset{\sim}{\gamma}[k_1\alpha_1\phi(t)]-\underset{\sim}{\gamma}[k_1\varepsilon_L(t)]+2/3\varepsilon_L(t) \qquad (4.1.1)$$

$$2\gamma[k_1A_1(t)] + 8/3A_1(t) - 2\gamma[k_2A_2(t)] - 2A_2(t) =$$

$$3\gamma[(k_1\alpha_1+D_T)\phi(t)]+\gamma[(\ell_2(t)-k_1)\varepsilon_L(t)]+2/3\varepsilon_L(t) \qquad (4.1.2)$$

$$2v_1\gamma[k_1A_1(t)]-4/3v_1A_1(t)+2v_2\gamma[\ell_2A_2(t)]+v_1\gamma[k_1\varepsilon_L(t)]+v_2\gamma[n_2\varepsilon_L(t)]$$

$$+4/3v_1\varepsilon_L(t)=\gamma\ \sigma_L(t)+3v_1\gamma[k_1\alpha_1\phi(t)]-v_2\gamma[D_L\phi(t)] \qquad (4.1.3)$$

where both the matrix bulk modulus k_1 and the fiber elastic constants may be temperature dependent and therefore dependent upon the time history of the applied temperature.

During the first time step evaluation, the hereditary integral operator reduces to simply

$$\gamma x(o)=g(o)x(o) \qquad (4.1.3)$$

and the problem reduces to an elastic analysis with the initial, t=o, loadings and the initial matrix shear compliance, g(o). Subsequent time steps are evaluated utilizing a trapezoidal numerical integration scheme where the hereditary integral operator is evaluated as

$$\gamma x=C_1+C_2x(t_i) \qquad (4.1.4)$$

The constants C_1 and C_2 are functions of the prior time history of $x(t)$ and $\phi(t)$. This yields three linear simultaneous equations for $A_1(t_i)$, $A_2(t_i)$, and $\varepsilon_L(t_i)$. Thus the problem is easily solved.

4.2 Finite Element Analysis

Implementation of the thermo-viscoelastic matrix constitutive relations within the finite element analysis is accomplished in the same manner as in the CCA model. The viscoelastic strain (or stress) is simply treated as a pre-stress in the analysis.

The form of the thermoviscoelastic constitutive relations for the matrix were defined in equations 2.3 with the shear hereditary integral defined in equation 2.13. Utilizing the compliance form, an elemental constitutive matrix is formulated

$$\{\varepsilon\} = [D]^{-1}\{\sigma\}$$

where $[D]^{-1}$ is viscoelastic and the compliance matrix $[D]^{-1}$ is comprised of terms involving the inverse of the matrix bulk modulus operating on the isotropic part of the stress tensor σ_{kk} and the shear compliance operator $\underline{\gamma}$ operating on the deviatoric part of the stress tensor s_{ij}. Evaluation of the compliance matrix at any time step is therefore evaluated using the same simple trapezoidal integration scheme as in the CCA model yielding

$$\{\varepsilon(t_i)\} = [D_i]^{-1}\{\sigma(t_i)\} + \{t_p\} \qquad (4.2.1)$$

This form is equivalent to equation 4.1.4 and the constant matrix $[D_i]^{-1}$ and vector $\{t_p\}$ are functions of the prior time history of $\phi(t)$ and $\{\sigma(t)\}$. The constitutive relations 4.2.1 are then inverted to yield

$$\{\sigma(t_i)\} = [D_i]\{\varepsilon(t_i)\} + \{\sigma_p\}$$

The development from this point on is the standard method for obtaining the elemental stiffness and load vector for finite element analysis and will not be detailed.

5. PREDICTED COMPOSITE BEHAVIOR

The two analytical models developed in the previous sections have been utilized to predict the response of a unidirectional fiber composite with a thermo-viscoelastic matrix. The results of the analysis are presented in this section.

5.1 Material Properties and Finite Element Mesh

The fiber properties utilized are typical carbon fiber data and are given in table 1. It has been assumed that these properties are independent of temperature.

Matrix properties utilized in the analysis are given in table 1 and figures 6-9. These properties are derived from data obtained from [6] utilizing some basic assumptions and modifications. The data obtained from [6] included a complete characterization of the extensional properties of a 3502 matrix material system over the temperature range 303°K-423°K. The thermal expansion coefficient was found to be constant over this temperature range. Since two material constants are needed to define an isotropic material and only extensional data were given, Poisson's ratio at the reference temperature was assumed to be 0.3, thus defining the matrix initial, reference temperature properties. The time dependent portion of the matrix shear compliance, Δg, is shown in figure 6 and was computed directly from the data in [6] noting that all time dependent extensional response is due to shear alone.

Determining the temperature dependence of the matrix parameters g_o, h, V_1 and V_2 required modifications to the data from [6]. The data were obtained in a form which could not be used below the reference temperature, 303°K. Additionally, the two vertical shift functions, V_1 and V_2 were not well balanced when extrapolated beyond the temperature range over which they were obtained. These difficulties and the lack of a second matrix material constant as described above were remedied by modifying the form of the mathematical representation of all temperature dependent material parameters which were obtained from [6]. After modification, a single vertical shift function was obtained as the product of the functions V_1 and V_2 and this function was used to replace V_2 in equation 2.13 with V_1 set to unity. Finally, it was assumed that the matrix bulk modulus was independent to temperature. These modifications yielded the data shown in figures 7-9. While these data are no longer purely experimentally obtained, they do provide a consistent set of properties for analysis.

In both the CCA and finite element analytical models, the fiber volume fraction was set as 0.6. The finite element model of the repeating element of the periodic hexagonal array is shown in figure 10. The loadings for the finite element analytical model consisted of transverse extension and temperature. Boundary conditions for these analyses were discussed in section 3.4.

5.3 Axial Shear and Temperature Loading

The impetus for developing the analytical models described earlier was to determine the response of a unidirectional composite subjected to combined loading and cylic temperature. The temperature cycle of interest has extremes of 116°K and 589°K. Before performing the analysis corresponding to such a thermal cycle, isothermal analysis at the two extremes and at the reference temperature were performed.

The CCA model was utilized with a constant axial shear stress of 1 MPa at temperatures of 116°K, 303°K, and 589°K. The resulting axial shear strains are shown in figures 11 and 12. These data demonstrate the acceleration of creep strain and softening of the initial and time dependent portions of the composite response as temperature is increased. The axial shear strain at 116°K appears to be nearly constant. This phenomenon is a function of the horizontal shift, h, approaching 10^{-4} and the vertical shift, V_2, approaching 0.5 at this low temperature. The very small value of "h" causes the creep rate to decrease so that in effect, the creep strain after one hour is comparable to a reference temperature creep strain after 10^{-4} hours. The vertical shift factor at 116°K has the effect of nearly halving the creep strain response that would occur at 10^{-4} hours at room temperature. The combination of these two shift functions cause the resulting creep strain to appear constant in the figure.

The thermal strains produced by step temperature changes from the reference temperature to 116°K, 304°K and 589°K are shown in figure 13. The strains have been normalized with respect to the temperature change yielding secant thermal expansion coefficients at these three temperatures. These data demonstrate the same horizontal and vertical shift effects as were seen in the axial shear compliance. The strains at 304°K and 116°K are redrawn with an expanded scale in figure 14. The data again show the very low temperature response to be nearly constant in time.

Having determined the composite response to step mechanical and temperature loadings, an analysis was performed utilizing a constant axial shear stress of 1 MPa in conjunction with the temperature cycle shown in figure 15. The 24 hour period can be considered representative of the temperature cycle experienced by satellite in geosynchronous orbit.

The axial shear strain produced by this loading/cyclic temperature combination over 48 hours (2 periods) is shown in figure 16. The shear strain is seen to increase sharply during the initial 8 hours while the temperature is increasing. As the temperature drops, the strain decreases reflecting the increased matrix initial shear modulus at lower temperatures. The difference in shear strain at 24 and 0 hours is the accumulated creep strain after one temperature cycle.

During the second temperature cycle, the response is similar but the increase in creep strain is much smaller indicating that at some future time, the composite response may settle into purely periodic steady-state behavior.

The free thermal expansion strains produced by the cylic temperature are shown in figures 17 and 18. The evidence of viscoelastic behavior in these results is the non-zero strain at 24 and 48 hours where the temperature is at the reference temperature and no temperature change is applied. The increment of viscoelastic strain induced between 24 and 48 hours is seen to be much smaller than between 0 and 24 hours. As with the axial shear strain response, this would indicate that a steady state response may occur after further cycling.

The total effects of the thermoviscoelastic matrix material on pure thermal expansion is considerably smaller than for the mechanical, axial shear load. This is due to the more nearly hydrostatic matrix stress state induced by a temperature change alone.

5.4 Transverse Extension and Temperature Loading

The finite element analytical model was also used to predict the composite response under combined mechanical and cyclic temperature loading. The mechanical loading consisted of a 1 MPa transverse extensional stress while the temperature cycle was as shown in figure 15.

The composite strain response to this loading history is shown in figure 19. The magnitude of the applied stress is small and the overall composite response is dominated by free thermal expansion. When the free thermal expansion strains are removed, the effects of the thermoviscoelastic matrix on the composite response are readily apparent as seen in figures 20, 21, and 22. These figures persent the mechanical strain in loading direction, (x_2) in the direction perpendicular to the load (x_3) and in the axial direction (x_1), respectively.

The composite response without thermal expansion is seen to reflect the same phenomena that were present under axial shear and thermal cycling. The response during the second temperature cycle is similar to that exhibited during the first cycle but with a reduced total change in strain over the period of the cycle.

6. DISCUSSION AND RECOMMENDATIONS

The development of thermoviscoelastic analytical capabilities for unidirectional fiber composites has provided a means for realistic modelling of material behavior under cyclic loading and temperature environments. These capabilities have been demonstrated through the prediction of composite response under combined Heavieside mechanical load and cyclic thermal load. The results have demonstrated the significance of temperature dependence of the various material parameters in the matrix representation.

The predicted composite compliances under cyclic temperature have indicated that significant differences in response occur during the first and second temperature cycles while indicating that under subsequent cycling, the material response may become steady-state. This area will require further study to adequately define the material response to cyclic temperature.

The difficulties encountered in acquiring matrix thermoviscoelastic material properties and the sensitivity of the predicted composite behavior have demonstrated the need for further study of the proper form of the hereditary integral representation. This area also includes the representation of the various temperature dependent parameters within the hereditary integral.

The analytical models developed have proven the significance of and need for the thermorheologically complex matrix representation. The much more realistic matrix representation utilized here has provided a pioneering effort in the analysis of fiber composites.

REFERENCES

1. Z. Hashin, "Viscoelastic Behavior of Heterogeneous Media," J. Appl. Mech., 32, pp. 157-173, (1965).

2. Z. Hashin, "Viscoelastic Fiber Reinforced Materials," AIAA J., pp. 1411-1417, (1966).

3. Z. Hashin, Theory of Fiber Reinforced Materials, NASA CR-1974, (1972).

4. R. M. Christensen, Theory of Viscoelasticity, Academic Press, (1982).

5. R. A. Schapery, "Viscoelastic Behavior and Analysis of Composite Materials," in Composite Materials, 2, Sendeckyj, G. P. Ed. Academic Press, (1974).

6. B. D. Harper and Y. Weitsman, "Characterization of Thermo-Rheologically Complex Materials," To be published J. of Rheology.

7. Y. Weitsman, "On the Thermoviscoelastic Characterization of Adhesives and Composites," in Progress in Science and Engineering of Composites, Hayashi, T., et al., Eds, ICCM-IV, Tokyo, pp. 771-779, (1982).

8. Z. Hashin and B. W. Rosen, "The Elastic Moduli of Fiber Reinforced Materials," J. Appl. Mech., 31, pp. 223-232, (1964).

Table 1. Material Properties

Fiber Properties

$E_A = 231$ GPA

$E_T = 22.4$ GPa

$G_A = 22.1$ GPa

$G_T = 8.30$ GPa

$\nu_A = 0.30$

$\alpha_A = -1.33$ μm/m°c

$\alpha_T = 7.04$ μm/m°c

$V_F = 0.60$

Matrix properties at reference temperature (303°K)

$k = 3.58$ GPa

$g(o) = 0.606\ (\text{GPa})^{-1}$

$\nu = 0.30$

$\alpha = 30.0$ μm/m°K

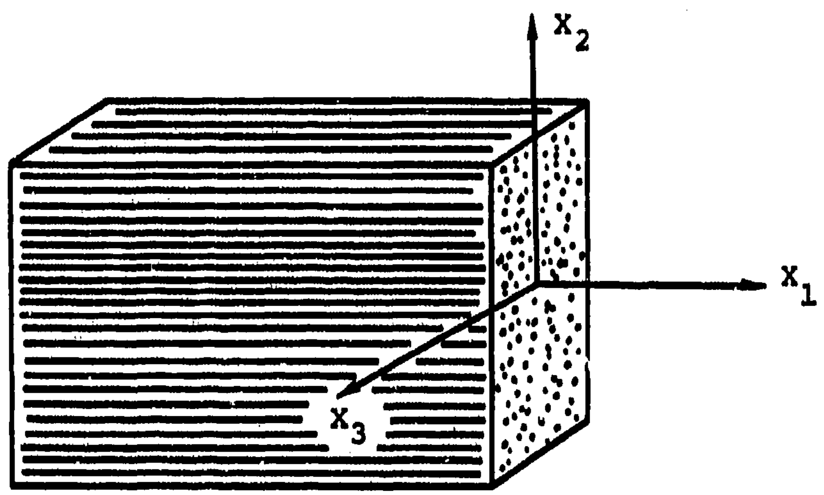

Figure 1. Unidirectional Fiber Composite

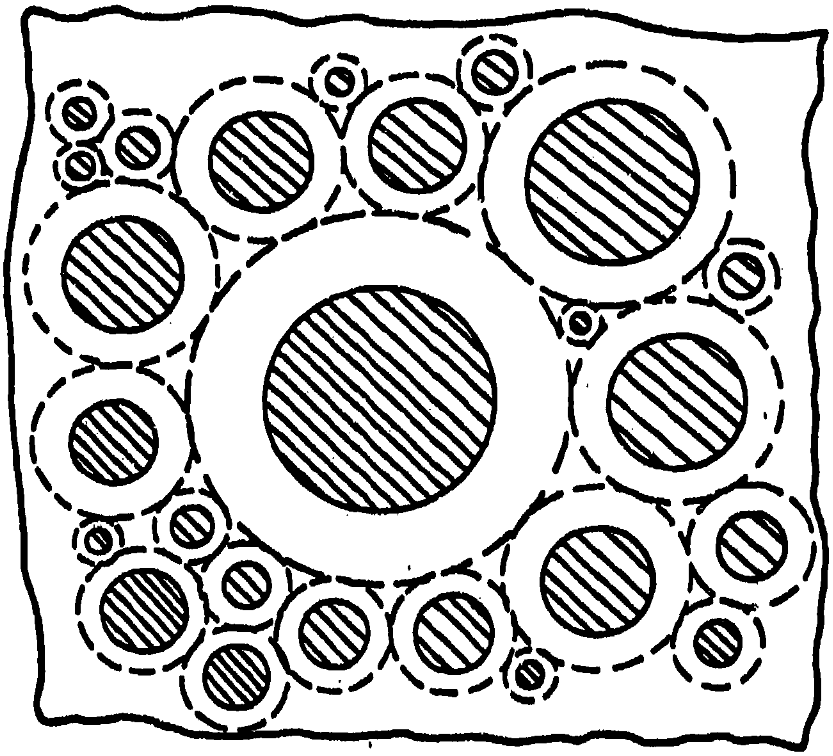

Figure 2. Composite Cylinder Assemblage

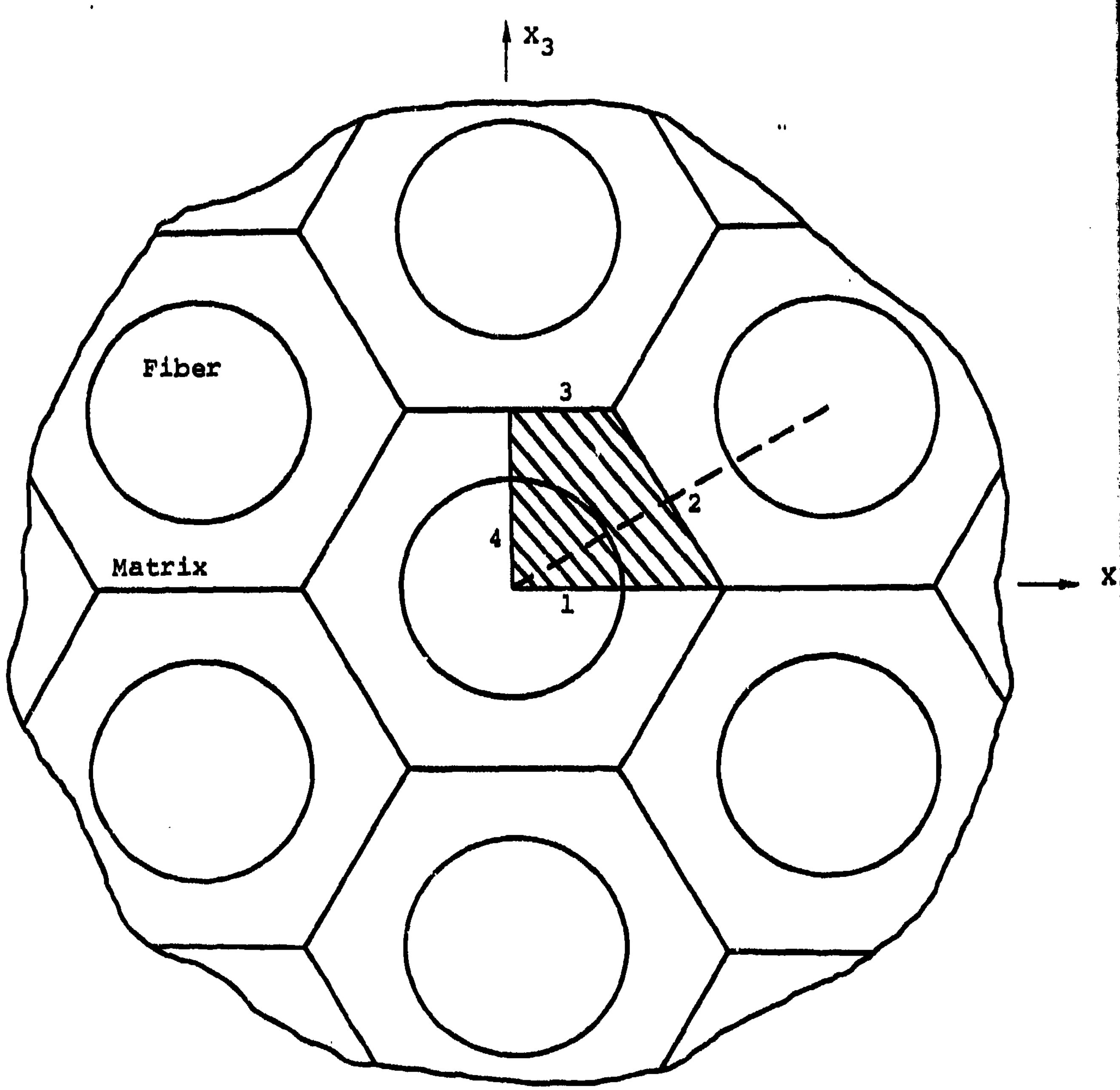

Figure 3. Periodic Hexagonal Array and Repeating Element

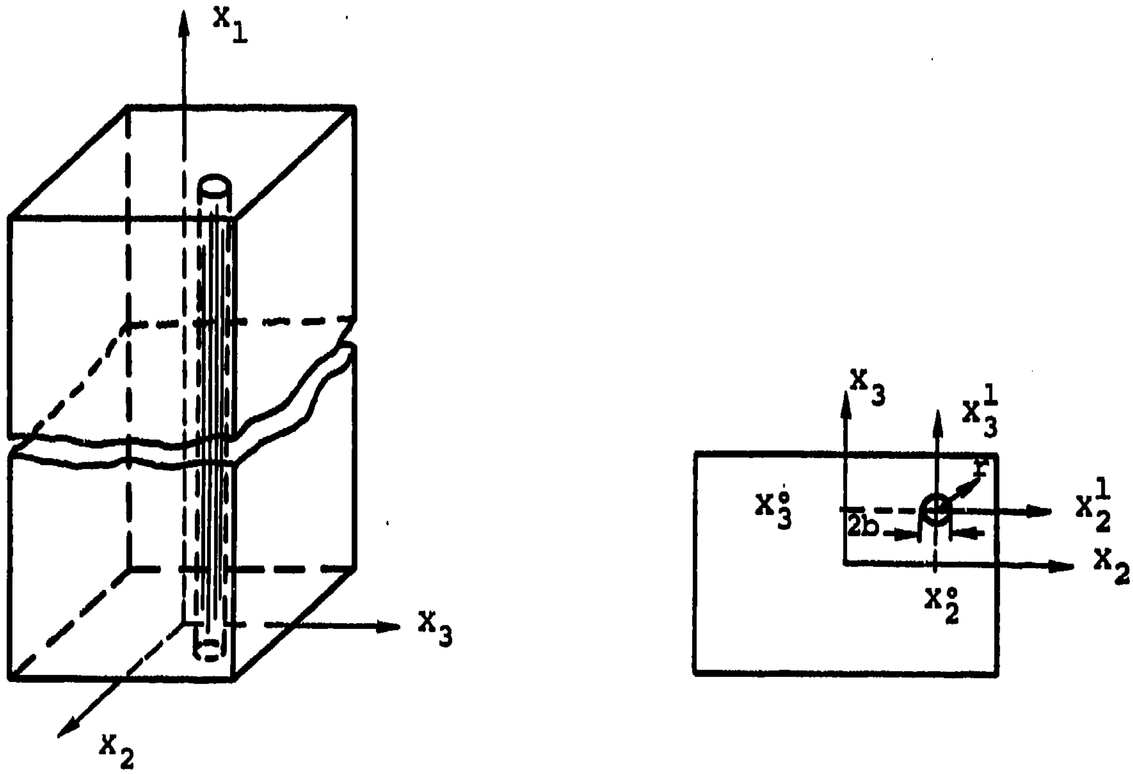

Figure 4. Cylindrical Element

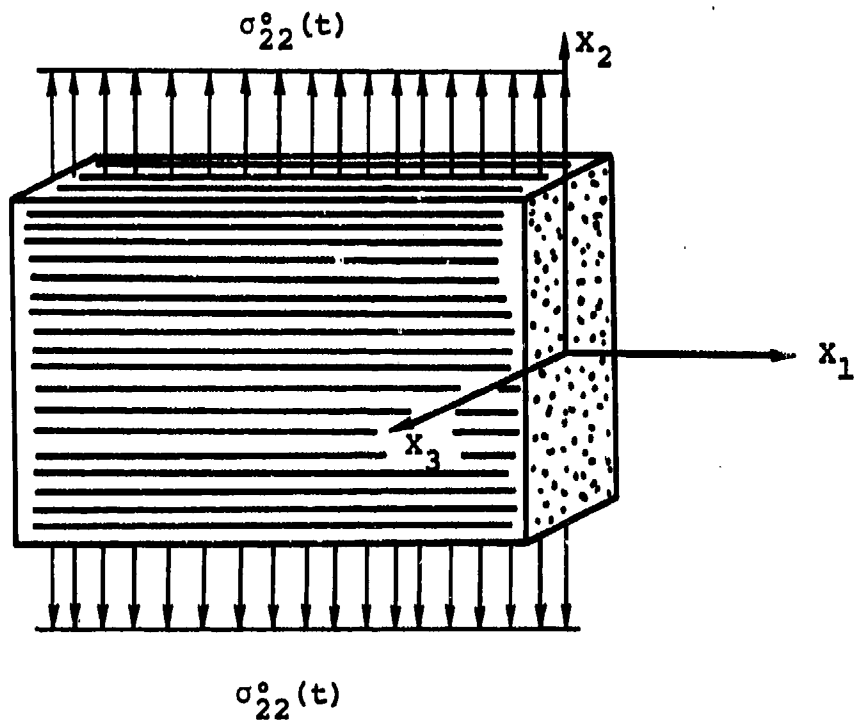

Figure 5. Transverse Stressing

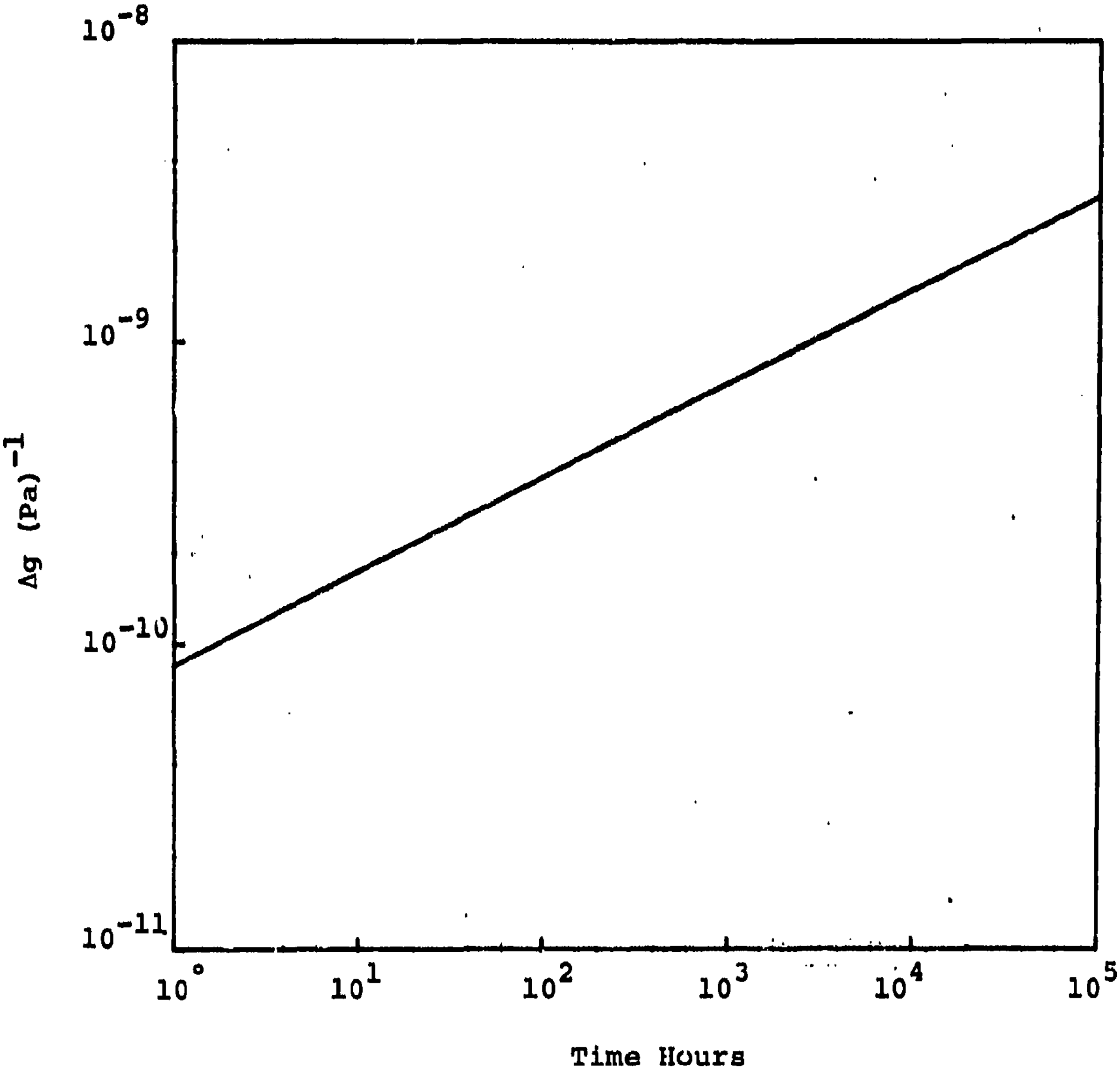

Figure 6. Time Dependence of Matrix Shear Compliance

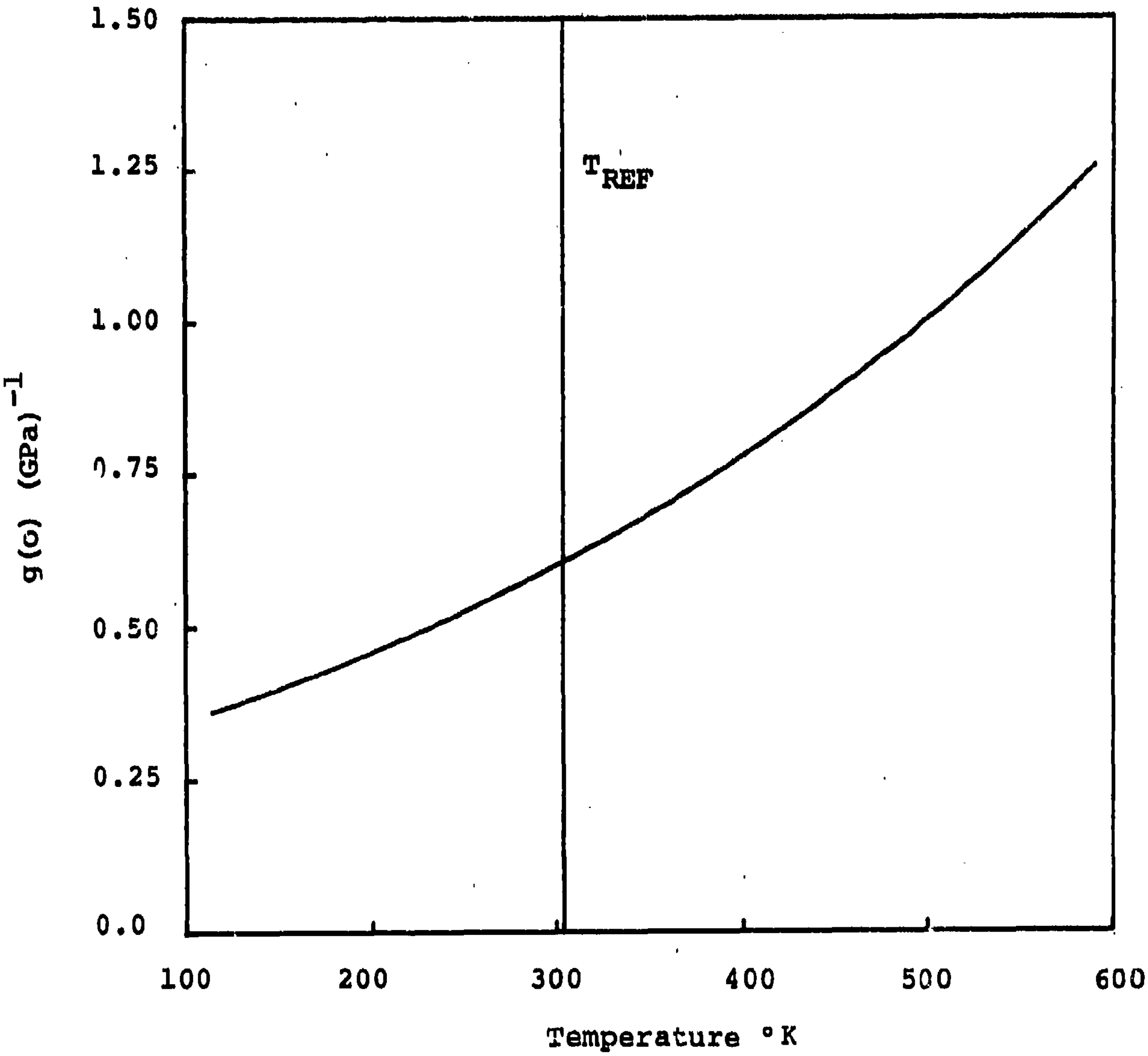

Figure 7. Temperature Dependence of Matrix Initial Shear Compliance

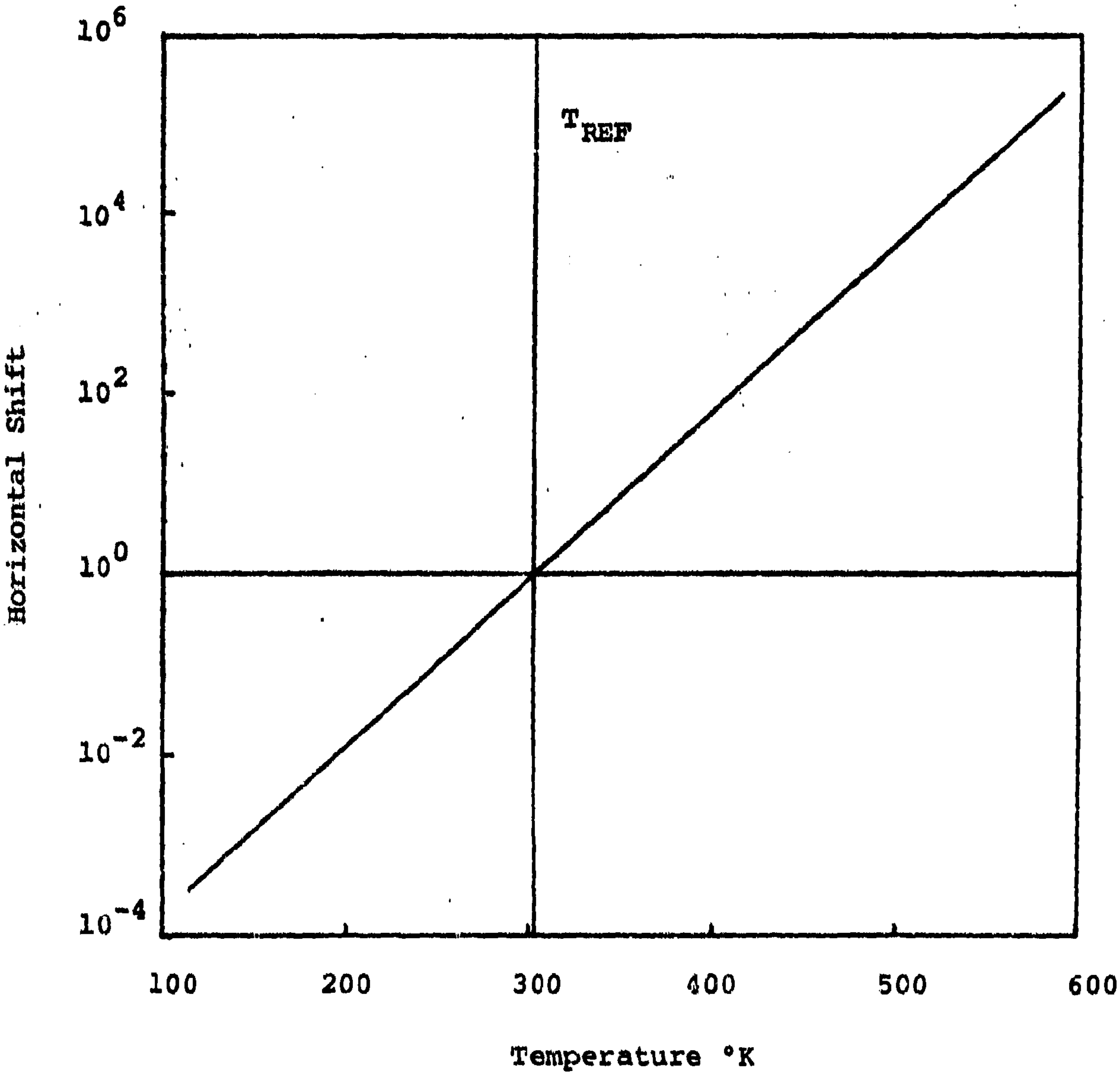

Figure 8. Temperature Dependence of Horizontal Shift Function

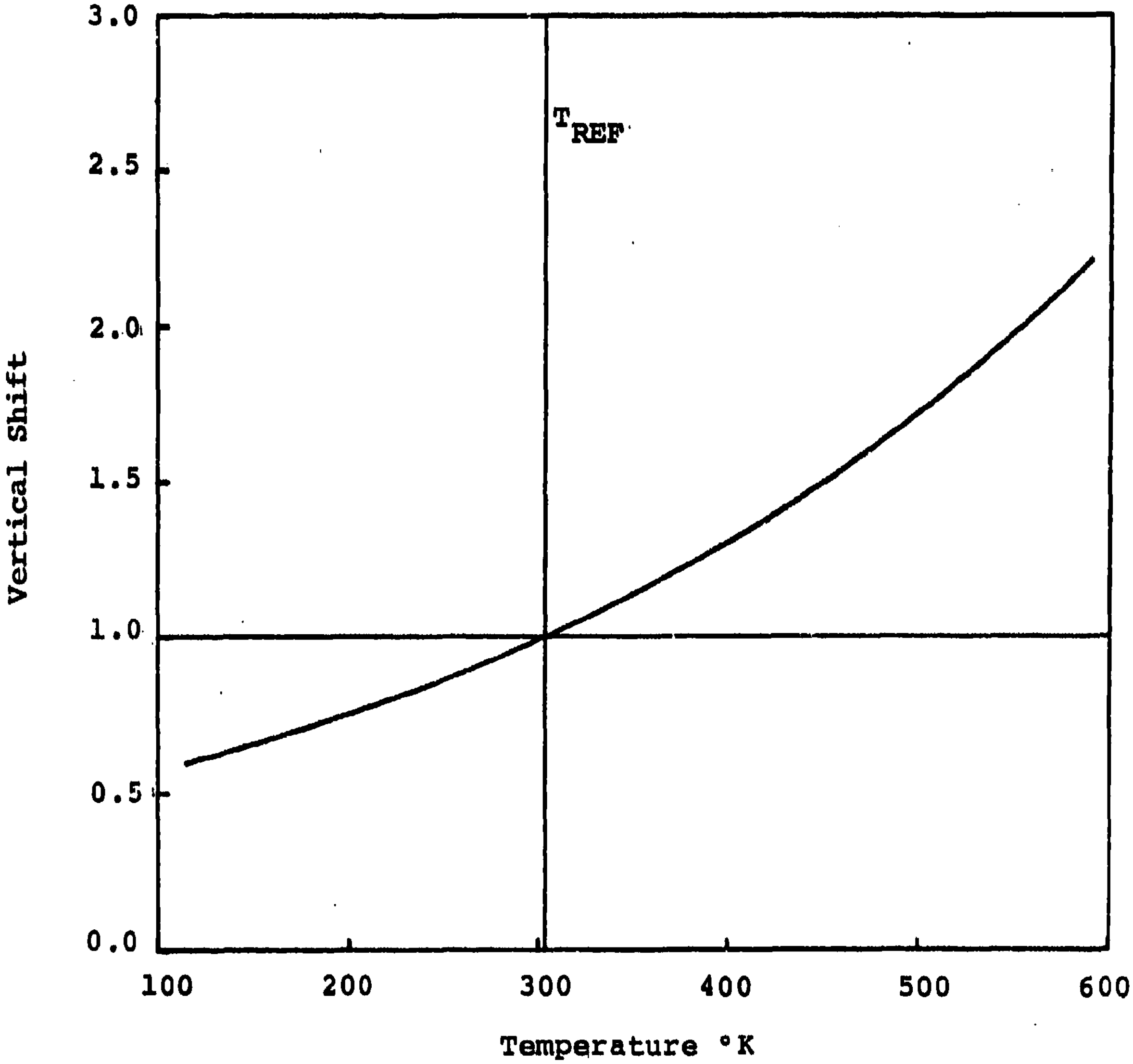

Figure 9. Temperature Dependence of Vertical Shift Function.

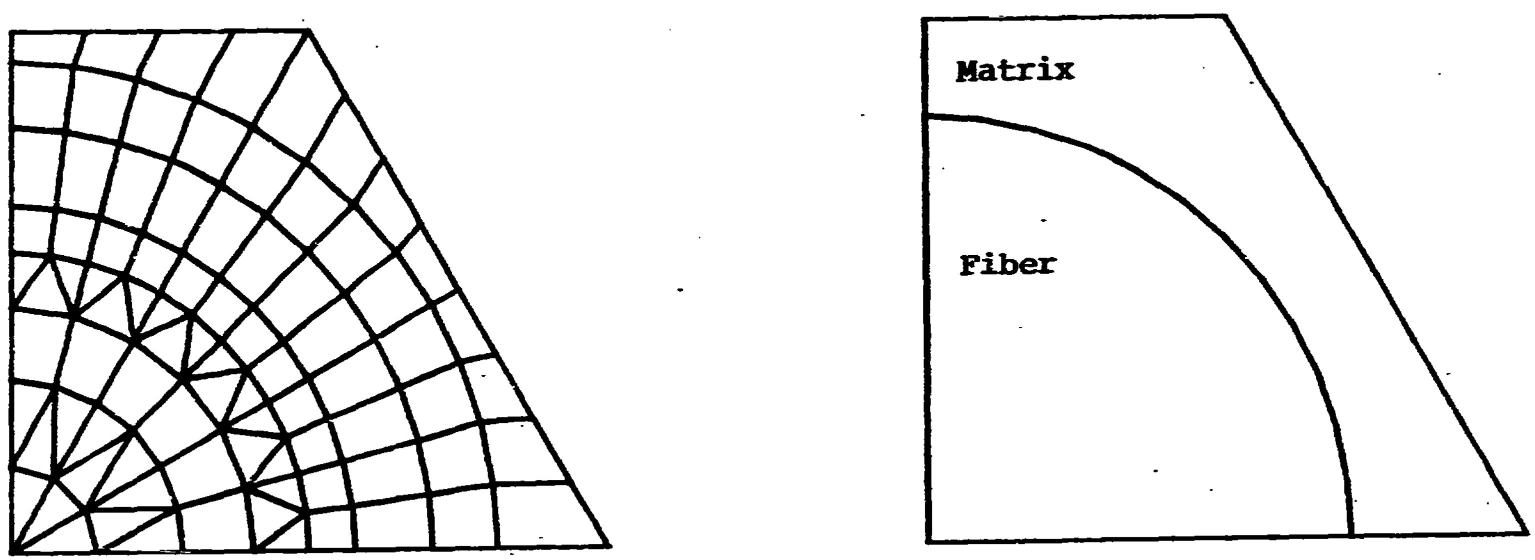

Figure 10. Finite Element Mesh, Hexagonal Array

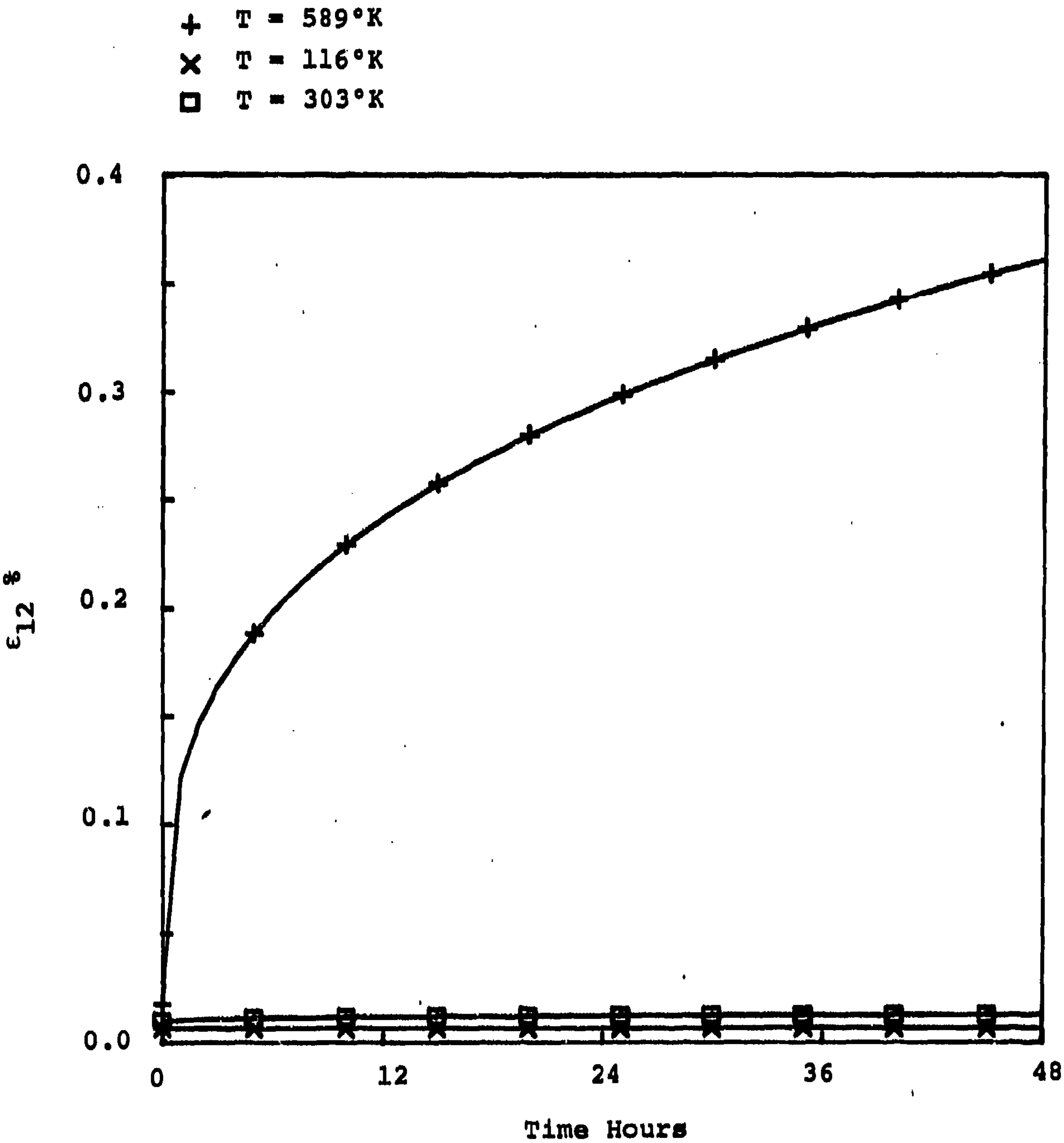

Figure 11. Axial Shear Strain for 1 MPa Shear Stress

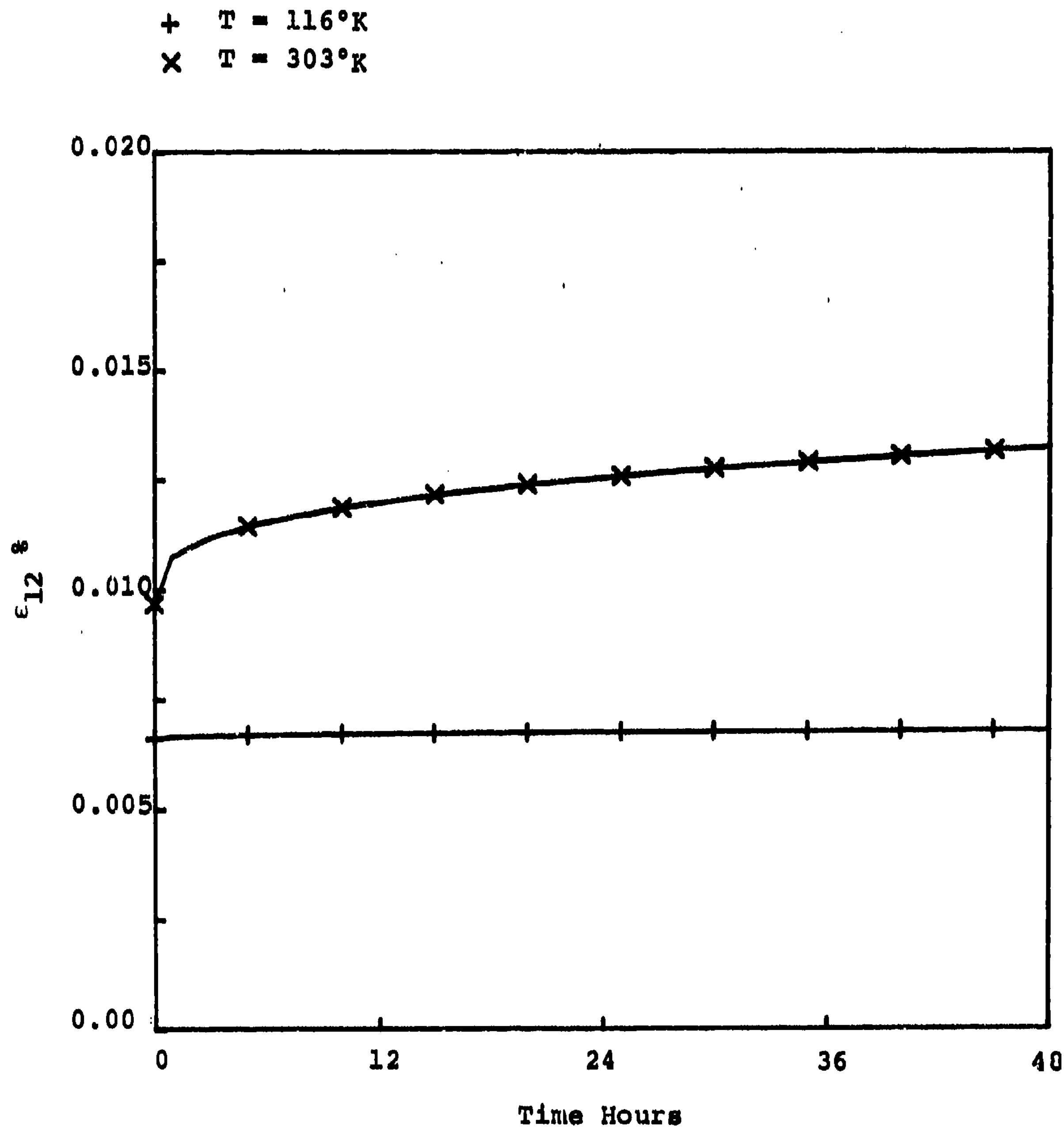

Figure 12. Axial Shear Strain for 1 MPa σ_{12} Stress

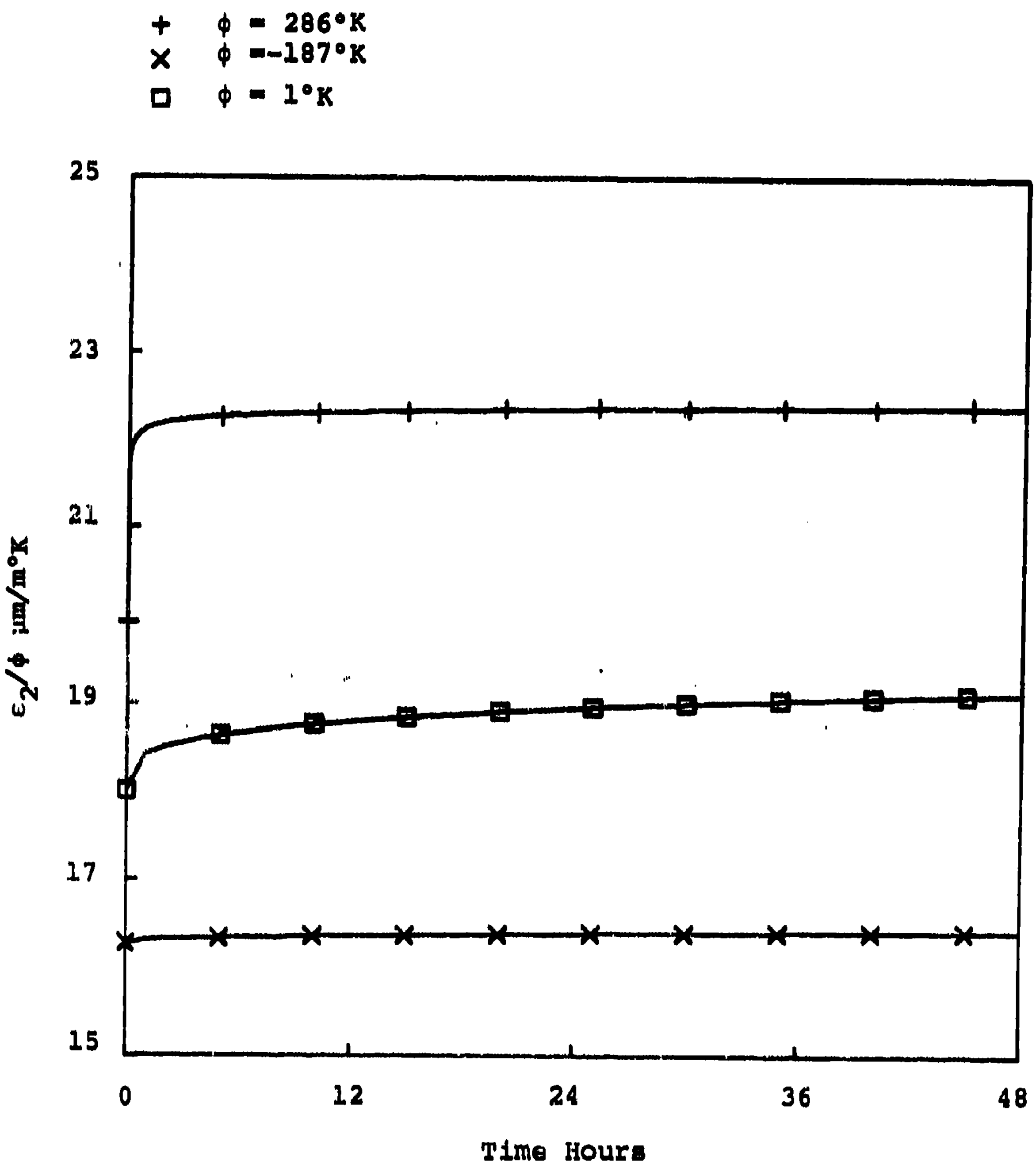

Figure 13. Transverse Thermal Strains Divided by Temperature Change

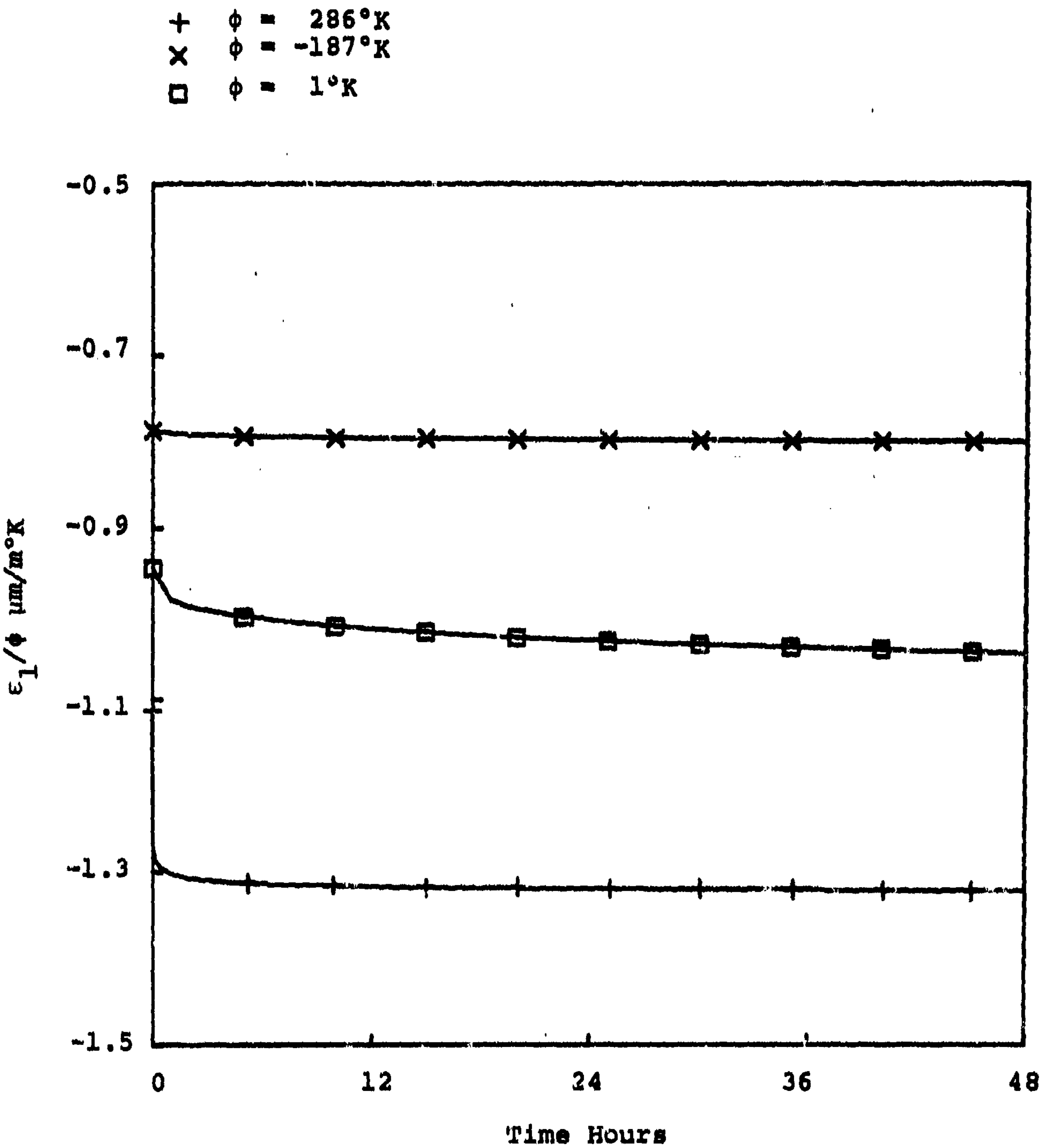

Figure 14. Axial Thermal Strain Divided by Temperature Change

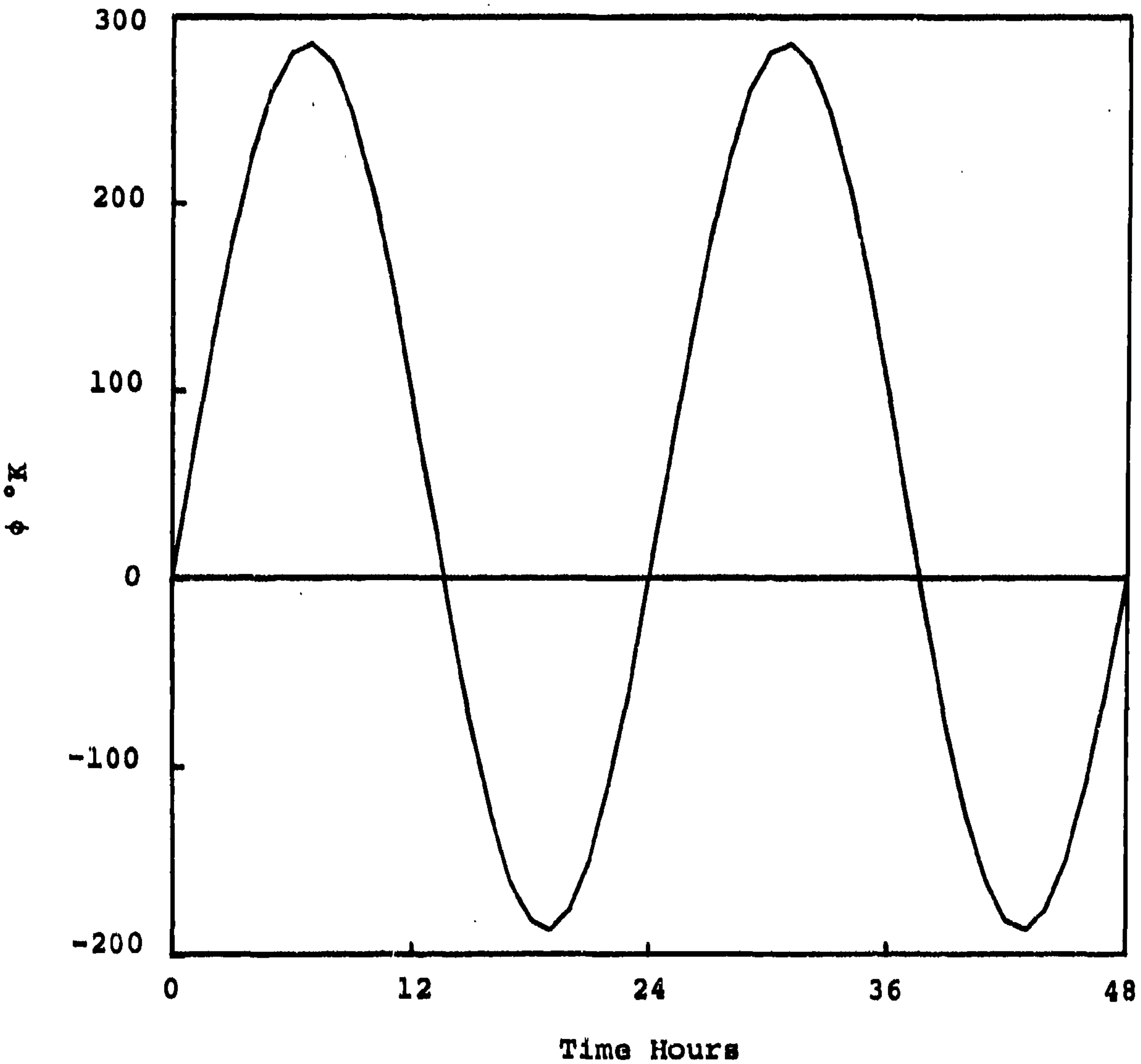

Figure 15. Temperature Cycle

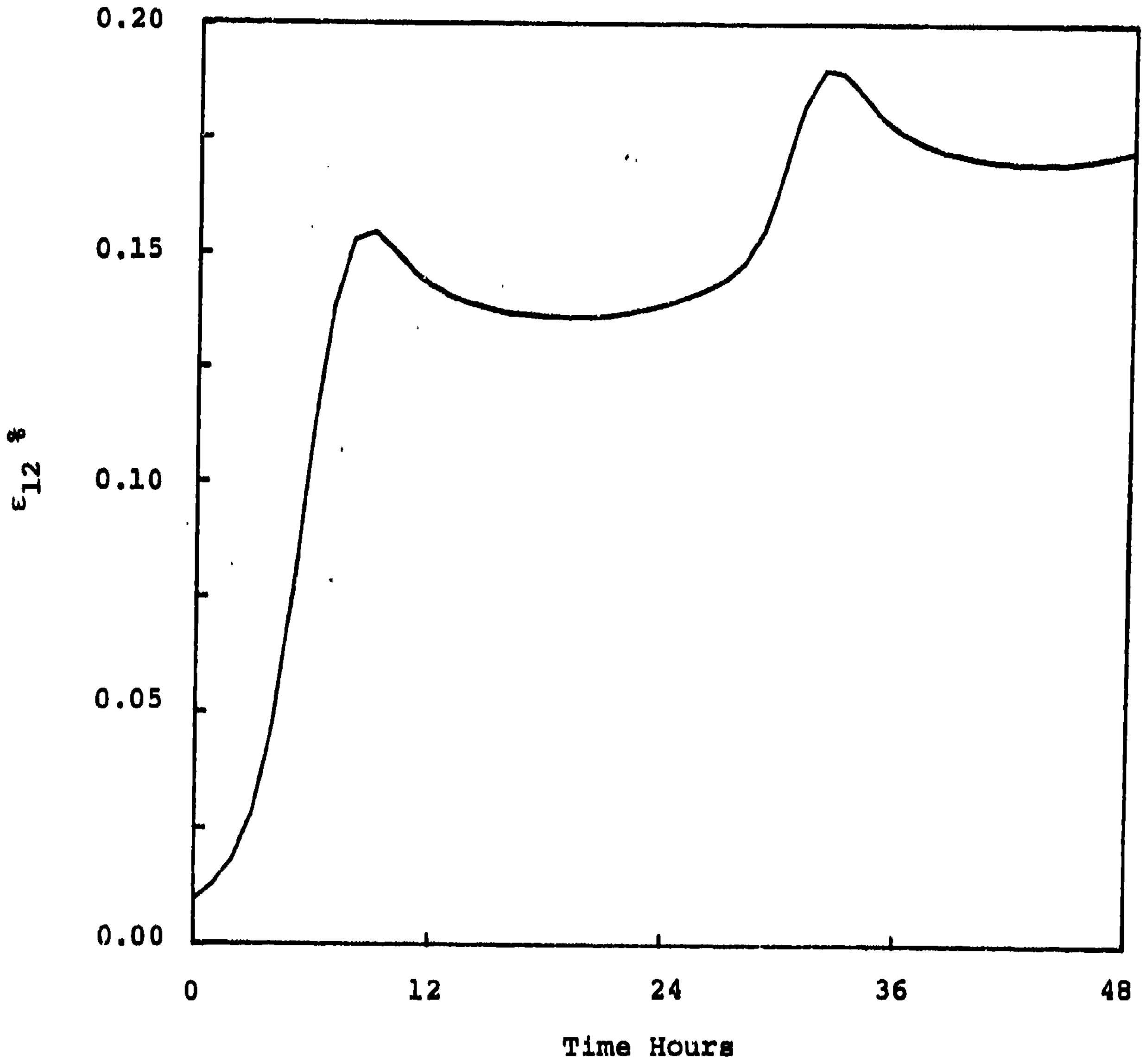

Figure 16. Axial Shear Strain Due to 1 MPa Stress and Cyclic Temperature

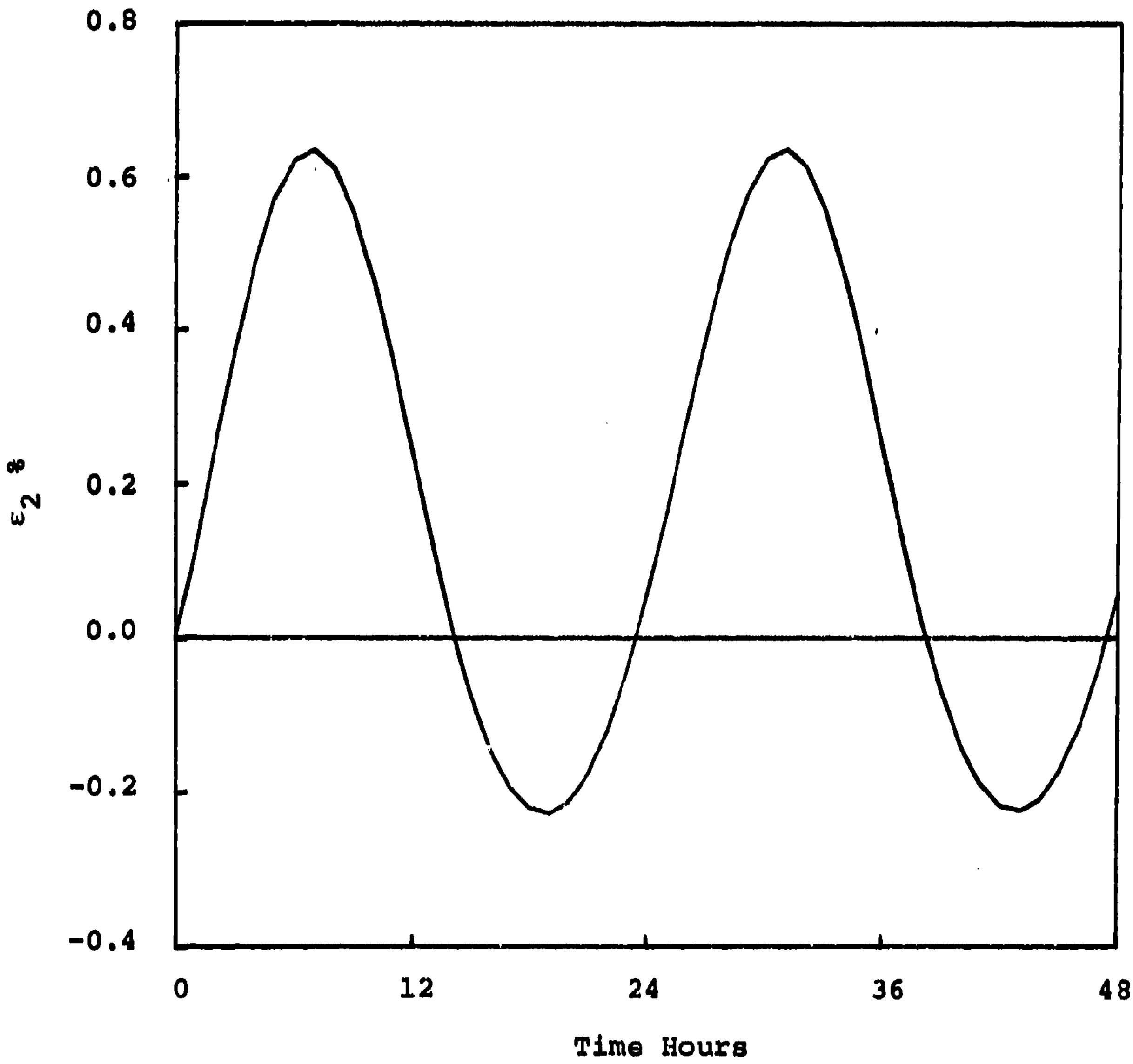

Figure 17. Free Thermal Expansion Strain for Cyclic Temperature

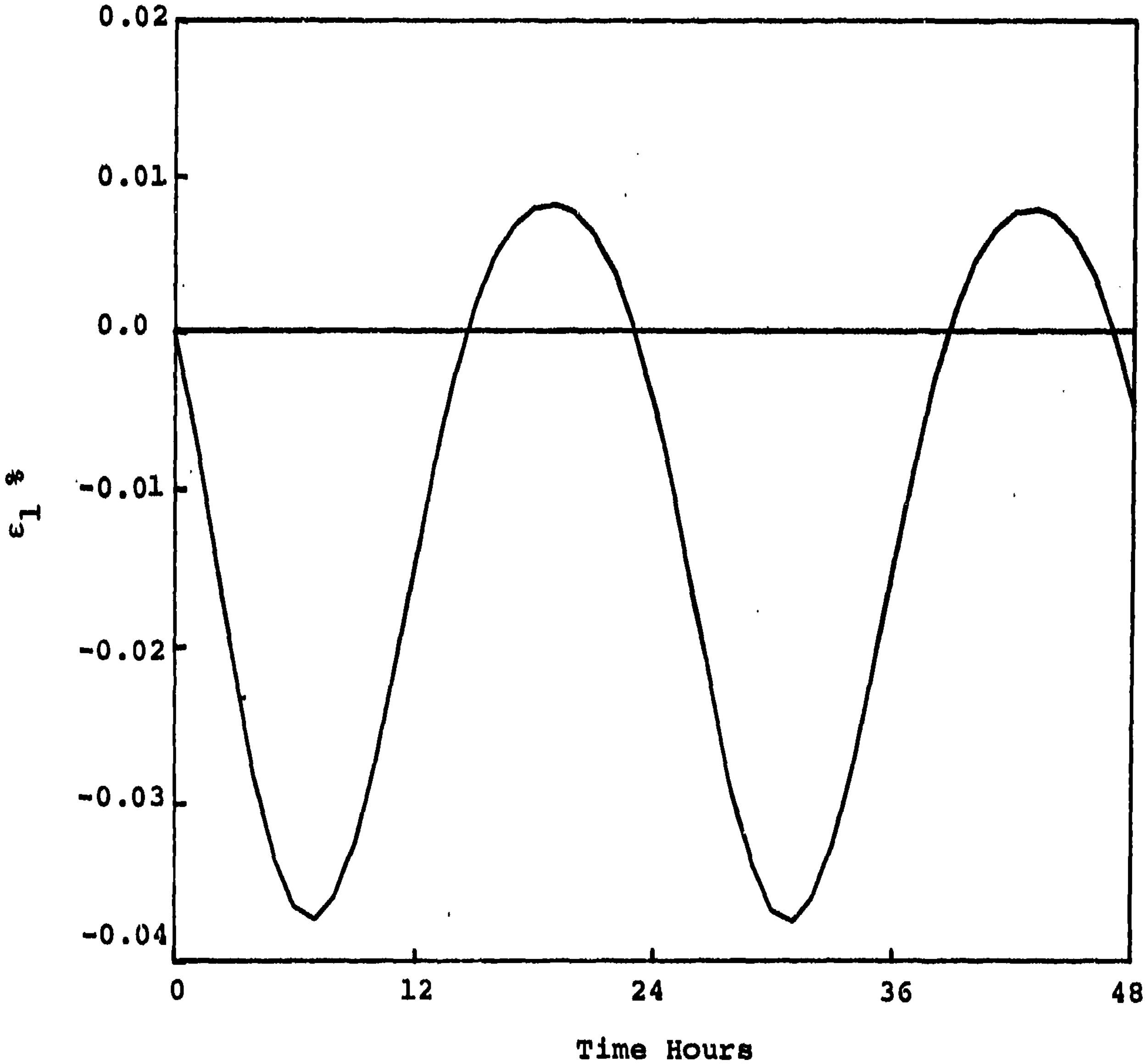

Figure 18. Free Thermal Expansion Strains Due to Cyclic Temperature

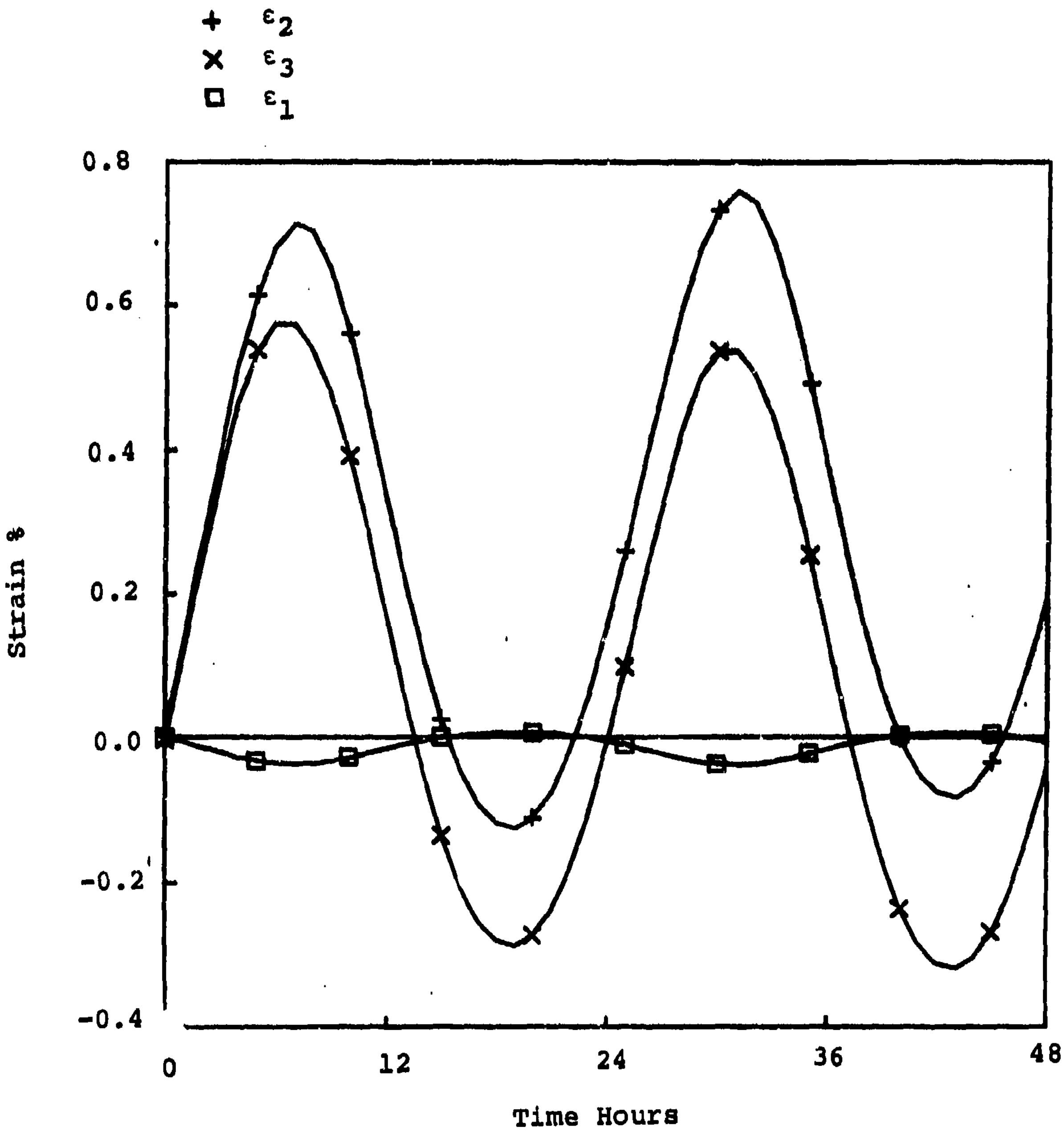

Figure 19. Strain History for 1 MPa σ_2 Stress and Cyclic Temperature

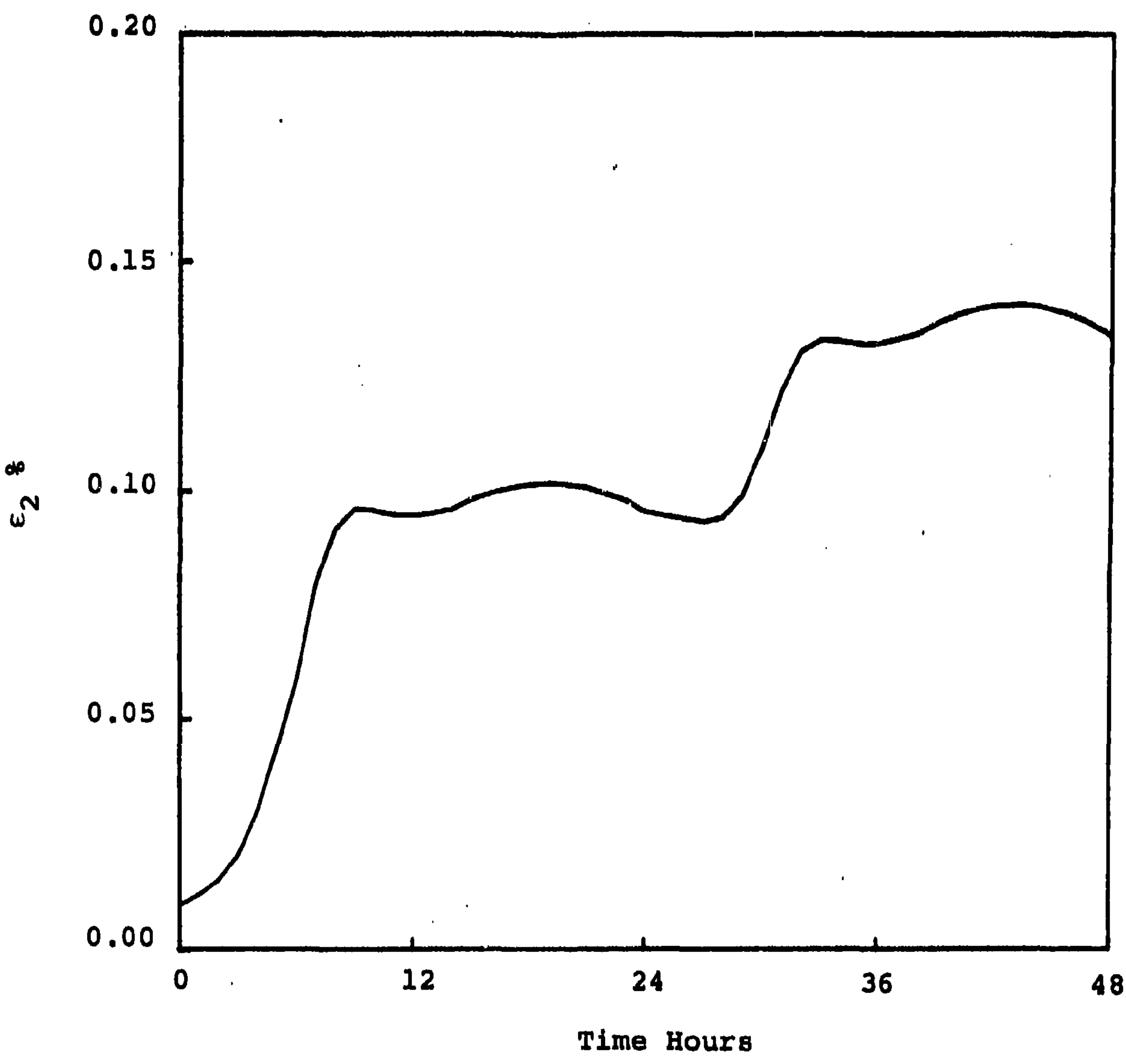

Figure 20. Mechanical Strain for 1 MPa σ_2 Stress and Cyclic Temperature

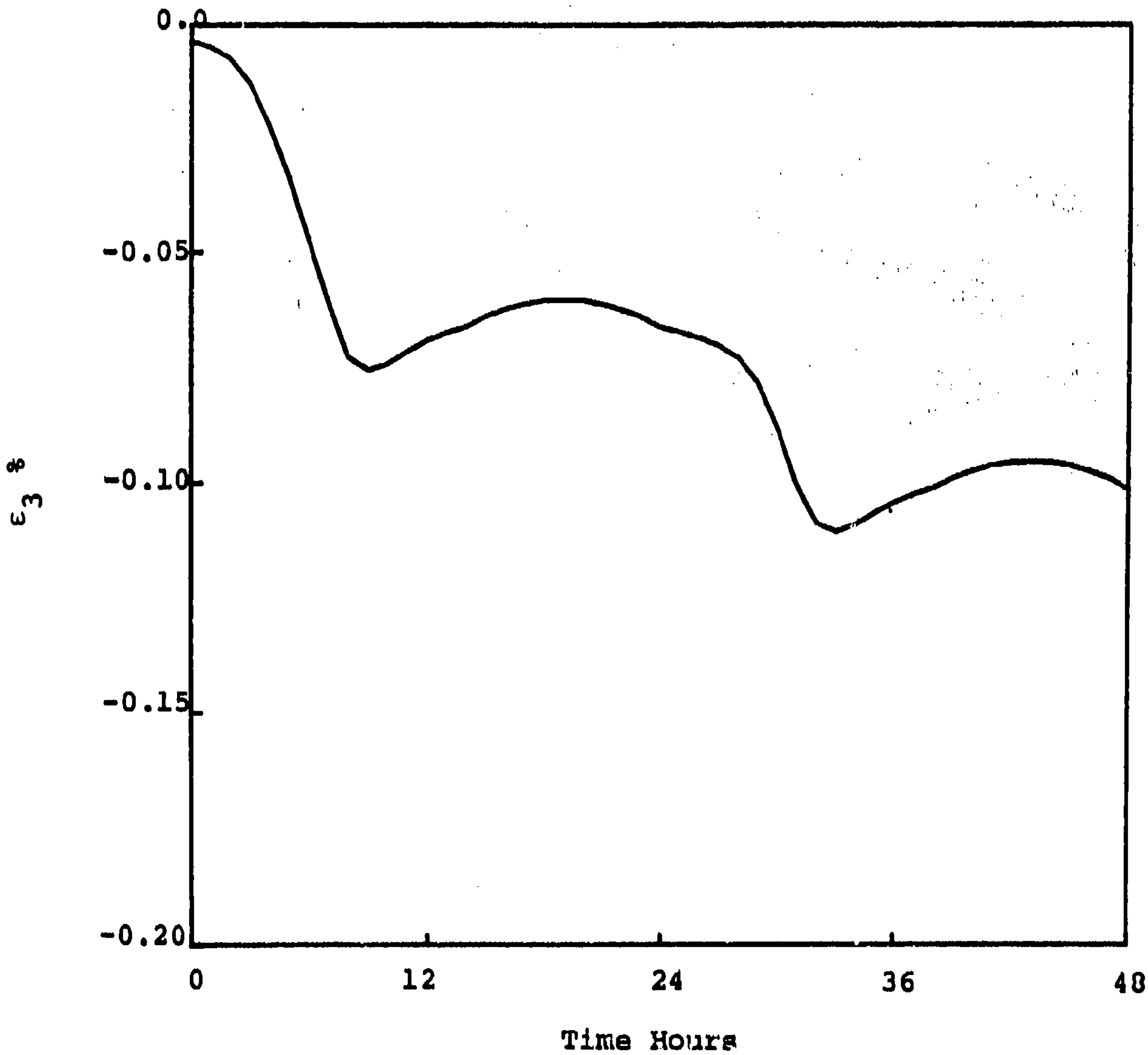

Figure 21. Mechanical Strain for 1 MPa σ_2 and Cyclic Temperature

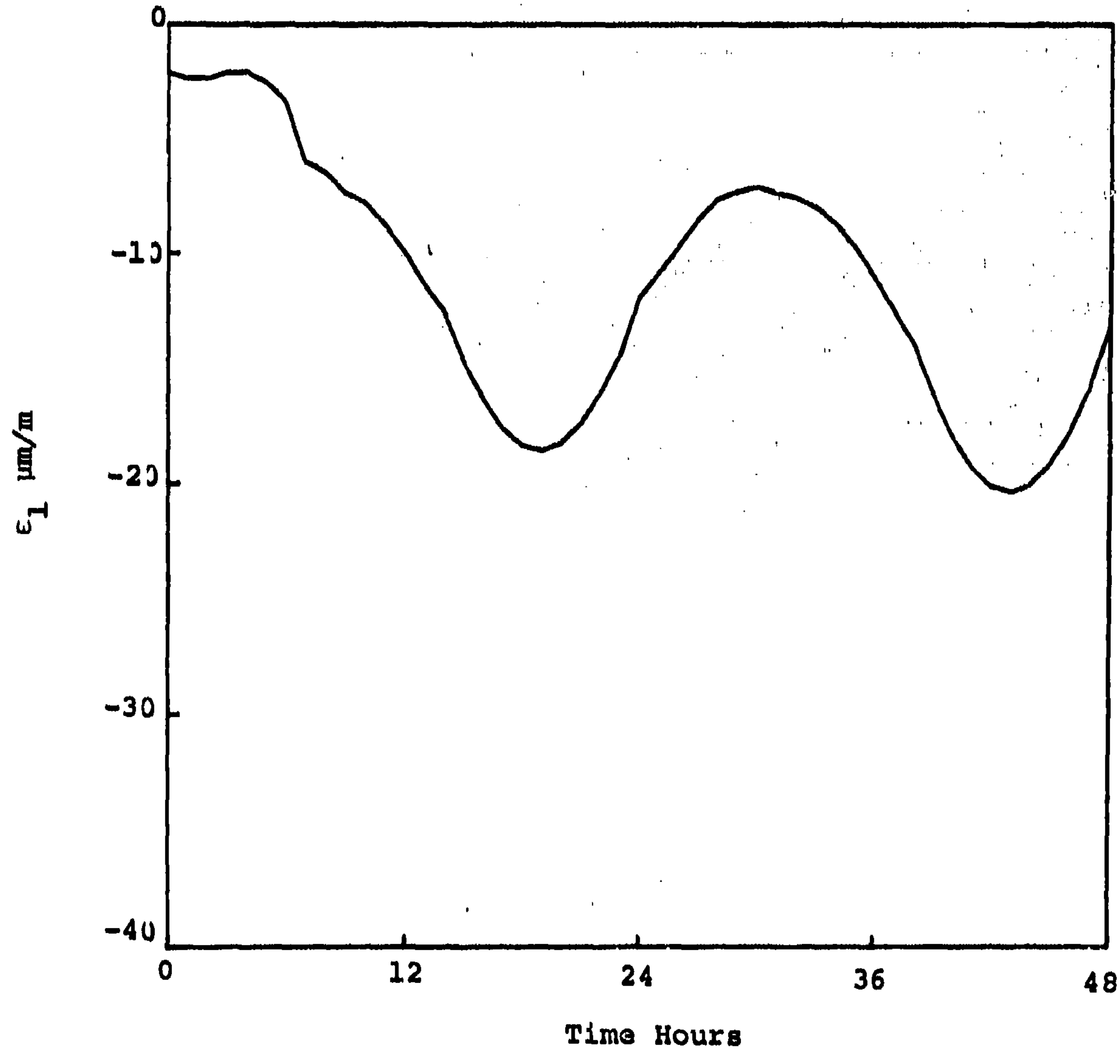

Figure 22. Mechanical Strain for 1 MPa σ_2 Stress and Cyclic Temperature