



ł

MICROCOPY RESOLUTION TEST CHART NATIONAL BUREAU OF STANDARDS-1963-A Ĺ

MRC modifical Summary Aspure #2774

PECENT RESEARCH IN REFERENCEMATE DESIGN FOR QUALITY INFORMATION WITH APPLICATIONS TO LOGISTICS

George E. S. Box



Mathematics Research Center University of Wisconsin—Madison 610 Walnut Street Madison, Wisconsin 53705

November 1984

(Received November 1984)

Approved for public release Distribution unlimited

Sponsored by

U. S. Army Research Office D. G. Sox 12211 Sesearch Tilangle Park Worth Carolina 27700



# 85 01 15 015

-REPRODUCED AT GOVERNMENT EXPENSE

# UNIVERSITY OF WISCONSIN-MADISON MATHEMATICS RESEARCH CENTER

### RECENT RESEARCH IN EXPERIMENTAL DESIGN FOR QUALITY IMPROVEMENT WITH APPLICATIONS TO LOGISTICS

George E. P. Box

Technical Summary Report #2774 November 1984

#### ABSTRACT

The success of Japanese industry in producing high quality products at low cost is cited. Consideration of certain aspects of scientific method leads to discussion of recent research on the role of screening designs in the improvement of quality. A projective rationale for the use of these designs in the circumstances of <u>factor sparsity</u> is advanced. In this circumstance the possibility of identification of sparse <u>dispersion</u> effects as well as sparse <u>location</u> effect is considered. A new method for the analysis of fractional <u>factorial designs is advanced</u>.

MTIS GRA&I MTIC TAP Merry upped Merry upped

Distribution/

Availability Codes

Sprcial

7910

0007 101070

BV....

Dist

AMS (MOS) Subject Classifications: 62K15

Key Words: Logistics, Quality Control, Scientific Method, Screening Designs, Factor Sparsity, Fractional Factorials, Dispersion Effects, Location Effects, Normal Plots, Bayesian Analysis

Work Unit Number 4 (Statistics and Probability)

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.



## RECENT RESEARCH IN EXPERIMENTAL DESIGN FOR QUALITY IMPROVEMENT WITH APPLICATIONS TO LOGISTICS

George E. P. Box

#### 1. LOGISTICS AND QUALITY CONTROL

A traditional philosophy of quality control has been to "inspect bad quality out" and indeed there are famous military standards that employ this philosophy. W. Edwards Deming (1982) has likened this to making toast according to the recipe "you burn it and I'll scrape it", and has urged the alternative philosophy of assuring that good quality has been <u>built in</u> to the product in the first place. In particular he attributes to the latter philosophy the success of Japanese industry in producing high quality products at low cost. A typical example of the dramatic consequences that have been attributed to these differences of approach are the air-conditioner defect rates shown in Table 1 and quoted by David Garvin (1983).

ſ

1

(In the	factory:	Assembly 3	line	defects per	100	units)	
				America	n	Jaj	panese
Total			••	63.5			0.95
Leaks			• •	3.1			0.12
Electrical			••	3.3		I	0.12

(In the field: Service call rate per 100 units under first year warranty coverage) American Japanese 10.5 0.6 Total . . . . . . . . . . . . . . . . . . 1.0 0.05 Compressors . . . . . . . . . . . . . . . . . . 1.4 0.002 Thermostats 0.5 0.028 Fan motors

TABLE 1. Defect rates in US and Japanese air conditioners

Similar comparisons have been made between defect rates in American and Japanese automobiles.

The same United States industry that makes air conditioners and motor vehicles also makes military hardware. It seems clear therefore that a major change in quality philosophy could produce a major improvement in the reliability of the Army's equipment. The philosophy of "building quality in" employs a policy of never ending quality improvement which may be typified in terms of the traditional statistical model

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.

where y is a quality characteristic believed to depend on a set of variables denoted by  $\chi_1$  whose identity is known, and e is the difference  $y - f(\chi_1)$ usually referred to as <u>error</u>. (Such "errors" are often somewhat arbitrarily imbued by the theoretician with properties of randomness, normality independence and homoscedasticity). In reality e is a function  $e(\chi_2)$  of a number of additional variables,  $\chi_2$  say, which affect the system but whose identity is usually unknown. In general, quality improvement is achieved by transferring elements of the unknown factor vector  $\chi_2$  into the known factor vector  $\chi_1$  as indicated below

 $y = f(x_1) + e(x_2)$ known unknown

The effect of such transfer is two-fold

- (i) to reveal effects of previously unknown factors which may then be adjusted to levels yielding higher quality and/or used to control the process.
- (ii) to remove variation previously caused by haphazard changes in these factors.

Some of the statistical techniques which contribute to this transfer are quality control charting (including Shewhart, Cusum, Pareto and Fishbone charts) and designed experimentation on line and off line (employing in different and appropriate contexts factorial, fractional factorial and orthogonal array designs, evolutionary operation and response surface methods).

#### 2. SCIENTIFIC METHOD AND QUALITY

Charting and experimentation are examples respectively of <u>passive</u> <u>surveillance</u> and <u>active intervention</u> both of which are important elements in scientific method which it is desirable to consider further.

Humans differ from other animals most remarkably in their ability to learn. It is clear that although throughout the history of mankind technological learning has taken place, until three or four hundred years ago change occurred very slowly. One reason for this was that in order to learn something - for example, how to make fire or champagne - two <u>rare events</u> needed to coincide: (a) an informative event had to <u>occur</u>, and (b) a person able to draw logical conclusions and to act on them had to be <u>aware</u> of that informative event.

Passive surveillance is a way of increasing the probability that the rare informative event will be constructively taken note of and is exemplified by quality charting methods. Thus a Shewhart chart is a means to ensure that

$$y = f(x_1) + e$$

-2-

possibly informative events are brought to the attention of those who may be able to discover in them an "assignable cause" (Shewhart 1931) and act appropriately.

Active intervention by experimentation aims, in addition, to increase the probability of an informative event <u>actually occurring</u>. A designed experiment conducted by a qualified experimenter can dramatically increase the probability of learning because it increases simultaneously the probability of an informative event occurring and also the probability of the event being constructively witnessed. Recently there has been much use of experimental design in Japanese industry particularly by Genichi Taguchi (Taguchi and Wu (1980)) and his followers. In off-line experimentation he has in particular emphasized the use of highly fractionated designs and orthogonal arrays and the minimization of variance.

.

In the remainder of this paper we briefly outline some recent research on the use of experimental design in the improvement of quality.

#### 3. USE OF SCREENING DESIGNS TO IMPROVE QUALITY

Table 2 shows in summary a highly fractionated two-level factorial design employed as a screening design in an off-line welding experiment performed by the National Railway Corporation of Japan (Taguchi and Wu, 1980). In the column to the right of the table is shown the observed tensile strength of the weld, one of several quality characteristics measured.

The design was chosen on the assumption that in addition to main effects only the two-factor interactions AC, AG, AH, and GH were expected to be present. On that supposition, all nine main effects and the four selected two-factor interactions can be separately estimated by appropriate orthogonal contrasts, the two remaining contrasts corresponding to the columns labelled  $e_1$  and  $e_2$  measure only experimental error. Below the table are shown the grand average, the fifteen effect contrasts, and the effects plotted on a dot diagram. When the effects are plotted on normal probability paper, thirteen of them plot roughly as a straight line but the remaining two, corresponding to the main effects for factors B and C, fall markedly off the line, suggesting that over the ranges studied, only factors B and C affect tensile location by amounts not readily attributed to noise.

If this conjecture is true, then, at least approximately, the sixteen runs could be regarded as four replications of a  $2^2$  factorial design in factors B and C only. However, when the results are plotted in Figure 1 so as to reflect this, inspection suggests the existence of a dramatic effect of a different kind - when factor C is at its plus level the spread of the

To facilitate later discussion we have set out the design and labelled the levels somewhat differently from Taguchi.

Kind of Welding Rods

Ē

ĩ

(

- Period of Drying
  - Welded Material
    - Thickness
      - Angle
- Opening
- Current
- Welding Method
- Preheating

Tensile

Factor

Factor			۵	Н	e1	ტ	ſĿ,	НЭ	AC	A	ല	АН	e1	AG	ט	ß	υ	strength
Column	Number	0	-	7	ัต	4	ъ	9	2	ω	6	10	=	12	13	14	15	kg/mm <sup>2</sup>
	F	+	1	1	+	1	÷	+		1	+	+	1	+	1		+	43.7
	7	+	+	ı	ł	t	I	+	+	1	1	+	+	+	+	I	1	40.2
	с	+	I	+	ı	I	+	ı	+	1	+	ı	+	+	1	+	ı	42.4
	4	+	+	+	+	1	ı	1	1	I	1	t	ł	+	+	+	+	44.7
	S	+	ł	I	+	+	1	ı	+	ı	+	+	I	ı	+	+	•	42.4
	9	+	+	ı	ı	+	+	ı	I	1	ı	+	÷	I	I	+	+	45.9
	7	+	ı	+	ı	+	I	+	I	1	+	I	+	ı	+	ı	+	42.2
Run	8	+	÷	+	+	+	+	+	+	I	ı	۱	1	1	1	I	ı	40.6
	6	+	ı	I	+	I	+	+	ı	+	ı	ı	+	ł	+	+	ł	42.4
	10	+	+	I	1	ı	1	+	+	+	+	ı	ł	I	I	+	+	45.5
	11	+	ł	+	I	ł	+	I	+	+	ı	+	ł	1	+	I	+	43.6
	12	+	+	+	+	1	I	ł	ł	+	+	+	+	١	I	I	ł	40.6
	13	+	I	ı	+	+	ı	1	+	+	ł	I	+	+	ı	1	+	44.0
	14	+	+	ı	1	+	+	ı	ı	+	+	ı	I	+	+	I	ł	40.2
	15	+	I	+	ı	+	I	+	1	+	I	+	I	+	1	+	I	42.5
	16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	46.5
Effect		43.0	.13	15	30	15	.40	03	.38	.40	05	.43	.13	.13	38	2.15	3.10	

-4-





.

þ

Ð

data appears much larger than when it is at its minus level. Thus, in addition to detecting shifts in location due to B and C, the experiment may also have detected what we will call a <u>dispersion effect</u> due to C. The example raises the general possibility of analyzing unreplicated designs for dispersion effects as well as for the more usual location effects.

(



Figure 1. Tensile data as four replicates of a 2<sup>2</sup> factorial design in factors B and C only.

<sup>&</sup>quot;Data of this kind might be accounted for by the effect of one or more variables other than B that affected tensile strength only at the "plus level" of C (only when the alternative material was used). Analysis of the eight runs made at the plus level of C does not support this possibility, however.

#### 4. RATIONALES FOR USING SCREENING DESIGNS

Before proceeding we need to consider the question, "In what situations are screening designs, such as highly fractionated factorials, useful?"

#### 4.1. Effect Sparsity

A common industrial problem is to find from a rather large number of factors those few that are responsible for <u>large effects</u>. The idea is comparable to that which motivates the use in quality control studies of the "Pareto diagram." (See, for example, Ishikawa (1976)). The situation is approximated by postulating that only a small proportion of effects will be "active" and the rest "inert". We call this the postulate of <u>effect</u> <u>sparsity</u>. For studying such situations, higly fractionated designs and other orthogonal arrays (Finney (1945), Plackett and Burman (1946), Rao (1947), Taguchi and Wu (1980)) which can screen moderately large numbers of variables in rather few runs are of great interest. Two main rationalizations have been suggested for the use of these designs; both ideas rely on the postulate of effect sparsity but in somewhat different ways.

#### 4.2. Rationale Based on Prior Selection of Important Interactions

It is argued (see for example Davies (1954)) that in some circumstances physical knowledge of the process will make only a few interactions likely and that the remainder may be assumed negligible. For example, in the welding experiment described above there were 36 possible two-factor interactions between the nine factors, but only four were regarded as likely, leaving 32 such interactions assumed negligible. The difficulty with this idea is that in many applications the picking out of a few "likely" interactions is difficult if not impossible. Indeed the investigator might justifiably protest that, in the circumstance where an experiment is needed to determine which <u>first order</u> (main) effects are important, it is illogical that he be expected to guess in advance which effects of <u>second order</u> (interactions) are important.

#### 4.3. Projective Rationale Factor Sparsity

A somewhat different notion is that of <u>factor</u> sparsity. Thus suppose that, of the k factors considered, only a small subset of unknown size d, <u>whose identity is however unknown</u>, will be active in providing main effects and interactions within that subset. Arguing as in Box and Hunter (1961) a two-level design enabling us to study such a system is a fraction of resolution R = d + 1 (or in the terminology of Rao (1947) an array of strength d) which produces complete factorials (possibly replicated) in every one of the  $\binom{k}{d}$  spaces of d = R - 1 dimensions. For example, we have seen that on the assumption that only factors B and C are important, the welding design could be regarded as four replicates of a  $2^2$  factorial in just those two factors. But because the design is of resolution R = 3 the same would have been true for any of the 36 choices of two out of the nine factors tested. Thus the design would be appropriate if it were believed that not more than two of the factors were likely to be "active".

	Columns	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	2 <sup>15-11</sup> 2 <sup>111</sup>	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
(b)	2 <sup>8-4</sup> IV	•	•		•			•	•			•		•	•	
(c)	2 <sup>5-1</sup>	•	•		•				•							•
(d)	24	•	•		•				•							

TABLE 3. Some alternative uses of the orthogonal array of Table 2.

For further illustration we consider again the sixteen-run orthogonal array of Table 2 and, adopting a roman subscript to denote the resolution R of the design, we indicate in Table 3 various ways in which that array might be used. It may be shown that

(a) If we associated the fifteen contrast columns of the design with fifteen factors, we would generate a  $2^{15-11}_{III}$  design providing four-fold replication of  $2^2$  factorials in every one of the 105 two-dimensional projections.

(b) If we associated only columns 1, 2, 4, 7, 8, 11, 13, and 14 with eight factors we would agenerate a  $2_{\rm IV}^{8-4}$  design providing two-fold replication of  $2^3$  factorials in every one of the 56 three-dimensional projections.

(c) If we associated only columns 1, 2, 4, 8, and 15 with five factors we would generate a  $2_V^{5-1}$  design providing a  $2^4$  factorial in every one of the four-dimensional projections.

(d) If we associated only columns 1, 2, 4, and 8 with four factors we would obtain the complete  $2^4$  design from which this orthogonal array was in fact generated.

Designs (a), (b) and (c) would thus be appropriate for situations where we believed respectively that not more than 2, 3, or 4 factors would be active. Notice that intermediate values of k could be accommodated by suitably omitting certain columns. Thus the welding design is a  $2_{III}^{9-5}$  arrangement which can be obtained by omitting 6 columns from the complete  $2_{III}^{15-11}$ . Notice finally that for intermediate designs we can take advantage of both rationales by arranging, as was done for the welding design, that particular interactions are isolated.

The designs give partial coverage for a larger number of factors, for example (Box and Hunter (1961)) 56 of the 70 four-dimensional projections of the  $2\frac{8-4}{1V}$  yield a full factorial in four variables.

A discussion of the iterative model building process by Box and Jenkins (1970) characterized three steps in the iterative data analysis cycle indicated below

identification — fitting — diagnostic checking

Most of the present paper is concerned with model identification - the search for a model worthy to be formally entertained and fitted by an efficient procedure such as maximum likelihood. The situation we now address concerns the analysis of fractional designs such as the welding design in the above context when only a few of the factors are likely to have effects but these may include dispersion effects as well as location effects.

#### 5. DISPERSION EFFECTS

We again use the design of Table 2 for illustration. There are 16 runs from which 16 quantities -- the average and 15 effect contrastr been calculated. Now if we were also interested in possible dispe ... effects we could also calculate 15 variance ratios. For example, in column 1 we can compute the sample variance  $s_{1-}^2$  for those observations associa er ith a minus sign and compare it with the sample variance  $s_{1+}^2$  for observations associated with a plus sign to provide the ratio  $F_1 = s_{1-}^2/s_{1+}^2$ . If this is done for the welding data we obtain values for  $lnF_i^*$  given in Figure 2(a). It will be recalled that in the earlier analysis a large dispersion effect associated with factor C (column 15) was found, but in Figure 2(a) the effect for factor C is not especially extreme, instead the dispersion effect for factor D (column 1) stands out from all the rest. This misleading indication occurs because we have not so far taken account of the aliasing of location and dispersion effects. Since sixteen linearly independent location effects have already been calculated for the original data, calculated dispersion effects must be functions of these. Recently (Box and Meyer 1984a) a general theory of location-dispersion aliasing has been obtained for factorials and fractional factorials at two levels. For illustration, in this particular example it turns out that the following identity exists for the dispersion effect F1, that is the F ratio associated with factor D and hence for column 1 of the design.

$$F_{1} = \frac{(2-3)^{2} + (4-5)^{2} + (6-7)^{2} + (8-9)^{2} + (10-11)^{2} + (12-13)^{2} + (14-15)^{2}}{(2+3)^{2} + (4+5)^{2} + (6+7)^{2} + (8+9)^{2} + (10+11)^{2} + (12+13)^{2} + (14+15)^{2}}$$
(1)

Now (see Table 2) 14 = B = 2.15 and 15 = C = 3.10 are the two largest location effects, standing out from all the others. The extreme value of  $F_1$  associated with an apparent dispersion effect of factor D(1) is largely

In this figure familiar normal theory significance levels are also shown. Obviously the necessary assumptions are not satisfied in this case, but these percentages provide a rough indication of magnitude.



B and (b) after elimination of location effects for Đ

-9-

accounted for by the squared sum and squared difference of the location effects  $\hat{B}$  and  $\hat{C}$  which appear respectively as the last terms in the denominator and numerator of equation (1). A natural way to proceed is to compute variances from the residuals obtained after eliminating large location effects. After such elimination the alias relations of equation (1) remain the same except that location effects from eliminated variables drop out. That is zeros are substituted for eliminated variables. Variance analysis for the residuals after eliminating effects of B and C are shown in Figure 2(b). The dispersion effect associated with C (factor 15) is now correctly indicated as extreme. It is shown in the paper referenced above how, more generally, under circumstances of effect sparsity a location-dispersion model may be correctly identified when a few effects of both kinds are present.

#### 6. ANALYSIS OF UNREPLICATED FRACTIONAL DESIGNS

Another important problem in the analysis of unreplicated fractional designs and other orthogonal arrays concerns the picking out of "active" factors. A serious difficulty is that with unreplicated fractional designs no simple estimate of the experimental error variance against which to judge the effects is available.

In one valuable procedure due to Cuthbert Daniel (1959, 1976) effects are plotted on Normal probability paper. For illustration Table 4 shows the calculated effects from a  $2_{IV}^{8-4}$  design used in an experiment on injection molding (Box, Hunter and Hunter, 1978, p. 399). These effects are plotted on normal probability paper in Figure 3.

=	-0.7	*	1	mold temp.
=	-0.1	+	2	moisture content
=	5.5	+	3	holding pressure
=	-0.3	+	4	cavity thickness
=	-3.8	+	5	booster pressure
=	-0.1	+	6	cycle time
=	0.6	+	7	gate size
=	1.2	+	8	screw speed
		= -0.7 = -0.1 = 5.5 = -0.3 = -3.8 = -0.1 = 0.6 = 1.2	= -0.7 + $= -0.1 +$ $= 5.5 +$ $= -0.3 +$ $= -3.8 +$ $= -0.1 +$ $= 0.6 +$ $= 1.2 +$	$\begin{array}{rcrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$

 $T_{9} = T_{1.2} = -0.6 + 1.2 + 3.7 + 4.8 + 5.6$   $T_{10} = T_{1.3} = 0.9 + 1.3 + 2.7 + 4.6 + 5.8$   $T_{11} = T_{1.4} = -0.4 + 1.4 + 2.8 + 3.6 + 5.7$   $T_{12} = T_{1.5} = 4.6 + 1.5 + 2.6 + 3.8 + 4.7$   $T_{13} = T_{1.6} = -0.3 + 1.6 + 2.5 + 3.4 + 7.8$   $T_{14} = T_{1.7} = -0.2 + 1.7 + 2.3 + 6.8 + 4.5$  $T_{15} = T_{1.8} = -0.6 + 1.8 + 2.4 + 3.5 + 6.7$ 

TABLE 4. Calculated effects from a  $2^{8-4}_{IV}$  design showing alias structure assuming three factor and higher order interactions negligible. Injection molding experiment.



1

..........

and a series

Figure 3. Normal plot of effects. Injection molding experiment.

An alternative Bayesian approach (Box and Meyer, 1984b) is as follows: Let  $T_1, T_2, \dots, T_n$  be standardized effects with

# $T_i = e_i$ if effect inert

$$T_i = e_i + \tau_i$$
 if effect active

$$e_{i} \neq N(0,\sigma^{2}), \tau_{i} \neq N(0,\sigma_{\tau}^{2}) k^{2} = \frac{\sigma^{2} + \sigma_{\tau}^{2}}{\sigma^{2}}.$$

Suppose the probability that an effect is active is  $\alpha$ .

Let  $a_{(r)}$  be the event that a particular set of r of the  $\nu$  factors are active, and let  $\mathbb{T}_{(r)}$  be the vector of estimated effects corresponding to active factors of  $a_{(r)}$ . Then, (Box and Tiao, 1968) with  $p(\sigma) \propto \frac{1}{\sigma}$  the posterior probability that  $\mathbb{T}_{(r)}$  are the only active effects is:

$$\mathbb{P}[\mathbf{a}_{(\mathbf{r})} | \mathbf{T}, \alpha, k] \propto \left[ \frac{\alpha k^{-1}}{1 - \alpha} \right]^{\mathbf{r}} \left\{ 1 - \left(1 - \frac{1}{k^2}\right) \frac{\mathbf{S}_{(\mathbf{r})}}{\mathbf{S}} \right\}$$

where  $S_{(r)} = \mathcal{I}'_{(r)}\mathcal{I}_{(r)}$  and  $S = \mathcal{I}'\mathcal{I}$ . In particular the marginal probability that an effect i is active give T,  $\alpha$  and k is proportional to

$$\sum_{\substack{a \ (r) \\ i \text{ active}}} \left[ \frac{\alpha k^{-1}}{1-\alpha} \right]^{r} \left\{ 1 - (1 - \frac{1}{k^{2}}) \frac{s(r)}{s} \right\}^{-\frac{v}{2}}.$$

A study of the fractional factorials appearing in Davies (1954), Daniel (1976) and Box, Hunter and Hunter (1978) suggested that  $\alpha$  might range from 0.15-0.45 while k might range from 5 to 15. The posterior probabilities computed with the (roughly average) values.  $\alpha = 0.30$  and k = 10 are shown in Figure 4(a) in which N denotes the probability (negligible for this example) that there are no active effects. The results from a sensitivity analysis in which  $\alpha$  and k were altered to vary over the ranges mentioned above is shown in Figure 4(b).

It will be seen that Figure 4(a) points to the conclusion that active effects are associated with columns 3, 5 and 12 of the design and that column 8 might possibly also be associated with an active factor. Figure 4(b) suggests that this conclusion is very little affected by widely different choices for  $\alpha$  and k. Further research with different choices of prior, with marginization with respect to k, and with different choices of the distribution assumptions is being conducted.

For three-level and mixed two and three level designs for example, this analysis is carried out after the effects are scaled so that they all have equal variances.



Figure 4(a) Welding experiment. Posterior probability that factor i is active  $(\alpha = 0.30, k = 10)$ .



Figure 4(b) Sensitivity analysis for posterior probability a = .15 - .45, k = 5 - 15.

#### 7. OTHER RESEARCH

i i i i

STREET SCAPES

Topics which are emphasized in Taguchi's approach to "off line quality control" are (a) reduction of variation by error transmission studies and (b) the choosing of a product design so that it is robust with respect to environmental variables.

These topics are also receiving attention in further research.

#### REFERENCES

Deming, W. Edwards (1982). Quality, Productivity and Competitive Position. M.I.T., Center for Advanced Engineering Study, Cambridge, MA 02139.

Garvin, David A. (1983). Quality on the Line. Harvard Business Review, Vol. 61, No. 5, p. 64-75.

Shewhart, W. A. (1931). THE Economic Control of Quality of Manufactured Product. Van Nostrand (Reprinted in 1981 by American Society for Quality Control.)

Taguchi, G. and Wu, Y. (1980). Introduction to Off-Line Quality Control. Central Japan Quality Control Association, Nagoya, Japan.

Ishikawa, K. (1976). Guide to Quality Control. Asian Productivity Organization, Tokyo.

Finney, D. J. (1945). The Fractional Replication of Factorial Arrangements. Annals of Eugenics, 12, 4, 291-301.

Plackett, R. L. and Burman, J. P. (1946). Design of Optimal Multifactorial Experiments. Biometrika, 23, 305-325.

Rao, C. R. (1947). Factorial Experiments Derivable from Combinatorial Arrangements of Arrays. J. Roy. Statist. Soc., B9, 128-140.

Davies, O. L. Editor (1954). The Design and Analysis of Industrial Experiments. London: Oliver and Boyd.

Box, G. E. P. and Hunter, J. S. (1961). The  $2^{k-p}$  Fractional Factorial Designs. Technometrics, 3, 311, 449.

Box, G. E. P. and Jenkins, G. M. (1970). Time Series Analysis, Forecasting and Control. San Francisco: Holden-Day.

Box, G. E. P. and Meyer, R. D. (1984a). Analyzing Two-Level Fractional Factorial Designs for Possible Dispersion Effects. Mathematics Research Center Technical Summary Report #2746, Madison, WI 53705.

Daniel, C. (1959). Use of Half-Normal Plots in Interpreting Factorial Two-Level Experiments. Technometrics, <u>1</u>, 4, 149. Daniel, C. (1976). Applications of Statistics to Industrial Experimentation. New York: Wiley. Estimate Later

. .

Box, G. E. P., Hunter, W. G. and Hunter, J. S. (1978). Statistics for Experimenters. New York: Wiley.

( )

ι.

Ĩ.

۰.

1

Ģ

Box, G. E. P. and Meyer, R. D. (1984b). An Alternative Approach for the Analysis of Fractional Designs. Mathematics Research Center Technical Summary Report, Madison, WI 53705.

Box, G. E. P. and Tiao, G. C. (1968). A Bayesian Approach to Some Outlier Problems. Biometrika, <u>55</u>, 119.



REFURI DUCUMENTATION FAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
. REPORT NUMBER 2. GOVT ACCESSION NO	. 3. RECIPIENT'S CATALOG NUMBER
2774	
TITIE (and Substitia)	5. TYPE OF REPORT & PERIOD COVERED
	Summary Report - no specific
RECENT RESEARCH IN EXPERIMENTAL DESIGN	reporting period
FOR QUALITY IMPROVEMENT WITH APPLICATIONS	6. PERFORMING ORG. REPORT NUMBER
TO LOGISTICS	
AUTHOR()	8. CONTRACT OR GRANT NUMBER(*)
George E. P. Box	
	DAAG29-80-C-0041
	10 DECEMAN EL ENENT DECLECT TASK
» PERFORMING ORGANIZATION NAME AND ADDREss Mathematics Research Center IIniversity of	AREA & WORK UNIT NUMBERS
All Walnut Street Wisconsin	Work Unit Number 4 -
Madisan Wissonsin 52706	Statistics and Probability
I. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
U. S. Army Research Office	November 1984
P.O. Box 12211	13. NUMBER OF PAGES
Research Triangle Park, North Carolina 27709	15
4. MONITORING AGENCY NAME & ADDRESS(II different from Controlling Office)	15. SECURITY CLASS. (of this report)
	UNCLASSIFIED
	UNCLASSIFILD
	154. DECLASSIFICATION/DOWNGRADING SCHEDULE
2 DISTRIBUTION STATEMENT (of the channel entered in Black 20. If different for	m Rened)
7. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different fro	m Report)
7. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different fro 8. SUPPLEMENTARY HOTES	m Report)
7. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different fro 8. SUPPLEMENTARY HOTES 9. KEY WORDS (Continue on reverse elde if necessary and identify by block number)	m Report)
<ol> <li>DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different fro</li> <li>SUPPLEMENTARY HOTES</li> <li>KEY WORDS (Continue on reverse elde if necessary and identify by block number)</li> <li>Logistics, Quality Control, Scientific Method, Sci Factor Sparsity, Fractional Factorials, Dispersion Effects, Normal Plots, Bayesian Analysis</li> </ol>	reening Designs, n Effects, Location
<ol> <li>DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different in Supplementary hotes</li> <li>SUPPLEMENTARY HOTES</li> <li>KEY WORDS (Continue on reverse elde if necessary and identify by block number) Logistics, Quality Control, Scientific Method, Sci Factor Sparsity, Fractional Factorials, Dispersion Effects, Normal Plots, Bayesian Analysis</li> <li>ABSTRACT (Continue on reverse elde if necessary and identify by block number) The success of Japanese industry in producing low cost is cited. Consideration of certain aspec leads to discussion of recent research on the role improvement of quality. A projective rationale for in the circumstances of <u>factor sparsity</u> is advance possibility of identification of sparse <u>dispersion</u> <u>location</u> effect is considered. A new method for to factorial designs is advanced.</li> </ol>	g high quality products at cts of scientific method e of screening designs in the or the use of these designs ed. In this circumstance the n effects as well as sparse the analysis of fractional



# FILMED

2-85

DTIC