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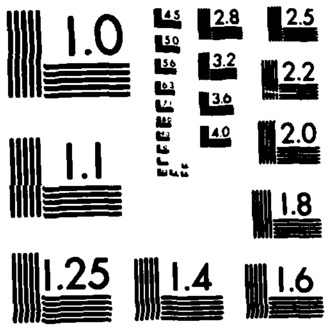

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A NOTE ON THE DISTURBANCE DECOUPLING PROBLEM FOR RETARDED SYSTEMS

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ABSTRACT

The disturbance decoupling problem for linear control systems is to design a feedback control law in such a way that the disturbances do not influence these outputs which are to be regulated. In this note we present a very simple solution to this problem for a rather general class of retarded functional differential equations with delays in the state variables.

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Work Unit Number 5 - Optimization and Large Scale Systems

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A

A NOTE ON THE DISTURBANCE DECOUPLING PROBLEM FOR RETARDED SYSTEMS

Ruth F. Curtain* and Dietmar Salamon**

1. INTRODUCTION

The disturbance decoupling problem (DDP) for finite dimensional systems is readily solved by using concepts such as (A,B) - invariant subspaces (Wonham [5]). In [1] Curtain has shown that a similar approach is also successful for certain classes of infinite-dimensional systems, namely those governed by partial differential equations. For retarded functional differential equations (RFDE) this approach is fraught with problems as discussed by Curtain in [2] and in [4] by Pandolfi who analyses the situation in some detail. He concludes that for retarded systems one needs an unbounded feedback control law. Even allowing for unbounded feedback there is no guarantee that the required maximal (A,B) - invariant subspace contained in ker D will exist. In view of these negative results concerning the DDP for retarded systems we feel that a positive result, no matter how simple, might help to shed some light on this important problem. Using a simple straightforward approach we give sufficient conditions for the solution of the DDP for a general class of linear RFDE's. This condition is generically satisfied if only those systems are taken into consideration which satisfy a certain necessary condition for the solvability of the DDP and for which the number of inputs is larger than the number of to be regulated outputs. The required feedback is indeed unbounded but easy to write down. Finally, we solve the DDP using output injection.

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2. DDP FOR RETARDED SYSTEMS

Consider the following retarded system

$$\dot{x}(t) = Lx_t + B_0u(t) + E_0d(t) \tag{2.1}$$

$$z(t) = D_0x(t) \tag{2.2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $x_t : [-h,0] \to \mathbb{R}^n$ is defined by $x_t(\tau) = x(t+\tau)$ for $-h \le \tau \le 0$, $u(t) \in \mathbb{R}^m$ is the control input, $d(t) \in \mathbb{R}^q$ is some disturbance, and $z(t) \in \mathbb{R}^k$ is the output to be regularted. We assume that L is a bounded linear operator from $H^1 = H^1[-h,0;\mathbb{R}^n]$ into $\mathbb{R}^n$ which can be represented in the form

$$L\phi = A_0\phi(0) + \int_{-h}^{0} A_1(\tau)\phi(\tau)d\tau \tag{2.3}$$

for $\phi \in H^1$. Of course, $E_0 \in \mathbb{R}^{n\times q}$, $B_0 \in \mathbb{R}^{n\times m}$, $D_0 \in \mathbb{R}^{k\times n}$. For the state space we choose $M^2 = \mathbb{R}^n \times L^2[-h,0;\mathbb{R}^n]$ so that the initial condition for (2.1) is

$$x(0) = \phi^0,\ x(\tau) = \phi^1(\tau),\ -h \le \tau \le 0 \quad , \tag{2.4}$$

with $\phi = (\phi^0,\phi^1) \in M^2$. Then the integrated version

$$\begin{aligned} x(t) = \phi^0 &+ \int_{-h}^{0} [A_1(\tau-t) - A_1(\tau)]\phi^1(\tau)d\tau \\ &+ \int_0^t [B_0u(s) + E_0d(s)]ds \\ &+ \int_0^t [A_0 + A_1(s-t)]x(s)ds \end{aligned} \tag{2.5}$$

of (2.1), (2.4) admits a unique solution $x(\cdot) \in C[0,T;\mathbb{R}^n]$ for every initial state $\phi \in M^2$, every input $u(\cdot) \in L^2[0,T;\mathbb{R}^m]$ and every disturbance $d(\cdot) \in L^2[0,T;\mathbb{R}^q]$. Here we have defined $A_1(\tau) = 0$ for $\tau \notin [-h,0]$. If $\phi^1 \in H^1$ and $\phi^0 = \phi^1(0)$, then the solution $x(\cdot)$ of (2.5) is in fact in $H^1[0,T;\mathbb{R}^n]$ and satisfies (2.4) and (2.1) for almost every $t \in [0,T]$.

The free motions of (2.5) are described by the solution semigroup $S(t) \in \mathcal{L}(M^2)$ which maps the initial state $\phi \in M^2$ into the corresponding state $(x(t),x_t) \in M^2$ of the free system $(u(\cdot) \equiv 0,\ q(\cdot) \equiv 0)$ at time $t \geq 0$ and is generated by the operator $A : \mathcal{D}(A) \to M^2$ defined by

$$A\phi = (L\phi^1, \dot{\phi}^1),\quad \mathcal{D}(A) = \{\phi \in M^2 \,|\, \phi^1 \in H^1,\ \phi^0 = \phi^1(0)\} \quad .$$

(Delfour [3]). In general the state $\hat{x}(t) = (x(t),x_t) \in M^2$ of (2.5) is described by the variation-of-constants formula

$$\hat{x}(t) = S(t)\phi + \int_0^t S(t-s)[Bu(s) + Ed(s)]ds \tag{2.6}$$

(Delfour [3]) where the operators $B : \mathbb{R}^m \to M^2$, $E : \mathbb{R}^q \to M^2$ are defined by $Bu = (B_0u,0)$, $Ed = (E_0d,0)$ for $u \in \mathbb{R}^m$ and $d \in \mathbb{R}^q$. This means that $\hat{x}(t)$ is a mild solution of the evolution equation

$$\begin{aligned} d/dt\,\hat{x}(t) &= A\hat{x}(t) + Bu(t) + Ed(t) \quad , \\ z(t) &= D\hat{x}(t) \quad , \quad \hat{x}(0) = \phi \quad . \end{aligned} \tag{2.7}$$

Of course the output operator $D : M^2 \to \mathbb{R}^k$ is given by $D\phi = D_0\phi^0$ for $\phi \in M^2$.

The disturbance decoupling problem is to design a feedback control of the form

$$u(t) = Fx_t = F_0x(t) + \int_{-h}^{0} F_1(\tau)\dot{x}(t+\tau)d\tau \tag{2.8}$$

with $F_0 \in \mathbb{R}^{m\times n}$, $F_1(\cdot) \in L^2[-h,0;\mathbb{R}^{m\times n}]$ such that the output $z(t)$ of the closed loop system (2.1), (2.2), (2.8) is independent of the disturbance $d(t)$.

We now prove our main result.

THEOREM 1

Suppose that

$$D_0E_0 = 0 \quad , \quad D_0B_0 \text{ is onto} \tag{2.9}$$

and choose $G_0 \in \mathbb{R}^{m\times k}$ *such that*

(2.10) $$D_0B_0G_0 = I \in \mathbb{R}^{k\times k} \quad .$$

Then the DDP for system (2.1), (2.2) is solved by the feedback control law

(2.11) $$u(t) = -G_0D_0Lx_t \quad .$$

In fact, the output of the closed loop system (2.1), (2.2), (2.11), (2.4) is given by

(2.12) $$z(t) \equiv D_0\phi^0 \quad .$$

Proof. First note that the closed loop system (2.1), (2.11) is of the same type as (2.1) and therefore gives rise to unique solutions $x(\cdot)$ in $H^1[-h,T;\mathbb{R}^n]$ corresponding to the initial condition (2.4) with $\phi \in \mathcal{D}(A)$. This solution satisfies $\dot{x}(t) = Lx_t - B_0G_0D_0Lx_t + E_0d(t)$ for almost every $t > 0$. This implies that $z(\cdot) \in H^1[0,T;\mathbb{R}^k]$ and

$$\dot{z}(t) = (I-D_0B_0G_0)D_0Lx_t + D_0E_0d(t) = 0$$

for almost every $t > 0$. Hence $z(t) \equiv z(0) = D_0\phi^0$ is independent of the disturbance $d(t)$ if $\phi \in D(A)$. In general (2.12) follows from the fact that $z(\cdot) \in C[0,T;\mathbb{R}^k]$ depends continuously on the initial state $\phi \in M^2$. □

REMARK

The condition $D_0E_0 = 0$ is necessary for the solvability of the DDP and the condition D_0B_0 being onto requires

(2.13) $$\operatorname{rank} B_0 > \operatorname{rank} D_0 = k$$

which means that the number of to be regulated outputs is less than or equal to the number of inputs. Furthermore, D_0B_0 is onto if and only if D_0 is onto and

(2.14) $$\ker D_0 + \operatorname{range} B_0 = \mathbb{R}^n \quad .$$

This condition is generically satisfied if (2.13) holds.

In [4] it has been shown that the DDP for (2.1), (2.2) is solvable if and only if there exists a subspace $V \subset M^2$ with the properties

(2.15) $$\text{range } E \subset V \subset \ker D \quad ,$$

(2.16) there exists a feedback law of the form (2.8) with $F \in L(H^1, \mathbb{R}^m)$ such that whenever $\phi \in V$ then the corresponding state $\hat{x}(t) = (x(t), x_t) \in M^2$ of the closed loop system (2.1), (2.8), (2.4) remains in V for all $t > 0$.

The second property may be referred to as semigroup feedback invariance and is equivalent to saying that V is invariant under the feedback semigroup $S_F(t) \in L(M^2)$ which is generated by the operator $A_F : \mathcal{D}(A_F) \to M^2$ given by

(2.17) $$A_F\phi = (L\phi^1 + B_0F\phi^1, \dot{\phi}^1), \ \mathcal{D}(A_F) = \{\phi \in M^2 | \phi^1 \in H^1, \ \phi^0 = \phi^1(0)\} \quad .$$

Theorem 1 shows that in our case the subspace V is given by

(2.18) $$V = \{\phi \in M^2 | D_0\phi^0 = 0\} = \ker D \quad .$$

In view of the nice result for the infinite dimensional DDP in terms of a maximal (A,B) - invariant subspace obtained in [1] it is interesting to reformulate our results in terms of the abstract Cauchy problem (2.7) associated with (2.1), (2.2). In [1] a subspace $V \subset M^2$ is called (A,B) - invariant if

(2.19) $$A(V \cap D(A)) \subset V + \text{range } B \quad .$$

In general, this concept is weaker than semigroup feedback invariance. In our case the subspace $\ker D$ is itself (A,B) - invariant provided that (2.14) is satisfied since then $\ker D + \text{range } B = M^2$. Therefore $\ker D$ is itself the maximal (A,B) - invariant subspace contained in $\ker D$ and Theorem 1 shows in addition that $\ker D$ is semigroup feedback invariant if (2.14) holds and if we allow for unbounded feedback.

COROLLARY 2

If (2.14) *holds then the subspace* $V = \ker D \subset M^2$ *is semigroup feedback invariant with respect to the abstract Cauchy problem* (2.7).

The following result has been established in [1] and [4].

LEMMA 3

If there exists a maximal semigroup feedback invariant subspace $V^*(\ker D)$ *contained in* $\ker D$, *then the DDP for* (2.7) *is solvable if and only if*

$$\text{range } E \subset V^*(\ker D) \quad . \tag{2.20}$$

So another approach to obtain Theorem 1 would be to combine Corollary 2 and Lemma 3. This complements the results in [1] on the DDP using bounded feedback.

Finally we would like to comment on another idea in [4], namely, to allow only subspaces of the special form

$$V(Q) = \{\phi \in M^2 \,|\, \phi^0 \in Q,\ \phi^1(\tau) \in Q,\ -h < \tau < 0\} \quad . \tag{2.21}$$

Pandolfi gave another sufficient condition for the solvability of the DDP for (2.1), (2.2) in terms of a semigroup feedback invariant subspace of the form (2.21). In our case Theorem 1 shows that $V(\ker D_0)$ is the maximal semigroup feedback invariant subspace of the form (2.21) contained in $\ker D$.

3. DDP BY OUTPUT INJECTION

Consider the retarded system

(3.1) $$\dot{x}(t) = Lx_t + E_0 d(t) + f(t) \quad ,$$

(3.2) $$y(t) = C_0 x(t) \quad ,$$

(3.3) $$z(t) = D_0 x(t)$$

where L, E_0, D_0 are defined as in section 2 and $C_0 \in \mathbb{R}^{p\times n}$. Then the DDP by output injection is to design a control law of the form

(3.4) $$f(t) = Ky_t = K_0 y(t) + \int_{-h}^{0} K_1(\tau)\dot{y}(t+\tau)d\tau$$

with $K_0 \in \mathbb{R}^{n\times p}$ and $K_1(\cdot) \in L^2[-h,0;\mathbb{R}^{n\times p}]$ such that the to be regulated output $z(t)$ of the closed loop system (3.1-4) is independent of the disturbance $d(t)$. This is the dual problem of the one discussed in section 2. Therefore we have the following dual result of Theorem 1.

THEOREM 4

Suppose that

(3.5) $$D_0 E_0 = 0, \; C_0 E_0 \text{ is injective}$$

and choose $H_0 \in \mathbb{R}^{q\times p}$ *such that*

(3.6) $$H_0 C_0 E_0 = I \in \mathbb{R}^{q\times q} \quad .$$

Then the DDP for system (3.1-3) *is solved by the following output injection control law*

(3.7) $$\begin{aligned} f(t) &= -LE_0 H_0 y_t \\ &= -A_0 E_0 H_0 y(t) - \int_{-h}^{0} A_1(\tau) E_0 H_0 \dot{y}(t+\tau)d\tau \quad . \end{aligned}$$

PROOF. The solution of (3.1), (3.2), (3.7) with initial state zero is in $H^1[-h,T;\mathbb{R}^n]$ and satisfies $\dot{x}(t) = L(I - E_0H_0C_0)x_t + E_0d(t)$ for almost every $t > 0$. Introducing the auxiliary variable $w(t) = (I - E_0H_0C_0)x(t)$ and taking into account (3.6) we obtain

$$\dot{w}(t) = (I - E_0H_0C_0)Lw_t \quad .$$

Hence (3.5) shows that $z(t) = D_0w(t)$ is independent of $d(t)$. □

REMARK

The condition $D_0E_0 = 0$ is necessary for the solvability of the DDP for system (3.1-3) and the condition on C_0E_0 being injective requires

(3.8) $$\text{rank } C_0 > \text{rank } E_0 = q$$

which means that the number of observable outputs is larger than or equal to the number of disturbances. Furthermore, C_0E_0 is injective if and only if E_0 is injective and

(3.9) $$\ker C_0 \cap \text{range } E_0 = \{0\} \quad .$$

This condition is generically satisfied in (3.8) holds.

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