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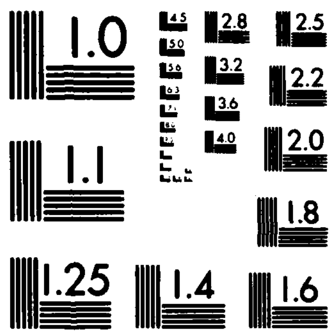
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FINAL REPORT

AFOSR GRANT 83-0319

NEW METHODS FOR NUMERICAL SOLUTION OF ONE CLASS OF STRONGLY NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS WITH APPLICATIONS IN HYDRODYNAMICS

VLADIMIR OLIKER and PAUL WALTMAN

The funding for the proposed research was initiated by AFOSR on September 1, 1983, for a period of one year. In early October, both investigators met with the Program Manager, Captain John P. Thomas, Jr. It was agreed that, of the general study originally proposed, the priority during the first year should be given to the investigation of the proposed numerical method for finding elliptic solutions of the Monge-Ampère equations, with the hope that remaining topics will be investigated during additional funding periods.

The particular topics of the investigation include:

1. Determination of the simplest classes of nonlinear equations of Monge-Ampère type and corresponding forms of boundary data to which the proposed numerical method can be applied;

2. Analytic formulation of the method in terms of the appropriate function spaces (this is a nontrivial task, because one deals here with cones of convex functions rather than spaces themselves, and the standard approximation theory is not readily applicable);

3. Theoretical investigation of the convergence rate and stability;

4. Development of stopping rules;

5. Construction of an algorithm suitable for a paralle computer and development of a computer code of the method;

6. A feasibility study;

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This effort studied a proposed numerical method for finding elliptic solutions of the Monge-Ampere equations, in the context of hydrodynamic and antenna design applications. Some computational experiments were performed on representative model problems. A theoretical estimate for the convergence rate of one part of the scheme was obtained. For the problem of reflector antenna design, the solvability of the corresponding linearized problem was demonstrated. A geometric interpretation of the solution was obtained which helps in understanding the problem.

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7. Use of the proposed method in combination with other methods in the following setting. It is well known that once a "good" initial approximation is found, there are many effective schemes to complete the computation. Hence, it was suggested in the proposal to investigate numerical schemes in which our method will be used as a way for providing a good initial guess followed by an application of a fast Newton-type method;

8. Comparison of performance on nonlinear problems of Monge-Ampère type of the proposed methods with other techniques presently in use.

We began the investigation by trying out our method on the "one dimensional version" of Monge-Ampère equation. Namely, we considered the two point boundary value problem (Dirichlet and Neumann boundary data) for the equation

$$y'' = f(x,y,y') \qquad x \in [a,b] \ ,$$

where f is subject to usual smoothness conditions assuring existence and uniqueness (cf. H. B. Keller, Numerical Methods for Two-Point Boundary Value Problems, Blaisdell, 1968). A computer code for this equation was written and tried on several examples in which the solutions grow quite fast (we tried the equation for the catenary surface). We did not carry out a detailed comparison with the shooting methods, but the numerical experiments gave indication that our procedure in such a situation will not be worse than shooting methods. We hope at some later time to return to this subject and carefully investigate it as the numerical solution of two point boundary value problems for ODE's is a subject of interest.

The simplest Monge-Ampère equation which we have investigated so far is of the form

$$\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial y^2} - \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 = \phi(x,y) \ . \qquad (*)$$

For this equation, considered in a convex domain, and continuous Dirichlet boundary data, one can prove convergence of the approximating sequence. As it is pointed out in the proposal, one has to solve first the "polyhedral" version of the equation and then investigate the convergence of the polyhedral solutions to the solution of (*), when the step size tends to zero. We succeeded in obtaining a theoretical estimate of the convergence rate for the first part of the procedure. We have also written a computer code for the method and did several computational experiments. Compared to the initially proposed algorithm, there were made several important improvements: a) it turned out quite useful to put a routine in the program to check the convexity of solution on each iteration step; b) we experimented with different grids and presently are using a hexagonal grid. For the equation (*) the method is computationally <u>feasible</u>. The algorithm has the advantage of being perfectly suitable for parallel computing since on each iteration all vertices can be processes independently. Thus, the items 1, 3, 4, 5, 6 have been partially completed.

Currently, we are working on extending the algorithm to include linear terms and terms of lower order of nonlinearity and Neumann boundary conditions. The particular form of such equation is

$$\frac{\partial^2 u}{\partial^2 x}\ \frac{\partial^2 u}{\partial^2 y} - \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 + a(x,y)\ \frac{\partial^2 u}{\partial x^2} + 2b(x,y)\frac{\partial^2 u}{\partial x \partial y}$$

$$+ c(x,y)\ \frac{\partial^2 u}{\partial y^2} + f(x,y,u,\ \frac{\partial u}{\partial x},\ \frac{\partial u}{\partial y}\ ) = 0 \ .$$

This class of equations includes as a special case the balance equation of the atmospheric dynamics.

In addition to the questions listed above, we have been working on the Monge-Ampère equation describing the problem of reflector antenna design. For the corresponding boundary value problem, there is no theoretical foundation (cf. B.S. Westcott, Shaped Reflector Antenna Design, Research Studies Press Ltd., 1983, p. 40). Since this problem involves an equation of Monge-Ampère type, and potentially is an important area where our numerical method can be applied, we have investigated it. At present, we can prove the solvability of the corresponding linearized problem. We also found a nice geometric interpretation of the solution which helps in understanding the problem.

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