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Final Report—Part II

METHODS OF COMPUTER-AIDED ANALYSIS OF NON-GAUSSIAN NOISE  
AND APPLICATION TO ROBUST ADAPTIVE DETECTION

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The combination of adaptive differential quantization with low-rank approximations to data matrices or estimated covariance matrices is believed to be a new and effective method for multivariable, robust, adaptive detection.

Methods of Computer-Aided Analysis of Non-Gaussian Noise  
and Application to Robust Adaptive Detection

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**Abstract**

We present a methodology for the modeling of certain non-stationary and non-gaussian random time series data with application to weak signal detection. Some components of the noise, which give it its non-gaussian characteristics, can be individually modeled, synthesized and subtracted to provide a gaussian residual. Further, it is shown that this process can also be carried out when signals are present.

The proposed methodology is applied to some Arctic Acoustic data using a combination of adaptive differential quantization and adaptive signal estimation algorithms based on singular-value-decomposition of a data matrix which we have developed.

The combination of adaptive differential quantization with low-rank approximations to data matrices or estimated covariance matrices is believed to be a new and effective method for multivariable, robust, adaptive detection.

### Introduction

A frequent problem with data, if one is considering detection of signal components or estimation of signal parameters is the need to formulate probability distributions for the data. Usually the underlying physical system responsible for noise is partially unknown and difficult to characterize in detail. One approach is to estimate the probability distributions directly from the data, but this can be difficult to do because of non-stationarity and the short duration of important events in the data.

We assume that the noise can be considered to be a mixture of non-stationary, high amplitude non-gaussian components plus a low amplitude gaussian stationary component. The methodology that we propose for modeling the data is to identify, categorize, model, and remove the non-gaussian components in a piece-wise fashion based on their ease of separability from the background noise and signals. This approach to modeling and processing complicated and non-stationary data is similar to that of Middleton [9] where it is suggested if the data is a mixture of the above form, then one way of dealing with the non-gaussian interference is to estimate and then null or subtract out the strong non-gaussian interference prior to signal processing and reduce the problem to one of the background gaussian noise or signal. Liu and Nolte [10] have shown that when the noise is gaussian and consists of a sum of a strong highly coherent component and a weak component of independent noise samples, then null steering is nearly optimal. The application of special smoothers and cleaners by Martin and



Thomson [2] for obtaining robust spectral estimates when the data is contaminated by outliers has provided motivation for our new use of adaptive differential quantization for robust detection.

Our proposed technique of iterative processing converts the problem of dealing with a complicated non-stationary and non-gaussian multivariate distribution to a sequence of simpler modeling problems. A summary of the steps is as follows:

- 1) locate and identify the non-gaussian interference components
- 2) estimate and subtract out the non-gaussian components
- 3) perform signal processing (signal detection, spectral estimation) using the residual

We present experimental results where we apply this methodology to processing a set of single channel digitized Arctic undersea acoustic data. Voitch and Walk [1] have characterized this data set and hypothesize that the data can be modeled as a mixture of three components as follows:

- (1) weak stationary gaussian background
- (2) a number of strong non-stationary sinusoidal-like components which can occur randomly throughout the data
- (3) sporadic high intensity impulsive bursts

The non-gaussian elements of the mixture are components (2) and (3). We will verify the hypothesis that the Arctic acoustic data is of the above mixture form in two steps; first, by application of algorithms we have developed for high resolution spectral estimation and adaptive signal estimation, to estimate and remove the non-gaussian interference components, such as the Taft-Kumarasan (T-K) method of improved linear

prediction [5,6,7] or an improved Prony method [8] to estimate parameters of the sinusoidal-like and exponentially damped sinusoid components, data-adaptive estimation of low rank signals using singular-value-decomposition of a data matrix [4], and adaptive differential quantization for isolating and removing impulses; and secondly, applying various statistical tests (skewness, kurtosis, and tests for normality) to the data, estimated interference and residual. Further, we will show that a weak signal which is injected into the non-gaussian interference of the Arctic acoustic data can be readily recovered using our proposed methodology and these algorithms. The paper will be presented in four parts as follows:

- I) The identification of the components responsible for the non-gaussian characteristics of the Arctic acoustic data.
- II) Modeling, estimation, and removal of the non-gaussian interference.
- III) Testing the background or residual for normality.
- IV) Recovery of a weak signal in the non-gaussian interference.

#### I. The Identification of the Components Responsible for the Non-Gaussian Characteristics of the Arctic Acoustic Data Set

By synchronized viewing of (a) the acoustic waveform (b) the time-local kurtosis and (c) the time-local power density spectrum, one quickly becomes aware that the two primary components responsible for the non-gaussian characteristics of the Arctic Acoustic data set are (1) a component of short-time duration, impulsive waveform (see Fig. 2) and (2) a component of

narrow-band sinusoidal-like interference (see Fig. 1) that appears to be non-stationary, varying in amplitude and frequency over time. These two components will be considered separately. Note that we will refer to the narrow-band components as tones throughout the remainder of the paper.

#### A) Impulsive Component

The impulsive waveforms can be described as short time duration ( a few milliseconds), high amplitude (with respect to the surrounding background noise) bursts that can have considerable variability in their structure (see Fig. 2 and Fig. 3 ).

The sections of the data that have impulsive components present appear to be characterized by a high coefficient of kurtosis [1], that is, much greater than 3.0 . The kurtosis for a gaussian random process is 3.0 .

#### B) Tonal Component

These regions are characterized by short-time power spectra that appear to consist of line components (see Fig. 1), often appearing to be harmonically related. Also the tonal or narrow-band regions generally feature a low kurtosis [1] (less than 3.0) . As noted by Veitch and Wilk [1] , the kurtosis of a random phased sinusoid is 1.5 and many of the tonal regions have a kurtosis of about 2.5 or lower. This implies that the tonal regions can perhaps be modeled as mixture of discrete sinusoids.

The tonal duration is variable, it can be on the order of minutes or seconds. The tonal regions also appear to be non-stationary, with the number of line components, frequencies, and amplitudes often varying considerably over time intervals on the order of seconds (see Fig. 1).

It appears that the non-gaussian regions can be located and identified

on the basis of high kurtosis and/or power spectra containing strong line components.

## II. Modeling, Estimation and Removal of the Non-Gaussian Interference Components

In this section we present a three-step methodology for (1) modeling the non-gaussian interference components, (2) estimating the parameters of the non-gaussian components and (3) removing the non-gaussian interference components .

### A) Tonal Component

From the results of section I , it is conjectured that the tonal interference can be locally modeled as a sum of random phased sinusoids or equivalently, that the tonal interference is strongly low rank. This implies that techniques for low rank interference removal [4] can be used to recover the background noise or signal intact in the vicinity of tonals.

The tonal components appear to be non-stationary , varying both in frequency and power over time intervals on the order of the resolution scale of the spectrogram (see Fig. 1). However, of greater importance, in terms of modeling and estimation is the local stationarity of the tonals. If the tonals can be approximated as being locally stationary, that is, fixed in parameter over short-time intervals, then techniques developed for data-adaptive estimation of low rank signals can be used to estimate and remove the tonals.

For the experimental work, we consider a tonal region whose power spectra has only one line component present, but is representative of the other tonal regions that have more than one line component present. This to

avoid the difficulties of estimating the number of line components present. It is conjectured that the single tonal results would be representative of the other multiple tonal regions.

To determine the stationarity of the single tonal, that region was partitioned into 64 sample blocks (6.5 milliseconds) and assuming that the tonal is actually sinusoidal in nature, an improved Prony method [8], was used to estimate the frequency, amplitude, and phase of the tonal in each block. Also the power of the residual or background noise remaining after the tonal is reconstructed using sinusoidal model and then subtracted out was also estimated. The results were then plotted (see Fig. 4).

They indicate that the frequency is slowly varying but the power appears to fluctuate over short time periods. Therefore, the tonal can only be considered as approximately stationary over short time intervals. This implies the need for short data length techniques. We now consider the suitability of using a sinusoidal model for the tonal.

The techniques, developed for data adaptive estimation of low-rank signals based on singular-value-decomposition of a data matrix [4] will be used to estimate the tonal and subtract it out. A data vector length of 64 samples was used with the data being arranged in the form of a backward predictor matrix (Hankel matrix) with predictor order 48 assuming a rank 2 interference.

The suitability of the sinusoidal model or low-rank assumption can be judged first, by spectral analysis and secondly, by computing the kurtosis of the unprocessed data, estimated tonal interference and residual. As stated previously in section I the tonal regions have a low kurtosis. If

the tonals are sinusoidal , the estimated tonal should have a kurtosis of about 1.5 and if the background noise is gaussian, the residual should have a kurtosis of about 3.0 . This will be covered in more detail in Section III for determining the normality of the residual data or background noise.

The power spectral plots of unprocessed tonal data, estimated tonal data , and residual clearly show that the tonal component has been subtracted out (see Fig. 5 for power spectral plots of two data records from the tonal region). Further, kurtosis of the estimated tonal interference is low, about 1.8 (see Fig. 6) and residual data kurtosis is generally close to 3.0 (see Fig. 6). This implies that there is considerable justification for assuming that the tonals are sinusoidal in nature.

#### B) Impulsive Component

Martin and Thomson [2] pointed out the degradation of conventional power spectra and covariance structure estimators when data is contaminated by outliers. Similar loss in performance can be expected in conventional signal detectors. We propose two techniques for processing the impulsive bursts (outliers) . The first technique, for bursts to which this is applicable, is to model the impulsive bursts as exponential signals (see Fig. 9). The second technique is to model the impulsive burst as an outlier cluster.

If the burst can be modeled as an exponential signal, then the same low rank interference removal techniques [4] as applied to the tonals can be also used here. This implies that if we consider weak signal detectability in the presence of the transient, we can estimate and remove the transient

while leaving the weak signal essentially intact. To process impulsive bursts that fit the outlier cluster model, we use a modified form of adaptive differential quantization to locate and smooth the outlier contaminated data points. This technique is similar to the adaptive linear predictive type smoother-cleanser used by Martin and Thomson [2] for pre-processing outlier contaminated data to obtain robust power spectra estimates.

To obtain a preliminary determination of whether or not the above models are suitable, a visual inspection of acoustic waveform plots of the transients was performed. This revealed that some of impulsive bursts do indeed appear to be potentially exponential in nature (see Fig. 3b, Fig. 3d, and Fig. 3f), being characterized by a single main peak and smooth sinusoidal-like transition in the leading and trailing sections. However, there are also bursts that appear to be complex, with several apparent main peaks and a discontinuous structure (see Fig. 3a, Fig. 3c, and Fig. 3e) fitting the outlier cluster model.

To verify the exponential model, backward and forward linear prediction with low rank improvement [5,6,7] was applied to the trailing and leading sections of the apparent exponential transients respectively (see Fig. 9 and Fig. 10).

The eigenvalue spread of the estimated data covariance matrix (see Fig. 10b and Fig. 10f) of the leading and trailing sections of the impulsive burst indicated that the data is strongly low-rank (Note: if real-valued exponential signals are present in the data, then the rank of the data covariance matrix is equal to twice the number of exponential signals

present [5] if the exponential signals are not of zero complex frequency). The prediction-error filter zeros were also computed and plotted (see Fig. 10g).

The angular locations of the prediction-error filter zeros outside the unit circle on the  $z$ -plane determines the exponential frequencies and the inverse of the zero's radius determines the damping factor [6]. The zero positions appear to also indicate that the transients are exponential in nature.

The exponential type transients were then estimated and subtracted out using the techniques of data adaptive estimation of low-rank interferences with the singular-value-decomposition of a data matrix [4]. Plots of the transient estimates (Fig. 10e and Fig. 10h) and of the residual data (Fig. 10d and Fig. 10i) remaining after the transient was subtracted out indicated that the transient was well modeled as exponential.

To process bursts that are of the impulse cluster model, we use a form of adaptive differential quantization [3]. First, we can express the observed data using a similar model as did Martin and Thomson [2].

$$y_n = x_n + v_n \quad (1)$$

where  $y_n$  is the observed data,  $x_n$  is the true process data and  $v_n$  are outliers.

The rate of change of the true process  $x_n$  is defined as

$$\Delta_n = x_n - x_{n-1} \quad (2)$$

The rate of change  $\Delta_n$  (2) can be viewed as an crude estimate of the innovations process. The adaptive differential quantization algorithm is given below :



First, obtain robust estimates for standard deviation of the process rate of change  $\Delta_n$  (2) and  $x_n$  are obtained using median type estimators based on N previous samples as follows:

$$\text{Dev}[\bar{\Delta}_n] \approx \bar{\Sigma}_n = \text{median} ( |\bar{\Delta}_{n-N}|, |\bar{\Delta}_{n-N+1}|, \dots, |\bar{\Delta}_{n-1}| ) \quad (3)$$

$$\text{Dev}[x_n] \approx \bar{V}_n = \text{median} ( |y_{n-N}|, |y_{n-N+1}|, \dots, |y_{n-1}| ) \quad (4)$$

$$\text{where } \bar{\Delta}_n = y_n - y_{n-1} \quad (5)$$

Denote the delta modulator output as  $\bar{x}_n$ , then

1) If previous data sample is not contaminated by an outlier then

$$\bar{x}_n = W_1(\bar{\Delta}_n) + y_{n-1} \quad (6)$$

where

$$W_1(\bar{\Delta}_n) = \begin{cases} \bar{\Delta}_n & \text{if } -\tau_1 \bar{\Sigma}_n \leq \bar{\Delta}_n \leq \tau_1 \bar{\Sigma}_n \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

and  $\tau_1$  is a threshold constant.

2) If the previous data sample was contaminated by an outlier, then we can not make a reliable prediction and hence determine a bound on  $y_n$  using the robust estimate of the standard deviation of  $x_n$ ,  $\Delta_n(y)$ . In this case

$$\bar{x}_n = W_2(y_n) \quad (8)$$

where

$$W_2(y_n) = \begin{cases} y_n & \text{if } -\tau_2 \bar{V}_n \leq y_n \leq \tau_2 \bar{V}_n \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

and  $\tau_2$  is the threshold constant. In our experimental results,  $\tau_1$  and  $\tau_2$  are set assuming that  $x_n$  and  $\Delta_n$  are gaussian and zero mean.

The adaptive delta modulator essentially functions as a first order

linear predictor by estimating confidence bounds on the range of the next observed data sample  $y_n$  using robust estimates of the deviation of rate of change  $\Delta_n$  and  $x_n$  based on  $N$  previously observed data samples. If  $y_n$  is outside the predicted bound and  $y_{n-1}$  was not outlier contaminated, then replace it with the value of the previous data point  $y_{n-1}$ , otherwise if  $y_{n-1}$  is outlier contaminated, then a burst of outliers is assumed and we set  $y_n=0$ . It can be shown that if the first autocorrelation lag of the data is sufficiently large and the remaining lags are small in respect to the variance of  $x_n$ , then this replacement scheme should yield good results assuming no further information on the correlation structure of  $x_n$  and the outliers  $v_n$ .

The adaptive differential quantizer was applied to both types of transients. The plots of the transients (see Fig. 2) before and after processing indicated that differential quantizer functioned well and was robust in smoothing and removing the transients.

### III Testing the Background or Residual for Normality

It is conjectured that the background or residual data remaining after the tonal and transient components are removed is gaussian or approximately gaussian. To test the hypothesis that the residual data is gaussian, the coefficient of kurtosis statistics and the Kolmogorov-Smirnov test for normality was used.

We will first consider a tonal region. A tonal region with only a single line component was selected for simplicity.

For testing purposes, a block of 100 consecutive data records (records

#1680-1779) was selected from that region (1024 samples/record).

The narrow-band component was then estimated and subtracted out using the data adaptive low-rank signal estimation technique based on the singular-value-decomposition of a data matrix [4]. A data vector length of 64 samples was used with the data being arranged in the form of a backward predictor matrix (Hankel matrix) with predictor order of 48 assuming a rank 2 interference.

The unprocessed tonal data and the residual remaining after the tonal had been estimated and subtracted out was partitioned into 50 blocks of 2 records each. From the two record block, only every 12'th data sample was used to obtain independence between successive samples yielding a total of 170 independent samples. The Kolmogorov-Smirnov test for normality was then applied to the independent 170 sample set obtained from each two record block.

The level of acceptance statistic for the hypothesis that the tested distribution is gaussian was plotted along with the coefficient of kurtosis for each data record. (see Fig. 6 and Fig. 7) .

The plots (Fig. 6 and Fig. 7) clearly show that the level of acceptance for the gaussian hypothesis is high and the kurtosis is much closer to 3 (gaussian process has kurtosis of 3) after the tonal component had been removed. For comparison, the same statistical tests were applied on another 100 record block (records # 4601-4670) of data that was tonal and burst free. Veitch and Wilk [1] noted that this block of data appeared to be gaussian. Also, variance plots of the residual (see Fig. 8) seem to indicate that the background noise levels are approximately stationary.

A set of records containing transients was obtained on the basis of high kurtosis (greater than 4) from the data set. The records were then processed with adaptive differential quantization to smooth and remove the impulsive bursts. The normality of records was judged by the coefficient kurtosis before and after processing.

The results (see Fig. 2) indicate that the transient is primarily responsible for the high kurtosis and that after smoothing and removal, the kurtosis of the data record is much closer to 3 or nearer to gaussian.

Although the experimental work was limited, these preliminary results do tend to support the hypothesis that the background noise is gaussian or approximately gaussian even in the presence of the tonals and transients.

#### IV) Recovery of a Weak Signal in the Non-Gaussian Interference

We will show that a weak signal can be recovered from both types of non-gaussian interference, namely tonals and impulsive bursts, using our methodology. To demonstrate that the tonal can be estimated and subtracted out with little distortion to a weak background signal, a weak sinusoidal signal was injected in vicinity of tonal. We use the same single tonal region as earlier in section II. The tonal section was partitioned into blocks of 64 samples each as before. The blocks were then Fourier transformed before and after signal injection (see Fig. 11). Next the tonal was estimated and subtracted out using the technique of low-rank interference removal with the singular-value-decomposition of data matrix [4] as described previously. The tonal estimate and residual data was also Fourier transformed before and after signal injection. The Fourier

transforms of the residual clearly indicate that the weak signal can be recovered with little distortion (see Fig. 11).

For impulsive bursts, we show that if the burst is exponential, then we can estimate and subtract out the burst in a similar manner as in the tonal case to recover the weak signal. If the burst is not exponential, then we can use adaptive differential quantization to minimize the effect of the burst prior to detecting the weak signal using conventional techniques.

A weak sinusoidal signal was injected in the locality of an exponential type transient. The transient was Fourier transformed before and after signal injection (see Fig. 12). Next the transient was estimated and subtracted out using the technique of low-rank interference removal with the singular-value-decomposition of a data matrix [4]. The transient estimate and residual data was also Fourier transformed before and after signal injection. The Fourier transforms of the residual clearly indicate that the weak signal can be recovered with little distortion (see Fig. 12 ).

Next, a record containing an impulsive burst that does not fit the exponential model (outlier cluster) was chosen. A weak sinusoidal signal was injected and the record was then processed with adaptive differential quantization. Fourier transforms of the record before and after processing clearly indicate the improvement attained (see Fig. 13) when removing the burst.

#### IV) Conclusion

The results of our application of the piece-wise modeling and processing methodology to the Arctic acoustic data clearly demonstrated the

simplicity of this approach. To successfully apply this methodology to other types of non-stationary and non-gaussian data of the mixture form, it is only necessary to categorize and model the non-gaussian components in terms of their local effect on the desired signal processing application ( spectral estimation, detection etc. ) rather than formulating a general probabilistic model.

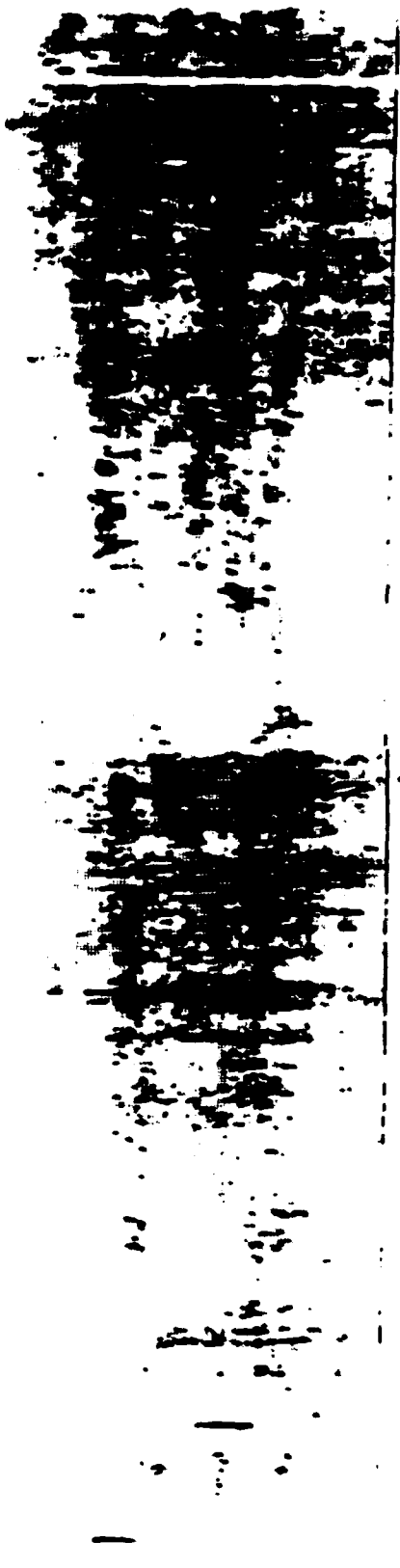
It is also noted that the results obtained in modeling and processing the Arctic acoustic data could be further improved by using more sophisticated models for the non-gaussian components. An example of a more sophisticated model would be to model the narrow-band components as a chirp or frequency modulated signals rather than approximating it as sinusoidal over short-time intervals. The advantage is that we could work over longer time intervals, hence improving the signal-to-noise ratio.

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2500

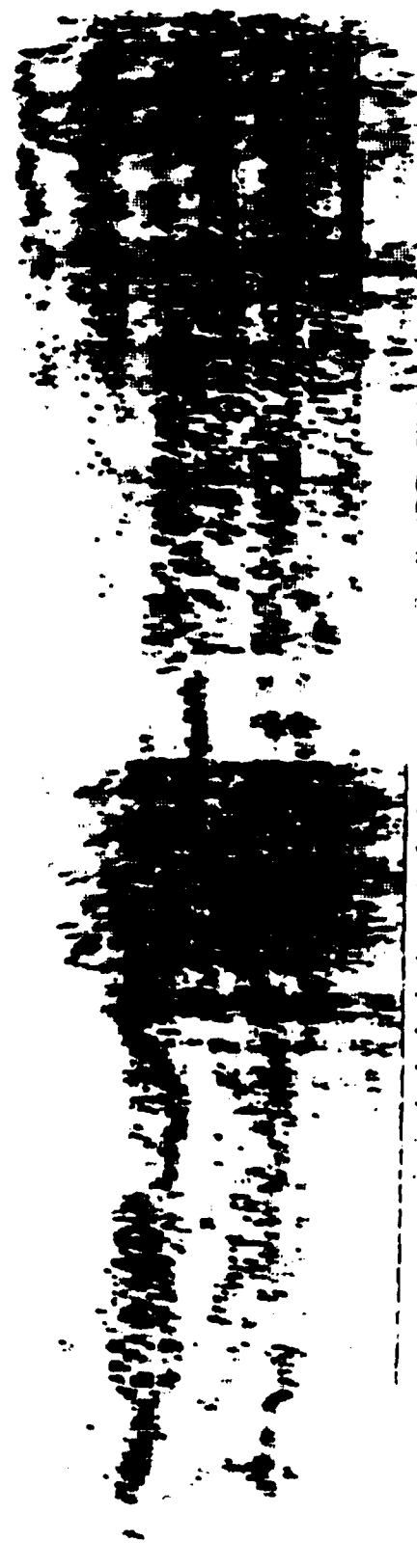
FREQUENCY HZ



56.9

2500

FREQUENCY HZ



56.9

Fig. 1) SPECTROGRAM OF ARCTIC SEA NOISE DATA SET  
RECORDS 1-2500 (256 SECONDS)

112.8



2500

FREQUENCY Hz

0



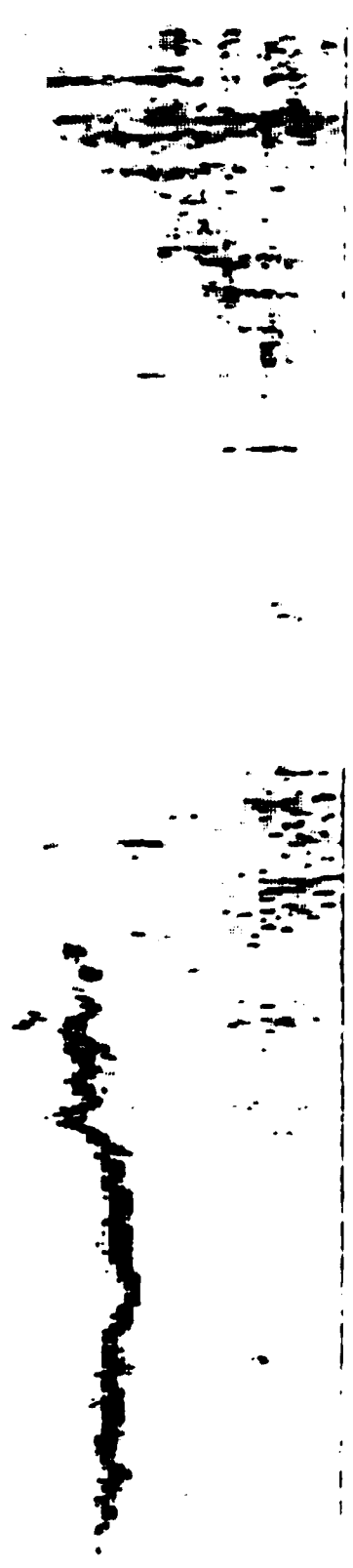
113.8

170.7 sec

2500

FREQUENCY Hz

0



170.7

227.6 sec

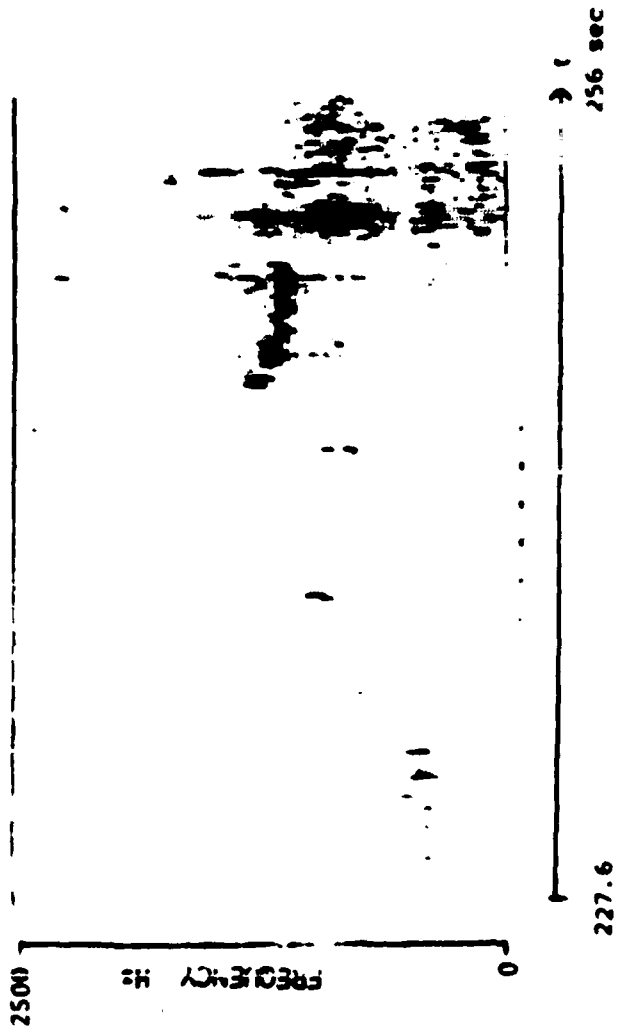


Fig. 2) Waveform plots of records containing impulsive bursts before and after processing by adaptive differential quantization.

Note: The coefficient of kurtosis is defined as

$$c_k = \frac{\mu_4}{(\mu_2)^2}$$

where

$$\mu_1 = \frac{1}{1024} \sum_{n=1}^{1024} x_n \quad \mu_2 = \frac{1}{1024} \sum_{n=1}^{1024} (x_n - \mu_1)^2$$

$$\mu_4 = \frac{1}{1024} \sum_{n=1}^{1024} (x_n - \mu_1)^4$$

and  $x_n$ , for  $n=1,2,\dots,1024$  are samples of the particular noise record.



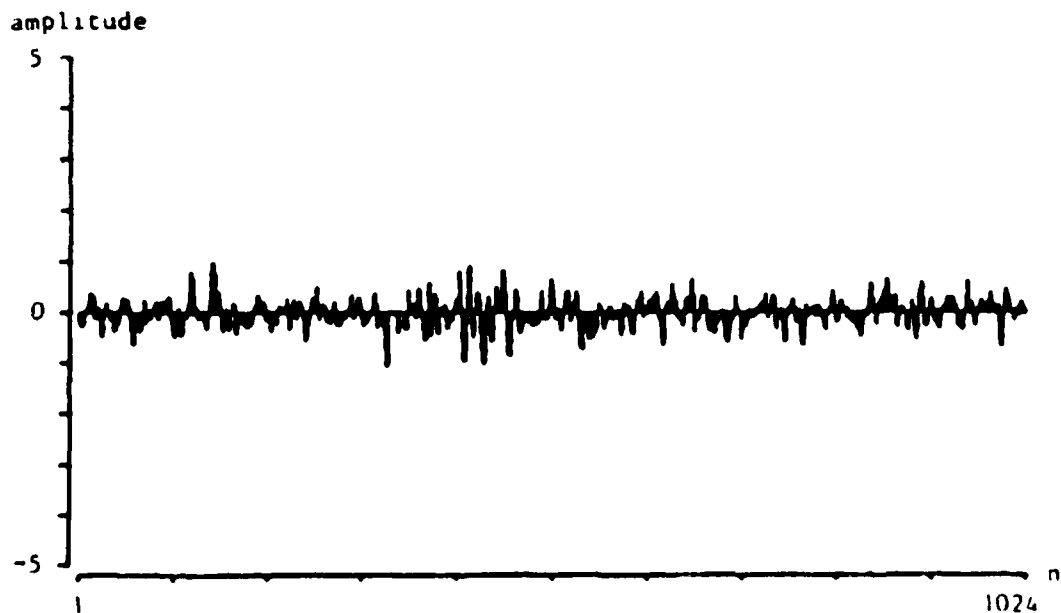
a) Record #1362: coefficient of kurtosis = 28.2



b) Record #1362: Processed by adaptive differential quantization where  $T_1 = 3.76$ ,  $T_2 = 2.51$ , and  $N = 101$ .  
coefficient of kurtosis = 3.19



c) Record #2066: coefficient of kurtosis = 29.52



d) Record #2066: Processed by adaptive differential quantization where  $T_1 = 3.76$ ,  $T_2 = 2.51$ , and  $N = 101$ . coefficient of kurtosis = 3.91



e) Record #2177: coefficient of kurtosis = 13.4



f) Record #2177: Processed by adaptive differential quantization where  $T_1 = 3.76$ ,  $T_2 = 2.51$ , and  $N = 101$ . coefficient of kurtosis = 3.8



g) Record #2220: coefficient of kurtosis = 22.97



h) Record #2220: Processed by adaptive differential quantization where  $T_1 = 3.76$ ,  $T_2 = 2.51$ , and  $n = 101$ .  
coefficient of kurtosis = 3.36



i) Record #2236 coefficient of kurtosis = 19.55

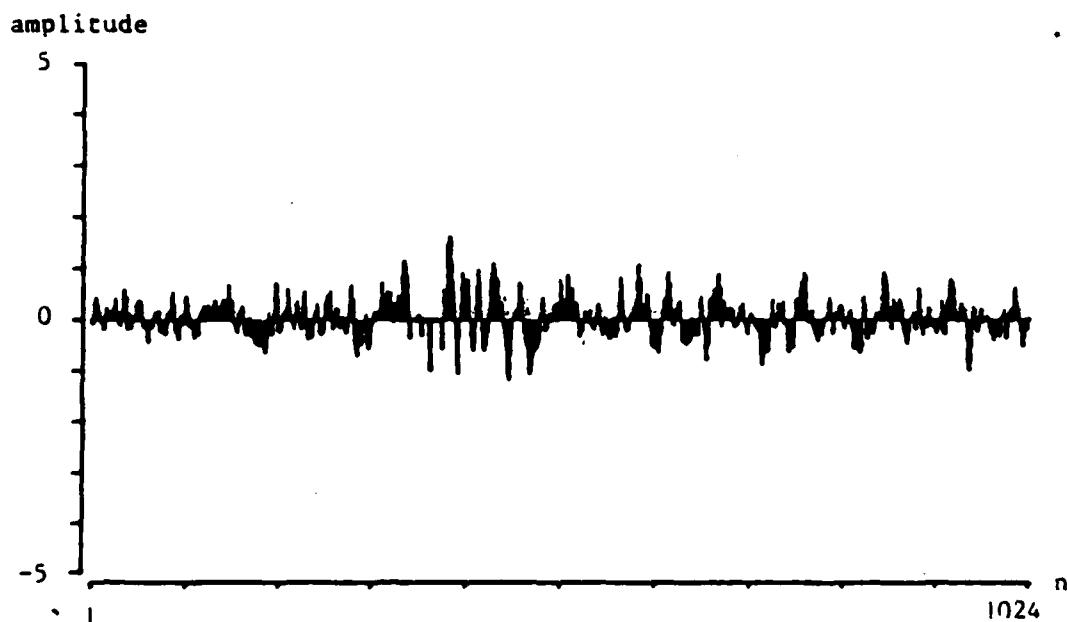


j) Record #2236: Processed by adaptive differential quantization where  $T_1 = 3.76$ ,  $T_2 = 2.51$ , and  $N = 101$ .  
coefficient of kurtosis = 3.87.



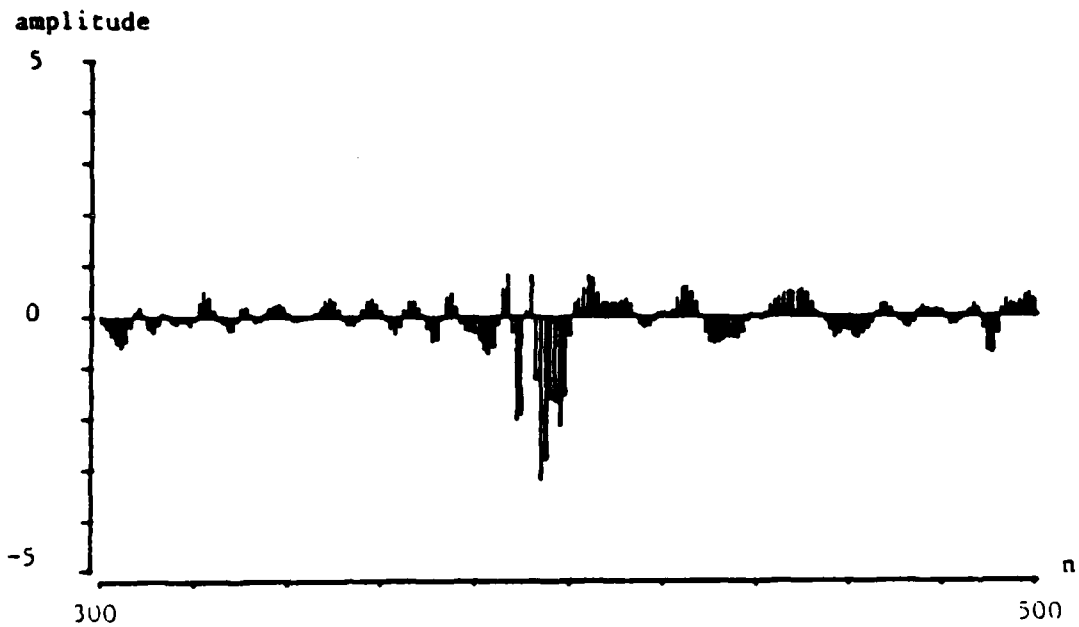


k) Record #2248: coefficient of kurtosis = 18.49

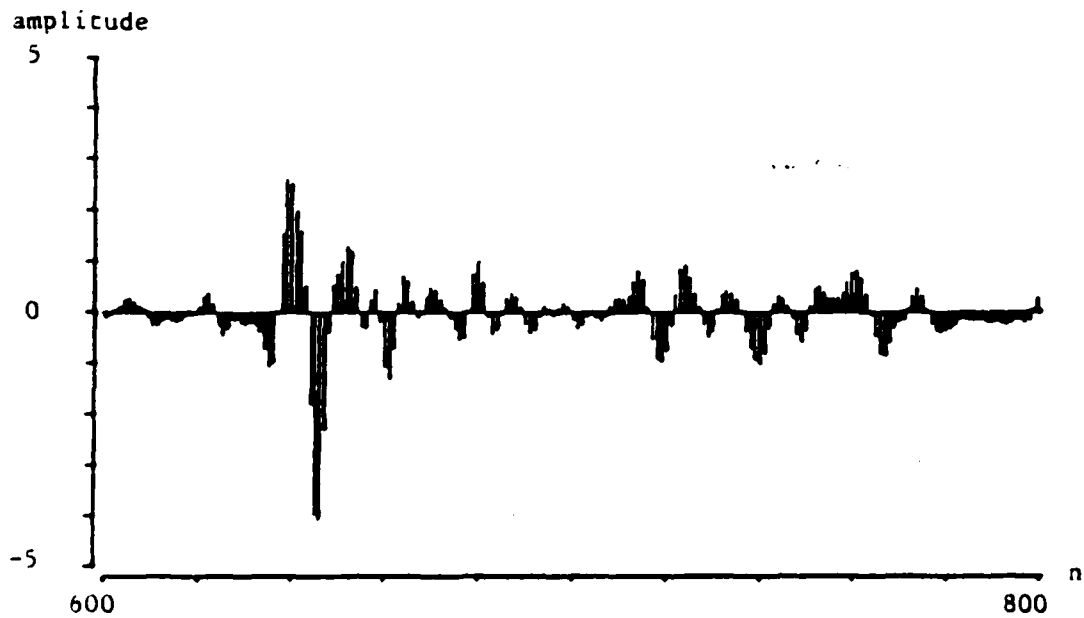


l) Record #2248: Processed by adaptive differential quantization  
where  $T_1 = 3.76$ ,  $T_2 = 2.51$ , and  $N=101$ .  
coefficient of kurtosis = 3.87

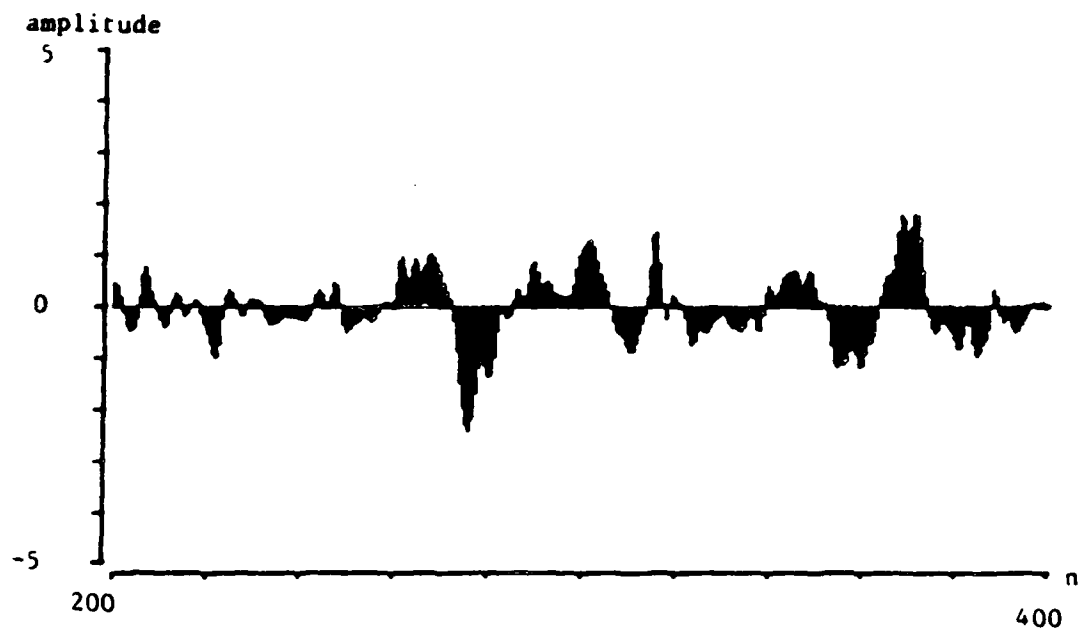
Fig. 3) Expanded scale plots of some impulsive burst waveforms.



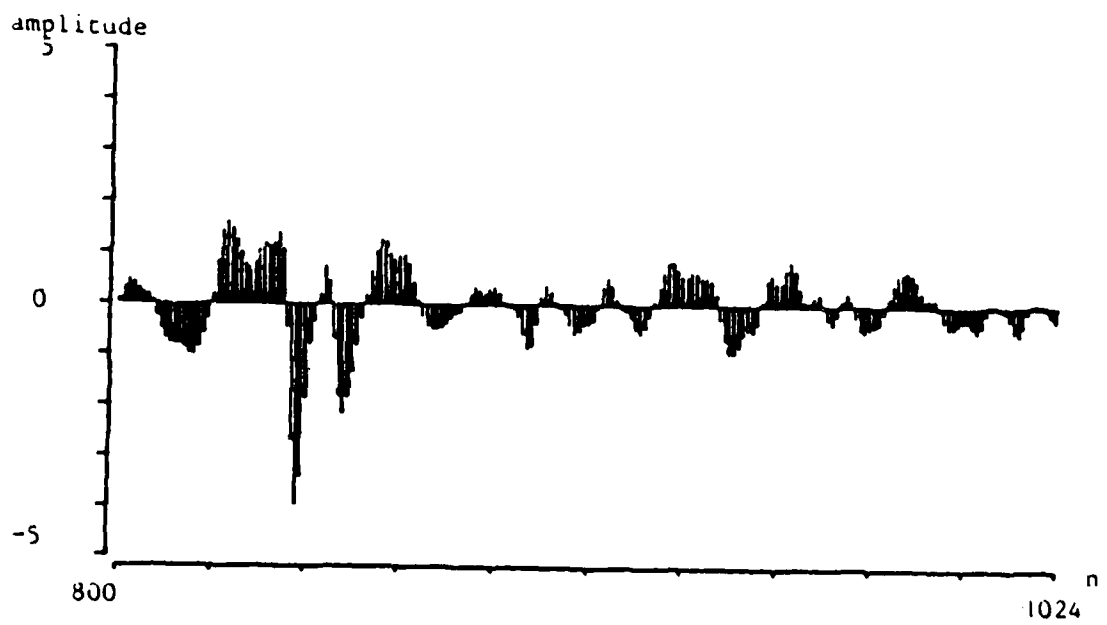
a) Record #1362



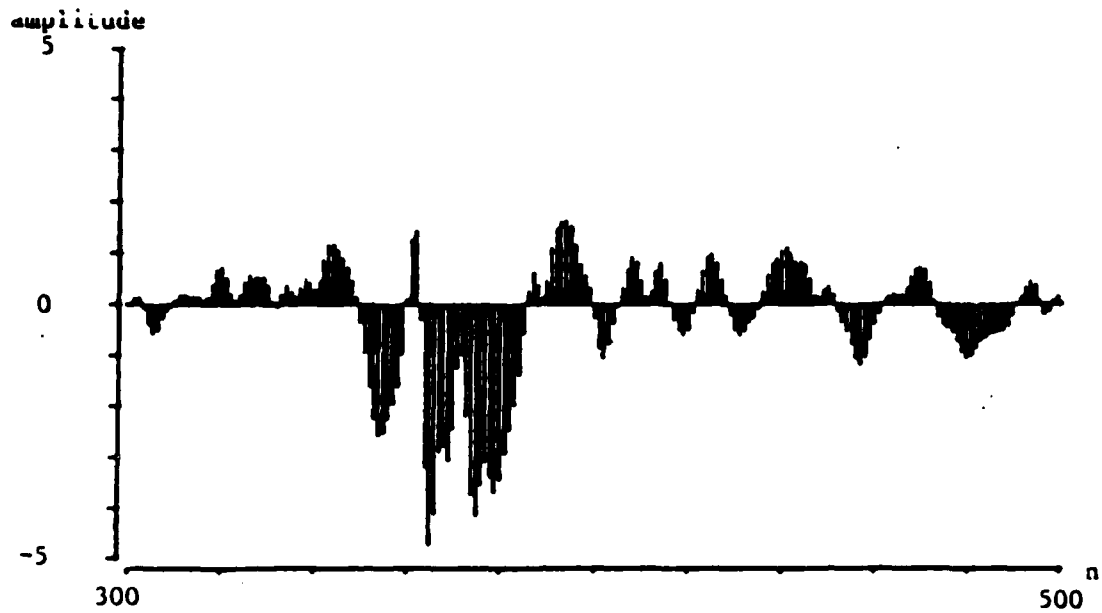
b) Record #2066



c) Record #2220



d) Record #2236

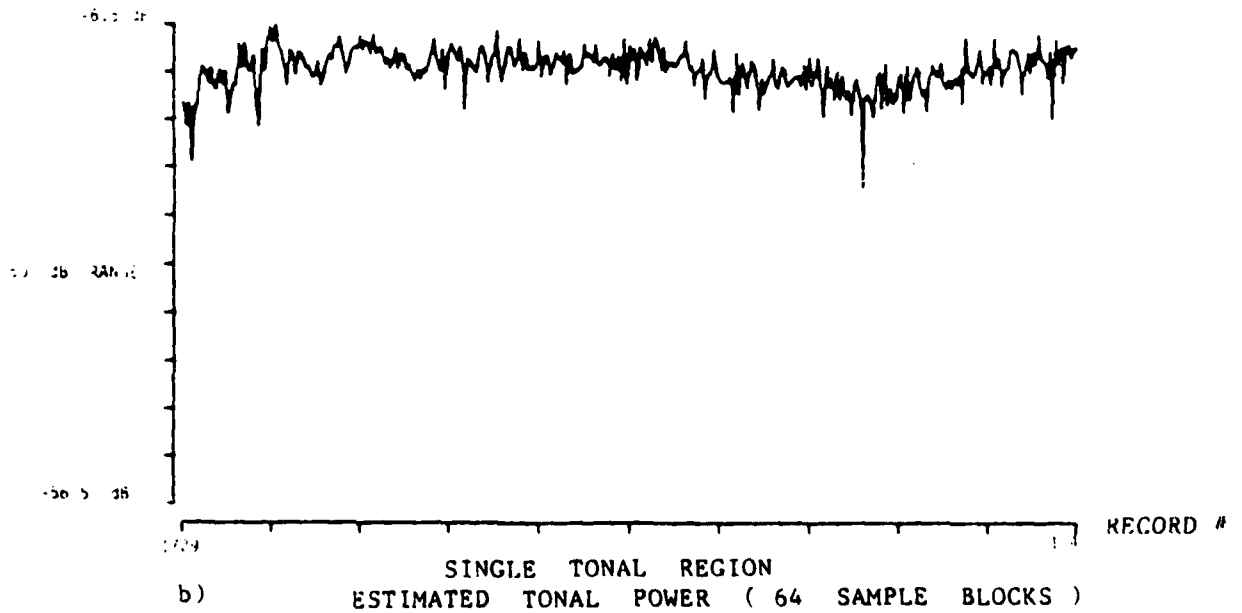
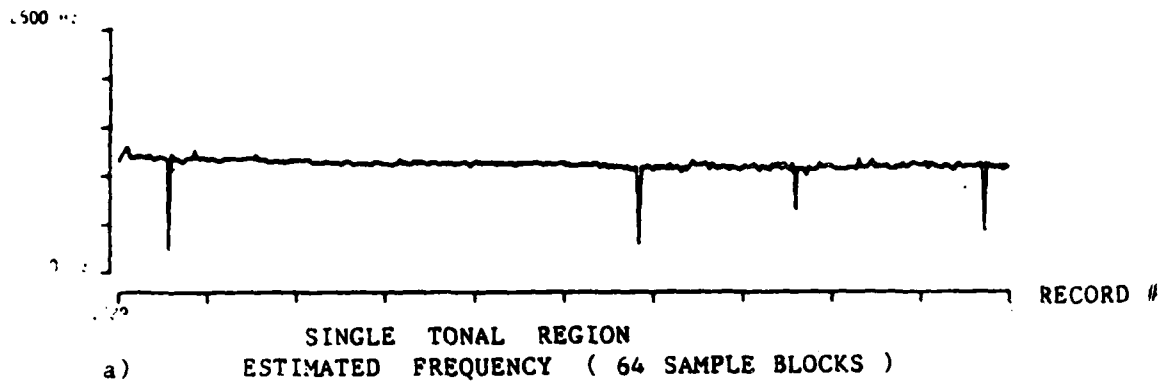


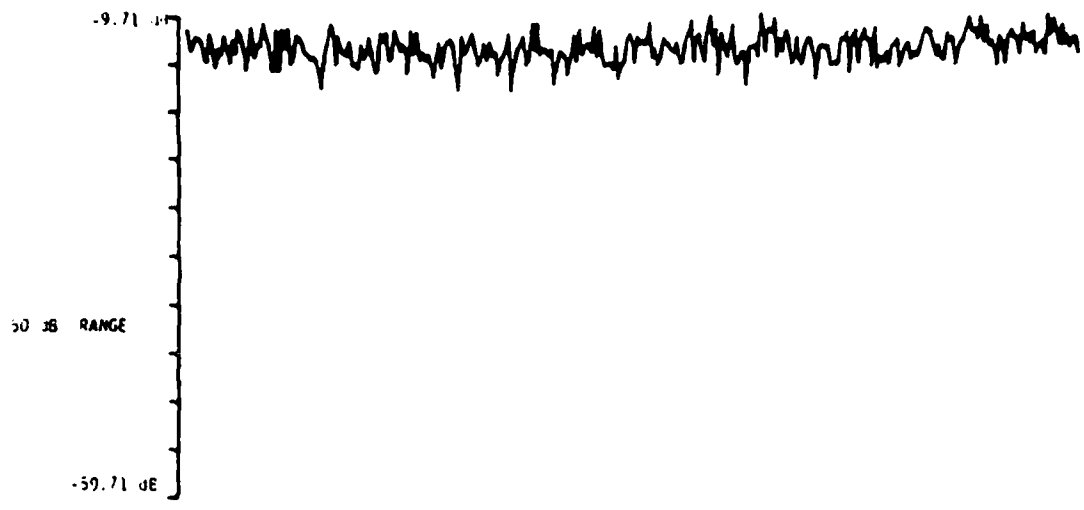
e) Record #2248



f) Record #5546

Fig. 4) Estimation of the tonal frequency, tonal power, and background noise level over a short time interval ( 64 samples ) using an improved Prony method.





c) ESTIMATED BACKGROUND NOISE POWER ( 64 SAMPLE BLOCKS )

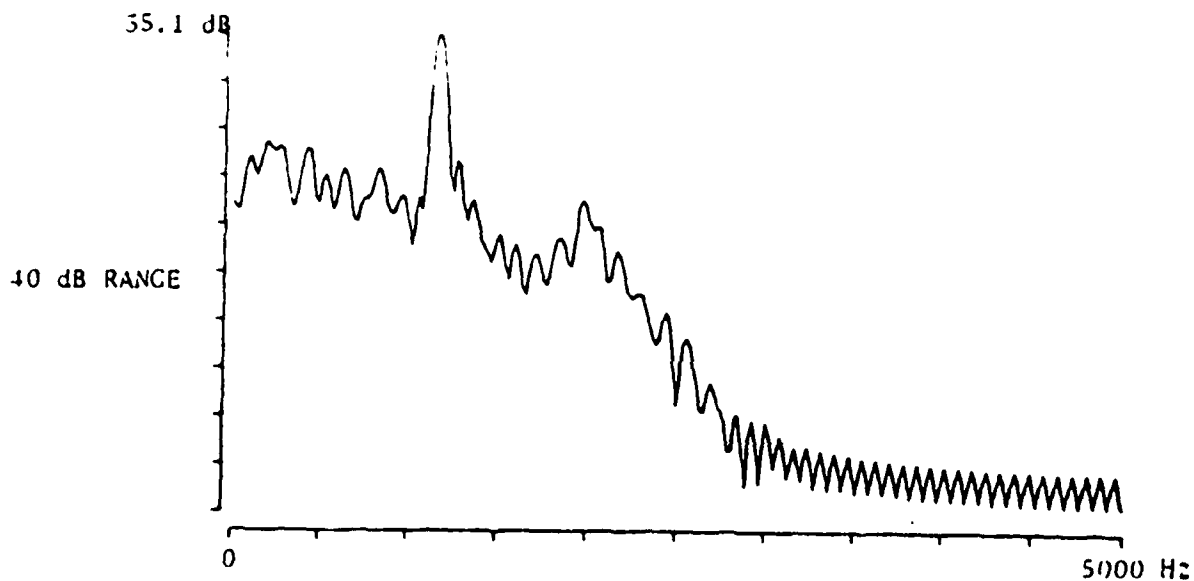
Fig. 5) AVERAGED PERIODOGRAM OF THE UNPROCESSED DATA, ESTIMATED TONAL AND RESIDUAL OBTAINED USING THE DATA-ADAPTIVE SIGNAL ESTIMATION ALGORITHM BASED ON SVD FOR RECORD #1739 .

The periodogram is calculated as follows:

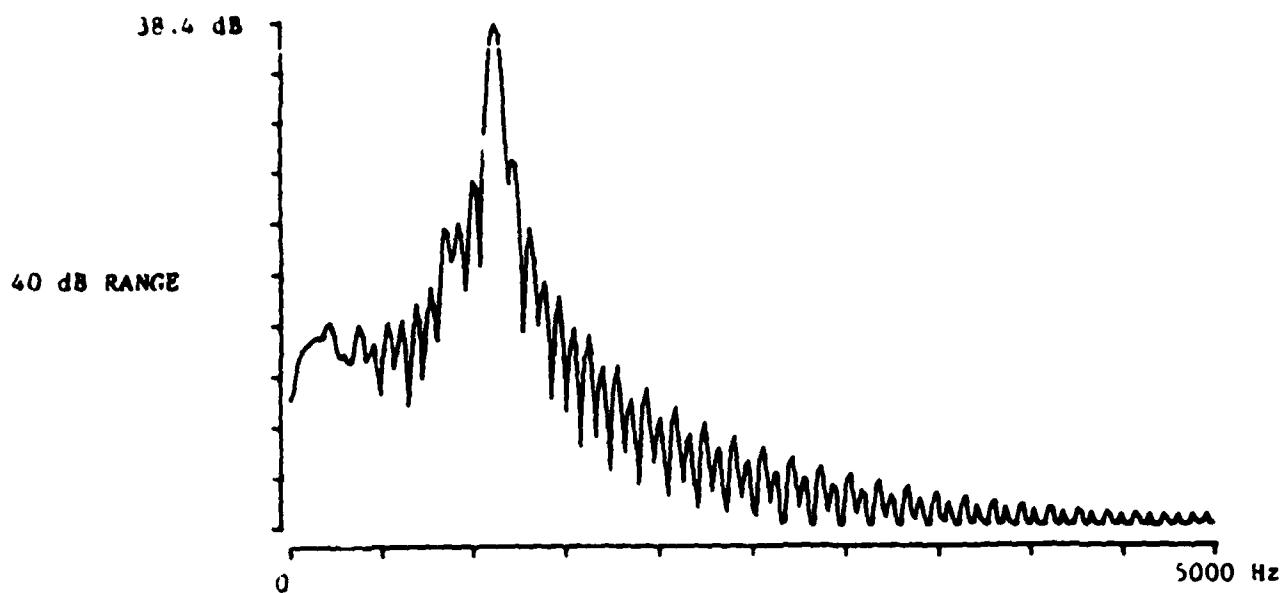
$$|Y(e^{j\omega})|^2 = \frac{1}{8} \sum_{j=1}^8 |X_j(e^{j\omega})|^2 \quad \text{where} \quad X_j(e^{j\omega}) = \sum_{k=1}^{128} x_{k+128(j-1)} e^{-j\omega k}$$

with  $\omega$  being evaluated at  $2\pi k/128$  intervals for  $k=0,1,\dots,128$  .

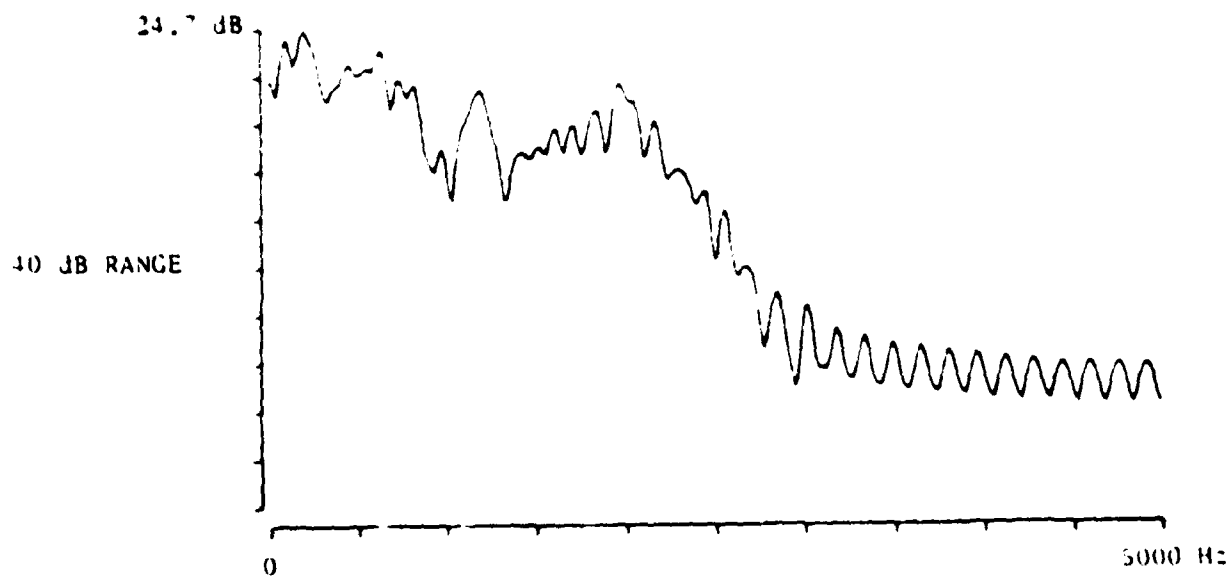
Note:  $x_k$  are the data samples .



a) AVERAGED PERIODOGRAM OF UNPROCESSED DATA



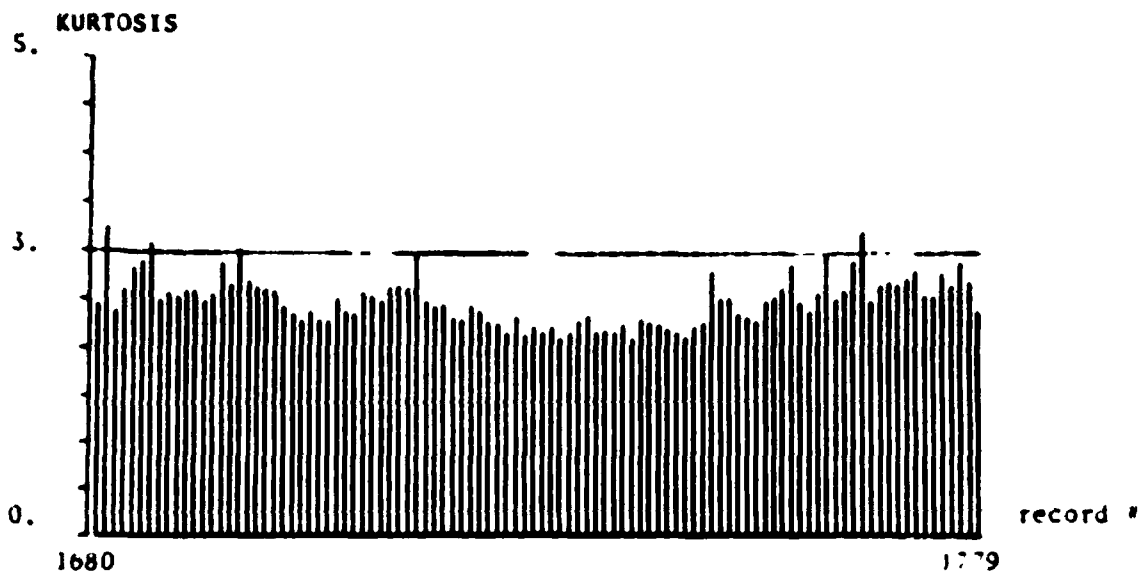
b) AVERAGED PERIODOGRAM OF ESTIMATED TONAL



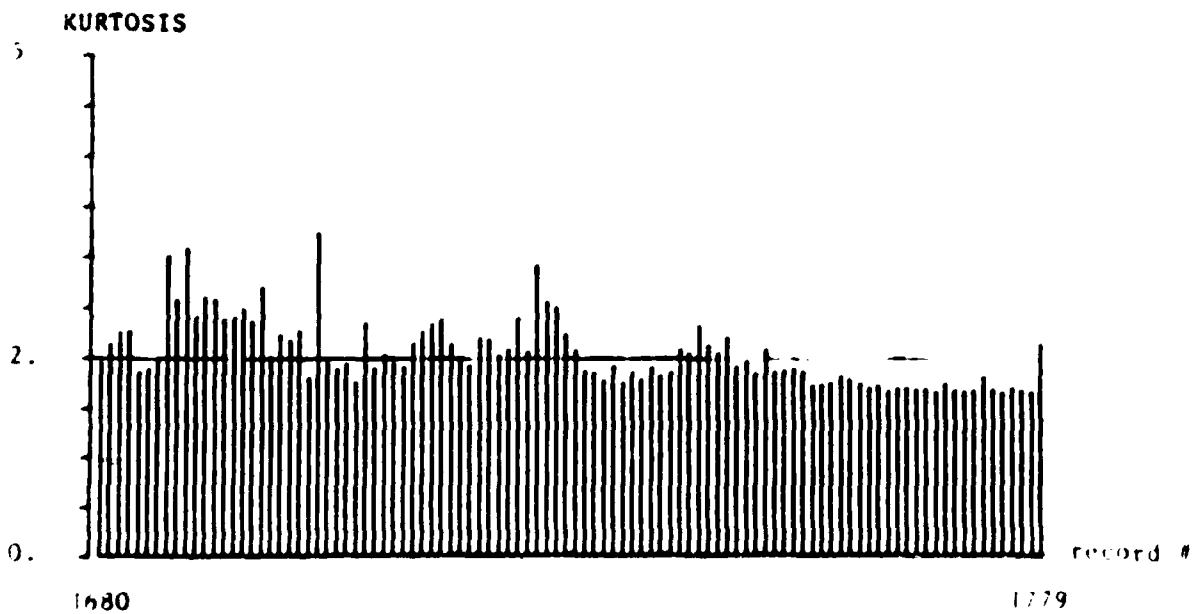
c) AVERAGED PERIODOGRAM OF DATA RECORD AFTER TONAL HAS BEEN ESTIMATED AND SUBTRACTED OUT



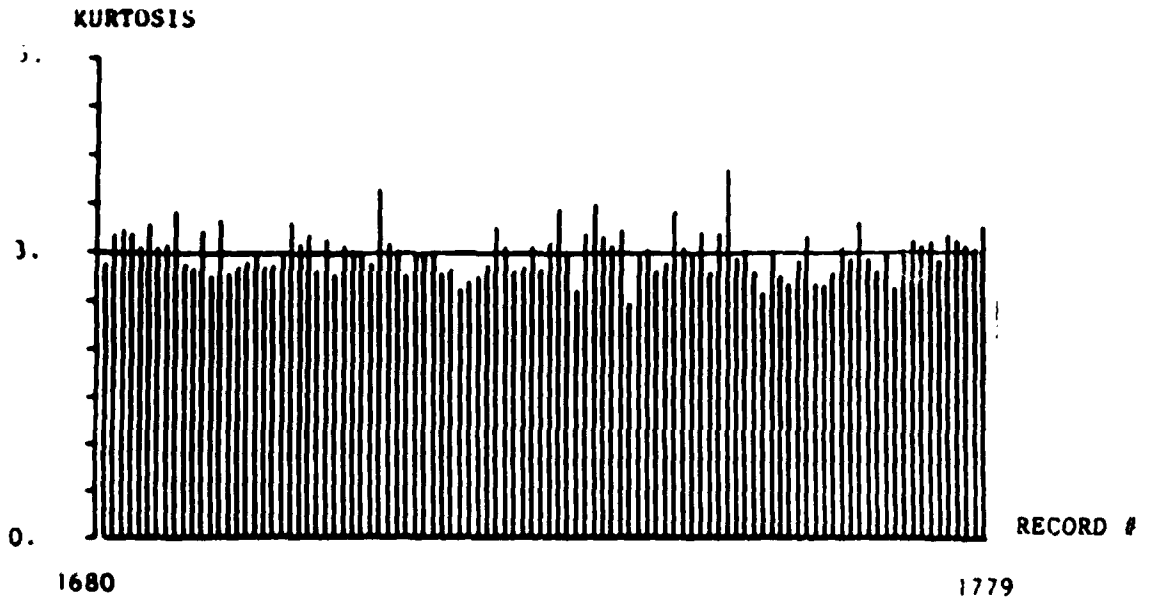
FIG. 6) COEFFICIENT OF KURTOSIS FOR DATA RECORDS 1680-1779 .  
THE COEFFICIENT OF KURTOSIS IS CALCULATED FOR 1024  
SAMPLE BLOCKS.



a) UNPROCESSED DATA



b) ESTIMATED TONAL



c) RESIDUAL

Fig. 7) THE RESIDUAL DATA LEFT AFTER THE TONAL HAS BEEN ESTIMATED AND SUBTRACTED OUT AND A REGION WITH NO APPARENT TONAL OR IMPULSIVE COMPONENTS SELECTED FOR COMPARISON ARE TESTED FOR NORMALITY USING THE KOLMOGOROV-SMIRNOV TEST.

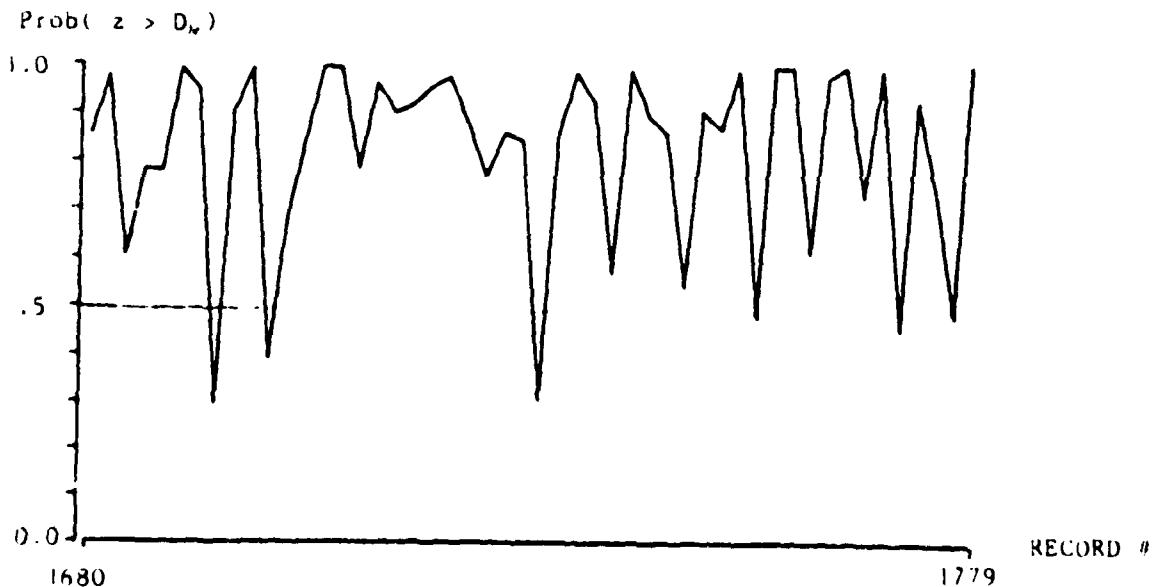
THE 100 RECORD REGION IS PARTITIONED INTO 2 RECORD BLOCKS (2048 SAMPLES) WITH EACH BLOCK BEING SEPARATELY TESTED FOR NORMALITY. AN INDEPENDENT SAMPLE SET IS OBTAINED BY TAKING EVERY 12'th SAMPLE FROM BLOCK, YIELDING A TOTAL OF 170 SAMPLES. THE SAMPLE SET IS NORMALIZED TO ZERO-MEAN AND UNIT VARIANCE AND THE CUMMULATIVE DISTRIBUTION  $\hat{F}_N(Y)$  IS ESTIMATED FOR THE DATA. THE KOLMOGOROV-SMIRNOV DISTANCE STATISTICS ARE THEN COMPUTED ASSUMING THE SAMPLE SET IS UNIT NORMAL ( $F_N(Y)$ ).

$$D_N^- = \sup_v \{ F_N(v) - \hat{F}_N(v) \}$$

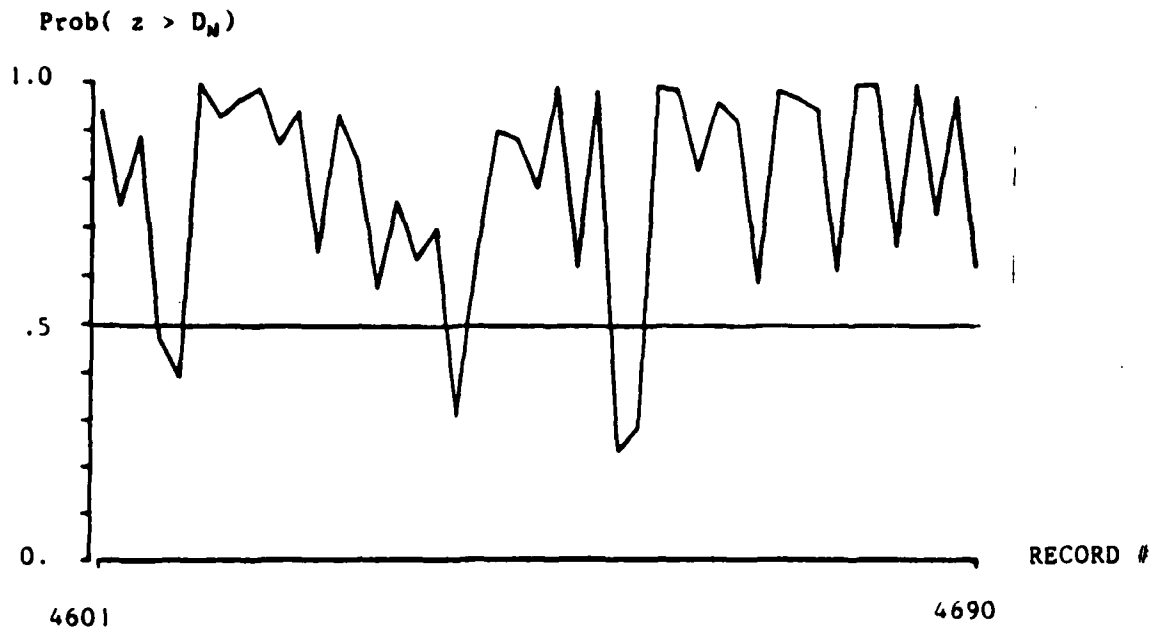
$$D_N^+ = \sup_v \{ \hat{F}_N(v) - F_N(v) \}$$

$$D_N = \max(D_N^+, D_N^-)$$

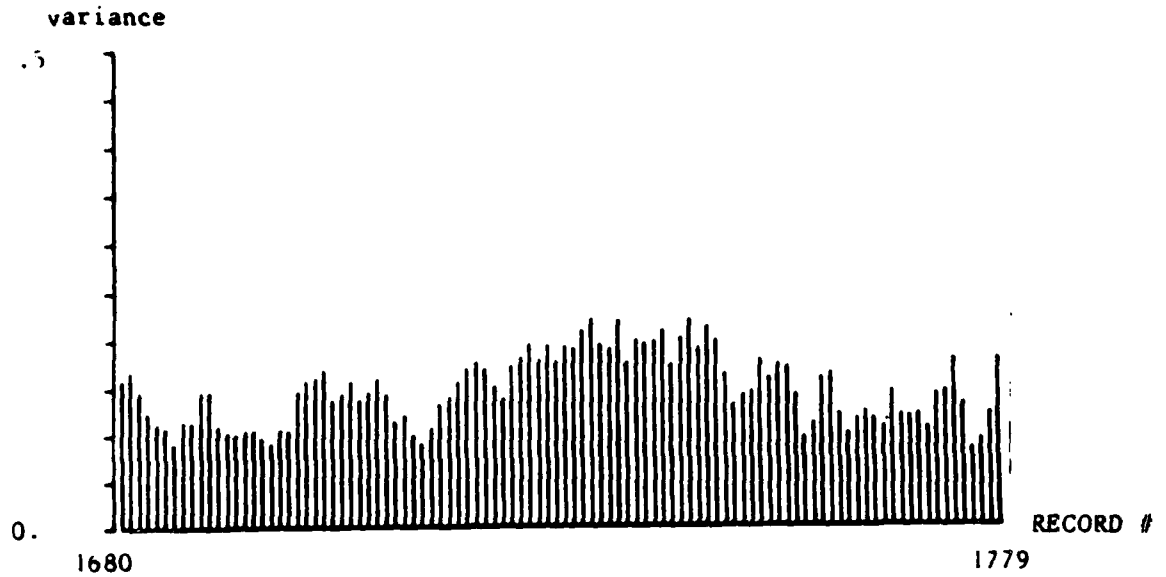
FROM FORMULAS,  $\text{Prob}(z > D_N)$  IS CALCULATED



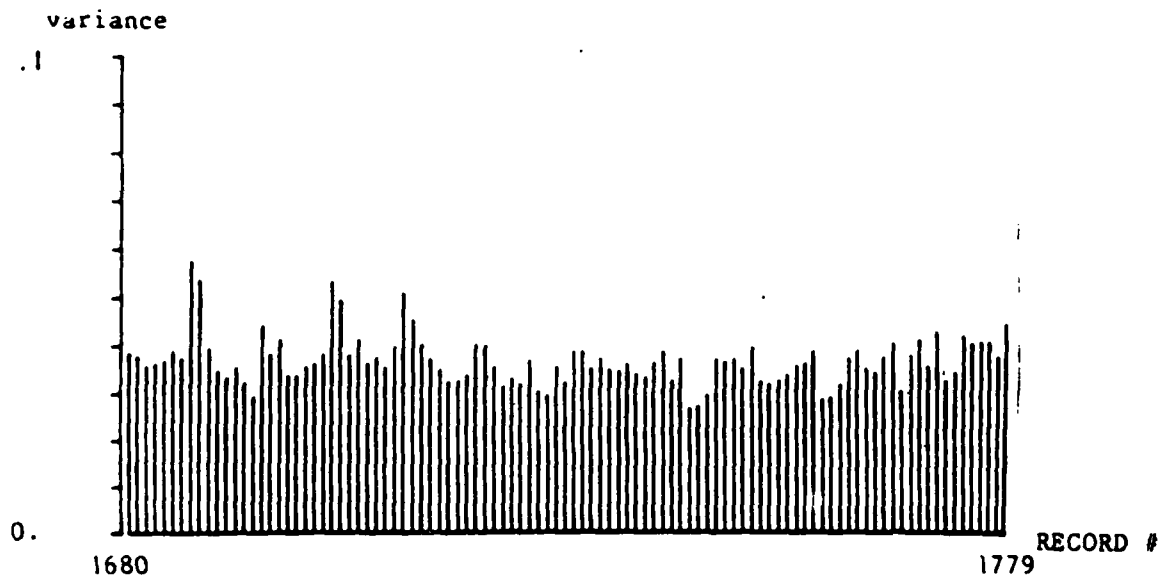
a) RESIDUAL DATA



b) DATA FREE OF TONAL AND IMPULSIVE  
CONTAMINATION



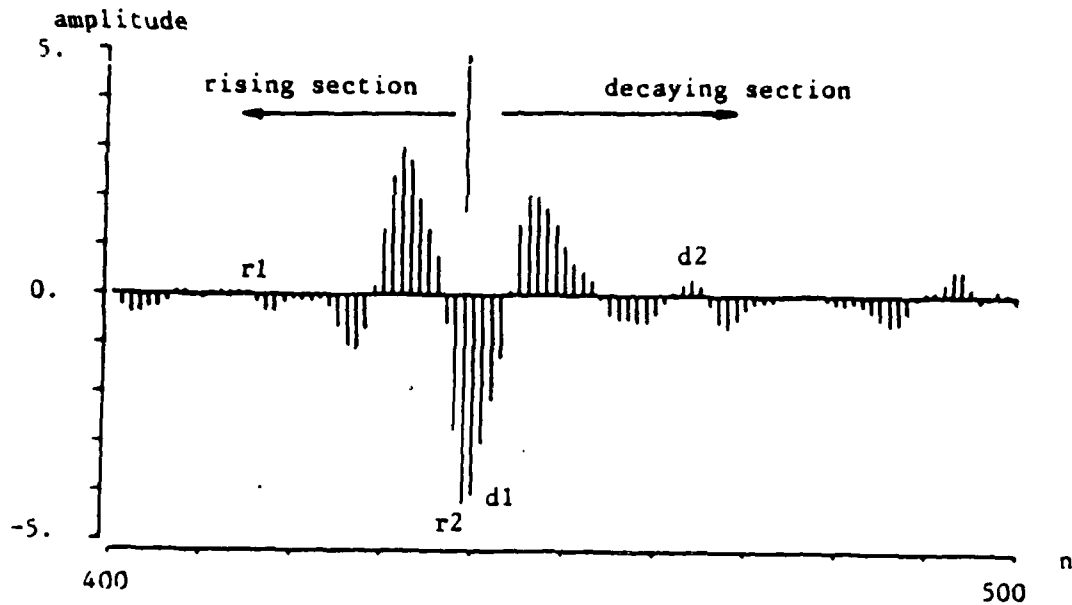
a) UNPROCESSED TONAL DATA



b) RESIDUAL

Fig. 8) THE VARIANCE OF THE TONAL REGION AND THE RESIDUAL REMAINING AFTER THE TONAL COMPONENT HAS BEEN ESTIMATED AND REMOVED .

NOTE THAT THE VARIANCE IS COMPUTED PER RECORD (1024 SAMPLES) .



CLOSE UP OF IMPULSIVE BURST FROM RECORD #64.B

BURST IS SPLIT INTO SECTIONS -- EACH SECTION IS MODELED SEPARATLY

A) RISING SECTION

$$x(n)_r = \sum_{k=1}^{M_r} A_{rk} e^{(j\omega_{rk} + \sigma_{rk})n} + w(n)_r \quad \text{FOR } n=r1, r1+1, \dots, r2$$

B) DECAYING SECTION

$$x(n)_d = \sum_{k=1}^{M_d} A_{dk} e^{(j\omega_{dk} + \sigma_{dk})n} + w(n)_d \quad \text{FOR } n=d1, d1+1, \dots, d2$$

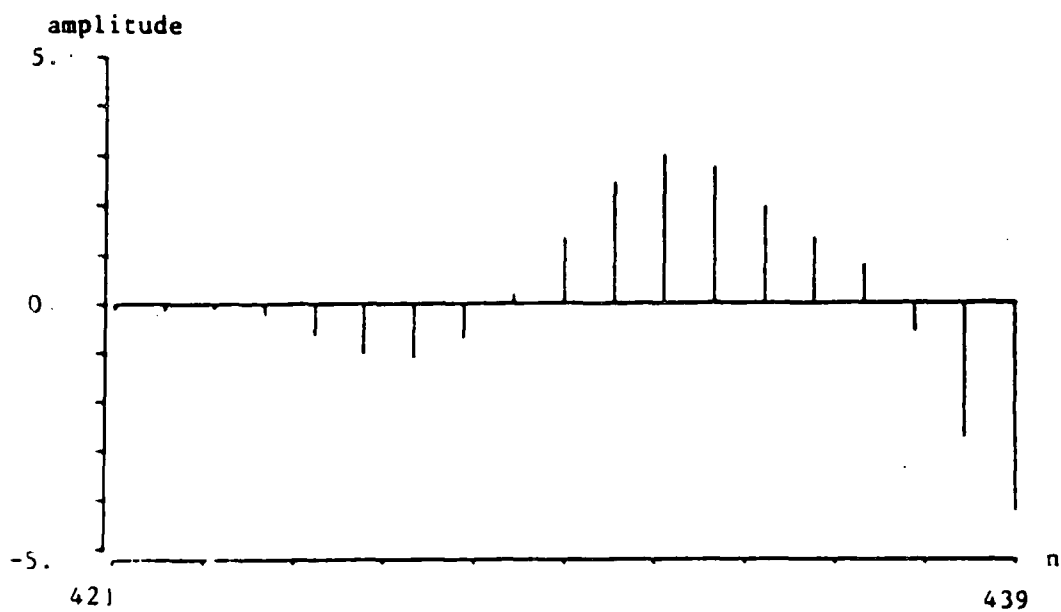
WHERE  $w(n)_r$  AND  $w(n)_d$  IS THE BACKGROUND NOISE AND/OR SIGNAL

BURST IS ESTIMATED AND SUBTRACTED OUT USING THE TECHNIQUE OF DATA ADAPTIVE SIGNAL ESTIMATION BY SINGULAR VALUE DECOMPOSITION OF A DATA MATRIX

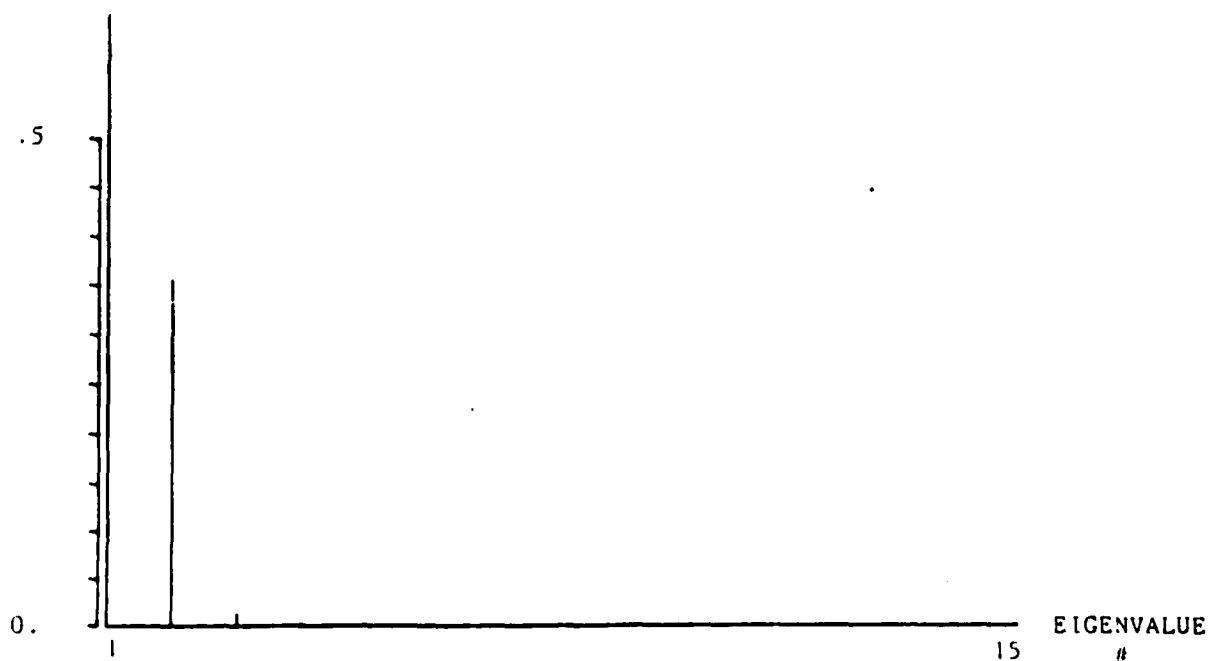
THE RANK OF THE DATA MATRIX IS EQUAL TO THE NUMBER OF EXPONENTIALS PRESENT

Fig. 9) MODELING IMPULSIVE BURSTS AS EXPONENTIALS

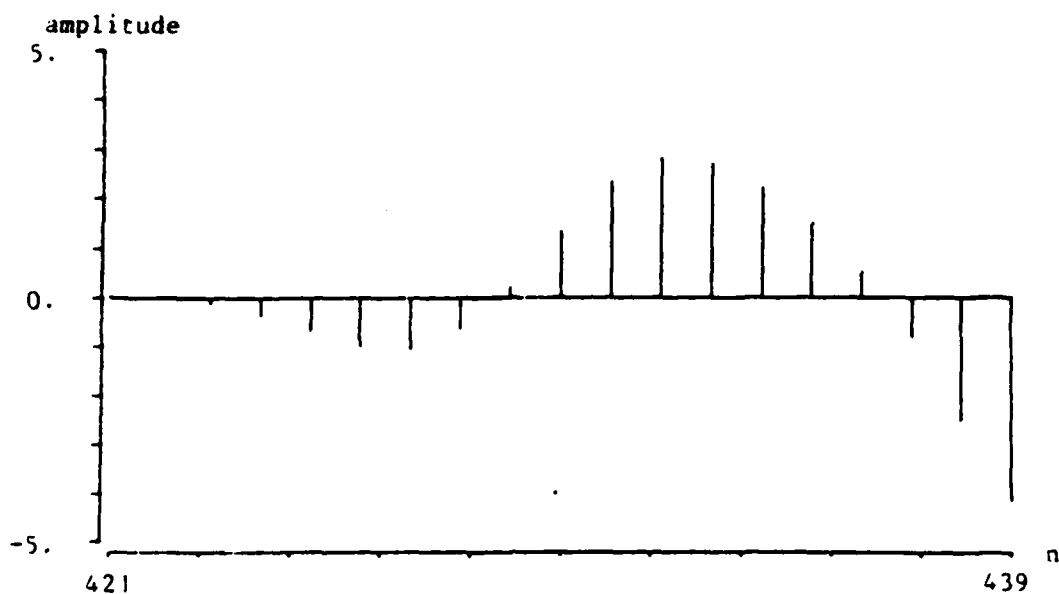
Fig. 10) MODELING AN IMPULSIVE BURST AS EXPONENTIAL



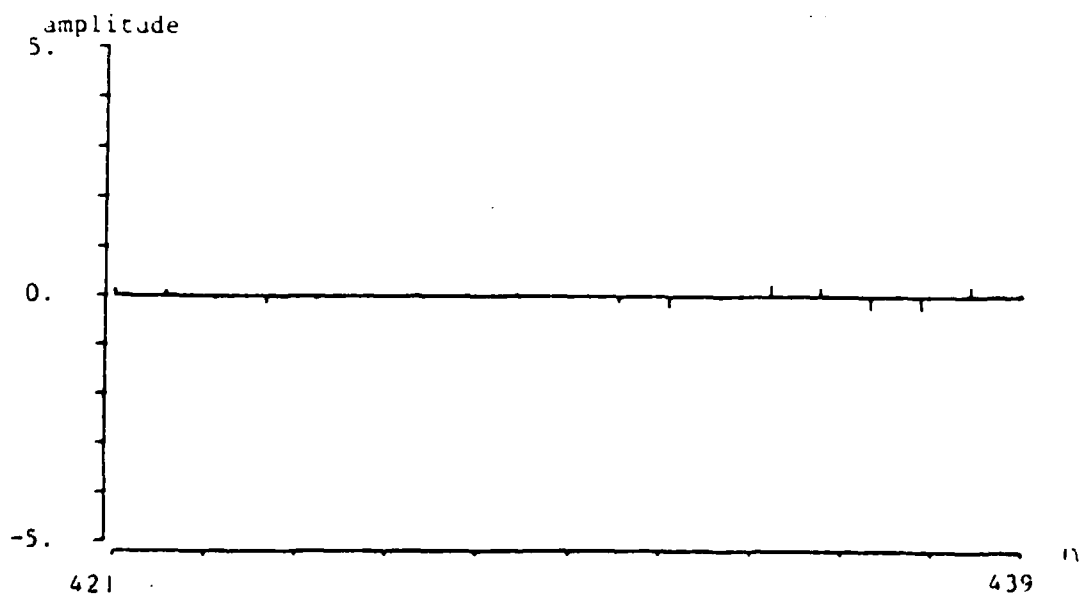
a) RECORD #64.B RISING SECTION OF BURST



b) NORMALIZED EIGENVALUES OF ESTIMATED CORRELATION MATRIX (normalized in respect to the trace)  
FORWARD LINEAR PREDICTION:  $N=20$ ,  $L=15$   
RISING SECTION OF BURST RECORD #64.B  
SAMPLES 421-439

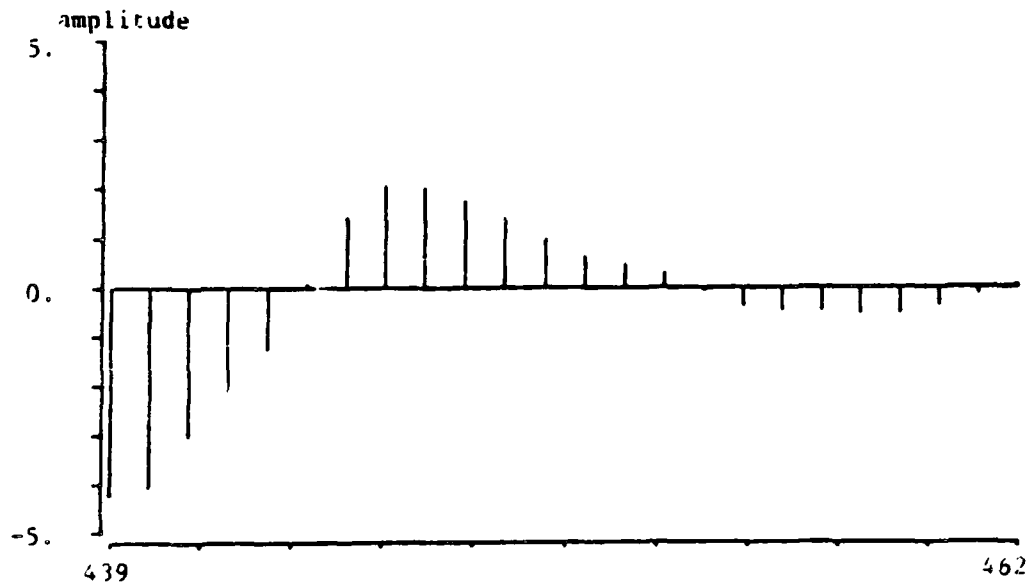


c) RANK 2 RECONSTRUCTION  
 FOWARD LINEAR PREDICTION: N=20, L=15  
 RISING SECTION OF BURST RECORD #64.B

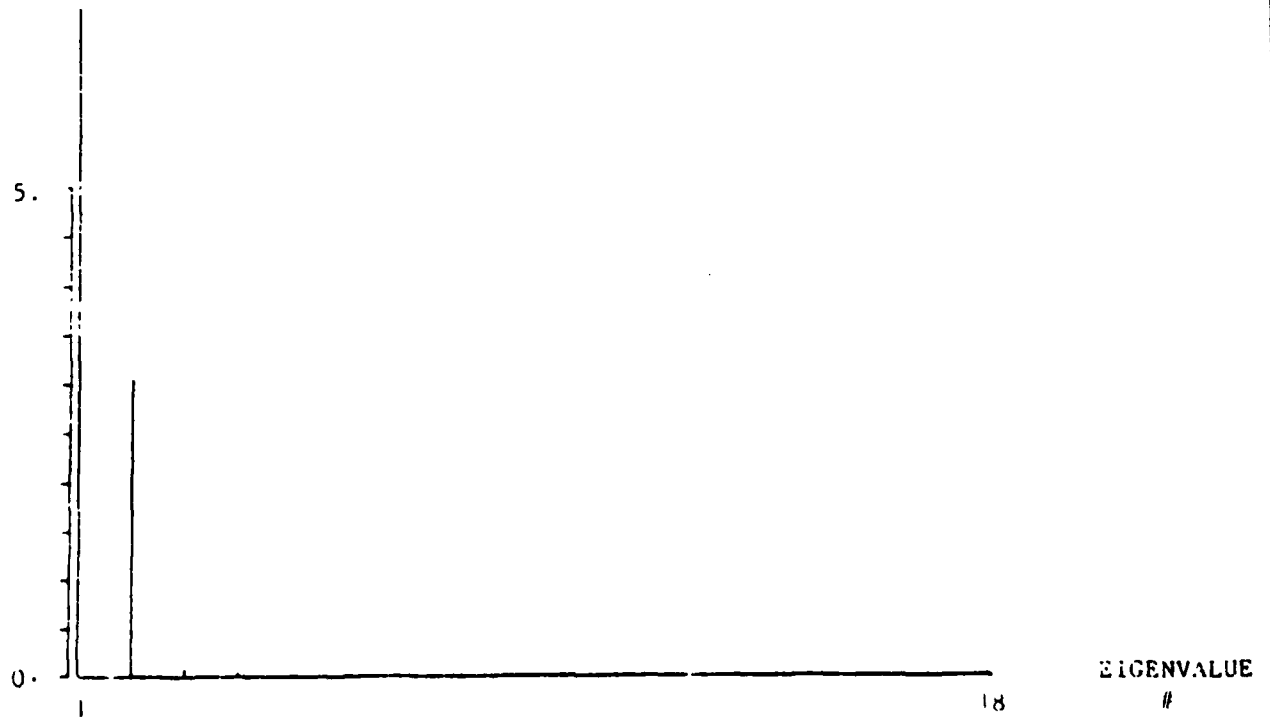


d) RANK 2 RECONSTRUCTION ERROR  
 FOWARD LINEAR PREDICTION: N=20, L=15  
 RISING SECTION OF BURST RECORD #64.B

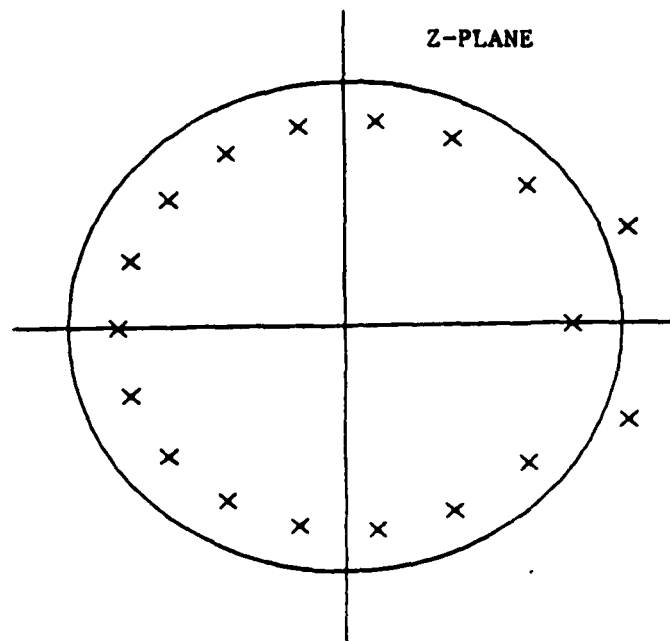




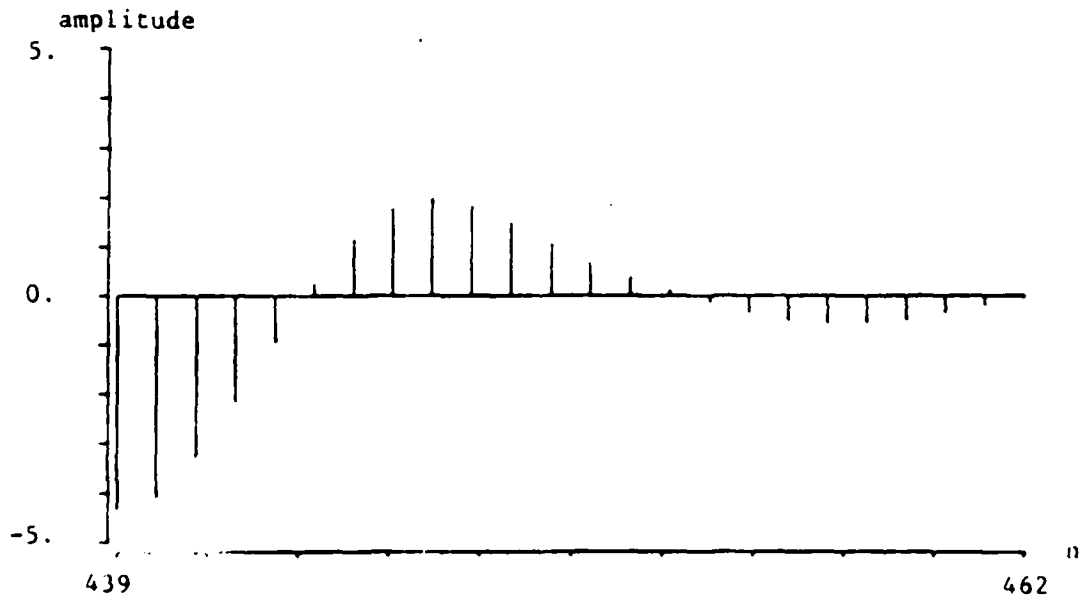
e) RECORD #64.B DECAYING SECTION OF BURST



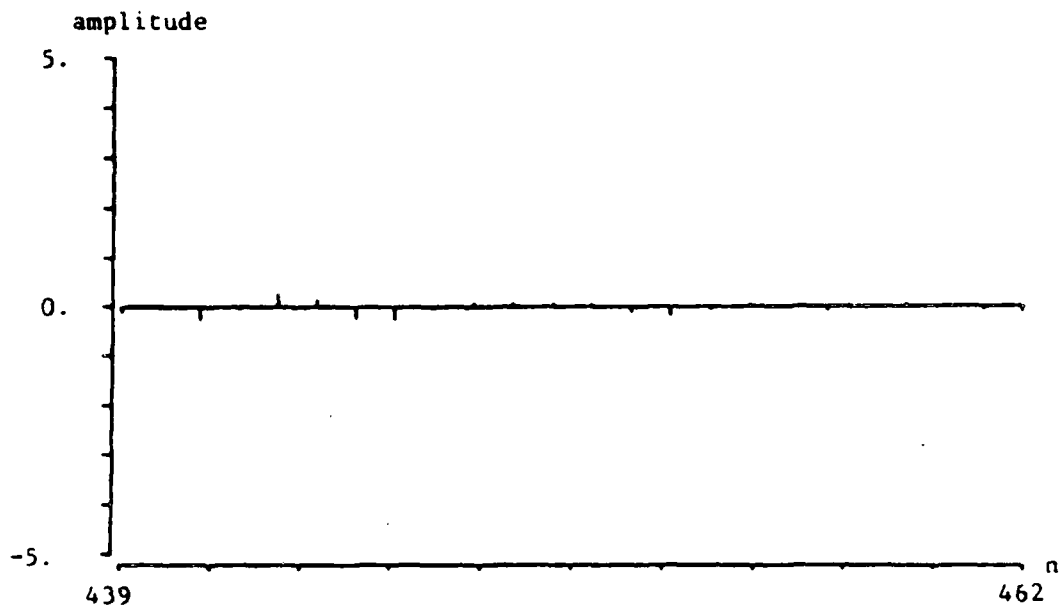
f) NORMALIZED EIGENVALUES OF ESTIMATED CORRELATION MATRIX (normalized in respect to the trace)  
 BACKWARD LINEAR PREDICTION:  $N=25, l=18$   
 DECAYING SECTION OF BURST RECORD #64.B  
 SAMPLES 439-462



- g) PREDICTION ERROR FILTER ZEROS ABOUT UNIT CIRCLE  
 BACKWARD LINEAR PREDICTION:  $N=25, L=18$  RANK 2 SOLUTION  
 DECAYING SECTION OF BURST SAMPLES 439-462 RECORD #64.B

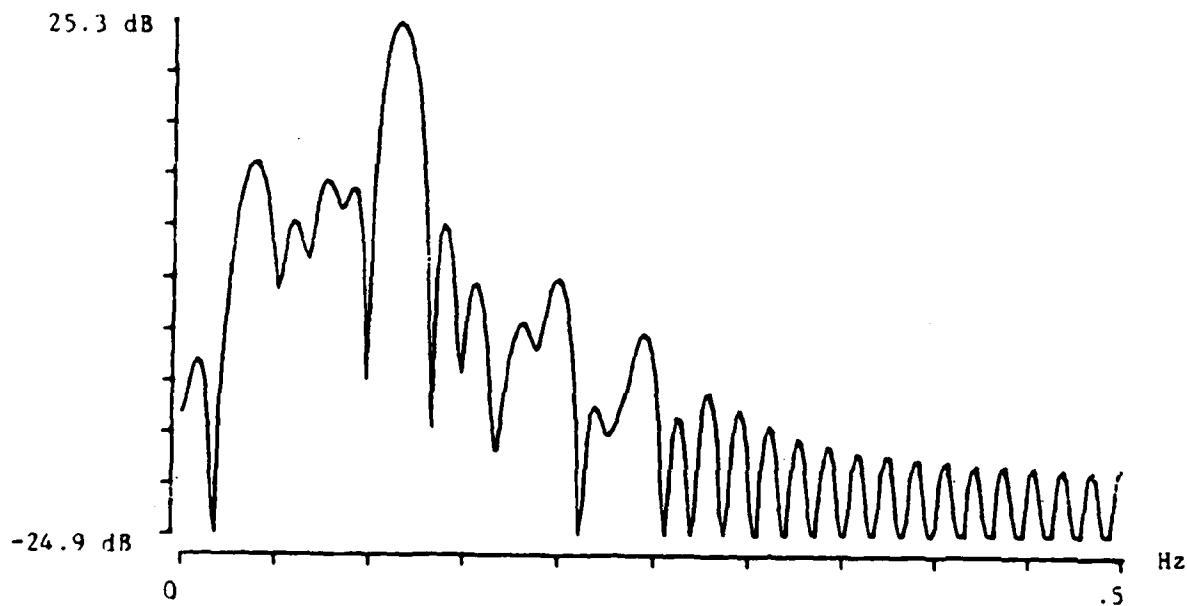


- h) RANK 2 RECONSTRUCTION  
 BACKWARD LINEAR PREDICTION:  $N=25, L=18$   
 DECAYING SECTION OF BURST RECORD #64.B

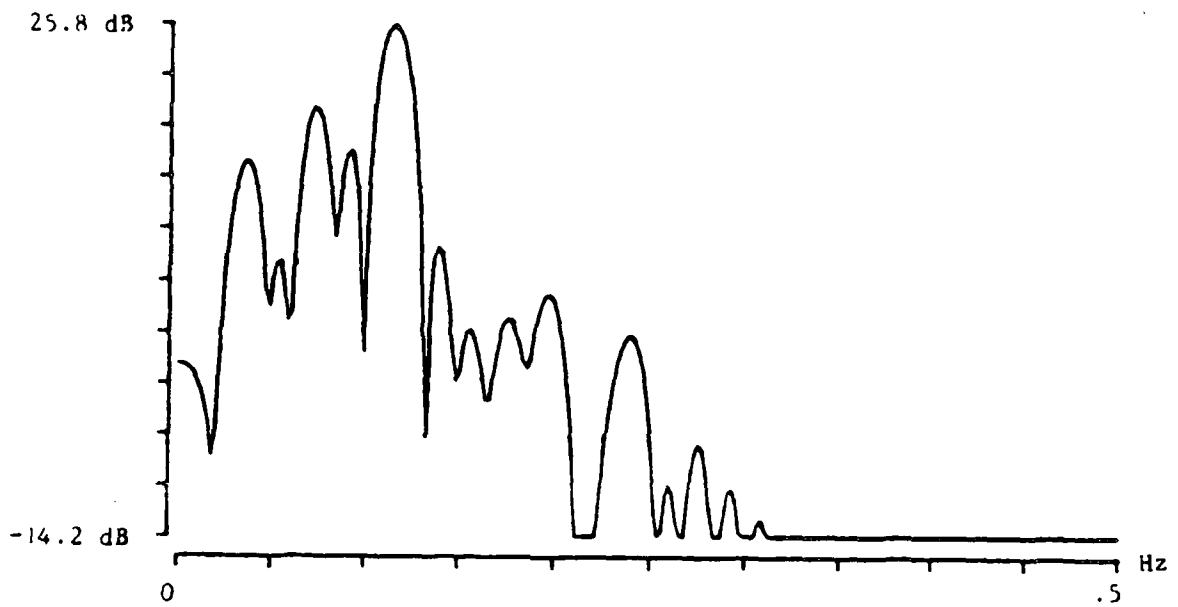


- i) RANK 2 RECONSTRUCTION ERROR  
BACKWARD LINEAR PREDICTION:  $N=25$ ,  $L=18$   
DECAYING SECTION OF BURST RECORD #64.B

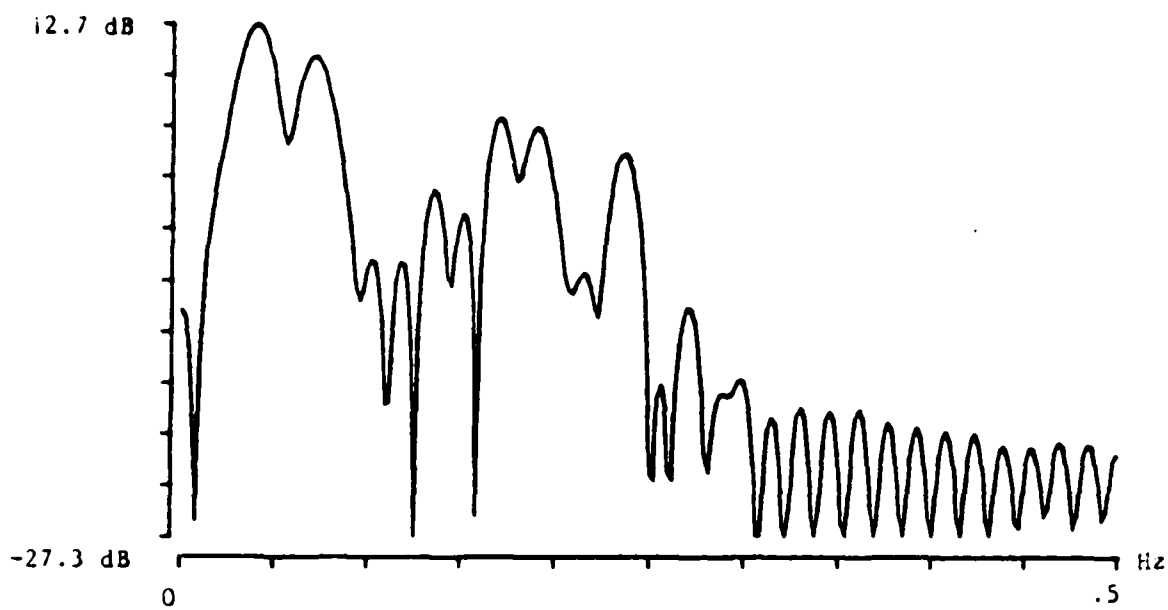
Fig. 11) DETECTING A WEAK SIGNAL IN THE PRESENCE OF TONAL INTERFERENCE BY FIRST ESTIMATING AND REMOVING THE INTERFERENCE AND THEN FOURIER TRANSFORMING THE RESIDUAL . 64 SAMPLE TONAL DATA VECTOR FROM RECORD #1756 SAMPLES 1-64 . THE SIGNAL USED IS  
 $s(n) = .274 \cos(2\pi(.07) (n-1)) \quad n=1,2,\dots,64$



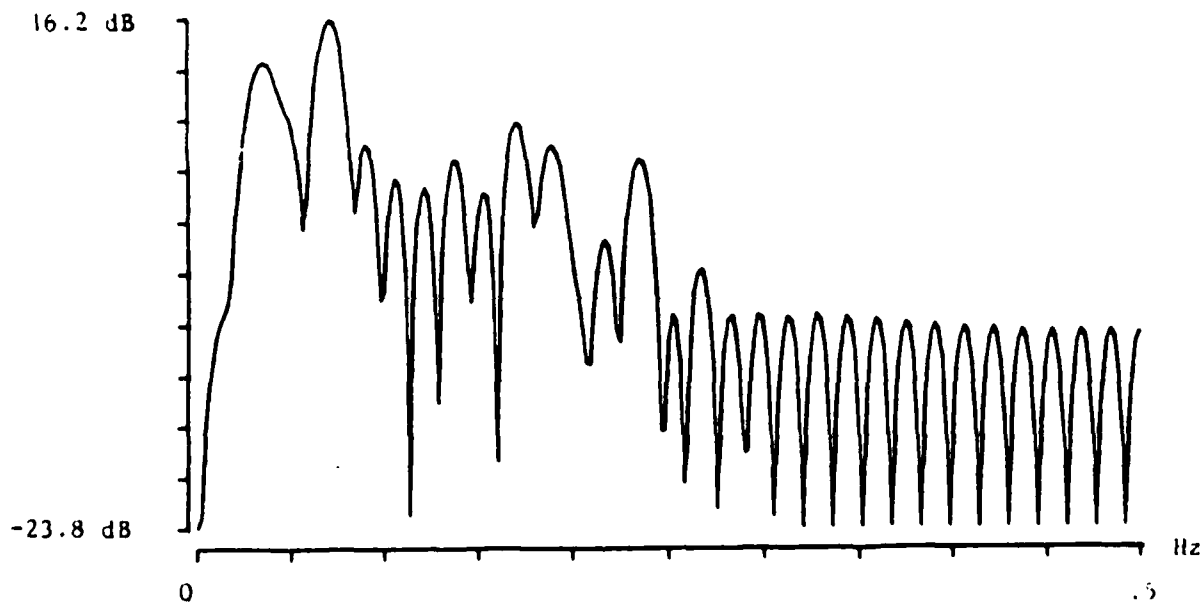
a) FOURIER TRANSFORM OF UNPROCESSED DATA VECTOR



b) FOURIER TRANSFORM OF UNPROCESSED DATA VECTOR  
WITH WEAK SIGNAL INJECTED



c) FOURIER TRANSFORM OF RESIDUAL  
WEAK SIGNAL NOT INJECTED IN DATA VECTOR

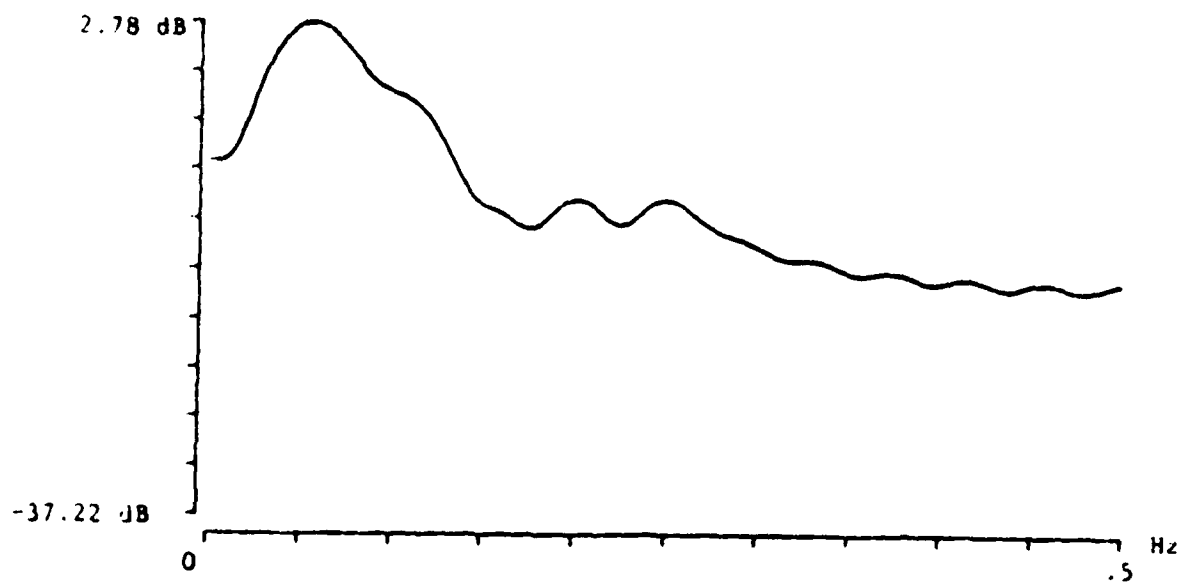


d) FOURIER TRANSFORM OF RESIDUAL  
WEAK SIGNAL INJECTED IN DATA VECTOR

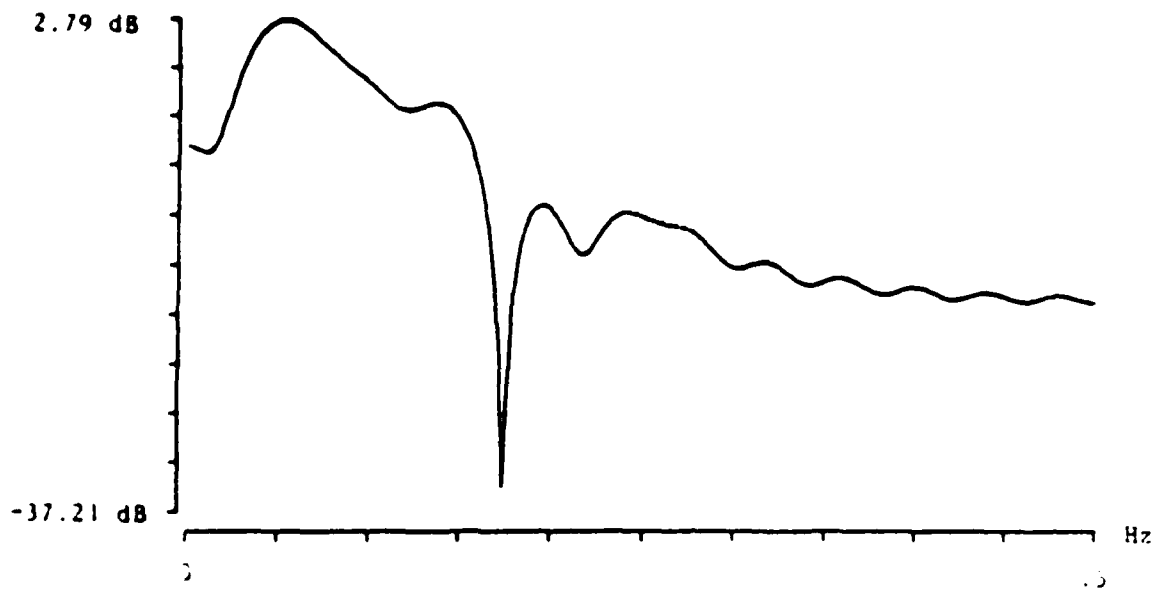
Fig. 12) DETECTING A WEAK SIGNAL IN THE PRESENCE OF AN IMPULSIVE BURST BY FIRST ESTIMATING AND REMOVING BURST AND THEN FOURIER TRANSFORMING THE RESIDUAL.

DATA VECTOR USED IS THE DECAYING SECTION OF BURST FROM RECORD #64.B SAMPLES 439-462

THE WEAK SIGNAL THAT IS INJECTED IS  
 $s(n) = .5 \cos(2\pi(.15)(n-439))$  for  $n=439, 440, \dots, 462$

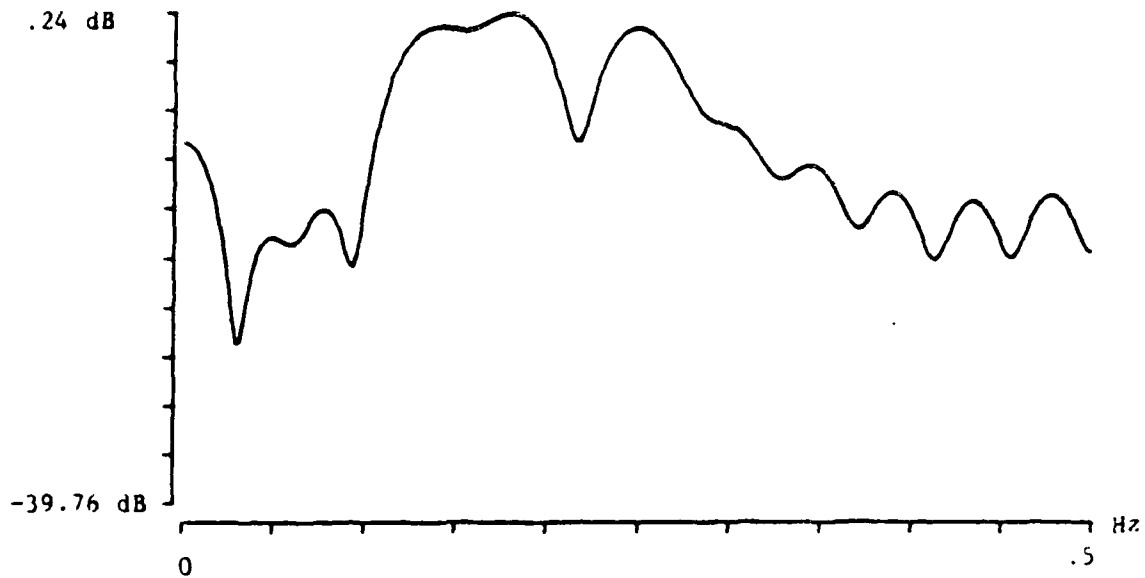


a) FOURIER TRANSFORM OF UNPROCESSED DATA VECTOR

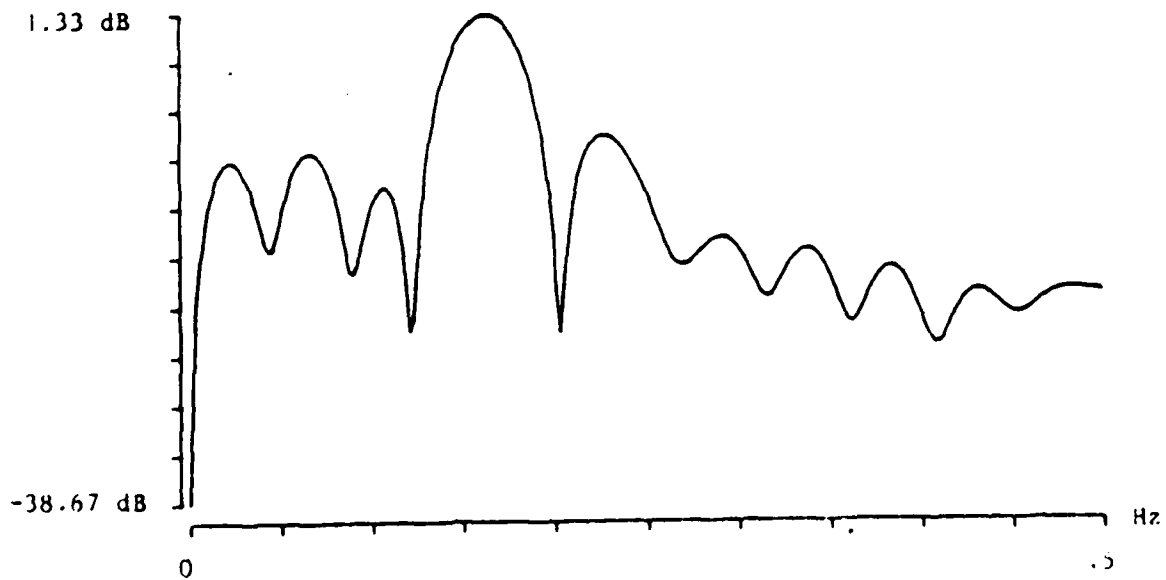


b) FOURIER TRANSFORM OF UNPROCESSED DATA VECTOR  
WITH WEAK SIGNAL INJECTED



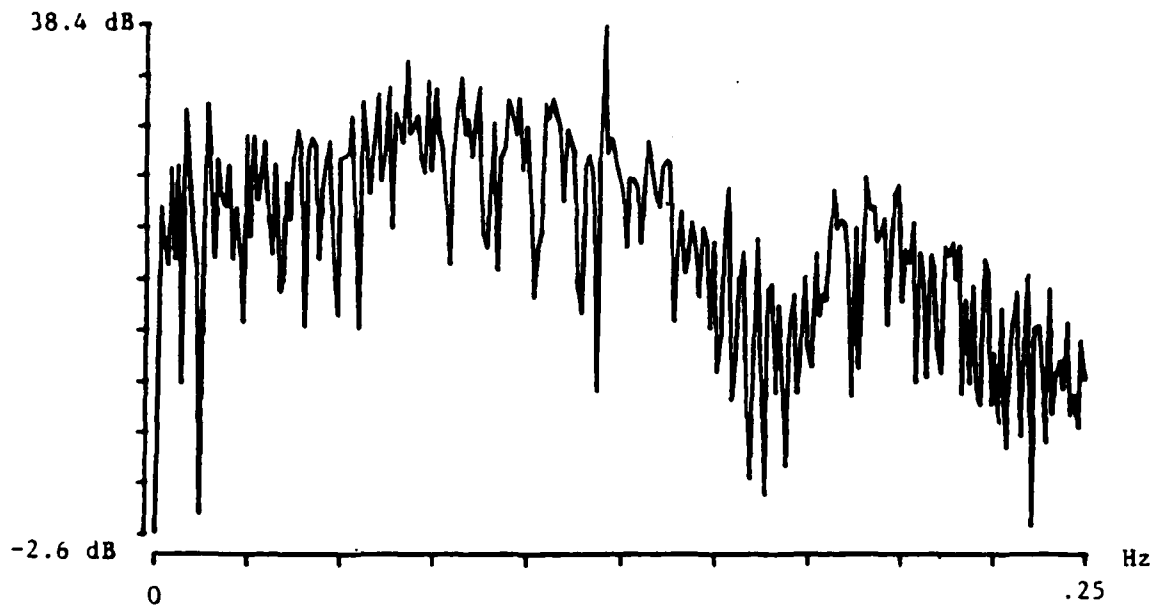


c) FOURIER TRANSFORM OF RESIDUAL  
 NO WEAK SIGNAL INJECTED  
 BACKWARD PREDICTOR DATA MATRIX: N=25,L=18

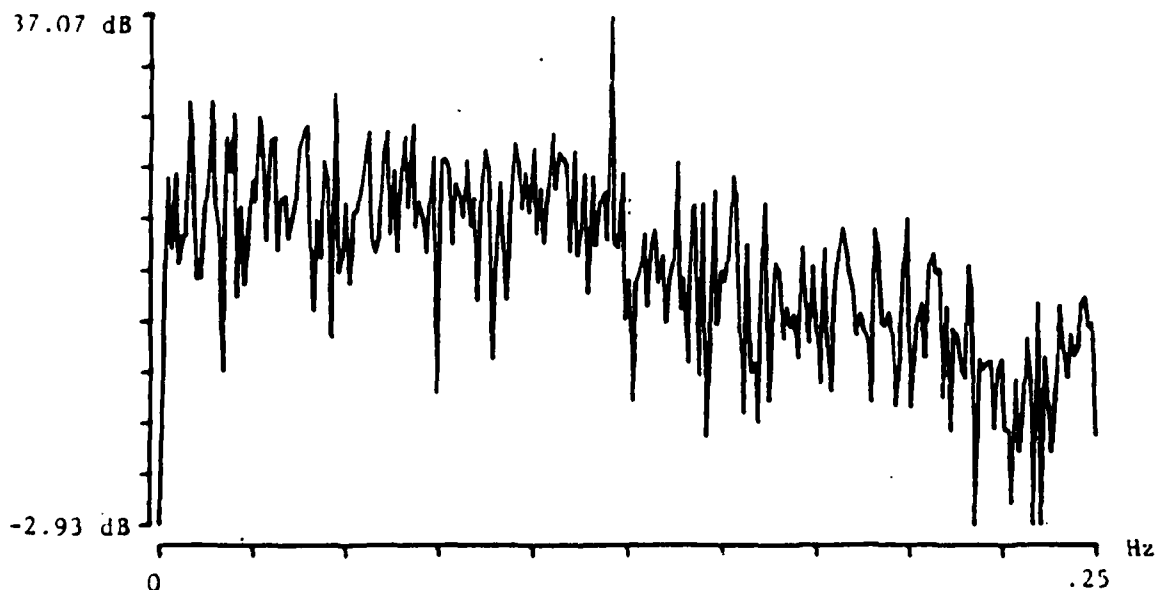


d) FOURIER TRANSFORM OF RESIDUAL  
 WEAK SIGNAL INJECTED  
 BACKWARD PREDICTOR DATA MATRIX: N=25,L=18

Fig. 13) DETECTING A WEAK SIGNAL IN THE PRESENCE OF AN IMPULSIVE BURST BY FIRST REMOVING THE BURST USING ADAPTIVE DIFFERENTIAL QUANTIZATION AND THEN FOURIER TRANSFORMING THE CLEANED DATA. THE SIGNAL IS  $s(n) = .149\cos(2\pi(.12)(n-1))$  for  $n=1,2,\dots,1024$



a) FOURIER TRANSFORM OF UNPROCESSED RECORD #2066 WITH WEAK SIGNAL INJECTED



b) FOURIER TRANSFORM OF RECORD #2066 WITH WEAK SIGNAL INJECTED PROCESSED BY ADAPTIVE DIFFERENTIAL QUANTIZATION.  $T_1 = 3.76$ ,  $T_2 = 2.51$ , and  $N = 101$

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