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We derive an approximate probability distribution for the SNR (signal-to-noise ratio) of an improved adaptive detector in near rank-1 gaussian noise where the filter weights are computed using the principal eigenvectors of the estimated noise covariance matrix. The noise consists of two components, a strong rank-1-covariance interference component plus white noise. Computer simulation is used to verify the approximating SNR distribution and show that it

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We use this distribution to show the improvement possible when using filter weights based on just the principal eigenvectors rather than the full inverse of the estimated sample covariance matrix when the noise covariance is near rank 1. For example we compare the expected value of SNR for our improved method and the conventional adaptive detector based on the inverse of the estimated covariance matrix. We find that for twenty-five samples of data and an INR value of 10 dB, the expected value of SNR for our new method is better than the comparison method. Other statistics can also be obtained from the probability density.

The main advantage of our principal-component method of adaptive detection, in comparison with the method based on the inverse of an estimated covariance matrix, is that much less data is required to produce a given, needed level of SNR with high probability.

Final Report—Part I

ON THE PROBABILITY DENSITY OF SIGNAL-TO-NOISE RATIO IN AN IMPROVED ADAPTIVE DETECTOR

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Abstract

We derive an approximate probability distribution for the SNR (signal-to-noise ratio) of an improved adaptive detector in near rank-1 gaussian noise where the filter weights are computed using the principal eigenvectors of the estimated noise covariance matrix . The noise consists of two components, a strong rank-1-covariance interference component plus white noise. Computer simulation is used to verify the approximating SNR distribution and show that it is accurate even for small sample size and low interference-to-noise ratios (INR).

We use this distribution to show the improvement possible when using filter weights based on just the principal eigenvectors rather than the full inverse of the estimated sample covariance matrix when the noise covariance is near rank 1 . For example we compare the expected value of SNR for our improved method and the conventional adaptive detector based on the inverse of the estimated covariance matrix. We find that for twenty-five samples of data and an INR value of 10 dB , the expected value of SNR for our new method is better than the comparison method. Other statistics can also be obtained from the probability density.

The main advantage of our principal-component method of adaptive

detection, in comparison with the method based on the inverse of an estimated covariance matrix, is that much less data is required to produce a given, needed level of SNR with high probability.

CHAPTER ONE

(A) Introduction

(A1) Detection with Known Covariance

Let us consider the problem of detecting a signal in gaussian noise where both signal and noise are assumed to be real valued, the two hypotheses are

$$H\phi: \quad x = n \qquad \text{(noise only)} \tag{1.00}$$

$$H_1: \quad x = n + s \quad \text{(signal present)} \tag{1.01}$$

where x is the vector of the observed data; n is a zero-mean gaussian random vector with covariance matrix $E(nn^T) = N$; and s is the signal to be detected. The dimensions of x, n, and s are $\rho x1$.

Under these conditions it can be shown that a likelihood ratio test statistic [4] is

$$z = x^T N^{-1} s \tag{1.02}$$

Equivalently, the test statistic z can be considered as the inner product of x and weight vector w.

$$z = x^T w \tag{1.03}$$

where $w = N^{-1}s$. (1.04)

Brooks and Reed [4] have shown that selecting a weight vector according to formula (1.04) is the same as maximizing the signal-to-noise ratio (SNR). We can express this as maximizing the quadratic ratio

$$\text{SNR} = \frac{w^T s s^T w}{w^T N w} \tag{1.05}$$

The weight vector that maximizes this ratio (see [1,2,4]) is

$$w = N^{-1} s \tag{1.06}$$

The maximum SNR is

$$SNR_{max} = s^T N^{-1} s \tag{1.07}$$

(A2) Standard Adaptive Detection with Unknown Covariance

In practice we do not know the noise covariance matrix and so it is usually estimated from the data. The maximum-likelihood estimate of the noise covariance matrix [7] is

$$\tilde{N} = \frac{1}{K} \sum_{j=1}^{K} n_j n_j^T \tag{1.08}$$

where the observed noise vectors, n_j ,are $\rho x1$ vectors which are mutually independent and gaussian distributed with zero mean and covariance matrix N. Following standard methods one can obtain an estimate of the weight vector by inverting the sample covariance matrix [1,2,3] as follows:

$$\tilde{w} = \tilde{N}^{-1} s \tag{1.09}$$

To measure the effectiveness of this estimate of w [1], we can use a normalized SNR which is formed by (a) replacing w by $\tilde{w}$ in formula (1.05) and (b) dividing by the maximum SNR value of formula (1.07) . The resulting normalized SNR is given by the formula

$$\tilde{SNR} = \frac{\tilde{w}^T s s^T \tilde{w}}{(\tilde{w}^T N \tilde{w})(s^T N^{-1} s)} \tag{1.10}$$

This ratio is always between zero and one

$$0 \le \tilde{SNR} \le 1$$

It is a random variable because the estimated weight vector $\tilde{w}$ is a function of the observed data. Reed, Mallet,Brennan [1] have derived the distribution of the $\tilde{SNR}$ for the above weight vector estimate based on the inversion of the sample covariance matrix. The distribution is Beta with expected value

$$E(\tilde{SNR}) = \frac{K + 2 - \rho}{K + 1} \qquad (1.11)$$

where K is the number of vectors used to estimate the covariance matrix and ρ is the matrix order .

From the expected value of $\tilde{SNR}$ formula (1.11), it is roughly seen that if the sample size K is small with respect to the matrix order, then the estimate of w is likely to be poor [3], since $E(\tilde{SNR})$ is small.

(A3) Improved Adaptive Detection for Unknown, but Approximately-Low-Rank Covariance

If N is approximately low rank, it can be shown theoretically through eigenvalue and eigenvector perturbation analysis (Wilkinson [9]) and experimentally [10] that the principal eigenvalues and eigenvectors of $\tilde{N}$ are relatively stable, but the small, near zero eigenvalues and their corresponding eigenvectors fluctuate widely with perturbation.

These results suggest that it should be possible to increase the probability of obtaining a high value of SNR by using estimates of w based on the principal eigenvectors of $\tilde{N}$. Let us look at w by decomposing it in terms of the eigenvectors of the underlying true covariance matrix, N . The eigen decomposition of N is $N = V \Lambda V^T$ where V is the matrix of eigenvectors and Λ is the diagonal matrix of eigenvalues. The k'th eigenvector of N is denoted by v_k .

Then the desired weight vector of (1.04) can be written as

$$w = N^{-1}s = \frac{1}{\lambda_1}(s^T v_1)v_1 + \frac{1}{\lambda_2}(s^T v_2)v_2 + . . + \frac{1}{\lambda_\rho}(s^T v_\rho)v_\rho \quad (1.12)$$

This formula shows the origins of the difficulty in using the estimated weight vector of (1.09). The right-hand-side eigenvectors and eigenvalues of (1.12) would then be replaced by the corresponding estimated values. The fluctuations in the small eigenvalues can then cause large errors, because their reciprocals are used in the computation of the weight vector. Next assume that $\lambda_2 = \lambda_3 = . . ., = \lambda\rho$ and consider the case in which λ_1 is much larger than the other eigenvalues. Let us consider the limiting weight vector as $\lambda_1 \to \infty$.

$$\lim_{\lambda_1 \to \infty} w = \frac{1}{\lambda_2}\left[(s^T v_2)v_2 + (s^T v_3)v_3 + . . . + (s^T v_\rho)v_\rho\right] \quad (1.13)$$

Neglecting the scale factor $1/\lambda_2$ which does not affect SNR, we can express w as

$$\lim_{\lambda_1 \to \infty} w = s - (s^T v_1)v_1 \quad (1.14)$$

This form of the weight vector can be intrepreted as a null-steerer, because it eliminates any component which is proportional to a specific vector, in this case v_1 . Liu and Nolte [5] have shown that when the noise covariance matrix N is near rank 1 and the component of the signal lying in the direction of v_1 is not strong, then the performance of the weight vector of (1.14) is about the same as the optimum weight vector of (1.04). For the case of unknown, but approximately low-rank covariance, motivated by the above considerations, we propose the use of the following weight vector:

$$\tilde{w} = s - (s^T\tilde{v}_1)\tilde{v}_1 \tag{1.15}$$

where $\tilde{v}_1$ is the principal eigenvector of $\tilde{N}$. In previous work we proposed this weight vector for adaptive detection and evaluated its performance through simulation [15] . Here we study the performance analytically in order to explain the results of the simulation.

Since this estimate is based on the principal eigenvector of $\tilde{N}$, and the principal eigenvectors are relatively unaffected by perturbation [10], it should be superior to the conventional estimator based on the inversion of the sample covariance matrix.

Using the null steerer form of the weight vector formula (1.15) and an approximation for the principal eigenvector (of the estimated covariance matrix $\tilde{N}$) obtained through one iteration of the power method, we will derive an approximate expression for the distribution of $S\tilde{N}R$, formula (1.10) under the conditions of near rank-1 noise and large sample size. Below is a brief outline of the steps we will take;

(1) Transform $S\tilde{N}R$ formula (1.10) into new coordinates based on the eigenvectors of N, that is

$$S\tilde{N}R = \frac{\sum_{j=1}^{\rho} \sum_{k=1}^{\rho} d_j \; d_k \; c_j \; c_k}{\sum_{m=1}^{\rho} \frac{c_m^2}{\lambda_m} \sum_{n=1}^{\rho} \lambda_n \; d_n^2} \tag{1.16}$$

with $c_k = v_k^* \, s$ (1.17)

$$d_k = v_k^T\tilde{w} = v_k^T \left(s - (s^T\tilde{v}_1)\tilde{v}_1 \right) \tag{1.18}$$

and where λ_m and v_k are the eigenvalues and eigenvectors of N, respectively.

(2) Obtain an approximation for the principal eigenvector of $\tilde{N}$, $\tilde{v}_1$, by taking one iteration of the power method [11] using v_1 as the starting vector.

$$\tilde{v}_1 \approx \bar{v}_1 = \tilde{N}v_1 \tag{1.19}$$

where $\bar{v}_1$ denotes our approximation to $\tilde{v}_1$.

(3) Expand $\bar{v}_1$ using the eigenvectors of N as a basis.

$$x = V^T\bar{v}_1 \tag{1.20}$$

and

$$\bar{v}_1 = Vx \tag{1.21}$$

Asymptotically, the expansion scale factors in vector x are gaussian as the covariance sample size tends toward infinity.

(4) Using the expansion (1.21) of $\bar{v}_1$ in place of $\tilde{v}_1$ in formula (1.18), substituting (1.18) in (1.16) and approximating the distribution of x by a gaussian distribution, then as the noise tends toward rank 1, $S\tilde{N}R$ formula (1.16) reduces to a ratio of quadratics that are a function of one random variable.

$$S\tilde{N}R \approx \frac{\alpha_1 + \alpha_2\,\beta + \alpha_3\,\beta^2}{\alpha_4 + \alpha_5\,\beta + \alpha_6\,\beta^2} \tag{1.23}$$

where the α_k, k=1,2..6 are constants and β is a ratio of two asymptoticaly gaussian and uncorrelated random variables.

(5) Finally using formula (1.23), the distribution of β (the distribution of ratios of gaussians [6] is available in closed form),

and standard univariate random variable transformation theorems, the approximate distribution of $\widetilde{SNR}$ can be readily obtained. The paper will be presented in three main parts; (1) Reduction of $\widetilde{SNR}$ assuming near rank-1 noise and large sample size, (2) Finding the approximate probability density of $\widetilde{SNR}$ using the reduced form, and finally, (3) experimental results .

CHAPTER TWO

I) Reduction of $\widetilde{SNR}$ assuming near rank 1 noise and large sample size.

Coordinate Transformation of $\widetilde{SNR}$

We start the derivation by re-expressing the signal and weight vectors in terms of the eigenvectors of the N, recalling that the eigenvectors of N form a basis. This coordinate transformation makes the problem more tractable analytically. Performing the coordinate transformation we obtain

$$s = \sum_{k=1}^{\rho} c_k \, v_k \tag{2.00}$$

$$\tilde{w} = \sum_{k=1}^{\rho} d_k \, v_k \tag{2.01}$$

where the v_k's are the eigenvectors of N. Now substitute formulas (2.00) and (2.01) and the eigen-decomposition of N into $\widetilde{SNR}$, formula (1.10).

$$\widetilde{SNR} = \frac{\displaystyle\sum_{j=1}^{\rho} \sum_{k=1}^{\rho} d_j \, d_k \, c_j \, c_k}{\displaystyle\sum_{m=1}^{\rho} \frac{c_m^2}{\lambda_m} \sum_{n=1}^{\rho} \lambda_n \, d_n^2} \tag{2.02}$$

where λ_j are the eigenvalues of N. The d_j's are random variables that are a function of $\tilde{w}$. So far no approximations have been used. We simply have a different expression for $\widetilde{SNR}$.

The quadratic forms in formula (2.02) that involve the random scale factors d_j can be thought of as linear combinations of the products $d_j d_k$. With this in mind, we will proceed by first determining the approximate form or distribution of the products $d_j d_k$ under the asymptotic conditions as $\lambda_2 \rightarrow 0$ and $K \rightarrow \infty$ (near rank-1 covariance and large sample size) . If we refer back to formula (1.18), it can be seen that the sole contribution to the random component of random variable d_j is from $\tilde{v}_1$, the principal eigenvector of the estimated sample covariance matrix. Therefore, the next step is to determine the approximate distribution (as $\lambda \rightarrow 0$ and $K \rightarrow \infty$) of $\tilde{v}_1$. From this point onward we assume that the noise is gaussian and strongly rank one, that is, the noise covariance matrix is approximatly rank 1. To simplify the analysis, the principal eigenvalue of N, λ_1 is fixed at 1 without loss in generality. Also, assume that the remaining eigenvalues are much smaller than 1 and equal.

$$\lambda_1 = 1 >> \lambda_2 = \lambda_3 = . . . = \lambda_\rho \tag{2.03}$$

The form of the weight vector that we will be using is the null steerer (1.15) .

$$\tilde{w} = s - (s^T \tilde{v}_1)\tilde{v}_1 \tag{2.04}$$

<u>Approximating the Principal Eigenvector of $\tilde{N}$</u>

To estimate $\tilde{v}_1$, the principal eigenvector of $\tilde{N}$, we use a three-step procedure. First take one iteration of the power method [11] using v_1 (recalling that v_1 is the principal eigenvector of N) as the starting vector. We then represent the resulting vector, $\tilde{N}v_1$, in terms of the principal eigenvectors of the true covariance matrix N as follows

$$\tilde{N}\, v_1 = V\, x \tag{2.05}$$

in which the columns of V are the corresponding eigenvectors of N. Finally, we form our estimate of $\tilde{v}_1$, which we denote by $\bar{v}_1$, by normalizing the above vector to unit length

$$\bar{v}_1 = [\, 1 \,/\, (x^T x)^{1/2}\,]\, V\, x \tag{2.06}$$

where x is a ρx1 vector with elements $\{x_k\}$ containing the expansion scale factors.

We have shown in a previous paper [10] that if the true covariance matrix of the noise, N , is approximatly rank 1 , then even for small sample sizes of 16 or 32 observations, $\tilde{v}_1$ lies in nearly the same direction as v_1 (see Fig. 1). The rate of convergence of the power method [11] is proportional to the magnitude of the ratio of the principal eigenvalue to the second eigenvalue and since the starting vector is the principal eigenvector, v_1 , of the underlying population covariance, it follows that in the near rank 1 case convergence should be rapid , implying that $\bar{v}_1$ should be an accurate estimate of $\tilde{v}_1$.

In appendix A we show that the scale factors (elements of x) are asymptotically multivariate gaussian and independent as follows:

$$x_1 \sim N(\, 1\, ,\, 2/K\,) \tag{2.07}$$

$$x_k \sim N(\, 0\, ,\, \lambda_2/K\,) \quad \text{for } k=2,3\ldots\rho \tag{2.08}$$

Determining the Weight Vector Expansion Coefficents d_k

Let us now find the expansion coefficients d_k of formula (2.01) . To obtain the d_k coefficents, pre-multiply (1.15) by the k'th eigenvector of N.

$$d_k = v_k^T \tilde{w} = v_k^T s - (s^T \tilde{v}_1)(v_k^T \tilde{v}_1) \tag{2.09}$$

Note that $c_k = v_k^* s$. Then substitute $\bar{v}_1$ (2.06) in place of $\tilde{v}_1$ in the weight vector expression (2.09) and expand

$$d_k = c_k - \left[\frac{\left[\sum_{j=1}^{\rho} c_j x_j \right]}{\left[\sum_{j=1}^{\rho} x_j^2 \right]^{1/2}} \right] \left[\frac{x_k}{\left[\sum_{j=1}^{\rho} x_j^2 \right]^{1/2}} \right] \tag{2.10}$$

$$d_k \approx c_k - \frac{c_1 x_1 x_k}{x_1^2 + \varepsilon} - \frac{x_k y}{x_1^2 + \varepsilon} \tag{2.11}$$

where

$$y = \sum_{j=2}^{\rho} c_j x_j \tag{2.12}$$

$$\varepsilon = \sum_{j=1}^{\rho} x_j^2 \tag{2.13}$$

Before we form the products $d_j d_k$, it is noted that the normalization terms in d_k (2.11) are simplified to a function of x_1 as $\lambda_2 \to 0$

since the variance of the x_j's (for $j=2,3\ldots\rho$) are proportional to λ_2. This corresponds to the near rank 1 case .

$$\lim_{\lambda_2 \to 0} \frac{1}{x_1^2 + \varepsilon} = \frac{1}{x_1^2} \tag{2.14}$$

The right-hand side of (2.14) is a good approximation of the normalization term for sufficently small λ_2 . Replace the normalization terms in formula (2.11) by the approximation (2.14) .

$$d_k \approx c_k - \frac{c_1 x_k}{x_1} - \frac{x_k y}{x_1^2} \tag{2.15}$$

<u>Reduction of the Products $d_j d_k$ Under Near Rank-1 and Large Sample Condition</u>

Using the formula (2.15) and assuming that the error due to the simplification of the normalization term (2.14) is neglible, we next compute $d_j d_k$.

$$d_j\, d_k \approx c_j c_k - \frac{c_1 c_j x_k}{x_1} - \frac{c_1 c_k x_j}{x_1} - \frac{c_j x_k y}{x_1^2}$$
$$- \frac{c_k x_j y}{x_1^2} + \frac{c_1^2 x_j x_k}{x_1^2} + \frac{2 c_1 x_j x_k y}{x_1^3} + \frac{x_j x_k y^2}{x_1^4} \tag{2.16}$$

We now determine the behavior of the products $d_j d_k$ (2.16) in the near rank-1 ($\lambda_2 \rightarrow 0$) and large sample case ($K \rightarrow \infty$). This can be done through calculation of the expected values and variances of the individual random terms in (2.16). From these calculations (see Appendix B) it can be seen that the variance and expected value of the i'th random term (for j,k≠1,1) in (2.16) is roughly proportional to the ratio $(\lambda_2/K)^{zi}$ where z_i is some integer number. This implies that as $\lambda_2 \rightarrow 0$ and $K \rightarrow \infty$, the term(s) (in formula (2.16)) having a variance proportional to the least power of λ_2/K will contribute the dominating variance to $d_j d_k$ (2.16). It follows that under these conditions, the random component of $d_j d_k$ should be accurately approximated by the dominating random term in (2.16) in respect to variance.

The approximate expressions for the products $d_j d_k$ are obtained using the above methodology (with the exception of d_1^2, which can be simplified algebraicly). We have

For j=1 , k=1

$$d_1^2 \approx \tilde{D}_{11} = \frac{y^2}{x_1^2} \tag{2.17}$$

for j=1, k≠1

$$d_1\, d_k \approx \tilde{D}_{1k} = \frac{-\, c_k\, y}{x_1} \tag{2.18}$$

Finally , for $j=2,3\ldots\rho$

$k=2,3\ldots\rho$

$$d_j\, d_k \approx \tilde{D}_{jk} = c_j\, c_k - c_1 \frac{\left[c_j\, x_k + c_k\, x_j \right]}{x_1} \tag{2.19}$$

where the approximating form of $d_j d_k$ is denoted as $\tilde{D}_{jk}$. It is assumed that $c_k \neq 0$ in the above formulas. For sufficently small λ_2 and reasonable values of K, the $\tilde{D}_{jk}$'s should be good approximations to the $d_j d_k$'s . We note that formulas (2.17) , (2.18) and (2.19) are functions of random variables which are ratios of gaussian random variables.

Simplification of $\tilde{SNR}$ with $\tilde{D}_{jk}$

Substituting $\tilde{D}_{jk}$ (formulas (2.17) (2.18) (2.19)) in place of $d_j d_k$ in formula (2.02) and through algebraic manipulation of the numerator and denominator terms, we can transform $\tilde{SNR}$ (2.02) to the desired form, formula (1.23)

$$\text{where} \quad \beta = \frac{\sum_{j=2}^{\rho} c_j\, x_j}{x_1} = \frac{y}{x_1} \tag{2.20}$$

Start by simplifying the numerator of $\tilde{SNR}$. From (2.02) the numerator (denoting the numerator as n) is

$$n = \sum_{j=1}^{\rho} \sum_{k=1}^{\rho} d_j d_k c_j c_k \tag{2.21}$$

Separate the numerator double summation in terms of $d_1{}^2$, d_{1k} (for $k=2,3\ldots\rho$) and $d_j d_k$ (for $j,k=2,3\ldots\rho$) .

$$n = c_1^2 \, d_1^2 + 2c_1 \sum_{k=2}^{\rho} c_k d_1 d_k + \sum_{n=2}^{\rho} \sum_{m=2}^{\rho} d_n d_m c_n c_m \tag{2.22}$$

Through (1) substitution of $\tilde{D}_{jk}$ (formulas (2.17) (2.18) (2.19)) in place of $d_j d_k$ in formula (2.22), (2) interchanging the order of summation in the last term of n (2.22) and factoring, and finally (3) the collection of equivalent terms , the numerator (2.22) can be written as

$$n \approx \left[\sum_{n=2}^{\rho} \sum_{m=2}^{\rho} c_n^2 c_m^2 \right] + \frac{c_1^2 \quad y^2}{x_1^2} - 4 \, c_1 \left[\sum_{k=2}^{\rho} c_k^2 \right] \frac{y}{x_1} \tag{2.23}$$

The numerator is now in the desired form. We simplify the denominator in a similar manner. From (2.02) the denominator (denoting the denominator as d) is

$$d = \left[\sum_{j=1}^{\rho} \frac{c_j^{\,2}}{\lambda_j} \right] \left[\sum_{k=1}^{\rho} \lambda_k d_k^2 \right] \tag{2.24}$$

As stated previously , we assume that

$\lambda_1 = 1$

and

$\lambda_2 = \lambda_3 = \lambda_4 = \ldots, = \lambda_p$

Now separate the denominator summation in terms of d_1^2 and d_j^2 (for $j=2,3...\rho$) and the λ_2 and $1/\lambda_2$ scale factors.

$$d = c_1^2 d_1^2 + \sum_{j=2}^{\rho} \sum_{k=2}^{\rho} c_k^2 d_j^2 + \frac{1}{\lambda_2} \sum_{k=2}^{\rho} c_k^2 d_1^2 \qquad (2.25)$$

$$+ \lambda_2 \sum_{j=2}^{\rho} c_1^2 d_j^2$$

Then as before, through (1) substitution of $\tilde{D}_{jk}$ (formulas (2.17) (2.18) (2.19)) in place of $d_j d_k$ in (2.25), (2) interchanging the order of summation in the second and fourth terms of (2.25) and factoring, and finally (3) the collection of equivalent terms, the denominator can be written as

$$d = \left[\sum_{k=2}^{\rho} c_k^2/\lambda_2 + c_1^2 \right] \frac{y^2}{x_1^2} + \left[\sum_{j=2}^{\rho} \sum_{k=2}^{\rho} c_k^2 c_j^2 \right] + \lambda_2 c_1^2 \left[\sum_{k=2}^{\rho} c_k^2 \right]$$

$$- 2 c_1 \left[\left[\sum_{k=2}^{\rho} c_k^2 \right] + \lambda_2 c_1^2 \right] \frac{y}{x_1} \qquad (2.26)$$

Using the simplified forms of the numerator (2.23) and denominator (2.26) and forming their ratio , the approximate $\widetilde{SNR}$ is

$$\tilde{SNR} \approx \frac{\alpha + a_1\beta + a_2\beta^2}{\alpha + \Delta + a_3\,\beta + a_4\beta^2} \tag{2.27}$$

where
$$a_1 = -4\,c_1 \left[\sum_{k=2}^{\rho} c_k^2\right] \tag{2.28}$$

$$a_2 = c_1^2 \tag{2.29}$$

$$\alpha = \sum_{j=1}^{\rho} \sum_{k=1}^{\rho} c_j^2\, c_k^2 \tag{2.30}$$

$$\Delta = \lambda_2 c_1^2 \left[\sum_{k=2}^{\rho} c_k^2\right] \tag{2.31}$$

$$a_3 = -2\,c_1 \left[\left[\sum_{k=2}^{\rho} c_k^2\right] + \lambda_2 c_1^2\right] \tag{2.32}$$

$$a_4 = \left[\left[\sum_{k=2}^{\rho} c_k^2 / \lambda_2\right] + c_1^2\right] \tag{2.33}$$

The $\tilde{SNR}$ is now a function of one random variable, β . We have obtained the form of $\tilde{SNR}$ that we wanted. The density of $\tilde{SNR}$ can now be readily be obtained using standard univariate transformation theorems for functions of random variables.

Validity of Approximations

Although earlier we stated that the reduced formulas for $\tilde{D}_{jk}$ (2.17), (2.18), and (2.19) only hold for $c_j \neq 0$ for $j=1,2\ldots\rho$ and $c_k \neq 0$

for $k=1,2...\rho$, our above result (2.27) is valid even when some of the c_j's and c_k's are zero or very small. This is because the $\tilde{D}_{jk}$'s associatted with the relatively large non-zero c_j's and c_k's contribute the dominating variance in formulas (2.23) and (2.26) . By comparison, the $\tilde{D}_{jk}$'s associated with the zero or very small cj's and c_k's contribute generally little variance to (2.23) and (2.26) under the near rank-1 and large sample size condition.

The accuracy of the approximation for $\tilde{SNR}$ (2.27) can be determined through the error calculations in Appendix B.

II) Determining the approximate probability density of $\widetilde{SNR}$ using the reduced form.

In section I it was shown that when the noise covariance matrix is almost rank 1 and the sample size is sufficently large , the $\widetilde{SNR}$ reduces to a function of a single random variable β that is approximatly a ratio of gaussians. Obtaining the density of $\widetilde{SNR}$ is now a simple two step procedure:

1) Determine the density of β .

2) Using standard univariate transformation theorems, obtain the density of $\widetilde{SNR}$ with from density of β .

Determining the Density of β

Recall from section I that β (2.20) is

$$\beta = \frac{\left[\sum_{j=2}^{p} c_j x_j\right]}{x_1} \tag{3.00}$$

In appendix A it was shown that the x_k's for reasonable sample size K tend toward gaussian, therefore the random variable β can be approximated as a ratio of independent gaussian random variables. From appendix A the normal approximations were shown to be:

$$x_1 \sim N(1,\ 2/K) \tag{3.01}$$

$$x_k \sim N(0,\ \lambda_2/K) \text{ for } k=2,3,\ldots p \tag{3.02}$$

We can therefore consider β as a ratio of two gaussian random variables

, x and u as follows:

$$\beta = x/u \tag{3.03}$$

where

(3.04) (3.05)

$$x = \sum_{j=2}^{\rho} c_j\, x_j \quad , \qquad u = x_1$$

The means and variances of x and u are

$$E[x] = \bar{x} = 0, \quad E[u] = \bar{u} = 1 \tag{3.06}$$

$$\sigma_x^2 = \lambda_2 \left[\sum_{j=2}^{P} c_j^2 \right] / K \qquad \sigma_u^2 = 2/K \tag{3.07}$$

Kanter [6] has derived a general form of the probability density for the ratio of two gaussian random variables , using the same notation as before we have

$$p(\beta) = \left[\frac{b^2}{\sqrt{\pi}\gamma}\right] \exp\left[-\phi(\tilde{s}^2 + 1)\right] / (s^2(\beta) + 1) \tag{3.08}$$

$$\cdot \left[z(\beta)\ \mathrm{erf}(\ z(\beta)\ )\ \exp(\ z^2(\beta)\ ) + \frac{1}{\sqrt{\pi}}\right]$$

where $a = \sigma_x$, $b = \sigma_u$ (3.09)

r = correlation coefficient $0 \leq r < 1$ (3.10)

$$\gamma = ab\ \sqrt{(1-r^2)} \tag{3.11}$$

$$\phi = \bar{u}^2 / \ 2b^2 \tag{3.12}$$

$$s(\beta) = (b^2\beta - rab)/\gamma \tag{3.13}$$

$$\tilde{s} = (b^2(\bar{x}/\bar{u}) - rab)/\gamma \tag{3.14}$$

$$z(\beta) = \sqrt{\phi}\ \ (\ \tilde{s}\ s(\beta) + 1\ )\ /\ \sqrt{(\ s^2(\beta) + 1\ )} \tag{3.15}$$

Noting that the correlation coefficent is zero ($r=0$) since x_1 and x_k for $k=2,3\ldots\rho$ are approximatly independent, we can now obtain the density of β directly through substitution.

$$p(\beta) = [\sqrt{2} / \sqrt{\pi}\lambda_2\Gamma)] \, [\exp -K/4] \; / \; [(2/\lambda_2\Gamma)\, \beta^2 + 1] \tag{3.16}$$

$$\cdot \left[\left[\frac{\sqrt{K} \;/\; 2}{\sqrt{[(2/\lambda_2\Gamma)^2\beta + 1\,]}}\right] \operatorname{erf} \left[\frac{\sqrt{K} \;/\; 2}{\sqrt{[(2/\lambda_2\Gamma)^2\beta+1]}}\right] \exp \left[\frac{K \;/\; 4}{(2/\Gamma)\; \beta^2 + 1} \right] + \frac{1}{\sqrt{\pi}} \right]$$

$$\text{where} \quad \Gamma = \sum_{j=2}^{\rho} c_j^2 \tag{3.17}$$

We have obtained the density for β. At this point we can derive the approximate density of $\tilde{SNR}$.

Finding the Approximate Density of $\tilde{SNR}$

To calculate the approximate density of $\tilde{SNR}$, we need the use of the following random variable transformation theorem [8]

Theorem A: Given a function $g(\beta)$, where β is a random variable with density $p(\beta)$, the density of $g(\beta)$ can be found using $p(\beta)$ as follows: we solve the equation $y=g(\beta)$ and denote its real roots by $\beta_1, \beta_2 \ldots \beta_n$. Clearly, the values of the roots depend on y. Then the density of y is given by the formula

$$f(\,y\,) = \frac{p(\,\beta_1\,)}{|\,g'(\,\beta_1\,)\,|} + \; . \; . \; . + \frac{p(\,\beta_n\,)}{|\,g'(\,\beta_n\,)\,|} \tag{3.18}$$

$$\text{where} \quad g'(\beta) = \frac{d\, g(\beta)}{d\beta} \tag{3.19}$$

and

$$g(\beta) = \frac{\alpha + a_1\beta + a_2\beta^2}{\alpha + \Delta + a_3\beta + a_4\beta^2} \tag{3.20}$$

Since $g(\beta)$ is a ratio of quadratics, it follows that there are two real roots. Multiplying both sides of formula (3.20) by the denominator of $g(\beta)$ and collecting terms we have

$$y\alpha + y\Delta + ya_3\beta + ya_4\beta^2 = \alpha + a_1\beta + a_2\beta^2 \tag{3.21}$$

$$(ya_4 - a_2)\beta^2 + (ya_3 - a_1)\beta + (\alpha[y-1] + y\Delta) = 0 \tag{3.22}$$

Note that formula (3.22) is in the form of a standard quadratic, therefore the standard root formula for quadratic equations can now be applied to obtain the roots of $g(\beta)$. The two roots are

$$\beta_{1,2} = \frac{(a_1 - ya_3) \pm \left[(ya_3 - a_1)^2 - 4(ya_4 - a_2)\ (\alpha[y-1] + y\Delta\ ) \right]^{1/2}}{2(ya_4 - a_2)} \tag{3.23}$$

Next we determine the derivative of $g(\beta)$ with respect to β. Differentiating we get

$$\frac{dg(\beta)}{d\beta} = (a_1 + 2a_2\beta)\ (\alpha + \Delta + a_3\beta + a_4\beta^2)^{-1} \tag{3.24}$$

$$-(\alpha + a_1\beta + a_2\beta^2)\ (a_3 + 2a_4\beta)\ (\alpha + \Delta + a_3\beta + a_4\beta^2)^{-2}$$

We can now obtain the approximate density of $\widetilde{SNR}$ through direct substitution using theorem A formula (3.18), the density of β (3.16), the roots $\beta_{1,2}$ (3.23) and the derivative $g'(\beta)$ (3.24). Denoting the density of $\widetilde{SNR}$ as f(y) , we get

$$f(y) \approx \frac{p(\beta_1)}{|g'(\beta_1)|} + \frac{p(\beta_2)}{|g'(\beta_2)|} \tag{3.25}$$

We recall that β_1 and β_2 are functions of y. The dependence on y is given in formula (3.23). The density f(y) exists over the region where β_1 and β_2 are real .

<u>Asymptotic Moments of $\widetilde{SNR}$ when $\lambda_2 \to 0$</u>

The formulas we have derived for the probability density of $\widetilde{SNR}$ (3.25) are complicated and one would probably have to resort to numerical evaluation of the integrals to obtain the first and second moments of $\widetilde{SNR}$. However, in the extreme case when $\lambda_2 \to 0$, it can be shown that asymptotically as $K \to \infty$

$$\widetilde{SNR} \xrightarrow{d} 1 - \left[\frac{(1 - c_1^2)}{K} \right] z^2 \tag{3.26}$$

where $z \sim N(0,1)$ and "d" implies convergence in distribution. Using (3.26) we can readily obtain the expected value and variance of $\widetilde{SNR}$.

$$E[\ \tilde{SNR}\ ] = \frac{K - (1 - c_1^2)^2}{K} \tag{3.27}$$

$$\mathrm{Var}[\ \tilde{SNR}\ ] = \frac{2\ (1 - c_1^2)^4}{K^2} \tag{3.28}$$

Proof: First, it was shown in Appendix A that the numerator and denominator of β (3.00) are asymptotically gaussian (see Appendix A for detail). Through application of a theorem for the asymptotic distribution of a function of asymptotically gaussian random vectors [Theorem B, pg. 124,14] , it is seen that β (3.00) itself is asymptotically gaussian as $K \rightarrow \infty$ as follows:

$$\beta \xrightarrow{d} \left[\lambda_2 \sum_{j=2}^{\rho} c_j^2 \right]^{1/2} z \tag{3.29}$$

where $z \sim N(0,1/K)$. Next, without loss in generality set the signal power to unity, that is

$$\sum_{j=1}^{\rho} c_j^2 = 1 \tag{3.30}$$

The summation in (3.29) can now be rewritten as a function of c_1

$$\beta \xrightarrow{d} \left[\lambda_2 (1 - c_1^2) \right]^{1/2} \tag{3.31}$$

substitute (3.31) in place of β in formula (2.27) and let $\lambda_2 \to 0$ (note that c_1 is constrained to be $c_1^2 \leq 1$ in formula (3.31)).

$$\tilde{SNR} \xrightarrow{d} \frac{1}{1 + (1 - c_1^2) z} \tag{3.32}$$

Finally using (3.32) and [Theorem B, pg. 124,14] (asymptotic distribution of functions with vanishing first order derivatives) we get the asymptotic distribution of $\tilde{SNR}$ as $K \to \infty$.

These formulas should generally give a good estimate of the moments of $\tilde{SNR}$ even for the general case when λ_2 is close to zero and K is large. Using the formulas for the expected value of $\tilde{SNR}$ for the low rank filter weight estimate formula (3.27) and formula (1.11) when the filter weights are based on the inversion of the sample covariance matrix, we can roughly determine the number of samples needed to attain equivalent performance (in respect to the expected values of $\tilde{SNR}$ for both methods) using the conventional filter weight estimate (1.09) compared against the improved rank-1 filter weight estimate (1.15) . Denoting the number of samples used to estimate the noise covariance in the low rank method as K_I and the sample size for the conventional detector as K_C, to attain equivalent performance , K_C must be

$$K_C \approx K_I \left[(\rho - 1)/(1 - c_1^2) \right] - 1 \tag{3.33}$$

CHAPTER FOUR

III) Experimental Results

Computer simulation results for the distribution of $\widetilde{SNR}$ are presented for three cases; (1) rank-1 interference plus white noise, (2) a highly correlated 1'st order autoregressive noise component plus white noise, and (3) when the interference data being used to estimate the interference covariance matrix is contaminated by signal. Further, for the rank-1 interference plus white noise case, we show the improvement of the low rank filter weight estimates over the conventional estimate (based on the inversion of the estimated sample covariance matrix).

The notation is the same as in sections I and II. The rank-1 interference plus white noise gaussian noise vector is generated as follows :

$$n_k = \sum_{j=1}^{\rho} w_{kj} \bar{r}_j \tag{4.00}$$

where the scale factors w_{kj} are zero-mean and independent gaussian random variables with variance

$$\mathrm{Var}[\ w_{k1}\ ] = 1 + \sigma^2 \tag{4.01}$$

and for $j=2,3...\rho$

$$\mathrm{Var}[\ w_{kj}\ ] = \sigma^2 \tag{4.02}$$

The vectors $\bar{r}_j$ $(j=1,2...\rho)$ are the normalized eigenvectors of the matrix R. The elements of R are

$$r_{ij} = \alpha^{|i-j|} \qquad \text{for} \quad i=1,2...\rho \tag{4.03}$$

$$j=1,2...\rho$$

The matrix R corresponds to the covariance of a first order AR process with correlation α, therefore the noise vector n_k corresponds roughly to a snapshot of a 1'st order autoregressive noise component plus white noise if $\sigma^2 \gg \gamma_j$ (for $j=2,3...\rho$) where the γ_j are the eigenvalues of R. The interference-to-noise ratio is defined as

$$INR = 10\log_{10}(1 / \rho\sigma^2) \tag{4.02}$$

where ρ is the covariance matrix order.

The signal vector we shall use is

$$s_k = \cos(2\pi f (k-1)) \quad \text{for } k=1,2...\rho \tag{4.03}$$

where f is the frequency.

For our computer simulation, we set $\alpha=.9999$, $\rho=20$, $K=25$ (covariance sample size). Three hundred independent trails were performed with INR being varied from 10dB to 15dB and the signal frequency varied from .03 to .06 . The noise covariance matrix was estimated using the maximum-liklihood estimate formula (1.08). Scattergrams (see Fig. 2) show that the $\widetilde{SNR}$ approximation is accurate . Next, histograms with confidence bounds were generated using the simulated data. The confidence bounds were calculated by noting that the frequency count in a particular histogram bin follows the binomial distribution and then using the DeMoiv̄re-Laplace theorem [8] to estimate the standard deviation of the frequency count,

$$\sigma_k = \sqrt{(n f_{rk} (1 - f_{rk}))} \tag{4.04}$$

where n is the number of trials and f_{rk} is the expected frequency for the k'th histogram bin obtained using the derived $\widetilde{SNR}$ density (3.25) . Figure 3 shows that the experimental data fits the $\widetilde{SNR}$ density (3.25) well.

Next we show that the probability density formula (3.25) for $\widetilde{SNR}$

can be also used to predict the performance of the detector when the interference is a mixture of highly correlated 1'st order AR process noise and white noise. The noise vector n_k is generated using (4.00) and the same parameters as before except

$$\mathrm{Var}[\, w_{kj} \,] = \gamma_j + \sigma^2 \quad \text{for } j=1,2\ldots\rho \tag{4.05}$$

where γ_j $(j=1,2\ldots\rho)$, are the eigenvalues of matrix R, and the value of the signal frequency f is .16 . In evaluating the $\widetilde{\mathrm{SNR}}$ density formula (3.25) , we assume that λ_2 is proportional to σ^2 , that is,

$$\lambda_2 = \frac{\sigma^2}{\gamma_1 + \sigma^2} \tag{4.06}$$

or equivalently that $\gamma_k=0$ for $k=2,3\ldots\rho$. Histograms (see Fig. 4) show that we get relatively good agreement with the predicted distribution .

When the noise data is contaminated by the signal that is being received, that is,

$$\tilde{n}_k = n_k + s_0 s$$

where s_0 is a scalar, we can still attain reasonable performance using the rank-1 filter weight estimate . This is shown through experimental results. The signal-to-white-noise ratio (SWNR) of the contaminating signal is defined as

$$\mathrm{SWNR} = 10\log_{10}(\, s_0^2 \,/\, 2\sigma^2 \,) \tag{4.07}$$

Using the same methodology and parameters as in the first set of experiments (α=.9999, INR=10dB, K=25) and with signal frequency f=.047, we generate the signal contaminated noise data using (4.06) and SWNR=-15dB . The histogram of the experimental data (see Fig. 5) and statistics indicate even with signal contamination, we can still attain relatively high values of $\widetilde{SNR}$.

By evaluating the formula for the expected value of $\widetilde{SNR}$ (1.11) and the density formulas derived by Reed, Mallet, Brennan [1,formula 17] for the conventional adaptive detector based on the inverse of the sample covariance matrix and evaluating the density of our improved detector using the $\widetilde{SNR}$ density formula (3.25) , it can be seen that for even small sample sizes (see Fig. 6,7), the filter weights based on the principal eigenvector yield superior results in this example.

Below, the approximate formulas that were derived in Chapter 3 for the expected value of $\widetilde{SNR}$ (3.27) and variance (3.28) are evaluated and compared against the computer simulation results obtained earlier. It can be seen from the below results, that formulas (3.27) and (3.28) are accurate.

a) mean and standard deviation of $\widetilde{SNR}$ calculated using the approximate formulas (3.27) and (3.28) when INR=10 dB, α=.9999, K=25

$$f=.03 : E[\widetilde{SNR}]=.962 , Dev[\widetilde{SNR}]=.0542 \tag{4.08}$$

$$f=.04 : E[\widetilde{SNR}]=.965 , Dev[\widetilde{SNR}]=.0500 \tag{4.09}$$

$$f=.05 : E[\widetilde{SNR}]=.960 , Dev[\widetilde{SNR}]=.0566 \tag{4.10}$$

$$f=.06 : E[\widetilde{SNR}]=.963 , Dev[\widetilde{SNR}]=.0524 \tag{4.11}$$

b) mean and standard deviation of $\widetilde{SNR}$ obtained through computer simulation based on 300 independent trials and using the same parameters as before

$$f=.03 : E[\widetilde{SNR}]=.965 , Dev[\widetilde{SNR}]=.0442 \tag{4.12}$$

$$f=.04 : E[\widetilde{SNR}]=.965 , Dev[\widetilde{SNR}]=.0419 \tag{4.13}$$

$$f=.05 : E[\widetilde{SNR}]=.967 \quad , Dev[\widetilde{SNR}]=.0438 \tag{4.14}$$

$$f=.06 : E[\widetilde{SNR}]=.967 \quad , Dev[\widetilde{SNR}]=.0398 \tag{4.15}$$

CHAPTER FIVE

Conclusion

The formulas we have derived are useful over a reasonable range of interference-to-noise ratios and signal types. For near rank-1 noise, the null steerer generally provides good performance over conventional detectors based on sample covariance matrix inversion. Further, these results should give insight into determining optimal adaptive detectors and their performance for higher rank near singular noise.

The reader may question why we did not use the known asymptotic distribution of the principal eigenvector of a Wishart matrix [13] in deriving the $\widetilde{SNR}$ probability density. Through experimentation , we have determined that the power method approximation for the principal eigenvector of the estimated sample covariance matrix is much more accurate for small sample sizes than the extreme asymptotic distribution.

APPENDIX A

Determining the Form and Distribution of x_k

We will compute the expansion scale factors which are defined in formula (2.05) and show that the x_k are asymptoticaly gaussian. Recall that the eigen-decomposition of N is

$$N = V \Lambda V^T \tag{a1.0}$$

Neglecting its length , the principal eigenvector of $\tilde{N}$ is approximated by $\tilde{N}v_1$ and it can be represented using the matrix V of column vectors which are the eigenvectors of the true covariance matrix N as follows:

$$\tilde{N} v_1 = V x \tag{a1.1}$$

in which x is the ρx1 column vector of the scale factors.

$$x = [x_1 \; x_2 \; . \; . \; . \; . \; x_\rho]^T \tag{a1.2}$$

Then

$$x = V^T \tilde{N} v_1 \tag{a1.3}$$

Next, v_1 can be rewritten as

$$v_1 = V b \tag{a1.4}$$

where $b = [1 \; 0 \; 0 \; . \; . \; . \; 0]^T$. Note that the dimensions of b are ρx1 . Now substitute formula (a1.4) into (a1.3) .

$$x = V^T \tilde{N} V b \tag{a1.5}$$

To obtain the distribution of x we need to first find the distribution of $V^T\tilde{N}V$. Use a theorem of Wishart matrices (from [7]), recalling that $\tilde{N}$ is Wishart [13] . For additional discussion and use of the Wishart distribution of estimated covariance matrices see (Anderson [7]), (Goodman [16]), (Reed, Mallet, Brennan [1]) and (Capon and Goodman [17]).

Theorem B: If $\tilde{N} \sim W_\rho(N,K)$, then $V^T \tilde{N} V \sim W_\rho(V^T N V , K)$.

In this notation, $Y \sim W_\rho(X,K)$ means that the ρxρ matrix Y is Wishart distributed with expected value X and K degrees of freedom.

Therefore

$$V^T \tilde{N} V \sim W_\rho(V^T N V , K) \tag{a1.6}$$

The distribution parameters of (a1.6) can be simplified by substituting the eigen-decomposition of N in place of N.

Then

$$V^T \tilde{N} V \sim W\rho(\Lambda , K) \tag{a1.7}$$

where Λ is the diagonal matrix of eigenvalues of N. We now have the distribution of $V^*\tilde{N}V$. Anderson [7] has shown that the sample covariance matrix is asymptotically gaussian distributed as the sample size increases. The equivalent can be stated for Wishart matrices as K $\rightarrow\infty$. The theorem from Anderson [7] is re-stated below for the case of Wishart matrices.

Theorem C: If $\tilde{\Sigma}$ is Wishart $W_\rho(\Sigma , K)$, then the asymptotic distribution of

$$X = \frac{1}{\sqrt{K}} \left[\tilde{\Sigma} - \Sigma \right] \tag{a1.8}$$

is normal with mean 0 and covariances

$$E[x_{ij} x_{kl}] = \sigma_{ik} \sigma_{jl} + \sigma_{il} \sigma_{jk} \tag{a1.9}$$

where x_{ij} is the i,j'th element of matrix X (a1.8) and σ_{ij} is the i,j'th covariance from Σ .

Let

$$\tilde{\Sigma} = V^T \tilde{N} V \tag{a2.0}$$

then

$$x = \tilde{\Sigma} b \tag{a2.1}$$

It can now be seen that x is simply the first column of $\tilde{\Sigma}$. By application of theorem C directly to formula (a1.5) and using formula (a1.9), it can be easily shown that x is asymtotically multivariate gaussian with mean

$$E[\, x^T \,] = \bar{x}^T = [\, 1 \;\; 0 \;\; 0 \; . \; . \; . \; 0 \,]^T \qquad \text{(a2.2)}$$

and covariance matrix

$$\text{(a2.3)} \qquad E[\, (x-\bar{x})(x-\bar{x})^T] = \begin{bmatrix} 2/K & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \lambda_2/K & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & 0 & \lambda_2/K & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot & \cdot \\ \cdot & & & \cdot & \cdot & 0 & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 & \lambda_2/K & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & \lambda_2/K \end{bmatrix}$$

of dimension $\rho x \rho$ and where K is the number of vectors used in estimating the covariance matrix $\tilde{N}$. Note that the elements of x are asymptoticaly uncorrelated, therefore independent. Further , for reasonable sample size we can consider the distribution of x to be approximatly multivariate gaussian with the above parameters.

APPENDIX B

Means and Variances of $\tilde{D}_{jk}$ and the Approximation Errors

In appendix B we will first compute the error of approximating the products $d_j d_k$ (formula (2.16)) by the corresponding asymptotic results $\tilde{D}_{jk}$ (formulas (2.17) (2.18) (2.19)) and then the respective means and variances of $\tilde{D}_{jk}$ and the approximation errors for the near rank 1 covariance matrix and large sample case. The approximation error Δ_{jk} is given by

$$\Delta_{jk} = \tilde{D}_{jk} - d_j\, d_k \qquad \text{(b1.0)}$$

Through substitution of formulas (2.16), (2.17), (2.18), and (2.19) into formula (b1.0) we get (it is noted that $\Delta_{11}=0$):

For $j=1$, $k=2,3...\rho$ the error is

$$\Delta_{1k} = \frac{-c_1\, x_k\, y}{x_1^2} - \frac{x_k\, y^2}{x_1^3} \qquad \text{(b1.1)}$$

and finally for $j=2,3,...\rho$

$k=2,3,...\rho$

$$\Delta_{jk} = \frac{c_j\, x_k\, y}{x_1^2} + \frac{c_k\, x_j\, y}{x_1^2} - \frac{c_1^2\, x_j\, x_k}{x_1^2} - \frac{2\, c_1\, x_j\, x_k\, y}{x_1^3} - \frac{x_j\, x_k\, y^2}{x_1^4} \qquad \text{(b1.3)}$$

We will now calculate the mean and variance of the approximations $\tilde{D}_{jk}$ (formulas (2.17) (2.18) (2.19)) and the approximation error terms Δ_{jk} (formulas (b1.1) (b1.2) (b1.3)) . Since $\tilde{D}_{jk}$'s and Δ_{jk}'s are sums of ratios of uncorrelated random variables, we need the use of the following formulas for eatimating the mean and variance of ratios of two random variables. Given uncorrelated random variables u and x, then

$$E\left[\frac{x}{u}\right] \approx \frac{E[x]}{E[u]} \quad (b1.4) \qquad \text{and} \qquad Var\left[\frac{x}{u}\right] \approx \frac{\sigma_x^2}{E^2[u]} \qquad (b1.5)$$

These approximations (b1.4) and (b1.5) are generaly accurate when

$$E[\ u\ ] \neq 0 \quad (b1.6) \quad , \quad E[\ u\ ] >> \sigma_u \qquad (b1.7)$$

and
$$\frac{\sigma_u}{E[\ u\ ]} << \frac{\sigma_x}{E[\ x\ ]} \qquad (b1.8)$$

where $\sigma_x{}^2$ and $\sigma_u{}^2$ are the variances of x and u respectively. Note that although the moments of these ratios might not exist in the strict theoritical sense, these estimates (formulas (b1.4) and (b1.5)) should give an good indication where the bulk of the probability density mass of the ratio x/u lies (using the Tchebycheff inequality [8]). Further, note that the above conditions (b1.6), (b1.7), and (b1.8) are essentially satisfied in the case that we are considering, near rank 1 covariance and large sample size . Using formulas (b1.4) and (b1.5) and the gaussian approximations for the x_k's as derived in appendix A we get

Note that throughout the mean and variance results we shall use

$$\Gamma = \sum_{n=2}^{\rho} c_n^2 \qquad (b1.9)$$

We now calculate the expected value and variance of the approximations

$\tilde{D}_{jk}$ (formulas (2.17), (2.18) and (2.19)) and the error terms Δ_{jk} (formulas (b1.1) and (b1.3)) using formulas (b1.4) and (b1.5) .

1) $\tilde{D}_{11}$

$$E[\ \tilde{D}_{11}] \approx \frac{\lambda_2\ \Gamma}{K\ [\ 1 + 2/K\ ]} \quad \text{(b2.0)} \qquad , \qquad Var[\ \tilde{D}_{11}\ ] \approx \frac{2\ \lambda_2^2\ \Gamma^2}{K^2\ [\ 1 + 2/K\ ]^2} \quad \text{(b2.1)}$$

2) $\tilde{D}_{1k}$: $k=2,3\ldots\rho$

$$E[\ \tilde{D}_{1k}] \approx 0 \qquad \text{(b2.2)}$$

$$Var[\ \tilde{D}_{1k}] \approx \frac{\lambda_2\ c_k^2 \sum_{n=2}^{\rho} c_n^2}{K} \qquad \text{(b2.3)}$$

3) $\tilde{D}_{jj}$: $j=2,3\ldots\rho$

$$E[\ \tilde{D}_{jj}] \approx c_j^2 \qquad \text{(b2.4)}$$

$$Var[\ \tilde{D}_{jj}] \approx \frac{4\ \lambda_2\ c_1^2\ c_j^2}{K} \qquad \text{(b2.5)}$$

4) $\tilde{D}_{jk}$: $j \neq k$ for $j=2,3\ldots\rho$

$k=2,3\ldots\rho$

$$E[\,\tilde{D}_{jk}] \approx c_j\, c_k \tag{b2.6}$$

$$\mathrm{Var}[\,\tilde{D}_{jk}] \approx \frac{\lambda_2\, c_1^2\, [\, c_j^2 + c_k^2\,]}{K} \tag{b2.7}$$

The mean and variance of the error Δ_{jk} is

1) Δ_{1k} : $k=2,3\ldots\rho$

$$E[\,\Delta_{1k}] = \frac{-\lambda_2\, c_1\, c_k}{K\,[\,1 + 2/K\,]} \tag{b2.8}$$

The variance of the terms of Δ_{1k} are

$$\mathrm{Var}\left[\frac{c_1\, x_k\, y}{x_1^2}\right] \approx \frac{\lambda_2^2\, c_1^2\,(\,c_k^2 + \Gamma\,)}{K^2\,[\,1 + 2/K\,]^2} \tag{b2.9}$$

$$\mathrm{Var}\left[\frac{x_k\, y^2}{x_1^3}\right] \approx \frac{\lambda_2^3\,(\,12\, c_k^2\, \Gamma + 3\, \Gamma^2\,)}{K^3\,[\,1 + 6/K\,]^2} \tag{b3.0}$$

2) Δ_{jj} : $j=2,3\ldots\rho$

$$E[\,\Delta_{jj}\,] \approx \frac{\lambda_2\, c_1^2 - 2\,\lambda_2\, c_k^2}{K\,[\,1 + 2/K\,]} + \frac{\lambda_2\,(\,2\, c_k^2 + \Gamma\,)}{K^2 + 12\, K\,[\,1 + 1/K\,]} \tag{b3.1}$$

The variance of the terms Δ_{jj} are

$$\mathrm{Var}\left[\frac{c_1^2\ x_j^2}{x_1^2} \right] \approx \frac{2\ \lambda_2^2\ c_1^4}{K^2\ [\ 1 + 2/K\ ]^2} \tag{b3.2}$$

$$\mathrm{Var}\left[\frac{2\ \ c_j\ x_j\ y}{x_1^2} \right] \approx \frac{4\ \lambda_2^2\ c_j^2\ (\ c_j^2 + \Gamma\ )}{K^2\ [\ 1 + 2/K\ ]} \tag{b3.3}$$

$$\mathrm{Var}\left[\frac{2\ \ x_j^2\ y}{x_1^3} \right] \approx \frac{4\ \lambda_2^3\ (\ 12\ c_j^2 + 3\ \Gamma\ )}{K^3\ [\ 1 + 6/K\ ]^2} \tag{b3.4}$$

$$\mathrm{Var}\left[\frac{x_k^2\ y^2}{x_1^2} \right] \approx \frac{\lambda_2^4 \left[(24c_j^4 + \Gamma)\ (72c_j^2 + 9\Gamma) - (2c_j^2 + \Gamma)^2 \right]}{K^4 \left[1 + (12/K)\ (\ 1 + 1/K\ ) \right]^2} \tag{b3.5}$$

3) Δ_{jk} : $j \neq k$ for $j=2,3\ldots\rho$

$k=2,3\ldots\rho$

$$E[\ \Delta_{jk}] = \frac{-2\ \lambda_2\ c_j\ c_k}{K\ [\ 1 + 2/K\ ]} + \frac{2\ \lambda_2^2\ c_j\ c_k}{K^2 + 12\ K\ [\ 1 + 1/K\ ]} \tag{b3.6}$$

The variance of the terms of Δ_{jk} are

$$\mathrm{Var}\left[\frac{c_1^2\ x_j\ x_k}{x_1^2} \right] \approx \frac{\lambda_2^2\ c_1^4}{K^2\ [\ 1 + 2/K\ ]^2} \tag{b3.7}$$

$$\mathrm{Var}\left[\frac{c_j\, x_k\, y}{x_1^2}\right] \approx \frac{\lambda_2^2\, c_j^2\, (\, c_k^2 + \Gamma\,)}{K^2\, [\, 1 + 2/K\,]^2} \tag{b3.8}$$

$$\mathrm{Var}\left[\frac{c_k\, x_j\, y}{x_1^2}\right] \approx \frac{\lambda_2\, c_k^2\, (\, c_j^2 + \Gamma\,)}{K^2\, [\, 1 + 2/K\,]^2} \tag{3.9}$$

$$\mathrm{Var}\left[\frac{2\, x_j\, x_k\, y}{x_1^3}\right] \approx \frac{4\, \lambda_2^3\, \left[\, 2\, (\, c_j^2 + c_k^2\,) + \Gamma\, \right]}{K^3\, [\, 1 + 6/K\,]} \tag{b4.0}$$

$$\mathrm{Var}\left[\frac{x_j\, x_k\, y^2}{x_1^4}\right] \approx \frac{\lambda_2^4\, \left[\, c_k^2\, c_j^2 + (\, c_j^2 + c_k^2\,)\, \Gamma\, +\, \Gamma^2\, \right]}{K^4\, \left[\, 1 + (12/K)\, [\, 1 + 1/K\,]\, \right]} \tag{b4.1}$$

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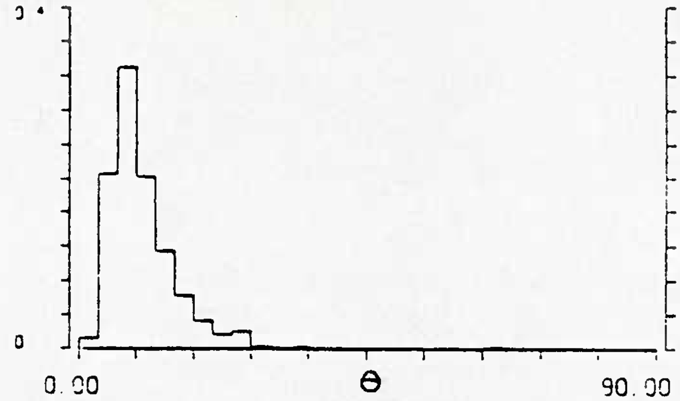
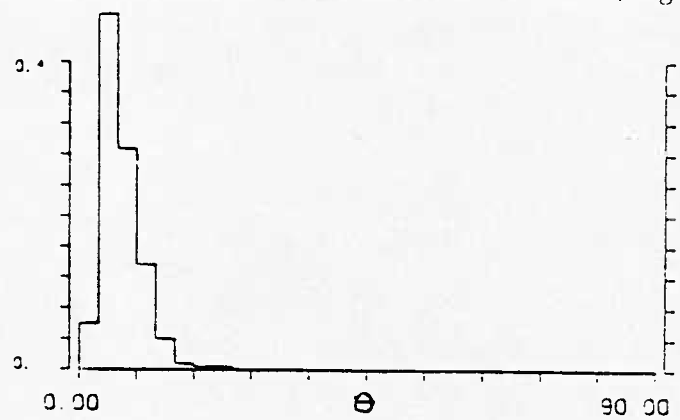


a) Eigenvector #1 : K=16
mean=9.8, standard deviation=5.59 (degrees)



b) Eigenvector #1 : K=32
mean=6.57, standard devation=3.1 (degrees)

Fig. 1)

Histograms based on 500 independent trials of the magnitude of the angular perturbation of the principle eigenvector of $\tilde{N}$. $\tilde{N}$ (1.08) was generated with K independent realizations from a first-order autoregressive process.

$$n(j)_k = .9n(j-1)_k + w(j)_k \qquad \text{for } j=1,2\ldots 10$$

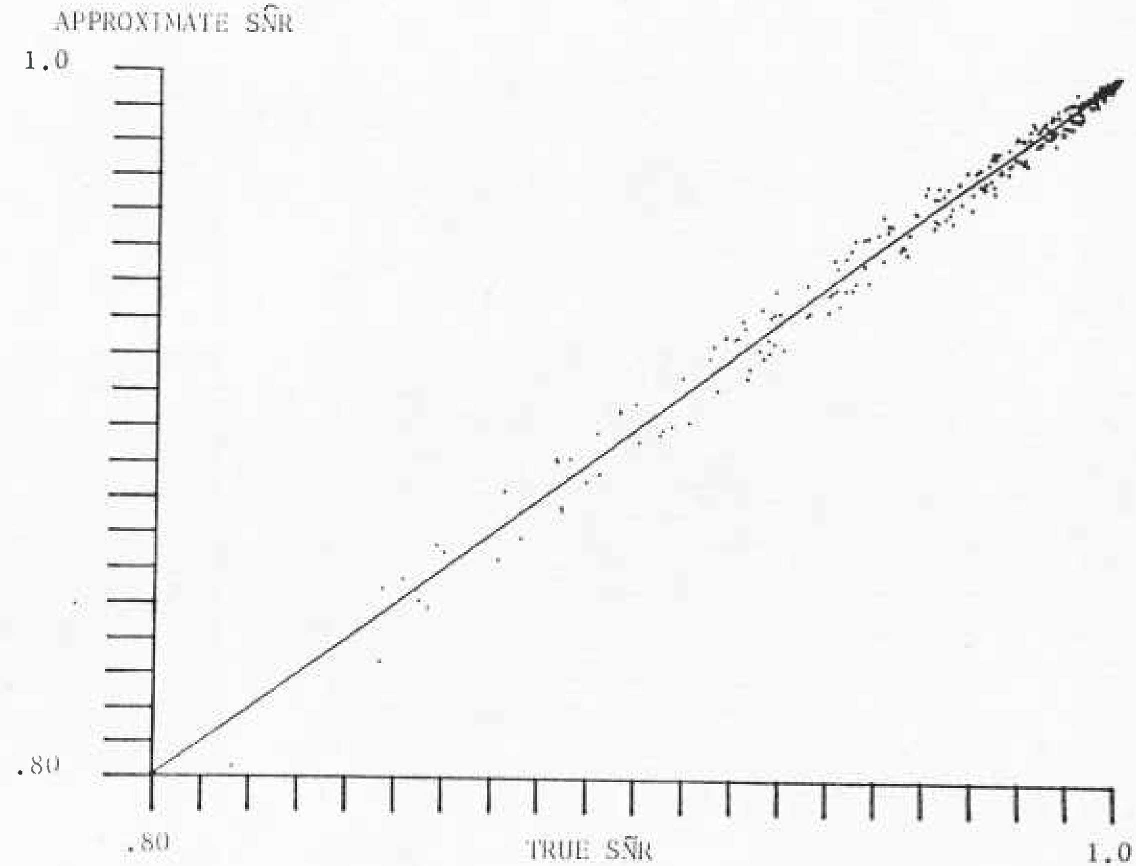
where $w(j)_k$ are independent, zero-mean, unit variance gaussian random variables.

The magnitude of the angle between the principle eigenvector of $E(\tilde{N})$ and $\tilde{N}$ is

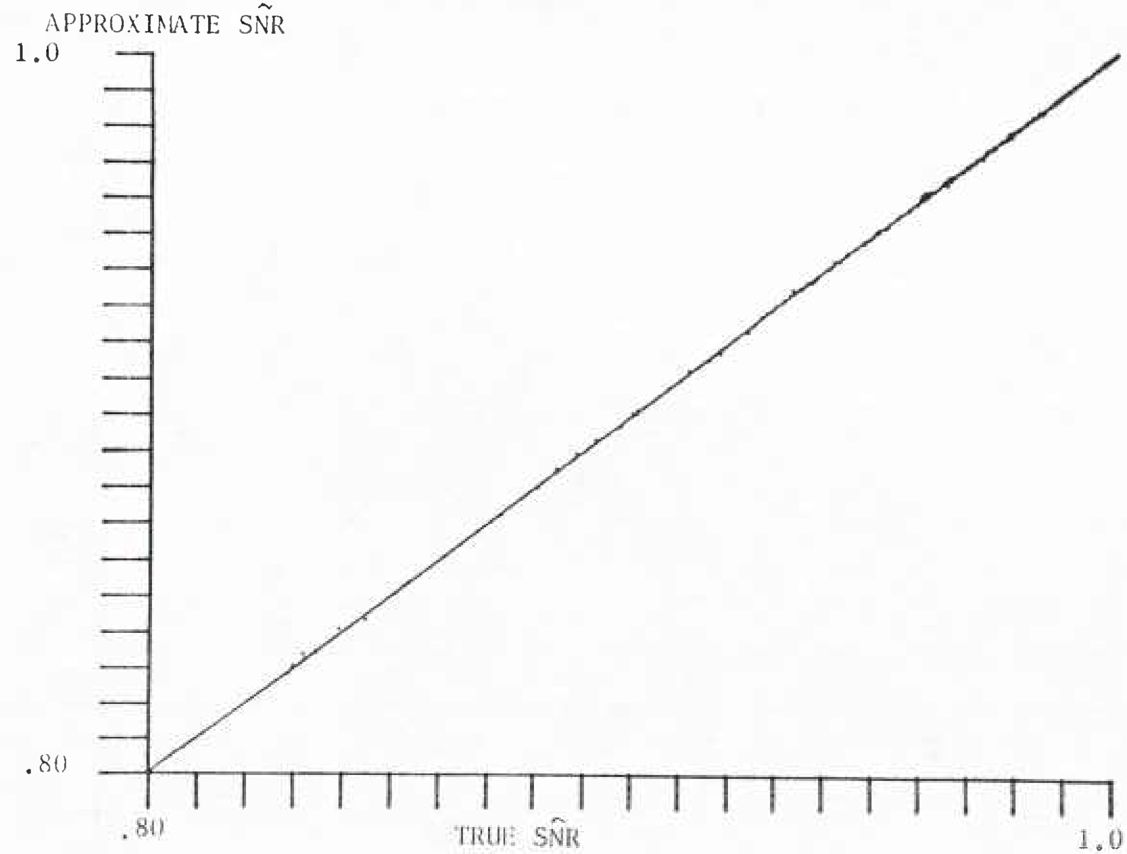
$$\theta = (180/\pi)\cos^{-1}|\tilde{v}_1^T v_1|$$

Fig. 2) SCATTERGRAMS OF TRUE SNR FORMULA (1.10) VS. APPROXIMATE SNR FORMULA (1.23) BASED ON 300 INDEPENDENT TRIALS. THE EXPERIMENTAL PARAMETERS ARE:

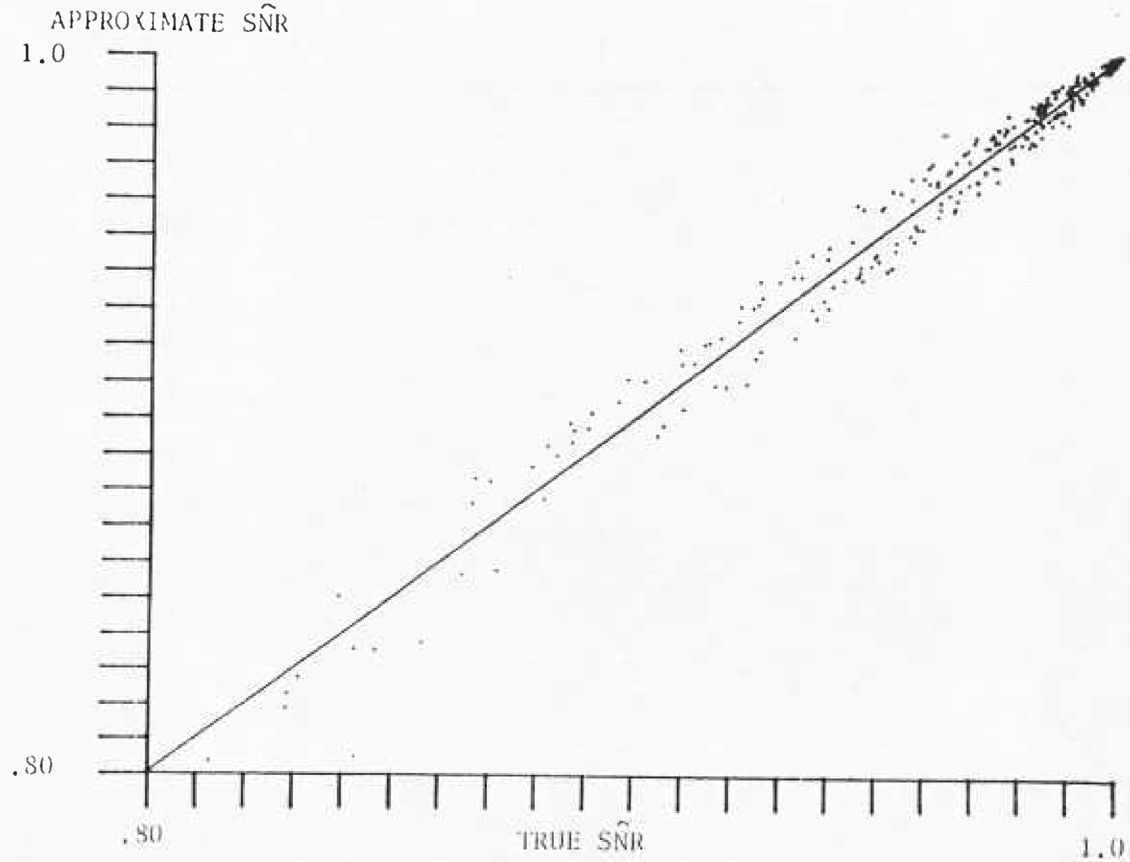
α=.9999 , ρ=20 (COVARIANCE MATRIX ORDER)
K=25 (COVARIANCE SAMPLE SIZE)



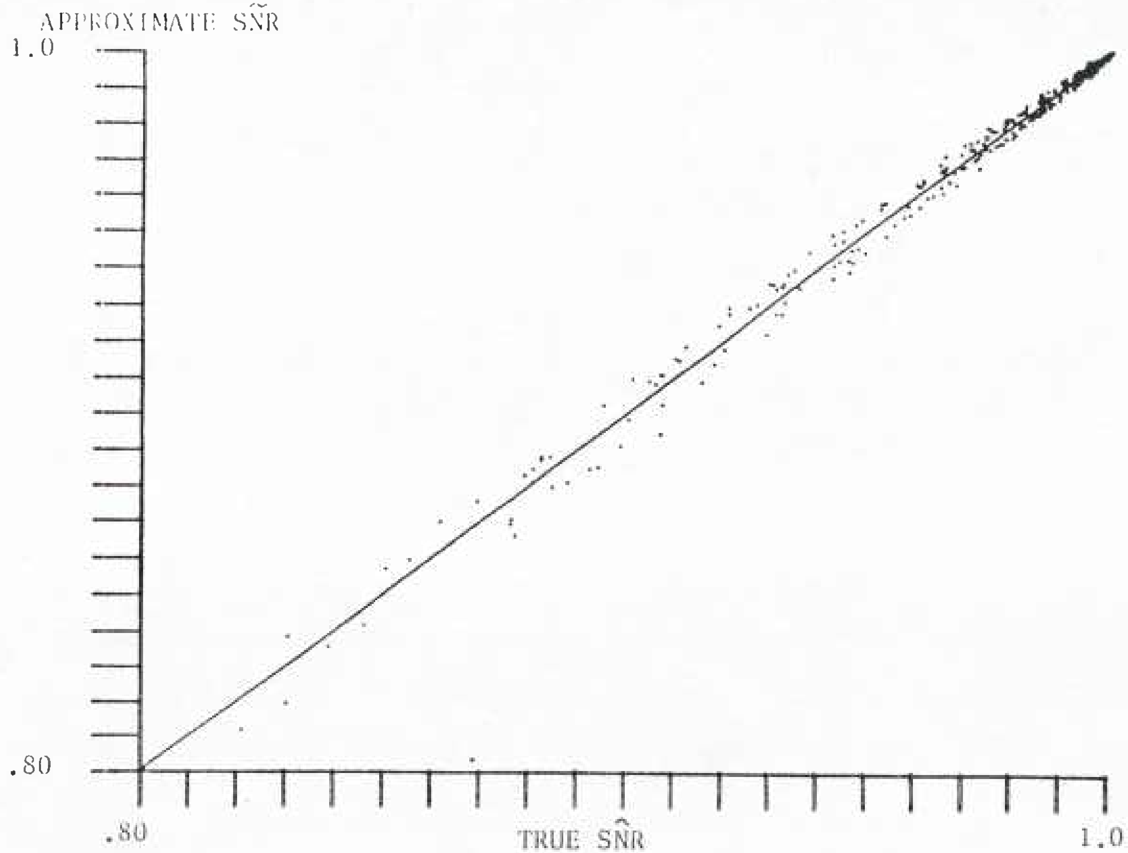
2a) INR=10 dB , SIGNAL FREQUENCY: f=.06



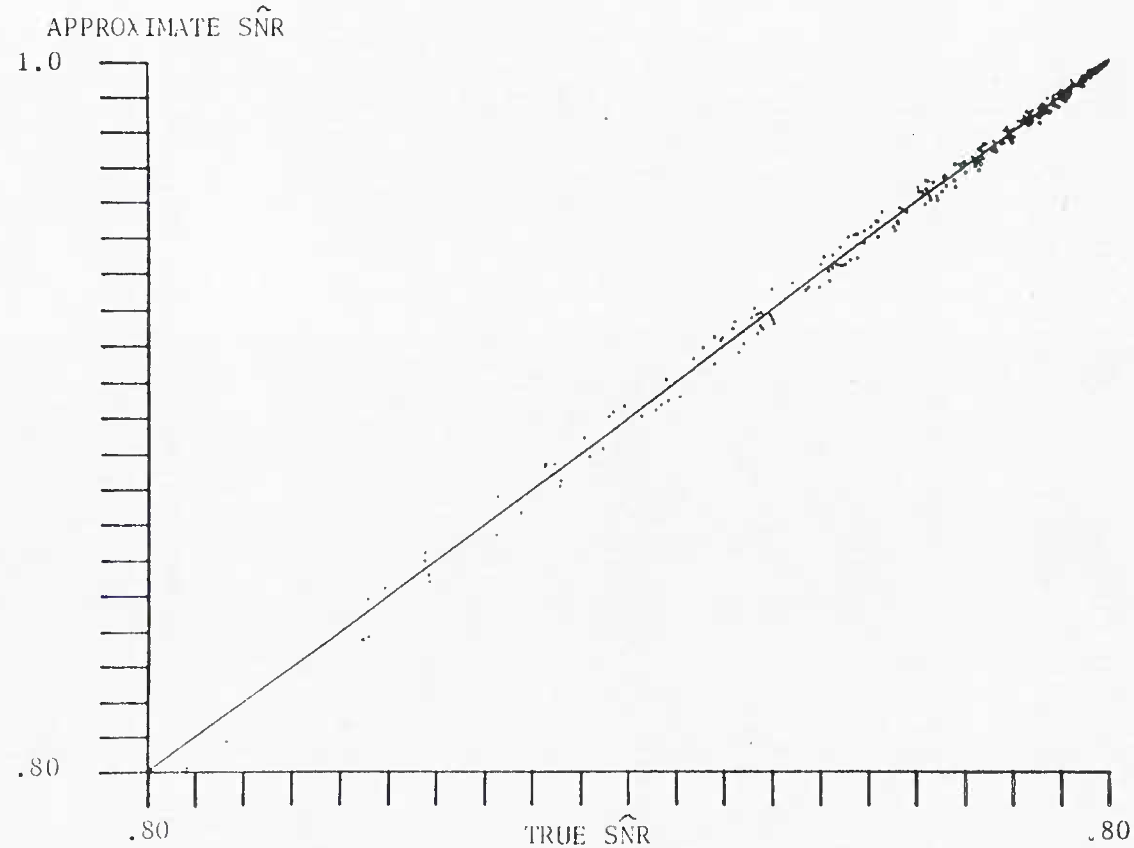
2b) INR=10 dB , SIGNAL FREQUENCY: f=.05



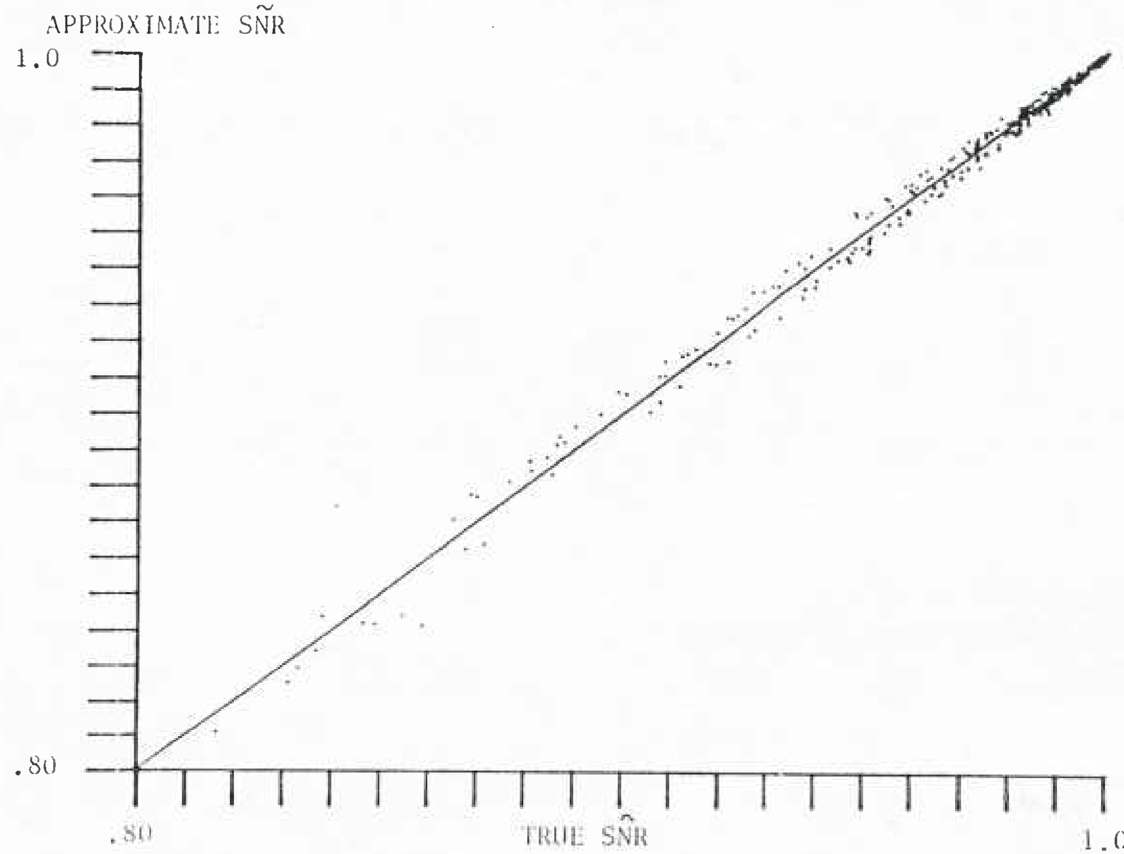
[illegible]) INR=10 dB , SIGNAL FREQUENCY: f=.04



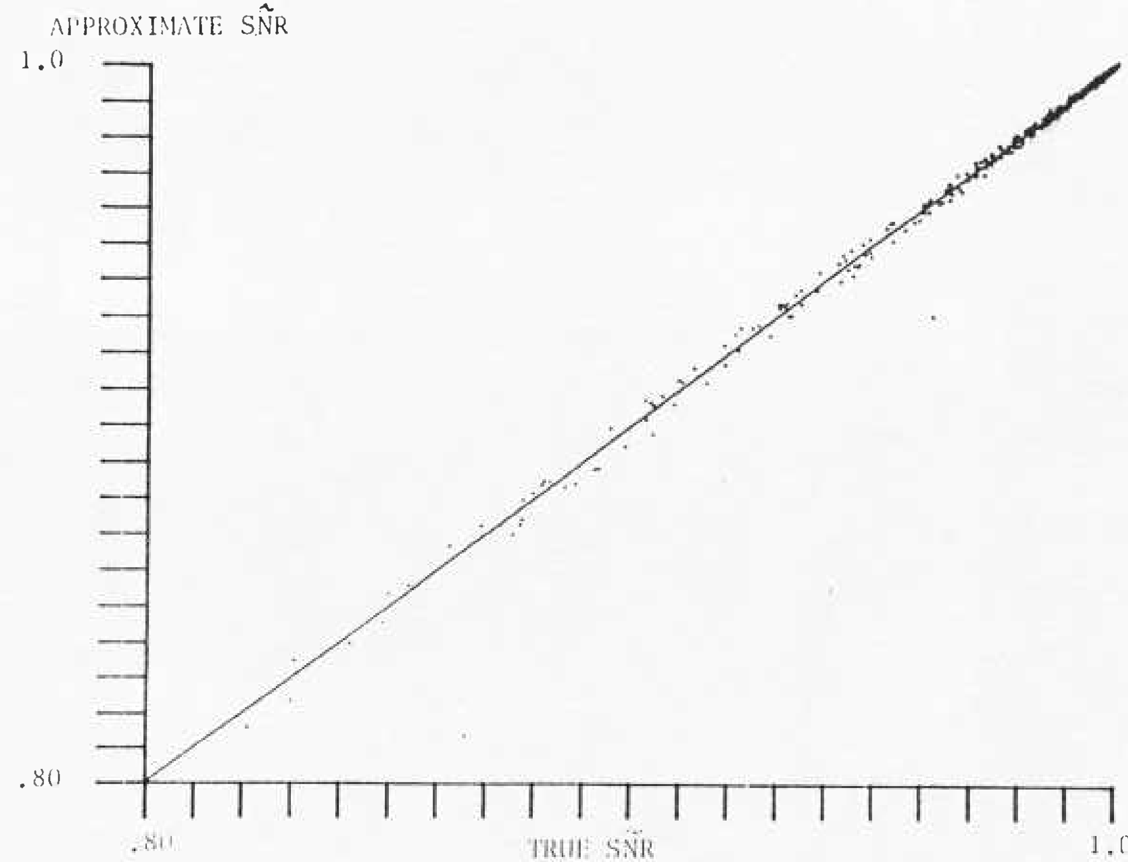
2d) INR=10 dB , SIGNAL FREQUENCY: f=.03



2e) INR=15 dB , SIGNAL FREQUENCY: f=.06

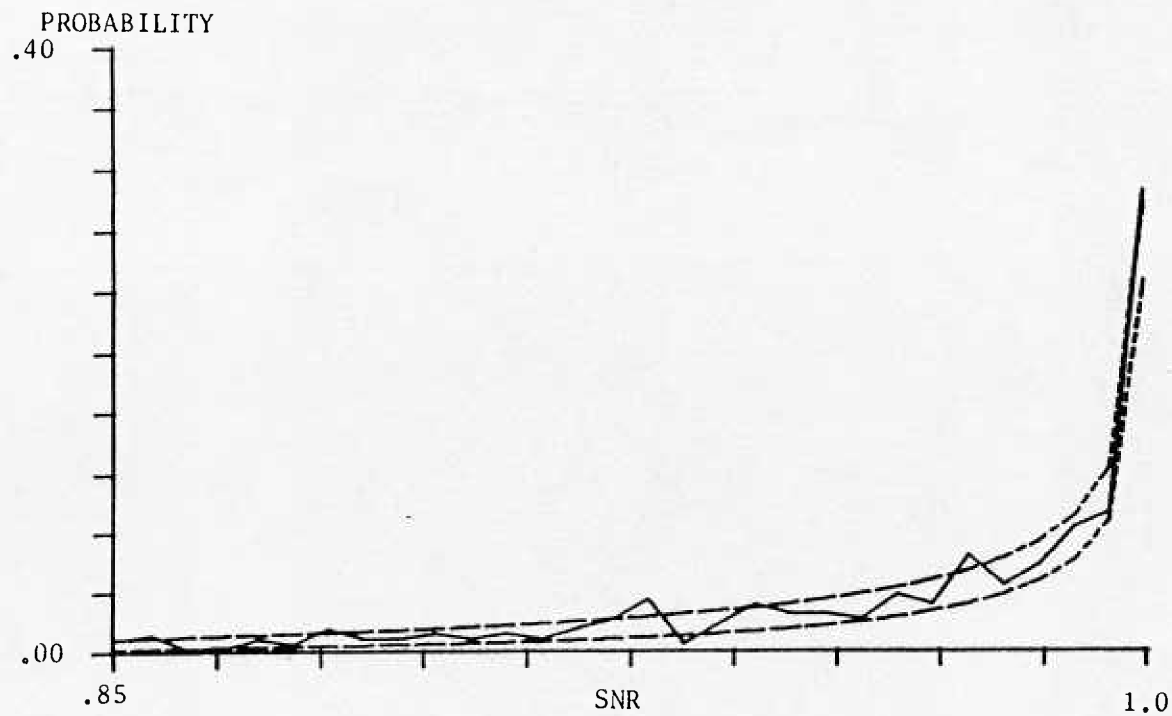


2(f) CNR=15 dB , SIGNAL FREQUENCY: f=.04



2g) INR=15 dB , SIGNAL FREQUENCY: f=.03

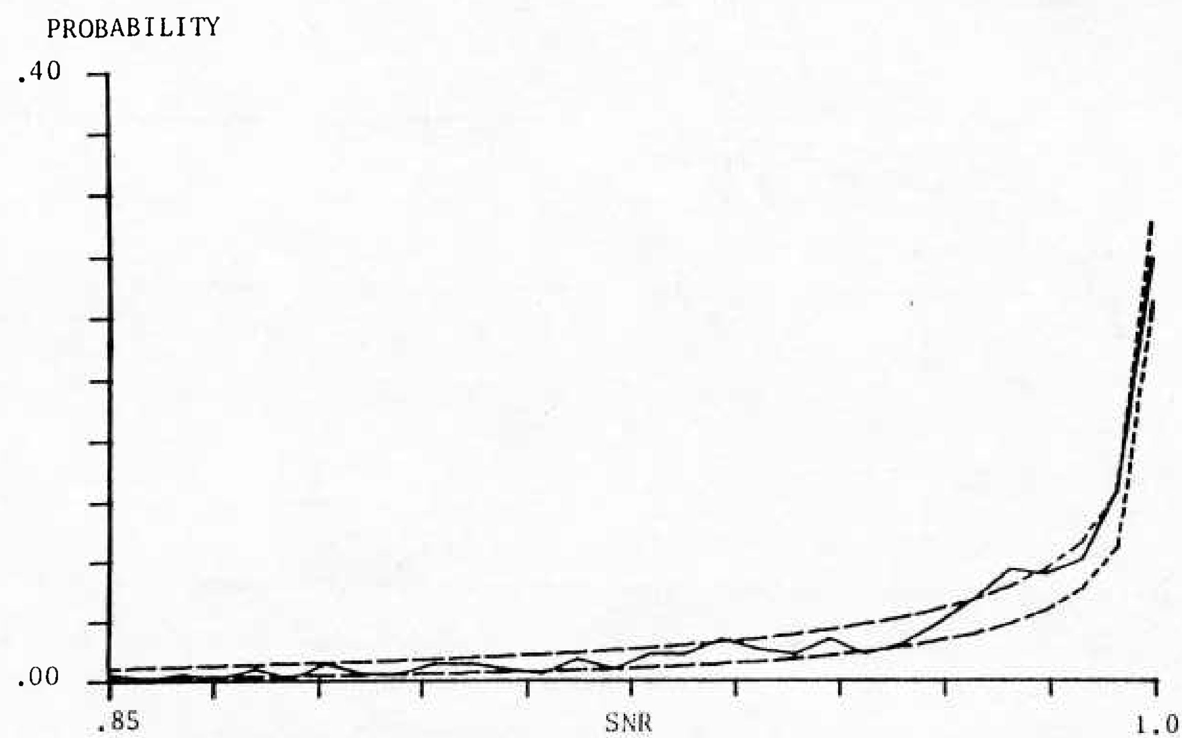
Fig. 3) HISTOGRAMS OF SNR FORMULA (1.10) BASED ON 300 INDEPENDENT TRIALS . BOUNDS ARE PLACED ON THE EXPERIMENTALY OBTAINED BIN FREQUENCIES , ASSUMING THAT SNR HAS THE DERIVED PROBABILITY DENSITY (3.25) AND THEN CALCULATING THE EXPECTED VALUE AND ESTIMATED STANDARD DEVIATION OF THE BIN FREQUENCY. HISTOGRAMS ARE GENERATED USING 30 EQUALLY SPACED BINS. THE EXPERIMENTAL PARAMETERS ARE:
α=.9999 , ρ=20 (COVARIANCE MATRIX ORDER)
K=25 (COVARIANCE SAMPLE SIZE)



3a) INR=10 dB, SIGNAL FREQUENCY: f=.06
THE STATISTICS OF SNR ARE:
MEAN = .967
STANDARD DEVIATION = 3.98E-2

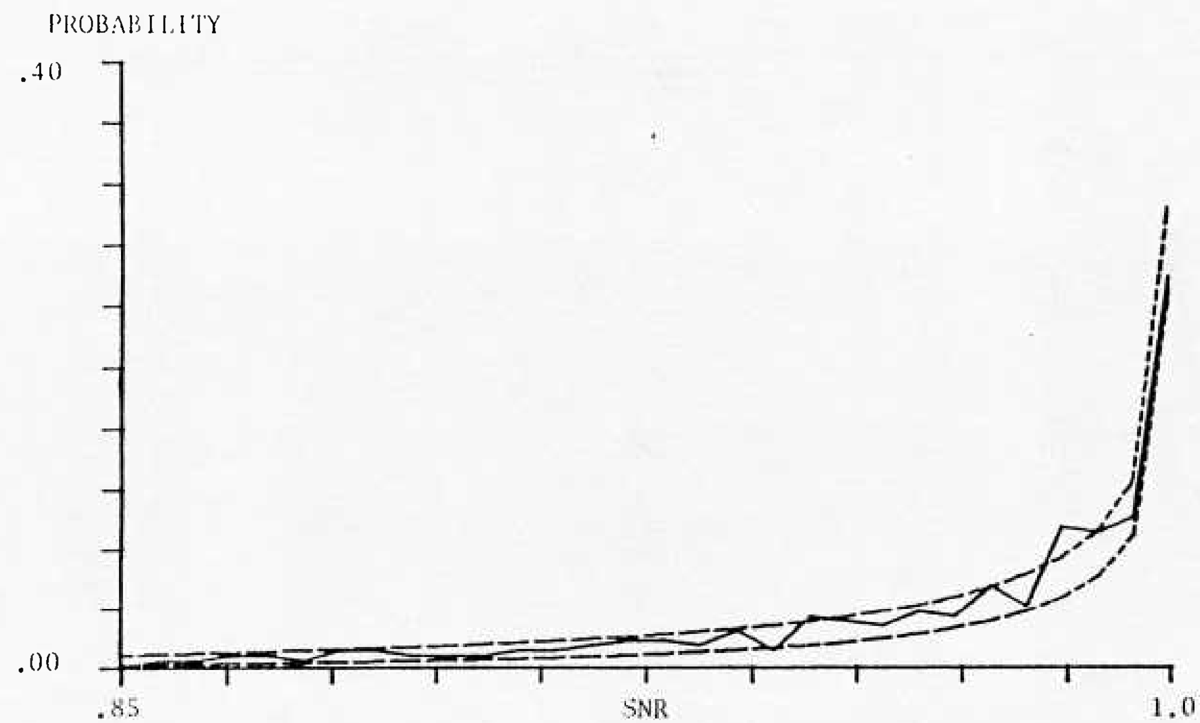
——— EXPERIMENTAL SNR

--------- CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION



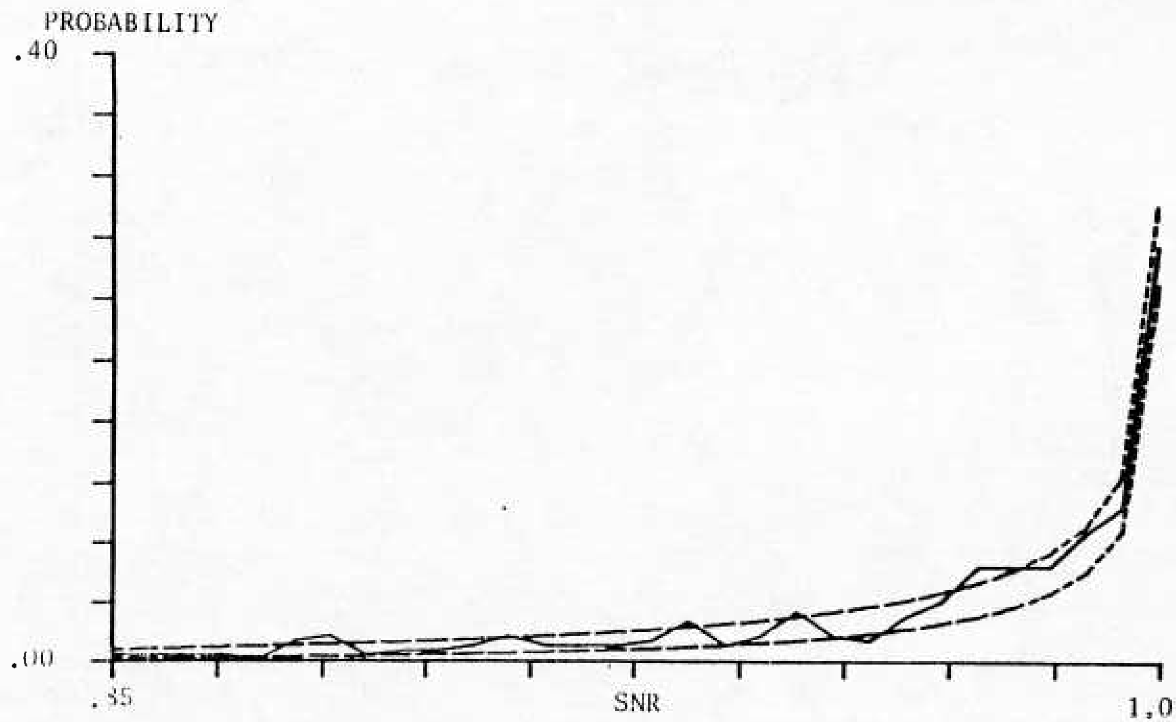
3b) INR=10 dB , SIGNAL FREQUENCY: f=.05
THE STATISTICS OF SNR ARE:
MEAN = .967
STANDARD DEVIATION = 4.38E-2

——— EXPERIMENTAL SNR
-------- CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION



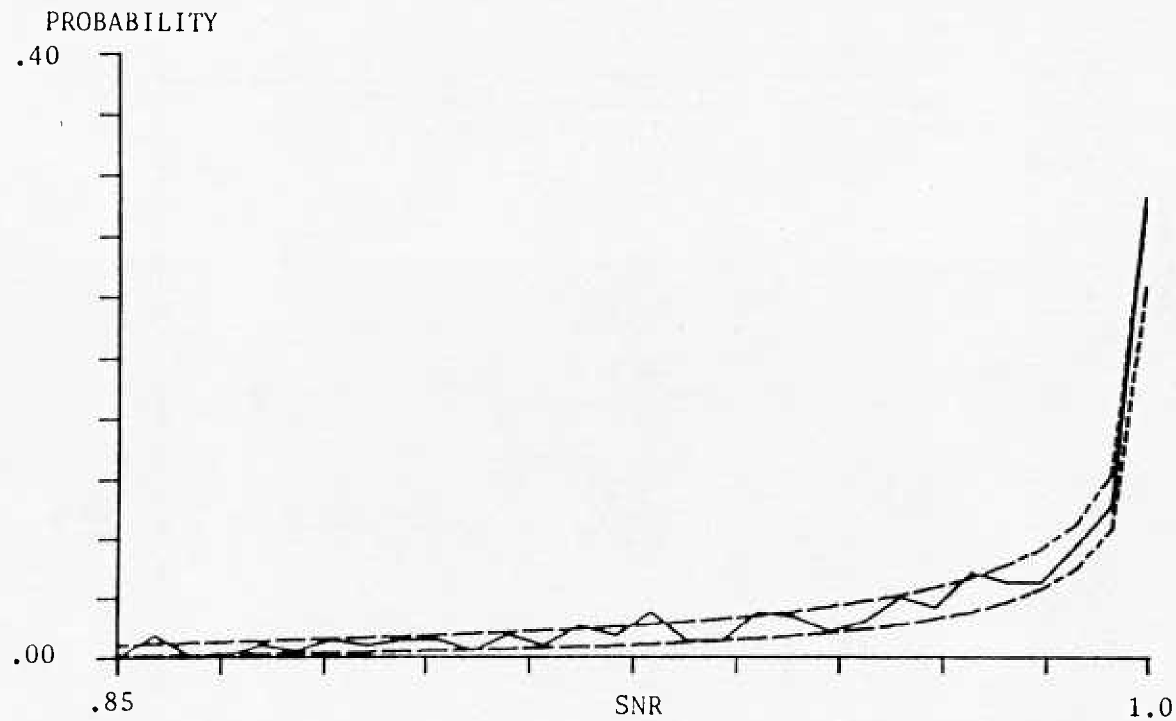
3c) INR=10 dB , SIGNAL FREQUENCY: f=.04
THE STATISTICS OF SNR ARE:
MEAN = .965
STANDARD DEVIATION = 4.19E-2

——— EXPERIMENTAL SNR
---------- CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION



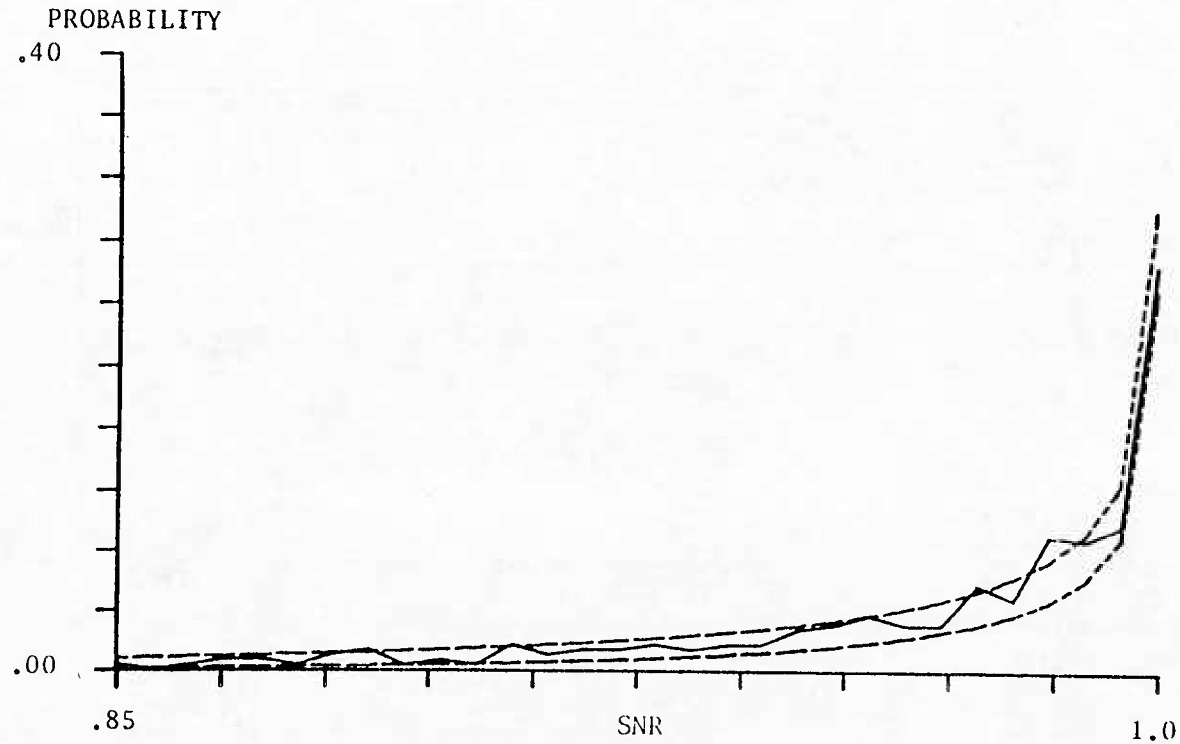
3d) INR=10 dB , SIGNAL FREQUENCY: f=.03
THE STATISTICS OF SNR ARE:
MEAN = .965
STANDARD DEVIATION = 4.42E-2

——————— EXPERIMENTAL SNR
- - - - - - - - - - CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION



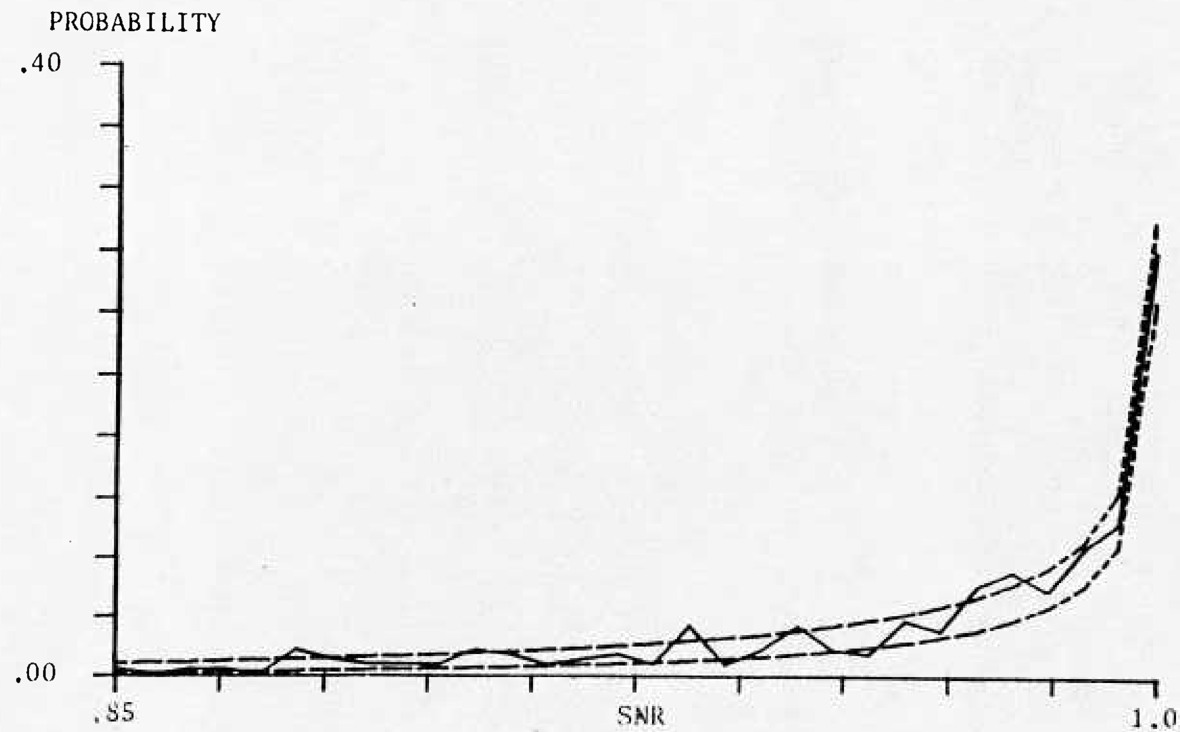
3e) INR=15 dB , SIGNAL FREQUENCY: f=.06
THE STATISTICS OF SNR ARE:
MEAN = .967
STANDARD DEVIATION = 3.97E-2

——— EXPERIMENTAL SNR
--------- CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION



3f) INR=15 dB , SIGNAL FREQUENCY: f=.04
THE STATISTICS OF SNR ARE:
MEAN = .966
STANDARD DEVIATION = 4.18E-2

——————— EXPERIMENTAL SNR
- - - - - - - - - - CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION



3g) INR=15 dB , SIGNAL FREQUENCY: f=.03
THE STATISTICS OF SNR ARE:
MEAN = .965
STANDARD DEVIATION = 4.40E-2

———— EXPERIMENTAL SNR
--------- CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION

Fig. 4) HISTOGRAMS OF SNR FORMULA (1.10) BASED ON 300 INDEPENDENT TRIALS FOR THE CASE WHEN THE NOISE CONSISTS OF A 1'st ORDER AUTOREGRESSIVE COMPONENT PLUS A WHITE NOISE COMPONENT. BOUNDS ARE PLACED ON THE EXPERIMENTALY OBTAINED BIN FREQUENCIES, ASSUMING THAT SNR HAS THE DERIVED SNR DENSITY (3.25) AND THEN CALCULATING THE EXPECTED VALUE AND ESTIMATED STANDARD DEVIATION OF THE BIN FREQUENCY. HISTOGRAMS ARE GENERATED USING 30 EQUALLY SPACED BINS. THE EXPERIMENTAL PARAMETERS ARE:

$\alpha=.9999$, $\rho=20$ (COVARIANCE MATRIX ORDER)
$K=25$ (COVARIANCE SAMPLE SIZE)

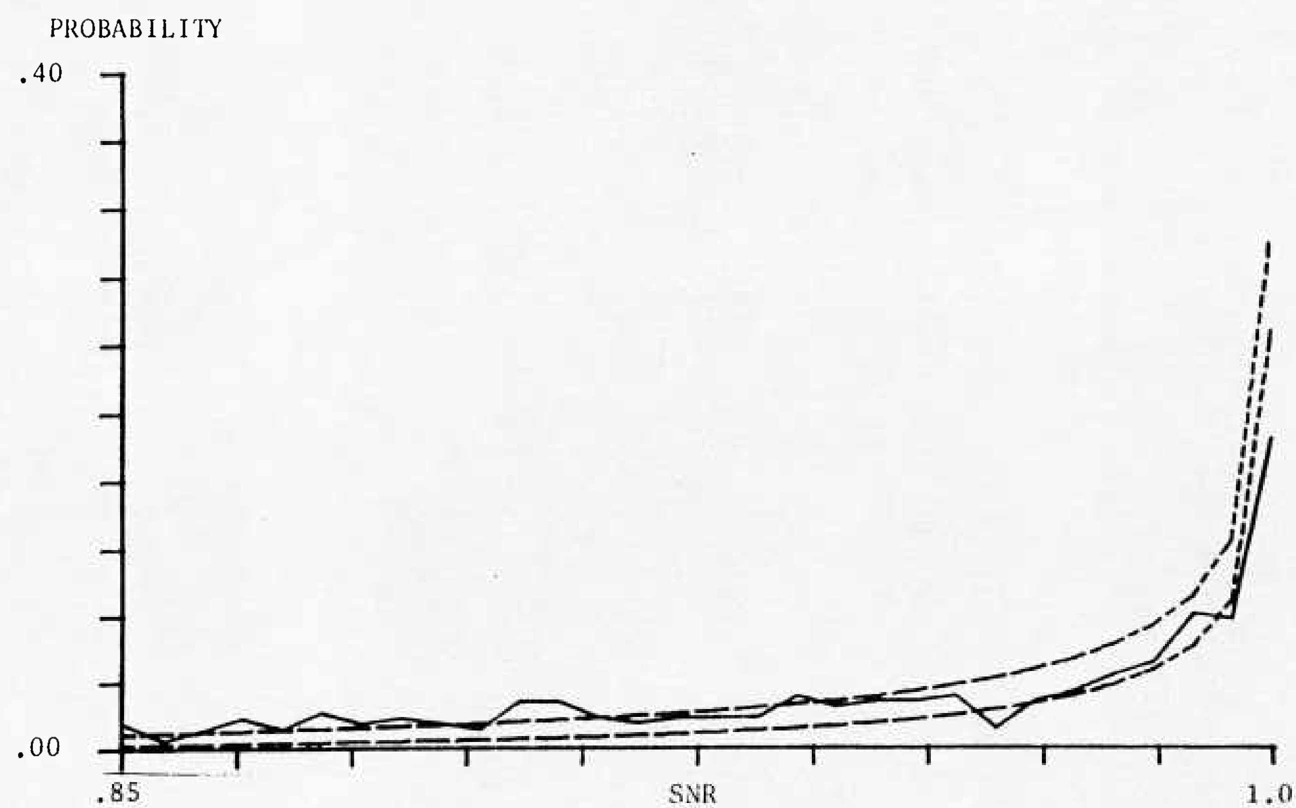


Fig. 4) 1'st ORDER AR NOISE PLUS WHITE NOISE
INR=5 dB , SIGNAL FREQUENCY: f=.16
THE STATISTICS OF SNR ARE:
MEAN = .93
STANDARD DEVIATION = 7.76E-2

——— EXPERIMENTAL SNR
--------- CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION

Fig. 5) HISTOGRAMS OF SNR FORMULA (1.10) BASED ON 300 INDEPENDENT TRIALS FOR THE CASE WHEN THE NOISE BEING USED TO ESTIMATE THE COVARIANCE MATRIX IS CONTAMINATED BY SIGNAL. BOUNDS ARE PLACED ON THE EXPERIMENTALY OBTAINED BIN FREQUENCIES, ASSUMING THAT SNR HAS THE DERIVED DENSITY (5.25) AND THEN CALCULATING THE EXPECTED VALUE AND ESTIMATED STANDARD DEVIATION OF THE BIN FREQUENCY. HISTOGRAM IS GENERATED USING 30 EQUALLY SPACED BINS. THE EXPERIMENTAL PARAMETERS ARE:

$\alpha=.9999$, $\rho=20$ (COVARIANCE MATRIX ORDER)
K=25 (COVARIANCE SAMPLE SIZE)

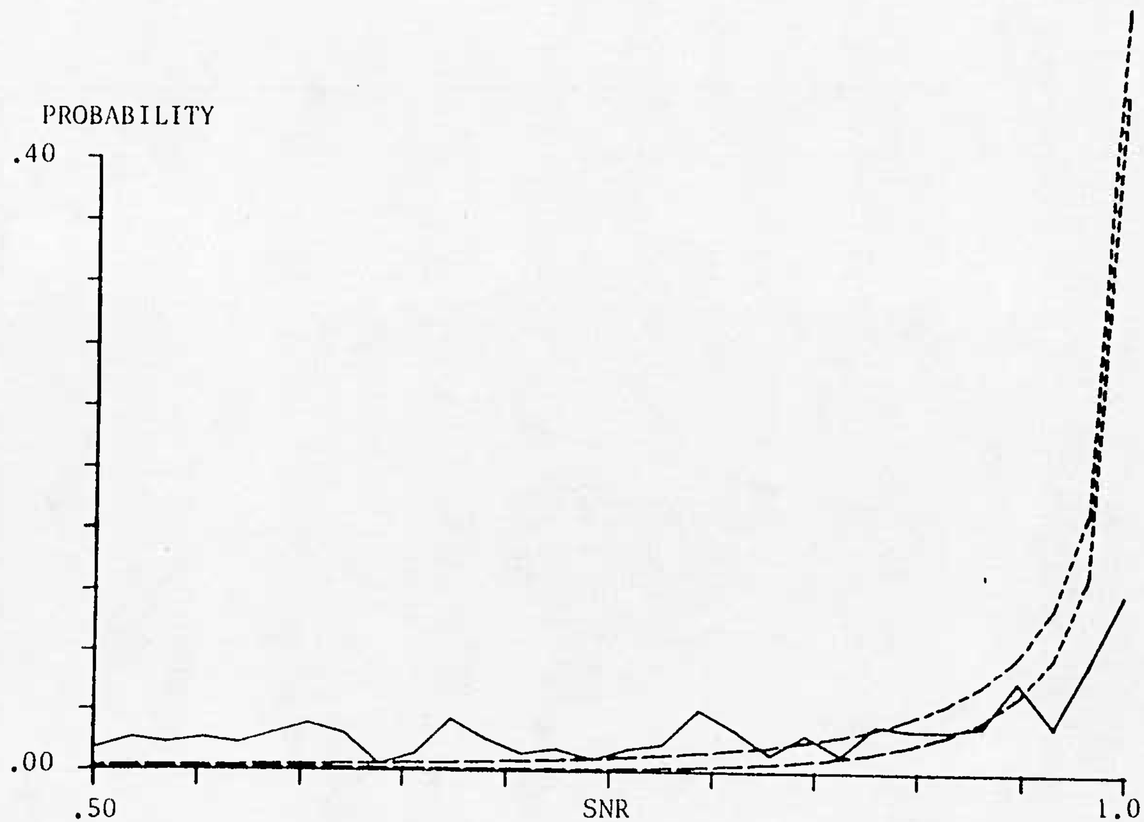


Fig. 5) SIGNAL CONTAMINATED NOISE
INR=10 dB, SWNR=-15 dB
SIGNAL FREQUENCY: f=.045
THE STATISTICS OF SNR ARE:
MEAN = .72
STANDARD DEVIATION = .23

——— EXPERIMENTAL SNR
---------- CONFIDENCE BOUND FOR EACH HISTOGRAM BIN BASED ON THE ESTIMATED STANDARD DEVIATION ABOUT THE EXPECTED BIN FREQUENCY USING THE BINOMIAL DISTRIBUTION

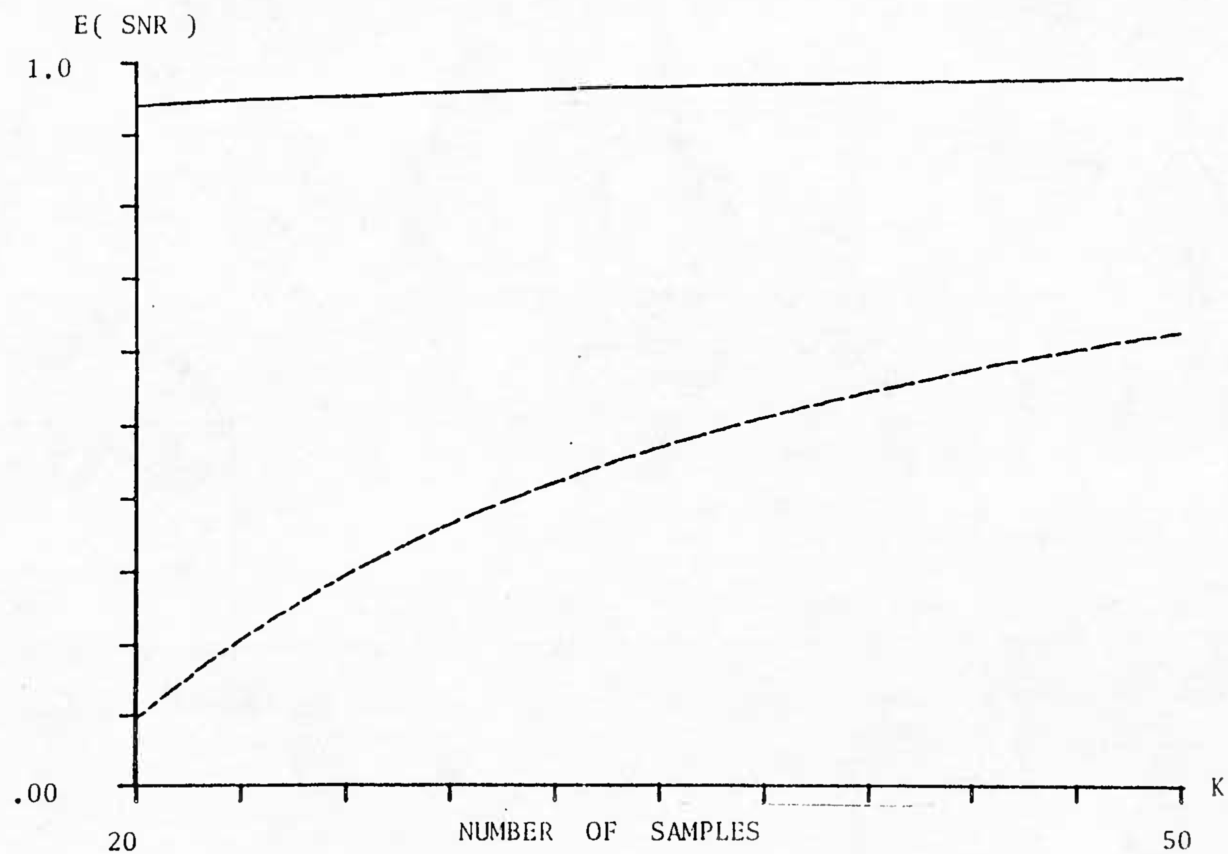


Fig. 6a) THEORETICAL EXPECTED VALUE OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 dB, f=.03 , ρ=20 , α=.9999

——— USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENVECTOR (1.15)

------- USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

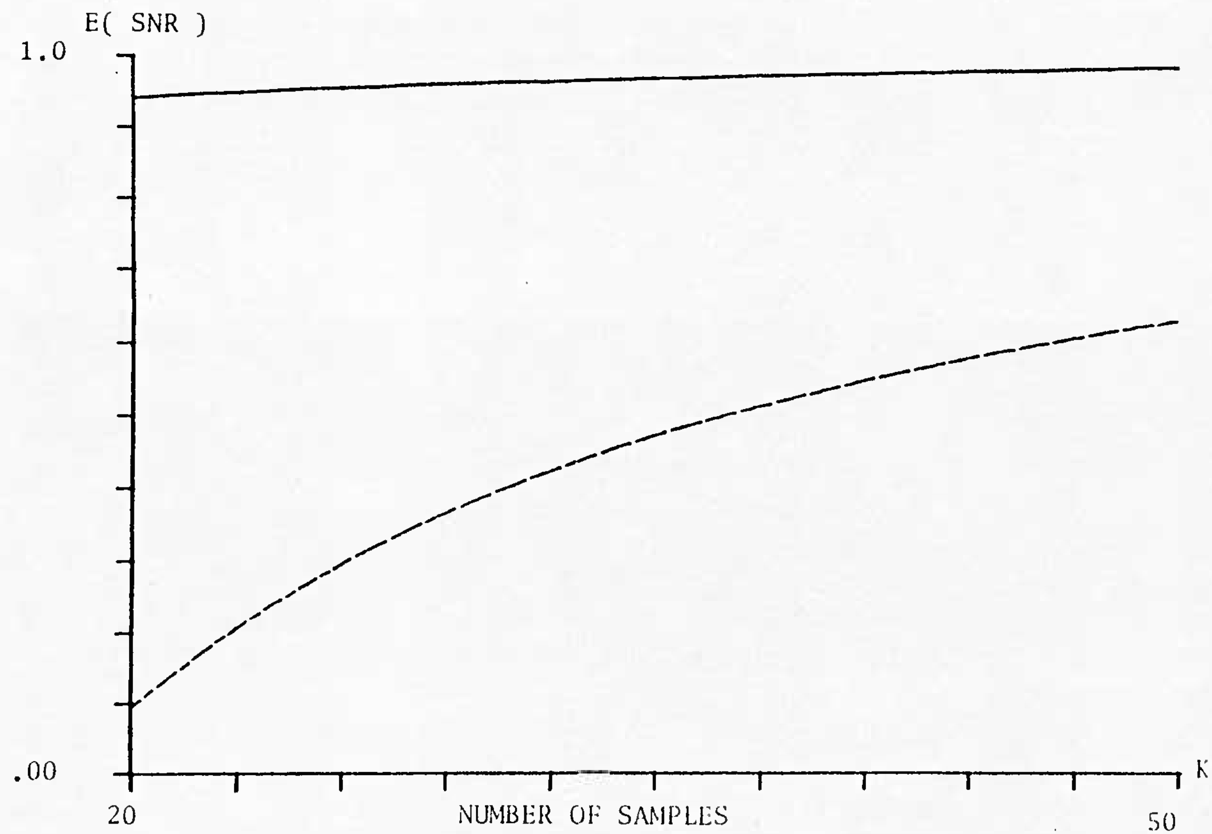


Fig. 6b) THEORETICAL EXPECTED VALUE OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 dB , f=.04 , ρ=20 , α=.9999

——— USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENVECTOR (1.15)

------ USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

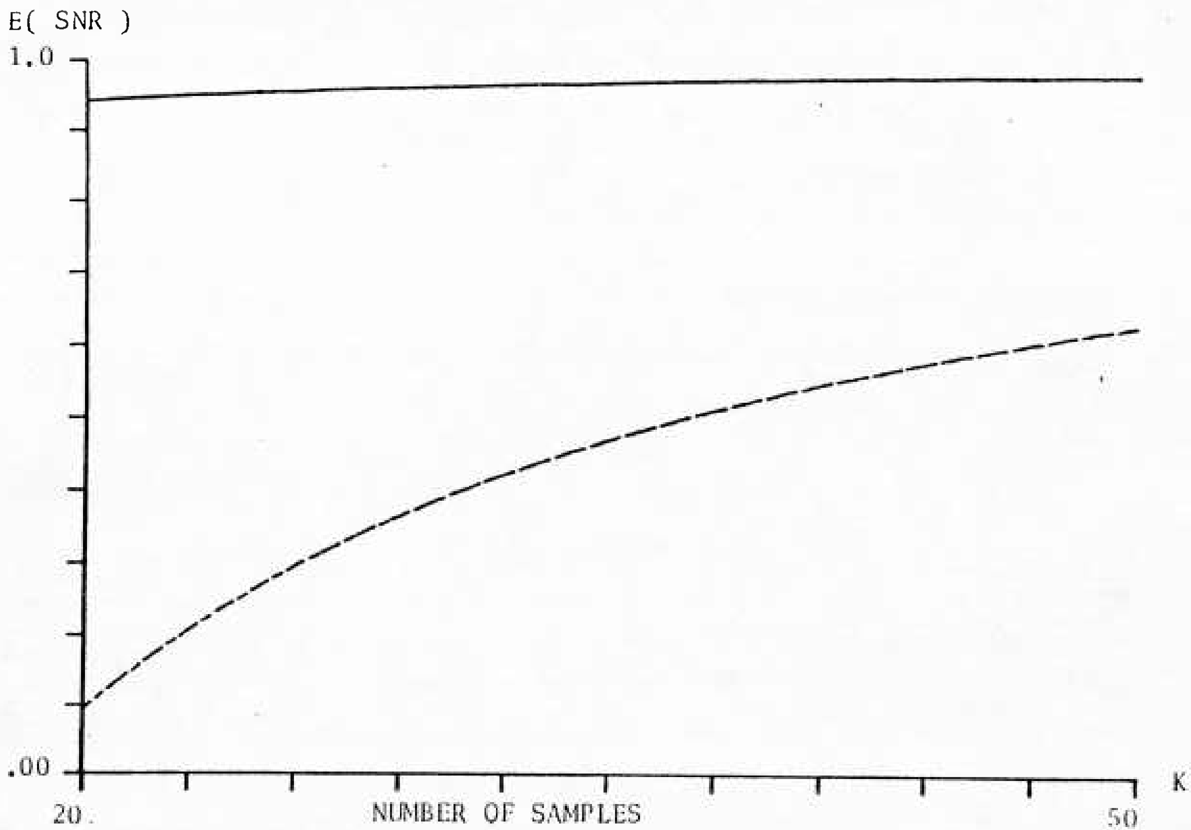


Fig. 6c) THEORETICAL EXPECTED VALUE OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 dB, f=.05 , ρ=20 , α=.9999

——— USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENVECTOR (1.15)

------ USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

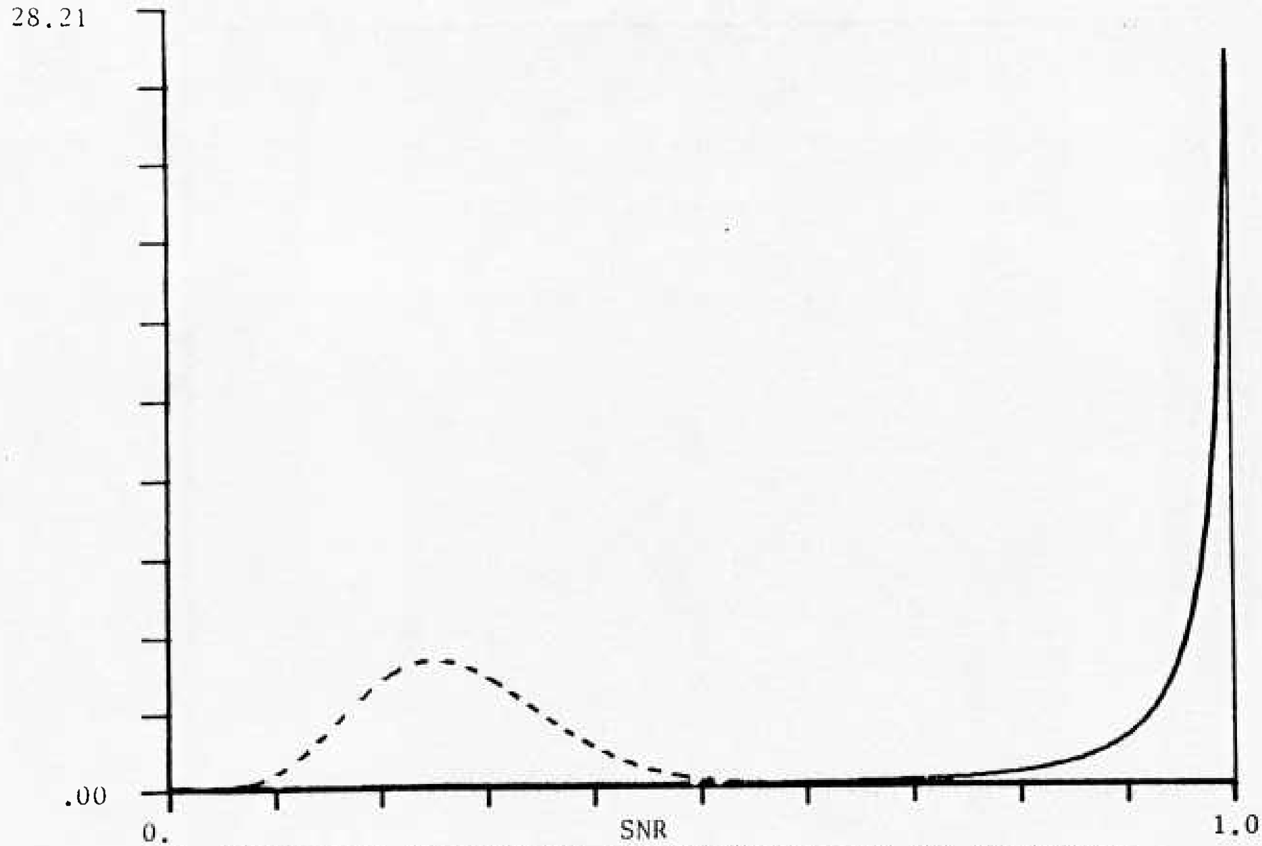


Fig. 7a) THEORETICAL PROBABILITY DENSITY OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 dB, f=.03 , K=25 , ρ=20, α=.9999

——— USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENVECTOR (1.15)

------ USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

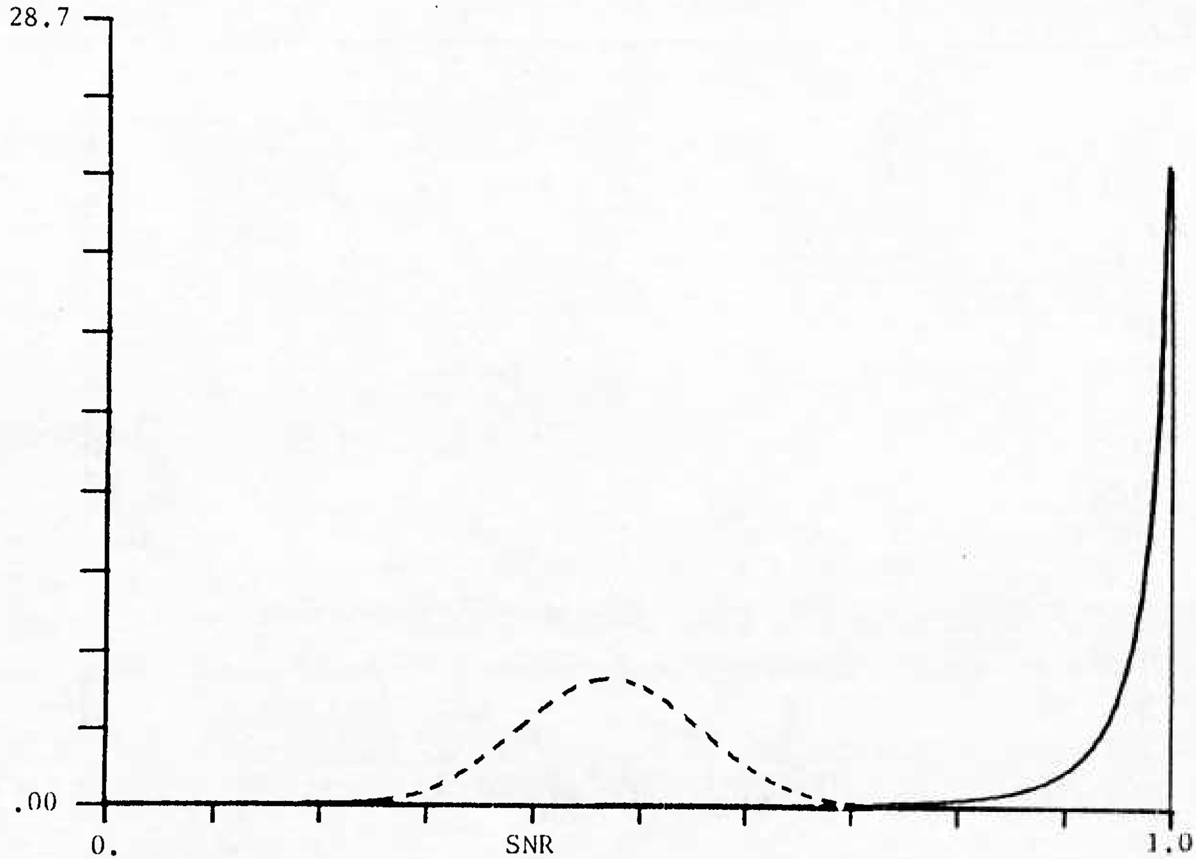


Fig. 7b) THEORETICAL PROBABILITY DENSITY OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 DB , $f=.03$, $K=35$, $\rho=20$, $\alpha=.9999$

——— USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENVECTOR (1.15)

------ USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

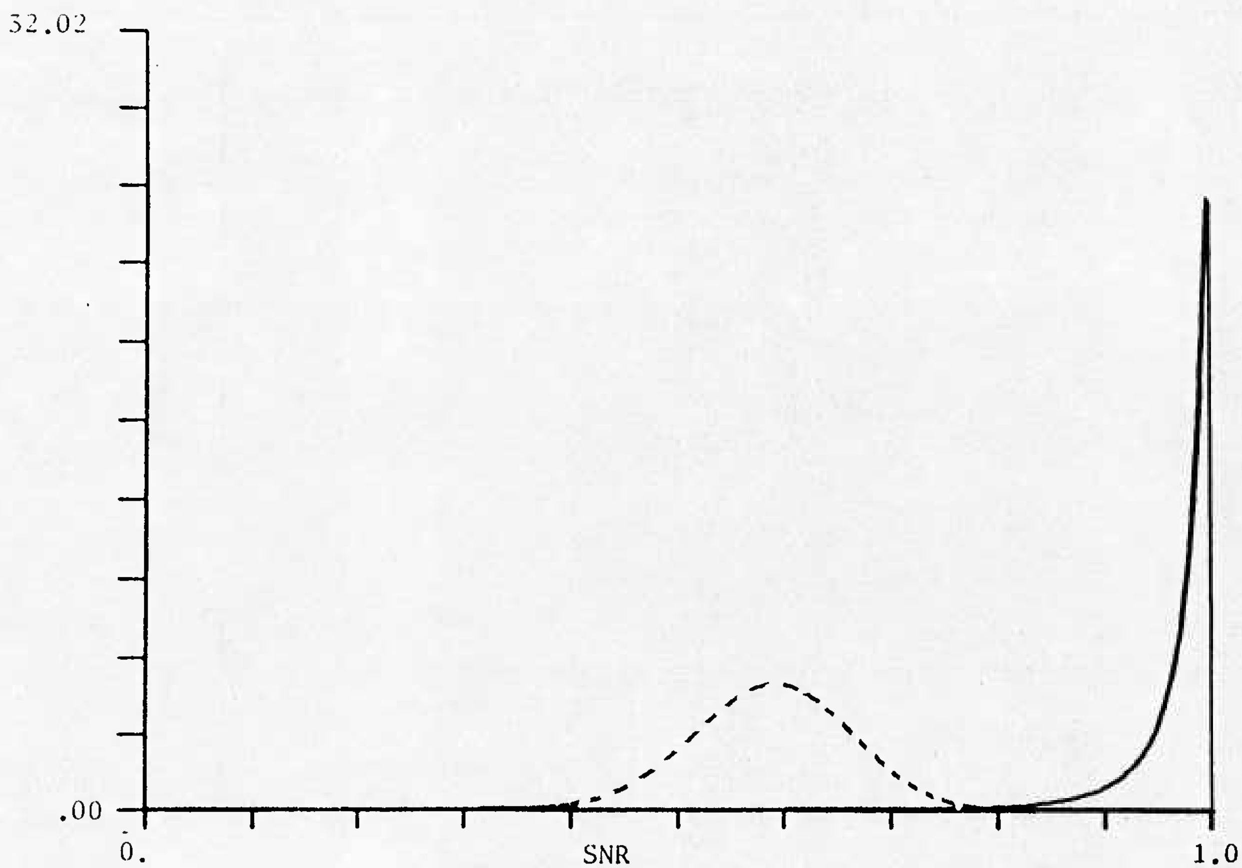


Fig. 7c) THEORETICAL PROBABILITY DENSITY OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 dB , f=.03 , K=45 , ρ=20 , α=.9999

——— USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENEVCTOR (1.15)

----- USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

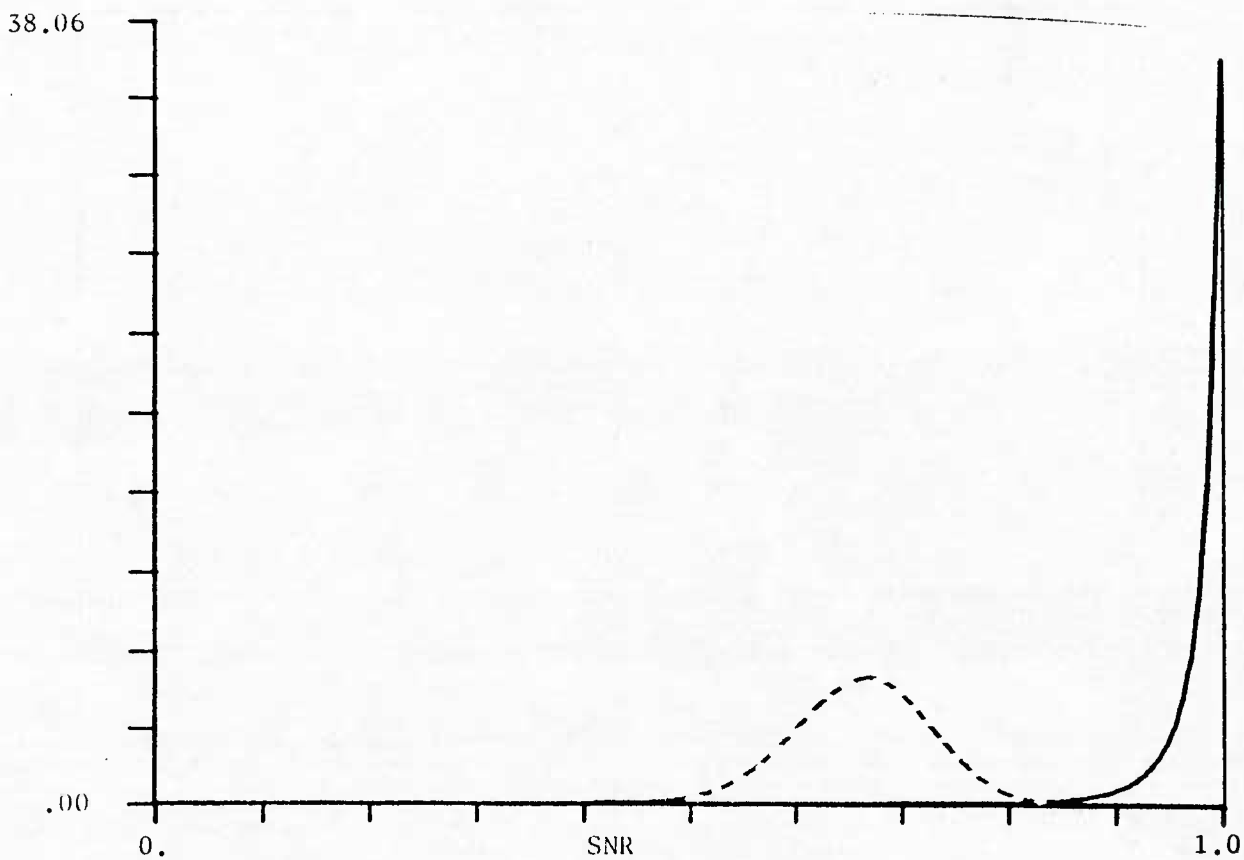


Fig. 7d) THEORETICAL PROBABILITY DENSITY OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 dB , f=.03 , K=55 , ρ=20 , α=.9999

______ USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENVECTOR (1.15)

\- - - - - USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

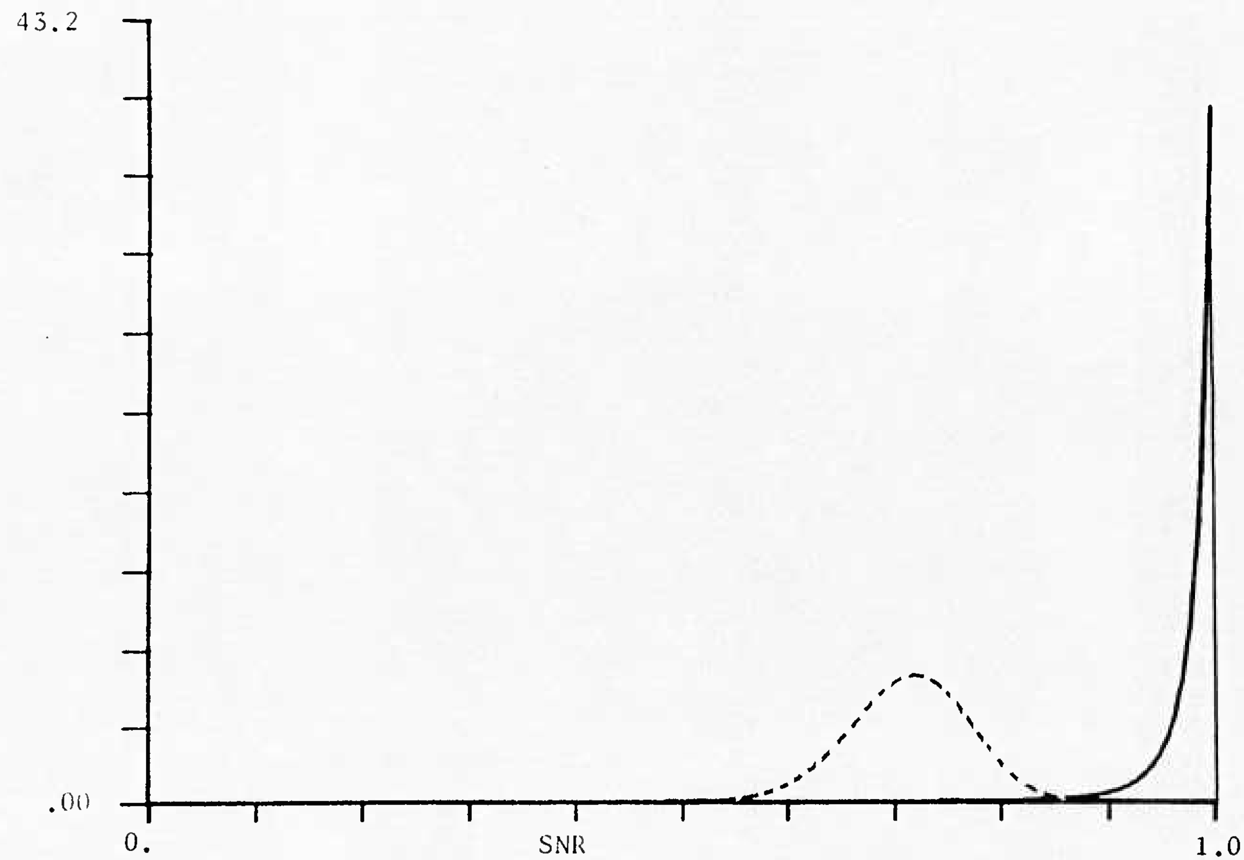


Fig. 7e) THEORETICAL PROBABILITY DENSITY OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 dB , f=.03 , K=65 , ρ=20 , α=.9999

——— USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENVECTOR (1.15)

------ USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

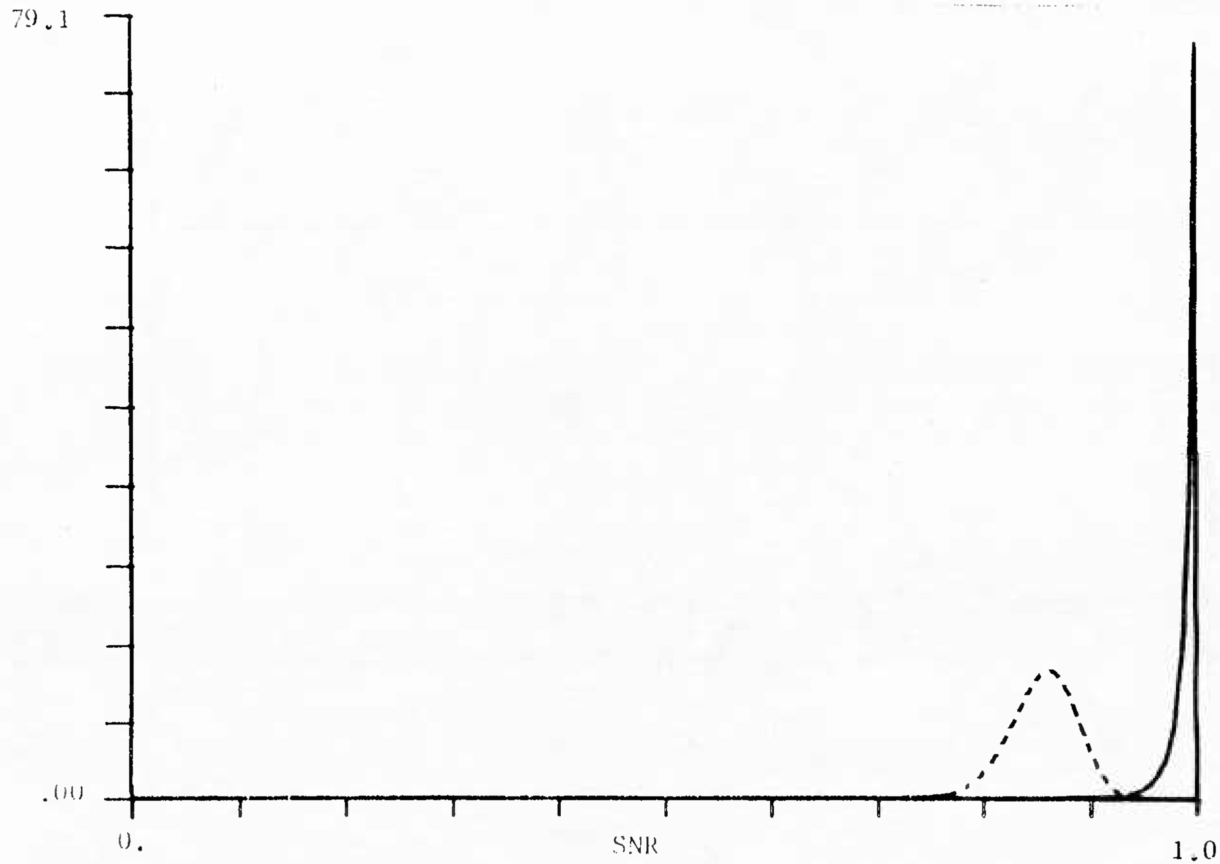


Fig 7f) THEORETICAL PROBABILITY DENSITY OF SNR FOR TWO METHODS OF DATA DEPENDENT SNR IMPROVEMENT : INR=10 dB , f=.03 , K=130 , ρ=20 , α=.9999

——— USING FILTER WEIGHTS BASED ON THE PRINCIPAL EIGENVECTOR (1.15)

- - - - - USING FILTER WEIGHTS BASED ON THE INVERSE OF THE ESTIMATED COVARIANCE MATRIX (1.09)

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