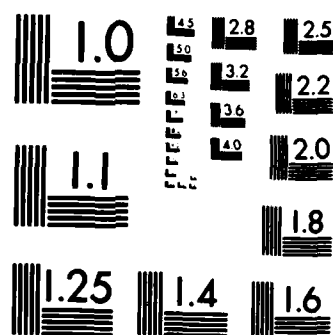END

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MODELING LOCALIZED STRESS
FIELDS IN COMPOSITE LAMINATES

Final Scientific Report

by

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19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

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Stress Concentration
Interlaminar stresses

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

A structural model is developed using Reissner's variational principle to analyze stress gradient problems in laminated composite plates. A statically admissible stress field, which has explicit dependence on the thickness coordinate, is selected for each layer. Both interlaminar traction and displacement continuity are enforced between layers. This model is similar to an earlier one developed by Pagano, but has four dependent variables less per layer than Pagano's model. It is shown that the model can duplicate Pagano's results for the same number of subdivisions through the thickness.

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The uniform axial extension of a bidirectional symmetric laminate is used as the example problem for comparison to Pagano's theory and to illustrate the methodology. *Originator-supplied keywords include: Sy... ... ... and ... ... stresses*

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1 INTRODUCTION

Classical lamination theory [1,2] ignores the thickness stress
components with respect to the in-plane stress components, and models a
laminate as an equivalent single layer with the familiar A, B, and D
stiffness matrices. It is a direct extension of classical plate theory
based on the Kirchhoff hypotheses. This theory is adequate for many
engineering problems. However, there are problems where thickness
effects are important and, consequently, classical lamination theory is
inadequate.

Laminated plates manufactured from advanced filamentary composite
materials, like graphite-epoxy, are susceptible to thickness effects
because their effective transverse shear moduli are significantly
smaller than the effective elastic modulus along the fiber direction.
The concept of a "thin" plate, based solely on geometric parameters
obtained from analyses of homogeneous metallic plates, must be carefully
considered for these materials. The higher order plate theory of
Whitney and Sun,[3] for example, is based on accounting for thickness
shear and thickness normal deformations, although the laminate is treated
as an equivalent single layer.

Refined plate theories which integrate the lamina properties
through the plate thickness into an equivalent single layer theory
provide improved global response estimates for deflections, vibration
frequencies, and buckling loads in this class of materials. However,
thickness stress response in localized regions of geometric, load, and

material discontinuity require more detailed analysis. For example, interlaminar stresses which are significant near the straight free edge of tensile coupons,[4] near cutouts,[5] and near supported edges,[6] are neglected in the formulation of equivalent single layer theories. Although interlaminar stresses may be small with respect to in-plane stress components, so is the thickness strength of advanced composite laminates significantly smaller than their in-plane fiber direction strength. Hence, a theory which incorporates interlaminar stress response is necessary to implement a rational delamination failure criteria. A minimum requirement of such a theory would be a ply-by-ply analysis. Such theories have been developed by Seide,[7] Srinivas,[8] and Pagano.[9] Of course, the more detailed the theory the more difficult and/or cumbersome it is to obtain solutions to specific problems. Elasticity solutions, which model each ply as a homogeneous anisotropic material with effective moduli, are complicated for practical laminates, and are fraught with difficult issues such as the existence of stress singularities[10] and, if they exist, how to effectively incorporate them into the analysis. The approximate ply-by-ply structural theories are more tractable with respect to elasticity, but obliterate singularities. Nevertheless, such approximate theories should provide improved estimates of interlaminar stresses for delamination studies.

2 OBJECTIVE

The objective of this research is to develop a structural model for the static stress response of laminate composite plates in the vicinity

of stress risers. The laminated plate model shall include all stress components in general, and satisfy both interlaminar traction and displacement continuity. The mathematical model developed to meet these criteria should be amenable to numerical solution, and hence provide a stress analysis tool for delamination studies and for stress concentration problems in which three-dimensional effects are important.

3 STATUS OF THE RESEARCH

A mathematically linear laminated plate model was developed using Reissner's variational principle. The laminae are assumed to be linear elastic, anisotropic, and homogeneous. The model includes interlaminar stresses as dependent variables and satisfies both interlaminar traction and displacement continuity. By assuming a statically admissible stress field which has explicit dependence on the thickness coordinate z in each layer, the field equations are reduced to partial differential equations in the reference plane coordinates x and y. For the general case of the static response of a laminated plate there would be 19N equations and dependent variables, where N is the number of layers through the thickness. The methodology of approach, and details of the mathematical model for a $[0/90]_s$ laminate subject to uniform axial extension, are documented in Reference 11.

The laminated plate model is similar to one developed by Pagano[9] in 1978, but the present model has 4N dependent variables and equations less than Pagano's model. The reduction in the size of the mathematical model relative to Pagano's model is achieved by a different assumption

for the normal stress component in the thickness direction for each layer. As shown in Reference 11, the mathematical model can duplicate Pagano's solutions for the problem of uniform axial extension of a $[0/90]_s$ laminate with traction free lateral surfaces. Thus the laminated plate model should be computationally more efficient with respect to Pagano's model.

4 PUBLICATIONS

4.1 Report; see Reference 11.

4.2 Reference 11 was submitted for publication in <u>Composite Structures</u>, an international journal, June 1984.

5 PROFESSIONAL PERSONNEL

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6 INTERACTIONS

6.1 Spoken paper

"Modeling the Stress Field in Laminated Composite Plates Near Discontinuities", by E. R. Johnson, April 10, 1984 at the 1st Annual Technical Review for the Center for Composite Materials and Structures, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, April 9-11, 1984.

6.2 Seminar and Discussions

"Modeling the Stress Field in Laminated Composite Plates Near Discontinuities", by E. R. Johnson, presented at the Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio, April 30, 1984. Discussions followed with Drs. N. J. Pagano, S. R. Soni, V. B. Venkayya, N. S. Khot, and G. P. Sendeckyj.

7 REFERENCES

1. E. Reissner and Y. Stavsky, "Bending and Stretching of Certain Types of Heterogeneous Aeolotropic Elastic Plates," J. of Appl. Mech., Vol. 28, p. 402 (1961).

2. S. B. Dong, K. S. Pister and R. L. Taylor, "On the Theory of Laminated Anisotropic Shells and Plates," J. Aero. Sci., Vol. 28, p. 969 (1962).

3. J. M. Whitney and C. T. Sun, "A Higher Order Theory for Extensional Motion of Laminated Composites," J. Sound and Vibration, Vol. 30, pp. 85-97 (1973).

4. R. B. Pipes and N. J. Pagano, "Interlaminar Stresses in Composite Laminates Under Uniform Axial Extension," J. Composite Materials, Vol. 4, pp. 538-548 (1970).

5. J. B. Whiteside, I. M. Daniel and R. E. Rowlands, "The Behavior of Advanced Filamentary Composites Plates with Cutouts," Air Force Flight Dynamics Laboratory Technical Report AFFDL-TR-73-48, Wright-Patterson Air Force Base, Ohio (1973).

6. T. L. Waltz and J. R. Vinson, "Interlaminar Stresses in Laminated Cylindrical Shells of Composite Materials," AIAA Paper No. 75-755, presented at the 16th Structures, Structural Dynamics and Materials Conference, Denver, CO, May 27-29, 1975.

7. Paul Seide, "An Improved Approximate Theory for the Bending of Laminated Plates," in proc. of symposium honoring E. Reissner, University of California San Diego, La Jolla, CA, June 23, 1978.

8. S. Srinivas, "A Refined Analysis of Composite Laminates," J. Sound and Vibration, Vol. 30, pp. 495-507 (1973).

9. N. J. Pagano, "Stress Fields in Composite Laminates," Int. J. Solids and Structures, Vol. 14, pp. 385-400 (1978).

10. S. S. Wang and I. Choi, "Boundary-Layer Effects in Composite Laminates: Part I - Free Edge Stress Singularities," J. Appl. Mech., Vol. 49, pp. 541-548 (1982).

11. E. R. Johnson and B. L. Kemp, "Modeling the Stress Field in Laminated Composite Plates Near Discontinuities," Center for Composite Materials and Structures Report CCMS-84-09, and Virginia Polytechnic Institute and State University Report VPI-E-84-21, Blacksburg, VA 24061, 1984.

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