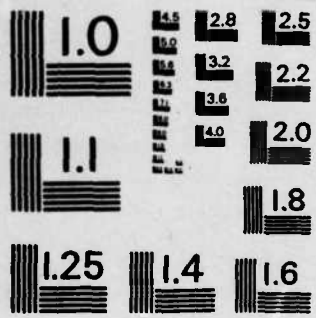
AD-A148 444 ON MODELING THE PERFORMANCE AND RELIABILITY OF MULTI-MODE COMPUTER SYSTEM..(U) DUKE UNIV DURHAM NC DEPT OF COMPUTER SCIENCE V G KULKARNI ET AL. NOV 84 1/1

UNCLASSIFIED CS-1984-6 AFOSR-TR-84-1098 DAAG29-84-K-0045 F/G 9/2 NL



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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) CS-1984-6	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 84-1098
6a. NAME OF PERFORMING ORGANIZATION Duke University	6b. OFFICE SYMBOL (If applicable)
7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research	
6c. ADDRESS (City, State and ZIP Code) Department of Computer Science Durham NC 27706	7b. ADDRESS (City, State and ZIP Code) Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR	8b. OFFICE SYMBOL (If applicable) NM
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-84-0132	
8c. ADDRESS (City, State and ZIP Code) Bolling AFB DC 20332-6448	10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
61102F	2304	A5	

11. TITLE (Include Security Classification)
ON MODELING THE PERFORMANCE AND RELIABILITY OF MULTI-MODE COMPUTER SYSTEMS

12. PERSONAL AUTHOR(S)
V.G. Kulkarni*, V.F. Nicola and K.S. Trivedi

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Technical	FROM ____ TO ____	NOV 84	20

16. SUPPLEMENTARY NOTATION
*University of North Carolina, Chapel Hill.

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
MAJ Brian W. Woodruff	(202) 767- 5027	NM

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AFOSR-TR- 84-1098

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CS-1984-6

Accession For		
NTIS GRA&I	X	
DTIC TAB	☐	
Unannounced	☐	
Justification		
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Distribution/		
Availability Codes		
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On Modeling the Performance and Reliability of

Multi-Mode Computer Systems*

V. G. Kulkarni†, V. F. Nicola‡ and K. S. Trivedi‡

Abstract

We present an effective technique for the combined performance and reliability analysis of multi-mode computer systems. A reward rate (or a performance level) is associated with each mode of operation. The switching between different modes is characterized by a continuous time Markov chain. Different types of service-interruption interactions (as a result of mode switching) are considered. We consider the execution time of a given job on such a system and derive the distribution of its completion time. A useful dual relationship, between the completion time of a given job and the accumulated reward up to a given time, is noted. We demonstrate the use of our technique by means of a simple example.

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* This work was supported in part by Air Force Office of Scientific Research under grants AFOSR-84-0132 and AFOSR-84-0140, by the Army Research Office under contract DAAG29-84-K0045 and by the National Science Foundation under grant MCS-830200.

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1. *Introduction*

We consider a model for the combined evaluation of performance and reliability of a multi-mode computer system. Performance (e.g., throughput, response time, instruction execution rate) changes from mode to mode and a mode change occurs in response to an event such as a failure or a repair. The stochastic process representing the modes (structure-states) and mode changes can be thought of as a reward process by associating a reward (performance index) with each mode [4,10]. We can then study the the distribution of the accumulated reward until time t by time domain methods[10] or by transform techniques[4,14].

The authors who have taken such a system-oriented view do not consider the effect of a fault occurring during the execution of a program. A task(job or program)-oriented view of such a sytsem recognizes the fact that it is possible for a system failure to occur before the completion of a task [7] and that even if the task is completed, its completion time is likely to be different from its execution time in a given mode [3,5,12]. The job in service is interrupted with each mode change and the type of service -interruption interaction depends upon the mode just entered. For example, the occurrence of a fault during the execution of a job preempts the job and a later system recovery may allow the job to resume from the point of interruption (the preemptive-resume (*prs*) discipline) or the job may have to be repeated from the beginning. In the latter case, the repeated job may have the identical work requirement as the original preempted job (the preemptive-repeat-identical (*pri*) discipline) or a different work requirement sampled from the same distribution (the preemptive-repeat-different (*prd*) discipline).

The purpose of this paper is to develop a model that unifies and extends the efforts of these two groups of researchers. In particular, we show that if all interruptions are of the preemptive-resume type then the completion time of a given task and the accumulated reward until a given time are dual measures, so that the distribution of one of them allows us to compute the distribution of the other. In fact, our model is even more general - in that both acyclic (closed or non-repairable) and cyclic (open or repairable) systems are modeled.

Our model provides an exact analysis of the completion time distribution of a program (job) executing in a multi-mode system. It is also possible to incorporate the effect of queueing in our model. If the time spent in each structure-state is large compared with the interarrival and processing times of jobs, then we can use steady- state performance measures as reward rates for each structure-state. Such approximate decomposition methods have been considered by several authors [4,7,10,15]. If the assumption of a wide separation between the structure-state holding times and job processing times does not hold, then a more complex analysis is required [1,5,12].

We develop the basic model in the next section. In sections 3, 4 and 5 , we consider the individual cases where all structure-states are of the same type, that is, preemptive-resume, preemptive-repeat-identical, or preemptive-repeat-different, respectively.

2. *The Basic Model*

Consider a single server (e.g., a computer) serving a single job (e.g., a program). The job is characterized by its work requirement, B. For example, the work requirement of a computer program can be measured in terms of the number of instructions to be executed. We assume that B is a random variable with cumulative distribution function $G(x) = P(B \leq x)$ and *LST* $G^{\sim}(s) = E(e^{-sB})$. To avoid trivialities we assume $G(0+) = 0$.

The rate at which the server performs work is assumed to change with time according to the following model: At any time the server is in one (and only one) of the $n+1$ states (modes) numbered $0,1,2,\ldots,n$. In state i the server performs work at rate $r_i \geq 0$, $1 \leq i \leq n$, work units per unit time (e.g., the instruction execution rate). The state

0 is an absorbing "failure" state, i.e., once the server is in state 0, it stays there forever and the work rate in this state is zero (r_0=0). We allow absorbing non-failure states among the states 1,...,n with reward rates greater than zero so that if the server enters such a state, the job will eventually complete. Let $Z(t)$ be the state of the server at time t. $\{Z(t), t \geq 0\}$ is called the ***structure-state process***. We shall assume that the structure-state process is a stochastic process with piecewise constant paths with finite number of jumps in finite intervals of time. Furthermore, the structure-state process is assumed to be independent of the work requirement B of the job.

The states $i = 1,2,...,n$ are classified as (i) ***prs***: preemptive-resume, (ii) ***pri***: preemptive-repeat-identical or (iii) ***prd***: preemptive-repeat-different.

The following quantities have been analyzed before in the literature for some special $\{Z(t), t \geq 0\}$ processes:

I. ***The job completion time (T(x)):*** defined to be the total time the server takes to complete a job that requires x units of work. T denotes the unconditional completion time of a job that requires a random amount of work, say B units. Gaver[5] studied the distribution of the r.v. T for a system subject to one type of failure and repair, in which the operating state is Markovian and the failure state is semi-Markovian. Nicola[12] extended Gaver's model to allow for mixed types of failures and repairs. Castillo and Siewiorek [3] condsidered a system with two types of failures in which the preemptive-repeat type failure could occur during the repair-time of the preemptive-resume type failure.

II. ***The probability of dynamic failure (η):*** defined to be the probability that the system fails before the job is completed, i.e. the server enters state 0 before completing B units of work[7].

III. ***The cumulative reward upto time t ($Y(t)$):*** defined to be the total amount of work done by the system up to time t. Y is the total accumulated work during the system's lifetime; it is the limit of $Y(t)$ as $t \to \infty$. The r.v. $Y(t)$ was first studied by Puri [14] for Markovian $Z(t)$ processes. Meyer[10] and Donatiello and Iyer[4] studied the distribution of $Y(t)$ for an acyclic Markovian $Z(t)$ process. Beaudry [2] studied the r.v. Y for a Markovian $Z(t)$ process, while Osaki and Nishio [13] studied the r.v. Y for a semi-Markovian $Z(t)$ process.

To present a unifying view of the quantities defined above, we introduce the cumulative measure, $W(t)$, defined as follows: Suppose that at time $t = 0$ the server starts processing a job with infinite work requirement. $W(t)$ is the amount of useful work completed by the server until time t (thus, excluding the work done prior to the last visit to a *pri* or a *prd* state). The following properties of the cumulative measure, $W(t)$, are immediately obvious:

(i) $W(0) = 0$,

(ii) $Z(t) = i \Rightarrow dW(t)/dt = r_i$,

(iii) If there is a transition in the structure-state process at time t and $Z(t+) = i$, then $W(t+) = 0$ if i is a *pri* or a *prd* state and $W(t+) = W(t-)$ if i is a *prs* state.

Typical sample paths of the structure-state process and the cumulative measure, $W(t)$, are shown in figure 1, for the following case: Set of states = $\{0,1,2,3\}$, state 1 is *prs* with $r_1 = 1$, states 2 and 3 are *pri* or *prd* with $r_2 = 2$ and $r_3 = 0$, state 0 is the absorbing failure state.

The following theorem shows how the quantities T, η, $Y(t)$ and Y can be related to each other via the cumulative measure, $W(t)$.

Theorem 1.

(i) $T = \min\{t \geq 0 : W(t) = B\}$.

(ii) The dynamic failure probability, $\eta = P(T = \infty)$.

(iii) If all states are *prs*, then

$$P(Y(t) \le x) = 1 - P(T(x) < t)$$

and

$$P(Y \le x) = 1 - P(T(x) < \infty).$$

Proof: (i) Let T be the job completion time. It is clear that

$$\{T > t\} \Leftrightarrow \{W(u) < B, \text{ for } all \ 0 \le u \le t\},$$

since $W(u)$ represents the useful work done upto time u. As $W(t)$ has piecewise continuous paths with only downward jumps, T is given by (i).

(ii) It is clear that

$$\{\text{ Dynamic Failure }\} \Leftrightarrow \{\text{ system fails before job completion }\}$$
$$\Leftrightarrow \{W(t) < B \text{ for } all \ t \ge 0\} \Leftrightarrow \{T = \infty\}.$$

Hence $\eta = P(T = \infty)$.

(iii) Let $T(x) = \min\{t \ge 0 : W(t) = x\}$. If all states are *prs*, then

$$\{Y(t) > x\} \Leftrightarrow \{W(t) > x\} \Leftrightarrow \{T(x) < t\}.$$

Hence

$$P(Y(t) > x) = P(T(x) < t). \qquad Q.E.D.$$

It is apparent from the above theorem that

$$T = \min\{t \ge 0 : W(t) = B\} \tag{2.1}$$

is the unifying random variable. This paper is devoted to the study of this random variable. Define the following distribution functions:

$$F_i(t,x) = P(T \le t \mid B = x, Z(0) = i), \ x \ge 0, \ 1 \le i \le n,$$

$$F(t,x) = P(T \le t \mid B = x), \ x \ge 0,$$

$$F_i(t) = P(T \le t \mid Z(0) = i), \ 1 \le i \le n,$$

$F(t) = P(T \le t)$

and the corresponding *LST*s (Laplace Stieltjes Transforms),

$$F_i^{\sim}(s,x) = E(e^{-sT} \mid B = x, Z(0) = i), \quad x \ge 0, 1 \le i \le n, \tag{2.2}$$

$$F^{\sim}(s,x) = E(e^{-sT} \mid B = x), \quad x \ge 0, \tag{2.3}$$

$$F_i^{\sim}(s) = E(e^{-sT} \mid Z(0) = i), \quad 1 \le i \le n, \tag{2.4}$$

$$F^{\sim}(s) = E(e^{-sT}). \tag{2.5}$$

From the independence of $\{Z(t), t \ge 0\}$ and B it follows that

$$F^{\sim}(s,x) = \sum_{i=1}^{n} F_i^{\sim}(s,x) P(Z(0) = i), \quad x \ge 0, \tag{2.6}$$

$$F_i^{\sim}(s) = \int_0^{\infty} F_i^{\sim}(s,x) dG(x), \quad 1 \le i \le n, \tag{2.7}$$

$$F^{\sim}(s) = \sum_{i=1}^{n} F_i^{\sim}(s) P(Z(0) = i). \tag{2.8}$$

From equations (2.6) - (2.8) it is clear that the conditional *LST*s $F_i^{\sim}(s,x)$ are of central importance to the analysis of T. In order to obtain explicit formulae for $F_i^{\sim}(s,x)$ it is necessary to make some further assumptions about the structure-state process. In the remaining paper we make the assumption that $\{Z(t), t \ge 0\}$ is a time homogeneous continuous time Markov chain (CTMC). The results derived here can be extended in a straight forward manner to the case when the structure-state process is assumed to be semi- Markov. Let q_{ij}, $1 \le i \ne j \le n$, be infinitesimal transition rate from state i to j and q_{i0} be the absorbing failure rate from state i. Let $Q = [q_{ij}]$, $1 \le i, j \le n$, be the n by n generator matrix where $q_i = \sum_{j=0, j \ne i}^{n} q_{ij} = -q_{ii}$. Note that row sums of Q are ≤ 0. We mention one property of the CTMC for future reference. Define

$$H = \min\{t \ge 0 : Z(t) \ne Z(0)\} \tag{2.9}$$

as the holding (or sojourn) time in the initial state. Then we have

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($^{\sim}$) denotes *LST*, i.e., the Laplace transform of a probability density function.

$$P(H \le x, Z(H+) = j \mid Z(0) = i) = \frac{q_{ij}}{q_i}(1-e^{-q_i x}), \quad (i \ne j). \tag{2.10}$$

In the next section we treat the case where all states $i = 1,2,..n$ are preemptive-resume (*prs*) and in sections 4 and 5 we consider the case where all states are preemptive-repeat (*pri* and *prd*, respectively). The mixed cases where some states are *prs* and some are *pri* or *prd* have been studied in [8].

3. *The Preemptive-resume Case*

In this section we assume that the states $1,2,..,n$ are all preemptive-resume states. Note that state 0 does not have to be classified since it is a failure state. Theorem 2 below gives a method of computing the conditional *LST*s defined by equation (2.2). First, some notation:

$$F_i^{\sim *}(s,u) = \int_0^\infty e^{-ux} F_i^{\sim}(s,x)dx \ , \quad 1 \le i \le n, \tag{3.1}$$

$$\underline{F}^{\sim *}(s,u) = [F_1^{\sim *}(s,u), F_2^{\sim *}(s,u),...,F_n^{\sim *}(s,u)]^T, \tag{3.2}$$

$$R = diag[r_1,r_2,...,r_n] \, , \tag{3.3}$$

$$\underline{r} = [r_1,r_2,...,r_n]^T, \tag{3.4}$$

where the superscript T denotes transpose.

Theorem 2. $F_i^{\sim *}(s,u)$, for $1 \le i \le n$, is given by

$$F_i^{\sim *}(s,u) = \frac{r_i}{s+r_i u+q_i} + \sum_{\substack{j=1 \\ j \ne i}}^{n} \frac{q_{ij}}{s+r_i u+q_i} F_j^{\sim *}(s,u), \ 1 \le i \le n. \tag{3.5}$$

Proof: Conditioning on the sojo[illegible] time H in the initial state we get

$$E(e^{-sT} \mid H=h, B=x, Z(0)=i) = \begin{cases} e^{-sx/r_i}, & \text{if } h \ge x/r_i \\ e^{-sh} \sum_{\substack{j=1 \\ j \ne i}}^{n} \frac{q_{ij}}{q_i} F_j^{\sim}(s,x-r_i h), & \text{if } h < x/r_i \end{cases}$$

(*) denotes the Laplace transform of a function

Unconditioning yields

$$F_i^{\sim}(s,x) = \int_0^{\infty} E(e^{-sT} \mid H = h, B = x, Z(0) = i) q_i e^{-q_i h}\, dh$$

$$= e^{-(s+q_i)x/r_i} + \sum_{\substack{j=1 \\ j \neq i}}^{n} q_{ij} \int_0^{x/r_i} e^{-(s+q_i)h} F_j^{\sim}(s, x - r_i h) dh$$

Multiplying both sides by e^{-ux} and integrating we get equation (3.5). *Q.E.D.*

Equation (3.5) can be put in a matrix form as follows:

$$[sI + uR - Q]\, \underline{F}^{\sim *}(s,u) = \underline{r},$$

where I is the identity matrix. As it is well known that $[sI + uR - Q]$ is invertible, we get

$$\underline{F}^{\sim *}(s,u) = [sI + uR - Q]^{-1}\, \underline{r} \; . \tag{3.5a}$$

A direct inversion with respect to s yields

$$d_t\, \underline{F}^{*}(t,u) = e^{(Q-uR)t}\, \underline{r} \; .$$

After integration and some manipulations, we get

$$\underline{F}^{*}(t,u) = \frac{1}{u}[I - e^{(Q-uR)t}]\, \underline{e} \; . \tag{3.6}$$

We now describe how we can use the above theorem to compute $F_i^{\sim}(s,x)$. Using Cramer's rule we can write

$$F_i^{\sim *}(s,u) = A_i(s,u) / C(s,u)$$

where $C(s,u) = det[sI + uR - Q]$ and $A_i(s,u)$ are appropriate n by n subdeterminants of the augmented matrix $[sI + uR - Q; \underline{r}]$. It is obvious that both $A_i(s,u)$ and $C(s,u)$ are polynomials in s and u. Hence one can use partial fractions to invert $F_i^{\sim *}(s,u)$ with respect to u. Let $d = |\{i : r_i > 0\}|$, i.e. d is the number of states in which work rate is positive. Then $C(s,u)$ is a d-degree polynomial in u for a fixed value of s. Let $-u_1(s), \ldots, -u_d(s)$ be the roots of $C(s,u)$. In the special case when these roots are distinct, we can write

$$F_i^{\sim *}(s,u) = \sum_{j=1}^{d} \frac{A_{ij}(s)}{u+u_j(s)}, \qquad 1 \le i \le n \ , \tag{3.7}$$

where

$$A_{ij}(s) = \lim_{u \to -u_j(s)} \frac{A_i(s,u)}{C(s,u)}(u + u_j(s)), \qquad 1 \le j \le d. \tag{3.8}$$

Inverting with respect to u, we get

$$F_i^{\sim}(s,x) = \sum_{j=1}^{d} A_{ij}(s)\, e^{-u_j(s)x}, \qquad 1 \le i \le n. \tag{3.9}$$

Hence from equation (2.4)

$$F_i^{\sim}(s) = \sum_{j=1}^{d} A_{ij}(s)\, G^{\sim}(u_j(s))\, , \qquad 1 \le i \le n, \tag{3.10}$$

(recall that $G^{\sim}(s) = \int_0^\infty e^{-sx} dG(x)$), and

$$F^{\sim}(s) = \sum_{j=1}^{d} [\sum_{i=1}^{n} \pi_i A_{ij}(s)]\, G^{\sim}(u_j(s)). \tag{3.11}$$

where $\pi_i = P(Z(0) = i)$, $1 \le i \le n$.

It is interesting to note that the *LST* of T for a given s is simply a linear combination of the *LST* of B evaluated at $u_1(s),\ldots,u_d(s)$.

Now, assuming that state 0 is reachable from every other state, the probability of dynamic failure can be computed easily from Theorem 1 as

$$\eta = P(T = \infty) = 1 - \lim_{s \to 0} F^{\sim}(s). \tag{3.12}$$

The following corollary indicates how the *LST* of the cumulative reward $Y(t)$, for a given t, can be obtained from the $F_i^{\sim *}(s,u)$ functions.

Corollary 1. For a given $t \ge 0$, let $Y(t)$ be the cumulative reward upto time t. Let

$$Y_i(x,t) = P(Y(t) \le x \mid Z(0) = i),$$

$$Y_i^{\sim}(u,t) = E(e^{-uY(t)} \mid Z(0) = i)$$

and

$$Y_i^{\sim *}(u,s) = \int_0^{\infty} e^{-st}\, Y_i^{\sim}(u,t)\, dt \quad .$$

Then

$$Y_i^{\sim *}(u,s) = \frac{1}{s}(1 - uF_i^{\sim *}(s,u)) \ , \quad 1 \le i \le n. \tag{3.13}$$

Proof: Part (iii) of Theorem 1 implies that

$$P(Y(t) < x \mid Z(0) = i) = P(T(x) > t \mid Z(0) = i).$$

Now,

$$Y_i^{\sim *}(u,s) = \int_0^{\infty} e^{-st} E(e^{-uY(t)} \mid Z(0)=i)dt$$

$$= \int_{t=0}^{\infty} e^{-st} \int_{x=0}^{\infty} e^{-ux} d_x P(Y(t) \le x \mid Z(0)=i)dt$$

$$= \int_{x=0}^{\infty} e^{-ux} \int_{t=0}^{\infty} e^{-st} d_x P(Y(t) \le x \mid Z(0) = i)dt$$

$$= \int_{x=0}^{\infty} e^{-ux} d_x[\int_{t=0}^{\infty} e^{-st}[1-P(T(x) \le t \mid Z(0) = i)]dt]$$

$$= -\int_{x=0}^{\infty} e^{-ux} d_x\, F_i^{\sim}(s,x)/s = [1 - u\, F_i^{\sim *}(s,u)]/s. \qquad Q.E.D.$$

Using equation (3.5a), we can write in a matrix form

$$\underline{Y}^{\sim *}(u,s) = [sI + uR - Q]^{-1}\, \underline{e} \tag{3.13a}$$

with

$$\underline{Y}^{\sim *}(u,s) = [Y_1^{\sim *}(u,s), Y_2^{\sim *}(u,s), \ldots, Y_n^{\sim *}(u,s)]^T.$$

A direct inversion with respect to s yields

$$\underline{Y}^{\sim}(u,t) = e^{(Q-uR)t}\, \underline{e} \ . \tag{3.14}$$

We end this section with a simple example.

Example 3.1. The switching server

Consider a system that operates in two modes each with a different work rate, say r_1 and r_2 for modes "1" and "2", respectively. The system switches between the two modes according to a Poisson process at different rates, say λ and μ from modes "1" and "2", respectively. A total system failure may occur at any mode of operation at different rates, say λ_0 and μ_0 for modes "1" and "2", respectively. The CTMC representing the switching server is shown in figure 2. In the case where a total system failure may not occur, i.e. $\lambda_0 = \mu_0 = 0$ and if $r_2 = 0$ then the switching server model reduces to the completion time model of job execution in a system subject to breakdowns and repairs considered by Gaver [5].

In this example we consider the case in which both states 1 and 2 are of the preemptive-resume type. We note that if we set $\mu = 0$ in this example we obtain the reward model of a two processor system considered by Meyer [10]. In our example the Q matrix is

$$Q = \begin{bmatrix} -\lambda' & \lambda \\ \mu & -\mu' \end{bmatrix}$$

where $\lambda' = \lambda + \lambda_0$ and $\mu' = \mu + \mu_0$. Then from Equation (3.5a)

$$\begin{bmatrix} F_1^{\sim *}(s,u) \\ F_2^{\sim *}(s,u) \end{bmatrix} = \begin{bmatrix} s + r_1 u + \lambda' & -\lambda \\ -\mu & s + r_2 u + \mu' \end{bmatrix}^{-1} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}.$$

Solving for $F_1^{\sim *}(s,u)$ and $F_2^{\sim *}(s,u)$ we get

$$F_1^{\sim *}(s,u) = \frac{r_1 r_2 u + r_1(s+\mu') + r_2\lambda}{(s+\lambda' + r_1 u)(s+\mu'+r_2 u) - \lambda\mu}$$

$$F_2^{\sim *}(s,u) = \frac{r_1 r_2 u + r_2(s+\lambda') + r_1\mu}{(s+\lambda'+r_1 u)(s+\mu'+r_2 u) - \lambda\mu}.$$

Hence, using eq. (3.9) we get

$$F_1^{\sim}(s,x) = A_{11}(s)\exp(-u_1(s)x) + A_{12}(s)\exp(-u_2(s)x),$$

$$\tilde{F_2}(s,x) = A_{21}(s)exp(-u_1(s)x) + A_{22}(s)exp(-u_2(s)x)$$

where

$$u_1(s) = [r_1(s+\mu')+r_2(s+\lambda')+\sqrt{(r_1(s+\mu')-r_2(s+\lambda'))^2+4\lambda\mu r_1 r_2}]/(2r_1r_2)$$

$$u_2(s) = [r_1(s+\mu')+r_2(s+\lambda')-\sqrt{(r_1(s+\mu')-r_2(s+\lambda'))^2+4\lambda\mu r_1 r_2}]/(2r_1r_2)$$

$$A_{11}(s) = (r_1(s+\mu') + r_2\lambda - r_1r_2u_1(s))/[(u_2(s) - u_1(s))r_1r_2]$$

$$A_{12}(s) = (r_1(s+\mu') + r_2\lambda - r_1r_2u_2(s))/[(u_1(s) - u_2(s))r_1r_2]$$

$$A_{21}(s) = (r_2(s+\lambda') + r_1\mu - r_1r_2u_1(s))/[(u_2(s) - u_1(s))r_1r_2]$$

$$A_{22}(s) = (r_2(s+\lambda') + r_1\mu - r_1r_2u_2(s))/[(u_1(s) - u_2(s))r_1r_2]$$

Then

$$\tilde{F}(s) = [\pi_1A_{11}(s)+\pi_2A_{21}(s)]\tilde{G}(u_1(s))+[\pi_1A_{12}(s)+\pi_2A_{22}(s)]\tilde{G}(u_2(s)) \,..$$

And η, the probability of dynamic failure, is given by $1 - \tilde{F}(0)$.

From corollary 1, we have

$$\tilde{Y}_1^{*}(u,s) = \frac{1}{s}[1 - u\,\tilde{F}_1^{*}(s,u)]$$

$$= \frac{(s+\lambda')(s+\mu') + r_2u(s+\lambda_0) - \lambda\mu}{s[(s+\lambda'+r_1u)(s+\mu'+r_2u)-\lambda\mu]}$$

$$= \frac{B_{10}(u)}{s} + \frac{B_{11}(u)}{s+s_1(u)} + \frac{B_{12}(u)}{s+s_2(u)}$$

where

$$s_1(u) = \frac{1}{2}[(\lambda' + r_1u + \mu' + r_2u) + \sqrt{(\lambda'+r_1u-\mu'-r_2u)^2 + 4\lambda\mu}]$$

$$s_2(u) = \frac{1}{2}[(\lambda' + r_1u + \mu' + r_2u) - \sqrt{(\lambda'+r_1u-\mu'-r_2u)^2 + 4\lambda\mu}]$$

$$B_{10}(u) = \frac{\lambda'\mu' + r_2u\lambda_0 - \lambda\mu}{s_1(u)s_2(u)}$$

$$B_{11}(u) = \frac{(\lambda'-s_1(u))(\mu'-s_1(u)) + r_2u(\lambda_0-s_1(u)) - \lambda\mu}{s_1(u)[s_1(u)-s_2(u)]}$$

$$B_{12}(u) = \frac{(\lambda'-s_2(u))(\mu'-s_2(u)) + r_2u(\lambda_0-s_2(u)) - \lambda\mu}{s_2(u)[s_2(u)-s_1(u)]}$$

Inverting with respect to s, yields

$$Y_1^{\sim}(u,t) = B_{10}(u) + B_{11}(u)e^{-s_1(u)t} + B_{12}(u)e^{-s_2(u)t}.$$

In a similar manner we can compute $Y_2^{\sim}(u,t)$. We note that the above *LST*s can be inverted in this case to obtain the distribution function of $Y(t)$ as an infinite sum of Bessel functions owing to the occurrences of radicals in the expresions of $s_1(u)$ and $s_2(u)$. However, in the case that μ=0 (as considered by Meyer), the radicals disappear and the inversion is relatively easy (as derived in [4] for arbitrary number of processors).

4. *The Preemptive-repeat-identical Case.*

In this section we assume that all states are preemptive repeat-identical. The main result is given in the following:

Theorem 3. The conditional *LST*s $F_i^{\sim}(s,x)$, $1 \le i \le n$ as defined in equation (2.2) satisfy the following simultaneous equations:

$$F_i^{\sim}(s,x) = e^{-(s+q_i)x/r_i} + \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{q_{ij}}{(s+q_i)}(1-e^{-(s+q_i)x/r_i})F_j^{\sim}(s,x),\ 1 \le i \le n. \qquad (4.1)$$

Proof: Conditioning on the holding time H in the initial state we have

$$E(e^{-sT} \mid H=h, B=x, Z(0)=i) = \begin{cases} e^{-sx/r_i} & ,\text{if } h \ge x/r_i \\ e^{-sh} \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{q_{ij}}{q_i} F_j^{\sim}(s,x), & \text{if } h < x/r_i \end{cases}$$

Unconditioning yields equation (4.1). *Q.E.D.*

Solving equations (4.1) we get $F_i^{\sim}(s,x)$, for $1 \le i \le n$. Then equations (2.7) and (2.8) can be used to compute $F^{\sim}(s)$. Finally, $\eta = 1 - F^{\sim}(0)$.

Example 4.1. Consider the switching server of example 3.1, except now we assume that states 1 and 2 are preemptive-repeat-identical.

Equations (4.1) become:

$$(s+\lambda')F_1^{\sim}(s,x) = (s+\lambda')e^{-(s+\lambda')x/r_1} + \lambda(1-e^{-(s+\lambda')x/r_1})F_2^{\sim}(s,x)$$

$$(s+\mu')F_2^{\sim}(s,x) = (s+\mu')e^{-(s+\mu')x/r_2} + \mu(1-e^{-(s+\mu')x/r_2})F_1^{\sim}(s,x).$$

Solving the above equations we get

$$F_1^{\sim}(s,x) = (s+\mu')[s+\lambda')a + \lambda(1-a)b]/\Delta$$

$$F_2^{\sim}(s,x) = (s+\lambda')[s+\mu')b + \mu(1-b)a]/\Delta$$

where $a = exp(-(s+\lambda')x/r_1)$, $b = exp(-(s+\mu')x/r_2)$ and $\Delta = (s+\lambda')(s+\mu') - \lambda\mu(1-a)(1-b)$. $F_i^{\sim}(s)$, for i = 1,2 and $F^{\sim}(s)$ can be obtained from equations (2.7) and (2.8).

5. *The Preemptive-repeat-different Case*

Here we consider the case, where all structure-states of the process are preemptive-repeat-different (*prd*).

The following theorem holds

Theorem 4. The *LST*s $F_i^{\sim}(s)$, for $1 \le i \le n$, as defined in equation (2.4) satisfy the following simultaneous equations

$$F_i^{\sim}(s) = G^{\sim}((s+q_i)/r_i) + \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{q_{ij}}{(s+q_i)}[1-G^{\sim}((s+q_i)/r_i)]F_j^{\sim}(s), \quad 1 \le i \le n. \tag{5.1}$$

Note that when $r_i \to 0$, $G^{\sim}((s+q_i)/r_i) \to 0$, since G (0+) = 0 and hence $\lim_{s\to\infty} G^{\sim}(s) \to 0$.

Proof: Conditioning on the work requirement B of the job to be executed and on the holding time H in the initial state we get

$$E(e^{-sT} \mid B=x, H=h, Z(0)=i) = \begin{cases} e^{-sx/r_i}, & \text{if } h \ge x/r_i \\ e^{-sh} \sum_{\substack{j=1 \\ j \ne i}}^{n} \frac{q_{ij}}{q_i} F_j^{\sim}(s), & \text{if } h < x/r_i \end{cases}$$

Note that if a structure state transition occurs before the job is completed then a different job with independent and identical distribution is restarted.

Now, unconditioning on B (the job's work requirement) yields

$$E(e^{-sT} \mid H=h, Z(0)=i) = \int_{x=0}^{r_i h} e^{-sx/r_i}\, dG(x) + \int_{x=r_i h}^{\infty} e^{-sh} \sum_{\substack{j=1 \\ j \ne i}}^{n} \frac{q_{ij}}{q_i} F_j^{\sim}(s)\, dG(x)$$

Unconditioning on H (the holding time in the initial state), yields equation (5.1). *Q.E.D.*

Solving equations (5.1) we get $F_i^{\sim}(s)$, for $1 \le i \le n$. Equation (2.8) can be used to get $F^{\sim}(s)$. The dynamic failure probability (η) follows immediately

$$\eta = P(T=\infty) = 1 - F^{\sim}(0).$$

Note that the preemptive-repeat-different case with a constant (or deterministic) work requirement of a job ($B=x$) corresponds to the preemptive-repeat-identical case.

Example 5.1. Again we consider the switching server of example 3.1 with the states 1 and 2 being preemptive- repeat-different. From equations (5.1) we have

$$F_1^{\sim}(s) = G^{\sim}((s+\lambda')/r_1) + \frac{\lambda}{(s+\lambda')}[1 - G^{\sim}((s+\lambda')/r_1)]F_2^{\sim}(s)$$

$$F_2^{\sim}(s) = G^{\sim}((s+\mu')/r_2) + \frac{\mu}{(s+\mu')}[1 - G^{\sim}((s+\mu')//r_2)]F_1^{\sim}(s)$$

It follows that

$$F_1^{\sim}(s) = \frac{G^{\sim}((s+\lambda')/r_1) + (\frac{\lambda}{s+\lambda'})(1 - G^{\sim}((s+\lambda')/r_1))G^{\sim}((s+\mu')/r_2)}{[1 - (\frac{\lambda}{s+\lambda'})(\frac{\mu}{s+\mu'})(1 - G^{\sim}((s+\lambda')/r_1))(1 - G^{\sim}((s+\mu')/r_2))]}$$

$$F_2^{\sim}(s) = \frac{G^{\sim}((s+\mu')/r_2)+(\frac{\mu}{s+\lambda'})(1-G^{\sim}((s+\mu')/r_2))G^{\sim}((s+\lambda')/r_1)}{[1-(\frac{\lambda}{s+\lambda'})(\frac{\mu}{s+\mu'})(1-G^{\sim}((s+\lambda')/r_1))(1-G^{\sim}((s+\mu')/r_2))]}$$

$F^{\sim}(s)$ can be obtained from equation (2.8).

6. *Conclusions and Extensions*

We have developed a unified model for the combined evaluation of performance and reliability of multi-mode computer systems. This allows us to compute both system-oriented measures (such as the accumulated reward) and task-oriented measures (such as the completion time and the dynamic failure probability) from a single model. We model preemptive-resume and preemptive-repeat interactions between task execution and mode change (failure/repair) events. It is clearly of interest to allow mixed preemptive-resume and preemptive-repeat interactions in the same model. This and other extensions have been studied and reported recently [8]. The techniques developed in this paper can be exetended to the case where the structure-state process is a semi-Markov process.

References

[1] Baccelli, F. and Trivedi, K.S., "Analysis of M/G/2 - Standby Redundant System," *Proc. 9th IFIP International Symposium on Computer Performance Modeling*, College Park, MD., 1983.

[2] Beaudry, M.D., "Performance-Related Reliability for Computing Systems," *IEEE Transactions on Computers*, June 1978, pp. 540-547.

[3] Castillo, X. and Siewiorek, D.P., "A Performance-Reliability Model for Computing Systems," *Proc. 1981 Int. Symp. on Fault-Tolerant Computing*, Portland, ME, June 1981, pp. 187-192.

[4] Donatiello, L. and Iyer, B., "Analysis of a Composite Performance Reliability Measure for Fault-Tolerant Systems," IBM Research Report, RC-10325, January 1984.

[5] Gaver, D.P., "A Waiting Line with Interrupted Service, Including Priorities," *J. Royal Statistical Society*, Series B24, 1962, pp. 73-90.

[6] Gay, F.A. and Ketelson, M.L., "Performance Evaluation for Gracefully Degrading Systems," *Proc. Ninth Annual International Conference on Fault-Tolerant Computing*, 1979.

[7] Krishna, C.M. and Shin, K.G., "Performance Measures for Multiprocessor Controllers," *Proc. 9th IFIP International Symposium on Computer Performance Modeling*, College Park, MD., 1983.

[8] Kulkarni, V. G., Nicola, V. F. and Trivedi, K. S., "A Unified Model for Performance and Reliability of Fault-Tolerant/Multi-Mode Systems," Submitted, August 1984.

[9] Laprie, J.-C., "Trustable Evaluation of Computer Systems Dependability," invited paper, in: G. Iazeolla, P. Courtois and A. Hordijk (eds.), *Mathematical Computer Performance and Reliability*, Elsevier Science Publishers B.V. (North-Holland), 1984.

[10] Meyer, J.F., "Closed-Form Solution of Performability," *IEEE Transactions on Computers*, Vol. C-31, No. 7, July 1982, pp. 648-657.

[11] Ng, Y.-W. and Avizienis, A., "A Reliability Model for Gracefully Degrading and Repairable Systems," *Proc. 1977 Int. Symp. on Fault-Tolerant Computing*, Los Angeles, CA, June 1977, pp. 22-28.

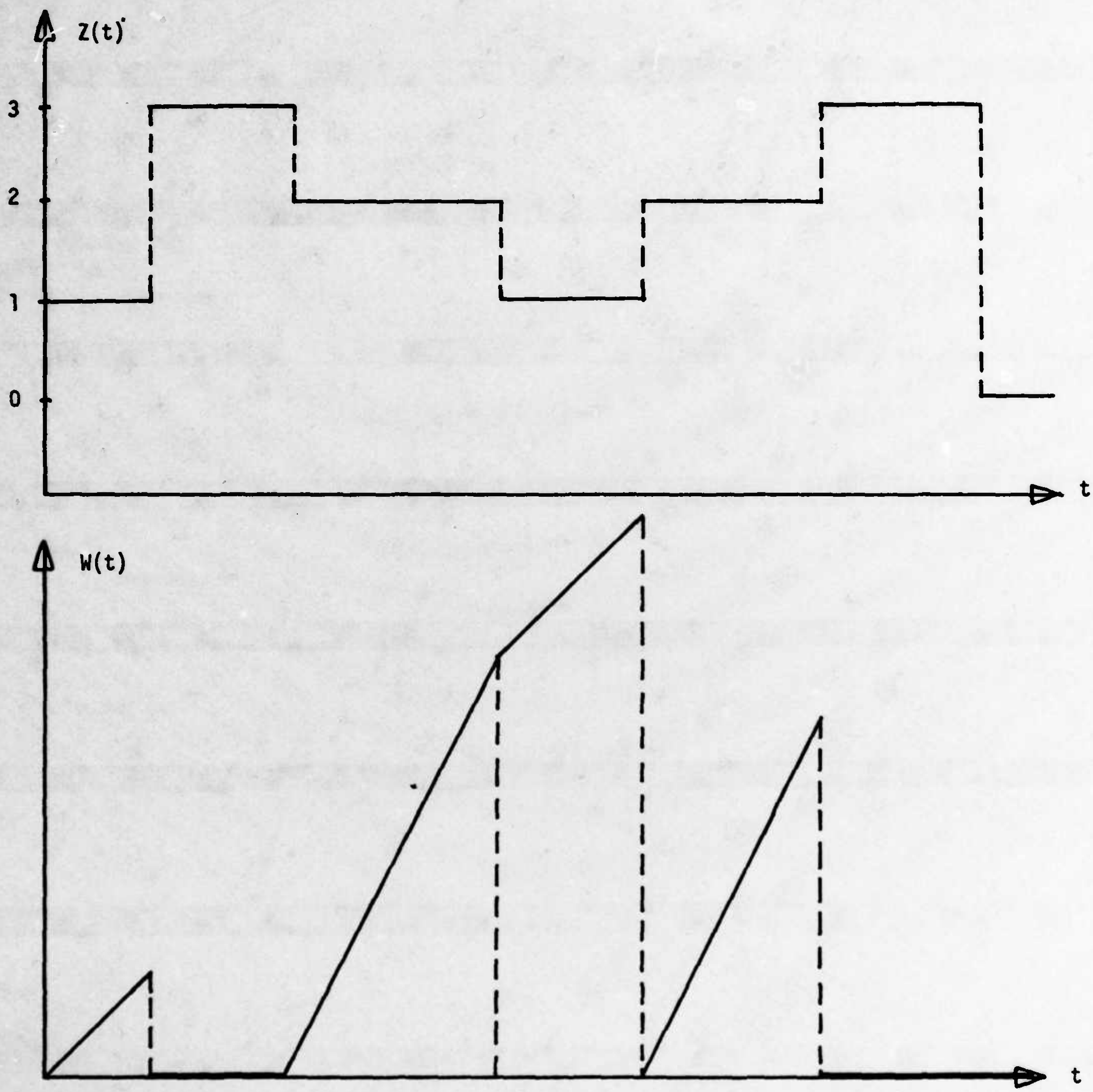
[12] Nicola, V. F., "A Single Server Queue with Mixed Types of Interruptions," EUT Report 83-E-138, Eindhoven University of Technology, The Netherlands, 1983.

[13] Osaki, S. and Nishio, T., "Reliability Evaluation of Some Fault-Tolerant Computer Architectures," *Lecture Notes in Computer Science*, Springer-Verlag, Berlin, 1980.

[14] Puri, P. S., "A Method for Studying the Integral Functionals of Stochastic Processes with Applications: I. Markov Chain Case," *J. Appl. Prob. 8*, 1971, pp. 331-343.

[15] Trivedi, K.S., *Probability and Statistics with Reliability, Queueing and Computer Science Applications*, Prentice-Hall, Englewood Cliffs, N.J., 1982.

[16] Trivedi, K. S., "Modeling and Analysis of Fault-Tolerant Systems," invited paper, *International Conf. on Modeling Techniques and Tools for Perfrmance Analysis*, May 16-18, 1984, Paris.



Typical Sample Paths of Z(t) and W(t)

Figure 1

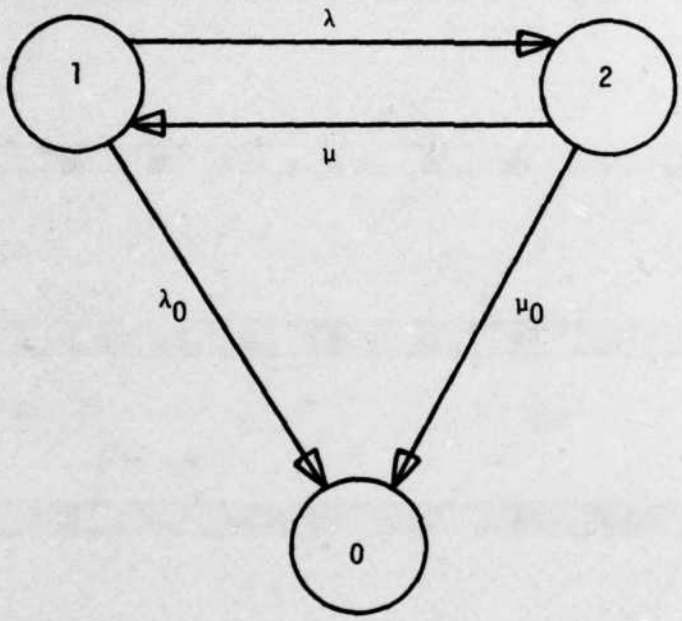


Figure 2

The Switching Server

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A direct inversion with respect to s yields

$$\underline{Y}(u,t) = e^{(Q-uR)t}\,\underline{e} \; . \tag{3.14}$$

$$F_2^{\sim *}(s,u) = \frac{r_1 r_2 u + r_2(s+\lambda') + r_1 \mu}{(s+\lambda'+r_1 u)(s+\mu'+r_2 u) - \lambda\mu}.$$

Hence, using eq. (3.9) we get

$$F_1^{\sim}(s,x) = A_{11}(s) exp(-u_1(s)x) + A_{12}(s) exp(-u_2(s)x).$$

$$B_{11}(u) = \frac{(\lambda'-s_1(u))(\mu'-s_1(u)) + r_2 u(\lambda_0-s_1(u)) - \lambda\mu}{s_1(u)[s_1(u)-s_2(u)]}$$

$$B_{12}(u) = \frac{(\lambda'-s_2(u))(\mu'-s_2(u)) + r_2 u(\lambda_0-s_2(u)) - \lambda\mu}{s_2(u)[s_2(u)-s_1(u)]}$$

Solving equations (4.1) we get $F_i^{\sim}(s,x)$, for $1 \le i \le n$. Then equations (2.7) and (2.8) can be used to compute $F^{\sim}(s)$. Finally, $\eta = 1 - F^{\sim}(0)$.

Example 4.1. Consider the switching server of example 3.1, except now we assume that holding time H in the initial state we get