WORKSHOP ON

UNSTEADY SEPARATED FLOW

UNITED STATES AIR FORCE ACADEMY
AUGUST 10-11, 1983

SPONSORED BY: AFOSR
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**ABSTRACT**

This report documents the proceedings of a "Workshop on Unsteady Flow Separation" held at the US Air Force Academy on 10-11 August 1983. This two day program was comprised of 27 presentations on a wide variety of topics ranging from fundamental concepts to potential applications. Flows with time dependent boundary conditions leading to global separated flow structures were highlighted.
COMPONENT PART NOTICE

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AFOSR/FJSRL/U. COLORADO
WORKSHOP ON UNSTEADY SEPARATED FLOWS

Organizer: Capt. Michael S. Francis, Ph.D.
AFOSR/NA
Bolling AFB, DC 20332

Coordination and Documents: Marvin W. Luttges, Ph.D.
Professor and Acting Chairman
Aerospace Engineering Sciences, Box 429
University of Colorado
Boulder, Colorado 80309

Coordination and Arrangements: Lt. Hank Helin
Frank J. Seiler Research Laboratory
U.S. Air Force Academy
Colorado Springs, Colorado 80840

Technical Assistance Acknowledgements

Michael C. Robinson
Laurie Beeman
Nancy Searby
Randy Jones
JoAnne Button
Terri MacGregor

Preparation of Final Publication

JoAnne Button
Erika Shuck
LIST OF ATTENDEES

Professor Holt Ashley
Dept. of Aeronautics & Astronautics
Durand 369
Stanford University
Palo Alto, CA 94305
(415) 497-4136

Mr. Morton Cooper
Flow Research Company
1320 Fenwick Lane, Suite 401
Silver Spring, MD 20910
(301) 589-5790

Dr. Robert Bass
P.O. Drawer 28510
6220 Culebra Road
Southwest Research Institute
San Antonio, TX 78284
(512) 684-5111 Ext. 2326

Dr. Eugene Covert
Dept. of Aeronautics & Astronautics
Massachusetts Institute of Technology
Cambridge, MA 02139

Dr. Dale Berg
Sandia National Laboratory: Div. 1636
P.O. Box 5800
Albuquerque, NM 87185
516-52-2054
(505) 844-1030

Dr. Steven Crow
Poseidon Research
1299 Ocean Avenue, Suite 821
Santa Monica, CA 90401

Dr. Lawrence Carr
NASA - Ames Research Center
Mail Stop 227-8
Moffett Field, CA 94035

Dr. Atlee Cunningham
General Dynamics - Ft. Worth Division
P.O. Box 748
Fort Worth, TX 76101
(817) 732-4811 Ext. 4418

Dr. Frank Cai'ta
united Technologies Research Ctr.
Silver Lane
East Hartford, CT 06108
(203) 727-7355

Dr. Frank A. Dvorak
Analytical Methods Inc.
2047 - 52nd Avenue, N.E.
Redmond, WA 98052
(206) 643-9090

Dr. Tuncer Cebeci
McDonnell Douglas Aircraft Co.
3855 Lakewood Blvd.
Long Beach, CA 90846
(213) 593-8322

Dr. John Eaton
Dept. of Mechanical Engineering
Stanford University
Palo Alto, CA 94305
(415) 497-1971

Dr. David Chou
Dept. of Mechanical Engineering
University of New Mexico
Albuquerque, NM 87106

Dr. Lars Ericsson
Lockheed Missiles & Space Co.
D/10-10, Bldg. 157
Sunnyvale, CA 94088
(408) 756-1396

Dr. C.-Y. Chow
Aerospace Engineering Sciences
University of Colorado
Campus Box 429
Boulder, CO 80309
(303) 492-7907

Dr. Hermann Fasel
Dept. of Aerospace & Mechanical Engin
University of Arizona
Tucson, AZ 85721
(602) 621-6119
Professor Peter Freymuth  
Aerospace Engineering Sciences  
University of Colorado  
Campus Box 429  
Boulder, CO 80309  
(303) 492-7611

Major Eric Jumper  
Dept. of Aeronautics & Astronautics  
Air Force Institute of Technology  
Wright-Patterson AFB, OH 45433  
(513) 255-2998

Dr. Mohamed Gad-El-Hak  
Flow Industries, Inc.  
21414 - 68th Avenue South  
Kent, WA 98032  
(206) 872-8500

Dr. Donald A. Kennedy  
Aerospace Engineering Sciences  
University of Colorado  
Campus Box 429  
Boulder, CO 80309  
(303) 492-7633

Dr. Wilbur Hankey  
AFWAL/FIMM  
Wright-Patterson AFB, OH 45433  
(513) 255-2455

Mr. Dennis Koga  
Dept. of Mechanics & Mechanical Engin.  
Illinois Institute of Technology  
Chicago, IL 60616  
(312) 567-3213

Lt. H. Heiin  
FJSRL/NH  
USAF Academy, CO 80840  
(303) 472-3122

Dr. Marvin W. Luttges  
Aerospace Engineering Sciences  
University of Colorado  
Campus Box 429  
Boulder, CO 80309  
(303) 492-7613

Dr. W.B. Herbst  
MBB-Flugzeuge (GMBH)  
Military Aircraft Division  
Postfach 80 11 60  
800 Munchen 80, FRG (Germany)  
89 - 6000581

Dr. Brian Maskew  
Analytical Methods, Inc.  
2047 - 152nd Avenue, N.E.  
P.O. Box 3786  
Redmond, WA 98052  
(206) 643-9090

Dr. C.M. Ho  
Dept. of Aerospace Engineering  
University of Southern California  
Los Angeles, CA  
(213) 743-6560

Dr. Ken McAlister  
NASA - Ames Research Center  
Mail Stop 215-1  
Aeromechanics Laboratory  
Moffett Field, CA 94035  
(415) 965-5892

Dr. Ed James  
Vehicle Research Corporation  
650 Sierra Madre Villa, Suite 100  
Pasadena, CA 91107  
(213) 351-4202

Mr. Peter Lorber  
Dept. of Aeronautics & Astronautics  
Massachusetts Institute of Technology  
Cambridge, MA 02139

Dr. Ed James  
Vehicle Research Corporation  
650 Sierra Madre Villa, Suite 100  
Pasadena, CA 91107  
(213) 351-4202

Mr. James B. Johnson  
Southwest Research Institute  
San Antonio, TX 78284  
465-70-7845
Dr. Hermann Viets  
College of Engineering, Rm. 151  
P.O. Box 6101  
West Virginia University  
Morgantown, WV 26506  
(304) 293-4821

Major John Walker  
FJSRL/NH  
USAF Academy, CO 80840  
(303) 472-3122

Dr. Robert Whitehead  
Office of Naval Research, Code 432F  
800 No. Quincy Street  
Arlington, VA 22217  
(202) 696-4404

Dr. James C. Wu  
School of Aerospace Engineering  
Georgia Institute of Technology  
Atlanta, GA 30332  
(404) 894-3028

Dr. Robert Woodcock  
AFWAL/FIGG  
Wright-Patterson AFB, OH 45433  
317-22-7393  
(513) 476-2076

Dr. John Yates  
Aeronautical Research Assoc. of Princeton  
P.O. Box 2229  
Princeton, NJ 08540  
(609) 452-2950 Ext. 244

Dr. James D. Wilson  
AFCSR/NA  
Bolling AFB, DC 20332  
(202) 767-4935
AFOSR/FJSRL/U. COLORADO
WORKSHOP ON UNSTEADY SEPARATED FLOWS
- PROGRAM -

Wednesday, 10 August 1983

8:00 AM  Welcome
  Introductory Remarks

SESSION 1 - APPLICATIONS
8:15 AM  "Supermaneuverability," W. Herbst (Invited Presentation)
9:15    "Wing Rock Flow Phenomena," L. Ericsson
9:45    Coffee Break
10:00    "Aerodynamic Agility Resulting from Energetic Separated Flows," D. A. Kennedy

SESSION 2 - AERODYNAMIC CHARACTERISTICS OF TIME DEPENDENT MOTIONS - I
11:00    "Unsteady Stall Penetration of an Oscillating Swept Wing," F. Carta
11:30    Lunch (NCO Club - buses leave at front of building)
1:00 PM   "Correlation of Lift and Boundary-Layer Activity on an Oscillating Lifting Surface," R. Bass, J. Johnson, and J. Unruh
1:30    "A Visual Study of a Delta Wing in Steady and Unsteady Motion," Mohamed Gal-el-Hak, C.-M. Ho, and R. Blackwelder
2:00    "Comparative Visualization of Accelerating Flows around Various Bodies, Starting from Rest," P. Freymuth, M. Palmer, and W. Bank
2:30    Coffee Break

SESSION 3 - ANALYTICAL AND NUMERICAL MODELS - I
3:45    "Unsteady Aerodynamic Loading on an Airfoil due to Vortices Released Intermittently from its Upper Surface," C.-Y. Chew and C.-S. Chiu
4:15    "Calculations of Oscillating Airfoil Flow Fields via the Navier-Stokes Equations," S. J. Shimrot

SESSION 4 - UNSTEADY BOUNDARY LAYER SEPARATION - I
4:45    "Some Structural Features of Unsteady Separating Turbulent Shear Flows," R. Simpson
5:15    "Can the Singularity be Removed In Time-Dependent Flows?" T. Cebeci, A. Khatib, and S. M. Schinke
Session 4 (Continued)

5:45 "On the Shedding of Vorticity at Separation," D. Tellonis, D. Mathioulakis, and M. S. Cramer

6:15 Return to Motels (buses leave front of building)

Thursday, 11 August 1983


9:00 "Passive and Active Device-Controlled Unsteady Separated Flowfields," H. Nagib, D. Koga, and P. Reisenthel


10:00 Coffee Break

SESSION 5 - ANALYTICAL AND NUMERICAL MODELS - II


11:15 "Preliminary Results from the Unsteady Airfoil Model USFAR2," J. Strickland, J. W. Oler and B. J. Im

11:45 Lunch (NCO Club - buses leave at front of building)

SESSION 6 - UNSTEADY BOUNDARY LAYER SEPARATION - II

1:15 PM "Experiments on Controlled, Unsteady, Separated Turbulent Boundary Layers," W. C. Reynolds and L. N. Carr

1:45 "Genesis of Unsteady Separation," C.-M. Ho


2:45 "Unsteady Leading Edge Separation," E. C. James

3:15 Coffee Break

3:30 "Natural Unsteadiness of a Separation Bubble Behind a Backward Facing Step," J. K. Eaton and A. L. Alving


4:30 Open Discussion

Closing Remarks
Supermaneuverability

Dr. W. B. Herbst
MESSERSCHMITT-BÖLCKOW-BLOHM GMBH
Munich, Germany

Abstract:

Supermaneuverability is defined as the capability of a fighter aircraft to execute tactical maneuvers with controlled sideslipping and at angles of attack beyond maximum lift. This paper deals particularly with post stall maneuverability at zero sideslip since this element of supermaneuverability is relatively unknown. The analysis is based on optimum control calculation of simplified maneuvers and on extensive manned and computerized close air combat simulation. This analysis explains the tactical advantage observed during combat simulations and leads to the definition of conventional maneuver duty cycle which is consistent with conventional air combat maneuvers with all aspect weapons. Reference is made to earlier studies about maneuvers with thrust vectoring and thrust reversal. Finally, requirements are given for the necessary level of thrust-to-weight ratios and control power including indications of technical solutions.

1. Introduction

There are three different concepts (fig. 1) of improving maneuverability by means of tilting engine thrust:

a) Inflight thrust reversal
b) Thrust vectoring
c) Post Stall maneuvering (PST)

(a) has been considered as a deceleration device which permits to slow the aircraft down rapidly into a speed regime of better turn performance, however, it does not directly contribute to maneuver performance in terms of improving a change of the direction of flight.

(b) has been discussed in conjunction with configurations, such as the BAe Harrier. Thrust vectoring offers an additional degree of freedom to establish a maneuver state. However, it requires engine exit momentum to point at the aircraft e.g. over the vectoring range, which is fairly incompatible with afterburner installation. A noticeable improvement of air combat capability has been demonstrated.

(c) is the subject of this paper. The engine is fixed to the aircraft fuselage and thrust vectoring is always in line with the e.g. The only difference to a conventional aircraft is the requirement for large angles of attack in excess of maximum lift angle of attack. Post Stall flight conditions have already been demonstrated, however, tactical PST-maneuvers require a level of controllability far beyond that of contemporary aircraft. Also, the tactical advantage was unknown.

There is similarity between the concepts (b) and (c). For both concepts the gain in sustained turn performance is small and limited to extreme flight conditions. The tactical advantage is based on short term and highly instantaneous maneuvers and the achievement of small radii of turn. (b) could even include (c) if the aircraft were allowed to exceed the stall limits. In a PST-maneuver, however, the advantage of thrust vectoring is marginal and may not justify the overall design penalties.

The tactical advantage of PST-maneuvering in short range air combat depends on the weapons used. There is a change in combat maneuver characteristics caused by all aspect capability of new weapons. In head-on situations particularly instantaneous turn performance combined with small turn radii has been found decisive. The new weapons - including radar controlled guns in combination with sideslipping flight modes - will initiate further improvements of air combat capability by means of further improved energy maneuverability. New maneuver modes - such as PST-maneuvering - may be the only way of achieving substantial improvements in close combat effectiveness.

2. Tactical results of air combat simulations

PST maneuvering has been the subject of manned and computerized dual- and multiple combat simulations. Significant observations are the following maneuver characteristics:
Actually, ever the larger i-range, only a somewhat remained to any significant degradation of tactical success due to PST-maneuvers and a observed during speed regime. Any maximum limits. There was less time at limit g-loadings.

As an overall result it was found:
- exchange ratio in dual combat with equal weapons against an opponent of equal conventional maneuver performance is about 2:1.
- exchange ratio in multiple combat remains to be largely dependent on the number of opponents, however, can be significantly improved. For example in [4] a single PST-capable fighter was able to neutralize two conventional opponents. Computer simulations [7] involving a larger number of opponents are showing an increase of the relative advantage of PST-maneuvering (Fig. 2).

A large amount of tactical data has been gathered in more than 1000 simulated engagements of 3 different manuever combat simulators flown by 5% operational pilots of three airforces. Still, it remains somewhat difficult to precisely understand the advantage. Sometimes the advantage is attributed to fuselage pointing over the larger i-range. Actually, only a relatively small number of gun shots have been observed during PST-maneuvers and a missile firing limitation to $\gamma = 30^\circ$ did not lead to any significant degradation of tactical success [1].

The majority of firing opportunities occurs right after finishing a PST-maneuver and returning to the conventional flight regime. PST-capability, therefore, must be interpreted as a maneuvering scheme rather than a gimbaling device.

In general, the pay-off is based on a trade of loss of energy versus positional and time advantage. A proper PST-maneuver preceding an engagement provides a decisive time advantage at a memetary expense of energy. In multiple engagements the loss of energy seems to be compensated by a roll-rate advantage, according to [4]. In order to convert from a right hand turn to a left hand turn against an alternate target a conventional aircraft would have to unroll, roll and re-load. With PST capability an aircraft could roll around the velocity vector at constant - very high - angle of attack. This capability was developed during

![Diagram](attachment:image.png)

**Fig. 2** Evaluation of Super-maneuverability in dual and multiple air combat with all opponent weapons. Results of computer simulations. a = 1.7, b = 3.7, c = actual case.

As a result, the aircraft with PST-capability was able to dictate the tactical course of the engagement. Missile capability did allow the conventional opponent(s) to disengage. Overall speed and loadfactor was observed to be lower than in conventional air combat. In particular, there was less time at limit g-loadings.

1. MANEUVER ANALYSIS

For any flight condition (aircraft attitude, velocity vector, power setting, altitude) the maneuver state (longitudinal and lateral acceleration) is a function of the sum of all forces, aerodynamic forces, engine inlet and exit momentum. The analysis shall be limited to zero sideslip conditions for PST-maneuvering. Fig. (3) and (4) are special cases of many such diagrams for an infinite number of flight conditions. An aircraft maneuver would be a sequence of maneuver states at continuously changing flight conditions. Fig. (3) and (4) indicate...
that a turn rate advantage beyond maximum lift angle of attack as compared to "corner speed" can be expected at very low speeds.

- that a significant turn radius advantage can be achieved at angles of attack well beyond maximum lift.

- that the high angle of attack part of an advantageous PST-maneuver would have to penetrate a relatively low speed regime (0.05 > M > 0.2).

- that no significant advantage could be expected from PST-maneuvers limited to moderate angles of attack (θ ≤ 30°).

Performance of a PST-maneuver is an optimum control problem with angle of attack and bank angles as independent variables (thrust was found to be maximum throughout the maneuver for best results). Pay-off function in real combat would be an earlier firing opportunity and to deny counterfire. In order to get better problem transparency, minimum time maneuvers with defined starting and end conditions have been calculated in [9]. There was an advantage of exceeding maximum lift angle of attack and instantaneous penetration into the PST regime whenever certain geometrical constraints had to be satisfied.

One of the most applicable analytical maneuvers, for example, is that of a 180° change of heading with the additional constraint of returning to
Aircraft with PST Capability, $\alpha_{\text{max}} = 90^\circ$
Comparison of maneuver cycles for minimum time maneuvers. Results of trajectory optimizations.

Simulated air combat engagements (both manned and computer simulations) have been investigated with regard to common characteristics of PST-maneuvers. Fig. (10) shows trajectories of a typical initial phase engagement of two opponents with equal conventional performance. Aircraft 1 is limited to maximum lift, aircraft 2 has PST-capability up to 70° angle of attack including certain control power in pitch and yaw. Both aircraft are equipped with the same all aspect weapon. The associated time history of turn rate vs. speed and angle of attack is plotted in Fig. (11) and Fig. (12). Starting at high speed, same altitude, and a typical positional offset opponents are pulling maximum g and then slow down to best instantaneous turn rate by means of gaining altitude. At the same time a smaller radius of turn would help to get the opponent into own weapon off-bore sight cone and to keep the opponent from achieving the same objective. A properly scheduled penetration into the PST-regime enhances both the slow down and the minimum radius part of the combat maneuver without a significant loss of turn rate.

As a result aircraft 2 achieves its firing opportunity at a time still outside of aircraft 1 firing cone. It is important to note that this firing opportunity occurs shortly after the aircraft has returned into the normal
flight regime. During the PST-maneuver the weapon was even pointing away from the target.

Fig. (13) is another representation of the

Fig. 10 Typical air combat engagement of a PST fighter against a conventional - limited fighter with all aspect weapons. Result of computer simulations.

![Graph showing Angle of Attack over Elapsed Maneuver Time](image)

**Angle of Attack [°]**

*Elapsed Maneuver Time [s]*

![Graph showing Angle-of-Attack for typical air combat engagement](image)
Aircraft 2 performs the PST-section of the maneuver (second 8 - 15) far outside aircraft 1's weapon range in its backward look angle sector and is getting a first firing opportunity in the 23. second being in a safe beam position relative to the target. There is active firing time without counterfire for several seconds until the opponent would pass each other almost head on. The pilot of aircraft 2 is never looking into his opponent firing cone until his first missile hits the target. He recovered the energy lost during the PST-maneuver right after the attack against aircraft 1 (fig. 14). Average speed of the PST-aircraft observed throughout many simulated dual and multiple combat engagements is only 5 - 10% lower as compared to the conventional opponents. Obviously, the aircraft is vulnerable during the high angle of attack phase of a PST-maneuver, however, the pilot has always the option to restrain if the momentary loss of speed constitutes a tactical disadvantage (provided conventional performance does not suffer too much from its incorporation into the overall design).

PST-maneuvers are consistent with the general dynamic characteristics of air combat with all aspect weapons [10]. There is the same

![Diagram of typical air combat engagement](image)

**Fig. 13** Typical air combat engagement. Relative positions of opponents and their aspects and firing opportunities. Time intervals in sec. Result of computer simulations.

**Fig. 14** Energy management in a typical air combat engagement. Result of computer simulations.
overall left hand maneuver cycle of the turn rate vs. speed time history with the addition of a short term right hand cycle excursion into the PST-regime.

4. PST regime and requirements

Limits are given by controllability, engine power and structural constraints (fig. 15). Actual usage, however, is dictated by tactical advantages. For high performance aircraft (thrust-to-weight > 1.0) there is a low speed regime of possible sustained maneuver. In combat, however, there are no sustained maneuvers (constant maneuver states); excess thrust is always used to re-accelerate or to gain altitude. As a general rule PST-capability requires

- sufficient control power in pitch, roll and yaw at mach numbers as low as 0.1 and incidence up to 70°.
- High angle of attack compatibility up to 70° at machnumbers as high as 0.6 (4000 m altitude) for example with regard to aircraft stability or air intake flow.
- Thrust/weight ration > 1.0 fig. (16) shows the time advantage of the optimized PST-maneuver (fig. 5) at different thrust-to-weight ratios. With decreasing engine power the observed maximum values of angle to attack are becoming smaller. At thrust-to-weight ratios of less than 0.6 there is no tactical advantage in exceeding maximum lift even if the capability is available.
- Sufficient control power, particularly in pitch and yaw.

- PST-maneuveres are characterized by high pitch rates and rotation in yaw and roll at the same time. Coordinated flight with zero sideslip requires rotation around the velocity vector. Since the pilot does not recognize the velocity vector the control system has to be mechanized accordingly. A lateral stick input would have to produce more yaw and less roll at increasing angle of attack. This caused some confusion with pilots during simulated combat, because a pilot tends to use body axis as reference.

As a result of manned combat simulations a requirement for velocity vector roll acceleration was developed. It is plotted in fig. (17) as a function of angle of attack for a speed of M = 0.2 at 6000 m altitude. A translation in body axis motion shows that for a conventional aircraft the demand for roll could marginally be satisfied with aerodynamic controls, however, for pitch and yaw a control augmentation would be required. For example, a 10° conical deflection of the jet exhaust would suffice if the nozzle actuation meets certain dynamic requirements. Such a nozzle would have to be integrated in the aircraft control system. It was found that nozzle control is an enhancement of handling characteristics and thus combat effectiveness even in the conventional flight regime beyond 10° angle of attack.

5. Summary

The capability of exceeding maximum lift angle of attack - Post Stall (PST) maneuvering - can improve future close air combat effectiveness to a degree unachievable by conventional performance. The tactical advantage is attributed to a combination of fairly high turn rates and small turn radii at PST flight conditions in all aspect weapon environment. PST maneuvers are short period and highly instantaneous and constitute a trade of short term loss of energy against positional.
Fig. 17 Control power requirements at PST conditions. Statistical results of manned combat simulations.

Advantage. PST maneuvers are an extension of conventional combat maneuvers. PST maneuverability requires an aircraft to be compatible with angles of attack up to 70° at Mach numbers up to \( M = 0.6 \), a thrust-to-weight ratio of at least 1.0 and a level of control power at Mach numbers as low as \( M = 0.1 \) which cannot be achieved by aerodynamic means alone.

6. References

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ABSTRACT

Flow mechanisms that can generate wing-rock type oscillations are described. It is shown that the slender wing-rock phenomenon, the limit cycle oscillation in roll observed for very slender delta wings, is caused by asymmetric leading edge vortices and that vortex breakdown can never be the cause of it as it has a damping effect. For that reason slender wing rock is only realized for delta wings with more than 74° leading edge sweep for which asymmetric vortex shedding occurs before vortex breakdown. For straight or moderately swept wings the flow mechanism causing wing rock is two-dimensional in nature, closely related to the dynamic stall phenomenon. Pointed forebodies provide a third flow mechanism, asymmetric vortex shedding sensitive to body motion, which can generate a rocking motion of a slender vehicle unless it is completely axisymmetric.

INTRODUCTION

The steadily increasing demands on performance expose present day aerospace vehicles to unsteady flow fields which generate highly nonlinear aerodynamics that exhibit significant coupling between longitudinal and lateral degrees of freedom. The complex vehicle dynamics are caused by separated flow effects of various types, which have largely eluded theoretical description. Consequently, the designer is to a large extent dependent upon existing experimental capabilities for dynamic testing, where dynamic support interference adds complexity to the already complicated separated flow characteristics. Thus, it can be rather difficult to obtain a true description of non-linear pitch-yaw-roll coupling phenomena such as wing rock and nose slice. In the present paper existing experimental results are examined to obtain a description of the underlying fluid mechanics.

DISCUSSION

Recent systematic experiments performed by Nguyen et al. provide the information needed to fully describe the fluid mechanic phenomenon leading to slender wing rock. The phenomenon is similar in many aspects to the limit cycle oscillation in pitch observed on blunt cylinder-flare bodies. Thus, the roll oscillations of an 80° delta wing are self-excited and build up to a limit cycle amplitude as shown in Fig. 1, an oscillatory behavior very similar to that observed for blunt-nosed cylinder-flare bodies. The roll oscillation, 

\[
\alpha_{\text{eff}} = \arctan(\tan \alpha \cos \phi)
\]

(1)

\[
\beta_{\text{eff}} = \arctan(\tan \alpha \sin \phi)
\]

(2)

It can be seen that the main effect is the induced sideslip. Thus, one can compare the \( C_{\ell}(\alpha) \)-characteristics for a slender delta wing with the \( C_{\phi}(\alpha) \)-characteristics for a cylinder-flare body. In both cases the characteristics change in a discontinuous fashion, and are likely to be associated with hysteresis, both phenomena typical for the effects of separated flow. The aerodynamic stiffness, \( C_{\ell} \) in Fig. 2a and \( C_{\phi} \) in Fig. 2b, increases dramatically when the discontinuity is encountered. With or
without associated hysteresis. Both discontinuous changes of the aerodynamic characteristics are associated with convective time lag effects\(^1\). This causes the statically stabilizing effects to become dynamically destabilizing, as is illustrated in Fig. 3 for the cylinder-flare body. The separation layer impacting on the flare at time \(t\), when \(\alpha(t) = 0\), was generated by the nose at a time increment \(\Delta t\) earlier, when the angle of attack was \(\alpha(t-\Delta t) > 0\). Thus, a residual flare force \(\Delta \tau\) influences the motion, and consequently, is undamping.

Thus, the statically stabilizing separation-induced flare force is dynamically destabilizing. It is shown in Refs. 7 and 8 how the unsteady aerodynamics measured for large amplitude oscillations \((\pm \alpha)\) around the undamped trim point deviates from static aerodynamic characteristics when accounting for the convective time lag effect (Fig. 4).

A sudden change of the leading edge vortex on a delta wing is also associated with convected time lag effects, as is illustrated by the experimentally observed formation of the leading edge vortex\(^9\) (Fig. 5). Accounting for this time lag provided good prediction of the vortex-induced unsteady aerodynamics of slender delta wings\(^1\) (Fig. 6), and should also permit the effect of the discontinuous aerodynamics associated with the vortex asymmetry to be predicted.

Two types of separation-induced discontinuities occur for the slender delta wing. One is caused by the breakdown of the leading edge vortices. It is the three-dimensional equivalent to airfoil stall. For very slender delta wings another discontinuous change of the aerodynamics can occur before vortex breakdown due to asymmetric leading edge vortices. The asymmetric vortex phenomenon has been studied extensively in the case of slender bodies of revolution\(^1\) and has been observed also on slender delta wings\(^1\) (Fig. 6). Vortex asymmetry occurs before vortex breakdown only for non-dimensional delta angles \(\alpha \leq 18^\circ\) (Fig. 7). In order to use Fig. 7 to explore the effects of sideslip, \(\alpha\), and roll angle, \(\beta\), an effective apex half-angle is formulated as follows (for small angles, \(\alpha < 15^\circ\), \(\beta < 15^\circ\)).

\[
\begin{align*}
\tilde{\alpha}_A &= \alpha - \Delta \alpha \\
\Delta \theta_A &= \tan \alpha \sin \beta \\
\Delta \theta &= J \cos \beta
\end{align*}
\]

The \(C_L\) evaluated from static measurements for a 70° delta wing\(^1\) shows the effects of both vortex burst and vortex asymmetry (Fig. 8). The first break in the \(C_L\)-characteristics at \(\alpha = 20^\circ\), is caused by vortex burst on the windward wing half (Fig. 9). Eqs. (3a) and (3b) give \(\alpha = 24.3\) for \(\alpha = 20^\circ, \beta = 4\) (\(C_L\) evaluated for \(\beta = \pm 4\)). The loss of lift due to vortex burst on the right wing-half causes the observed decrease of the derivative magnitude \(C_{L''}\) (Fig. 9). At \(\alpha = 35^\circ\) one obtains \(\alpha = 15.1\), for \(\beta = \pm 4\), giving vortex lift-off on the leeward wing half (Fig. 9). This causes a loss of lift which results in an increase of the magnitude of the rolling moment derivative, \(C_{M_{\beta}}\), in agreement with the experimental results (Fig. 8).

Even more erratic \(C_L\)-characteristics have been measured on an 86° swept delta wing\(^9\) (Fig. 10a). The expected behavior, following that for the 82.5° swept delta wing, \((\alpha = 7.5°)\) is indicated by a solid line. The zig-zag behavior of the dotted line connecting the experimental results can be understood if one studies the \(C_L\)-characteristics (Fig. 10b). It appears that between \(\beta = 0\) and \(\beta = 5\) the vortex asymmetry switches several times, with vortex lift-off alternating between the two wing halves. A static hysteresis of \(\beta = \pm 5\) is indicated.

Experimental results\(^6\) demonstrate that wing rock starts before vortex breakdown (Fig. 11), and that wing rock is associated with a loss of the time-average lift\(^6\) (Figs. 11 and 12). It is, of course, to be expected that the "lift-off" of one of the leading edge vortices\(^1\) (Fig. 6) will cause a loss of lift. Thus, wing rock is caused by the vortex-asymmetry and not by the vortex breakdown. Figures 13 and 14 illustrate the fluid mechanical reasons for this. At an \(\alpha - \beta\) combination where vortex asymmetry occurs, the wing half with the lift-off vortex loses lift and "dips down", rotating around the roll axis (Fig. 13). As a result of the increasing roll angle \(\beta\), the effective apex angle \(\tilde{\alpha}_A\) is increased, Eqs. (3a) and (3b), and the vortex attaches again. This produces a restoring rolling moment, the positive aerodynamic spring needed for the rigid body oscillation in roll (Fig. 1). Due to the convective time lag effect discussed earlier the wing is dynamically unstable in roll until the amplitude has reached the limit cycle magnitude, at which the damping on both sides of the discontinuity suffices to balance the undamping induced by it, as is illustrated by the cylinder-flare results in Fig. 4. According to Fig. 13 wing rock should start occurring for an 80° delta wing \((\alpha = 10°)\) at \(\beta = 27°\), which is in excellent agreement with experimental results\(^1\).
Thus, the discontinuity introduced by the vortex asymmetry has all the characteristics needed for the limit cycle oscillation in roll. Figure 14 demonstrates that vortex breakdown is lacking these characteristics. If for some reason, due to external disturbances for example, the vortex burst becomes asymmetric, as is sketched in Fig. 14, the resulting net loss of lift on one wing half will cause it to "dip down". This increases $\phi$ and thereby $\partial_1$, Eqs. (3a) and (3b), causing this wing half to penetrate further into the vortex burst region, and no switch to a restoring moment occurs. The opposite wing half gets out of the vortex burst region, generating increased lift that adds to the statically destabilizing rolling moment. Thus, no restoring moment, no positive aerodynamic spring, is generated and no rigid body $\omega$-oscillation is possible. If the positive spring is provided by the structure, as in the case of elastic vehicle dynamics, the dynamic effect of the vortex breakdown would be dynamically stabilizing, damping as the vortex burst is also associated with time lag effects. Thus, vortex breakdown has aerodynamic characteristics completely opposite to those needed to cause slender wing rock.

Although vortex burst cannot cause wing rock, it is involved in the cases of wing rock observed for very high angles of attack $\alpha > 35^\circ$ (Fig. 15). Figure 15 illustrates the fluid mechanics for $\alpha = 42^\circ$ and $\phi = 10^\circ$, for which a limit cycle amplitude of $\omega = 12^\circ$ has been measured. At $\omega = 0$, $\partial_1 = 10^\circ$, asymmetric vortex burst exists. The wing half with the largest lift loss dips down, increasing $\omega$ and $\partial_1$. When $\omega = 0$ for $\omega = 11^\circ$ and the vortex-induced lift on the opposite leading edge wing half was reduced dramatically or lost completely, a restoring rolling moment is generated. Because of the integrated damping effects discussed earlier in connection with Fig. 13 the amplitude $\omega$ has to exceed $\omega = 11^\circ$ substantially before net zero damping is reached and the limit cycle oscillation called wing rock is established.

Whereas Nguyen et al. measured no wing rock for their $\phi = 27^\circ$ wing at $\phi = 20^\circ$ for the same leading edge sweep (Fig. 16). This early wing rock occurrence is probably, as the authors suggest, caused by the centerbody used on their model (see inset in Fig. 16). The smaller limit cycle amplitude $\omega = 12^\circ$ (Fig. 18) compared to $\omega = 34^\circ$ which was the limit cycle amplitude measured by the Nguyen et al. (Fig. 22), is probably due to the lesser vortex-induced loads existing at the lower angle of attack $\phi$.

Nguyen et al. showed that the oscillations in roll damped down to zero amplitude if the $\phi = 30^\circ$ delta wing ($\phi = 10^\circ$) was yawed to $\phi = 10^\circ$ at $\phi = 27^\circ$ (Fig. 17). This is, of course, to be expected as the windward wing half has $\omega = 15^\circ$, Eqs. (2), (3a) and (3b), leaving it outside of the boundary for asymmetric vortex shedding (Fig. 13), whereas the leeward wing half with $\omega = 5^\circ$ remains inside the region for vortex-asymmetry. Thus, neither wing half crosses the boundary, and the wing-rock-inducing discontinuity is never encountered. Correspondingly, oscillations in yaw are possible, as is the case for $\phi = 0$ according to the experimental results (Fig. 18).

It was noted by the authors in Ref. 18 that the normal force measured during wing rock was below that measured in static tests. Thus, at $\phi = 20^\circ$ the mean or time average normal force is $C_{N0r} = 0.64$ for $\psi = 14^\circ$ (Fig. 16) whereas the static data showed $C_{N0r}$ to vary from $C_{N0r} = 0.80$ to $C_{N0r} = 0.65$ when $\phi$ increased from $\phi = 0^\circ$ to $\phi = 15^\circ$. As the static data show no rolling moment at $\phi = 0$ for $\phi = 32^\circ$, it is obvious that the vortices stayed symmetric in the static case, whereas in the dynamic tests vortex-asymmetry must have been present to cause the wing rock. The likely reason for this anomaly is the large centerbody. Whereas a thin splitter plate of similar height has been found to trigger early vortex asymmetry also in static tests by forcing asymmetric stagnation flow conditions on the topside, the lateral extent of the centerbody in Ref. 18 apparently allowed symmetric vortex formation in the static test. As a matter of fact, the wing rock motion was not self-induced at $\phi = 20^\circ$ but had to be started at a higher angle of attack, in which case it would persist when the angle of attack was reduced to $\phi = 20^\circ$. Even at $\phi = 30^\circ$ the vortices remained symmetric for 15 seconds (Fig. 12). This cannot, however, explain the big difference observed at $\phi = 30^\circ$ in the dynamic test with $\phi = 30^\circ$, $C_{N0r}$ varied between $C_{N0r} = 0.66$ and $C_{N0r} = 0.6$ whereas the static test gave $C_{N0r} = 1.28$ and $C_{N0r} = 0.8$ for $\phi = 0$ and $\phi = 30^\circ$, respectively. In this case it is the early vortex burst observed in the dynamic tests that is the likely reason for the additional lift loss.

In regard to the usage of the results in Fig. 7, which are obtained for asymmetric flow conditions, $\phi = 0^\circ$, for the asymmetric flow conditions discussed in Figs. 9, 13, 14, and 15, the following needs to be said. Whereas vortex burst is only affected by the presence or absence of the vortex on the opposite wing half, the asymmetric vortex shedding is very
dependent upon the "crowding" of the companion vortex. It is the strength of the vortex, represented by \( a \) in Fig. 7, and the closeness of the opposite vortex, represented by \( a' \) in Fig. 7, which determine whether or not "lift-off" of the vortex will occur. In a first approximation the effect of side slip on the "closeness parameter" can be neglected. That is

\[
\left( a' \right)_{\text{EFF}} = \frac{\left( a' \right)_L + \left( a' \right)_R}{2}
\]

It is shown in Ref. 19 that the vortex strength and associated aerodynamic loads are determined by the parameter \( \alpha/a' \) rather than by \( a' \) alone. Consequently, the indicated changes of \( \alpha' \) in Fig. 13 should be substituted by changes \( \alpha/a' \). That is, the changes would occur in the vertical rather than in the horizontal plane. The conclusions would, however, be the same in regard to the effects of roll angle \( \phi \).

In Ref. 21 a simple analytic method is presented, which can predict the limit cycle amplitude for the wing rock oscillations measured by Nguyen et al.

**WING ROCK OF NON-SLENDER WINGS**

A completely different flow mechanism is the cause of wing rock of straight or moderately swept wings. It is closely related to dynamic stall. The experimental results--in Fig. 14 illustrate that plunging oscillations of an airfoil can be undamped in the stall region. It is shown in Ref. 23 that this will be the case if the stall is associated with a significant loss of lift. It is the "leading-edge jet", the moving wall/wall jet analogy discussed in Ref. 24 (Fig. 15), that produces the negative aerodynamic damping in plunges shown in Fig. 14.

If an aircraft is perturbed when flying close to stall, the down-rolling wing half will experience the upstream moving wall effect illustrated for the down-stroke in Fig. 15. As it promotes separation, the loss of lift is increased and then the static lift loss, more the higher the plunging rate \( \dot{Z} \). This generates a rolling moment that drives the motion, i.e., it is undamping. The delayed stall due to low pressure downstream of the opposite wing (upstroke in Fig. 15) will add to the undamping rolling moment.

Thus, the induced effects of the local plunging velocity \( Z \) will drive the wing in roll. What stops the wing rolling motion to produce wing rock? Eq. (1) and Fig. 14 give the answer.

When the roll angle \( \phi \) has been increased enough to cause \( \delta_{gpy} \) to decrease below \( \delta_{\text{STALL}} \), the flow will reattach to generate the lift needed to produce a restoring rolling moment; the aerodynamic spring needed for wing rock, as was described earlier. The flow reattachment is associated with time lag effects\(^5\), creating negative aerodynamic damping to be added to the "leading edge jet" effect discussed earlier. Thus, the condition for wing rock exists also for a conventional wing.

Associated with wing-rock is the oscillation in yaw called nose slice. The down-rolling wing half will move back due to the stall-induced drag increase. The increased leading edge sweep angle will promote flow reattachment, thus reinforcing the aerodynamic spring of the wing rock. For a straight or moderately swept wing, the side slip due to nose slice will dominate over the roll-induced side slip. \( \delta_{gpy} \) in Eq. (2), as \( a' \) is small. The opposite is true for a slender delta wing where the drag increase due to nose slice is small and \( \delta_{gpy} \) is the dominant side slip component (\( a' \) is large in Eq. (2)). Thus, one expects the coupling between wing rock and nose slice to be weak for highly swept wings and strong for straight or moderately swept wings.

**BODY ROCK**

That a pointed nose of a slender body can provide a third mechanism for wing rock, or body-rock, was demonstrated recently\(^2\). Wings and tail surfaces could be removed from the model of an advanced aircraft without stopping the rocking motion. Obviously, it must be the vortices shed from the pointed forebody that supplied the driving mechanism for this body rock motion. It has been established that the formation of asymmetric body vortices can be dominated by the body motion\(^27\), and that the vortex that is not lifted off moves inboard to remain very close to the surface near the centerline of the body\(^28\), 29 (Fig. 16). Placing the cockpit in the inset sketch of Fig. 16 and considering the data by Fidler\(^20\) (Fig. 17), one starts to see what this third flow mechanism is.

It is shown in Ref. 31 that at a critical Reynolds number negative Magnus lift of large magnitude will be generated at very modest rotation rate on a circular cylinder. It is described in Ref. 27 how this flow phenomenon, which is caused by moving wall effects on boundary layer transition, can explain the results in Fig. 17. That is, the direction of even a very slow rotation determines the direction of
the vortex asymmetry. Based upon these results, one obtains the picture sketched in Fig. 18 for the vortex-induced effects on the cockpit.

At $t = t_1$ the body receives a rotational perturbation as indicated. The upstream moving wall effect causes transition to move ahead of flow separation, thereby changing the separation from subcritical to supercritical type indicated in Fig. 18. The downstream asymmetric vortex close to the body generates suction on the cockpit, thereby driving the rolling motion. At $t = t_2$, the cockpit has rotated to a position where it interferes with the flow separation, triggering a change from supercritical to subcritical type separation. This generates a restoring rolling moment, which will reverse the roll direction at some time between $t = t_2$ and $t = t_3$. At $t = t_3$ the roll rate is large enough to cause transition, changing the flow separation from subcritical to supercritical and generating a driving rolling moment at $O = 0$, as is indicated for $t = t_3$. The situation is somewhat similar to that for slender wing rock discussed earlier. That is, a switch of vortex asymmetry generates the aerodynamic spring, and the associated time lag generates negative aerodynamic damping. The difference is that in the present case of body rock the undamping is amplified by moving wall effects.

The experimentally observed body rock was obtained on a model that only had the roll degree-of-freedom (DOF). For an aircraft in free flight the asymmetry will generate the largest effect in the yaw DOF. That is, the motion will be nose-slice-dominated with relatively weak feedback from the roll DOF illustrated in Fig. 18. However, for vortex interaction with an aspect this third flow mechanism may become much more significant in regard to its impact on pitch-yaw-roll coupling.

It should be noted that the third flow mechanism exists only in a limited range of $M$ and $Re$. Of course, the critical Reynolds number range may be relatively wide, as the transition induced separation asymmetry moves towards the nose tip on a pointed ogive or cone as the Reynolds number is increased (Fig. 19).

CONCLUSIONS

An analysis of wing rock phenomena has shown the following:

- Slender wing rock is caused by asymmetric leading edge vortices.
- Vortex breakdown has a damping effect on the roll oscillations and can never cause wing rock.
- Thus, slender wing rock will only occur for delta wings with more than 74° leading edge sweep.
- Wing rock of a conventional wing can occur at stall if the stall causes an abrupt lift loss.
- Body rock can be generated on body alone by asymmetric vortex shedding from a slender nose if the body is not completely axisymmetric.

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Fig. 1 Time History of Wing Rock at $\alpha = 27^\circ$
for an $80^\circ$ Delta Wing (Ref. 6)

Fig. 2 Nonlinear Aerodynamic Characteristics

Fig. 3 Effect of Time Lag

Fig. 4 Nonlinear Aerodynamics of a Blunt Cylinder - Flare - Body (Ref. 7)

Fig. 5 Leading Edge Vortex Development on a
Delta Wing (Ref. 10)
Fig. 7 Effect of Wing Rock on Lift (Ref. 6)

Fig. 6 a = A Boundaries for Vortex Asymmetry and Vortex Burst (Ref. 16)

Fig. 8 Development of Wing Rock for an 80° Delta Wing (Ref. 18)

Fig. 9 Wing Rock Caused by Asymmetric Vortices.
Fig. 10 Effect of Vortex Burst on Lateral Stability.

Fig. 11 Time History of Wing Rock, \( \alpha = 30^\circ \) and \( \mu = 20^\circ \) (Ref. 18)

Fig. 12 Roll Response of an 80° Delta Wing \( \alpha = 25^\circ \) (Ref. 69)

Fig. 13 Effect of Sideslip on Roll Ratio of an 80° Delta Wing (Ref. 69)
\[ \bar{c} = 0.136; \Delta \zeta = 0.153, M = 0.4 \]

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**Fig. 14** Airfoil Damping in Plunging Oscillations (Ref. 22)

**Fig. 15** "Leading Edge Jet" Effect

**Fig. 16** Effect of Asymmetric Vortex Shedding on Side Force and Normal Force
Fig. 17 Effect of Spinning Nose Tip on Vortex-Induced Side Force (Ref. 30)

Fig. 18 Vortex-Induced Load on Aircraft Cockpit

Fig. 19 Oil Flow Pictures of Pointed Ogive at $\alpha = 20^\circ$ (Ref. 33)
Abstract

Forced unsteady flows are examined from the point of view of potential application of new devices which are made possible by the improved understanding of these flows. In particular, flows are examined which require no external driver but obtain their energy from the free-stream velocity. Other flows require no moving parts at all to generate the unsteadiness.

Introduction

Unsteady separated flows may appear as a consequence of the motion of the surface of the body or as a result of the unsteadiness in the flow over that surface. In many cases, when confronted by such flows, the objective is to eliminate or at least control the separation.

The present research program has taken a somewhat different perspective. The objective of the program is to examine the dynamics of flows which are forced to become unsteady and to specifically consider the possibilities for the application of such flows as well as determining their basic structure. In addition to the question of identifying devices which can drive a flow unsteady in a suitable way, it is necessary to determine the composition and intensity of the large scale structures created in the periodic flow. The latter point has led to techniques which have some utility in determining the vortex structure of quasi-steady three-dimensional flows.

When considering possible techniques to produce an unsteady periodic flow from a nominally steady flow, the basic requirements are apparent:

1. The device that produces the flow perturbation must be simple and inexpensive as well as easy to build.
2. It should be possible to control the device and its power to drive it.

Some interests

Under the present research program, two nozzles have been employed to generate the time dependent flow from a nominally steady flow. The first is a series of purely hydraulic oscillating jet nozzles which satisfies both of the requirements cited above. Such a nozzle produces a jet which oscillates from side to side and results in large scale structures which in the flow visualization results of Figure 1) which have an important effect on the flowfield. Although these nozzles can be employed to control boundary layers, most of the effort has been applied to free jets. The jet is controlled by the fluidic feedback system wrapped around the body of the jet nozzle in Figure 2. An alternate version employing a different fluidic feedback has been found to be capable of producing an unsteady jet even if the phase of the jet flow is different from that of the surroundings.

Figure 1. Smoke flow visualization of a fluidically oscillating jet.

Figure 2. Schematic of the fluidically oscillating jet nozzle.
Another embodiment of a fluidically controlled jet nozzle is shown in Figure 3. In this case the dynamic orientation of the jet is determined by a pair of rotating valves located on either side of the nozzle exit. Since the jet is always inclined toward the closed valve, this nozzle has the advantage that the phase position can be controlled in addition to the frequency. A disadvantage is that the control valves must be externally driven.

![Figure 3. Schematic of the rotor vortex generator.](image)

The second sector is a can shaped rotor shown in Figure 4. As the rotor turns, and as long as the freestream velocity is greater than the tip speed of the rotor, the appearance of the rotor tip will result in the production of a vortex behind the rotor. The flowfield thereby produced has been proved and found to have application to controlling flow on aircraft wings (Figure 5b), improving mixing in a dump combustor (Figure 3b), and controlling the flow in a diffuser (Figure 3d).

![Figure 4. Schematic of the rotor vortex generator.](image)

Current Efforts

The design embodied by the unsteady diffuser experiment (Figure 5d) can be adapted to the rotor jet experiment shown in Figure 6. The rotors are located at both sides of the nozzle exit and can be operated in any phase relationship to each other to produce different jet flows. Operated in an in-phase condition, the rotors tip appear similar in nature and the resulting flow is a usual jet. Operating the rotors in an out of phase condition results in a jet which tips the side to side in a manner similar to that of the diffusor oscillating jet (Figures 1 and 2). In this case the jet vortex structure downstream is even stronger since it is formed by the rotors. The resulting flow shows the jet oscillate. The vortex structure of the field may be shown by transforming the vortex to a plane cut as shown in Figures 6 and 7. The cut is made at an angle and the vortex structure is identified.
Another problem of current interest is the application of the rotor vortex generator to reduce the drag of a bluff body vehicle. An example is a van in which the objective is to reduce the drag but to affect the volume of the van in a minimal way. Thus, simply streamlining the van has severe limitations. The method proposed is to employ the rotor to turn the flow to reduce the drag but to affect the volume of the van in a minimal way. Thus, streamlining the van has severe limitations. The size of the wake behind the van has severe limitations. The structure of the flowfield is continuously measured by the hot-wire probe. Since the structure of the flowfield generated by the rotor of Figure 4 is coherent and synchronously varying with respect to the position of the rotor upstream, the velocity is sampled with respect to the position of the rotor. Thus a complete "time picture" of the structure is obtained as it passes the probe. In addition, the coherency of the flow structures allows the signals from several successive rotations to be averaged to improve the signal to noise ratio and reduce the effects of turbulence on the measurements. The signals from the hot wire are processed and recorded by the microcomputer through an analog to digital converter.

The velocity of the air at any given point in the flowfield is continuously measured by the hot-wire probe. Since the structure of the flowfield generated by the rotor of Figure 4 is coherent and synchronously varying with respect to the position of the rotor upstream, the velocity is sampled with respect to the position of the rotor. Thus a complete "time picture" of the structure is obtained as it passes the probe. In addition, the coherency of the flow structures allows the signals from several successive rotations to be averaged to improve the signal to noise ratio and reduce the effects of turbulence on the measurements. The signals from the hot wire are processed and recorded by the microcomputer through an analog to digital converter.

The shaft of the rotor in Figure 4 has been fitted with a light chopper disk of two tracks. One track has a single hole used to indicate the start of the rotor orbit and the second track has thirty-six equally spaced holes (every 10°) used to indicate the position of the rotor in its orbit. These two tracks are "read" by light emitting diodes (LED), photo transistor pickups. The resulting signals are passed through an interface which provides signal conditioning to prepare the signals for the microcomputer.

A third area of current interest is the dominance of an initial rectilinear vortex structure near a wall generated by the rotor shown in Figure 6. A schematic of the dynamics which such a structure undergoes is shown in Figure 7. Even the violent eddy-like eddy behind a rapidly changing wall-motion structure near the wall the structure dominates the flow and should lead to a drag reduction since it substantially reduces the size of the wake behind the vehicle.

**Figure 7. Rotor jet flowfield seen in a moving coordinate system.**

The instrumented system has been designed to resolve the instantaneous velocities of the micro-turbulent flow structures in the vicinity of the hot-wire probe. The output of the hot-wire probe is transmitted to a high-speed storage oscilloscope through a differential amplifier to improve the signal to noise ratio and reduce the effects of turbulence on the measurements. The signals from the hot wire are processed and recorded by the microcomputer through an analog to digital converter.
screw. The position accuracy of the system has been found to be within ±1 turn of the lead screw or about 0.030 inches. The bottom indicator has a reproducibility of better than 0.010 inches.

A computer program to record, process, and store the data from the hot-wire probe has been written and tested. The program takes 100 samples of each of the two wires in the probe for each of the 36 positions of the rotor. This data is averaged and reflected through the calibration curves for the hot wires to yield 36 two component velocities for two wires. The data is displayed on the computer console and stored as a disk file for further analysis. The procedure above is then repeated for each probe position. In this manner, the complete time profile of the convected structure is obtained along the probe path. The probe step size in the right direction is programmable.

The remainder of this paper will concern itself with the fact that an external driver or power source is not needed to turn the rotor shown in Figure 4. This result improves the utility of the rotor and additional potential applications are discussed.

The Self-Powered Rotor

The results and potential applications of the rotor device cited in the sections concerned with past and current efforts were based on a rotor requiring an external source of power to turn and generate the vorticity. With proper design, however, the rotor motion can be self induced with the required power being drawn directly from the freestream velocity. Perhaps more interesting than the fact that the motion is self induced is the direction of rotation. As shown in Figure 4, the self induced direction of rotation is counterclockwise. Therefore, an increase in the rotational speed will also increase the strength of the resulting vortices.

The basic mechanism which drives the rotor in a direction which opposes the freestream velocity is shown schematically in Figure 4. The relative pressure distribution is shown on the exposed portion of the rotor for the given configuration. Although there is a stagnation pressure effect near the upstream side of the rotor, the remainder of the upper surface is dominated by a low pressure distribution. One might say that the rotor "flies" in the freestream. From the distribution, it is clear that an integration yielding the moment about the axis of the rotor will result in a counterclockwise moment.

Figure 10a: Flowfield produced by the rotor in Figure 4.

Figure 10b: Flowfield produced by the rotor in Figure 4 shown in a moving coordinate frame.

A typical result obtained by this system is shown in Figure 10a for the flowfield produced by the rotor in an instantaneous position rotated 180° in the clockwise direction from that shown in Figure 4. No vortex is evident in the data until it is transformed into a moving coordinate system. The transformed data is shown in Figure 10b as the presence of the vortex structure is that the acceleration of the transformed plane can be by trial and error matched by experiment or by calculation. The result obtained in Reference 32, the results corresponding to the data shown in Figure 3 are presented in Reference 32.
The startup operation of the unpowered rotor device is as follows: Even if the rotor is well balanced and relatively free to turn (i.e., low frictional resistance), at low freestream velocities the rotor remains in a static orientation. As the freestream velocity is increased, the rotor assumes the position shown in Figure 11b with the cusp shape directly above the shaft. A further increase in the freestream velocity results in an oscillation or rocking of the rotor about this position. The amplitude of the oscillations increases with freestream velocity until the rotor begins to turn in the counterclockwise direction. It was never observed to turn in the clockwise direction.

It should be emphasized that the results of Figures 12a and b are singular results for this particular rotor shape and for the frictional resistance of this bearing configuration. A change in the rotor shape or the frictional resistance would change the value of the curve but is not expected to affect the functional dependence.

From the point of view of potential application, the fact that the rotor can be employed without the need for an external driver is significant. It will allow more widespread use of the concept behind the device, namely the potential use of unsteady flows to achieve results superior to those found with nominally steady flows. All of the rotor-based devices discussed in the preceding sections will be positively affected, from the point of view of application, by the possibility of self-induced rotation.

Other Potential Applications

A wide range of potential applications exists for unsteady flow devices. Since an external driver is unnecessary to power the rotor vortex generator, the potential applications for this particular technique have improved. The following examples are all related to flow control near a solid wall and are not meant to be at all inclusive. Many other embodiments of these concepts will appear in the future.

![Figure 12a. Self-driven rotor speed as a function of freestream velocity.](image)

![Figure 12b. Non-dimensional self-driven rotor speed as a function of freestream velocity.](image)

For the rotor concept, a wall of vortices is shown in Figures 13a and b where the vortices have been placed in opposite sense at a parallel jet exit. In either case, the large-scale vortices produced in the jet will allow the flow to remain attached to the wall in a situation where it otherwise would separate. In other, the jet vortices applied in both sides of the jet simultaneously, but it is felt that this will be inconsistent with the boundary layer.
From the point of view of the influence on the flow, the concept suggested in Figure 11a is likely to be superior. However, supporting the required structure within the geometrical constraints of the nozzle may become difficult. From this point of view, the suggestion of Figure 11b may be more easily achieved.

**Figure 14. Schematic of a rotary valve geometry applied to a wall jet.**

Another mechanical concept which allows the exit of the phase mode in addition to the momentum is shown in Figure 14, and is related to the jet flute shown in Figure 1. In this case, the nozzle must be designed such that the flow remains attached to the wall when the rotary valve is in the open position shown. Upon closing the rotary valve, the jet detaches from the wall and attaches to the outer lip of the nozzle. The result is a wall jet which flops up and down and produces an intermittent flow at the surface. Although the phase mode is controllable, the rotary valve cannot be externally driven.

An alternate embodiment of the rotary valve concept is shown in Figure 15. As before, the valve is applied to a wall jet configuration in Figure 15. A hybrid valve, combining control ports A and B, is also illustrated. This valve has two different control modes, one of which is a direct control and the other a two-port control of vortex rotation.

**Figure 15. Schematic of a fluidic device producing an intermittent flapping.**

**Analyses.**

In order to determine the effect of the unstable flow in the field, the coherent vortex structures need to be identified. This will then allow the trajectories of the structures to be followed and thus link the structures to the time averaged effects. Two techniques have been developed and examined in detail. The first involves a Fourier transformation of the entire flow field. More recently, another analysis has been based upon decomposing the flow velocities into various components including rotation, translation, expansion and shear as shown in Figure 16. The rotational movement has proven to be more useful in locating vortex structures because it yields a definite pattern when these structures are in the field. An example is shown in Figure 16, where the vortex structures were found in the field behind the vortex generator of Figure 15. The closed regions indicate a vortex in this lower left hand grid right portions of the field and correspond well to the results of flow visualization.

Vortex structures are defined as vortex in the flow field and three dimensions, either three space dimensions or space dimensions at any time. These effects are best in the same constraint conditions as used in Section 1. However, the flow field analysis is more complex differences in parameter used. However, an effort is made to generate the structures in three dimensions in lateral dimensions.


Recent studies have shown that

The results of these studies have shown that

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Pitching angle</td>
<td>Deg</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Angle of attack</td>
<td>Deg</td>
</tr>
<tr>
<td>( C_D )</td>
<td>Drag coefficient</td>
<td></td>
</tr>
<tr>
<td>( C_L )</td>
<td>Lift coefficient</td>
<td></td>
</tr>
<tr>
<td>( C_T )</td>
<td>Thrust coefficient</td>
<td></td>
</tr>
<tr>
<td>( F_D )</td>
<td>Drag force</td>
<td>N</td>
</tr>
<tr>
<td>( F_L )</td>
<td>Lift force</td>
<td>N</td>
</tr>
<tr>
<td>( F_T )</td>
<td>Thrust force</td>
<td>N</td>
</tr>
</tbody>
</table>

The pressure coefficient distribution, Eq. (3), is given by

\[ C_{p,\text{dist}} = \frac{F_p}{\frac{1}{2} \rho V^2 S} \]

where

- \( F_p \) is the total pressure force
- \( \rho \) is the air density
- \( V \) is the free-stream velocity
- \( S \) is the reference area

The Coriolis effect on the vortex

The Coriolis effect on the vortex is given by

\[ \frac{d^2 \psi}{dt^2} + \nabla \cdot (\nabla \times \psi) = 2 \Omega \times \nabla \times \psi \]

where

- \( \psi \) is the vorticity
- \( \Omega \) is the angular velocity
DESCRIPTION OF THE DATA BASE

The data base for the present study was obtained from aerodynamic experiments performed in oscillating tunnel-spanning wings (OTW) in both constant and sine swept configurations in the United Technologies Research Center (UTRC) Variable Speed Tunnel. The aerodynamic forces were taken normal to the wing leading edge of an NACA 0012 airfoil. A detailed description of the test program can be found in Ref. 1, and an amplification of the material discussed here is presented in Ref. 2.

The parameters for this test were the wing sweep angle, \(\alpha\), the amplitude of pitching motion, \(\delta\), the sine sweep of attack, \(\alpha_s\), the pitching frequency, \(f_p\), and the approach Mach number, \(M_\infty\). The effect of sweep was accounted for in the normalization process by reducing all data to the component in the course of \(M_\infty\) normal to the wing span.

\[ M_\infty = \frac{M}{\cos \alpha} \]

where \(\alpha\) is the chordwise direction.

Pressure data were obtained for alveolar angles and the carrier sweep set at the values of \(\alpha = 30^\circ, 45^\circ, 60^\circ\) and \(\alpha_s = 0^\circ, 2^\circ, 4^\circ, 6^\circ, 8^\circ, 10^\circ\). For each value of \(\alpha\) and \(\alpha_s\), the pressure data were averaged for each of the values of \(a\) and \(\alpha_s\). All tests were carried out in a plane perpendicular to the spanwise. Tests were carried out over a range of frequencies such that the swept-wing Reynolds numbers are

\[ R_e = \frac{U \cdot L}{v} \]

where \(U\) is the free stream velocity, \(L\) is the span of the wing, and \(v\) is the kinematic viscosity.

A dimensional pressure coefficient is

\[ C_p = \frac{p - p_\infty}{\frac{1}{2} \rho U^2} \]

where \(p\) is the pressure coefficient, \(p_\infty\) is the free stream pressure, \(\rho\) is the density of the air, and \(U\) is the free stream velocity.

The pressure data were then converted to the relative motion of the swept airfoil.

\[ \Delta = 
\begin{cases} 
9^\circ & \text{if } \alpha = 30^\circ \\
12^\circ & \text{if } \alpha = 45^\circ \\
15^\circ & \text{if } \alpha = 60^\circ 
\end{cases} \]

\[ M_\infty = 0.30 \quad \delta = 0.125 \]

\[ \alpha_s \quad \delta \quad \frac{\alpha_s}{\alpha} \]

\[ (\alpha, \alpha_s) \quad \text{Table of Values} \]

\[ \text{Fig. 1: PRESSURE DILT, LEADING EDGE} \]

\[ \text{Fig. 2: PRESSURE DILT, TRailing EDGE} \]

\[ \text{Fig. 3: PRESSURE DILT, LEADING EDGE} \]
For this target plot a set of constant wave contours (isotrans) are constructed in the region as shown in Fig. 16. The numbers that are inserted in each contour are values of constant reduced frequency, and the direction of the arrow at contour extremes identifies the direction of the gradient, with a single estimate of the overall wave speed.

Here it concentrates in the first four contours of the plot, which is the region of the chart. The second segment of the chart location where the wave speed reduces is the overall chart location. The second segment is the overall chart location. This set of contour segments always have an increasing wave speed as the chart is plotted from the first to the third segment. If you are interested in the overall wave speed as the chart is plotted from the first to the third segment, for example, is plotted time is the plot.

\[ \omega_c = f \cdot P + \frac{k}{c} \cdot \frac{\Delta t}{\Delta y} \]

In general, the leeward contour estimates are a function of the halving wave speed, and wave segments of different slopes. The end points of the first segment are labeled by \((\theta_1, \theta_2, \theta_3)\) and those of the second segment by \((\theta_4, \theta_5, \theta_6)\). Hence, there are actually two distinct regions within the first 10 percent of the chart that have different wave speeds. For this particular example, these values are \((\theta_1, \theta_2, \theta_3) = (1.15, 1.02, 0.12)\) and \((\theta_4, \theta_5, \theta_6) = (0.12, 0.02, 0.04)\), and with \(t = 0.25\), application of the wave speed Eq. (1) yields \(\omega = 0.13\) (first segment), \(\omega_2 = 0.04\) (second segment), and \(\omega_3 = 0.03\) overall. Furthermore, Fig. 14 of Ref. 1 for the entire chart shows that another discontinuity can be defined in the 10 percent chart station, separating the first segment from the second segment of the chart, indicating additional small changes along the chart. This first contour point is labeled by \((\theta_1, \theta_2, \theta_3) = (1.15, 0.84, 0.12)\). In addition, another wave segment can be defined by \((\theta_4, \theta_5, \theta_6) = (0.12, 0.02, 0.04)\) overall. It is seen that the wave speed increases in the region of high pressure gradient, which is the location characterized by the location of constant level contours lines in Fig. 10. Conversely, the wave travels are \(1.15\) over the second segment \(\omega_2\) where the dominant pressure gradient is lower, and utilizing its highest speed over the last segment \(\omega_3\) where pressure gradient is least.
FIG. 3 EFFECT OF SPEED ANGLE ON WAVE SPEED AT SEVERAL CHORDWISE LOCATIONS FOR M_s = 0.30 AND \( \beta = \theta \)

It is clear that there is a substantial effect of speed angle on both wave speeds. The upper left panel shows a negligible difference, while the right hand panels are the two average wave speeds, \( V_{W1.1} \) over the first 15% of the chord, and \( V_{W0.1} \) over the entire chord. (The latter is repeated from the upper left panel of Fig. 1.) It is clear that there is a substantial effect of speed angle on the local wave speed over the forward 15% of the chord, whereas examination of the panels for \( V_{W1.1} \) and \( V_{W0.1} \) over the entire chord show no significant differences. Virtually all of the same effect is observed to occur in the after portion of the chord. Lower left panel, however, is somewhat different, with a wave speed increase that is quite similar. This result is representative of all cases studied. The pressure wave is associated with the formation and subsequent transport of vortices, and results could imply that the latter effect may be independent of a number of conditions. Virtually no significant changes occur in the chord and its effect is deemed to be constant.
The steady state stall angle based on the zero moment slope, \( \alpha_{ss} \), is unaffected by sweep changes, consistent with Fig. 3 or Ref. 2 which is based on the data of Ref. 5. A comparison of the steady state stall angle with the zero frequency intercept of the least squares vortex inception line shows a two degree difference at \( \beta = 30^\circ \), and a reasonably good correlation at \( \beta = 1^\circ \).

A careful study of Fig. 2 reveals little or no effect of sweep angle on the least squares vortex angle. Specifically, if any constant value of \( \beta \) there is no horizontal trend of \( \alpha_v \) till there is only matter, this is an observation that is directly supported by the effect of pitch on \( \alpha_v \), which is altered next.

FIG. 4 EFFECT OF MACH NUMBER ON VORTEX INCEPTION ANGLE BASED ON INITIAL WAVE PROPAGATION
FIG. 5: TYPICAL MOTION TIME HISTORIES FOR THREE MEAN ANGLES OF ATTACK
The values of $a$ for the intersections of the $\phi = \frac{\pi}{2}$ line with the three motion curves in Fig. 5 are noted on the figure and are seen to vary from 0.056 to 0.021.

A direct examination of the effect of mean angles of attack variations on $\phi$ with reduced-frequency effects included is afforded by a return to the linear least-squares fit of Eq. (5). The scatter of individual data points may be substantiated by evaluating the equation in the form

$$ c = a - a_0 - ak $$

where $c = 0$ for an exact linear fit. In Fig. 6 a plot of this error function, $c$, versus $a_0$ illustrates the irregularity of the scatter, and thus directly to the observation that the vortex inception angle is independent of the pitch rate and $a_0$ is fixed. With the exception of a few isolated points, the error, $c$, is generally bounded below $|c|$ of 0.05 deg. and never exceeds 1.0 deg.

A question must be raised at this point on the validity of the relationship of the vortex inception angle to $a_0$ to the various dynamic stall measurements that have been cited in the literature. For example, "Altman's 1962 report (Ref. 7) several authors have shown that the dynamic stall angle increases with both increasing reduced frequency and pitch rate, yet at the same time, the vortex inception angle is independent of the pitch rate and $a_0$ is fixed. With the exception of a few isolated points, the error, $c$, is generally bounded below $|c|$ of 0.05 deg. and never exceeds 1.0 deg.

The apparent inconsistency lies in the fact that the data are plotted versus $\phi$ in Fig. 7 or versus $\phi$ in Fig. 8. The data are now in keeping with the results in Fig. 6 and the original apparent linear data fit is now a parabola.

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FIG. 3 CORRELATION OF RELATIVE VORTEX INCEPTION ANGLE WITH $\alpha_y$

FIG. 4 STEADY-STATE LIFT COEFFICIENT FOR NACA 0012 AIRFOIL AT $M = 0.40$

It has been found that this displacement of the lift response is fully associated with the stall of the airfoil. The time-mean component of the lift response has moved to its closer alignment than shown in Fig. 1. The results are shown in the form of Fig. II for the $15$ degree angle of attack case where Fig. III is repeated for Fig. II and plot II is omitted as the time-mean component. The resultant lift of change in the relative positioning of these lines indicates that the entire displacement is a time-dependent, metastable phenomenon. A similar analysis of these lines has been performed in Figs. II and III, in both of the instantaneous and the time-mean cases. The time-mean component has been corrected for the stall of the airfoil. This is clearly illustrated in Fig. II, where the lift/dep. of time displacement is accounted for. The remaining comparison in Fig. II (plots I through III) shows the addition of higher harmonic terms contributes to evoking the complete natural response without significant influence on the lift displacement at the tip of the airfoil.
Due to a factor of 1.3 to 1.5. This represents a lift failure above stall for the nose low-lift situation which has been shown to be sufficiently roll below stall.

- The order has a small effect on vortex inception angle and stall angle with a decrease in both angles as the increase.

- Lateral incision is substantially independent of angle of attack with the exception at 3 deg angle of attack.

- Lift coefficients are small differences associated with near supercritical to the region of the plate at or the 15 percent chord. Same trends are substantiated for case for both mean angles forward of this chord location.

- The vortex incision angle is independent of lift and also slightly smaller as the angle of attack increases.

- Angle of attack is very small effect on the total integrated lift response is very strongly determined by the first harmonic term.

**REFERENCES**


SIMULTANEOUS FLOW VISUALIZATION AND UNSTEADY LIFT MEASUREMENT ON AN OSCILLATING LIFTING SURFACE

Robert L. Bass**, James E. Johnson**, and James F. Unruh†

Abstract

Boundary layer and trailing edge flow activities were recorded using hydrogen bubble flow visualization techniques on an oscillating lifting surface in a two-dimensional water tunnel. Simultaneously with flow documentation, unsteady lift was measured over a range of reduced frequencies from 0.5 to 10. Unsteady loads using classical, inviscid theories were predicted for the experimental conditions investigated. Reduced frequency bands exhibiting poor agreement between experiment and theory were identified and a correlation to observed flow phenomena was accomplished. The results support the utilization of a separate viscous model near the trailing edge coupled with an inviscid flow field model to predict unsteady loads. The results further show that for certain reduced frequency bands, classical inviscid solutions may be applicable and adequate.

Nomenclature

b 1/2 chord length
C Chord length
C\theta Oscillatory lift coefficient
C_Lmax Two-dimensional steady state lift curve slope
C_L Oscillatory lift
C_I Reduced frequency (\omega/V)
t Time
V Free stream velocity
\delta Distance along chord from leading edge
\alpha Instantaneous angle of attack
\sigma Oscillatory angle of attack amplitude
\alpha_m Mean angle of attack
\delta_l Boundary layer thickness
\delta_w Boundary layer displacement thickness
L Width of lifting surface
\nu Fluid kinematic viscosity
\nu_c Circular frequency
\rho Fluid density
\phi Phase shift in Theodorsen's function

Introduction

The dynamics of the boundary layer (viscous effects) have an important bearing on lifting surface unsteadiness and hydrodynamic behavior. Inviscid theories are not adequate in many practical applications in their prediction of unsteady loads on flutter inception.

Improved theories of lifting surface dynamic performance which account for real fluid effects are required to advance the state of unsteady aerodynamics. This realization has led numerous researchers\(^ {1-2}\) to re-evaluate assumptions used in inviscid unsteady aerodynamic theories. Both analytical and experimental efforts have been undertaken. Most studies have evaluated the applicability of the Kutta condition in unsteady flows. Significant experimental activity has been undertaken to study trailing-edge loading and flow patterns on oscillating lifting surfaces. Theoretical efforts have also been initiated to include viscous corrections to inviscid solutions to allow better prediction of oscillatory loads. These research efforts have been accomplished primarily in 1974-1980.

In the composite, these efforts, which are discussed in Reference 10, represent both analytical and experimental research investigating the effects of viscosity on unsteady loads on oscillating lifting surfaces. The experimental efforts show that for many cases, viscous (real fluid) effects drastically alter the trailing-edge conditions on oscillating lifting surfaces and, thus, the classical Kutta condition is not maintained. The experimental work represents measurements of trailing-edge pressures and flowfield patterns under various oscillatory conditions. In some cases low reduced frequencies have been utilized and in other cases high reduced frequencies. Theoretical work has replaced the Kutta condition with auxiliary conditions which appear appropriate relative to the viscous flow phenomena at the trailing edge which alter the associated circulation.

In recent years, unsteady fluid dynamics has received considerable attention as it relates to unsteady separated flows, and there is a wealth of literature in high Reynolds number dynamic stall and unsteady viscous-inviscid interaction on oscillating airfoils.\(^ {11-18}\) Current interest in high angle of attack supermaneuverability aircraft has emphasized the need to advance the state of knowledge in unsteady separated flows. Also recent interest in low speed (Reynolds number) drone aerodynamics\(^ {19-21}\) has required a re-evaluation of unsteady separated flow activity at condition where viscous effects on separation phenomena and associated loads can result in significantly different lifting surface response than would be encountered at the Reynolds numbers associated with conventional or high speed aircraft. Regardless of the application, the protected operation of advanced lifting surfaces with active load controls in separated flow environments requires a significant advancement in unsteady separated flow technology.

Objective

The objective of this study is to expand the state of knowledge about the effects of unsteady boundary layer activity (viscous effects) on the loads of oscillating lifting surfaces. These objectives were accomplished through visualizing the boundary layer activity in a 2-D lifting surface harmonically pitching at the 4 Hz in a two-dimensional water tunnel. Simultaneous with boundary layer observation and documentation, unsteady lift was measured for a wide range of reduced frequencies 0.5 to 10. These measurements were conducted with the flow
Flow Visualization Equipment

Hydrogen bubbles were utilized to provide flow visualization for this study. A grid of platinum wire was placed upstream of the test model to provide visualization of the stream lines. In addition, hydrogen bubble generators were imbedded at different locations on the model surface to allow direct injection of tracers into the oscillating foil boundary layer. Photography of the hydrogen bubble tracers was successfully accomplished with a 16mm high-speed motion picture camera with zoom lens. The water tunnel test section viewing window was marked with a vertical line for a tracer reference point. The developed film was analyzed frame-by-frame on a Vanguard Motion Analyzer where flight trajectories of hydrogen bubbles were easily determined. The film analyzer was configured to computerize the coordinates of motion of the flow tracers. The computerized space-time histories of individual bubbles were used to establish flow separation and reattachment locations and reverse flow activity during a foil oscillation cycle. These data were used for recording qualitative information on boundary layer activity and were not used for load determination.

Dynamometer Design

A dynamometer, based upon the work of Epperson and Pengelley^, was used for measuring the dynamic lift coefficient on the oscillating model. The dynamometer, shown in Figure 1, is classified as an external dynamic balance which can resolve two forces (vertical and horizontal) and one moment about a given axis perpendicular to the plane containing the two forces. Electric resistance strain gages attached to the structural elements of the A-frame flexures mounted back-to-back were incorporated in a Wheatstone bridge circuit in such a manner that the bridge output was proportional to the applied external moment.

The airfoil section was supported by a shaft connected to an external dynamometer unit located on each side of the tunnel test section. The dynamometer units were attached to a clevis that was allowed to rotate upon shaft bearings mounted on the tunnel side walls. Oscillatory motion was imparted by a hydraulic servos wylinder located on the lower structure of the test section. A unique pressure balanced seal was fabricated which absorbs no load but inhibits the flow of liquid from the test section to the surroundings.

Model Design

A NACA 16-012 airfoil section molded of Resolin compound in two halves with a chord 3.5 cm long was used for the model. An oscillation shaft was positioned at the quarter chord point. The shaft was hollow and carried electrical wires from the surface hydrogen bubble generators to the power source. Surface generators were made of gold-plated aluminum chips, 0.019 cm by 0.635 cm imbedded flush in the surface of the hydrofoil. Figure 2 shows the model cross-section with the bubble generator locations and center coordinates.

Flow Visualization Results

Table 1 summarizes the test conditions employed. Flow visualization was obtained using a tungsten filament lamp, a 500-watt light bulb, and a Weston Type 920 illuminometer.
HYDROGEN BUBBLE GENERATOR LOCATIONS

At the lowest frequency tested (k = 0.45), the boundary layer remained attached to the foil until approximately the 3/4 chord where the boundary layer separated from the foil surface. During an oscillation cycle fluid was pumped into the separation zone along the foil trailing edge as the angle of attack increased to the maximum value of 6.8°. As the foil reversed direction and proceeded to lower angles of attack, fluid was pumped out of the separation zone into the wake until the minimum angle of attack of 4.8° was reached.

Increasing reduced frequency to k = 0.55 resulted in leading edge separation at X/2b = 0.17. The boundary layer reattached at approximately 40% of the chord and separated again at about 56% of the chord. Activity in the separation zone at the trailing edge of the foil was similar to the k = 0.45 case.

Increasing k to 0.65 resulted in leading edge separation at X/2b = 0.15 with a slight oscillation of the leading edge separation point during a foil pitching cycle. The boundary layer reattached at approximately 60% of the chord with recirculation oscillating between 6.3° < θ < 7.6° during a pitch oscillation with

The results showed that the boundary layer separation angle of attack increased to the maximum value of 6.8°. As the foil reversed direction and proceeded to lower angles of attack, fluid was pumped out of the separation zone into the wake until the minimum angle of attack of 4.8° was reached.

Increasing reduced frequency to k = 0.65 resulted in leading edge separation at X/2b = 0.15 with a slight oscillation of the leading edge separation point during a foil pitching cycle. The boundary layer reattached at approximately 60% of the chord with recirculation oscillating between 6.3° < θ < 7.6° during a pitch oscillation with
Table 1  Model test conditions

<table>
<thead>
<tr>
<th>Run</th>
<th>Freestream velocity, $V$, cm/s</th>
<th>Oscillation frequency, $f$, Hz</th>
<th>Reduced frequency, $k = 2\pi f / V$</th>
<th>Reduced velocity, $\lambda$</th>
<th>Angle of attack, deg</th>
<th>Reynolds No*. $Re = 2\pi V / \nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HV 1</td>
<td>9.82</td>
<td>0.132</td>
<td>0.648</td>
<td>54</td>
<td>5.85 - 0.95 sin 0.83</td>
<td>16,105</td>
</tr>
<tr>
<td>HV 2</td>
<td>3.90</td>
<td>0.147</td>
<td>1.80</td>
<td>155</td>
<td>5.825 - 0.95 sin 0.925</td>
<td>138</td>
</tr>
<tr>
<td>HV 3</td>
<td>16.23</td>
<td>0.151</td>
<td>0.445</td>
<td>2.25</td>
<td>5.775 - 1.05 sin 0.947</td>
<td>26,620</td>
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<tr>
<td>HV 4</td>
<td>12.51</td>
<td>0.144</td>
<td>0.552</td>
<td>0.81</td>
<td>5.80 - 1.0 sin 0.904</td>
<td>20,650</td>
</tr>
</tbody>
</table>

1) Velocity variation

| HV 1| 4.33                           | 0.387                          | 4.28                            | 0.574                    | 5.9 - 0.9 sin 2.43     | 7,105                         |
| HV 2| 4.18                           | 0.329                          | 3.76                            | 0.266                    | 6.05 - 0.95 sin 2.078  | 6.865                         |
| HV 3| 4.07                           | 0.316                          | 3.54                            | 0.102                    | 6.05 - 0.95 sin 5.269  | 6.70                          |
| HV 4| 3.69                           | 0.243                          | 3.13                            | 0.319                    | 5.05 - 0.95 sin 1.514  | 6.090                         |

2) Frequency variation

3) Mean angle of attack variation

| HV 1a| 3.99                           | 0.416                          | 5.24                            | 0.191                    | 1.95 - 1.05 sin 2.74    | 6,545                         |
| HV 2a| 4.18                           | 0.602                          | 6.89                            | 0.145                    | 5.60 - 1.0 sin 3.788    | 6,860                         |
| HV 3a| 4.56                           | 0.575                          | 6.04                            | 0.166                    | 7.65 - 1.05 sin 3.635   | 7,485                         |
| HV 4a| 4.34                           | 0.633                          | 6.96                            | 0.143                    | 1.55 - 1.05 sin 3.981   | 9,204                         |
| HV 5a| 4.70                           | 0.649                          | 6.90                            | 0.145                    | 2.25 - 1.05 sin 4.081   | 7,385                         |

4) Pitch amplitude variation

| HV 1a| 4.99                           | 0.503                          | 4.09                            | 0.244                    | 4.25 - 1.55 sin 1.61    | 9,655                         |
| HV 2a| 4.76                           | 0.435                          | 3.91                            | 0.256                    | 4.25 - 2.45 sin 2.952   | 9,450                         |
| HV 3a| 3.78                           | 0.446                          | 3.69                            | 0.274                    | 4.20 - 4.8 sin 2.363    | 9,245                         |

* Based on theoretical value for an equivalent flat plate of chord 2b

Table 2  Model boundary-layer observations

<table>
<thead>
<tr>
<th>Flow regime</th>
<th>Run</th>
<th>$k$</th>
<th>$f$, Hz</th>
<th>Observed boundary-layer activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>HV 1</td>
<td>0.45</td>
<td>2.2</td>
<td>16.2</td>
</tr>
<tr>
<td></td>
<td>HV 2</td>
<td>0.55</td>
<td>1.8</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>HV 3</td>
<td>0.65</td>
<td>1.5</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>HV 4</td>
<td>0.85</td>
<td>1.2</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Table 3  Model boundary-layer observations

<table>
<thead>
<tr>
<th>Flow regime</th>
<th>Run</th>
<th>$k$</th>
<th>$f$, Hz</th>
<th>Observed boundary-layer activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>HV 2a</td>
<td>0.85</td>
<td>1.2</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>HV 3a</td>
<td>1.5</td>
<td>0.56</td>
<td>4.0</td>
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<tr>
<td></td>
<td>HV 4a</td>
<td>3.1</td>
<td>0.32</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>HV 1a</td>
<td>4.3</td>
<td>0.26</td>
<td>4.3</td>
</tr>
<tr>
<td></td>
<td>HV 2a</td>
<td>4.8</td>
<td>0.10</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>HV 3a</td>
<td>10.9</td>
<td>0.09</td>
<td>4.9</td>
</tr>
</tbody>
</table>

* Based on theoretical boundary-layer activity on the flat plate of chord 2b

*2 At $X = 0.75$, a distinct vortex shedding from the leading edge was observed with the jet oscillation frequency equal to the pitch oscillation frequency.

*3 During an oscillation cycle, flow structure in the separation region for the first time was identified to the described for $X = 0.15$.

*4 Increasing the angle of attack resulted in a wider separation region and pronounced activity on the separation zone.
unsteady lift measurements were made. Figure 5 shows a comparison of theory and experiment for oscillatory lift coefficients \(C_{L, osc} \) versus reduced velocity \((1/k)\). Also shown on this figure are the various flow regimes (see Table 2) where different distinct boundary layer activity was observed during the flow visualization.

\[
\begin{align*}
\text{Fig. 5 & Comparison of theory and experiment for oscillatory lift coefficients } C_{L, osc} \text{ vs. } 1/k.
\end{align*}
\]
C(k) = C(k)e^{j\phi}.

A phase shift between the virtual mass and oscillatory lift contributions to C(k) was introduced to synthesize changes in the Kutta condition.

From the data shown in Figure 5, it can be seen that agreement between theory and experiment is good in those regions where boundary layer activity was well behaved. For 1/k < 0.75 (k > 1.33) good agreement between theory and experiment is noted. Also, for 1/k > 2 (k < 0.5), reasonable agreement is noted. However, at the knee of the theoretical curves (0.75 < 1/k < 2.0), significant deviation between measured and predicted oscillatory lift coefficient is noted. This flow regime represents conditions where the greatest degree of boundary layer activity is recorded wherein significant separation and reattachment of the boundary layer was observed. Also, the flow near the trailing edge was separated with flow alternating around the trailing edge region as the foil oscillated through its maximum and minimum angle of incidence. Since such trailing edge flow conditions will significantly affect the circulation around the oscillating foil in comparison to that predicted from classical inviscid theory, lift predictions were also carried out for various values of k. As noted in Figure 5, a 30 phase lag in C(k) provides improved agreement between theory and experiment.

Conclusions and Recommendations

The presented simultaneous unsteady loads measurements and boundary layer flow visualization provide additional insight into the effects of virtual unsteady loads on oscillating lifting surfaces, and additional insight into the flow conditions at the different reduced frequencies. The results in this paper show that:

1. The Kutta condition, based on qualitative flow visualization, was violated for many of the observed oscillating foil test conditions.
2. Observed boundary layer phenomena supports the hypothesis that unsteady boundary layer activity is responsible for poor agreement between current theoretical predictions and experimental data for predicting nonsteady loads and flutter.
3. The largest disagreement between unsteady load theory and experiment occurs in the reduced frequency range 0.75 < 1/k < 2.0, which corresponds with the most pronounced boundary layer activity (i.e., flow separation, flow around the trailing edge, etc.).
4. Introducing a phase lag in the circulation function provided better agreement between theory and experiment.

The experimental work presented in this report supports the hypothesis of a separate virtual mass effect, identified with the phase lag in C(k), which can be coupled with an inviscid flow field model to provide unsteady load predictions. The results of this study should be compared to the theory of Takagi and becoming a tool for obtaining information about frequency band classical inviscid solutions may be applicable and adequate.

Acknowledgements

The authors of this paper wish to express their appreciation to Dr. H. Norman Abramson who first suggested this work. Also, we wish to express our gratitude to Mr. C. M. Wood for his dedicated work in conducting the experiments, Mr. Victoriano Hernandez for his skillful art work on the figures, and Mrs. Adeline K. Raeke for typing the text.

References


A VISUAL STUDY OF A DELTA WING IN STEADY AND UNSTEADY MOTION

Mohamed Gad-el-Hak
Chih-Ming Ho*
Ron F. Blackwelder**
Flow Research Company
Kent, Washington 98032

Abstract

Two delta wings with a leading edge sweep of 45° and 60° were studied in a towing tank at chord Reynolds number up to 3.5x10⁶. The wings were pitched about the quarter chord point through typical angles of attack of 15° ± 15°, with a reduced frequency in the range of 0 to 1. In the steady state flow, dye visualizations revealed the existence of a shear layer near the leading edge that rolls up and forms discrete vortices parallel to the leading edge. These vortices were observed to pair at least once as they were convected downstream. Similar phenomena were observed in the unsteady case, except that the vortices shed from the leading edge were modulated and altered by the unsteady motion, which was an order of magnitude lower in frequency. In general, the unsteadiness delayed separation and promoted hysteresis similar to results obtained in unsteady two-dimensional airfoils.

Nomenclature

AR aspect ratio
β root chord
f pitching frequency
K reduced frequency, -fc/Rc
Re chord Reynolds number, Re=U/C
W wing semispan
T time (sec)
U* towing speed
α angle of attack
γ apex angle
θ angle between vortex cores
η perturbation wavelength
κ kinematic viscosity

1. Introduction

In a steady flow, a traditional two-dimensional airfoil experiences stall at large angle of attack. The separation on the upper surface reduces the leading edge suction peak and subsequently the lift decreases. The stall angle for most airfoils is around 12°. The aerodynamic effects of a delta wing are considerably different. The leading edge suction peaks predicted by potential theory do not exist (Jones and Cohen, 1961). Instead, two smooth suction peaks inbound of the leading edges are detected. The lift is mainly contributed by these two peaks which are produced as the flow separates on the leeward side of the wing and forms a pair of stationary leading edge vortices. Therefore, the lift on a delta wing is created by the separated vortical structures rather than by the attached flow near a convex surface. These vortices exist at angles of attack as large as 30° or more. The lift keeps increasing with α until the vortex breaks down. Hence, a delta wing is a good means to obtain high lift at large angle of attack.

By using air bubbles for visualization in a water channel, Elle (1938) showed that the separation vortices have very concentrated cores. Furthermore, the angle between the two cores, γ, is not a sensitive function of the angle of attack. The ratio between γ and the apex angle, β, is always between 0.6 and 0.7, but the vortex cores lift away from the wing surface with increasing α. Fink and Taylor (1967) investigated the pressure distribution on a wing with 0° ± 20°. The suction peaks of the spanwise pressure distribution at several chordwise locations always occurred at α = 20° of the semispan from the center for all tested angles of attack, 5° < α < 30°. In other words, the suction peaks were located under the vortex cores. A more detailed visualization (Fink, 1967) showed that there is a counter rotating vortex associated with each primary separation vortex. The existence of the counter rotating vortex could also be inferred from the total head survey.

Under many practical situations, e.g. fast maneuvering of an aircraft, the flow is not steady. The unsteady aerodynamic properties are significantly different from those in steady flow. For a two-dimensional airfoil, the lift, drag and moments experience large hysteresis during the execution of one cycle (Allister and Carr, 1978 & McCroskey, 1982). A large separation vortex develops near the leading edge and convects along the chord. High level surface pressure fluctuations are produced. On the unsteady delta wing the information about the time evolving vortices is very limited (Lambourne et al., 1969). This is one of the main reasons for this work.

2. Experimental Approach

2.1 Models and Test Conditions

Two delta wings with a leading edge sweep of 45° and 60° were used in the present investigation. The root chord of both wings was 25 cm, and the chord Reynolds number varied in the range of 2.5x10⁶ to 1.3x10⁷. Fig. 1 is a sketch of the 45° delta wing, which had a NACA 0012 profile at each spanwise section. The wing was made of two aluminum pieces with grooves on the inner surface of each for dye injection and storage. The 60° delta wing had a flat surface with a sharp leading edge.
Fig. 1 Schematic of the 45° Delta Wing.

The four-bar mechanism shown in Fig. 2 was used to align and to pitch the delta wing around the desired position along the chord. In the experiments reported herein the wing was pitched around the 1/4 chord position. The mean angle of attack could be set from 0° to -45°. A Boston Ratiotrol motor derived the four-bar linkages to produce approximately sinusoidal oscillations of amplitude ±10° and ±15° about a given mean angle of attack. The reduced frequency, \( K = \frac{f c}{u_{in}} \), was varied in the range of 0 to 3, and a digital readout displayed the instantaneous angle of attack of the wing.

Fig. 2 Photograph of the Pitching Mechanism.

2.3 Flow Visualization

Food color and fluorescent dyes were used in the present investigation. The food color dyes were illuminated with conventional flood lights. The fluorescent dyes were excited with sheets of laser projected in the desired plane. To produce a sheet of light, a 5 watt argon laser (Spectra Physics, Model 164) was used with a mirror mounted on an optical scanner having a 720 Hz natural frequency (General Scanning, Inc.). A sine-wave signal generator, set at a frequency equal to the inverse of the camera shutter speed, derived the optical scanner to produce light sheets approximately 1 mm thick.

Side views of the flow field were obtained using a vertical sheet of laser in the x-y plane at \( z = 10 \text{ cm} \) (40% of the root chord) and a stationary camera outside the tank as shown in Fig. 3. End views were obtained using a vertical sheet of laser in the y-z plane at \( x = 20 \text{ cm} \) (80% of the root chord). The camera was stationary and by necessity outside the towing channel; thus the view was from a 45° oblique angle and the horizontal scale was contracted by about 30 percent.

Fig. 3 Definition Sketch.

2.2 Towing Tank System

The wings used in the present investigation were towed at speeds in the range of 10 to 100 cm/sec through the water channel described by Hasse and co-workers (1961). The towing tank is 18 m long, 1.2 m wide, and 0.9 m deep. The pitching mechanism was rigidly mounted on a carriage that slides on two tracks mounted on top of the towing system. During towing, the carriage was supported by an oil film which insured a vibrationless tow, having an equivalent free-stream turbulence of about 0.4 percent.

Five sheets or five lines were seeped into the boundary layer through a system of slots and holes on the suction side of the wing. The slots were 0.2 cm wide and were milled at a 45° angle to minimize the flow disturbance. The holes were 0.1 mm diameter and were spaced at 1 cm center to center.

The wing was then placed in the flow field by laying several thin, horizontal sheets prior to towing the wing. The five sheets remained thin, about 1 mm in
thickness, due to the inhibition of vertical motion caused by introducing a weak saline stratification in the tank. The dye layers remained quiescent until disturbed by the flow field on and around the wing. Thus, the boundary layer flow as well as the potential flow could be observed since the dye layers existed in both flow regions.

3. Experimental Results

3.1 Steady State Flow

Dye visualization techniques were used to observe the flow on a delta wing at fixed angle of attack in the range of 0° to 45°. The trailing edge separation could be seen even at zero angle of attack. As \( \alpha \) increased above 5°, two stationary vortices occurred near the leading edge as found by other investigators. At the interface between these large primary vortices and the external potential flow, a thin shear layer was formed by the velocity difference between the free stream and the separated region. The shear layer was unstable and small secondary vortices were generated. In the present operating conditions about five small vortices rolled around the primary vortex/potential flow interface. At angles of attack above 10°, the separated region was sufficiently thick that the smaller secondary vortices were at least one diameter removed from the wing. Under these conditions, the secondary vortices merged in a pairing process as they were convected around the edge of the primary vortex, as shown in Fig. 4. In this side view using a vertical sheet of laser, the 45° delta wing was at angle of attack \( \alpha = 10° \), the chord Reynolds number was \( R_c = 2.5 \times 10^6 \), and the flow was from left to right. The pairing process appeared to be quite similar to the one observed in plane mixing layers (Winant and Browand, 1974). The physical mechanism for generating the small vortices in the delta wing case is believed to be the same as that in a free shear layer (Brown and Roshko, 1974). In the delta wing case, however, the flow field is more complicated because of the complex geometry and the non-planar velocity field.

![Flow Pairing](image1)

At smaller angles of attack, the vortex merging seemed to be inhibited by the presence of the wing surface. At larger attack angles, intense mixing made it more difficult to observe the pairing process. It is not clear from the present experiments whether the primary vortex only caused the secondary vortices by setting up the initial shear layer, or if it was possibly the result of several mergings of the secondary vortices.

Similar vortex formation and pairing were observed in the pitching case. However, the process was modulated by the lower frequency oscillation of the wing as shown in the next section.

3.2 Unsteady Flow

The flow visualization results of the 45° sweep delta wing undergoing a sinusoidal pitching motion are presented in here and in Flow Research Film No. 55 (available on request). On the suction side of the pitching delta wing, the leading edge separation vortices executed a grow-decay cycle during one period. Fig. 6 is a side view of the 45° wing undergoing the pitching motion \( \alpha(t) = 15 + 5 \sin(0.4t) \), at chord Reynolds number \( R_c = 2.5 \times 10^6 \) and reduced frequency \( K = 0.5 \). Both the upward and downward motions are shown side by side for the angles of attack of 10°, 12°, 14°, 16°, 18° and 20°. At a particular attack angle, the flow patterns were very different during the upward and downward motions. The hysteresis loop clearly existed. The discrete vortices formation and pairing is more readily observed in the upward part of the pitching cycle, and is modulated by the lower frequency oscillation of the wing as shown in the next section.

![Flow Visualization](image2)
Fig 6  Side View of the Pitching Delta Wing
Fig 7  Top View of the Pitching Delta Wing
Fig. 7 shows top view of the wing undergoing the pitching motion \( \alpha(t) = 15 + 15 \sin(0.8t) \), at chord Reynolds number \( R_c = 2.5 \times 10^4 \) and reduced frequency \( K = 1.0 \). Both the upward and downward motions are shown side by side for the attack angles \( \alpha = 0°, 5°, 10°, 15°, 20°, 25° \) and \( 30° \). The three dye slots on the left side of the wing are closer to the leading edge as compared to the ones on the right side. During the up stroke, the separation first started across the whole trailing edge at \( \alpha = 2° \). As the angle of attack increased, the separation propagated upstream from the two corners at the trailing edge toward the apex. The propagation speed along the leading edge was approximately equal to the flow speed (10 cm/sec). At \( \alpha = 25° \), the separation front reached the apex and the separation vortices were fully developed. During the down stroke, the vortices became smaller and diminished in size at about \( \alpha = 10° \), but the flow was still separated near the trailing edge.

Fig. 8 shows an end view of the pitching wing obtained using a vertical sheet of light in the \( v-z \) plane at \( x = 0.8 c \). The view is from \( 45° \) oblique angle and the flow is from the plane of the photograph. In this run the wing underwent the pitching motion \( \alpha(t) = 15 + 10 \sin(0.2t) \), at chord Reynolds number \( R_c = 2.5 \times 10^4 \) and reduced frequency \( K = 0.25 \). The up stroke at the angles of attack \( \alpha = 5°, 10°, 15° \) and \( 20° \) is shown in the figure. The dye was released only from the slots on the left side of the wing. Hence, only one of the leading edge separation vortices is marked. The vortex grows as the angle of attack is increased. In the cine film from which the frames shown in Fig. 8 were obtained, several discrete vortices rolled around the primary vortex and merged in a pairing process as mentioned above.

The dye layers technique was used to visualize the boundary layer flow as well as the potential flow around the wing. The horizontal dye layers were excited with vertical sheets of laser in the \( v-z \) and \( v-y \) planes. When the wing passed through the illuminated dye layers, the local motion of the fluid could be inferred from the deformation of the layers. In the end views, the dye layers near the center portion of the span moved downward during the up stroke as the separation vortex grew, indicating a strong downwash in that region. During the down stroke a second blob of dye appeared next to the separation vortex and nearer to the center portion of the span. As the angle of attack decreased, the separation vortex diminished while the second blob of dye grew in size. The nature of this motion is not clear at present, but it could be a separation bubble near the trailing edge similar to the one observed by Kinkelmann and Barlow (1980) on a small aspect ratio rectangular wing at constant angle of attack.

**Effects of the Reduced Frequency**

Lower reduced frequencies were also tried in order to analyze the effect of the reduced frequency on the hysteresis loop. The lowest numbers used frequency \( K = 1.0 \) in the present study. For \( K = 1.0 \) and 1.5, hysteresis was still observed. However, the "hysteresis" that was observed at lower reduced frequencies.
Fig. 10 is an end view at the same run conditions depicted in Fig. 9, but at an attack angle \( \alpha = 20^\circ \) during the down stroke. Both the separation vortex and the second blob of dye mentioned in the previous section appear in the photograph.

\[ \alpha = 20^\circ \text{ (decreasing)} \]

Fig. 10 End View at High Reduced Frequency.

A tentative explanation for the distinct change at \( K \geq 3 \) is as follows. The reduced frequency can be viewed as the ratio of the chord to a "perturbation" wavelength: \( K = \frac{\pi c}{2U_m} = \frac{c}{\lambda} \). In other words, the perturbation wavelength is equal to about the chord length at \( K = 3 \). As a matter of fact, the reattachment point can be seen near the trailing edge in Fig. 9. After the flow is reattached, a thin shear layer with intense vortices forms and a strong vortex counter rotating with respect to the attached wing circulation was shed from the trailing edge. The induced velocity of this vortex kept the separation region on the wing. Based on these arguments, it appears that a parameter based on the chord length and the perturbation wavelength is more appropriate to describe the flow than the conventional reduced frequency.

4. Conclusions

Two delta wings with a leading edge sweep of 45° and 60° were studied in a towing tank at chord Reynolds number up to 3.5x10^6. The wings were pitched about the quarter chord point with sinusoidal oscillations of amplitude \( \pm 5^\circ \), \( \pm 10^\circ \) and \( \pm 15^\circ \) about a given mean attack angle that varied in the range 0° to 45°. Food color and fluorescent dyes were used to visualize the flow field and around the wings. Sheets of laser excited the fluorescent dye to yield detailed flow information in the desired plane.

The steady state flow field was studied at angles of attack of 0° to 45°. Separation was observed near the trailing edge on the centerline at small attack angles as \( \alpha \) increased above 5°. Leading edge separation occurred forming two large scale stationary vortices approximately parallel to the leading edge as found by previous investigators. In addition, the dyed shear layer near the leading edge was observed to roll up and form discrete vortices along approximately straight lines emanating from the wing apex, similar to the ones commonly observed in free shear layers. These vortices rolled around the primary vortex potential flow interface, and merged in a pairing process much the same as observed in plane mixing layers.

Similar phenomena were observed in the unsteady case, except that the vortices shed from the leading edge were modulated and altered by the unsteady motion which had an order of magnitude lower frequency. The leading edge separation vortices executed a grow-decay cycle during one pitching period. As the angle of attack increased, the first observable separation was along the trailing edge of the wing. As \( \alpha \) increased further, the separated region increased by moving from the wing tips along the leading edge toward the apex. At large angle of attack, the potential flow field had a downward component near the center of the span indicating strong downwash in this region. As \( \alpha \) decreased from its maximum value, at least one and often two large separated regions appeared on the wing as a result of the unsteadiness. In general, the unsteadiness delayed separation and promoted hysteresis similar to results obtained on unsteady two-dimensional airfoils. As the oscillation frequency increased, the formation of the separated region appeared further downstream. At a reduced frequency \( K = 1 \), the streamlines remained parallel to the wing surface until high attack angles. At \( K = 3 \), no large scale separation was observed on the wing; however, a strong vortex was shed at the trailing edge as \( \alpha \) passed through its maximum value.

References


Abstract

Separation, vortex formation and turbulent decay are visualized for accelerating air flow around various bodies, starting from rest. The bodies investigated were a cylinder, a sphere, a flat plate, a round plate and an NACA 0015 airfoil. All bodies had the same characteristic length (body diameter or chord length) and were subjected to the same flow acceleration resulting in the same Reynolds number and allowing for a meaningful comparison.

Nomenclature

- c: characteristic length (body diameter, chord length)
- a: flow acceleration 2.44 m/sec²
- \( \nu \): kinematic viscosity of the air flow 0.18 cm²/sec
- R: Reynolds number \( 2^{1/2} c^{3/2} \nu^{-1} = 5200 \)
- \( \alpha \): angle of attack of airfoil or flat plate
- t: time
- \( t_c \): characteristic time scale \( c^{1/2} a^{-1/2} = 0.25 \) sec
- \( t_i \): time between acceleration startup (from rest) and the first movie frame shown in a figure
- \( t_f \): time between shown consecutive movie frames in a figure

Introduction

Visualization of starting flow around plates, cylinders and airfoils goes back to Prandtl (1905, 1927). Starting flow around a sphere was visualized by Honji (1972). More recent visualizations of unsteady flows have been reviewed by Taneda (1977) and most recent visualizations are due to Bouard and Coutanceau (1983) and Thomas (1983). A comparison of starting flow around even a limited number of different bodies is however hardly possible on the basis of past work. There are too widely differing techniques of visualization and photography, differences in flow configuration, water flow, airflow, accelerating and impulsive flow, different body dimensions, Reynolds number, time, visualized etc.

We compared these bodies under similar conditions in order to obtain insight into the similarities and differences of various unsteady flows. All bodies had similar dimensions, i.e. diameter or chord length and were subjected to the same acceleration of air flow. The same flow visualization and photographic techniques were used in all cases.

Experimental Setup

A large, low turbulence, open return wind tunnel was converted to unsteady operation as described by Freymuth, Palmer and Bank (1983). The tunnel started from rest, then maintained a nearly constant acceleration of 2.44 m/sec² (25% of gravity) for 5 seconds prior to power shutoff. The tunnel had a test section 0.9 m x 0.9 m across and was 20 m long with Plexiglas front and top walls allowing for photography.

We investigated a cylinder, a sphere, a flat plate, a round plate and an NACA 0015 airfoil. These bodies had the same diameter or chord length c = 15.2 cm; the Reynolds number was R = 5200.

A strip of liquid titanium tetrachloride (TiCl₄) was painted in flow direction on the body surface, releasing dense white fumes. The method is described in detail by Freymuth, Bank and Palmer (1983). Since smoke is released in the regions of vorticity production we assume that smoke patterns reveal vorticity patterns near the body and in the near wake behind the body, where differences in vorticity diffusion and smoke diffusion do not represent a problem. A problem may exist for low Reynolds number flow, far downstream, and when vortices of opposite sign are adjacent to each other (smoke density does not have a distinction in sign).

Smoke patterns were recorded for 3 sec with a 16 mm Bolex movie camera at 54 frames per sec and with a shutter speed of 1/500 sec. Because of space limitations, only a restricted number of frames of an acceleration sequence can be shown in the figures (additional frames can be obtained upon request from the authors). In the figure captions we denote the time between frames shown as \( t_i \) and the time from acceleration start to the first frame shown as \( t_f \).

Interpretation of these pictures can be accomplished by the reader in any desired detail. At the outset we will present some of our interpretations along with the photographs in the next section.

Comparative Flow Visualizations

In the figures of this section flow is always from left to right.

Figure 1 compares accelerated flow around a cylinder (left column) and around a sphere (right column), starting with the separation tongue from which the main vortex forms. Flow is initially quite similar for both cases. A separation tongue forms and develops into a main vortex first observed by Prandtl in 1904 behind a cylinder in a starting flow which initiated a counterrotating secondary flow located between the main vortex and the jet of discus vortex structure first observed by Prandtl in 1904 behind an infinitely started problem. Subsequently...
there is a weak tendency of the main vortex to decay into smaller ones prior to turbulent decay. For the sphere the secondary vortex is visible as a vortex ring on the sphere surface. Upon close examination quantitative differences exist between the two cases. Separation for the sphere occurs later (larger $t_c$) than for the cylinder but transition to turbulence proceeds faster, resulting in a smaller primary vortex. In the late stages of turbulent decay a turbulent vortex street develops behind the cylinder (first observed by Prandtl 1905) whereas the sphere wake remains rather symmetric and diffuse.

Figure 2 compares accelerated flow around a flat plate (left column), a round plate (middle column) and an NACA 0015 airfoil (right column), all placed perpendicular to the flow. For the round plate only the lower edge is visualized. While vortex development is qualitatively similar for the 3 cases the contrast to Figure 1 is considerable. The separation tongue which previously developed into the main vortex now starts formation of a vortex group consisting of up to ca. 10 small vortices, some of them in a stage of pairing. The 2 vortex groups behind the top and bottom edges of the body then interact while they become turbulent. Secondary flow is not established in these cases except near the round upper edge of the airfoil. Decay into turbulence is slower than in Figure 1 and is lowest for the round airfoil edge.

Figure 3 compares the symmetric NACA 0015 airfoil (left column) with the same airfoil in reverse (lunar) leading edge, round trailing edge and with the flat plate (right column), all at an angle of attack $\alpha = 20^\circ$. In all 3 cases vortex development is similar near the leading edge. As in case of the cylinder (Fig. 1) a triple structure forms consisting of the primary vortex, a secondary counter-rotating vortex and a tertiary vortex, which forms a beautiful pattern for the frame, left column. Also a second counter-rotating vortex is in evidence near the upper third frame, left column along with some faint indication of a small tertiary vortex behind the secondary vortex. The formation of even more elaborate patterns is prevented by turbulent decay with the primary vortex rolling on a turbulent large scale structure over the airfoil. For the reversed airfoil and for the flat plate the separation tongue is nearer the leading edge than for the regular airfoil and development of turbulence for the former case is more slow. For the airfoil, an additional mature vortex seems to form closer to the trailing edge which is swept into the wake. The airfoil surface is shiny and therefore one sees the global mirror image of the vortex structure in the airfoil in addition to the vortex structure itself. It should be mentioned that only visualizes the edge airfoil surface and flat plate surface because flowing along the bottom side remains invisible.

Concerning the vortex development at some height in Figure 4 where the airfoil, the flat plate and the disc all show vortex formation we have used a long exposure with a short flash at the same instant, a technique which was used by K. G. Obremski in the 1960's to study the effect of anode and cathode on the airfoil.
PREDICTION OF DYNAMIC STALL CHARACTERISTICS USING ADVANCED NONLINEAR PANEL METHODS

B. Maskew* and F.A. Dvorak**

Analytical Methods, Inc.
Redmond, Washington

Abstract

This paper presents preliminary results from a program of work in which a surface singularity panel method is being extended for modelling the dynamic interaction between a separated wake and a surface undergoing an unsteady motion. The method combines the capabilities of an unsteady time-stepping code and a technique for modelling extensive separation using free vortex sheets. Routines are being developed for treating the dynamic interaction between the separated wake and the solid boundary in an environment where the separation point is moving with time. The behavior of these routines is being examined in a parallel effort using a two-dimensional pilot version of the three-dimensional code. This will allow improvements in the procedures to be quickly developed and tested prior to installation into the main code.

The extended code is being coupled with an unsteady integral boundary layer method to examine the prediction of dynamic stall characteristics. The boundary layer code is accessed during the time-step cycle and provides the separation locations as well as the boundary layer displacement effect—the latter is modelled in the potential flow code using the source transpiration technique.

The preliminary results presented here include basic unsteady test cases for both the potential flow and boundary layer routines. Some exploratory separated flow calculations are included from a series of numerical studies on the stability of the calculation procedure. Correlations with experimental dynamic stall results have yet to be performed.

Introduction

Flow separation on the lifting surfaces of a vehicle at high angle of attack is always complicated by a certain degree of unsteadiness, but, when the vehicle itself is undergoing unsteady motion or deformation, or if it enters a different flow field rapidly, then the complexity of the separated flow is even greater, and culminates in the phenomenon of dynamic stall. If the angle of attack oscillates around the static stall angle, the fluid dynamic forces and moments exhibit large amounts of unsteadiness and a condition of negative dynamic damping often develops during part of the cycle. This can lead to the condition of flutter in a single degree of freedom oscillation rigid body motion. (Normally, in attacker flow, flutter only occurs when the body itself includes multiple degrees of freedom, e.g., combined bending and torsion of an aircraft wing.) During a rapid increase in angle of attack, the static stall angle can be greatly exceeded, resulting in excursions in the dynamic force and moment values that are far greater than their static counterparts. The consequences of dynamic stall are far-reaching and lead to such problems as wing drop, yaw (sometimes leading to spin entry), wing rocking and buffeting as well as stall flutter.

Although a great deal has been learned about dynamic stall characteristics—mainly through experimental observation—there is not at this time a completely satisfactory theoretical method (1), (2) for predicting the dynamic stall characteristic for new untested shapes even for the two-dimensional case. Moreover, quantitative correlations of experimental data on new geometries can be obscured by the effects of three-dimensional wind-tunnel interactions, wall interference and experimental uncertainties (3). In the present work a possible theoretical approach is examined for predicting dynamic stall characteristics. The approach combines an unsteady time-stepping method (4) with a steady inviscid/viscid iterative code (5) that includes an extensive separation model. The latter has proven very successful in the steady case. Both codes are applicable to general three-dimensional shapes and have been developed from the same basic panel method (6).

Extensive investigations of the dynamic stall characteristics of an airfoil oscillating in pitch have been reviewed by McCroskey (1), (2). In practical aerodynamic environments, the unsteadiness can be a combination of several motions. Unsteady motions other than pitching have been investigated by, among others, Liiva et al. (7), Lang (8), Lang and Francis (9), Maresca et al. (10), and Saxena et al. (11). In these references the phenomena of plunging, flap, spoiler and rectilinear oscillations were examined. There are very few theoretical approaches, however, and in a recent review, McCroskey (2) concludes that as the present time the engineer who needs answers should turn to one of the empirical correlation techniques, even though these are not completely satisfactory and only supply broad details of force and moment characteristics. The worst method of Ericsson and Reding is perhaps one of the most comprehensive of these methods. Their latest paper (12) incorporates a number of findings from the systematic experiments of Carr et al. (13).

Early theoretical approaches to dynamic stall addressed the deep stall aspect which is dominated by the passage of vortices shed from the vicinity of the leading edge. Haan (14) used a two-plane model of the airfoil. Later work by Baudu et al. (15) extended the basic model to thick sections using a panel method. The main drawback with the approach is that crucial assumptions regarding the location and time of vortex shedding have to be made in order to perform the calculations. Also, the results from (15) are sensitive to (a) the angle at which the vortex path leaves the surface; (b) the...
time at which vortex emissions terminate so as to start the reattachment process and (c) the viscous diffusion of the free vortices.

Calculations of the characteristics of the thin boundary layer in the attached flow regions of an oscillating airfoil using unsteady methods (16), (17), have demonstrated good qualitative agreement with experimental observations. However, one feature at least needs further examination in regard to improved modelling: it is reported (18), (19), that when incidence is increasing beyond the static stall angle, the location of zero skin friction in the turbulent boundary layer and the catastrophic separation can occur at different stations. Figure 1. Apparently, a long thin tongue of reversed flow precedes the main separation zone. This is not observed under quasi-steady conditions.

The present method goes beyond the capabilities of the earlier theoretical approaches in that both trailing-edge and leading-edge stall with vortex passage can be included in principle. The method, developed for the three-dimensional case, is applicable to arbitrary configurations and to general motions; i.e., not just pitch oscillation. In addition, because the basis of the method is a surface singularity panel code, a more reliable and direct coupling between the inviscid and viscous analysis is assured. Moreover, modelling of the separated zone in the trailing-edge region and more detailed treatment of the vortex/surface interaction should make the approach more viable for applications to dynamic separation problems.

Potential Flow Methods

General

Although remarkable advances are being made in flow field calculations using finite-difference and also finite-element methods, the surface integral approach using panel methods coupled with special routines for nonlinear effects still offers distinct advantages for many real flow problems. In particular, panel methods offer greater versatility for practical application to complicated configurations and are considerably more efficient in terms of computing effort. However, the concept of zonal modelling—in which a local Navier-Stokes analysis is coupled with a panel method—should not be overlooked. Ultimately, such a coupling should lead to an improved modelling of vortices (e.g., vortex cores, vortex dissipation and breakdown), thin viscous regions, local separation bubbles, and shock wave/boundary interactions.

Over the past decade, panel methods have seen a trend towards higher-order formulations (22), (25) and (26). At the outset it was argued that compared with the earlier low-order methods the more continuous representation of the surface singularity distribution should allow a reduction in panel density for a given solution accuracy and, hence, should lead to lower computing costs. No such benefits have appeared so far for the general three-dimensional case. In fact, preliminary investigations (27) have indicated that the prediction accuracy for problems with complicated interactions, such as vortex/surface or high curvature situations, depends more on the density of control points where the boundary conditions are enforced; the order of the singularity distribution plays a minor role. In the meantime, further developments of piecewise constant singularity panel methods, e.g., Morino (28), and AMI's Program VSAERO (6), are giving comparable accuracy to the higher-order methods at much lower computing costs.

The low computing cost of Program VSAERO makes it practical to apply the method to nonlinear problems requiring iterative solutions, e.g., wake relaxation for high-lift configurations, multiple-component problems and rotor cases, and viscous/inviscid calculations with coupled boundary layer analyses, including the case with extensive separation (3), and also time-stepping calculations for three-dimensional unsteady problems (4) that are beyond the scope of a harmonic analysis. This method, therefore, offers an attractive basis for a
practical tool aimed at predicting the aerodynamic characteristics of dynamic stall problems. At the outset, this code was being developed in two different directions; viz., one was for extensive separation modelling under "steady" conditions, while the other was for unsteady time-stepping calculations. These two capabilities, described in the section below, have now been brought together in one code.

**Separated Flow Model**

Under essentially steady conditions, the pressure distribution in a trailing-edge separation region is usually characterized by a constant pressure region extending back to the trailing edge followed by a short recompression region (e.g., (29)). A simplification of this characteristic is modelled in the two-dimensional CLMAX program (30), (31) using a pair of constant strength vortex sheets to enclose the separated region, Figure 2. The length of the sheets required a semi-empirical approach and the condition that the sheets be free-flying is satisfied in an iterative cycle in which segments of the sheet are aligned with calculated local flow directions. The method combines boundary layer and potential flow codes in an outer inviscid-viscous iteration cycle. The potential flow pressure distribution which includes the influence of the free vortex sheets is passed to the boundary layer routine which then supplies the separation points and the boundary layer displacement thickness distribution for the next iteration. The boundary layer displacement effect in the attached flow region is modelled by transpiration (i.e., source distribution) rather than by a displacement surface. The main advantage of the transpiration approach is that the matrix of influence coefficients in the panel methodology is essentially the same from one iteration to the next; only the wake condition changes.

The thin vortex sheet model of the upper separated shear layer was demonstrated by Young and Hoad (32) to be a reasonable representation of the flow as far back as the trailing edge. For example, a comparison from (32) of a Laser-velocimeter flow survey and a CLMAX program calculation is shown here in Figure 3. Downstream of the trailing edge the vortex sheet model becomes less representative of the flow, however, a later evaluation of a graded vorticity model over the recompression zone showed little effect on the airfoil solution. More detailed modelling of the recompression zone (such as, for example, the approaches used by Gross (33) or Zundt (34)) would be desirable in cases where the wake interacts closely with a downstream component.

A particular feature of the vortex sheet model, enclosing the region of low energy is that pressures can be calculated directly in the separated zone (30). This is an additional advantage over the displacement surface approach of Henderson (35) and over the source outflow model of Jacob (36). The CLMAX method generally gives very close agreement with experimental pressure distributions (30), (31). An initial extension of this method to the unsteady case is reported in (31) where quasi-steady solutions coupled with a phase shift model were used. Extension of the model to the three-dimensional case is reported in (37) for a stripwise model and in (38) for a more general treatment. The separation model has also been successfully installed in a transonic finite-difference code (39).

The same basic model is also applicable for the unsteady case; here, however, the assumption of constant vorticity is no longer valid. In fact, a dynamic wake model is essential and will be discussed below after the description of the unsteady formulation.
Here, \( r \) is the length of the vector from the surface element to the point, \( P \), and \( S-P \) signifies that the point, \( P \), is excluded from the surface integration. Equation (1) includes the contribution from the wake surface, \( W \).

The Dirichlet boundary condition is now applied in the interior region to render a unique distribution. In principle, any potential flow can be applied. However, the flow, \( f^w \), implied by Morino (28) and used by Johnson and Rubbert (25), has proven to be very reliable in practice. With this flow, Eq. (1) becomes

\[
0 = \iint_S \nabla \cdot \left( \frac{1}{r} \right) dS + 2\pi \rho
\]

and Eq. (2) becomes

\[
0 = \iint_{S-P} \nabla \cdot \left( \frac{1}{r} \right) dS + 2\pi \rho
\]

This is the basic equation of the method. It is solved for the unknown surface perturbation potential, \( \phi \), or surface doublet distribution, \( \lambda \), at a number of time steps as the configuration proceeds through the motion. The wake surface is transported at the end of each step using calculated velocities of points on the wake surface. The doublet distribution, \( \lambda_p = \lambda \), on each wake surface is known from solutions at earlier time steps. The unsteady Kutta condition

\[
\frac{\partial \lambda}{\partial t} + \nabla \cdot \lambda = 0
\]

is satisfied at points along each wake separation line at each time step.

At each time step the flow solution is determined with reference to the body-fixed frame. The incompressible pressure coefficient is, therefore, given by

\[
C_p = \left( \frac{\rho \beta}{\rho_0} \right)^2 V^2 + 2 \frac{\beta}{\rho_0} \lambda(V) V
\]

where \( V = h R - V \) is the instantaneous velocity of a point on the surface relative to a stationary reference frame, and \( V \) is given by Eq. (3).

Numerical Procedure. The general arrangement of the configuration is shown in Figure 4. The \( 1,2,3 \) coordinate system with unit vectors, \( e_1, e_2, e_3 \) is
fixed relative to the configuration. For symmetrical applications, the z-x plane is regarded as the plane of symmetry.

A numerical procedure has been assembled in a time-stepping mode to obtain the unsteady pressure distribution and forces and moments. The surface of the winn is represented by planar quadrilateral panels over each of which the doublet and source distributions are assumed constant. With this assumption, the surface integrals in Eq. (4) can be performed in the closed-form for each panel.

Equation (4) is then satisfied simultaneously at a point at the center of each panel. If there are N panels representing the configuration surface, Eq. (4) becomes:

$$\sum_{k=1}^{N} \left( \frac{w_{K}^{C} c_{JK}}{4\pi} + \frac{w_{J}^{S}}{4\pi} \right) - \frac{2m_{J}}{4\pi} + e_{J} = 0; \quad J = 1, N$$

where \( w_{K} \) is the unknown doublet value on panel \( K \).

$$E_{J} = \sum_{K=1}^{N} \left( -w_{K}^{C} c_{JK} + e_{J} - \frac{w_{J}^{S}}{4\pi} \right)$$

where \( w_{K} \), the number of panels in the wake, varies with time and \( V_{J} \) take their instantaneous values at each time step.

$$R_{J} = \sum_{K=1}^{N} w_{K}^{C} B_{JK}^{C}$$

is the source distribution due to rotation about the axis, \( h \), and

$$M_{J} = \sum_{K=1}^{N} w_{K}^{C} B_{JK}^{S}$$

are the components of a three-part source distribution due to the relative translation of the configuration and the onset flow. (Note: in a symmetrical case the \( y \)-component is zero.)

The quantities, \( B_{JK}^{C} \) and \( B_{JK}^{S} \), are the velocity potential influence coefficients for the constant source and doublet distributions, respectively, on panel \( K \), acting on the control point on panel \( J \). These include contributions from the image panel in the symmetrical case. Expressions for these influence coefficients have been given by Morino in [4] based on hyperbolic paraboloidal panels. Slightly different expressions are installed in the VSAERO code based on planar panels.

Equation (7) is solved by a direct method for \( N \leq 320 \) and by an iterative method for \( N > 320 \).

The surface pressure distribution is calculated using Eq. (6). The surface gradient of \( u \) is evaluated on each panel by differentiating a two-way parabolic fit through the doublet values on the panel and its four immediate neighbors. At the separation lines a simple differencing is applied for the gradients approaching the separation line.

The gradient of \( u \) with respect to time is evaluated by central differencing over two time steps; i.e.,

$$\frac{\Delta u}{\Delta t} = \left( \frac{u^{t+1} - u^{t}}{2\Delta t} \right)$$

For harmonic motions the real and imaginary pressures are obtained by Fourier analysis for the first harmonic based on solutions over a complete cycle. The calculations start with incidence \( \omega_{0} \) and a regular (i.e., steady) wake. Two iterations are performed to render the wake force free. An oscillatory doublet component based on a linearized solution is then superimposed along each wake line before starting the time-step model. Time-step calculations proceed over a half cycle before applying the Fourier analysis.

At each time step a new panel is formed at the head of each column of wake panels and all the existing wake panel corner points are convected downstream at the local velocity. Each wake panel keeps the doublet value it received at the time it was formed. This doublet value is based on the conditions at the separation line and satisfies Eq. (5). It is assumed that the shedding occurs at constant vorticity over the time interval, \( t \). In this way the doublet strength, \( \omega_{JK}^{t+1} \), on the new wake panel \( J \) is related to the strength, \( \omega_{JK}^{t} \), of the previous wake panel at the separation line by

$$\omega_{JK}^{t+1} = \omega_{JK}^{t} - \omega_{J}$$

where \( \omega_{J} \) is the resultant doublet value at the separation line.

Unsteady Separated Flow

The combined code for separated flow modeling in the unsteady case requires a more sophisticated treatment of the free-shear layer model than was used for the steady case. Velocities are still calculated at a set of outlets along each free shears, but in the unsteady case we now transport these points and their associated doublet values along;
the calculated velocity vectors for a small time interval, etc. In this way, as time progresses, a dynamic wake model is generated. At each step a new piece of free sheet is shed from the calculated separation point; the strength and size of this new segment is determined by the local upstream velocity condition. The location of the separation—calculated using an unsteady boundary layer code, see the next section—can now move with time.

It is convenient to regard the local vorticity (i.e., doublet gradient) on the free vortex sheets in two components; a streamwise component and a cross-flow component. The streamwise component is already force free and is related to the spanwise rate of shedding of circulation from the wing. The cross-flow vorticity component is associated with direct dumping of bound circulation from the separation line and must be transported with the local flow velocity in order to be force free. This cross-flow vorticity component—which was assumed constant with streamwise distance in the steady case—now varies along each streamline on the free sheets for two reasons: first, the vorticity value being convected onto the free sheet at each separation point is varying in time because of varying onset flow conditions and because of the changing separation locations. Secondly, stretching by the entire configuration of solid surfaces and free wake sheets. This stretching of the doublet distribution carried by the free sheets yields varying vorticity values when the downstream gradient is evaluated. In this way the free sheets can become highly distorted and centers of vortex roll-up may form. Special treatment of the sheets is therefore essential if numerical stability is to be maintained. Two routines are being evaluated in this work but they have not been fully implemented at this time.

(i) Vortex Amalgamation

To cater for vortex roll-up in a reasonable manner it is essential to include a vortex core model in which integrated vorticity is accumulated rather than to follow a detailed calculation of multiple turns of a vortex spiral. An amalgamation scheme similar to that of Moore (39) is being used. When the angle between neighboring segments representing a sheet exceeds a specified angle, the segment end points are merged to a new location at the centroid of their combined circulation. A number of such cores are allowed in the new routine to deal with complex motions. A viscous core expression can be applied to each vortex core when computing the field velocities.

(ii) Redistribution

Having performed the vortex-core amalgamation calculation along each free sheet the points defining the intermediate free sheets are redistributed with equal spacing in a manner similar to Fink and Som (40) and Sarukaya and Schopp (41). Portions of the sheet between amalgamated cores are treated independently here, Figure 5. This treatment, which is applied to both the sheet geometry and its doublet distribution, uses a biquadratic interpolation scheme based on surface distance along the sheet.

This routine should help stabilize the numerical calculations, especially in the initial part of each sheet when the separation location is varying with time. In the three-dimensional case the redistri-

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Fig. 5. Illustration of Multiple Vortex Core Amalgamation and Redistribution Scheme.

Fig. 6. Indicial Lift and Circulation for an Impulsively Started Two-Dimensional Flat Plate.

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Some recent refinements developed in the two-dimensional plate code have significantly reduced the computing requirement of these time-stepping calculations. For example, Figure 7 shows the effect on indicial lift of varying the number of time steps in the Wagner problem and demonstrates a rapid convergence.
The procedure has been tested also for the harmonic oscillation case. Earlier calculations required 80 time steps per cycle for a NACA 0012 oscillating in pitch about the quarter chord. These compared favorably with the Theodorsen flat plate function over a range of reduced frequency, Figure 8. The new calculations are also in good agreement but were performed with only 16 time steps per cycle.

Time-stepping calculations have also been performed for cases with prescribed extensive separations. The purpose of these calculations was to check the basic unsteady circulation shedding model in the potential flow code. For the first set of tests, the wake panels were simply transported at the onset flow velocity after the initial growth as determined from the surface conditions at separation. Several triangular shapes were considered, each starting impulsively from rest and proceeding forwards over 10 time steps for a total time of $t = \frac{U}{h} = 3.0$, where $h$ is the triangle base height. Separation was prescribed at the corners. Figure 10(a) shows the computed history of the drag coefficient from pressure integration for a 60 degree triangle with blunt face forward. A total of 40 panels was used to represent the triangle surface. The calculation was repeated in the presence of wind tunnel walls (also panelled) with a 10% blockage ratio. The indicated blockage correction is somewhat lower than that given by standard techniques. Figure 10(b) compares the computed pressure distributions for this triangle in and out of the tunnel. This "base" pressure has only a small variation and is quite close to experimental measurements. Figure 11 shows a summary of computed drag coefficient versus triangle semi-apex angle. The calculated values are slightly high in relation to the experimental data collected from several sources by Hemen in Aerodynamic Drag.
Finally, a test calculation was performed for a NACA 0012 section in a state of pitch from 10° to 30° with $\theta/2U_{\infty} = 0.176$. The calculation used 30 panels and 10 time steps. Separation points were prescribed and the motion was started impulsively.

![Diagram](image)

**Fig. 10.** Calculations on a Triangular Section Started Impulsively from Rest.

One further case was run for the 60° apex-forward triangle in free air with the full wake velocity calculation routine turned on but without the amalgamation and redistribution schemes at this stage. The calculated $C_D$ for this case falls below the experimental value. Figure 11. A series of computed wake shapes is shown in Figure 12. These are samples from a total of 40 time-step calculations. The total computing time for this case was 195 seconds on a PRIME 550 minicomputer—this is equivalent to less than 2 seconds of CRAY time. The solution should benefit from the numerical damping provided by the amalgamation and redistribution schemes described earlier.

![Diagram](image)

**Fig. 11.** Calculated Drag Coefficient of Two-Dimensional Wedges as a Function of Apex Angle from Rest.

Figure 13(a) shows a sample of the computed wake shapes and demonstrates a reasonable numerical behavior. Sample pressure distributions are shown in Figure 13(b). The passage of the leading-edge vortex is clearly shown. This is associated with a local region of reversed flow. These are preliminary test calculations aimed at exploring the numerical behavior of the calculation procedure and potential flow model. Future cases will include the coupled boundary layer calculation for precise time the separation point. At that time the calculated results will be compared with experimental data.
Fig. 12. Computed wake shapes for a 60° wedge started impulsively from rest.

(b) Calculated pressure distributions at two time steps.

Fig. 13. Calculated results for a NACA 0012 starting impulsively from rest and pitching from 10° to 30° at a rate of 3.0°/0.1 s.
\[ C_f = 2 \cdot \frac{1}{R} \]  
\[ L = 2\left[ 1 - K(H + 2) \right] \]  
\[ L = \text{functions of } \nu, \lambda, \text{and } K \]  
\[ K = \frac{-1}{2} \left( \frac{dU_e}{dx} \right) \]  
\[ \nu = \frac{K \cdot U_e \cdot U''_e}{U_e} \]

Calculation begins at the stagnation point and \( K \) takes the starting value, \( K_0 = 0.0855 \). The initial momentum thickness, \( \nu_{0,15} \), is \( [0.0855 \left( \frac{dU_e}{dx} \right)] \)

**Turbulent Boundary Layer Methods**

For unsteady turbulent boundary layers, the momentum integral equation and the entrainment equation are given by:

\[ \frac{1}{U_e} \frac{d}{dx} \left( U_e \left( U_e - \nu \right) \right) = C_E \]

where

\[ C_E = \frac{\lambda}{\lambda \cdot U_e} \]

**Laminar Boundary Layer Methods**

For laminar boundary layers, the momentum integral equation and the entrainment equation are given by:

\[ \frac{1}{U_e} \frac{d}{dx} \left( U_e \left( U_e - \nu \right) \right) = 0 \]

\[ \frac{1}{U_e} \frac{d}{dx} \left( U_e \left( U_e - \nu \right) \right) = 0 \]
\[ V_\tau = \frac{1}{k} \sqrt{-\frac{w}{U'_\tau}} \text{sgn}(w) \]  

(20)

and \( k = 0.41 \) is the von Karman constant.

The similarity solutions have shown that the entrainment coefficient, \( C_e \), can be expressed as

\[ C_e = C_{e S} - \frac{1}{U_e} \frac{36}{8\pi} \]  

(21)

where \( C_{e S} \) is the entrainment coefficient for the steady case as given by:

\[ C_{e S} = \sqrt{\frac{C_f}{2}} \left( 0.074G - 1.0957/G \right) \]  

(22)

Substitution of Eq. (21) into Eqs. (14) and (15) and the introduction of \( h^* = \frac{h}{h'} \) give

\[ \frac{1}{U_e} \frac{h'^*}{x} + \frac{u'}{x} = B_1 \]  

(23)

\[ \frac{h'^*}{x} + \frac{1}{U_e} \frac{h'^*}{x} = B_2 \]  

(24)

Equations (23) and (24) can be solved in various ways. Initial conditions at \( t = 0 \) and boundary conditions at the stagnation point (\( t = 0 \)) are sufficient to determine a solution in the region where the flow is attached, i.e., \( \tau > 0 \). In the present paper, the time derivatives are treated as forcing terms and the integration is performed in the \( x \)-direction using a Runge-Kutta method.

The boundary layer procedure has been tested against experiments and the calculations of other investigators. The results are shown in Figures 14 and 15. Figure 14(a), (b) and (c) shows the mean quantities (momentum thickness, skin friction and shape factor) for the experiment conducted by Cousteix (45) on a flat plate. The main free stream velocity is 22 m/sec and the motion is harmonic with respect to time with the frequency of 10 Hz. The present calculation (solid line) agrees very well with the calculation by Cousteix and the correlation with experiment is also very good.
Figure 15 shows the comparison with the calculation of Nash et al. (46) for a monotonically time-varying flow on a flat plate. The present calculation predicts the separation at the end of the plate when \( t = 0.682 \) as in the Nash et al. calculation. The overall results are in good agreement with their calculation.

![Graph](image)

(c) Mean Shape Factor.

**Fig. 14.** Concluded.

A preliminary calculation was performed for a NACA 0012 section oscillating in pitch about the quarter chord with \( \phi = 0.4 \). Figure 17 shows the predicted history of the separation location superimposed on the figure. The most forward separation reached \( \phi = 0.4 \) with a shear layer of about \( 10 \).
Finally, an experimental data case from (3) was run and the computed lift variation with 
flow rate compared with the measured data in Figure 1B. The
airfoil is a NACA 0012 and is oscillating in pitch
about the quarter-chord line with \( \phi = 8.1 + 4.9 \sin (0.2t) \), i.e., below the dynamic stall
onset. Reynolds number was \( 4 \times 10^5 \). This reduced fre-
quency condition is very close to the changeover
from a lead to a lag situation and so there is
only a small difference between the upswing and
downswing curves.

![Diagram of lift vs. angle of attack](image)

**Fig. 1A. Comparison of Calculated and Measured**
**Lift on a NACA 0012 Airfoil Oscillating**
in Pitch About the Quarter-Chord.

**Conclusions**

A system of routines has been developed to
handle unsteady three-dimensional inviscid flow
computation with an extensive separation model
and a steady boundary layer code. The routines
include treatment of the inviscid multipoint shock
wave and all the terms in the

\[ \frac{\partial}{\partial t} \left( \rho U \right) + \nabla \cdot (\rho U U) = 0 \]

equation for unsteady inviscid flows for situations involving
the onset of stall. A grid is generated using the

\[ \frac{\partial}{\partial t} \left( \rho U \right) + \nabla \cdot (\rho U U) = 0 \]

methodology. The results are compared with the

\[ \frac{\partial}{\partial t} \left( \rho U \right) + \nabla \cdot (\rho U U) = 0 \]

experimental data from the Advanced Airfoil

\[ \frac{\partial}{\partial t} \left( \rho U \right) + \nabla \cdot (\rho U U) = 0 \]

Sections at High Reynolds Numbers, VKI,

\[ \frac{\partial}{\partial t} \left( \rho U \right) + \nabla \cdot (\rho U U) = 0 \]


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Numerical Solutions of the Navier-Stokes Equations for Unsteady Separated Flows

Wilton L. Hankey
Air Force Wright Aeronautical Laboratories
Wright-Patterson Air Force Base, Ohio 45433

Introduction

Advances in computer hardware during the last decade have permitted the numerical solution of the Navier-Stokes equations. Original efforts concentrated on steady flow problems, however, more recently unsteady flows have been addressed. One of the best uses of the Navier-Stokes numerical programs is to solve separated flows due to the nonlinear character of the problem. For problems in linear aerodynamics more efficient computer programs exist. However, most separated flows are unsteady due to the fundamental instability of such flows. Navier's second theorem states that a velocity profile with an inflection point (C' = 0) is unstable. All separated flows have inflection points and hence have an unstable region. In these flows, small disturbances of certain wave length are amplified. In simple linear theories these disturbances attain infinite amplitude, however, a 'limit cycle' is achieved in nature. The time-dependent Navier-Stokes equations possess the capability to investigate unstable separated flows. This paper will review some of the progress accomplished over the last few years in this area.

Governing Equations

The time-dependent Navier-Stokes equations in a curvilinear coordinate system take the following form:

\[ \frac{	ext{d}}{	ext{d}t} \left( \rho \mathbf{u} \right) + \nabla \cdot \left( \rho \mathbf{u} \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f} \]

where

\[ \rho \mathbf{u} \] velocity

\[ p \] pressure

\[ \mu \] dynamic viscosity

\[ \mathbf{f} \] body force

1. Steady separated flows.
   Supersonic flight with steady boundary conditions.
   a. Missile at moderate angle of attack.
   b. Hypersonic cruiser at angle of attack.

2. Self-excited oscillations.
   Periodic solutions of unsteady flows with steady boundary conditions.
   a. Missile in a cylinder and missile.
   b. Weapons have cavity resonance.
   c. Buzz of a spike and inlet.

3. Forced Oscillations.
   Periodic solutions of unsteady flows with time dependent boundary conditions.
   a. Dynamic lift of an oscillating airfoil.
   b. Rotation of a flat plate.

Although other categories could also be addressed, these few cases were chosen to demonstrate the versatility of the numerical program and to indicate the present state-of-the-art in solving the Navier-Stokes equations. See Figures 1 and 2. These solutions averaged about one hour on a CRAY supercomputer.

Conclusion

By use of the time-dependent Navier-Stokes equations, simplifying assumptions are unnecessary to investigate many classes of unsteady separated flows. The projected advancement of computer capability over the next few years renders questionable the wisdom of supporting research on shorter term approximate methods for solving unsteady separated flows.

References


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Fig. 3 Flow over an Infinite Cylinder (Ref. 4)

Fig. 1 Cross Section at Angle of Attack (Ref. 2)

Fig. 4 Post-Tipped Nacelle Shape (Ref. 7)
Fig. 6  Computed Area Characteristics for
Rotating Plate (Ref. 19)
An unsteady flow analysis is made of the flow past a symmetric airfoil with identical vortices released intermittently from its upper surface. The vortex train is used to simulate the flow observed in the laboratory, which was produced by an oscillating or rotating spoiler or an oscillating can embedded in the airfoil surface. Based on numerical computations, the airfoil lift has a general behavior that it increases oscillatory with time, and seems to approach an asymptotic value as time increases indefinitely. The asymptotic lift is enhanced with increasing frequency at which vortices are released, and is only slightly influenced by changing the vortex-releasing position along the chord. The behavior of the lift is similar to that of the lift, but its magnitude in the latter problem. An analysis also indicates that it is more efficient to implement the vortex-released lift at higher angles of attack of the airfoil.

Introduction

Experiments have been conducted at the University of Colorado and elsewhere to study the effect of an oscillating or rotating spoiler, or an oscillating can, on the flow past an airfoil, which was produced by a rolling-up of vortex sheets resulting from boundary layer separation at the sharp tip of the spoiler or smart as sketched in Fig. 2, and treated as discrete potential vortices in our formulation based on an inviscid incompressible flow analysis. The symmetric airfoil configuration adopted in our computation is generated from a circle through Coanda transformation, and the effect of oscillating spoiler or rotating can is simulated by the intermittent appearance of discrete vortices of the same strength at a given location above the airfoil. In an unsteady flow about airfoil, a vortex sheet is expected to roll continuously from the sharp trailing edge. Attempts to the literature have not been successful yet, and the flow about the airfoil is computed by imposing the vortex sheet, or vortex sheet, on the boundary. The Kutta condition is determined along the body, and the lift and pitching moment are determined along the body. The effect of a vortex sheet on the flow about the airfoil is computed by imposing the Kutta condition at the trailing edge, and the lift and pitching moment are determined along the body.

When moving along the airfoil surface, the train of vortices not only makes the flow unsteady but also may cause significant changes in airfoil performance. While empirical data are not yet available for comparison, we are presenting here a theoretical analysis for computing the unsteady lift and drag of the airfoil. The lift and drag are computed by the vortex-lift method, and the vortex sheet is treated as discrete potential vortices in the formulation based on an inviscid incompressible flow analysis. The symmetric airfoil configuration adopted in our computation is generated from a circle through Coanda transformation, and the effect of oscillating spoiler or rotating can is simulated by the intermittent appearance of discrete vortices of the same strength at a given location above the airfoil. In a unsteady flow about airfoil, a vortex sheet is expected to roll continuously from the sharp trailing edge.
As shown in Fig. 1, the transformation
\[ z = -\frac{1}{2} (1 + \frac{1}{z}) \quad (1) \]
naps a circle, radius \( a \) (\( r < 1 \)) in the \( z = x + iy \) plane, centered at \( z = 1 - a \), into a symmetric Joukowski airfoil in the physical \( z \) plane, whose chord is slightly longer than 2. A free-stream velocity of magnitude \( u_0 \) and at an angle of attack \( \alpha \) is mapped without changing its orientation into a uniform stream of unit speed in the physical plane. For the convenience of formulation a \( \zeta = \zeta' + \zeta'' \) plane is introduced in Fig. 1, whose origin coincides with the center of the circle. The relations
\[ \zeta = \zeta' + \zeta'' \quad (2) \]
note for coordinate transformation between \( z \) and \( \zeta \) planes.

A discrete vortex of circulation \( k \), at a position \( \zeta \), outside the airfoil is transformed according to Eq. 1 into a vortex of the same strength at \( \zeta' \), outside the circle. To satisfy the boundary condition that the flow be tangent to the circle an to fulfill the requirement that the total circulation be conserved, a discrete vortex of strength \( k \), and another vortex of strength \( 2k \) are placed at \( \zeta' \) and the center of the circle, respectively. For each of the discrete vortices in the physical plane, be it a vortex generated by a spoiler or a vortex shed in the wake, a set of three vortices is placed in the transformed plane following the procedure just described. Suppose the initial vortex-street circulation about the airfoil is \( C \), which is determined by the surface angle of attack. If \( j \) vortices have been generated by a perturbing device in the unsteady flow field, a wake vortex is shed at any instant of time. Let this be the \( m \)th vortex whose position is \( \zeta_m \) in the transformed plane. Its circulation \( k_m \), determined by satisfying the Kutta condition that \( \zeta_m = \alpha \) be a stagnation point, has the expression
\[ k_m = \left[ -\frac{1}{2} \sin \frac{1}{2} \frac{m}{j} \left( \frac{1}{2} - \frac{1}{j} \right) \right] \quad (3) \]
\[ \frac{a_m - \alpha}{a_m} \]
\[ \frac{a_m - \alpha}{a_m} \]
This vortex will later move at the local fluid velocity without changing its circulation.

For numerical computations we choose \( a_m \) to obtain a 1% thick symmetric airfoil. The strength of vortices generated by a perturbing device is determined by the size of the linear, frequency of oscillation, Reynolds number, and other factors. It can be measured according to a method introduced by Kutta, Prandtl, and Lamb. However, because of insufficient empirical data corresponding to our concerned conditions, we arbitrarily pick a circulation value of 0.1% of the moment we have examined. This instantaneous strength represents a circulation \( k_m \), and if applied around an airfoil, would produce a lift coefficient of 0. For a given angle of attack \( \alpha \), the addition of vortexes to the flowfield and hence the circulation around the airfoil is given by
\[ k_m = \frac{a_m - \alpha}{a_m} \left( \frac{1}{2} - \frac{1}{j} \right) \left( \frac{1}{2} + \frac{1}{j} \right) \quad (3) \]
\[ \frac{a_m - \alpha}{a_m} \]
\[ \frac{a_m - \alpha}{a_m} \]

In the unsteady flow field, a wake vortex is shed at any instant of time. Let this be the \( m \)th vortex whose position is \( \zeta_m \) in the transformed plane. Its circulation \( k_m \), determined by satisfying the Kutta condition that \( \zeta_m = \alpha \) be a stagnation point, has the expression
\[ k_m = \frac{a_m - \alpha}{a_m} \left( \frac{1}{2} - \frac{1}{j} \right) \left( \frac{1}{2} + \frac{1}{j} \right) \quad (3) \]
\[ \frac{a_m - \alpha}{a_m} \]
\[ \frac{a_m - \alpha}{a_m} \]
\[ a_n, \text{ derived after taking a proper limit.} \]

\[ (u - iv) = \frac{ik}{z_n + \frac{1}{z_n - z_n}} \]

\[ = \frac{m}{j \alpha} \left[ \frac{1}{z_n - z_n^j} - \frac{1}{z_n - z_n^j} \right] \]

\[ \left( 1 - \frac{1}{z_n} \right) \]

\[ \Delta D = i \left[ 1 - i \cdot e^{-i \alpha} \right] \]

\[ = i \cdot e^{-i \alpha} \left[ k_j (u + iv) \right] \]

\[ = i \cdot e^{-i \alpha} \left[ k_j (u + iv) \right] \]

\[ \text{Results} \]

For the 17\% thick symmetric airfoil, computations have been made by varying angle of attack, chordwise position of the released vortices, and the period at which they are released.

At zero angle of attack and when vortex of circulation \( \Gamma \) are released with a period \( T = 6 \), at a height of \( 3.1 \) above the upper surface at the midchord position, the behavior of lift coefficient for unsteady flow.

\[ \text{At zero angle of attack} \]

\[ \text{Lift on the airfoil is plotted in Fig. 4. The sudden release of a vortex causes a negative lift on the airfoil. Lift increases with time in an oscillatory manner; the mean lift coefficient \( \bar{C}_L \) averaged within each oscillation, gradually approaches an asymptotic value after more than 10 vortices have been released. The variation of lift in one cycle has a shape that is almost identical to that in any other cycle, except that the entire cycle is shifted upward after each period.} \]

\[ \text{A representative lift curve within one cycle.} \]

At the beginning of the cycle when a vortex is just released, the lift has the lowest value. As this vortex reaches the vortex released in the previous cycle, it is added to a bundle of vortex core that forms the newly released vortex, as shown in the beginning of the first integration in Fig. 4. The shape of the entire lift curve is displayed by a map of the integral of the lift coefficient, which is the sum of the lift coefficients of all vortices released up to that point. This envelope of lift coefficient values is used to calculate the theoretical lift curve.
neighboring released vortices and stronger interactions between these vortices and the wake vortex sheet. It is interesting to note that all wake vortices have counterclockwise circulations. Within that period, lift climbs to a maximum and then drops to a minimum, both occurring when a released vortex leaves the trailing edge.

The variation of mean lift coefficient with time is plotted in Fig. 3 for different values of \( T \). Solid lines represent results for the arrangement that vortices are released at the midchord location. The plot reveals that for a shorter \( T \) when vortices are released at a higher frequency, a higher average lift can be generated on the airfoil. Each of the higher \( T \) curves approaches an asymptotic value, which increases with increasing frequency. The curves for lower values of \( T \) seem to have the same behavior, but a tremendous amount of computer time is needed to find their asymptotic lift values. For example, to reach \( t = 200 \) for the \( T = 2 \) curve, the CPU time is 2700 seconds on a Cyber 170-720 computer. No attempt has been made to extend the computation beyond that time. However, it should be pointed out that for \( T \) equals 2 or 1, the mean lift coefficient can become greater than 1.5, which is the static lift coefficient if the circulation of a released vortex is applied around the airfoil. For a period equal to or greater than 4, the mean lift cannot exceed this value.

The dashed and the dash-dotted lines in Fig. 3 are used to represent results obtained when the vortex-generating mechanism is at the 1/4-chord and 1/2-chord positions, respectively. In these two cases vortices are still released at the same initial height of \( 0.1 \) above the initial airfoil surface. The influence of changing short-chord position on lift is not too strong as revealed in the figure. Generally speaking, releasing vortices at the 1/2-chord position produces a decrease in average lift, whereas shifting lift upward to the 1/4-chord location has the opposite effect for \( T = 1 \) and \( T = 2 \).

Plotted in Fig. 4 are the initial values of lift coefficient for \( T = 1 \) and \( T = 2 \), respectively. The general pattern of lift variation is similar to that of the lift, but the magnitude in two orders smaller. The mean lift coefficient \( c_l \) is averaged over one period, and it appears negative for positives, respectively, for \( T = 1 \) and \( T = 2 \).

The effect of changing vortex angle of attack on mean lift coefficient is evident. Representative results obtained for the above vortices are shown in Fig. 6. The plot indicates that the lift varies in a manner similar to the lift coefficient. It is observed that a change in vortex angle of attack affects the vortex-induced lift and that the lift decreases with increasing angle of attack.

generated on a wing utilizing a properly designed vortex-triggering device installed on its upper surface. Despite the negligibly small drag shown in the result, it can be expected to be substantial in reality when vortices are generated by moving a spoiler into and out of the airfoil surface.

The strength of the released vortices is arbitrarily assigned in the present analysis. It actually should be determined more realistically from empirical data or from viscous flow computations. The latter approach is being taken by us in an attempt to solve the problem of an unsteady viscous flow past an oscillating spoiler.

Acknowledgment

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References


Fig. 1  Flow past a NACA 0012 airfoil with a spoiler at the quarter-chord position on the upper surface. Photograph was taken from the left rear side of the wind. Streak lines were traced out by smoke injected at different heights from a vertical column installed far upstream.

![Flow past a NACA 0012 airfoil with a spoiler at the quarter-chord position on the upper surface.](image)

Fig. 4  Initial variation of lift coefficient for \( \alpha = 0 \) and \( T = 5 \). Circulation of vortices released at the midchord position in \( \alpha \).

![Initial variation of lift coefficient for \( \alpha = 0 \) and \( T = 5 \). Circulation of vortices released at the midchord position in \( \alpha \).](image)
Circulation of vortices released in a representation of a phenomenon. Comparison of lift coefficient and parameters.
A NAVIER-STOKES CALCULATION OF THE AIRFOIL DYNAMIC STALL PROCESS

Stephen J. Shamroth
Scientific Research Associates, Inc.
Glastonbury, Connecticut 06033

Abstract

A time-dependent Navier-Stokes calculation procedure has been applied to the problem of an NACA 0012 airfoil oscillating in pitch to a low Mach number, high Reynolds number environment. The calculated results show many of the known physical features, including sudden suction surface separation, vortex shed at the leading and trailing edges and the return to attached flow at low incidences. Both the lift and moment coefficient curves show the expected features and the calculated wall pressure coefficients show strong correspondence to measured data.

Introduction

The unsteady flow field about an isolated airfoil is an important problem commonly encountered in a number of practical flow situations. These include airfoil flutter, vibration, buffeting and gust response, as well as the problem of airfoil dynamic stall. This latter problem, airfoil dynamic stall, is the subject of the present paper. Airfoil dynamic stall occurs in a variety of situations. One important flow which has initiated the present study is the helicopter rotor problem. As the helicopter blade travels through the rotor disc, the blade experiences a varying incidence angle. Over most of the disc the blade will be uninstalled (i.e., the flow will not contain any large separated regions leading to a decrease in blade lift or a generation of large blade moment coefficients); however, for example, in forward flight over the retreating portion of the disc large regions of separated flow may appear and this time history dependence of the problem makes dynamic stall prediction a particularly difficult task.

Despite its complex nature, airfoil dynamic stall has been the subject of a number of theoretical and experimental investigations over the past four decades. Experiment giving surface pressure data for a variety of flow conditions, including those of Gartt and Sallister, Wu and Newsome, and Sallister, et al. have examined the vortex shedding region of isolated and oscillating airfoils. Sallister, Carr and Mcardle and Chaang and Soderstrom have also investigated the blade region. In general, airfoil dynamic stall has been investigated using model tests in wind tunnels, and steady wind tunnel tests on isolated airfoils. In recent years, Sallister and Mcardle have reviewed some of the recent developments in the dynamic stall field, as well as some of the current problems and difficulties associated with dynamic stall. Some of the recent work on dynamic stall has focused on the development of a numerical method for calculating the unsteady flow field, and the use of this method to study the dynamic stall problem.

The present study is based on a Navier-Stokes calculation procedure, which has been applied to the problem of an NACA 0012 airfoil oscillating in pitch to a low Mach number, high Reynolds number environment. The calculated results show many of the known physical features, including sudden suction surface separation, vortex shed at the leading and trailing edges and the return to attached flow at low incidences. Both the lift and moment coefficient curves show the expected features and the calculated wall pressure coefficients show strong correspondence to measured data.

Analysis

The Coordinate System

The presence of bounding surfaces in the computational domain which do not fall upon coordinate lines presents significant difficulties for numerical techniques which solve the Navier-Stokes equations. If a bounding surface (such as the airfoil surface) does not coincide with a coordinate line, serious numerical errors may arise in the application of boundary conditions for the problem. A number of techniques have been developed to reduce these errors to an acceptable level. For example, Mehta and Soderstrom have used a number of techniques to reduce the numerical errors associated with the boundary conditions. In the present study, a number of techniques have been used to reduce the numerical errors associated with the boundary conditions.

The Coordinate System

The present study is based on a numerical method for calculating the unsteady flow field, and the use of this method to study the dynamic stall problem. Some of the recent work on dynamic stall has focused on the development of a numerical method for calculating the unsteady flow field, and the use of this method to study the dynamic stall problem.
The form of the equations expressed in the more common coordinate systems can be found in standard fluid dynamic texts. One possible approach for solving the equations in general nonorthogonal form is the strong conservation approach such as that used by Steger15. A second possible approach solves a set of equations in which the metric coefficients do not appear within derivatives. This has been termed the quasi-linear form by Shamoith and Hirtman16 and the chain rule conservation form by Hindman18. Although accurate results have been obtained with both forms of the equations, in cases where the Jacobian of transformation is independent of time the latter form is less sensitive to the precise manner in which the matrices are evaluated17,18. Therefore, the present effort utilizes the quasi-linear form. Discussion of coordinate systems having time-dependent Jacobians has been given by Thomas and Lombard19.

If the spatial variables are transformed from the Cartesian coordinates (x,y) to a new set of coordinates (r,θ) where

\[ x = (x,y) \rightarrow (r,θ) \rightarrow (x,y) \]  

(1)

The quasi-linear form of the equations becomes

\[ \begin{align*}
\frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} + \frac{\partial uv}{\partial y} &= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} \\
\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} &= \frac{\partial F_1}{\partial y} + \frac{\partial F_2}{\partial x}
\end{align*} \]  

where

\[ u, v, F_1, F_2 \] are density, u and v are velocity components, F_1 is pressure, and 1 and 2 in the terms denote the first and second pairs of variables, respectively. It is noted that the linearization of the equations required for this type of algorithms is one-step nonlinear scheme, since the term to be linearized is the derivative of the Jacobian of the transformation of coordinates, which is independent of time, and its effect is reflected in the time step. This is always the case when a fully implicit solution procedure is used. The form of the linearization technique is adopted from the work of Thomas and Lombard, who have shown that the use of a fully implicit solution procedure is required for the accurate solution of the governing equations as perturbations to the fundamental equations of the form governing equations are replaced by an implicit time difference approximation, optionally a backward difference or Crank-Nicolson scheme. Terms involving nonlinearity at the implicit time level are linearized by Taylor expansion in time about the solution at the known time level, and spatial difference approximations are introduced. The result is a system of multidimensional coupled (but linear) difference equations for the dependent variables at the unknown or implicit time level. To solve these difference equations, the Douglas-Gunn procedure for generating alternating-direction implicit (ADI) schemes as perturbations of fundamental implicit difference schemes is introduced. This technique lends to systems of coupled linear difference equations having narrow block-banded matrix structures which can be solved efficiently by standard block-elimination methods.

The method centers around the use of a formal linearization technique adapted for the integration of initial-value problems. The linearization technique, which requires an implicit solution procedure, permits the solution of coupled nonlinear equations in one space dimension (or the primitive degree of accuracy) by a one-step nonlinear scheme. Since no iteration is required to compute the solution for a single time step, and since only moderate effort is required for solution of the implicit difference equations, the method is computationally efficient, this effort is retained for multidimensional problems by using ADI techniques. Its method also economical in terms of computer storage, in the present form requiring only the two-levels of storage for each dependent variable. Furthermore, the ADI technique reduces multi-dimensional problems to sequences of one-dimensional problems, which are one-dimensional in the sense that easily solved narrow block-bordered matrices associated with one-dimensional cases are produced. The problem with the ADI technique is that of splitting error. The presence of splitting error may limit the accuracy of time step; however, as will be shown, the use of ADI technique for the steady state is not permitted, despite this potential problem. Therefore, the finding of the transient solution remains an urgent current investigation at ARMS.

Equations Considered

The equations considered here are the full compressible equations in the primitive variables. These equations are:

\[ \frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} + \frac{\partial uv}{\partial y} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} \]

\[ \frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} = \frac{\partial F_1}{\partial y} + \frac{\partial F_2}{\partial x} \]

\[ \frac{\partial p}{\partial t} + \frac{\partial (pu)}{\partial x} + \frac{\partial (pv)}{\partial y} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} \]

\[ \frac{\partial T}{\partial t} + \frac{\partial (Tu)}{\partial x} + \frac{\partial (Tv)}{\partial y} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} \]

in their conservative form, where

\[ u = \frac{\partial p}{\partial x} \]

\[ v = \frac{\partial p}{\partial y} \]

\[ F_1 = \frac{\partial p}{\partial x} + \frac{\partial T}{\partial x} \]

\[ F_2 = \frac{\partial p}{\partial y} + \frac{\partial T}{\partial y} \]
approximations inherent in them, and the solutions in usual manner at the boundaries through so-called "extraneous boundary conditions". If extraneous conditions are utilized, they should be chosen to be as reasonable as possible from a physical point of view. The hypothesis is adopted that if a solution can be obtained with the extraneous boundary conditions applied, then any defective influence of these extraneous conditions will result only from the approximations inherent in them, and the solutions should be assessed in this manner.

With these considerations in mind, the following approach was taken in setting boundary conditions for the airfoil flow field problem. The outer boundary was divided into four segments. These were an upstream boundary, a downstream boundary and two boundaries representing the wind tunnel walls. Consistent with a characteristic analysis, two physical boundary conditions are assumed for the upstream boundary and one for the downstream boundary. These represent items which could be set in a wind tunnel experiment and are the upstream stagnation pressure, the upstream flow angle and the downstream state pressure. The "extraneous boundary conditions" chosen were first derivative of density at the upstream boundary and second derivative of velocity components. It should be noted that the physical boundary conditions used, while clearly inappropriate for steady flow, may require further investigation in unsteady flows, particularly in regard to minimizing physically unrealistic wave reflections. These two boundary conditions are for the form of the current research code, present, and consideration of non-reflective boundary conditions for steady and unsteady flow was under consideration as well.

The final boundary conditions concern the solid wall of the wind tunnel walls. Numerical artificial dissipation was added in the solid walls to reduce spurious oscillations. A dissipation term was included in the solid wall boundary conditions, but upstream were used, first one-side difference of transverse momentum equation was applied to the solid surface. Second order upwind gradients were used. Although the presence of the artificial dissipation term was intended to damp out the spurious oscillations, it was found that this term was necessary only for the solid wall. For the fluid, only the artificial dissipation term was included.

The physical dissipation term for the solid wall was given by

$$\rho C D v = \frac{\nu}{\delta_{ll}}$$

where $\rho$ is the density, $C$ is the von-Karman damping factor, $\nu$ is the von-Karman constant and $\delta_{ll}$ is the boundary layer thickness, i.e.,

$$\delta_{ll} = \frac{0.14}{Re^{1/3}}$$

where $Re$ is the Reynolds number.

The artificial dissipation terms are difficult to include in the flow solver. It was found that the artificial dissipation term alone was not sufficient to suppress the oscillations, even when using the von-Karman damping factor and the von-Karman constant. Further, explicit addition of artificial dissipation term did not improve the accuracy for the high Reynolds number, it was necessary to use an artificial dissipation term in addition. More research is currently in progress.

Artificial Dissipation

The final item to be considered concerns the use of artificial dissipation. Since the calculations were at high Reynolds numbers, it was necessary to add "artificial dissipation" terms to suppress central difference spatial oscillations. Such "artificial dissipation" could be added via the spatial differencing formulation (e.g., added difference approximations for first derivatives) or by explicitly adding an additional dissipative type term. The present author found that the latter approach was more effective in suppressing oscillations without deteriorating solution accuracy. Various methods of adding artificial dissipation were investigated in detail. In the present research, the use of a high Reynolds number code to simulate dissipation terms was found to be more effective.

In all, the use of artificial dissipation was found to be effective in reducing spurious oscillations. However, further research is required to fully understand the effects of artificial dissipation on the solution accuracy. Additional research is currently in progress.
where \( \nu \) is the total or effective viscosity including both laminar and turbulent contributions, and \( Ax \) is the grid spacing. The dissipation coefficient \( d_y \) is non-negative and is chosen as the larger of zero and the local quantity \( \nu \left( \frac{\partial u}{\partial x} \right)^2 \). The dissipation parameter \( c \) is a specified constant and represents the inverse of the cell Reynolds number below which no artificial dissipation is added. The dissipation coefficient \( d_y \) is evaluated in an analogous manner and is based on the local cell Reynolds number \( Re_{cy} \) and grid spacing \( Ax \) for the \( y \)-direction and the specified parameter \( c_y \). It should be noted that recently calculations have been run with artificial dissipation added in the conservative form \( \nu \left( \frac{\partial u}{\partial x} \right)^2 \) and no significant difference between the forms was noted.

The question arises as to the values of \( c \) and \( c_y \) which should be chosen. Based upon the results of Ref. 2, as well as several other investigations for a variety of viscous subsonic and transonic flows it was concluded that setting \( c \) between 0.1 and 0.025 suppressed non-physical oscillations, gave solutions which were insensitive to the precise value of \( c \), and gave good agreement with data. Therefore, second order damping in the conservative form was used in the present calculations with \( c \) being set in the range between 0.1 and 0.05.

**Results**

The analysis described previously was applied to the flow about an NACA 0012 airfoil oscillating sinusoidally in pitch. The airfoil was immersed in a stream of Reynolds numbers equal to \( 2.08 \times 10^6 \) and Mach number equal to 0.30. The airfoil mean incidence was set at \( 12^\circ \) and oscillated with an amplitude of \( 8^\circ \) at a reduced frequency based upon semi-chord of 0.125. These conditions correspond to Run 51.035 of the data of St. Hilaire and Carta.

The calculation was run for slightly more than one cycle using a highly stretched grid with the first point away from the airfoil being \( 0.12 \times 10^{-5} \) chords from the airfoil. Approximately 950 time steps were required to proceed through a cycle for this reduced frequency. Higher reduced frequencies required fewer time steps. The calculation was initiated from a converged steady oscillation at \( 12^\circ \) incidence. The calculation was begun with the artificial dissipation parameter \( c \) being set to 0.05. During the high incidence downstroke portion of the calculation numerical problems appeared at \( \alpha = 19.7^\circ \), which required temporarily increasing artificial dissipation by raising the artificial dissipation factor, \( d_x \), to 0.3. It was kept at this value until \( \alpha = 14^\circ \) when it was dropped to 0.1. The numerical difficulties are believed to be related to mesh resolution and the temporary increase in artificial dissipation was adopted rather than adding more grid points simply due to economic necessity.

Comparisons between calculated and measured pressure distributions are given in Figs. 1-3 where the pressure coefficient \( c_p \) is defined as \( c_p = \frac{p - p_{\infty}}{\frac{1}{2} \rho U^2} \). As can be seen in Figs. 1 and 2, the agreement during much of the upstroke is very good. The excellent comparisons shown in Figs. 1 and 2 give evidence to the time-sequence of the calculation for the surface pressure. Further examination of the data and calculation shows that the calculated pressure distribution is good.
Incident indicated stall at 19.5°. Therefore, at incidence values greater than 17.3° the excellent quantitative agreement shown in Figs. 1 and 2 no longer is found. However, a strong qualitative agreement remained. For example, the data at 19.5° is presented with the calculation at α = 19.9°, δ < 0 in Fig. 3. Although these are at different values of α, they represent pressure distributions at approximately the same incremental time after stall is initiated; the distributions are remarkably similar, Fig. 3, as well as figures not presented here due to space limitations, indicate that although the calculation predicts stall to occur after the measured incidence, once stall occurs, when referenced to the time of stall, the calculated and measured pressure distributions become quite similar. The major discrepancy in the calculated and measured values appear to lie in the prediction of vortex initiation. This, in turn, is likely to be dependent on turbulence and transition modeling. Comparisons over the downstroke are given in Figs. 4 and 5. As shown in Fig. 5 by α = 8.5°, δ > 0, both measured data and calculation indicate the flow to be nearly recovered from the stall process. Obviously, the basic trends are in agreement as a strong qualitative comparison is shown between the calculation and the measured data. Overall, the detailed pressure distributions have good agreement with data.

Comparisons between measured and calculated lift and moment coefficients are given in Figs. 6 and 7. In viewing these figures it should be noted that lift and moment coefficients are integrated quantities. Relatively small differences in pressure distributions can lead to significant differences in lift coefficient and large differences in moment coefficients. Likewise, correspondence of the coefficients does not necessarily mean correspondence of surface pressure distribution. The predicted normal force coefficient is compared to the measured normal force coefficient in Fig. 6. As can be seen, the results are in good agreement over the upstroke. However, the measured lift begins to decrease at α = 19°.
whereas the calculated lift continues to increase until 20°. Both measurement and calculation show a precipitous drop near the maximum incidence. The major disagreement in the measured and calculated lift coefficient is over the low incidence half of the downstroke. The measured coefficient continued to decrease until \( \alpha = 12° \), whereas the calculated coefficient plateaued at \( \alpha \approx 16° \), indicating a somewhat more rapid recovery from stall. This agreement is regarded as reasonable with the major difference occurring at low incidence during stall recovery. A comparison between calculated and measured moment coefficient is shown in Fig. 7. As can be seen in Fig. 7, the current comparison shows good qualitative agreement, the major difference being the larger negative moment coefficient calculated than measured near \( \alpha_{\text{max}} \). Overall the agreement is quite good.

In addition to the surface pressure, plots of velocity vectors and vorticity contours were obtained at various times throughout the cycle. A detailed study of the plots and results show a clear pattern of the calculated stall process. During the initial portion of the downstroke the flow remains fully attached. At approximately 15.5° incidence no small leading edge and trailing edge separation bubbles appear on the suction surface. With increasing incidence the trailing edge separation point moves rapidly forward while the leading edge separation point remains small. By 18.5° the bubbles merge; however, they remain very thin and have only minimal effect on the outer nominally laminar flow. The separated zone remains very thin until a dramatic change occurs at approximately \( \alpha = 19° \); at this incidence a leading edge vortex is shed and the stall process commences. Selected plots are shown in Figs. 8 and 9. The breakdown of the suction side flow is evident in both the vector plot Fig. 8 and the vorticity plot Fig. 9. The data...
the appearance of the trailing edge counterclockwise vorticity appears to arise from two sources. The first source is the pressure surface boundary layer being convected around the trailing edge and up the suction surface by the large clockwise leading edge separation zone. The second source of this negative vorticity is a secondary boundary layer arising as the large clockwise separated zone is brought to a no-slip condition at the airfoil surface. By $\alpha = 16.1^\circ$, $\dot{\alpha} < 0$ the leading edge vortex has clearly broken away and the trailing edge vortex of opposite sign has increased in size and is moving somewhat upstream. This is demonstrated more clearly in the vorticity contour plot. By $\alpha = 16.5^\circ$, $\dot{\alpha} < 0$ the vortices are tending to interact and begin to move downstream. This process also has been discussed by Robinson and Luttges. Finally by $\alpha = 9.5^\circ$, $\dot{\alpha} < 0$ the flow has fully recovered.

**Concluding Remarks**

A Navier-Stokes calculation procedure has been applied to the problem of an isolated airfoil oscillating sinusoidally in pitch. The calculation procedure has solved the governing equations by an implicit technique with turbulence represented via a mixing length model. A highly stretched grid was used to resolve the turbulent boundary layer. Although further effort remains in regard to the turbulence model, boundary conditions, etc., the resulting calculation showed many of the observed flow field features, including the vortex shedding and separation processes. In addition, comparisons between predicted and measured surface pressure distribution showed strong qualitative agreement.

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**References**


Some structural features of unsteady separating turbulent shear flows are presented for practical Reynolds numbers and reduced frequencies for helicopter and turbomachinery flows. Upstream of detachment in moderate amplitude flows, the flow is quasi-steady, i.e., the phase-averaged flow is described by the steady freestream flow structure. Results presented here show that oscillation waveform and amplitude strongly influence the detached flow behavior. This is an important result since some practical unsteady flows have non-sinusoidal waveforms.

In spite of its importance, relatively little fundamental work has been done to describe the structure of unsteady separating turbulent shear layers. To this writer's knowledge, measurements of turbulent Reynolds stresses in an unsteady separating turbulent shear flow have been made only at SMU and by Cousteix et al. at ONERA in France. Parikh et al. and Jayaraman et al. at Stanford examined some cases with small amounts of near-wall reversed flow during part of a cycle, but with no flow detachment. The amplitude to mean freestream velocity ratio was about 0.3, 0.15, and 0.3 in these investigations, respectively. Direct measurements of the oscillating Reynolds shearing and normal stresses, surface shearing stress, and fraction of time backflow was present were made at practical Reynolds numbers and reduced frequencies by the SMU group.

As background for this paper, a brief summary is given in section II of the nature of a steady freestream separating turbulent boundary layer as determined by the work of Simpson et al. and Shilo et al. at SMU. The results from the sinusoidally unsteady separating turbulent shear layer of Simpson et al. are discussed in section III. Section IV presents the results from recent work that indicate that the oscillation waveform and amplitude strongly determine the behavior of a detaching turbulent shear layer.

II. SUMMARY OF THE NATURE OF A STEADY FRESTREAM SEPARATING TURBULENT BOUNDARY LAYER

The fraction of time with forward flow near the wall is a descriptive parameter in the flow detachment state. Detachment occurs with location of the time interval...
transient detachment (ITD) occurs with backflow 20% of the time; transient detachment (TD) occurs with backflow 50% of the time; and detachment (D) occurs where the time-average wall shear stress is zero.

Figure 1 shows a qualitative sketch of the second steady-freestream bottom wall turbulent separated flow studies at SMU and the locations of ID, ITD, and D when determined 1 mm from the wall. The mean flow upstream of ID obeys the "law of the wall" and the "law of the wake" as long as the maximum shearing stress |\(\bar{u}'v\) max| is less than 1.5 \(\bar{\tau}_w\), where \(\bar{\tau}_w\) is the local wall shearing stress. When |\(\bar{u}'v\) max| > 1.5 \(\bar{\tau}_w\), the Perry and Schofield mean velocity profile correlation and the law of the wall apply upstream of ITD. Up to one-third of the turbulence energy production in the outer region is due to normal stress effects, which modifies the relations between dissipation rate, turbulence energy, and turbulent shear stress that are observed further upstream.

Downstream of detachment, the mean backflow profile scales on the maximum negative mean shear rate \(\bar{\tau}_{w}\) and its distance from the wall \(L_1\).\(^1\) \(\bar{u}'v\)' law of the wall is not consistent with this result since \(\bar{u}'v\)' and \(\bar{\tau}_{w}\) increase with streamwise distance while \(\bar{u}'v\)' varies inversely with \(L_1\). \(\bar{u}'v\)' high turbulence levels exist in the backflow, with \(u'\) and \(v'\) fluctuations of the same order as \(\bar{u}'v\) becomes lower with increasing backflow and is about \(25\) lower in the outer region than for the upstream attached flow. Mixing length and eddy viscosity models are adequate upstream of detachment and in the outer region, but are physically meaningless in the backflow. \(\bar{\tau}_w\) never reaches zero, indicating that there is no location with no flow of the time. Normal and shear stress turbulent energy production in the outer region supply turbulence energy to the backflow by turbulence diffusion where it is dissipated. Large-scale turbulence energy production occurs in the backflow.

This turbulence energy diffusion and the wall near backflow are supplied intermittently by large-scale structures as they pass through the detached flow, as suggested by Figure 2. Scanning laser anemometer reveals a relatively coherent structure of the large eddies.\(^{14}\) The backflow does not come from far downstream. The frequency of passage \(n\) of these large-scale structures varies as \(U_e/n\) and is about an order of magnitude smaller than the frequency far upstream of detachment. Reynolds shearing stresses in the backflow must be modeled by relating them to the turbulence structure and not to local mean velocity gradients. The mean velocity profiles in the backflow are a result of time-averaging of the large fluctuations and are not related to the cause of the turbulence.

Commonly used turbulence models for attached flows do not work well for separated flows. Eaton\(^{15}\) discussed computations of the Simpson et al. flow that were made for the 1980-1981 AFSOR-HMTM-Stanford Conferences on Complex Turbulent Flows. The integral, eddy viscosity, and mixing length methods that were used were essentially attached boundary layer methods with some ad hoc change that would make the detached flow mean velocity profile look good. Unfortunately, the calculated Reynolds shearing stresses were much lower than the data. One 5 equation model was used that better predicted the shearing stress, but overpredicted the growth rate of the shear layer. Collins and Simpson\(^{16}\) also showed that when attached flow mixing length models were imposed on detached flows, low mixing lengths and low shear stresses must be used in order to get good mean velocity profiles.

III. SUMMARY OF THE NATURE OF A SEPARATING TURBULENT FLOW WITH MODERATE AMPLITUDE STATIONARY CHAOS

The experiments of Simpson et al.\(^5\) show that a periodic unsteady separating turbulent boundary layer at a streamwise distance Reynolds number of 471 x 10\(^6\), a practical reduced frequency of 0.61, and a ratio of the amplitude of the first harmonic to mean velocity outside the shear layer, \(A_{n,1}/U_0\), has both similarities and differences with steady freestream separating turbulent boundary layer at the same mean freestream conditions (Figure 3). Note that a through phase period of 0.5, while a direct sign for an ensemble average ordained at
given phase \( \phi \) of a cycle.) Figure 3 shows the
sinusoidal waveform \( U_e \). Upstream of where
intermittent backflow begins \( \gamma_{pu} < 1 \), the flow
and turbulence structure behave in a quasi-
steady manner.

After the beginning of detachment, large
amplitude and phase variations develop through
each flow and the structure is not quasi-steady.
Unsteady effects produce hysteresis in relationships
between flow parameters. As the free-
stream velocity during a cycle begins to
increase, the fraction of time that the flow
moves downstream \( \gamma_{pu} \) at a given phase of the
cycle increases and backflow fluid is washed
downstream. As the free-stream velocity nears
the maximum value in a cycle, the increasingly
unfavorable pressure gradient causes progressively
greater near wall backflow at downstream loca-
tions while \( \gamma_{pu} \) remains high at the upstream
part of the detached flow. After the free-
stream velocity begins to decrease, the
location where flow reversal begins moves
upstream. This cycle is repeated as the free-
stream velocity again increases.

Near the wall in the backflow region, the
ensemble-averaged velocity leads the free-stream
velocity by a large amount. The phase angle of
the periodic backflow velocity and \( \gamma_{pu} \) are
near independent of \( \gamma \) near the wall. The mean
backflow profile in terms of \( \gamma \) and \( \gamma_{pu} \) are
approximately the same as for the comparable
steady free-stream case. Thus, it appears that
the ensemble-averaged backflow near the wall
always flow a quasi-steady flow when utilized
\( \gamma_{pu} \) and \( \gamma \).

Downstream of detachment, \( \gamma_{pu} \) and \( \gamma \) are
slightly higher for the unsteady flow than the
steady flow, especially near the wall, where near
backflow occurs. The phase angle for \( \gamma_{pu} \) in the
backflow is progressively greater than the
free-stream velocity phase angle as the flow
moves downstream. The turbulence structure
progressively lags the ensemble-averaged flow
oscillations with \( \gamma_{pu} \) lagging \( \gamma \) in the backflow
by about 20%. The ratio \( \gamma_{pu} / \gamma \) increases from
about 0.5 upstream to detachment is about 0.2
downstream.

Near the wall, \( \gamma_{pu} \) is nearly in phase with
\( \gamma \), but since \( \gamma_{pu} \) and \( \gamma \) are
averaged backflow is greatest when \( \gamma_{pu} \) is low
and when \( \gamma_{pu} \) and \( \gamma \) are near maximum values. In
other words, \( \gamma \) in the backflow is nearly in
phase with \( \gamma_{pu} \). This is consistent with
the general observation from the steady flow
that \( \gamma_{pu} \) and \( \gamma \) are greater when there is more
near backflow.

The steady flow results show that
\( -\overline{uv} \sqrt{u^2 + v^2} \) decreases with decreasing \( \gamma_{pu} \). In
the steady flow, \( -\overline{uv} \) is greater with less ensemble-
averaged backflow or greater \( \gamma_{pu} \). In other
words, \( \gamma \) is lower and \( \gamma_{pu} \) is nearly in phase with
\( \gamma_{pu} \). As in the steady free-stream flow, \( -\overline{uv} \sqrt{u^2 + v^2} \) decreases with decreasing \( \gamma_{pu} \), although
there is some hysteresis and phase lag for the
unsteady flow.

Simpson and Shrivpasad reported measurements
for a reduced frequency of 0.90 also with
\( U_e / \gamma_{pu} = 0.3 \). The 0.90 reduced frequency has
the same qualitative behavior as the 0.60
reduced frequency case. Downstream of detach-
ment in both flows, the free-stream velocity and
the pressure gradient are in phase. In both
sinusoidally unsteady flows, the ensemble-
averaged backflow velocity profiles agree
with steady free-stream profiles for the same
\( \gamma_{pu} \) value near the wall when \( \gamma_{pu} \) is low.
However, the higher reduced frequency flow has
much larger hysteresis in ensemble-averaged
velocity profile shapes when \( \gamma_{pu} \) is low than the
lower reduced frequency. Larger and negative
values of the ensemble-averaged velocity
profile shape factor \( \lambda \) occur for this flow
during phases when the nondimensional backflow
is greater and \( \gamma_{pu} \) is low.
the experiments discussed above, an axial-compressor-type waveform, and a large amplitude oscillation, which were produced in the SMU wind tunnel. The parameter $R = (U_{\text{max}} - U_{\text{min}})/2U_{\text{e}}$ is 0.1, 0.238, and 0.75% for these cases, respectively. In each case examined the mean free-stream velocity $U_{\text{e}}$ was the same near the test-section throat (1.62m).

To survey the influence of waveform and amplitude on the flow, the surface skin friction at the test section throat and $R_{\text{min}}$ near the wall in the detaching flow zone were measured for these cases. Upstream of the strong adverse pressure gradient region, the mean unsteady skin friction factor $C_{f}$ was found to be independent of waveform shape and amplitude.

Table 1 shows $C_{f}$ in the near square of the free-stream velocity and, thus, includes the wind tunnel contributions to the oscillatory flow to the low dynamic pressure. Skin-friction measurements were made by a wax-mass surface instrument designed by Steeply et al. 3

This behavior is not fully surprising since the sinusoidal waveforms will upstream of the test section behave as a quasi-steady flow at practical reduced frequencies.

Figure 4 shows a mean profile at various $\alpha$ in the near-square of the free-stream velocity. The interpretation of this diagram is similar to that for sinusoidal flows, but with significant differences, since the free-stream velocity is constant, the backflow is washed down-stream, and the detached shear layer thickness increases as the freestream flow accelerates. Figure 5 shows the $R_{\text{min}}$ vs. $\alpha$ for all waveforms. The free-stream velocity increases and $R_{\text{min}}$ remains high in the unsteady oscillations and the observed trend in $R_{\text{min}}$ is similar to that of Figure 4, which is shown to be maintained as the freestream flow accelerates. Figure 6 shows $R_{\text{min}}$ vs. $\alpha$ for each phase of the oscillation for each amplitude, where $R_{\text{min}}$ decreases.

Figure 6 shows $R_{\text{min}}$ vs. $\alpha$ for the large amplitude oscillatory flow. During $20^\circ < \alpha < 120^\circ$, the backflow is completely washed out with $R_{\text{min}}$ becoming unity. As the freestream velocity reaches the maximum velocity, $R_{\text{min}}$ decreases in the downstream zone. Some second harmonic effects are evident for $120^\circ < \alpha < 240^\circ$. At larger $\alpha$, $R_{\text{min}}$ drops to very low values in the downstream zone and backflow occurs as far upstream as the test-section throat (1.62m). The quantitative differences between Figures 4 and 6 indicate that the oscillation amplitude influences the detached flow behavior.

Some mean velocity profiles for the large amplitude flow are shown in Figure 7. These profiles are much different in shape than those of the lower amplitude sinusoidal cases since large variations of the detached shear layer occur and the shapes of the ensemble-averaged profiles for each phase of the cycle are much different than those of the lower amplitude flows. Figure 8 shows profiles of the mean fraction of time that the flow moves downstream. The Figure 9 shows profiles of the phase angle of the first harmonic, unlike the lower amplitude sinusoidal waveform flows, there is little variation of this phase angle through the detached shear layer. This does not mean that this detached flow is quasi-steady, because higher harmonics and non-linear effects are important.
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REFERENCES


![Diagram of flow separation and boundary layers](image-url)
Figure 2. Freestream flow conditions for the steady free-stream flow of Simpson et al. (1931a) and the sinusoidal unsteady flow of Simpson et al. (1983). The solid line, steady flow, open symbols, sinusoidal unsteady flow on different days. Phase angles of time harmonics: solid line, freestream velocity; dashed line, pressure gradient (dashed line for. formal and.

\[
u = 
\begin{cases} 
   \nu_e & \text{for } n = 0 \\
   \frac{\nu_e}{n!} \cos(\omega t + \phi_n) & \text{for } n > 0 
\end{cases}
\]

Figure 3. Velocity waveform shapes: dashed line, sinusoidal case; --- , compressor-type case; solid line, large amplitude case.

Figure 4. Phase-averaged \(\nu\) flow for different pressure cases of a turbine for the compressor-type waveform.
Figure 6. Phase-averaged $Y_{pum in}$ vs. $X$ for different $\omega t$ phases of a cycle for the large amplitude waveform.

Figure 7. Mean velocity profiles $\bar{U}$ vs. $y$ for the large-amplitude flow: $\Delta$, 3.00 m; $\bigcirc$, 3.66 m; $\bigtriangleup$, 4.34 m. Closed symbols: directionally-sensitive laser anemometer data; open symbols: hot-wire anemometer data.

Figure 8. $\bar{Y}_{rms}$ vs. $\phi$ profiles for the large amplitude flow. Legend same as for Figure 7.

Figure 9. $\phi$ vs. $y$ profiles for the large amplitude flow. Legend same as in Figure 8.
Abstract

The evolution of unsteady boundary layers on oscillating airfoils is studied. The computational difficulties associated with the movement of the stagnation point as a function of space and time are solved by using a novel numerical scheme. Calculations are performed for pressure distributions typical of those found near the leading edge of airfoils. Results are presented for two cases. In the first, solutions are obtained for a flow with separation and with prescribed pressure distributions; they indicate that a singularity develops and is of the same type as that observed on a circular cylinder started impulsively from rest. In the second, results are obtained for the same flow and the viscous flow solutions are interacted with the external flow by using an inverse boundary-layer method. The interaction seems to remove the singularity, however, these results are preliminary and need to be checked and improved upon.

1.0 Introduction

In recent years some important developments have occurred in the theory of two-dimensional laminar boundary-layer flows. The crucial discovery, due to von Kármán and话题, is that the solution of the boundary-layer equations with boundary conditions corresponding to a circular cylinder started impulsively from rest develops a singularity. This, with a center velocity of the cylinder and a radius of region - 111 from the forward stagnation point. At this time the position of zero skin friction is very close to, but not coincident with, the singularity. This discovery was made by solving the governing boundary-layer equations in Lagrangian form subsequently, the existence of the singularity was confirmed by selecting the Eulerian form and by following a method of series truncation. The importance of this result is two-fold: First, it is of importance to the fundamental theory of laminar flows is indicated in indicating that an interactive theory is necessary to understand the evolution of the flow field at a finite time after the motion starts rather than many years after the free time. Secondly, it is clear that significant changes will occur in the flow field near the stagnation point of airfoils in relation to the phenomenon of turbulent flow. The physical processes which take place during the oscillation of the angle of attack of the airfoil are complicated and depend on the number of parameters. For example, an important characteristic is a large vortex that is formed near the surface at some stage in the cycle and causes stall to occur shortly afterwards. The occurrence of the vortex is probably associated with a breakdown of the unsteady boundary layer.

The purposes of the present research are to determine the relationship between unsteady separation and singularities in the solution, and to explore the possibilities of removing this singularity by interaction of the viscous and inviscid equations. In this paper we outline the current status of the work, report our progress and describe the problems which remain to be solved. So far we have examined the evolution of the boundary layer near the nose of an oscillating airfoil and found that, when the reduced frequency is of the same order as in the experiments on dynamic stall, the unsteady boundary layer ceases to behave in a smooth manner just downstream of separation and before one cycle has been completed as with the impulsively started circular cylinder, this irregular behavior signals the onset of a singularity in the solution of the boundary-layer equations. The equations and solution procedures used in this present study are described in Sections 2 and 3, respectively, and the results are presented and discussed in Section 4.

At present we are in the process of examining the link between this singularity and the external flow. Previous studies of steady interactive boundary layers have been reported for the leading-edge region of thin airfoils of the type considered here. If is the angle of attack, it was observed that the boundary layer near the nose is well behaved and unseparated if we is 1.16 and although there is significant adverse pressure gradient. At higher values of , however, separation occurred with an associated singularity which required the use of an interactive theory, with such a theory, calculation of flows with small separation presented no difficulties, but at higher values of , the solutions failed to converge, providing no steady-state solution. An important objective of our study of interactive unsteady flows is to determine whether a similar breakdown will occur, the equations and the solution procedures are similar to those of Sections 2 and 3 and the progress so far is described in Section 5.

1.0 Equations, Initial and Boundary Conditions

The boundary-layer equations for unsteady incompressible laminar flows on oscillating airfoils can be written as:
Usually the boundary-layer calculations for the above equations are performed for prescribed boundary conditions given by
\[ u(s,0,t) = 0, \quad v(s,0,t) = 0, \quad u(s,n_t) = u_e(s,t) \quad (3) \]

and we shall refer to this as the standard problem. In the interactive problem we determine \( u_e(s,t) \) partly from inviscid theory and partly from the pressure distribution resulting from the blowing velocity \( dU/ds \) induced by the boundary layer. Thus we write
\[ u_e(s,t) = u_0^e(s,t) + u_c(s,t) \quad (4) \]

where \( u_0(s,t) \) is the slip velocity at the airfoil according to inviscid theory and \( u_c(s,t) \) is related to the blowing velocity by a variation of the Hilbert integral
\[ u_c(s,t) = \frac{1}{2 \pi} \int_0^\infty \frac{u_e^*(r,t) \, dr}{s - r} \quad (5) \]

Strictly, Eq. (5) is valid only for straight walls but it can be generalized for any airfoil shape as discussed by Refs. 6 and 7. For unseparated boundary layers the effect of \( u_c(s,t) \) is generally weak but once separation occurs, its effect is enhanced significantly in the neighborhood of separation. That this must be so may be seen by noting that otherwise the integrand in Eq. (5) would develop a strong singularity at separation and cause the solutions to break down further downstream. As discussed in Refs. 7 and 8, it is sufficient to replace Eq. (5) by
\[ u_c(s,t) = \frac{1}{2} \int_0^\infty \frac{u_e^*(r,t) \, dr}{s - r} \quad (6) \]

where the prime denotes differentiation with respect to \( s \).

In the standard problem, the solution of boundary-layer equations requires that the external velocity distribution be specified. Since the present effort is directed toward any airfoil shape near the leading edge of the airfoil, a local model for the potential flow has been chosen in the place of a full-potential-flux code. We consider an ellipse with major axis 2a and minor axis 2ar(\( \epsilon \)-1) at an angle of attack \( \alpha \). The surface of the body is defined by
\[ x = a \cos \theta, \quad y = a \sin \theta \]

and with these definitions and to a first approximation, the external velocity around the ellipse can be deduced from inviscid flow theory to be
\[ u_e^*(r,t) = \frac{r}{\sqrt{1 + \epsilon^2}} \quad (7) \]

where \( u_e^*(r,t) \) denotes a dimensionless velocity, \( r = \sqrt{x^2 + y^2} \), the parameter \( \epsilon \) denotes a dimensionless distance related to the \( x \)- and \( y \)-coordinates of the ellipse by \( \epsilon = \frac{y}{x} \). The parameter \( \epsilon \) is related to the surface distance \( s \) by
\[ s = \frac{a \sqrt{1 + \epsilon^2}}{1 + \epsilon^2} \戳{(8)} \]

We next define a dimensionless distance \( n \) by
\[ n = \frac{\sqrt{R(1 + \epsilon^2)}}{2r(1 + \epsilon^2)} \]

with \( R = 2a u_w/\nu \), and a dimensionless stream function \( f \) by
\[ f(r,\epsilon, t) = \left( 1 + \frac{\epsilon^2}{2} \right) u_e^*(r,t) - \frac{1}{4} \left( 1 + \frac{\epsilon^2}{2} \right)^2 \]

Introducing these relations together with Eq. (8) into Eqs. (1) and (2), it can be shown that the continuity and momentum equations can be written as
\[ \epsilon \frac{d}{dr} \left( u_e^*(r,t) \right) = 1 + \frac{\epsilon^2}{2} \quad (10) \]

Here primes denote differentiation with respect to \( r \) and \( \epsilon = 1 + \frac{\epsilon^2}{2} \).

The boundary conditions for \( f \) and \( f' \) become
\[ f = f' = 0 \quad \text{at} \quad n = 0 \]

The definition of \( u_e^*(r,t) \) is given by
\[ u_e^*(r,t) = \frac{u_e^*(r,t)}{u_e^*(r,t)} = \frac{r + \epsilon \theta(t)}{1 + \epsilon^2} + \frac{u_c}{u_e^*(r,t)} \quad (10) \]

where with a prime denoting differentiation with respect to \( r \),
\[ u_e^*(r,t) = \frac{r + \epsilon \theta(t)}{1 + \epsilon^2} - \frac{u_c}{u_e^*(r,t)} \quad (13) \]

and the dimensionless displacement thickness is given by
\[ \theta(t) = \lim_{r \to 0} \left[ f - f^* (r, r, t) \right] \quad (14) \]

Substituting Eq. (14) into Eq. (13), and using Eq. (10), the edge boundary condition in Eq. (11) can...
be written as
\[ f' = \tau + r_0(t_1) + c \int_a^{\frac{a}{\tau}} \frac{\partial}{\partial r} \left( \frac{f(\tau, t_1)}{r} \right) \, dr \]

where
\[ c = \frac{1}{\sqrt{2 \pi (\tau + t)}^{1/2}} \]

To complete the formulation of the problem, initial conditions must be specified in the \((t, n)\) plane at some \(s = s_0\), either on the lower or upper surface of the airfoil as well as initial conditions in the plane on both surfaces of the airfoil. In the latter case, if we assume that steady-flow conditions prevail at \(t = 0\), then the initial conditions in the \((s, n)\)-plane can easily be generated for both surfaces by solving the governing equations for steady flow, which in this case, are given by Eq. (1) and by
\[ u \frac{\partial \psi}{\partial s} + v \frac{\partial \psi}{\partial n} = u_e \frac{\partial \psi}{\partial s} + v \frac{\partial \psi}{\partial n} \]

There is no problem with the initial conditions for Eqs. (1) and (17) since the calculations start at the stagnation point.

The generation of the initial conditions in the \((t, n)\)-plane are not so straightforward to obtain as is discussed in Section 3.1.

3.0 Solution Procedure

The solution procedure for the set of equations and boundary and initial conditions given in Section 2 can be achieved in two parts concerned, respectively, with the leading edge and downstream region. These two parts are considered in the following two subsections. In both cases the solution procedure makes use of Keller's box method, which is a two-point finite-difference scheme extensively used for the solution of parabolic partial-differential equations, as discussed by Bradshaw and co-workers.

3.1 Leading-Edge Region

The generation of the initial conditions in the \((t, n)\)-plane at \(s = s_0\) requires a special numerical procedure. Given, as we are, the complete velocity profile distribution on the previous time line, there is, in principle, no difficulty in computing values on the next time line by an explicit method, but if we wish to avoid the instability problems associated with such a method by using an implicit method, we are immediately faced with the problem of generating a starting profile on the new time line.

In order to explain the problem further, it is instructive to see what happens to the stagnation point as a function of time. For this purpose let us consider Eq. (7). If we choose \(r_0(t_1)\) to be of the form \(r_0(t_1) = A \sin(t_1)\), and let \(u_0 = u_0(1 + \cos(t_1))\), then Eq. (7) becomes
\[ u_0(r, t_1) = \frac{A \sin(t_1)}{1 + \cos(t_1)} \]

where \(\omega\) is related to the dimensional frequency by
\[ \omega = \frac{a t^2}{(1 + t) u_0} \]

Here \(r_0\) and \(A\) denote parameters that need to be specified. Since by definition \(u_0 = 0\) at the stagnation point, its location, \(r_s\), is given by
\[ r_s = r_0(1 + A \sin(t_1)) \]

and so the upper and lower surfaces of the airfoil as functions of time are defined, in particular, by \(r > r_s\) and \(r < r_s\). For example, let us take \(A = 1\), \(\omega = \pi/4\) and plot \(r_s/r_0\) in the \((t, n)\)-plane, as shown in Figure 1 for one cycle \((0 \leq t \leq 8)\).

![Fig. 1. Variation of stagnation point with time](image)

When \(t = 2\), the stagnation point \(r_s\) is at \(-2r_0\), when \(t = 6\) is at \(0\), etc. If \(r_s\) was fixed, we could assume that \(u = 0\) at \(r = r_s\) for all time and all \(n\), but this is not the case. It is also possible to assume that the stagnation point is coincident with zero \(u\)-velocity for a prescribed time. However, we should note that the stagnation point given by Eq. (19) is based on the vanishing of the external velocity. For a time-dependent flow, this does not necessarily imply that the \(u\)-velocity is zero across the layer for a given \(n\)-location and specified time. This point is substantiated by the results shown in Figure 2 taken from Ref. 10 and obtained with a novel numerical procedure called the characteristic box scheme. It is also evident from Figure 2 that flow reversals do occur due to the movement of the locus of zero \(u\)-velocity across the layer. This causes numerical instabilities which can be avoided by using either the zig-zag box or the characteristic box finite-difference scheme. The details of these numerical schemes have been reported in Ref. 9 and, with special reference to oscillating airfoils, in Ref. 10.

3.2 Downstream Region

A solution to the leading-edge region, obtained by the procedure of Section 3.1, may be used as initial conditions for the solution of the system of equations given by Eqs. (11) and (15) both with either standard or inverse procedures. In
Fig. 2. Velocity profiles in the immediate neighborhood of the stagnation line at different times for \( J = \pi/4 \) and \( A = 1 \). The dashed lines indicate the locus of zero u-velocity across the layer.

To solve the equations for both standard and inverse problems we use modified forms of Keller's box scheme. In the latter case we use the Hechul function formulation which treats the external velocity as an unknown.

The box scheme reduces Eq. (10) to a first-order system. With \( w, r \) and \( v \), respectively, we write

\[
\begin{align*}
 r' & = r \\
 v' & = v \\
 w' & = 0
\end{align*}
\] (20a, 20b, 20c)

and obtain

\[
\begin{align*}
 v' & = \frac{r}{1 + r^2} - v \frac{r}{1 + r^2} + (1 + r^2) \frac{w}{1 + r^2} \\
 & = r \left( \frac{r}{1 + r^2} - v \frac{r}{1 + r^2} + (1 + r^2) \frac{w}{1 + r^2} \right)
\end{align*}
\] (20d)

With this notation, the boundary conditions given by Eq. (11) can be written as

\[
\begin{align*}
 f + r & = 0 \quad \text{at} \quad \tau = 0 \\
 r & = (1 + z^2)^{1/2} w \quad \text{at} \quad \tau = \infty
\end{align*}
\] (21a, 21b)

For the standard problem \( w \) is known and is given by Eq. (18). A fourth boundary condition is required for the inverse problem and is obtained from Eq. (15). Introducing a discrete approximation to Eq. (15), it can be written in the form

\[
r_{e}(\tau) - C_{ij} e^{i}(\tau) = g_{i}
\] (22)

where \( C_{ij} \) is the matrix of interaction coefficients defining the relationship between the displacement thickness and external flow and the parameter \( g_{i} \) represents terms whose values are assumed to be known. It is given by

\[
 g_{i} = \sum_{k=1}^{m} c_{ik} (r_{k} - \tau) + \sum_{k=1}^{m} c_{ik} (r_{k})
\] (23)

The system of Eqs. (20) to (22) has been solved by the numerical procedure of ref. 9 for the standard and inverse formulations.

4.0 Nature of the Singularity for an Oscillating Airfoil

One phase of the calculations for the oscillating airfoil was carried out by choosing \( r_{0} = 1 \), \( A = -1/2 \) and \( \pi = 0.1 \). With these choices the maximum value of \( \tau_{max} \) defined by

\[
\tau_{max} = \tau_{0} (1 + \sin(\pi))
\] (24)

is sufficient to provoke separation with a strong singularity if the boundary layer were steady. At present \( r_{0}, A, \pi \) are being varied to examine their effect on the nature of singularity.

The unsteady flow calculations displayed in Fig. 1 show that the boundary layer eventually separates, the flow remaining smooth. However, just downstream of separation, it is evident that a singularity develops in the solution in the neighborhood of \( \tau = 0.12 \) and \( \pi = 308.75° \) and that it is not possible to continue the solution beyond this time without conceptual changes in the mathematical and physical formulation of the problem. While this is a satisfying conclusion, and may be interpreted as giving theoretical support to experimental observations of dynamic stall, it should be
treated with some caution. Boundary-layer singularities have been the subject of much controversy in recent years and it is clearly important to make sure that any irregularities in a computed solution are not creations of the numerical method used. We, however, feel confident that the calculations reported here are accurate and that the singularity is real.

Figure 3a shows that the variation of the displacement thickness is generally smooth except in the neighborhood of $\alpha = -1.12$ and for $\alpha = 108.75^\circ$. The first sign of irregularity is the steepening of the slope of $\tau_a$ when $\alpha = 300^\circ$. A local maximum of $\tau_a$ occurs at $\alpha = 3.12$ when $\alpha = 108.75^\circ$. When the same results are plotted for a displacement velocity, $(d/d\tau) (u_e^*)$, (Fig. 3b), we observe that the steepening of the displacement velocity near $\alpha = 2.12$ is dramatic. For example, for $\alpha = 300^\circ$, the peak is at $\tau = 2.125$, for $\alpha = 305^\circ$, it is at $\tau = 2.105$, for $\alpha = 305^\circ$, it is at $\tau = 2.09$ and finally for $\alpha = 308.75^\circ$, the peak moves to $\tau = 2.08$. It should be noted that the maximum value of displacement velocity moves towards the separation point with increasing $\alpha$; the same behavior will be shown to occur for the circular cylinder discussed below.

As shown in Fig. 3c, the wall shear parameter $f_w$ shows no signs of irregularity for $\alpha = 308.75^\circ$ but a deep minimum in $f_w$ occurs near $\tau = 2.15$, i.e. near the peak of $\tau_a$.

It is interesting and useful to compare the results presented in Fig. 3 for an oscillating airfoil with those obtained for a circular cylinder started impulsively from rest. This comparison lends support to the accuracy of the present calculation method and at the same time enables us to compare the characteristics of two distinctly different unsteady flows near the singularity location. The circular cylinder problem has been extensively studied as reported in Refs. 11 and 12 and the present results shown below are in close agreement with those of previous authors, but with subtle differences which may have important implications.

Figure 4 shows the results obtained by Cebeci for the circular cylinder problem. As in the case of the oscillating airfoil, the flow separates and remains smooth up to the separation point. However, just downstream of separation with increasing time, a singularity develops in the neighborhood of $\alpha = 112^\circ$ and $\alpha = 3.0$ and it was not possible to continue the boundary-layer calculations beyond this time and angular location. From Fig. 4a we see that while the variation of displacement thickness is smooth for values of $\alpha$ less than $108^\circ$, it begins to steepen dramatically thereafter. The same results are plotted in Fig. 4b to demonstrate that, as in Fig. 3b, the displacement velocity exhibits a maximum which increases rapidly with time. Again the maximum shift towards the location of separation with increasing time.

The results of local skin-friction coefficient calculations in Fig. 4c follow similar trends to those obtained for the oscillating airfoil. In both cases, the distributions pass through zero with no signs of irregularity and do not exhibit any breakdown before the time corresponding to the singularity.

The very careful calculations of van Dommelen and Shen are reproduced on Figures 5 and 6 for displacement thickness and velocity profiles, respectively. The corresponding displacement thickness results of Cebeci together with the new calculations of the velocity profiles are reproduced for
Fig. 4. Computed results of Cebeci for the circular cylinder. (a) Displacement thickness $\delta^*/L$. (b) Displacement velocity, $d/d^* (u_e^*)$. (c) Wall shear parameter, $f_w$.

Fig. 5. Comparison between the displacement thickness values obtained by van Dommelen and Shen (circles) and by Cebeci (solid line) for the circular cylinder. (a) $t = 2.0$, (b) $t = 2.5$, (c) $t = 2.75$. ($x$ is in radians.)

Fig. 6. Comparison between the velocity profiles obtained by van Dommelen and Shen (solid lines) and by Cebeci (symbols). (a) $7.75^\circ$, (b) $t = 7.984375$ (van Dommelen and Shen) and $t = 7.9875$ (present calculations).
comparison purposes. As can be seen from Fig. 5, the agreement between the sets of calculations for three values of $t = 2, 2.5$ and 2.75 is excellent. The velocity profiles of Fig. 6a, which correspond to a time $t = 2.75$ as in Fig. 5, are also in excellent agreement for various angular locations. In contrast, the calculated velocity profiles of van Dommelen and Shen at $t = 2.98375$ show differences from the present results obtained at $t = 2.9875$. The figure confirms the expected close agreement of the two sets of results at the two smallest angular locations, but significant differences at the two highest values. The trend is different in that the present results show that the location and the magnitude of the maximum negative velocity increases with angular location. Also the tendency for flattening of the velocity profiles in the vicinity of the singularity is not confirmed by the present results.

Figure 7 shows the velocity profiles obtained by Cebeci for two values of $t$ as a function of angular location. It is clear that the magnitude of negative velocity increases with angular location and suggests that as the singularity is approached, the magnitude of the negative velocity will tend to infinity.

Figure 8 allows comparison of the displacement velocities obtained by Cowley and by the present method for four values of time. We would expect, from the previous comparisons that the two sets of results would be in close accord at least for times up to 2.75. The figure shows the expected close agreement until the maximum value is approached. The discrepancies apparent at higher values of $t$ cannot readily be explained, and it should be noted that the location and time of singularity occur at different values of $t$; the results of Cowley and van Dommelen and Shen appear to agree in this respect. The reasons for these discrepancies are presently under investigation.

4.0 Effect of Interaction on the Singularity

The interaction procedure discussed in Section 3 has been applied to the flow problem previously examined in Section 4 with the standard method. The results are shown in Fig. 9 and are discussed below. In contrast to the standard problem which makes the implicit assumption of infinite Reynolds number, the interaction requires the specification of a finite Reynolds number. In addition, a thickness ratio $\tau$ has to be specified and, since the definition of $\tau$ involves $R$ and $t$, the calculations are performed for a specific value of $\tau$ (4.5 x $10^3$) for the present results. Other values of $\tau$ are being examined to determine the effect of combinations of Reynolds number and thickness ratio.

The present calculations were performed in the following way. For all values of time with $\tau t_0$ ranging from 0 to 360°, the standard method with the leading-edge region procedure of Section 3 was used to generate the initial conditions at a short distance from the leading edge, $\tau = 0.5$. With these initial conditions and for each value of $\tau t_0$, the inverse method was used to calculate the unsteady flow from $\tau = 0.5$ to $\tau = 5.5$, for the specified value of $\tau$. Since the system of equations is now elliptic, several sweeps in the $\tau$-direction were necessary to achieve a converged solution. Where flow reversal was encountered, as happens for values of $\tau t_0 > 270°$ and $\tau > 2$, up to three sweeps are required; where separation did not exist, a single sweep was sufficient. It is to be expected that the value of $\tau$ will influence the number of sweeps and, since it is linked to physical parameters, will affect the singularity and the size of the bubble.

Figure 9a shows the variation of displacement thickness $\tau^*$ and Fig. 9b the wall shear parameter $f^*$ as a function of nondimensional distance $\tau$ and time. It is evident that for values of $\tau = 2.5$, the solutions are well behaved. As expected, the displacement thickness $\tau^*$ increases with $\tau$ for all values of time and reaches a maximum around $\tau t_0 = 300°$ as a consequence of the change in the angle of attack. In the same range of $\tau$, the shear stress parameter decreases for all values of $\tau t_0$ and reaches a minimum corresponding to the maximum in displacement thickness.

For values of $\tau = 2.5$, the solutions remain well behaved until around $\tau t_0 = 290°$. The general trends are in accord with the expectations and there is negligible difference between the results obtained with the standard and interactive methods for values of $\tau t_0$ up to the maximum for which the standard method allowed solutions. The wiggles apparent in the solutions for high values of $\tau t_0$ remain to be explained. In particular, the influence of numerical parameters such as $\tau$ and $\tau(t)$ spacing together with the assigned value of $\tau$ need to be systematically explored. Nevertheless, it is important to note that the singularity no longer
Fig. 8. Comparison between the displacement velocity values obtained by Cowley (solid lines) and by the present method (dashed lines) for the circular cylinder. (a) t = 2.5, (b) t = 2.6, (c) t = 2.7, (d) t = 2.8.

Fig. 9. Effect of interaction on the variation of (a) displacement thickness $\delta^*$, (b) wall shear parameter $f^*$ for an oscillating airfoil with $f = 4.5 \times 10^{-3}$. Solid lines in the insert represent the results obtained by the standard method and dashed lines those by the inverse method.

exists and, in contrast to the standard method which allowed the solutions to be performed up to $t = 208.85^\circ$, the calculations with the inverse method were performed for a complete cycle. It is also important to note that the chosen value of $\phi$ implies a high Reynolds number ($10^6$) for a thickness ratio of $\sigma = 0.1$. Since typical leading-edge bubbles are associated with lower values of Reynolds number, we expect that interactive calculations can be performed over a wide range of angle of attack without problems associated with the singularity.

4.0 Acknowledgment

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7.0 References


Abstract

Steady and unsteady velocity components over a backward-facing circular arc are measured by Laser-Doppler Velocimetry. A periodic disturbance is added to the mean flow and the response of unsteady separation is investigated. Special attention is given to the distribution and the flux of vorticity.

1. Introduction

Aerodynamicists are interested in the phenomenon of separation because it controls the behavior of separated flows and therefore the loading of airfoils and phenomena like steady or unsteady stall. The study of unsteady separation has generated some controversies, mostly because of the widely varied tools of investigation and quite often because of confusing terminology. The significance of the phenomenon lies in its interaction with the entire flow field. Its local properties may explain the detailed structure of the flow but these should only be viewed as a step towards the understanding and eventually prediction of the global effects. In this paper we discuss our initial experimental efforts in this direction. We study how the vorticity generated in the boundary layer is convected and shed in the free stream. Moreover, we are interested in the interaction of the separated free-shear layer with the dead water and the rigid wall.

Experimental work on unsteady separation has been initiated in the past few decades following the pioneering work of Sears, "Moore" and "Gott". The experimental work of Vidal and Ludwig shows only to steady flow, and the work of Desbard and Miller is confined to high frequency oscillatory flow. Our knowledge at the beginning of the 1970's about this complex phenomenon was therefore remarkably narrow and seriously in need of more intensive experimental investigation. Some progress was achieved by methods of flow visualization and by hot-wire anemometer techniques. Such methods have already been employed by Schnaub et al., "Heller" and "Rutter". "Nayfeh" and "Felger" and "McRury" to study unsteady viscous flow phenomena, but the specific cases considered and the scale of the models were designed for a study of the whole flow field. In all those studies the boundary layers were so thin that it was essentially impossible to study the features of the unsteady boundary-layer and in particular, separation. The most recent effort "Carr" and "McRury" employed a variety of sensing devices, ranging from flow visualization methods (e.g., Schlieren) to pressure or velocity measuring methods (dial gauges, hot wire anemometers, etc.) to study the phenomenon of unsteady stall. In this study pressure and velocity measurements were carried out at various stations on the airfoil model and compared with flow visualization data.

The present research was initiated more closely the local behavior of unsteady separation. In the first phase of the work "Gilpin" and "Venturini", "Gilpin" and "Zanetti" emphasis was given to the visualization aspects of the flow. This was accomplished by flow visualization methods and water-glycerin mixtures to achieve thick boundary layers with not so small velocities. Steady flows over moving surfaces were first examined in an open channel. Unsteady effects, transient and oscillatory were later conducted in a closed water tunnel designed and constructed for this purpose, with two different systems of pressure disturbance generators.

More recently, Mezaris and Telionis obtained detailed LDA measurements over a rearward facing circular arc. In the work of Ref. 14, unsteadiness is introduced by a flap pulsating over the circular arc. The model employed in the present study is the same with the model of Mezaris and Telionis and will be described in the main body of the paper.

It is well known that the amplitude ratio is scaled for a flat plate by the frequency parameter \( k = xU/U \), where \( x \) and \( U \) are the frequency, the distance along the wall and the free stream velocity respectively (Lighthill). With increasing \( x \), or equivalently \( k \), the amplitude profiles have been proved analytically and experimentally (Telionis) to have smaller peaks as they approach the wall. The oscillatory part of the flow tends to become plug flow and the Stokes layer is confined closer and closer to the wall. This is not the case if the adverse pressure gradient increases with \( x \) (Ref. 14). It is well known that the effects being studied are far more than increases with \( x \) (Ref. 14). In fact, the data of "Venturini" indicate that as the amplitude of the outer flow.

In the present paper we introduce the disturbance in the incoming stream rather than through a pulsating flap. We then examine more carefully the flow in the vicinity of separation by a more powerful data acquisition system which permits frequency domain analysis, as well as Fourier series expansions of all variables. Moreover, the vorticity field is examined carefully. What is unique about the problem considered here is that the effects being studied are far more extensive than viscosity. It is well known that separation and the wake formation process is independent of the level of the outer velocity for a given range of the Reynolds number. In other words, as long as the geometry of the wall body and hence the potential flow does not change, the location of separation is fixed. In our case, for all steady cases corresponding to instantaneous values of the free stream the flows are identical. Instantaneous response of the wake is then the most interesting and visible effects. This is not the case when pulsating circular arcs are used, since a single flap pulsating over the incoming airfoil, introduces in single wave disturbances.
2. Facilities and the Model

To conduct the research reported in this paper, some modifications and additions were necessary. The basic task was to convert the tunnel to an unsteady water tunnel. Disturbances were introduced upstream of the settling chamber by a rotating vane (Fig. 1). For the past 5 years, our experimental work on unsteady aerodynamics was performed with a steady stream. Unsteadiness was introduced by dynamic motion of the model or part of it. Modifications were proposed and carried out to convert the tunnel to an oscillating tunnel. This, of course, required extra efforts to reduce the turbulence in the tunnel and recalibrate the facility. To control the mean flow, a rotating vane was installed immediately above the test section as shown in Fig. 1. This vane was coupled to a NELLER DC motor with variable speed control. The unit controls automatically the speed to within ±0.5% of the set value. It is also equipped with an optical encoder which can be interfaced directly with the laboratory computer.

A very significant factor in studies of unsteady aerodynamics is the amplitude of oscillation. In the knowledge of the present authors, in all facilities employing some mechanical method for control of the frequency of oscillation, the amplitude ends up being a function of the frequency. A separate system is necessary if one desires to control independently both the amplitude and the frequency of the oscillation. In our case this was accomplished by a bypass pipe and a bypass valve as shown in Fig. 1. The position of this valve controls the efficiency of the rotating vane. Charts of the performance of these controls have been included in an appendix to the report (Ref. 11).

Any periodicity externally added to a tunnel generates free-stream turbulence. Many existing unsteady flow facilities operate with turbulence levels of the order of 1%. Careful studies require much lower turbulence levels. To quiet the flow in our tunnel we studied the literature on honeycombs and screens and contacted personally an expert in the field (Nagib). As a result of our investigations, we installed in the settling chamber a second set of finer honeycombs and 3 sets of fine screens.

Typical results are shown in Tables 1 and 2, obtained with an LDV tracker and counter respectively. In this table, the tunnel speed and the bypass valve are controlled independently. It is surprising that the bypass system influences greatly the turbulence level, even though it is far upstream of the test section. We have also performed an exhaustive study of turbulence frequency spectra examples of which are included in Ref. 14.

The acceleration and deceleration of large masses of water induce fluctuations of pressure that may exceed the strength of the plexiglass structure. Moreover, the efficient operation of hydrogen bubbles requires pressure levels lower than the atmosphere. To control the pressure level in the tunnel, we have installed a water trap and piping which connects the system to a vacuum pump.

Experiments are conducted on a model similar to the model of Nagib and Telford. A consequence of the test section leads the flow between two slightly diverging flat plates to account for the two growing boundary layers. At the entrance of the convergence, the boundary layers growing on the walls of the tunnel are sucked. New boundary layers start developing at the flat portion of the model. A very short distance downstream of the
### Table 1: The effect of the bypass system and the tunnel speed on the turbulence level.

The speed of the tunnel is represented by the speed of the driving pump in control system values which correspond roughly to speeds of up to 3m/sec. In the vertical columns, Fo is the mean doppler frequency, RMS is the root mean square of the signal and TJ is the turbulence level. The by-pass opening is also marked in values of the control system. The starred rows correspond to tests with no extra honeycomb at the entrance of the test section. Data were obtained with a 3ISA tracker. This instrument displayed large discrepancies if the reading is not within a narrow band of the scale. The readings marked by a letter were repeated with a different scale and resulted in the following values of the turbulence level: (a) TJ = 0.65%, (b) RMS = 0.75 kHz, (c) TJ = 1.35%, (d) RMS = 2 kHz, (e) TJ = 0.25%, (f) RMS = 1.5 kHz, (g) TJ = 1.8%.

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<th>By Pass at 4</th>
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<td>Fo RMS TJ</td>
<td>Fo RMS TJ</td>
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*Table 2: The effect of the bypass system and the tunnel speed on the turbulence level.*
convergence, the boundary layer develops like a flat plate Blasius layer (Mazars and Tellonis). Further downstream, the top surface diverges to generate a region of adverse pressure gradient. However, the bottom plate is continued further downstream in this way the separated region is not affected by mirror image separation. It is recalled that flows about symmetric bodies, as for example a circular cylinder are controlled by the interaction of two shear layers with opposite signs of vorticity, which eventually results into periodic shedding of large scale vortices. The situation is different in the case of an airfoil at an angle of attack, whereby the separating flow over the suction side develops with little or no influence of the trailing edge vorticity. This is exactly the situation which is simulated by our \( \text{Eq} \).

Measurements are conducted on the diverging section which has the shape of a circular arc as shown schematically in Fig. 2.

![Fig. 2 The model and coordinates defining the mesh of measurements.](image)

3. Instrumentation and Data Acquisition

Our plans to measure vorticity by shifting the beams can be implemented much easier in the backward scattering mode. However, the most significant advantage here is the convenience and precision of traversing mechanisms. A special system was designed and constructed for traversing the measuring volume via mirrors. Such a system should meet some basic requirements: (i) it should allow very accurately controlled displacements; (ii) it should permit the displacement of the measuring volume in two directions, parallel and perpendicular to the flow; (iii) it should be controlled directly by the laboratory computer; (iv) it should be free of vibrations, and finally, (v) it should allow the rotation of the beams about their structure, so that components of the velocity in any direction can be measured.

The system we designed and constructed is shown schematically in Fig. 3. The path of the optics is marked by a linear traversing mechanism, which allows the entire system to move in the desired plane. The parallel beams are then reflected twice from mirrors as shown in the figure and pass through a lens to converge at the measuring volume. The upper mirror together with the lens translates in the vertical direction to facilitate motion of the measuring volume along the \( y \)-axis. Details of the mirror tower are shown in Fig. 4.

![Fig. 3 The LV System in a backward scattering mode showing the traversing mirrors.](image)

![Fig. 4 The traversing mechanism.](image)
the information is fed directly to the computer without the interference of digital to analog and analog to digital converters.

A major improvement in our experimental facilities has been the acquisition of a PDP-11 computer with all the necessary boards (D/A and A/D converters, digital or analog output, multiplexers, etc.). We have already prepared and tested software for all the necessary operations in our laboratory. Most of these tasks were performed before by an IBM 370 which of course is still available. However now we have the flexibility to control and define the way of manipulating our data in line. Moreover, we can feed in the data directly to an IBM 370 for further processing.

The laboratory computer is the heart of the entire system. It controls all other instruments. It displays the measuring values. It checks continuity of the signal. It receives the data, it manipulates them in the line and it stores them. The particular arrangement for the instant object is shown in Fig. 5, in outline. The part of the system is shown in Fig. 6. The computer executes conditional sampling of the pressure signal at a point and then orders the stepping motors to proceed to the next position. According to the specific application, the X counters are interrogated by the computer to ensure proper operation of the system. The software provides for input from encoders which would ensure that the micro-positioning commands have been executed properly. Moreover, the general condition of the tunnel will be recorded, namely speed, temperature of the helium, and frequency of the imposed oscillation.

4. Results & Conclusions

Data were obtained along a mesh aligned with the coordinate systems x-y-z [Fig. 7]. A total of 10 vertical stations were thus defined at a distance of 1 mm apart. Along each station the vertical increments varied between 0.125 mm and 1.0 mm. Data obtained in this way do not correspond to velocity profiles traditionally discussed in binary layer investigations since the plane that is not perpendicular to the surface of the body, where we found that the system x-y-z is defined [Fig. 7], is more important for the study of phenomena than those which are parallel to the separating flow shear layers. Therefore, velocity components are obtained, which allow for subsequent data in a more traditional way.
coordinate system. Raw and reduced data are presented plotted over a portion of the circular arc drawn to scale. Attention is focused on the region between \( x = 30 \) and \( x = 55 \) mm, downstream of the origin. Profiles are plotted against \( y \) coordinate stretched by a factor of 0.5 with respect to the \( x \)-coordinate. All quantities plotted are reduced by the value corresponding to the maximum at the first station. All the raw data are available on tape for any investigator who would care to request them.

The LDV signals were averaged over 10 samples to smooth out the small amounts of hydrodynamic and electronic noise. For a speed of 125,000, the velocity profiles of the \( u \)-component of the velocity are displayed in Fig. 7. It was estimated that the point of zero skin friction for this case is approximately at \( x = 55 \) mm. A good number of downstream stations were now obtained beyond separation as shown in this figure. This was possible only with shifting of the Doppler frequency which was not successful when the data of Ref. 14 were obtained.

Evidence was presented in Ref. 14 that, even for steady flow, the boundary layer remains attached for some distance beyond the point of skin friction. Our present investigation seems to corroborate this finding as documented in Fig. 8. In this figure we display the turbulence level profiles reduced by the free-stream turbulence. The turbulence level apparently stays low and in fact increases in the small recirculating region which extends from the point of zero skin friction at \( x = 55 \) mm until \( x = 100 \) mm.

Unsteady data were obtained for a driving frequency of 0.1 Hz. A signal from the rotating vanes was used to provide a trigger for ensemble averaging of the data. Information obtained in this way was stored in the form of waveforms. An example is shown in Fig. 9. These waveforms were Fourier analyzed to generate the mean value and the amplitude of all harmonics at each point in time in the form

\[
\bar{v}(t) = \sum_{n=1}^{\infty} V_n \sin(n \omega t + \phi_n)
\]

\( n \) are integers
profiles of the u-component of the velocity, mean and amplitudes respectively.

Fig. 9 Selected velocity waveforms at $x = 50\text{mm}$. Front to back correspond to $j = 9, 5.2, 4.8, 4.2, 4.0$ and $2.8\text{mm}$.

Fig. 10 Reduced mean velocity profiles at stations $x = 40$ through $60\text{mm}$.

Instantaneous profiles at each station and for any of the phase points for which data are stored have been plotted and studied. We selected to display here in Fig. 12 sixteen profiles equispaced within a period at the first upstream station, $x = 45\text{mm}$, at which the point of zero skin friction vanishes. At this station, the skin friction is always positive and its minimum value is zero. Another station of significance for the variation of zero skin friction is the location at which the skin friction is always negative but its maximum is equal to zero. This occurs approximately at the station, $x = 65\text{mm}$ [Fig. 13]. Incidentally, according to the definition of Despard and Hiller, this is the location of separation. However, this is valid for high enough frequencies for which the point of separation does not respond to the periodic fluctuations of the velocity.

To determine the normal component of the velocity, the component in a direction inclined by $30^\circ$ with respect to the $x$-axis, say $v'$, was measured. In terms of $v$ and $w'$ then the normal com-
component \( v \) can be calculated by a simple algebraic formula.

Fig. 11 Reduced velocity amplitude profiles at stations \( x = 40 \) through 300mm.

Of great interest for separating flows is the behavior of displacement thickness. In a large number of unsteady separation problems studied so far by other investigators, the separated region is confined in the axial direction. As a result, the displacement thickness goes through a maximum but then drops again. In our case the displacement thickness grows continuously with the axial distance, until it becomes meaningless to define such a quantity. The waveforms of the displacement thickness for the stations \( x = 40 \) to 300mm we show in Fig. 14. For the first station this quantity is about 130 out of phase with the free

Fig. 12 Velocity profiles at \( x = 45 \)mm for 15 values of the period.

Fig. 13 Velocity profiles at \( x = 55 \)mm for 15 values of the period.

Fig. 14 The waveforms of displacement thickness. The symbols \( \circ \), \( \bullet \), \( \ast \), \( \times \), \( \Phi \), \( \Phi \), \( \Phi \), \( \Phi \) correspond to stations 9 through 15 in this order. The period of oscillation is 5 sec.
ever, further downstream a peculiar behavior is observed. The minimum of the displacement thickness shifts upstream to about the quarter point of the period until \( x = 45 \text{mm} \). Downstream of this station the minimum and therefore the entire waveform of the almost sinusoidal signal shifts again towards the midpoint in the phase. This suggests that the station \( x = 45 \text{mm} \) is the origin of a wave-like phenomenon which propagates periodically in both directions, upstream and downstream. Incidentally, this station is the station of the uppermost position of zero skin friction. This phenomenon is more clearly illustrated in Figs. 15 and 16 where the displacement thickness is plotted versus the axial distance, with the period appearing as a parameter.

Fig. 15 Axial variation of displacement. The symbols \( \alpha, \beta, \phi, \chi, \delta \) correspond to times \( t \), \( t + \frac{1}{2} \), \( t + 1 \) through \( t + 4 \) in this order.

Fig. 16 Axial variation of displacement. The symbols \( \alpha, \beta, \phi, \chi, \delta \) correspond to times \( t \), \( t + \frac{1}{2} \), \( t + 1 \) through \( t + 4 \) in this order.

5. The Shedding of Vorticity

A classical relationship between circulation, \( \Gamma \), and vorticity, \( \omega \), dictates that

\[
\Gamma = \int \omega dA
\]

where \( A \) is the area contained by the contour of integration of the circulation integral. Howarth (see discussion in Ref. 24) has demonstrated that if the area is chosen appropriately, the rate of shedding of vorticity at separation becomes

\[
\frac{d\Gamma}{dt} = \int \omega dy
\]

which within the boundary layer approximation yields

\[
\frac{d\Gamma}{dt} = \frac{1}{2} U_e^2
\]

where \( U_e \) is the edge velocity at separation. This simple formula has been used extensively in the literature of discrete vortex dynamics. The validity of this formula and its possible improvement and extension to unsteady flow is the topic of an on-going investigation at PIP & SU.

In all the attempts to calculate the strength of the discrete vortex via Eq. (4), the position and initial velocity of the vortex are arbitrarily assigned. The present group is working on an interactive approach which will allow the calculation of this necessary information. In the present paper we have made the first steps towards measuring the flux of vorticity.

Our data have been employed so far to calculate the quantity \( u/\gamma \) which is equal to vorticity within the boundary-layer approximation. Instantaneous profiles of this quantity have been calculated and are plotted in Fig. 17 for 4 phases of the flow. A lot more phase values have been plotted but are omitted here due to space limitations. A careful inspection of Fig. 17 indicates the periodic lift-off of the shear layer. However, it is known that the second term of vorticity, the quantity \( v/\chi \) may not be negligible in the neighborhood of separation.

In the continuation of our research we plan to calculate the combined effect of the terms \( u/\gamma \) and \( v/\chi \). Moreover we intend to calculate the instantaneous moments of vorticity, namely the center of gravity of the vorticity and the vorticity flux which will generate information on the position, \( y_c \), magnitude, \( \omega_c \), and convective velocity \( U_c, \omega_c \) of a discrete vortex equivalent to the total vorticity shed by the boundary layer. These quantities can be calculated by the formulas

\[
\omega_c = \frac{1}{2} \frac{dy}{d\tau}
\]

\[
\frac{d\gamma}{d\tau} = -\frac{1}{2} U_e^2
\]

\[
\frac{d\sigma}{d\tau} = -U_e^2
\]

\[
\frac{d\omega}{d\tau} = -U_e^2
\]

\[
\frac{d\gamma}{d\tau} = -U_e^2
\]
Fig. 17. Vorticity profiles for 4 phases of the periodic oscillation. Top to bottom, $T/16$, $5T/16$, $9T/16$ and $13T/16$.

It is proposed here that the nascent vortices and their initial motion in discrete vortex dynamics should be determined in this way to eliminate the arbitrary assumption commonly employed in this theory. The present authors intend to calculate the instantaneous values of these quantities and compare with interacting boundary-layer calculations.

5. Conclusions

The data obtained so far in this investigation and presented here display some important trends in the developing of unsteady separating flows. Most interesting is the fact that the amplitude of oscillations increases further along the separating free shear layer. Vorticity is therefore pulled away from the wall but it is pulsating with a much higher amplitude than within the attached boundary layer. Work on this problem is currently being continued. More data are obtained, but the data already stored and presented here in raw form can be used to generate useful information. In an expanded version of this paper we intend to present data on the flux of vorticity and circulation and its ultimate fate once separated from the solid boundary.

Acknowledgement

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Optimal combinations of variables for maximizing airfoil.

Corroborative hotwire anemometry documented the oscillations can be four to five times greater.

Airfoil oscillation dynamics directly influenced synchronous leading and trailing edge vortices induced through unsteady flow separation. Airfoil oscillation dynamics directly influenced vortex initiation, development and traversing velocities. The results suggest the existence of optimal combinations of variables for maximizing both vortex strength and residence time over the airfoil.

Introduction

Much of the new impetus for research into unsteady separated flows has resulted from the potential use of large scale vortices for improving aerodynamic performance (McCrorey 1982, et al). The possible exploitation of large scale vortices as an additional energy source has already been demonstrated. Current research activities (Carr et al. 1977, McCrorey et al. 1975) have shown that the unsteady lift and moment coefficients produced via dynamic airfoil oscillations can be four to five times greater than steady state counterparts.

However, before a realistic utilization of such unsteady vertical structures is possible, many fundamental questions regarding the physics of vortex development in a temporally dependent flow field must be answered. The dependence of vortex initiation, development, strength and interaction with the airfoil are required as a function of airfoil geometry and oscillation dynamics in order to determine both the physics of development as well as defining the control and repeatability of vortex production. The present study focuses on the relations between the forcing variables (airfoil geometry and oscillation dynamics) and the resultant vortex developmental behavior.

Methods

Experiments were conducted on an NACA 0012, NACA 0015 and flat plate in the low turbulence 2' x 2' wind tunnel at the University of Colorado. Flow field measurements were obtained via flow visualization in conjunction with hotwire anemometry.

A solid aluminum NACA 0012 with 10" chord, a hollow-core aluminum NACA 0015 with six inch chord, and a 6-inch and 12-inch chord by 0.25 inch thick solid aluminum flat plate were used for the experiment. Stiffeners were placed along the span of the 12" flat plate to minimize flexing of the plate during rapid oscillation. The airfoils were driven with a 1/3 H.P. D.C. motor using a 6 to 1 gear reduction connected to a variable displacement scotch yoke. Sinusoidal oscillation rate could be held constant while the magnitude of the oscillation angle was changed by a radial bearing adjustment on the fly wheel of the D.C. motor. A rotating potentiometer on the fly wheel was used to determine the oscillation period and specific airfoil attitude during any portion of the rotation cycle.

Flow visualization was obtained using a smoke rake constructed of a NACA 0015 airfoil with 1/8-inch diameter tubes inserted in the trailing edge. The smoke rake was located in the settling chamber of the 2' x 2' wind tunnel in order to minimize disturbances to the test section. The rake could be moved vertically to optimize the position of the smoke lines. Dense smoke generated from heated Rosco fog juice was stored in a 55-gallon drum and delivered to the rake at modest pressure. The pressure and ensuing flow rates were adjusted to prevent smoke from being emitted as a turbulent jet.

Synchronization of data acquisition with the airfoil angle during oscillation was accomplished using a 0 to 5 volt ramp output from the potentiometer on the fly wheel which corresponded to the 0 to 2 oscillation of the airfoil. Voltage discrimination levels from 3 to 5 volts could be preset on the electronic trigger. When the selected voltage level was reached, a pulse was generated to trigger both the stroboscopic lamp for flow visualization and LSI 11/23 microprocessor for velocity measurements. All angle-dependent data collection could be synchronized with the proper phase angle of the airfoil. Synchronization signals were checked regularly by stroboscopic examination of airfoil position relative to a fixed micrometer scale.

Single and multiple phase locked stroboscopic (7u sec duration, point source) pictures were taken of the flow field using a 35 mm SLR camera and ASA 400 TR-X film developed at ASA 800. The phase-locked multiple exposure photographs highlighted the repetitiveness of...
flow field disturbances.

Velocity measurements were made using a conventional constant temperature 2-needle hotwire probe constructed of 0.0001-inch Wollaston wire. Using various linearizing circuits referenced elsewhere (Francis, et al., 1978), a 0 to 5 volt, full scale output was preset for the velocity of the mean flow. The hotwire probe was mounted on an orthogonally driven traverse mechanism in the wind tunnel test section.

Hotwire data acquisition and subsequent reduction were accomplished with an LSI 11/23 microprocessor. Upon receipt of the phase-locked trigger signal, the analog signal was digitized and stored at a sampling frequency of 1000 Hz. Data were collected over two complete oscillation cycles for each data sample and ten successive data samples runs were averaged.

RESULTS

The unsteady flow field produced by an airfoil driven through sinusoidal pitch oscillations was extremely complex due to the temporal and spatial interdependence on oscillation dynamics. Changes in any independent parameters altered the flow field. Such complexity did not prohibit highly reproducible flow structure synchronized with the airfoil oscillation. The most prominent structures were the (1) initiation and growth of a leading edge vortex, (2) the interaction between the leading edge vortex and the upper airfoil surface, and (3) the development of a trailing edge vortex. These structures were stable enough to permit repeatable visualizations using multiple exposure photographs and the analog averaged velocity histories using hot-wire anemometry when both were synchronized with specific phase angles of the oscillation cycle. The initiation, development, and shedding characteristics of these flow structures were studied in detail.

The general patterns of vortex formation remained quite constant over the extensive range of test conditions and airfoil geometries examined. Figure 1 depicts the vortex formation over an oscillating flat plate. At the flat plate approach the maximum angle of attack, a small structure was visualized about the flat plate leading edge at approximately 0.2 chord (Fig. 1A). Further into the oscillation cycle, this small structure rapidly grew in size and was readily identified as a leading edge vortex. Superimposed smokelines from successive phase-locked, multiple exposure photographs attested to the reproducible appearance and growth of this leading edge vortex. The presence of the leading edge vortex over the airfoil surface reattached what otherwise would have been separated flow under static test conditions. As the clockwise rotating vortex was about to shed into the wake (Fig. 1B), a vortex exhibiting counterclockwise circulation was initiated around the trailing edge from beneath to above the airfoil. Whereas, the presence of the leading edge vortex had reattached separated flow, the initiation and rapid growth of the trailing edge vortex resulted in complete 'cataclysmic' flow separation from the airfoil leading edge (Fig. 1C). It is evident near the airfoil leading edge where turbulent separation had occurred.

VORTEX STRUCTURE

Flow visualization and hotwire anemometry provided insight into the structure of the leading edge vortex. Multiple-exposure visualizations of a single vortex initiated over a flat plate Fig. 1A indicate an almost laminar

Fig. 1 - Vortex development over an oscillating 12° flat plate; Re 87,300; α = 15° ± 5°cos (T); osc pt. 0.75 c; K = 1.0; A-J correspond to 1/10 increments of the oscillation cycle.
rotation of outlying streamlines down and around a turbulent vortex circumference. The absence of smoke from the vortex core created a striking contrast between the turbulent vortex circumference and the laminar rotation induced in the potential field. The visualizations suggested the existence of a strong shear layer between the inner core and the vortex circumference. In addition, growth of the vortex from a separation tongue (see Freymuth, these proceedings) emanating from the boundary layer would have little smoke. Diffusion at the turbulent circumference of the vortex into the potential field suggests that a decrease in the induced velocity would correspond to an increase in radius away from the vortex core.

Anemometric measurements which give a time-dependent profile of the velocities of the vortex substantiated visualizations. A hot-wire probe positioned at various chord locations above and normal to the airfoil surface yielded velocity profiles at each location over successive oscillation cycles. Two representative cycles are indicated in the data of Fig. 2. The velocity perturbations increased as the probe was immersed first in the potential flow field (Fig. 2A) and later in the vortex. A peak velocity maximum was obtained with the probe located tangent to the vortex circumference (Fig. 2B). Further movement toward the vortex center produced a velocity minimum. The probe recorded the passage of the high velocity region of the vortex circumference and entered the slow moving inner core. The two maxima on either side of the velocity minima indicated the passage of the vortex as the probe entered and exited the inner core. Though only two oscillation cycles are shown, profiles at any location were quite reproducible when phase-locked to airfoil oscillation.

Multiple exposure photographs in Fig. 2 indicate the relative positions of the hot-wire probe and vortex which produced velocity maxima and minima peaks. In the far field, outside the vortex core (Fig. 2A), the velocity maxima occurred with the probe tangent to the vortex diameter. Similarly, inside the vortex core (Fig. 2E), the minima between the two velocity maxima was observed with the probe again tangent to the diameter and on a perpendicular bisector through the vortex center. A plot of these instantaneous peak velocities along this bisector line (Fig. 2A) shows the instantaneous velocity field induced by the passing vortex. Inside the core (Fig. 2A, 0.0 to 0.15c), the vortex behaves as a solid body rotation. Outside the viscous core, the velocity induced in the potential field diminishes as $1/R$ with $R$ being the effective vortex radius. This result agrees with the classic solutions derived by Oseen (1911) and Hamel (1916) as reviewed by Schlichting (1979) for the velocities induced by a vortex filament as a function of radial distance.

Repeated measurements across most test conditions and airfoil geometries indicated similar structures for any leading edge vortex. This similarity permitted a characterization to be made of the leading edge vortices based upon relative density and circulation velocity. The circumference diameter and the peak velocity obtained there were selected as measures to be used across various test conditions. These
MEAN ANGLE OF ATTACK

Mean angle of attack effects on vortex development; 6° NACA 3015 airfoil; Re 60,000; $\theta = 15^\circ$; osc. pt. 0.25c; $K = 0.5$; $F = 1$ correspond to mean angles of 0, 5, 10, 20 and 30 degrees respectively.

To create a leading edge vortex, it was necessary to oscillate airfoils or plates such that the critical stall angle was exceeded during some portion of the oscillation cycle. Thus, the static stall angle provided a good reference angle for determining whether or not a leading edge vortex would be produced. Figure 3 shows the dependence of vortex development on mean angle of attack. For $\alpha_m$ up to $5^\circ$ (Fig. 3 F = 1), the multiple exposure photographs indicated no leading edge vortices had been generated. A further increase in mean angle (Fig. 3 H, $\alpha_m = 10^\circ$) produced small leading and trailing edge vortices most evident in the wake. Leading edge vortex diameter increased 25% as the mean angle of attack was increased from $10^\circ$ to $15^\circ$. For $\alpha_m$ increments between $15^\circ$ to $30^\circ$, vortex size remained unchanged.

Vortex characteristics became quite sensitive to other test parameters at higher mean angles ($\alpha > 20^\circ$). With an oscillation angle of $\pm 5^\circ$ at a reduced frequency of $K = 0.25$, the leading edge vortex separated from the airfoil at approximately 30% chord. The detachment was not totally dependent upon the mean angle of attack. Duplication of the same geometric conditions but at a higher oscillation rate ($K = 1.0$) produced a vortex which remained attached shedding into the wake only at the airfoil trailing edge.

OSCILLATION ANGLE

The magnitude of the oscillation angle provided another reference condition for the production of synchronous vortices. Small oscillation angles ($K \leq 0.5$) at rapid oscillation rates ($K \geq 2.0$) were shown to effectively attach modestly separated flow. Instantaneous ($\tau_0$ sec duration) single exposure photographs documented the presence of vortex development similar to that shown in Fig. 1. Multiple exposure phase-locked photographs, however, showed no spatially and temporally synchronous vortices. Oscillation angles up to $\alpha_m = 3$ at $K$ values $0.25$ similarly produced little evidence of synchronous flow structures. However, further increases in either the reduced frequency parameter or oscillation amplitude produced repeatable disturbances. Hence, a threshold condition existed requiring a combination of dynamic parameters of sufficient strength to elicit repeatable flow structure.

REDUCED FREQUENCY

The reduced frequency parameter, oscillation angles, and oscillation axes directly affected the size, velocity profile and repeatability of vortex formation. Of these dynamic parameters, the reduced frequency ($K$) provided the most predictable alteration of the flow fields. From a series of multiple exposure photographs (12/cycle), the location of the vortex center was tracked as a function of time and fraction of the oscillation cycle. Across the Reynolds numbers ($60,000 - 140,000$) and oscillation angles ($3-5^\circ$) examined (Fig. 4), $K$ provided an accurate index of the vortex convection velocity. The straight line drawn through the data points established the average traversing speed across the airfoil at 30 to 40% of the free stream velocity.

The individual vortex positions ($Re = 60,000$, $K = 0.5$, and $K = 0.75$) reveal instances of momentary delays. These delays were most prominent as the airfoil reversed direction from the maximum angle of attack and pitched downward. The characteristic circulation velocity of the vortex circumference showed that, maximum delays were obtained from vortices exhibiting the
largest reference profile velocity maxima. As the airfoil approached the lowest positions, a rapid acceleration in traversing velocity ensued. Though the reduced frequency was a good measure of the average vortex position, it did not adequately predict either vortex delay or acceleration characteristics. All dynamic parameters as well as airfoil geometry were observed to affect vortex motion.

The flow field structures related to K changes ranged from weak, poorly synchronized vortices at low K values (Fig. 5A K<0.25), to a succession of compact, intense, and closely spaced leading edge vortices well synchronized to airfoil oscillation at high K values (Fig. 5E K>1.75).

At K values greater than 1.5, significant alterations in the developmental pattern of trailing edge vorticity occurred. The presence of multiple leading edge vortices over the airfoil surface displaced the formation of trailing edge vortices into the wake. The trailing edge vortex which had produced complete flow separation over the airfoil at lower K values (K < 1.25) generated no observable separation at high K values (Fig. 5 D & E). Thus, attached flow over the airfoil resulted throughout the entire oscillation cycle at elevated values of K > 1.5 across the other conditions tested.

Velocity measurements at three chord locations over the airfoil (Fig. 6) indicated a peak circulation, velocity increase of 16% as the reduced frequency was changed from 0.25 to 0.5. At these low K values, the vortex diameter changed slightly across test conditions with larger diameters occurring at a Reynolds number of 140,000. Both the characteristic circulation velocity and diameter were combined to obtain a relative circulation index. Though the characteristic maximum vortex velocity decreased in almost linear fashion across the airfoil chord, vortex diameter increased rapidly from 0.25 chord to the trailing edge. The net effect was a continual increase in relative circulation as the vortex passed from the leading to the trailing edge.

**OSCILLATION AXIS**

Similar to the reduced frequency parameter, the oscillation axis location directly affected the velocity magnitudes and repeatability of vortex development. In order to minimize the effects of airfoil geometry, a flat plate was oscillated about two different axis locations; 0.25 and 0.75 C. Figure 7 contrasts the vortex development between the two oscillation positions for otherwise duplicate test conditions. The diffuse smokelines in Fig. 7, plates A - E do not exhibit the highly repeatable characteristics observed with oscillations about 0.75C. Both plates A and F were taken at the same position in the oscillation cycle, T = 2 or the maximum angle of attack 20°. The leading edge vortex in plate A (oscillation axis 0.25C) appears much larger and further down the chord than in plate F (oscillation axis 0.75C). A better match between conditions based upon vortex diameter and placement over the airfoil occurs between A and G at 20° further into the oscillation cycle. This 20° phase shift produced a vortex size and location match plate for plate throughout the entire oscillation cycle. Changing the oscillation axis from 0.25C to 0.75C would appear to have delayed the development of the leading edge vortex by 20° of the oscillation period. A detailed analysis of vortex initiation and traversing velocity as a function of the dynamic parameters is presented later.

Velocity measurements (Fig. 6) showed dynamic alterations in both the circulation
Fig. 7 - Oscillation axes effect on vortex development; 12" flat plate; Re 70,000; \( \pm 15^\circ \pm 5^\circ \) osc. pt.; K 2.5; A-E and F-J correspond to 1/5 increments of the oscillation cycle velocities and vortex diameters with changes in oscillation axes and K. An average circulation velocity increase of 15% was obtained for reduced frequencies of 1.0, 1.5, and 2.5 by relocating the oscillation axis from 0.25 to 0.75 C. The vortex diameter decreased an average value of 40% between K values of 2.0 and 2.5 for a fixed oscillation position. Also, average vortex diameters decreased 40% to 50% with elevated values of K. Whereas the magnitude of the circulation velocities were directly proportional to increases in the oscillation axis and K, the size of the vortex diameter was inversely related.

A direct correlation existed between the magnitude of the circulation velocity and the repeatability of vortex generation. Flow visualizations which produced the most concise, temporally repeatable vortices also possessed the strongest circulation velocities. The magnitude of circulation velocity and vortex diameter to obtain relative circulation produced results similar to those obtained with the NACA 0012 airfoil (Fig. 6). The average circulation of the leading edge vortex increased as the vortex passed from the leading to the trailing edge of the flat plate used in these tests.

Sinusoidal oscillations of the flat plate about 0.75 C at elevated values of K (x 1.5) failed to produce the linear decay in velocity across the airfoil chord. A 25% decrease in velocity occurred from the leading edge to 0.25 C. A linear increase in vortex diameter was also indicated. The decrease was followed by a rapid acceleration of the circulation velocity at 0.5 C with a corresponding decrease in vortex diameter. Velocity decay commenced again from 0.7 C to the trailing edge. The velocity reduction followed by the rapid acceleration of the vortex over midchord was commensurate with the formation of a second vortex over the leading edge. Thus, it appears that strong interaction between successively generated vortices at high K values altered the normal vortex velocity decays.

Airfoil geometry altered the development of both leading and trailing edge vortices. Under static conditions, the flat plate exhibited flow separation at slightly lower mean angles of attack (x 10°) than the NACA 0012 and 0015 airfoils (x 9.3° & 11.4°, respectively). With an oscillation angle fixed at x 45°, the flat plate produced leading edge vortices at lower mean angles of attack than the two conventional airfoils. This result was not unexpected as it had been shown earlier that the static stall angle had to be exceeded in order to generate a leading edge vortex.

Under duplicate test conditions, the sharp leading edge of the oscillating flat plate produced better temporal and spatial, coherence than either the NACA 0012 and 0015 airfoils. It
least part of the key to this difference appeared to be the sharp leading edge of the oscillating flat plate. When either the NACA 0012 or 0015 airfoils were rotated 180° and oscillated about 0.75C with the sharp trailing edge forward, the leading edge vortices produced were qualitatively the same as those generated from a flat plate oscillated about 0.75C. The most intense, cohesive, and repeatable vortex structures were observed from the flat plate and reversed NACA 0012 and 0015 airfoils oscillated about 0.75C at the maximum limiting frequency of the oscillatory mechanism (-15 Hz).

Significant differences were observed, however, in trailing edge vortex development between the flat plate and reversed NACA airfoils. Passage of the leading edge vortex over the rounded trailing edge of the NACA 0012 airfoil always triggered a trailing edge vortex independent of reduced frequency. In contrast, the sharp trailing edge of both the flat plate and a conventionally oriented NACA 0012 airfoil produced trailing edge vortices at different positions in the oscillation cycle, relatively independent of the leading edge vortex location. For either the normal or reversed geometry, development of the trailing edge vortex at K ≤ 1.25 produced flow separation whereas K values in excess of 1.5 resulted in attached flow through the entire oscillation cycle.

**VORTEX INITIATION**

The identification of leading edge vortex initiation using flow visualization and hot-wire anemometry presents a unique problem. Though flow visualization may not necessarily indicate the initial build-up of vorticity, it is crucial in identifying a developed vortex structure. All test conditions which produced synchronized leading edge vortices revealed fully developed vortices by 0.1 to 0.2 chord. For analysis purposes, therefore, the 0.1 chord location of the leading edge vortex was selected as a spatial reference for vortex initiation. The composite Figure 9 cites the contribution of oscillation angle (1/2 C, mean angle (1), and the reduced frequency parameter (K) to the initiation of a leading edge vortex relative to the oscillation cycle. All data points were obtained from flow visualization and hot-wire anemometry using a 12 in. C flat plate and reversed NACA 0015 airfoil with 0 in. C. Oscillation of both devices was about 0.75C at K = 0.75C.

In Figure 9, it is clear that with low values of 1/2 C, a vortex formed during the upstroke of the oscillation cycle and increasing values of 1/2 C delayed the appearance of a vortex until or even beyond 2.0C. It is notable that this relation was asymptotic, further excursions beyond 1/2 C provided little additional delay in vortex initiation.

Also in Figure 9, increasing 1/2 C were correlated with the increasingly early appearance of vortices. At 1/2 C of 10°, a leading edge vortex was identified over 0.25 chord when both test devices were well into the downstream portion of the oscillatory cycle. As 1/2 C approached 20°, the vortices appeared during the upstroke portion. Hence, the effects of 1/2 C are opposite of each other with both generating asymptotic limits. Somewhat surprisingly, l seems a somewhat less effective variable than 1/2 C across the values tested.

Variations in reduced frequency parameter also produced the large alterations in vortex initiation. The index of initiation shows that increases in K delayed the appearance of leading edge vortices in almost linear fashion. The delay increased through K values of 1.25 with more modest delays occurring at higher K values.

**TRAVERSING OR CONVECTING VELOCITY**

It was noted earlier (Fig. 4) that momentary delays occurred in the traversing motion of the leading edge vortex passing from the leading to the trailing edge. Hot-wire measurements at 0.1 C increments over an oscillating flat plate for two different oscillating axes (0.5 and 0.75C) and three different values of K (1.0, 1.5, and 2.5) showed similar results (Fig. 10). The leading edge vortex did not traverse the airfoil surface with a uniform velocity after initiation. Vortices accelerated over the airfoil surface at different rates dependent upon the test conditions.

Plotting the vortex position as a function of the oscillation cycle may distort the dependence of vortex development on airfoil oscillation. When the same position data were plotted in real time (Fig. 11), the dynamics of vortex initiation and traversing velocity were observed independent of oscillation dynamics. The time required to complete one oscillation cycle for each of the three K values is indicated above the time scale in Fig. 11. Though the oscillation rate differed by a factor of 2.5, vortex initiation and position as a function of real time remained quite similar across conditions. In contrast, Fig. 10 shows the position data to oscillation cycle phases. The actual dependence of vortex initiation and position on real time seem to indicate a characteristic initiation and development time that is somewhat independent of the airfoil oscillation dynamics.

Other characteristics of vortex initiation and position, however, were observed to be dependent upon the airfoil oscillation dynamics. Regardless of oscillation axes (Fig. 11), increases in the reduced frequency parameter resulted in vortex traversing velocity
FIG. 10 - Vortex position as a function of oscillation cycle; 12" flat plate; Re 70,000; i > 15° ± 5°; osc. pt. 0.25 and 0.75 c.

FIG. 11 - Vortex position as a function of time; 12" flat plate; Re 70,000; i > 15° ± 5°; osc. pt. 0.25 and 0.75 c.

DISCUSSION

The observations made in the previously described tests may be organized into two general categories: (1) initiation, development and convection velocities of vortices and (2) inherent vortex characteristics. As will be noted later, vortex characteristics appear to relate to the convection velocities.

INITIATION:

Assuming that the static stall angle is exceeded at some time in the sinusoidal oscillation cycle, a, increments are related directly to earlier vortex occurrence during upward pitching of the lifting surface: the larger the a, the earlier in the cycle a vortex is initiated. Both a and K value increments delay the appearance of a vortex to later portions of the oscillation cycle. When the oscillation axis is moved back from 0.25 to 0.75 chord locations, vortex initiation is similarly delayed. Thus, earlier vortex initiation derives from an stall while later vortex initiation derives from any test condition which increases a values. Across the tests done in the present series of studies, vortex appearance occurred as early as midway through the upward pitching and as late as midway through the downward pitching of the oscillation cycle.

TRAVELING OR CONVECTING VELOCITY:

The traversing velocity (V/Vu) of the leading edge vortex was affected in a manner analogous to the initiation. Increments in both the reduced frequency parameter (K) and oscillation amplitude increased the vortex traversing velocity. Over the conditions tested, increased K resulted in linear changes, but oscillation amplitude produced velocities which approached an asymptotic limit of 0.4 V0 for Kr 2 6°. A similar asymptotic condition probably exists at higher reduced frequencies as well; for it is doubtful that the traversing velocity would exceed the local free stream velocity value. The traversing velocity was inversely related to increments in the mean angle of attack. A decrease of 33% in traversing velocity resulted from increasing the mean angle of attack from 10° to 30° for both the flat plate and reversed NACA 0015 airfoil. Hence, it appears that the reduction in vortex traversing can be directly attributed to an airfoil blockage effect. Conditions which produced greater
blockage of the free stream flow (greater \(k_0\)) for longer periods of time (reduced \(K\) values), resulted in slower traversing velocities. Vortices developing in the shadowed wake of the airfoil remained for longer periods over the airfoil.

**Relative Circulation:**

The size, strength, and repeatability of leading edge vortices were directly altered by changes in the oscillation axes and varying the oscillation rate. Higher peak circulation velocities with simultaneous decreases in vortex diameters were realized with either increased reduced frequency parameter or rearward movements of the oscillation axes. Through multiple exposure flow visualization and ensemble averaged hotwire signals, the concise, energetic and extremely coherent vortices (axis 0.75C, \(K > 1.0\)) were repeatable to within ±5% variation of induced velocity over 30 consecutive oscillation cycles.

When the peak velocity was combined with the corresponding vortex diameter, the relative circulation estimate remained relatively unchanged across conditions. The initial formation (vortex over 0.2 chord) value of the circulation appeared constant across \(K\) values and oscillation positions for the flat plate. As the vortex traversed the airfoil surface from leading to trailing edge, the relative circulation increased by a factor of 2.5. Hence, the vorticity appeared to build with time over the airfoil rather than decay with time after initiation. The magnitude of the relative circulation was inversely related to the oscillation rate. Though the peak velocity increased with \(K\), the decrease in the vortex diameter resulted in an overall decrease in the relative circulation at higher \(K\) values. For the flat plate oscillating at increased \(k_0\) values, approximately 25% as the reduced frequency was increased from 1.0 to 2.5. Although each vortex possessed a reduced circulation at higher \(K\) values, the total overall vortex influence about the airfoil was higher since multiple vortices resided over the airfoil at all times during an oscillation cycle.

The "temporal" component of this sinusoidally forced unsteady flow seems to provide a common thread between vortex initiation, size, strength and traversing velocity. Increasing the reduced frequency parameter decreases the amount of time available for vortex initiation and development. Since the initial formation of a leading-edge vortex is delayed at increased \(K\) values, a characteristic time of vortex initiation and formation was suggested and must be taken. Similarly, these high oscillation rates produce initial vortices with relatively high circulation velocities but smaller diameters and relatively constant circulation indices. These concise, energetic vortices tend to move across the airfoil at significantly higher traversing speeds. A characteristic development time as well as a vortex-vortex interaction was suggested. Lower reduced frequencies permit the airfoil to remain at increased angles of attack for longer periods of time. The duration of this blockage effect as described previously, directly affected vortex size and traversing speeds. Lower reduced frequencies permitted a greater period of time for the vortex to grow in size and remain over the airfoil surface in the wake region of the blocked flow. Since the relative circulation indices were quite constant, higher reduced frequencies resulted in vortices with higher circulation velocities and smaller diameters. Less time was available for the development of the vortex in the shadowed wake; hence, the vortices remained concentrated.

**Conclusion**

Harmonic oscillation of airfoils at angles of attack in excess of static stall were observed to produce complex interactive flow field structures. The most predominate of these were the formation of both a leading and trailing vortex which were temporally and spatially dependent upon the airfoil oscillation dynamics. These structures were sufficiently reproducible to permit multiple exposure flow visualizations and ensemble averaged hotwire anemometer profiles to be made phase locked to the airfoil oscillation. The presence of the leading edge vortex was observed to reattach otherwise separated flow at angles of attack which produced complete flow separation under non-oscillating conditions.

The dynamics of the airfoil oscillation directly influenced vortex initiation, development and traversing velocity. The oscillation rate (reduced frequency), mean angle of attack, oscillation angle, oscillation axes as well as the airfoil geometry were shown to directly alter the vortex circulation velocity and size, yet, left the relative circulation index quite constant. Optimal selection of the airfoil geometry and oscillatory parameters should permit maximum lift enhancement through increasing vortex residence time and circulation velocity over the airfoil surface. Much additional work remains to be done in two areas: (1) measuring the total vorticity field during airfoil oscillation and (2) examining the interaction of multiple vortices over the airfoil at elevated values of \(k_0\). Both these approaches have readily apparent explanatory and technical exploitation values.
BIBLIOGRAPHY


Abstract

The novel lift generation mechanism postulated by Weis-Fogh (1973) and evaluated by Lighthill (1973) was redefined and extended in terms of unsteady separated flows as described by Weis-Fogh. Other means of exploiting unsteady separated flows may exist also within the insect world. Many of these possibilities remain to be examined and to be analyzed in regard to generation, control and use.

Introduction

The publication of two papers concerned with hovering in the Chalcid wasp, Enoclerus fasciatus (Weis-Fogh, 1973; Lighthill, 1973) redefined a renewed interest in biological models of flight. Weis-Fogh had suggested the existence of a novel lift generation mechanism to explain hovering in the Chalcid wasp as well as other insects. Lighthill showed that such a novel mechanism could, in principle, account for the required level of lift generation during hovering. More recently, Maxworthy (1979) has shown that the postulated lift generation mechanism is in fact attained by a single, physical model. The insect's wing posture is crucial, and this posture is in turn crucial to the mechanism. The present report concerns the existence of such a novel mechanism as well as a number of other possibilities.

The flight nodes of a dragonfly include (1) lift generation and use by insects 
M. V. Lutgers, C. Somps, M. Kliss, and M. Robinson
Aerospace Engineering Sciences
University of Colorado
Boulder, Colorado 80309

UNSTEADY SEPARATED FLOWS: GENERATION AND USE BY INSECTS

Abstract

The novel lift generation mechanism postulated by Weis-Fogh (1973) and evaluated by Lighthill (1973) was redefined and extended in terms of unsteady separated flows as described by Weis-Fogh. Other means of exploiting unsteady separated flows may exist also within the insect world. Many of these possibilities remain to be examined and to be analyzed in regard to generation, control and use.

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Literature Review

The present report summarizes flight mechanisms in dragonflies that appear to exploit unsteady flows to achieve rather remarkable aerodynamics. We envision that such studies may provide a means of identifying crucial combinations of unsteady variables that are effective in both generating and using unsteady flows.

Dragonfly flight was studied both in unrestrained, normal specimens and in tethered, laboratory-tested specimens. In both instances, high-speed photography permitted the characterization of wing motions including stroke length, stroke angle, stroke frequency, angles of attack and phase angles for the tandem pair of wings. In the laboratory, such observations were coupled with force balance measures such that instantaneous correlations of wing motion and lift were obtained. Flow visualization also was obtained for tethered insects during elicited flight episodes. Simple oscillating plate models were used in a zero flow test to simulate at least some aspects of observed dragonfly aerodynamics.

The flight modes of a dragonfly include (1) aig-speed forward and upward maneuvers, (2) gliding or soaring maneuvers, and (3) hovering or the horizontal. Angles of attack of less than 1° and stroke frequencies of 25-36 Hz, wing stroke angles of 35° forward and 20° behind the root attachment as well as angles of 45° beneath and 60° above the horizontal, angles of attack of 1 to 50° and phase angles for the tandem wings of near 3 to 13°.

In tethered insects, a well-defined point of zero lift generation occurred once during each wing beat cycle of the tandem wings. The maximum lift generated at this point was approximately 20 and body weight. In contrast, unrestrained dragonflies routinely exhibit very stable hovering indicative of more continuous lift generation. Photographs of dragonflies exhibiting both modes of Flight revealed no remarkable changes in wing motion despite the vortices differences in lift generation.

Flow visualization in the vicinity of tethered dragonflies revealed two major types of structure. During episodes of high lift generation, smoke moves rapidly from head of the insect to form 45° angle vortices. The structure of the vortices is commonly connected vortexes that appear to stall at the wingtip in the vicinity of the stroke. In addition of more modest lift generation, the vortices exhibit large stationary vortexes surrounding the upper wing stroke quadrant.

As a working model of dragonfly flight mechanisms, summarizes the observed wing positions, angles of attack, flow fields and lift peaks we have been able to characterize. In essence, it appears that high angles of attack of the wings generate a local circulation exploitable to provide relatively high lift under hovering conditions. Angles of attack appear smaller if thrust is to be provided and if net flow is to be achieved.

In view of these observations, vertically-oriented flat plates of varying thicknesses were driven back and forth in a zero flow test condition to evaluate the effects of stroke length and stroke frequency. The results of these studies were quite clear. For very thin plates, stroke length dictated vortex size and stroke frequency had little effect other than a modest enhancement of flow structure cohesiveness. The addition of angularity to the flat plates resulted in net flow dominated by vortices. These observations are consistent with the notion that flow structure may be initiated by a wing and can be expected to persevere. This preserved local flow structure could be exploited by another wing or by the same wing on a later stroke.

Overall, these experiments indicate that unsteady flows may be used to support quite sophisticated insect flight maneuvers. No significant change in wing geometry is needed to achieve such flight and only modest alterations in dynamic wing stroke variables are required. The observations made here indicate that dragonflies use mechanisms quite different from those used by the Chalcid wasp, as described by Weis-Fogh. Other means of exploiting unsteady separated flows may exist also within the insect world. Many of these possibilities remain to be examined and to be analyzed in regard to generation, control and use.
configurations are often more simple in insects and muscle-elicited changes in configuration during various flight modes are more limited. Unlike birds, for example, many insects achieve excellent aerodynamics and maneuverability in the absence of a variety of deployable wing devices. Also, insects are able to accomplish a wide range of flight behaviors without the aid of an elaborate set of neural controls. Thus, attempts to understand biological flight are best focused upon these accomplished, but simple, flight practitioners.

Work focused upon the Chalcid wasp is limited by several considerations. First, the wasp is extremely small (≈0.5 cm) and exhibits do so, wing beat frequencies approach 300 Hz.翼

Because of these limitations with the Chalcid wasp, we have elected to study flight in the dragonfly. Weis-Fogh (1972) cited this insect as one which most probably had to utilize novel lift generation mechanisms. More recently, Horberg (1975) and Savage et al. (1979) have provided evidence that these insects produce higher amounts of lift than can be anticipated by usual steady state aerodynamic principles. Work on underlying neural control mechanisms (Pringle, 1963; Neville, 1960) indicates that the insects use central pattern generators to drive wing movements. These movements are subject to only minor (Alexander, 1982; Olberg, 1983) feedback modifications. Thus, the dragonfly appears to achieve excellent aerodynamic lift using standard steady state mechanisms and not depending upon elaborate wing control. The wing motions exhibited by dragonflies reveal highly stereotyped kinematics and fluid-wing interactions. So dedicated are the wing motions, wing musculature has but two cardinal motions, elevation and depression. Flexion and extension, characteristics of insects which retract or fold the wings when not in use, are muted from both the control and musculature of the dragonfly (Clark, 1940).

As an aside, it is notable that fossil evidence suggests that dragonflies may have survived the time of dinosaurs, essentially unchanged except for a modest increase in size. Presumably these insects when mute and eat while airborne have engaged in innumerable life and death struggles in the approximately 250 million years of existence. Obviously, those highly skilled fliers have, as a species, prevailed through this test of time. Humbled by this remarkable record of survival, we set out to investigate the inherent flight wisdom of the dragonfly.

Our studies were designed to determine (1) whether lift generation of the dragonfly is predictable. These insects inevitably depart rapidly at a 45° angle from the horizon and in the direction of body orientation. The insect net was simply moved to an intercept position. Within an hour of capture, the insects were in the laboratory being subjected to cooling (for next day testing) or to chloroform anesthetization (for handling purposes). Each insect was weighed fully measured and then tethered to a small wood beam using cyanoacrylate. A variety of tests ensued as described below.

To relate laboratory observations with natural flight behavior in the dragonfly, unrestrained flight of the dragonfly was observed in the habitat from which they were collected. Flight modes were recorded and simple aerodynamic values were estimated. In addition, the wing kinematics were recorded using telescopic photography (35 mm camera; 1/1000 shutter speed; 250 mm lens; 1000 ASA color film). Most photographs were taken as the dragonflies hovered near favorite reed-cop perch.

Lift measurements and simultaneous wing motion analyses were achieved with tethered dragonflies mounted to a one-dimensional strain-gauge force balance. The amplified force balance output was displayed on one channel of a CRT oscilloscope and a photodetector output displayed on the second channel. As the strobe (≈0.5 msec) illumination was triggered for photographing wing motion, the diode signal on the oscilloscope marked related instantaneous lift values. Alternatively, continuous Strobotach (Gen. Radio) output provided diode marking of lift values associated with videotaped, phase-related wing motions. From a large number of elicited flight episodes visualized for 21 dragonflies, it was possible to describe both the wing kinematics and the associated, phase-related lift values. In many instances, it was necessary to place a mark on the rear wings of the dragonflies to assure subsequent discrimination of the front from the rear wings. In some instances, the cyanoacrylate cement used for tethering spread to legs of the insects or to other thoracic structures. These specimens were not used since the spread might have altered flight patterns and lift generation. In general, the insects could be tested over numerous flight episodes for periods of at least two hours. Altered wing motions and decreased lift generation were used as indicators of deteriorating flight behavior.

Flow visualization was achieved during dragonfly flight episodes. Heated kerosene smoke was delivered (<10 cm sec -1 ) from a 1 cm diameter tube approximately 10 cm in front of the test specimens. The laminar smoke stream was positioned to intercept the median wing position approximately mid-span. Flow visualization was done in a 60 x 50 x 30 cm zero flow box constructed of clear east acrylic (Plexiglas). At about the middle of a flight episode lasting 3-5 sec, a stroboscopic photograph was taken. This procedure was repeated many times to produce a composite model of flow (smoke stream)-wing interactions. Based upon the above observations, an empirical model of wing motion and flow interactions was constructed to relate to the observed amount of lift generation. Also, a few simplified experiments with an oscillating flat plate were pursued to simulate some of the flow-
wing interactions observed during dragonfly testing. Finally, a novel means of testing dragonflies was developed to permit more detailed flow-wing visualizations. This novel procedure is described together with a projection of its potential in model studies of insect flight.

**Results**

Before turning our attention to the laboratory studies, a few of our observations on natural flight will be summarized (Table 1). After spending much of its development as a predatory water nymph, the dragonfly emerges as the flying adult form from a metamorphosis that takes place on a twig or reed that grows out of the water habitat of the nymph. As the wings are unfolding and drying, the adult, itself, is subject to predation by birds. Thereafter, dragonflies are not subject to heavy predation because both color and maneuverability provide excellent survival probability.

Flight in the adult dragonfly serves several biological functions and roles. During feeding, the dragonfly alternates between gliding, powered flight and aerobatics. During territory patrols, a prelude to mating, the dragonfly glides with an occasional powered wingbeat sequence to maintain altitude. In instances of startle or predation threat, episodes of escape behavior consist of powered flight upward at 45° angles mixed with dramatic aerobatics. Mating consists of male and female dragonflies flying in tandem with constant, albeit poorly coordinated, wing motion exhibited by one or both members of the union. In all of the above circumstances, periods of hovering occur. Hovering also occurs as a prelude to a dragonfly returning to a favored perch atop a reed.

Because of flight modes, the dragonfly exhibits escape, gliding and hovering. Escape consists of short duration (<5 sec) episodes of high amplitude, high frequency (<30 Hz) wingbeat cycles that propel the insect at speeds estimated at >5 m sec\(^{-1}\). Gliding is a forward, low angle flight rarely interrupted by wing motion. This mode continues for periods of 10 or more seconds. A relatively moderate speed of 1 m sec\(^{-1}\) is estimated. Hovering is stationary flight that shows remarkable spatial stability (<0.5 cm) and durations of >10 sec. Slow spatial corrections occur with movements in all planes and with high-speed movements, as well. We judged that transitions from one flight mode to another occurred in approximately 100 msec. During most flight modes, the dragonfly body remains oriented along the flight path and during hovering this horizon-oriented body position does not change regardless of upward, downward, sideward or backward movements.

During all of our dragonfly observations, only one flight instability appeared to occur—a roll instability. In hovering, the dragonfly maintained its position in space relative to a fixed landing site atop a reed, but often the body's roll axis gave way to a rotation of the body about its long horizontal axis. The net result, of course, was that the legs were sometimes rolled away from the landing site and further flight corrections were required.

Using high-speed film and natural lighting, we photographed many dragonflies during hovering. This photography provides a basis for comparing the wing kinematics of tethered dragonflies to those of dragonflies exhibiting natural flight. A few of these photographs are shown in Fig. 1. The wing positions are quite similar to those photographed previously (Daitor, 1975; Norberg, 1975). Also, the same wing motion characteristics are seen in our laboratory tests. This comparison is important since the lift generation measured in tethered specimens (whether using normal wing kinematics or not) was quite high. Undoubtedly, the wing-fluid interactions supporting such high lift are of prime importance but, in addition, it seems biologically unlikely that highly disturbed wing motions would be capable of high lift generation. In any event, the photographs of natural flight clearly show that the tethered dragonflies exhibit, at our present level of analysis, normal wing kinematics.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>Speed</th>
<th>Maneuvering</th>
</tr>
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<tbody>
<tr>
<td>1) Escape</td>
<td>&lt;5 sec</td>
<td>&gt;5 m sec(^{-1})</td>
<td>modest; upward, forward ± 45° angle from body axis</td>
</tr>
<tr>
<td>2) Gliding</td>
<td>&gt;10 sec</td>
<td>1 m sec(^{-1})</td>
<td>little; low negative angle glide path; occasional wing use to regain altitude</td>
</tr>
<tr>
<td>3) Hovering</td>
<td>&gt;10 sec</td>
<td>---</td>
<td>excellent but slow movement in virtually any direction in any plane</td>
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**Table 1. Observed natural flight behaviors of dragonflies.** Based on observed major flight characteristics exhibited during feeding, patrolling, territory displays, mating and predation avoidance, these general categories summarize the most reliably observed flight modes.
Examples of wing positions photographed for hovering dragonflies. The photographs were taken with a 1/30 sec shutter speed using a telephoto lens and natural light. Over 100 such photographs were used to document the similarities in wing kinematics between free flying and tethered dragonflies.

Lift generation measures had an average weight of \(1.4 \pm 1.5\) g. The area of the four wings averaged \(8 \times 1.1\) m. The typical wing loading was \(27 \pm 0.0\) g cm\(^{-2}\). Aspect ratios of the front wings were 9.5 and of the rear wings were 6.7. The dragonfly wing exhibits relatively thick venation that provides both structural reinforcement as well as local turbulence or roughness. The leading edge venation of the front wing is comparatively thick, while the trailing edge has a relatively thin venation. The trailing edge of the wings are more delicate and flexible than the leading edge - the outer 25% of the wing. At the outboard, the wing narrows to the point of attachment. A representative photograph of the dragonfly dragonfly wing is provided in Figure 1.
Simultaneous photographs of dragonfly wing motions and photodiode marked, lift generation. These frontal views of the dragonfly revealed wing stroke angles while side views (not shown) revealed caudal-rostral angles as well as the geometric angle of attack of the wings. In these photographs, the rear wings were marked to allow easier differentiation of front and rear wing angles.

Both wing motions and oscillographic records could be photographed simultaneously. A typical series of wing positions and associated lift traces are provided in Fig. 3. Using these photographs and Strobotach illuminated video tapes, it was possible to construct the summary of wing kinematics as shown in Fig. 4. As may be seen, the wing tips trace out an oval during each cycle. Beginning at the top of this oval, the wing tips move forward and backward, then upon reaching the bottom of the oval they twist upward while moving backward to return to the top of the oval. Both the front and rear wing of the tandem pair show similar motions, although the rear wing can lead the front one by as much as 150° in some instances, the wings are clearly 180° out of phase with each other, so it is difficult to determine which wing leads through a typical wingbeat cycle. The frequency of wingbeat cycle is approximately 25–35 Hz and at any instant is the same for both wings. Average wing tip velocities were about 40% larger during upstrokes (~170 cm/sec) than during downstrokes (~120 cm/sec).

The model of wing tip motion in Fig. 6 summarized approximate angles of attack associated with various phases of a typical wingbeat cycle. The angles indicated are shown for increments of time of about 5 msec (36). Angle are depicted on the upstroke than the downstroke. In these summaries, no attempt has been made to represent wing twisting along the span; however, a modest amount of twisting propagates from the root to the wing tip each time the wing changes direction from upstrokes to downstrokes and vice versa.

The lift generation recorded across all 21 specimens and over a dozen elicited flight episodes each revealed an instantaneous peak force during each complete wingbeat cycle (Fig. 6). The average amplitudes of such lift peaks was approximately 7 g f' 1 g' = 5.81 x 10^-2 N). Thus, at one instance during each wingbeat cycle, the tethered dragonflies produced lift forces of about 7 times the average body weight. Both Verheggen (1971) and Weis-Fogh (1973) had estimated that dragonflies must have a minimum lift.
Summary of geometric angles of attack throughout a wing beat. These angles are indicative of wing tip characteristics and do not indicate a small amount of spanwise twisting. Hypothesized flow and thrust vectors are also indicated. The angle representations are indicated over a typical 40 msec wingbeat cycle duration at approximately 5 msec intervals.

Anxious to determine how these wing kinematics produce such high lift, we arranged a flow visualization test for tethered dragonflies. Thick kerosene smoke delivered a short distance ahead of the dragonfly in the zero flow apparatus was rapidly drawn toward the insect, passed through the tandem wings and appeared as two discernible wakes behind the insect. With ambient illumination, the laminar stream of smoke appears to run into an eddying flow around the insect and to emerge as a split, turbulent wake. With stroboscopic illumination, however, the smoke shows a considerable amount of identifiable structure (Fig. 7). The upper wake shows the effects of vortices which seem to be decaying while the lower wake shows rather homogeneous smoke distribution as might be expected from a fully turbulent flow. Video tape records suggested the presence of vortex structures near the wing surface but due to the changing winglet motions it was not possible to document such flow structure photographically. As was quite clear from the lift graph and the figure on

Fig. 6 Summary of lift generation during a typical wingbeat cycle. All wingbeat cycles have been normalized to the indicated time scale to correct for variations in wingbeat frequency. As indicated, lift peaks of approximately 7 g are generated once during each wingbeat cycle. Such peaks give rise to inordinately high apparent C\textsubscript{T} values.
2 Stroboscopic flow visualizations of wake structures about a dragonfly during a tethered flight episode. Smoke was delivered immediately ahead of the dragonfly wing at a low velocity.

Encouraged by these findings, we sought a way to better understand the flow-wing interactions employed by the dragonfly to achieve high lift. Two approaches were tried: (1) the dragonfly was exploited to become an automation model of its flight kinematics and (2) an oscillating flat plate model was used for dynamic flow visualization. Both approaches are in their infancy but both appear to have good potential.

The automation dragonfly approach employs electrical stimuli delivered directly to thoracic cardiovascular systems to elicit flight kinematics. Appropriately placed electrodes can yield stimulus amplitude-dependent control of wingbeat amplitude and frequency-dependent control of wingbeat frequency. Thus, the crucial parameters of the wing kinematics can be made to operate in a quite mechanical fashion. Interestingly, the dragonfly may at any time override the electrical stimuli and initiate flight kinematics of varying frequencies. The dragonflies tested to date rarely elect to disrupt our automation procedures in this way. The result is a reproducible visualization of wing-fluid interactions at any phase, in what appears to be, a rather normal wingbeat cycle. The preliminary results of this approach are encouraging. As may be seen in Fig. 8, the single front wing (others removed to provide flow simplicity) elicits reproducible vortices during certain portions of the driven wingbeat cycle. Since the visualizations represent 48 stroboscopic exposures taken at the same time delay between a stimulus pulse and the onset of illumination, both the automation characteristics of the wing motion and the reproducible structures of the flow are evident. We have not analyzed the wing-flow interactions fully but we can definitely cite the presence of unsteady energized flows produced by dragonfly wing kinematics. Such flows are characteristically seen in the complete four wing automation model as well. The size, shape and position of these structures, as suggested by the visualizations, make it clear that the flow-wing interactions are remarkably different from those postulated for the Weis-Fogh lift generation mechanism [cf., Weis-Fogh, 1972; Mankiewy, 1975; Mankiewy, 1981].

**Fig. 8** Multiple exposure flow visualizations about an automation model of the tethered dragonfly. All but one forward wing have been removed. The vortex formation is clearly documented for a variety of wing-flow interactions throughout a portion of the wing stroke cycle. The driven wing kinematics are quite similar to those of the unrestrained and tethered (nondriven) dragonflies.
In our second model approach employing an oscillating flat plate, we simply sought a didactic indication of flow-plate interactions using a variety of thin plates. In these tests the effects of oscillation amplitude, oscillation rate and plate thickness were visualized. Anemometry data were collected in a single dimension using an overheated miniature thermister. The experimental parameters were quickly reduced to those which produced cohesive vortex structures. Within this parameter range vortex cohesiveness was rated from the flow visualizations and then plotted in regard to the appropriate production parameters. The results were strikingly consistent (Fig. 9). Very cohesive structures occurred with extremely thin plates (≤ 0.5 mm). As plate thickness increased, both oscillation frequency and oscillation amplitude improved flow cohesiveness. As plate thickness decreased, oscillation frequency had a large effect on flow cohesiveness but stroke length had little effect (Fig. 10). The addition of small amounts of angularity to the oscillating plate did not change this relation. But in instances of angularity that exceeded 20-25°, a definite net flow was induced in the flowfield.

Overall, these observations substantiate an informal bias obtained from flow visualizations about dragonfly wings. First, the flow structure is exceedingly cohesive and, secondly, wingbeat amplitude has little effect upon such structures. The use of plate angles indicates that the oscillating dynamics (that induce an increased or decreased angle relative to flow) can enhance vortex production and cohesiveness. It appears that dragonflies may be able to use similar flow control mechanisms. Other characteristics of this model system will be presented elsewhere (Bilinski-Lutiges, in preparation). Some of the nondimensional relationships are intriguing in and of themselves.

**Discussion**

The relationships between dragonfly lift generation and wing kinematics have shown the presence of high lift once during each wingbeat cycle. Also, the total amount of lift generation measured was quite large. These observations are inconsistent with steady-state aerodynamics and indicate that the dragonfly must employ unsteady or energized separated flows to achieve lift. Such direct empirical verification of the need to employ alternative lift generation mechanisms has not been achieved for other hovering insects.

The wing kinematics of the dragonfly differ significantly from those reported for the Chalcosia wag (Mell-Fogh, 1973). Also, the front and rear wings operate independently in an "unstabled" configuration and the dragonfly body remains horizontal throughout different flight modes.
hovers, escapes and glides. These facts suggest that the dragonfly must utilize unsteady flows and related lift generation mechanisms which differ substantially from those of the Chalcid wasp. The flow visualization studies corroborate such a difference. Visualized vortex-dominated flows are immediately adjacent to the wing's mid-span. They persist in this spatial relation to the wing over an appreciable amount of the wingbeat cycle. And, these flows are lateralized such that each side of the insect's wing interacts with a local flowfield that is somewhat independent of the flowfield on the other side. It is tempting to speculate that the lateralization of flowfields underlies the roll instabilities seen in the natural flight of dragonflies.

One major conclusion may be drawn from the above observations: dragonflies use unsteady mechanisms that differ in many ways from those used by the Chalcid wasp. Further, it now appears likely that other biological organisms could utilize yet other unsteady flow characteristics to achieve lift and flight behaviors. This is not a deterrent to attempting to understand and, perhaps, emulate the use of such flows. Rather, it is encouraging that many exploitation possibilities may exist: each optimized for a range of different aerodynamic needs.

For us, our work has just begun. Given the data at hand, we must determine how the dragonfly produces the unsteady separated flows we have visualized. We must determine how these flows are controlled. And, we must determine how such flows interact with the dragonfly wings to produce the remarkable lift values we have documented.

References


THEORETICAL STUDY OF NON-LINEAR UNSTEADY AERODYNAMICS OF A NON-RIGID LIFTING BODY

J. C. Wu**, N. L. Sankar**, and H. Hu Chen*
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

AD-P004 169

Abstract

A recently developed general theory of aerodynamics is utilized in an investigation of non-linear unsteady flow problems involving a non-rigid lifting body. Previous theoretical and experimental efforts have demonstrated that, under certain restrictive circumstances, unsteady flows can be approximated by small departures from steady or quasi-steady behavior. The addition of unsteady phenomena to steady ones reasonably describe such flows. The resulting linearized equations describing the flows are often amenable to mathematical treatment. A large body of valuable literature has been developed over the years dealing with various aspects of linear unsteady flows. Most of the fundamental concepts of unsteady flows that are adequately described by the linear theory are well-understood. In contrast, in the domain of non-linear unsteady flows, where strong unsteady effects invalidate the linear simplifications, the mathematical and experimental difficulties attendant to a rigorous treatment of unsteady aerodynamic problems are immense. Many of the essential and unique features of the non-linear unsteady flows are not well-understood today.

In recent years, there has been a rapid growth of research activity in non-linear unsteady aerodynamics. Most of the current research topics in non-linear unsteady aerodynamics are motivated by applications in turbomachines, marine propellers, helicopter rotors, etc., where strong flow unsteadiness is an intrinsic part of the overall behavior. The minimization of alleviation of large adverse effects caused by flow unsteadiness is of primary concern in these applications. A number of researchers have, however, undertaken research efforts aiming at the employment of unsteady aerodynamic forces advantageous in the design of future aerospace vehicles. It is worthy of note that the design of aircraft of the past and present generations are conceptualized mainly within the context of steady and quasi-steady aerodynamics. Since the production of large unsteady forces is invariably associated with flows in the non-linear domain, an improved understanding of non-linear unsteady aerodynamic phenomena is prerequisite to the advantageous utilization of such forces.

1. Introduction

The subject of unsteady aerodynamics has been, for more than half a century, an active field of fluid dynamics research. Previous theoretical and experimental efforts have demonstrated that, under certain restrictive circumstances, unsteady flows can be approximated by small departures from steady or quasi-steady behavior. The addition of unsteady phenomena to steady ones reasonably describe such flows. The resulting linearized equations describing the flows are often amenable to mathematical treatment. A large body of valuable literature has been developed over the years dealing with various aspects of linear unsteady flows. Most of the fundamental concepts of unsteady flows that are adequately described by the linear theory are well-understood. In contrast, in the domain of non-linear unsteady flows, where strong unsteady effects invalidate the linear simplifications, the mathematical and experimental difficulties attendant to a rigorous treatment of unsteady aerodynamic problems are immense. Many of the essential and unique features of the non-linear unsteady flows are not well-understood today.

In recent years, there has been a rapid growth of research activity in non-linear unsteady aerodynamics. Most of the current research topics in non-linear unsteady aerodynamics are motivated by applications in turbomachines, marine propellers, helicopter rotors, etc., where strong flow unsteadiness is an intrinsic part of the overall behavior. The minimization of alleviation of large adverse effects caused by flow unsteadiness is of primary concern in these applications. A number of researchers have, however, undertaken research efforts aiming at the employment of unsteady aerodynamic forces advantageous in the design of future aerospace vehicles. It is worthy of note that the design of aircraft of the past and present generations are conceptualized mainly within the context of steady and quasi-steady aerodynamics. Since the production of large unsteady forces is invariably associated with flows in the non-linear domain, an improved understanding of non-linear unsteady aerodynamic phenomena is prerequisite to the advantageous utilization of such forces.

In previous papers**, detailed and rigorous derivations of the general aerodynamic theory and preliminary results of a theoretical study of the vortex/airfoil interaction problem have been presented. In the present paper, important concepts related to the application of this general theory are reviewed. In particular, it is shown that, with this general theory, the total air load on a solid body can be divided into several components, each representing the contribution of a distinct physical process. This distinguishing feature of the general theory can be utilized in the establishment of a reasonable understanding of the detailed mechanism of production of aerodynamic forces. It is anticipated that such an understanding will in time provide a rational basis for the alleviation, control, or utilization of large non-linear aerodynamic forces in various applications.

The contents of the present paper are centered upon the theoretical treatment of the Weis-Fogh problem. Extensive computational efforts, however, are being carried out concurrently with theoretical studies. In fact, computational results have provided in the past and are continually providing important physical insights to unsteady aerodynamic problems. Numerical procedures utilized in the present research program emphasize integral-representation formulations of the viscous flow equations. This formulation permits the solution field to be confined to the vortical region of the flow, whereas the non-vortical regions are treated analytically. The solution field co be confined to the vortical region of the flow, whereas the non-vortical regions are treated analytically. The resulting numerical procedure is particularly well-suited for computing high Reynolds number external flows. For these flows, the vortical regions, which must encompass boundary layers, recirculating flows and wakes, comprises only a small part of the total flow field. The calculation of vortex region can be carried out independently, thus eliminating the difficulties of simultaneously accounting for the various length scales present in the flow. At present,
parametric studies of two-dimensional flows can be performed economically using widely available computers such as the CDC-6600 computer. Three-dimensional computations have been carried out for several flows involving simple boundary geometries. The amounts of computation required for these flows are not unreasonable. Computed results for the unsteady flow problems discussed in this paper and in Reference 3 will be presented in future articles. In this connection, it is worthy of note that the general viscous theory of aerodynamics used in the present study relates the aerodynamic forces and moments acting on lifting bodies to their vortical environments. The general theory and the integral-representation approach are therefore ideally suited for one another in a combined theoretical and computational research program.

Previous applications of the general viscous theory of aerodynamics are concerned with flows past rigid lifting bodies. The Weis-Fogh motion considered in this paper, however, involves two wings attached and yet moving relative to one another. The present study is, in this context, a precursor to a more comprehensive study of the unsteady aerodynamics of flexible (non-rigid) lifting bodies. It has been recognized for a number of years that unsteady aerodynamics of flexible lifting bodies is an inherent to aquatic propulsion and flight of animals. Obviously, a reasonable understanding of the physical mechanisms of generation of unsteady aerodynamic forces accompanying large amplitude motions of flexible lifting surfaces is beyond its biological significances, of decisive import to novel designs of airborne vehicles utilizing large unsteady forces.

2. Vorticity Dynamics

Unsteady incompressible motions of viscous fluid are governed by the law of mass conservation and Newton's laws of motion. The mathematical statements of these laws are familiarly expressed, in terms of the velocity vector \( \mathbf{v} \) and the pressure \( p \), as the continuity and the Navier-Stokes equations. It is, however, advantageous to introduce the concept of vorticity vector \( \omega \) defined by

\[
\omega \times \mathbf{v} = \mathbf{f}
\]

and to consider the vorticity transport equation

\[
\frac{\partial \omega}{\partial t} + (\mathbf{v} \cdot \nabla) \omega = (\mathbf{v} \times \mathbf{v}) + \frac{\partial \mathbf{p}}{\partial x} - \frac{\partial \tau_{ij}}{\partial x_i}
\]

There are several major advantages in the use of the concept of vorticity. In the first place, the remarkable success of the well-known circulation theory in predicting the lift force implies that the vorticity of the circulation is accountable for forces exerted by the fluid on aerodynamic surfaces. Secondly, it is well known that viscous effects are present only in the vortical part of the flow. This fact suggests that it is possible, by studying flows about solid body, to confine the solution to the vortical region of the flow through the use of the vorticity concept. Thirdly, the concept of vorticity permits the overall flow problem to be decomposed into a kinematic aspect and a kinetic aspect. This decomposition facilitates the identification of important physical processes associated with various types of flows. These major advantages offered by the use of the vorticity concepts have been emphasized by the present authors in studies of unsteady aerodynamic problems. In particular, the first advantage has yielded the general viscous theory of aerodynamics and the second advantage the integral representation approach for computing viscous flows. The third advantage has been utilized previously in studies of various steady and time-dependent flow problems. This advantage is briefly reviewed below.

The kinematic aspect of the viscous flow problem is described by Eq. (1) and the continuity equation. This aspect expresses the instantaneous relationship between the velocity field and the vorticity field. The kinetic aspect of the problem is described by Eq. (2). This aspect is concerned with the redistribution of vorticity in the fluid through various kinetic processes. In studies of unsteady flows, it is convenient to follow the kinetic development of the vorticity field in the fluid. A knowledge of the vorticity field is needed in the solution of the vorticity transport equation. This velocity field is kinematically a function of the vorticity field. The vorticity transport equation is non-linear and its mathematical analysis presents great difficulties. It is, however, possible to obtain a significant amount of understanding about the kinetic processes involved in unsteady flows without detailed mathematical analyses.

Consider a finite solid body immersed in an infinite incompressible fluid with uniform viscosity. The solid body is initially at rest in the fluid which is also at rest. Subsequent prescribed motion of the solid body induces a corresponding unsteady motion of the fluid. It has been shown that vorticity is neither created nor destroyed in the interior of the fluid domain. Vorticity, however, is continually being generated at the solid boundary in contact with the fluid following the initiation of the solid motion. This vorticity spreads into the interior of the fluid by the process of viscous diffusion and, once there, is transported away from the solid surface by both convection and diffusion. Since the transport of vorticity by convection is a finite rate process and that by diffusion is effectively finite rate, the vortical region of the flow is of finite extent at any finite time level after the initiation of the solid motion. Outside the vortical region, the flow is irrotational and therefore inviscid. If the Reynolds number is not small, then the effective rate of viscous diffusion is much smaller than that of convection. Therefore, a large region of the fluid, ahead and to the side of the solid, is free of vorticity and is inviscid.

The general pattern of unsteady flow development can be briefly described as follows. As a consequence of the solid motion relative to the fluid, vorticity is generated continually at the fluid/solid interface. Once generated, the vorticity moves along the solid surface as long as the flow remains attached. That is, since the effective rate of viscous diffusion is much smaller than that of convection, the vorticity generated on the solid surface cannot penetrate far into the interior of the fluid domain before being carried downstream by the fluid motion. A thin layer of vorticity adjacent to the solid boundary is therefore present. This layer is simply the well-known boundary layer. The vorticity within the boundary layer continually moves downstream with the fluid and, at the same time, is continually being replenished through the generation of vorticity on the solid surface. This process of replenishment is present in both steady and unsteady flows. In steady flows, the replenishment process and the vorticity transport process balance one another and the vorticity distribution in the boundary layer is independent of time in a reference frame attached to the solid. In unsteady flows, the replenishment and transport processes do not balance one another and the vorticity distribution in the boundary layer is time-dependent. In both steady and unsteady flows, because the boundary layers are thin, it is often convenient to represent the vorticity in the layers by vortex sheets.

The representation of a boundary layer by a vortex sheet in the present context does not imply an inviscid fluid assumption. Rather, the representation...
approximates the location of the vorticity across the boundary layer by a given point adjacent to the solid surface. The strength of the concentrated vortex sheet simply the integrated vorticity across the boundary layer. The vortex sheet moves along the solid surface. The distinction between the inviscid assumption and the present approximation is not merely a matter of semantics. While the two concepts often lead to the same conclusions in analyses and computation, the present approximation is based on a viscous flow viewpoint and experiences no conceptual difficulties associated with previous inviscid theories.

The vorticity in the boundary layer essentially leaves the vicinity of the solid surface through several possible avenues. If no massive separation of the flow occurs on the solid surface, then the vorticity in the boundary layer eventually feeds into a wake layer. This occurs, for example, in the case of a thin airfoil at a small angle of attack. The two boundary layers at the two sides of the airfoil in this case merge at the trailing edge, with both layers feeding vorticity into the wake layer. In steady flows, the total flux of vorticity entering the wake is zero. In unsteady flows, a net flux of vorticity enters the wake. Since the boundary layers are thin, the wake layer, which is a continuation of the boundary layers, is also thin initially. As the vorticity layer moves away from the solid through the convective process, viscous diffusion produces only a slow growth in the thickness of the wake layer. In consequence, it is reasonable in many applications to represent the wake layer also by a vortex sheet. The wake layer is usually unstable. The velocity field associated with the vorticity in the wake layer causes the wake layer to "roll-up." If the roll-up process occurs at a large distance from the solid, then it is reasonable to represent the rolled-up vorticity by a single vortex filament in analyzing the flow near the solid. If the roll-up process occurs near the solid, however, then detailed structure of the rolled-up vorticity may be necessary. In any event, the total strength of the rolled-up vorticity needs to be known in order to determine correctly the aerodynamic forces acting on the solid. For three-dimensional flows, the vorticity in the boundary layer leaves the vicinity of the solid surfaces also through the formation of tip vortices which usually roll up.
by Eqs. (3) are generally not included in inviscid analyses. Equation (4) states that the aerodynamic force acting on the solid bodies is composed of two contributions. The first term on the right side of Eq. (4) gives the contribution of the time variation of the first moment of the vorticity. The second term gives the contribution of the vorticity of the fluid displaced by the solid bodies. Equation (5) states that the moment of aerodynamic force is composed of contributions of two terms, a contribution of the total second moment of the vorticity field and a contribution of the moment of inertia. The inertia terms in Eqs. (6) and (7) of course vanish in cases where the solid body experiences no acceleration. As discussed earlier, it is convenient to divide the overall unsteady flow problem into its kinetic and kinematic aspects. The general theory described here relates the unsteady aerodynamic forces and moments acting on the solid bodies to the kinetic development of the vorticity field. The task of analyzing the kinetcs and the kinematics of the flow remains.

It is clear from the general theory, that all the information about aerodynamic forces and moments are contained in the time-dependent vorticity environment of the lifting body. No information about the potential field surrounding the vorticity region is needed in the theory. Under certain restrictive circumstances, it is possible to specify the vorticity field approximately without actually solving the vorticity transport equation. The general theory described above then permits the unsteady aerodynamic forces and moments to be determined in a straightforward manner.

It needs to be emphasized, even at the risk of appearing repetitive, that the general theory described above is exact in that it is an exact consequence of the viscous flow equations. This fact, however, does not inhibit the introduction of approximations to the equations given in this section. As discussed in Section 2 of this paper, the word "approximation" is used here to indicate that the precise distribution of the vorticity in the fluid is compromised in exchange for convenience in the evaluation of unsteady aerodynamic forces and moments. Through this approximation, the general theory offers an opportunity of establishing important physical insight to the mechanisms of generation of large unsteady aerodynamic forces. This opportunity is available even with relatively imprecise approximations of the vorticity distribution. Under circumstances where the underlying approximations can be accurately approximated, the general theory leads to accurate predictions of unsteady aerodynamic forces and moments in both the linear and the non-linear domains.

The approximations to the general theory are conceptually different from the inviscid fluid assumption which is the basis of classical theories. Since a truly viscous fluid does not exist in nature and since the limit of vanishingly small viscosity is distinct from a zero viscosity, the success of the inviscid theories must depend upon the auspicious circumstance that inviscid conclusions coincide with certain approximations of viscous conclusions. The general viscous theory has been shown9, 10, to yield, at various levels of approximation, well-known conclusions of classical inviscid theories.

For high Reynolds number external flows containing no appreciable regions of separation, as that discussed in Section 2, the vorticity distribution in the fluid is accurately represented by vortex sheets and vortex filaments. The integrals in Eqs. (1), (4), and (5) over the region $R$, then reduces to integrals over surfaces in three-dimensional flows and over lines in two-dimensional flows. Under these circumstances, the analyses become considerably simpler. To determine the aerodynamic force, the following approximation of Eq. (6) may be used:

$$\mathbf{f} = - \frac{d}{dt} \mathbf{W} \cdot \mathbf{d} \mathbf{a} \cdot \mathbf{v} \cdot \mathbf{d} \mathbf{S}$$

where $W$ is the solid surface (or lines) including tip vortex sheets and starting vortex where they exist, and $S'$ is a surface enveloping the solid surface $S$ and at infinitesimal distance from $S$. The distinction between $S$ and $S'$ is conceptually important. With the approximation discussed, the vortex sheet represents the boundary layer vorticity which is in the fluid domain. The velocity of the vortex sheet is different from the solid surface velocity.

In Eq. (6), the vortex sheet on $S'$ approximates the boundary layer adjacent to the solid surface. The vortex strength $v$ on $S'$ is therefore the integrated vorticity across the thickness of the boundary layer. To the accuracy of the boundary layer approximation, the vorticity is the negative of the normal derivative of the velocity component in the direction tangent to the solid surface. One therefore obtains, upon integrating the vorticity along the normal direction,

$$v(s) = - v(s, s) + v(s, 0)$$

where $s$ in the boundary layer coordinate tangential to the solid surface, $v$ is the tangential velocity component, $s$ is the normal coordinate at the edge of the boundary layer and $0$ is the normal coordinate on the solid surface. If $v(s, 0) = 0$, then $v$ is simply the negative of the boundary layer edge velocity.

Equation (7) is easily generalized to three-dimensional applications. According to Eq. (7), the vortex sheet on the solid surface represents a discontinuity in tangential velocity between the solid and the inviscid flow surrounding the boundary layer. This discontinuity is consistent with the discontinuity in tangential velocity of the boundary layer, which possesses a finite albeit small thickness, by a sheet. This approximation of course is not suitable for the kinetic transport of vorticity within the boundary layer. The approximation, nevertheless, is well-suited for the computation of aerodynamic forces. In many applications, distribution of the vortex strength on the solid can be computed without actually performing boundary layer calculations. For example, if the vorticity distribution in the detached part of the flow is known, then the strength of the vortex sheet on $S'$ is uniquely determined, as discussed in Ref. 5, without computing the detailed flow within the boundary layer. It is mentioned in passing that the concept described in Ref. 5 is similar to that used in the panel/vortex lattice methods currently receiving a great deal of attention within the aerodynamics community. In many existing panel codes, fictitious source-sink distributions over $S$ are used. It is not difficult to show that, however, that these source-sink distributions are equivalent to vortex distributions over $S$. The vortex distributions over $S$, as discussed earlier, are approximations of real vorticity in the boundary layer. Computationally, the use of vortex distributions over $S$ is as convenient as the use of source-sink distributions.

Also, in vortex lattice methods, the vortices in the interior of the fluid domain are usually allowed to convect but not to diffuse. This restriction, however, is not necessary and can be removed in viscous computations.

Once the vortex distributions over $S$ and $W$ are computed, Eq. (6) immediately gives the aerodynamic force $f$. The use of Eq. (6) clearly offers distinctive advantages over the prevailing surface pressure-shear stress integration method since, with Eq. (6), both the unsteady drag and the lift can be evaluated in a straightforward manner directly from the vortex.
distributions.

In general, the vortical region in the fluid is composed of a vortical system near the solid bodies and a vortical system trailing the solid bodies. The near vortical system in general contains attached boundary layers and detached recirculating flows. The vorticity in the near vortical system represents the vorticity shed from the near vortical system at previous time levels. Shortly after the initiation of the motion of a solid body, the vorticity region is confined to thin layers near the solid body. In the case of a lifting body, a concentrated dose of vorticity, i.e., a starting vortex, leaves the vicinity of the body shortly after the motion's onset. The average velocity of this starting vortex is initially one half of the freestream velocity. This fact is consistent with the well-known Wagner effect and can be shown by analyzing the vorticity distribution in the boundary layer as it leaves the solid body's trailing edge. With increasing time, the starting vortex moves in the general downstream direction, becomes diffused, and approaches the velocity of the freestream. Between the starting vortex and the near vortical system is the remainder of the trailing vortical system which, for convenience, is called the vortical wake. The line of demarcation between the near vortical system and the vortical wake need not be delineated precisely. The division of the overall vortical system into its several components is extremely useful since, with the general viscous aerodynamic theory, the contributions of each of these components to the aerodynamic force and moment can be considered individually. For example, with Eq. (4), the integral over $R_1$ can be written as the sum of four integrals over, respectively, the unsteady boundary layers, the recirculating regions, the vortical wake, and the starting vortex. By separating the overall aerodynamic force into contributions by the several flow components, considerable physical insights can be developed.

In the case of a high Reynolds number flow containing no appreciable recirculating regions, Eq. (6) shows that the overall aerodynamic force is composed of four contributions. The first contribution, represented by the vortex moment integral over $S^*$, is due to the development of the unsteady boundary layer. The second and third contributions are due to, respectively, the movements of the wake and the starting vortex, and are represented by the vortex moment integral over $W$. The fourth contribution is due to the solid body acceleration.

The preceding discussion concerning the aerodynamic force is applicable also to the moment of aerodynamic force. In the case of a high Reynolds number flow containing no appreciable recirculating regions, Eq. (5) yields an expression showing terms of vortex sheet strengths over $S^*$ and $W$ and the effects of solid body acceleration.

4. Weis-Fogh Mechanism

It is well-known that, according to inviscid theories, steady lift force acting on an airfoil is proportional to the circulation around the airfoil. For an airfoil initially at rest and is set into motion impulsively, the circulation is developed through the shedding of a starting vortex. That is, because of the need to conserve total vorticity, the acquisition of a circulation around an airfoil is accompanied by the release of a starting vortex. The circulation around the starting vortex is equal in magnitude and opposite in sense to the circulation acquired by the airfoil. The process of vortex shedding is not difficult to understand in the context of viscous flow. Indeed, the starting vortex is a direct consequence of the unsteady boundary layer activities around the airfoil. Therefore, it is not surprising that understanding the process of vortex shedding in the context of an inviscid fluid, i.e., a fluid with a zero viscosity rather than a vanishingly small viscosity.

Weis-Fogh observed that certain types of insects, e.g., Encarsia Formosa, with a pair of wings pivoted together at their trailing edge, rotate their wings individually in opposite directions. The rotation generates a circulation about each wing. If the two wings are identical in shape and their rotational speed is identical, then the circulation magnitude of each wing is equal to that of the other wing. The senses of the circulations of the two wings are opposite to one another. Because of symmetry, no trailing edge shedding of vortices occurs. In fact, if vortices were shed from the trailing edges of the two wings, they would annihilate one another because they are of opposite senses.

Weis-Fogh described the above mentioned mechanism of generating circulation as the "fling" phase of a fling-clap cycle. At the beginning of the cycle, the two wings are close to each other. The wings' leading edges separate from one another as the wings fling apart, i.e., rotate about their trailing edges and open up into a V shape. After reaching a certain opening angle, the two wings break apart and move in opposite directions around the body of the insect. The wings eventually flip and return to their initial position through a clap motion.

During the fling phase, the wings rotate about the points $A_1$ and $A_2$, with a angular velocity $\Omega = \dot{\alpha}$

where $\alpha$ is the half angle of the opening of the wing pair. The motion is symmetric about the line $\dot{EF}$. The fluid infinitely far from the wing pair is stationary. Each of the two wings acquires a circulation during the fling phase. Since the two circulations of the two wings are equal in magnitude and opposite in sense, the total circulation about the two wings is zero. At the moment of breaking apart of the wings, each wing possesses a circulation suitable for generating lift during its subsequent motion. It is easy to see that if the range $0 < \theta < \alpha$ corresponds to the fling phase, then the range $\pi - \alpha < \theta$ corresponds to the clap phase of the Weis-Fogh motion.

Lighthill emphasized the absence of the trailing-edge shedding of vortices in the Weis-Fogh motion. He indicated that it is remarkable that the Weis-Fogh mechanism works "for a fluid of zero viscosity not simply in the limit of vanishing viscosity". He presented a two-dimensional inviscid analysis which showed that the circulation around each wing during the fling phase is proportional to the wings' angular velocity $\dot{\alpha}$ and to a function of the opening angle $\alpha$ of the wings. He expressed this function in the form of an integral and presented disputed results for this function. Edwards and Cheng recently extended Lighthill's work and presented a closed form expression for the wing's circulation during the fling phase.

Although vortices are not expected to be shed at the trailing edges of the two wings during the fling phase, the rotation of the wing edge shedding of vortices. Lighthill examined this leading edge separation phenomenon and concluded that its effect on the wing's circulation is weak.
experiments by Maxworthy\textsuperscript{11}, however, showed a substantial effect of the leading-edge separation. Results of a recent study by Edwards and Cheng\textsuperscript{12} are in general agreement with Maxworthy's observation\textsuperscript{13}.

Weis-Fogh suggested that the opening of the wing pair causes the necessary circulation to be generated immediately and thus avoiding any delays in the build-up of the maximum lift required by the well-known Wagner's effect. Also, in the case of the conventional wing, substantial work must be done by the wing on the fluid to provide the kinetic energy associated with the starting vortex. In consequence, the wing experiences a large unsteady drag immediately after the start of its motion. To the present authors' knowledge, although the problem of generation of circulation on the wing pair has received the attention of previous investigators, the question of unsteady lift, unsteady drag force, and power expenditure experienced by the wings during the fling phase of the Weis-Fogh motion have not yet been resolved. Answers to these questions are important in establishing a comprehensive understanding of non-linear unsteady aerodynamics and in future utilization of large unsteady aerodynamic forces.

For the problem under consideration, the processes of generation and transport of vorticity described in Section 2 of this paper lead to an unsteady boundary layer wherever the flow near the solid surface is essentially tangential to the surface. The thickness of this boundary layer is comparable to the diffusion length, $l = \frac{ck}{2\sin x}$, where $x$ is the time duration of the fling phase of the Weis-Fogh motion. If this diffusion length is much smaller than the chord of the wings, then the boundary vorticity is accurately approximated by a vortex sheet. For the present problem, the flow near the lower surface of the wing is expected to remain attached throughout the fling phase. The flow is, however, expected to separate from the leading edge, resulting in leading-edge vortex shedding.

Following Lighthill\textsuperscript{14}, the problem is reformulated in a conformally mapped plane, $\zeta = \epsilon + i \eta$, through the use of a Schwarz-Christoffel transformation. A closed form expression for the transformation function presented by Edwards and Cheng\textsuperscript{12} is used. The transformation function is

$$ z = x + i y = \frac{1}{2} \frac{\zeta^3}{\sqrt{\eta^2 - 1}} \left( 1 - \frac{\zeta}{\eta^2} \right)^{1/2} $$

The stream function vanishes on the line of symmetry $\eta = \eta_1 F$ and also infinitely far from the wing.

It is simple to show that, with the potential function $\phi$, the complex potential, $W = \phi + i \psi$, expressed in terms of $\zeta$ below satisfies these conditions for the stream function:

$$ W = -\frac{1}{2\sin \frac{\pi}{2}} \left( (\zeta + 1)^{2-3i\eta} (\zeta - 1)^{2i\eta} - (\zeta - 1)^2 - 2(\zeta - 2i) \zeta \right) $$

(12)

In Eqs. (12) and the following equations, the subscript $1$ is used to indicate the omission of the leading-edge vortex shedding phenomena. The complex velocity in the $\zeta$-plane, $V = dw/d\zeta$, is given by

$$ V_{\zeta} = -\frac{1}{\sin \frac{\pi}{2}} \left[ (\zeta + 1)^{2-3i\eta} (\zeta - 1)^{2i\eta} - (\zeta - 1)^2 - 2(\zeta - 2i) \zeta \right] $$

(13)

(\zeta + 1 - 2i) \eta \right] $$

The complex velocity in the $\zeta$-plane is given by the right side of Eq. (13) divided by the derivative $dz/d\zeta$. It is, however, more convenient to use a transformed version of Eq. (6) in studies of aerodynamic forces acting on the wings.

In the present problem, the wing's cross-sectional area is negligibly small. The last integral in Eq. (6) therefore vanishes. The contributions of the unsteady boundary layer and the wake can be treated individually. One has therefore, with the wake, i.e., the shed vortices, omitted,

$$ F_{ix} = F_{iy} = i \frac{d}{dt} \int_{\gamma} \gamma_{i} ds $$

(14)

where $F_{ix}$ and $F_{iy}$ are respectively the horizontal and the vertical components of aerodynamic force acting on the wing pair due to the motion of the wing pair.

Because of symmetry, the vortex strength on the right wing is equal in magnitude and opposite in sense to the vortex strength on the left wing. The lift force $L_i$ acting on each wing is therefore one half of the total vertical force $F_y$ and is given by the time variation of the vortex moment on a single wing. One therefore has

$$ L_i = -\frac{1}{2} \frac{d}{dt} \int A_{BA_2} \times \gamma_{i} d\zeta $$

(15)

where the integration is over both the upper and the lower surfaces of the left wing.

Since the wing has no tangential velocity on its surface, the last term in Eq. (9) vanishes. The vortex strength $\gamma_{i}$ is therefore simply the negative of the tangential velocity at the fluid side of the vortex sheet. Since $dz = d\zeta (dz/d\zeta)$, Eq. (20) can be rewritten as

$$ L_i = -\frac{d}{dt} \int_{1}^{1} x \gamma_{i} d\zeta $$

(16)

where $\gamma_{i}$ is the negative of the $\zeta$-component of velocity, given by Eq. (13), on the $\zeta$-axis.

On the $\zeta$-axis, for the interval $-1 < \zeta < 1$, one obtains from Eqs. (9) and (13) the following expressions

$$ x = \sin \left[ k \left( \zeta \right) \left( 1 - \zeta \right)^{1/2} \right] $$

(17)

and

$$ \gamma_{i} = -\frac{1}{2} \frac{d}{dt} \left[ (\zeta + 1)^{2-3i\eta} (\zeta - 1)^{2i\eta} - (\zeta - 1)^2 - 2(\zeta - 2i) \zeta \right] \cos 2a $$

(18)

Placing Eqs. (17) and (18) into Eq. (16), one obtains,
after performing the integration,

$$L_1 = \rho c^3 \frac{d}{dt} \left[ \Omega f_1(\alpha) \right]$$  \hspace{1cm} (9)

where

$$f_1(\alpha) = \frac{3\pi}{4} a_0 - 2\pi \frac{a}{\alpha} + 3\alpha^2 / 2$$  \hspace{1cm} (20)

Equation (21) can be rewritten as

$$\frac{d}{dt}\left( \Omega f_1(\alpha) \right)$$  \hspace{1cm} (21)

where \( \Omega \) is the angular acceleration of the wing.

Equation (21) states that the unsteady lift component \( L_1 \) has two contributions. One contributor is the angular acceleration of the wing, its effect being directly proportional to the angular acceleration. The other contributor is the angular velocity of the wing, its effect being proportional to the square of the angular velocity. Both contributions are functions of the opening angle \( \alpha \), of the wing pair, as given by \( f_1(\alpha) \) and its derivative.

For convenience, the horizontal force acting on the wing \( A, B, A_2 \) is designated as an unsteady drag. This drag cannot be determined from the vorticity moment on a single wing alone. The total horizontal force acting on the wing pair is zero because of symmetry. The two wings are hinged together at their trailing edges. In Fig. 3 is shown a free body diagram for the wing, with \( N_1 \) and \( S_1 \) representing respectively the normal and tangential components of the unsteady aerodynamic force acting on the wing. \( H_1 \) is the reaction at the hinge, i.e., the force exerted by the wing \( A, C, A_2 \) on the wing \( A, B, A_2 \). Because of symmetry, this reaction is directed horizontally. The free body diagram shows that the unsteady lift \( L_1 \) is the sum of the vertical components of \( S_1 \) and \( N_1 \) and is the negative of \( H_1 \).

The unsteady normal force \( N_1 \) acting on each wing is given by

$$N_1 = \int_{A_1}^{B} p_1 \, ds - \int_{B}^{A_2} p_1 \, ds$$  \hspace{1cm} (22)

where \( p_1 \) is the pressure acting on the wing. Since the boundary layer cannot support a significant pressure difference across the layer, it is permissible to let this pressure \( p \) be the pressure at the boundary layer's outer edge, where the vorticity is negligibly small and the flow is potential. From the inviscid momentum equation, one obtains

$$\frac{\rho}{2} \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = 0$$  \hspace{1cm} (23)

At the edge of the boundary layer, the normal velocity \( \mathbf{v} \) is negligible small compared to the tangential velocity. The tangential velocity magnitude is equal to the strength \( \gamma \) of the vortex sheet representing the boundary layer. Using this information, one obtains from Eqs. (22) and (23),

$$N_1 = \rho \frac{d}{dt} \int_{A_1}^{B} \frac{\partial}{\partial t} \mathbf{v} \cdot ds$$

$$= 2 \int_{A_1}^{B} \frac{d}{dt} \mathbf{v} \cdot ds$$  \hspace{1cm} (24)

where \( r \) is the distance from the origin in the \( \xi \)-plane.

Using Eqs. (22) and (23), it can be shown that

$$\frac{N_1}{\rho c^3} = \frac{d}{dt} [\Omega f_1(\alpha)] \csc \alpha - \Omega^2 \gamma(\alpha) \cot \alpha$$  \hspace{1cm} (25)

where

$$g_2(\alpha) = \pi c^2 \frac{\csc^2(2 \alpha \gamma / \pi)(1 - 2\gamma^2)/2}{\nu}$$  \hspace{1cm} (26)

Equations (21) and (25) gives the following expressions for the unsteady tangential force and the unsteady drag

$$S_1 = \frac{\rho c}{2} \frac{d}{dt} [\Omega f_1(\alpha)] \csc \alpha - \Omega^2 \gamma(\alpha) \cot \alpha$$  \hspace{1cm} (27)

$$D_1 = \frac{\rho c^3}{4} \frac{d}{dt} [\Omega f_1(\alpha)] \csc \alpha - \Omega^2 \gamma(\alpha) \cot \alpha$$  \hspace{1cm} (28)

The normal forces can be expressed in the form

$$N_1 = \frac{\rho c}{4} \int_{A_1}^{B} [\Omega f_1(\alpha)] \csc \alpha - \Omega^2 \gamma(\alpha) \cot \alpha$$  \hspace{1cm} (29)

where the subscript \( n \) indicates the fact that the functions \( f_n(\alpha) \) and \( \gamma_n(\alpha) \) are related to contributions to \( N \).

The forces \( L_1 \) and \( D_1 \) are easily expressed in forms similar to Eq. (29). The tangential force \( S_1 \) is independent of the angular acceleration and is proportional to the angular velocity, as is shown in Eq. (27). The physical significance of this rather remarkable feature of the tangential force is not yet explained. It is clear, however, that this tangential force is due to the well-known leading edge suction effect which is also present in steady flows past airfoils at non-zero angles of attack. In the case of a thin airfoil represented by an infinitesimally thin flat plate, this suction force can be determined through a limiting process. The procedure just described for the Weis-Fogh problem when applied to the flat plate, at an angle of attack and a free stream velocity \( u_w \), gives a suction force value of \( -p u_w \sin \alpha \), where \( \Gamma \) is the circulation (boundary layer vortex) around the plate. This suction force corresponds to a zero drag as is expected for steady inviscid flows.

The functions \( f_1, f_2, f_3, \ldots \) are represented graphically in Figures 4, 5, 6 and 7. It is worthy of note that all these functions are easily expressible in terms of the two functions \( f_1 \) and \( \gamma_1 \) and the derivatives of these two functions.

It is of interest to determine the power expenditure needed to maintain a prescribed motion of the wing. The power requirement \( P_1 \) for the case of no leading-edge vortex shedding is given by

$$P_1 = \int_{A_1}^{B} \rho c^3 \frac{d}{dt} [\Omega f_1(\alpha)] \csc \alpha - \Omega^2 \gamma(\alpha) \cot \alpha$$  \hspace{1cm} (30)

where

Since, on the wing, \( v_n = \pi r \), where \( r \) is the distance from the origin, one has, in the \( \xi \)-plane.

$$P_1 = \rho c \int_{A_1}^{B} \Omega f_1(\alpha) \frac{d}{dt} [\csc \alpha - \Omega^2 \gamma(\alpha) \cot \alpha]$$  \hspace{1cm} (31)

Placing Eqs. (9), (12), and (13) into Eq. (31), one obtains, after considerable manipulations,

$$\rho c \int_{A_1}^{B} \frac{d}{dt} \left[ \frac{\pi}{4} \gamma(\alpha) \right]$$  \hspace{1cm} (32)

where
\[ f_p(a) = \frac{1}{2} \left( 1 - \frac{2a}{b} \right)^{1/2} + \frac{4a^2}{b^2} \left( 1 - \frac{2a^2}{b^2} \right)^{1/2} \]

Equation (33) can be expressed as
\[ \frac{P}{\rho c^2} = 2 \alpha \frac{f_p(a)}{a^3} + \alpha^3 \frac{f_p(a)}{a^3} \]

where \( \alpha \) is the derivative of \( f_p(a) \) with respect to \( a \).

The function \( f_p(a) \) and \( \alpha \) are shown graphically in Figure 8. The function \( f_p(a) \) is positive, as expected. The function \( \alpha \) is always negative. This suggests that, during the flapping phase of the Weis-Fogh motion, if the wing is moving at a constant velocity, then the fluid performs work on the wing and not the other way around! It is likely that this apparent paradox is a consequence of the omission of the leading-edge vortex shedding phenomena.

5. Leading-Edge Vortex Shedding

The analyses of the preceding section omits the presence of vortices shed from the wing's leading edge. Using the present general viscous theory of aerodynamics, the effects of the shed vortices on the aerodynamic forces, moments, and power requirements can all be expressed as functions of the distribution (location and strength) of shed vortices.

To demonstrate the use of the general theory, consider a distribution of shed vorticity \( \omega \) in the \( \zeta \)-plane. Because of symmetry, one has \( \omega_\zeta = \omega_\zeta (\bar{\zeta}) \) in the \( \zeta \)-plane. A complex potential \( W_z \) associated with this vorticity distribution exists in the region free of shed vorticity. This complex potential is
\[ \Phi = 2 \pi \int_{R_w} \ln \left( \frac{\zeta - \zeta_0}{\zeta - \zeta_0} \right) \omega_\zeta dR \]

where \( R_w \) is the region occupied by the shed vorticity in the upper complex \( \zeta \)-plane.

The real part of \( W_z \) vanishes on the \( \zeta \)-axis. The complex potential \( W_z = W_z + W_z \) therefore satisfies the boundary conditions for \( \phi \) stated in Section 4. This complex potential \( W_z \) is given by
\[ W_z = k \int_{R_w} \omega_\zeta (\zeta_0) dR \]

Equation (35) states that the effects of the shed vortices can be considered in a piecewise manner. That is, the region of integration \( R_w \) can be divided into segments and the effects of vorticity within each segment studied individually and the results summed. To illustrate this process, consider the effects of vorticity in a small region, say \( R_{0w} \), around the point \( \zeta = \zeta_0 \).

The total vorticity in this small region is designated \( \delta \zeta \). Consider this total vorticity to be a vortex located at \( \zeta = \zeta_0 \). By reason of symmetry, there exists a vortex of strength \( -\delta \zeta \) at \( \zeta = \bar{\zeta} \), as shown in Fig. 2. The complex potential \( W_z \) for this vortex pair is
\[ W_z = -\frac{\pi}{2} \ln \left( \frac{\zeta - \zeta_0}{\zeta - \bar{\zeta}} \right) \]

In terms of \( \zeta_0 \) and \( \bar{\zeta} \), through Eq. (9).

The vortex strength \( \delta \zeta \) at the wing in the \( \zeta \)-plane is simply the negative of the \( \zeta \)-component of velocity on the \( \zeta \)-axis for \(-1 < \zeta < 1\). This velocity component is easily obtained from Eq. (36). The increment lift \( \delta L \) due to the vortex pair \( \delta \zeta \) and \( -\delta \zeta \) is related to \( \delta \zeta \) through
\[ \delta L = \rho \frac{d}{d\zeta} \int_{-1}^{1} \zeta \delta \zeta d\zeta \]

It can be shown that
\[ \int_{-1}^{1} \zeta \delta \zeta d\zeta = \int_{-1}^{1} \omega_\zeta dR \]

where \( \omega_\zeta \) is the \( \zeta \)-coordinate of the vortex \( \delta \zeta \) and \( k \) is a function of \( \omega_\zeta \). It is clear from Eqs. (37) and (38) that the increment lift \( \delta L \) is dependent upon the vortex strength \( \delta \zeta \) the movement of \( \delta \zeta \) in the \( \zeta \)-direction, and the opening angle of the wing pair through \( k \). If all the shed vorticity in the flow is reasonably represented by a single pair of vortices with known strength and location, then Eqs. (37) and (38) permit the lift due to shed vorticity to be evaluated simply. In general, it is possible to represent the shed vorticity by a series of pairs of vortices. The lift increment due to each pair of vortices can be determined using Eqs. (37) and (38). The sum of all the lift increments and \( \delta L \) given by Eq. (39) is the total lift acting on a Weis-Fogh wing during the flapping phase. Indeed, with a distribution of shed vorticity, the total lift \( L \) is obviously the sum of \( L_1 \) and \( L_2 \), the contribution of the shed vorticity to the lift is given by
\[ L_2 = \int_{R_w} \beta N \omega_\zeta (\zeta_0) dR \]

The total normal force \( N \) and the total power requirements \( P \) are expressible in terms of the potential function and vortex strength in forms identical to Eqs. (24) and (31). These equations, the potential function \( \phi \) is the sum of \( \phi_1 \) and \( \phi_2 \), the real part of \( W_z \). The form of Eqs. (24) and (31) states that the contributions of \( \phi_1 \) and \( \phi_2 \) to the normal force and the power can be individually evaluated and added together to give the contributions of \( \phi \). The total vortex sheet strength \( \gamma \) is the sum of \( \gamma_1 \) and \( \gamma_2 \), the vortex sheet strength due to \( W_z \).

The vortex sheet strength, however, appears in Eqs. (24) and (31) as squared terms. In consequence, the contribution of the vortex sheet \( \gamma \), which represents the unsteady boundary layer on the wing, is not simply the sum of the contributions of \( \gamma_1 \) and \( \gamma_2 \). Therefore, one has
\[ N = N_1 - N_2 - \int_{R_w} \gamma_1 \zeta \zeta_0 dR \]

Similarly, one has
\[ P = P_1 + P_2 - \int_{R_w} \gamma_1 \zeta \zeta_0 dR \]

Closed form expressions for \( N \) and \( P \) have been obtained by the present authors. Expressions for \( S \) and \( T \) are easily obtained from those for \( L \) and \( N \). Because of the length limitation of the present paper, discussions of these results are postponed for the future.
6. Concluding Remarks

The present work has yielded useful results for the lift and the drag acting on a wing undergoing the flapping and the clap phases of the Weis-Fogh motion. Closed-form expressions have been obtained for the unsteady lift, drag, and the power expenditure of the Weis-Fogh wing. These results have led to the identification of major contributors to unsteady aerodynamic forces acting on the wing and the power requirement to sustain the prescribed wing motions.

It should be pointed out that, compared with aerodynamics of rigid lifting bodies, the Weis-Fogh problem is substantially more difficult to treat. For the rigid body problem, the use of the general viscous theory, as stated as Eqs. (3), (4) and (5) in this paper, is particularly convenient since the overall vortical system can be divided into components and the contribution of each component to airloads can be studied individually. Even though the unsteady flow may be in the non-linear domain, the linear addition of the individual contributions gives the correct total aerodynamic force and moment acting on the rigid body. The line of action of the aerodynamic force and consequently the power expenditure are easily obtainable from the total force and moment. For the Weis-Fogh motion, the total force and moment acting on the wing pair can be determined also in a manner similar to that for the rigid body problem. The force and moment acting on each individual wing, however, cannot be evaluated using Eqs. (4) and (3), excepting that the lift on each wing, because of symmetry, happens to be one half of the total lift on the wing pair. In the present paper, the unsteady total normal force acting on the wing and the power expenditure of the wing are determined from the pressure distribution on the wing. The drag force and the leading-edge suction force are determined from the lift and the normal force. Since the pressure field is not a linear function of the vorticity field, the total unsteady force and the power expenditure for each wing is not a simple sum of individual contributions of the several vortical flow components. Eqs. (41), for example, shows that the total normal force is composed of three terms. The terms $N_1$ and $N_2$ are respectively the individual contributions of the wing motion and the shed vorticity (wake). In addition to these two contributions, there is a third contribution due to the interaction of the first two contributors. This third contribution is expressed in the form of an integral in Eq. (42). The integrand of this integral contains the product of $\gamma_1$ and $\gamma_2$. $\gamma_1$ and $\gamma_2$ are the vortex sheets representing the vorticity in unsteady boundary layers resulting from, respectively, the wing motion and the shed vorticity. From Eqs. (42), it is clear that the wing motion and the shed vorticity both contribute to the power expenditure of the wing. In addition, there is a third contribution to the power expenditure due to the interaction of the first two contributors.

The Weis-Fogh problem is a special case of the problem of non-linear unsteady aerodynamics of non-rigid lifting bodies. The present study is, in this context, a precursor to a more comprehensive study of unsteady aerodynamics of flexible lifting bodies. It is anticipated that the development of a routine capability for predicting unsteady aerodynamic behavior in the non-linear domain will require extensive and persistent efforts over a number of years. The work described in this article represents only a few initial steps in search of this capability. In that context, the results of the present study are encouraging in that the recently developed general viscous theory of aerodynamics is shown to be well suited for the theoretical study of unsteady aerodynamic problems, including those associated with flexible lifting bodies.

Acknowledgements

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References

Figure 1. Weis-Fogh Motion

Figure 2. Wings in Transform Plane

Figure 3. Free Body Diagram

Figure 4. Weis-Fogh Lift Functions

\[ \frac{L_1}{\rho c^3} = \pi f_{L_1} + \Omega^2 g_{L_1} \]

Figure 5. Weis-Fogh Drag Functions

\[ \frac{D_1}{\rho c^3} = \pi f_{D_1} + \Omega^2 g_{D_1} \]

Figure 6. Weis-Fogh Normal Force Functions

\[ \frac{N}{c^3} = \pi f_{N_1} + \Omega^2 g_{N_1} \]
Figure 7. Weis-Fogh Leading-Edge Suction Functions

\[
\frac{S}{\rho c^3} = \frac{2}{\pi} \alpha S_{s1}
\]

Figure 8. Weis-Fogh Power Functions

\[
\frac{p_l}{\rho c^2} = \frac{2}{\pi} \alpha P_{pl}(\alpha) \cdot \frac{3}{\pi} P_{pl}(\alpha)
\]
THEORETICAL INVESTIGATION OF DYNAMIC STALL USING A MOMENTUM INTEGRAL METHOD

E.J. Jumper
and
J.E. Hitchcock
Air Force Institute of Technology
Wright-Patterson AFB, Ohio

Abstract

An analytical study into the gust response of an airfoil is presented. The momentum-integral equation for steady flow is extended into the unsteady flow regime to predict the behavior of an airfoil that experiences a constant-rate-of-change of angle-of-attack gust. The von Karman-Pohlhausen method of integration is successfully modified to incorporate the additional transient flow terms: the equation of closure necessary to do this is also presented. Finally, computation of the flow about a Joukowski airfoil using the new equations is performed and the results are presented and discussed. It will be shown that these results are in agreement with existing experimental data.

In 1932, Max von Kramers published the results of wind-tunnel experiments simulating a wing encountering a constant-rate-of-change of angle-of-attack gust. His results demonstrated that the wing encountered stall at higher angles of attack than in the static-stall case and the maximum coefficient of lift, C_max, was increased a proportionate amount to the increased stall angle of attack. Since that time, a number of empirical studies have been undertaken in the general area of unsteady flow, but referred to as dynamic stall.

Gusts, however, have concentrated on the constant-amplitude of the angle-of-attack, i.e., gust. This work is the work of Deereus and Kuebler, which was a flow visualization study and more recently by Daley which was a combination flow visualization-pressure measurement study. These studies quantified only the stall angle of attack as a function of the wind and the constant i. A summary of the results of references 2 and 3 is shown in Figure 1. It is interesting that the Kramer results and the results of references 2 and 3, while agreeing in trend, do not agree in the extent to the dynamic-stall effect; the effect exhibited in reference 3 is approximately twice that of Kramer's experiment. It is also noted that these two experiments differed in what is probably a critical way. The Kramer experiment had the wind fixed in Newtonian space with a rotating free stream, while the Deereus and Kuebler and Daley experiments used a rotating wind rather than a constant velocity free stream.

Little analytic work has been performed on understanding the physics of the phenomena. That which has been in the area of full Navier-Stokes solutions which amount to computational experiments. Such numerical experiments lead to information which is similar to that obtained from experiments like those mentioned above, but because of inclusion of all possible effects, the solutions are of limited use in understanding the critical physics of the phenomena that may be contributing the effect.

Because of their simplicity, integral methods have been helpful in understanding the interplay of the physical phenomena leading to stall. Until now, however, these methods could not be applied to unsteady flow because the closure equation necessary to perform the integrations was not available. This paper presents an extension of the integral method to unsteady flow, i.e., Kramers' results presented are the results of a study using the method to analyze a Joukowski airfoil for the case equivalent to that of the Kramers experiment where the wing remains fixed in Newtonian space.

II. Integral Method Extension to Unsteady Flow

It can be shown that the momentum-integral equation for the boundary layer in unsteady flow is given by:

\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{F} \]

where \( \rho \) is the density, \( \mathbf{u} \) is the velocity, \( p \) is the pressure, \( \mu \) is the viscosity and \( \mathbf{F} \) is the force per unit mass. The term \( \nabla^2 \mathbf{u} \) is the Laplacian of the velocity and the term \( \rho \mathbf{F} \) is the force per unit mass of the airfoil. Thus, the form of the equation is similar to that for steady flow, and the only difference lies in the last term. However, it is not necessary to assume that the flow is steady in order to make use of the momentum-integral equation. In fact, it is possible to solve the equation exactly for arbitrary unsteady flows.

Figure 1 - Summary of data for increase in stall vs. reduced angular rate, taken from reference 3.

Fig. 1 - Summary of data for increase in stall vs. reduced angular rate, taken from reference 3.

...
The introduction of the transient terms into the momentum-integral equation requires an additional equation which, until now, has not been known. This closure equation, however, may be arrived at as follows for the case in which transients are not too severe. First the observation is made that the displacement thickness is related to the boundary layer thickness by

$$ \delta_1 = C_1 \delta $$

Assuming $C_1$ is slowly varying in time,

$$ \frac{\partial \delta_1}{\partial t} = \frac{\partial \delta}{\partial t} $$

Further, for all laminar flows the boundary layer thickness is related to the potential flow by

$$ \frac{1}{\sqrt{U}} = C_1 \sqrt{\frac{1}{V_x}} $$

so that,

$$ \frac{\partial}{\partial t} \left( \frac{1}{\sqrt{U}} \right) = C_1 \frac{\partial}{\partial t} \left( \frac{1}{\sqrt{V_x}} \right) \left( \frac{1}{\sqrt{V_x}} + \frac{1}{\sqrt{V_y}} \right) $$

(12)

such that the velocity profile is

$$ \frac{U}{U_0} = \left( \frac{2}{3} \right) \left( \frac{1}{V_x} - \frac{1}{V_0} \right) \left( \frac{1}{V_x} + \frac{1}{V_y} \right) $$

(13)

Eq (13) may now be used to evaluate $\delta_1$ and $\gamma$ from Eqs (2) and (3)

$$ \frac{\delta_1}{\gamma} = \frac{2}{10} \frac{1}{120} $$

(14)

and

$$ \frac{\delta_1}{\gamma} = \frac{2}{375} \frac{1}{405} - \frac{1}{2} $$

(15)

These equations, Eqs (14) and (15) may be combined to yield

$$ \frac{\delta_1}{\gamma} = \frac{2}{10} \frac{1}{120} \left( \frac{2}{375} \frac{1}{405} - \frac{1}{2} \right) $$

(16)

Further, Eq (16) may be used to evaluate the right hand side of eq (19).

$$ \frac{\delta_1}{\gamma} = \frac{2}{10} \frac{1}{120} \left( \frac{2}{375} \frac{1}{405} - \frac{1}{2} \right) $$

(17)

The functions $\gamma$ and $\delta_1$ may be written in terms of a new parameter, $\pi$, which is an explicit function of $\eta$

$$ \delta_1 = \frac{2}{375} \frac{1}{405} - \frac{1}{2} $$

(18)

such that

$$ \pi = \frac{\delta_1}{\gamma} $$

(19)

and

$$ \pi, \delta_1 \pi = \pi, \delta_1 $$

(20)

Eq (18) may now be written in terms of these two functions as

$$ \delta_1 = \delta_1 \delta_1 \left( \frac{2}{375} \frac{1}{405} - \frac{1}{2} \right) $$

(21)

where $\pi = \delta_1 / \delta_1$ we further define a function as

$$ \pi, \delta_1 \pi = \pi, \delta_1 \delta_1 $$

(22)

Eq (21) may be expanded in such that

$$ \pi = \delta_1 / \delta_1 $$. 

(23)
and

\[ K = ZK \frac{\partial U}{\partial x} + \frac{1}{U} \frac{\partial U}{\partial t} \]  

(24)

Eqs (12), (18), (19), (20), (22), (23), and (24) now constitute a modified set of equations similar to the steady state equations which may be stepwise numerically integrated in a manner equivalent to the von Karman-Pohlhausen method.

III. Application to Constant-\( \alpha \) Gust

The method outlined above was applied to the case of a 1\% thick symmetrical Joukowski airfoil experiencing a constant-\( \alpha \) gust. The unsteady potential flow field was solved for the Joukowski airfoil to provide the flow parameters \( U_c, \alpha_c, \beta_c, \) and \( \phi \) needed for the momentum-integral-method solution of the boundary layer. The transient potential field was approximated as pseudo-steady by neglecting the starting vortices, an effect which we suspect is small but which is the subject of a separate investigation to be published at a future date.

In order to determine the stall angle, separation at the quarter chord was defined as stall for the purposes of this investigation. The angle of attack at stall was determined for a number of flow conditions and constant \( \alpha_c \), and the results plotted as \( \alpha_c \) vs the reduced angular rate, where \( \alpha_c \) is determined as the angle at which the steady state case and the reduced angular rate, \( \alpha' \), is given by

\[ \alpha' = \frac{\alpha_c}{U_c} \]  

(25)

where \( U \) is the chord. The results are shown in Figure 3.

IV. Comparison to Experiment

Although the Kramer experiment used a different airfoil, the similarity of the leading edge geometries leads one to suspect that the results should be generally comparable. The effect shown in Figure 2 was compared to the results of reference 1 as follows. Kramer's result may be written as

\[ C_{\text{max dyn}} = C_{\text{max st}} + 0.36 \frac{C_{\text{\alpha}}}{U_c} \]  

(26)

where \( \alpha \) is in radians per second. The results of Figure 2 may be written as

\[ \alpha_{\text{stall dyn}} = \alpha_{\text{stall st}} + 0.096 \frac{C_{\text{\alpha}}}{U_c} \]  

(27)

If written for \( \alpha \) in radians per seconds. Since these results are not in terms of \( C_{\text{\alpha}} \), they are not directly comparable to Eq (26). However, if we assume that the effect of increasing \( \alpha_c \) to simply displace the \( C_{\alpha} \) vs \( \alpha_c \) curve by a comparable amount (see Figure 3), then \( C_{\text{\alpha}} \) may be arrived at by simply multiplying by the slope of the un stalled \( C_{\alpha} \) vs \( \alpha_c \) curve. This assumption is similar to that of the pseudo-steady-state assumption in that the effect of the starting vortices is assumed negligible.

The comparison to Kramer's result, Eq (26), is remarkable.

![Graph](image)
In regard to the first, there is still work to be done in establishing the limits to the applicability of the assumptions leading to Eq (8), but the tools for such a study are available in the solution itself, and work in this direction is already underway. Sensitivity studies carried out to date have shown that the worst case errors incurred from these assumptions change the slope of the curve of Figure 2 by less than 10%. In regard to the second, we now have available in the unsteady momentum-integral method a tool to help understand the pieces of physics which are at play in delaying the stall, and although not detailed here, it is clear from plotting the various modified Pohlhausen parameters from the solution that the transient term in the external flow is balancing the unfavorable pressure gradient to delay separation. Much more is to be learned from further exploitation of the unsteady method because of the fact that our solution is not at all limited to constant-$\alpha$ cases.

Finally, it should be mentioned that we have also begun work on trying to adapt the method to a non-Newtonian boundary. It is too early to say whether this work will be successful, but the intention is to address the discrepancies in the experiments so graphically demonstrated by a comparison of Figures 1 and 2.

References


PRELIMINARY RESULTS FROM THE UNSTEADY AIRFOIL MODEL USTAR2

J. H. Strickland*
J. W. Oler**
E. J. Im
Texas Tech University
Lubbock, Texas 79409

Abstract

Preliminary results from an Unsteady Airfoil analysis in 2 dimensions have been obtained from a computer code (USTAR2) developed by the present authors. This computer code is based upon an analysis which utilizes a doublet panel method to model the airfoil surface, an integral unsteady boundary layer scheme to model the viscous attached flow, and discrete vortices to model the detached boundary layers which form the airfoil wake region. This model has been used to successfully predict steady lift and drag coefficients as well as pressure distributions for several airfoils with both attached and detached boundary layers. In addition, calculations have been made for a limited number of cases for both attached and detached unsteady flow situations. These calculations are compared in a cursory way with experimental data to point out some of the strengths and weaknesses of the present formulation.

Introduction

Unsteady aerodynamics is an important phenomenon which has been studied more intensively in recent years. These studies have been made in connection with applications pertaining to helicopters, axial flow turbines and compressors with inlet distortions, vertical axis wind turbines, and missiles and fixed-wing aircraft undergoing rapid maneuvers. Lifting surfaces subjected to time-dependent freestream velocity or time-dependent body motions may, in some cases, also have significant stall regions on their surfaces.

A number of approaches have been taken with regard to the prediction of unsteady stalled airfoils. Most of these approaches have been reviewed by McCroskey10. In general, these approaches range from empirical models11,6 to models based on the Navier-Stokes equations12,9. The empirical models are generally applicable to small sinusoidal pitch oscillations about some relatively low angle of attack. The Navier-Stokes solutions tend to consume large amounts of computer time and are usually limited to low Reynolds number solutions. Unfortunately, there are few models which can be considered as representing something in between the extreme cases of almost total empiricism and the lengthy more exact solutions of the Navier-Stokes equations. There have been several boundary layer codes developed which can be used to predict some of the behavior associated with unsteady stall13,14. There are also been models of the potential flow behavior related to the shedding of leading edge vorticity as typified by the work of Ham12. More recently, Katz13 simulated the unsteady separated flow over a thin cambered airfoil. The models of both Ham and Katz required empirical information regarding the appearance and position of the separation point.

USTAR2 Analysis

Recently, the present authors formulated and began development on an analytical model7 which is potentially capable of predicting dynamic effects for stalled and unstalled airfoils undergoing arbitrary airfoil motions. This model does not require input of airfoil section data and may thus be used to examine arbitrary airfoil shapes. An Unsteady Airfoil model in 2 dimensions based on this analysis has been implemented via a computer code (USTAR2). Execution times for this code are short when compared to Navier-Stokes solutions and little empiricism is required. In order to validate this analysis, however, comparison between USTAR2 predictions and experimental data must be made for a number of cases. This paper presents some of the preliminary comparisons.

The USTAR2 model is a two-dimensional incompressible formulation which is based on state-of-the-art methods with some extensions. The potential flow regions near the airfoil are modeled using the doublet panel analysis of Ashley14 which consists of a Green's function representation of the potential flow resulting from the motion of the airfoil and the presence of associated trailing wakes. This potential flow is based upon the Laplace equation for the velocity potential 

\[ \nabla^2 \Phi = 0 \]  

(1)

which is valid for both steady and unsteady flow. By Green's theorem, a solution to (1) may be represented by integrals over the 'boundaries' of the flow where these boundaries are replaced by surfaces across which potential jumps occur. These surfaces, as depicted in Figure 1, are represented by the airfoil surfaces and the wake sheets which spring from the trailing edge and any separation point. With Green's theorem, the disturbance potential at any field point \( r \), due to the airfoil and wake surfaces, may be written as

\[ \Phi(r) = \int_{S_{Airfoil}} \frac{1}{2} \left[ \frac{1}{r} \right] dS + \int_{S_{Wake}} \frac{1}{2} \left[ \frac{1}{r} \right] dS \]  

(2)
where \( \phi \) and \( \psi \) are doublet distributions on the airfoil and wake surfaces \( S \) and \( W \) respectively, \( n \) is the distance normal at the source point, and \( R \) is the distance separating the field point and source point. Boundary conditions include a kinematic surface tangency condition given by

\[
\frac{\partial \phi}{\partial n} + (\hat{u}_s - \hat{u}_n) \cdot n = 0 \quad \text{on } S
\]

where \( \hat{u}_s \) and \( \hat{u}_n \) are the freestream and airfoil surface velocities respectively and \( n \) is the outward normal to the airfoil surface. An additional boundary condition is the "trailing edge flow condition" which, in the present model, requires that the flow direction at the trailing edge be along the trailing edge bisector. Equations (2) and (3) are solved for \( \phi \) and \( \psi \) by first eliminating \( \psi \) to form a single equation in \( \phi \). The airfoil and wake surfaces are then discretized to form a set of linear algebraic equations in terms of the unknown \( \phi \). The potential \( \psi \) is then obtained from (2). The pressure distribution around the airfoil is obtained from the unsteady Bernoulli equation. Details of this analysis are presented in reference 14.

Figure 1. Schematic of Boundary (Vortex) Sheets for Unsteady Pitching Motion with Separation

The primary function of the boundary layer analysis is to predict the presence and location of any boundary separation point on the airfoil surface. The pressure gradient and edge velocity distributions, which are used in boundary layer calculations, are obtained from the potential flow model which may include sheets of vorticity shed from the boundary layer separation point. Therefore, a strong coupling between the boundary layer analysis and potential flow analysis exists for separated flow situations. The turbulent boundary layer analysis used in the present work is essentially that due to Lyrio, Fazliger, and Kline15. This unsteady integral technique gives excellent results for the steady flows of Tillman, Harring, and Norbury; Stratford, Samuel and Houbert (see Coles and Hirst, reference 17); Kim16; Stapp and Strickland19; and Vignarath (see Kim, reference 18). More importantly, this method predicts the unsteady boundary layer data of Karlson and Houdeville; et al.21 and compares well with the finite difference methods at McCroskey and Phillips22 and Singleton and Nash23.

An abundance of data for steady flow over airfoils exists and can be used to check the ability of the USTAR2 model to predict such flows. One such case is shown in Figure 2 where the data of Sheldahl and Klimas26 are compared with USTAR2. Unsteady data are, on the other hand, relatively scarce. In the present paper the unsteady experimental data consists of some recently obtained data for an airfoil undergoing a pitch up motion from zero angle of attack. In each case the pitching rate, \( \omega \), remains constant. These data consist of airfoil surface pressure data, flow visualization data, and surface hot-wire data. All data were obtained in the USAF Academy low-speed 2 ft x 3 ft wind tunnel. The experimental arrangements will be briefly described.

Figure 2. Lift and Drag Coefficients for a NACA 0015 Airfoil with Re = 8 x 10^6

Airfoil surface pressure data were obtained by Francis, Hersee, and Retelle27. Details of the test setup can be found in that reference. A computer controlled pitch oscillator was used to impart constant \( \omega \) pitching to a 5-inch chord NACA 0012 airfoil. Pressure taps were located at 19 positions along the surface of the airfoil. Approximately 25 repetitions of each case were run so as to obtain ensemble averages of the surface pressure coefficients at the pressure ports. These data were then used to obtain lift and drag coefficients for the airfoil as a function of time.

Flow visualization data were obtained by Walker, Melin, and Strickland28. In this work, the flow around a 5-inch chord NACA 0015 airfoil was examined using a smoke wire placed across the
tunnel upstream of the pitching airfoil. The wire was placed in a plane normal to the axis of rotation of the pitching airfoil. A smoke producing oil (Roscollne) was coated on a 0.005 inch diameter tungsten wire which was in turn heated electrically to produce a large number of smoke streaks in the flow. These streaks have a rather uniform spacing due to the regular spacing of smoke material droplets which are formed when the wire is coated. The smoke was illuminated by a high intensity strobe light placed downstream of the airfoil. The proper sequencing of airfoil pitch commands with strobe light and smoke wire triggering was accomplished by computer control.

A 35 mm camera looking along the pitch axis was used to record the visual data.

In the work by Walker, Helin, and Strickland, a NACA 0015 airfoil with a 6-inch chord was instrumented with an array of hot-wires. As indicated in Figure 3, seven hot-wires were mounted on the upper surface of the airfoil (suction side). The hot-wire sensing elements (TSI-10 hot films) were soldered to pairs of number 9 sewing needles which protruded above the airfoil surface approximately 0.20 inches. The needle supports in turn were mounted in electrically insulated plugs which were machined flush with the airfoil surface. A TSI model 1050 hot-wire anemometer system, along with an in-house linearizer, were used to obtain velocity signals. Approximately 25 repetitions of each case were run to obtain ensemble averages of the velocity signal from each probe.

Discussion of Results

Lift and drag data are shown in Figure 4 for a NACA 0012 airfoil pitching up from a zero angle of attack at a non-dimensional pitching rate \( \Gamma \) of 0.089. The non-zero lift coefficient at zero angle of attack is due to the so-called "pitch circulation." The slope of the lift curve in this sub-stall region can be seen to be considerably less than for the steady case due to the downwash on the airfoil produced by the vortex sheet springing from the trailing edge. The results from a simple analysis given in Reference 29 are seen to agree with the experimental lift and drag very well up to and a little beyond the stall angle which is approximately 10 degrees.
The wake geometry obtained from the USTAR2 analysis is compared in Figure 6 to flow visualization data. The vortex sheets obtained in the USTAR2 analysis basically represent "streaklines" in that most of the fluid particles which make up these sheets were either injected into the flow at the leading or trailing edge. While exact comparisons between visual and calculated results are difficult to make, it does appear that the predicted large scale vortex is not as tightly rolled up as that indicated by flow visualization. This again indicates that the discrete wake vortex cores are growing at excessive rates in the USTAR2 analysis.

Velocities obtained from three of the seven surface mounted hot-wire probes are shown for a particular case in Figure 7. Near the nose of the airfoil (7% chord) the velocity measured by the probe rises until the probe becomes immersed in the separated boundary layer which occurs at an angle of attack of about 19°. The probe is located above the airfoil surface about 3% of a chord length. The calculated edge velocity drops much sooner since the separated vortex sheet passes over the 7% chord position almost immediately after the boundary layer separates from the nose. Therefore, agreement between experiment and analysis is at least qualitatively correct at the 7% chord position. For this particular flow situation, the experimental work indicates that a significant region of reversed flow first appears at an angle of attack of about 2% to 3% degrees somewhat downstream of the nose (20% chord). The magnitude of the reversed flow reaches a maximum at about the 60% chord and has a value of about 1.5 times the freestream velocity. Further downstream (87% chord), the reverse flow due to the initial large scale vortex passage is reduced and occurs at a higher angle of attack. As can be seen from Figure 7, results obtained from the USTAR2 analysis at 60% chord are in poor agreement with the experimental data above an angle of attack of 25 degrees. The prediction of reversed flow due to the vortex passage lags that of the measured data by a considerable length of time. The immediate reasons for this lag are not clear, but may also be associated with an incorrect discrete vortex core growth rate.
Conclusions

Preliminary results from the USTAR2 analysis are encouraging, but indicate that additional requirements are necessary for satisfactory predictions at large angles of attack for the cases studied. In-depth correlations between the surface pressure data, flow visualization data, and surface velocity data should be made in order to more completely understand the constant $\frac{r}{2}$ flows used in this paper to test the validity of the USTAR2 analysis.

Acknowledgement

The authors would like to thank Sandia National Laboratory, Albuquerque, New Mexico for their past and present support on contracts 74-1218 and 52-3727 as well as acknowledging the AFOSR support given to one of the authors in the form of an AFOSR/SCREE Summer Faculty Fellowship.

References


EXPERIMENTS ON CONTROLLED, UNSTEADY, SEPARATED TURBULENT BOUNDARY LAYERS

William C. Reynolds
Stanford University

Rangarajan Jayaraman
IBM Scientific Center

Lawrence W. Carr
NASA-Ames Research Center

Abstract

Experiments on turbulent boundary layers subjected to controlled unsteadiness have been performed in a special water channel. The flow is steady in the development section upstream of the unsteady test section, where the boundary layer is subjected to an oscillating adverse free-stream velocity gradient sufficient to induce flow reversal near the wall. Measurements of the mean, oscillatory, and turbulence components of the streamwise velocity in the boundary layer indicate that the mean velocity and mean turbulent stresses are unaffected by the oscillation, whereas the periodic components of these quantities are strongly dependent upon frequency. At low frequencies the boundary layer behaves quasistatically; at intermediate frequencies the boundary layer behavior correlates with the Strouhal number based on the length of the unsteady region; and at high frequencies the outer region of the boundary layer behaves as a Stokes layer described by laminar equations. Reverse flows occur in this Stokes layer, but the boundary layer remains thin and hence attached to the surface. Although the boundary layer would separate from the surface at zero frequency, separation is prevented by rather slow oscillations, and hence unsteadiness can be used as a means for separation control.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U )</td>
<td>amplitude of ( \delta )</td>
</tr>
<tr>
<td>( K )</td>
<td>( (-U_{\infty})/(dU_{\infty}/dx) ), acceleration parameter</td>
</tr>
<tr>
<td>( L )</td>
<td>length of unsteady region, ( 0.61 ) m</td>
</tr>
<tr>
<td>( S )</td>
<td>( \sqrt{\pi/\alpha} ), Stokes layer thickness, ( m )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>phase of ( \delta )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( \nu/\alpha ), momentum thickness Reynolds number</td>
</tr>
<tr>
<td>( \beta_{u} )</td>
<td>( u/(x-x_{0})/\alpha )</td>
</tr>
<tr>
<td>( \beta_{r} )</td>
<td>( r/\alpha )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>streamwise velocity, ( m/s )</td>
</tr>
<tr>
<td>( \tilde{\sigma} )</td>
<td>periodic component of ( \sigma )</td>
</tr>
<tr>
<td>( \bar{\sigma} )</td>
<td>mean component of ( \sigma )</td>
</tr>
<tr>
<td>( \bar{\sigma} )</td>
<td>phase average of ( \sigma )</td>
</tr>
<tr>
<td>( U_{*} )</td>
<td>local free-stream velocity, ( m/s )</td>
</tr>
<tr>
<td>( U_{0} )</td>
<td>free-stream velocity at start of the unsteady region, ( m/s )</td>
</tr>
<tr>
<td>( X, X' )</td>
<td>streamwise coordinate, ( m )</td>
</tr>
<tr>
<td>( x_{0} )</td>
<td>( (x-x_{0})/L )</td>
</tr>
<tr>
<td>( y )</td>
<td>distance from surface, ( m )</td>
</tr>
<tr>
<td>( \delta )</td>
<td>boundary layer thickness (Coles), ( m )</td>
</tr>
<tr>
<td>( \nu )</td>
<td>kinematic viscosity, ( m^{2}/s )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>frequency, ( rad/s )</td>
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Introduction and Objectives

Unsteady turbulent boundary layers have many important applications. Prediction of the behavior of such boundary layers requires models, which need physical insight for their development and sound data for their evaluation. The Stanford unsteady boundary layer research program is designed to meet this need.

The objective of this program is the development of understanding of unsteady turbulent boundary layers and the collection of definitive data for the development and evaluation of predictive models. This paper summarizes results pertinent to this workshop; for full details see Ref. (1).

Experimental System

The experiments are performed in a special water channel shown in Fig. 1. Water was chosen as the working fluid for ease of flow visualization, laser anemometry, and control of unsteadiness over a wide range of pertinent frequencies.

A unique feature of our apparatus is the maintenance of steady flow in the region upstream of the unsteady flow test section. This makes the boundary layer entering the test section a standard, two-dimensional, flat-plate turbulent boundary layer, and gives the modeler a very well known inflow condition. The unsteady flow takes the form of oscillation (or step change) in the free-stream velocity gradient between zero (flat plate) and an adverse condition.

In order to maintain the desired steady flow upstream of the unsteady test section, the channel flow rate must be held constant. This is accomplished by presenting a constant resistance to the flow and by fixing the inlet and discharge pressures. The fixed resistance is provided by slots in the discharge plate (osc. plate in Fig. 1), and a constant-head tank and open jump fix the pressures.

The unsteady behavior in the test section is treated by control of the way in which the fluid leaves the test section. A bleed plate on the channel wall opposite the test surface and similar plate at the end of the recovery section can be used to remove all or part of the flow. By oscillating the slotted flow-control plate back and forth, the relative amounts of flow extracted through these two bleed plates is oscillated, while maintaining the total flow fixed, and this provides the controlled unsteadiness. With a uniform exit bleed from the wall opposite the test boundary layer, the free-stream velocity seen by the test boundary layer decreases linearly in the downstream direction, and with no opposite-wall bleed the free-stream velocity remains uniform. Hence, the (initial) behavior of the free-stream velocity seen
by the test boundary layer as shown in Fig. 2. The actual variation deviates slightly from this design target, as we shall discuss shortly.

The inlet section was carefully developed to produce a very two-dimensional, quiet inlet flow. Boundary layer suction removes the nozzle boundary layers, and suction from the wall opposite the test surface is used to maintain a uniform free-stream velocity over the test section in the developing region. The side-wall boundary layers are removed at the entrance to the test section, the intent being to maintain two-dimensionality of the flow in the unsteady test section and downstream recovery region (see Ref. 1 for documentation).

Measurements of the streamwise velocity component have been made using a single-component DISA Laser Doppler Anemometer with tracker. The LDA signals are prefiltered to remove bias noise and then examined in real time with a dedicated MINC microcomputer. The mean, turbulence, and periodic components of the signal are extracted, and then these are decomposed into three components, and is the turbulence. The time-average is extracted by averaging over a large number of cycles at the same phase angle. See Ref. 1 for procedural details.

Ref. 1 and its associated microfiche contain approximately 1000 graphs and tables of data from these experiments. A digital tape for use by modelers is also available.

Free-Stream Dynamics

Figure 3 shows the time-average free-stream velocity in the test region over the range of frequencies. Note that the free-stream velocity gradient is linear as intended and essentially independent of frequency. The acceleration parameter corresponding to this gradient is

\[
K = \frac{dU}{\delta} = -1.95 \times 10^{-7}
\]

This represents a strong adverse-pressure gradient for which, in a steady flow, one would expect the development of a separation.

An important observation is that the boundary layer remained detached during all of these unsteady experiments. However, when we set the frequency to zero (at the mean free-stream gradient shown in Fig. 3) a separation builds up over a period of a minute or so, and the boundary layer leaves the surface and never reattaches in the test section. Therefore, we could not conduct experiments at zero frequency for this case (we could not initialize for the high-amplitude case). It is interesting that separation is prevented in experiment at the lowest operating Strouhal numbers.
Figure 4 shows the amplitude of the unsteady component in the free stream. Note that the amplitude grows linearly from the start of the test section, as per the design target (Fig. 2). However, unlike the design target the phase of this variation is not constant in the streamwise direction. Figure 5 shows the phase lag; note that the lag decreases downstream in the test section, and over the second half of the unsteady region is very small at all frequencies. These phase lags are probably not important in discussing the qualitative aspects of the boundary layer behavior, but probably should be considered by modelers comparing a prediction to the data.

**Time-Averaged Behavior**

Figure 6 shows the time-average velocity profiles in the boundary layer near the end of the unsteady section. The scales are staggered to prevent confusion; when these data are laid on top of one another, very little difference can be seen, indicating that the mean velocity profile is essentially unaffected by the oscillation frequency.

Figure 7 shows the time-average of the streamwise component of Reynolds stress, normalized on the local free-stream velocity at the outer edge of the boundary layer, near the end of the unsteady section. Again we see that the time-average turbulence profiles are essentially all the same, indicating no significant effect of the oscillation on the time-averaged turbulence.

The observation that the time-average flow is not significantly affected by the oscillation is consistent with other recent observations in unsteady boundary layers. It also explains why prediction methods based on steady flow models often do very well in predicting transitory stall phenomena.

**Unsteady Component**

Figure 8 shows the amplitude of the unsteady component at the end of the test section, normalized on the free-stream velocity. Here a significant variation with frequency is seen. Note that at low frequencies the amplitude is greater in the boundary layer than in the free stream, by over a factor of 2 at 0.1 Hz. At high frequencies the amplitude is uniform over most of the layer, indicating a slug-like motion. Note the log-linear region at low frequencies, suggestive of retained law-of-the-wall behavior in this range.

Figure 9 shows the phase of the unsteady component. Note particularly the high-frequency region, where the outer region of the boundary layer shows in phase with the free stream, and there is a rapid change in phase near the surface. The phase lead near the surface approaches 90° as the frequency is increased; it is interesting that this is the theoretical phase lead for a laminar boundary layer.

The phase-averaged velocity profiles, for St = 0.1, can be computed from the time-average of periodic components at any desired phase (directly) by phase-averaging the data. At low frequencies these profiles show the reversal near the wall. However, as the frequency is increased the flow near the wall periodically reverses in this region near the surface.

Figure 10 shows the phase-average profiles at 2 Hz over a portion of the cycle. Note that the lowest three curves display reversed flow near the wall. At this frequency and location the flow reversal occurs from about 130° to 210°, and is greatest at about 165°, consistent with the phase lead shown in Fig. 9.

**Flow Regimes**

Analysis of the governing equations and the data suggests that there are three regimes of flow in unsteady boundary layers. These can be expressed in terms of the time scale for the oscillation,

\[ T_0 = \frac{1}{\omega} \]

the time scale for the unsteady region,

\[ T'_0 = (x - x_0) / U_0 \]

and the time scale for the large eddies

\[ T_e = \frac{L}{U_0} \]

The ratios of these time scales define the two pertinent Strouhal numbers,

\[ St_x = T'_0 / T_0 \]
\[ St_t = T_e / T_0 \]

When \( T_0 \) is long compared to \( T'_0 \) and \( T_e \), both the overall flow and the eddies have time to respond to the change, and the flow behaves quasi-statically. Since the zero-frequency flow here is a separation flow, and the oscillating flows are not, it would seem that this regime was not reached in the experiments, and hence that the quasi-static regime requires Strouhal numbers at least as low as

\[ St_x < 0.04 \]
\[ St_t < 0.2 \]

When \( T_0 \) is comparable with \( T'_0 \), the eddies have time to respond and the entire boundary layer should reflect the influence of the oscillation. As a result, integral quantities such as the displacement thickness vary throughout the cycle. The equations (Ref. 1) suggest that the behavior should correlate on \( St_x \). Figure 11 shows the displacement thickness as a function of \( St_x \), normalized by the amplitude of the free-stream fluctuation, for the full set of high-amplitude and low-amplitude oscillations. Note that for \( 0.5 < St_x < 3 \) the correlation on \( St_x \) is excellent, indicating that \( St_x \) is indeed the correct correlating parameter for this range of flows (a similar plot vs \( St_t \) does not collapse the data). Note also that higher frequencies the displacement thickness variation becomes independent of \( St_x \).

When \( T_e \) is short compared to \( T'_0 \), i.e., \( St_t \) is sufficiently high, the eddies do not have time to respond to the oscillation. Our data suggest that this occurs approximately for \( St_t > 0.5 \). It is in this region that the displacement thickness becomes independent of \( St_x \).

Analysis of the equations of motion (Ref. 1) suggest that at high frequencies the laminar Stokes equations should describe the unsteady component in the boundary layer, and we indeed find this to be the case. Figure 12 shows the amplitude of the periodic component, normalized on the free-stream velocity amplitude, plotted on the semi-log axis (to more clearly show the collapse of the data for a whole range of \( St_t \) and the excellent agreement of these data with the...
analytical solution of the Stokes equation. The same holds for the phase (Fig. 13).

We conclude that the flow reversal encountered at high frequencies is simply the Stokes layer response of the viscous region near the wall, and that the dynamics of this layer is not coupled significantly to the turbulence for \( \text{St} > 0.5 \).

Concluding Remarks

Three regimes of boundary layer behavior in unsteady flow have been identified, and boundaries for these regimes suggested. Important work remaining to be done includes direct measurement of the turbulent shearing stress in unsteady boundary layers and direct measurement of the wall shear stress. We are now beginning to conduct these measurements at Stanford, and in addition are exploring the interesting case of a sudden step change in more detail. We hope to extend our program to three-dimensional unsteady effects in the future.

Acknowledgements

Dr. Pradip Parikh did the detailed design of the experimental facility and conducted important preliminary experiments, and has provided much helpful advice throughout this program. The work is supported by the Army Research Office and by the Aeromechanics Laboratory, RTL (AVRADCOM); companion work contributing to the facility has been supported by NASA-Ames Research Center and by the National Science Foundation.

Reference

Figure 3. Time-average free-stream velocity distribution

Figure 4. Amplitude of the periodic component of the free-stream velocity

Figure 5. Phase of the periodic component of the free-stream velocity

Figure 6. Time-averaged turbulence profiles in the boundary layer

Figure 7. Time-averaged turbulence profiles in the boundary layer
Figure 1. Amplitude of the periodic component of velocity in the boundary layer.

Figure 2. Phase of the periodic component of velocity in the boundary layer.
Figure 11. Amplitude of the periodic component of the displacement thickness, normalized on the amplitude of the free-stream velocity oscillation, for all oscillatory experiments.

Figure 12. Amplitude of the periodic component in the Stokes layer, normalized on the amplitude of the free-stream velocity oscillation, for all high-frequency oscillatory experiments.

Figure 13. Phase of the periodic component in the Stokes layer, relative to the phase of the free-stream velocity, for all high-frequency oscillatory experiments.
GENESIS OF UNSTEADY SEPARATION

Ho, Chih-Ming*
University of Southern California
Los Angeles, California 90089-1454

Abstract

Unsteady separation is a problem of great technical importance but with little basic understanding. A very limited amount of experimental data is available because of the difficulties involved in measuring the temporally evolving separated flows. In this presentation we first examine the flow field of a downstream moving separation in detail and then we explore the possibility of applying the learned physical mechanism to the upstream moving separation problems and to the unsteady separation on a lifting surface. 

Introduction

The unsteady separation was recognized to be intrinsically different from steady separation in the mid 1950's. Rott, Sears and Moore pointed out that the vanishing of the wall shear stress could not be the criterion for unsteady separation. Instead separation should occur at the zero shear stress location in a coordinate system convected with the separation speed. This has been called the MRS criterion since then. The obvious problem in this criterion is that the separation speed is not known a priori. Large amounts of theoretical effort have been spent on this challenging topic. The most complete summary is the monograph by Teleionis. Extensive experimental works are available in studying the effects produced by unsteady separation on lifting surfaces.

Experimental investigations on the processes involved in unsteady separation however are very few. The main problem is that the velocity field needs to be well documented, and severe limitations exist in measuring the time evolving separated velocity field. Separation usually involves two or even three spatial variables. In the unsteady case, time is an additional variable. Therefore the measurements require the use of many probes as well as large and fast data storage systems. Furthermore, reverse flow occurs in most separations and the well developed hot wire anemometry cannot be used to measure the reverse flow. Therefore the laser Doppler velocimeter is needed. These difficulties significantly hamper the progress in the technically important unsteady separation problem.

In this presentation we will first discuss the unsteady separation which occurs in an incompressible flow. In this special situation, the instantaneous velocity and the surface pressure fluctuations were surveyed in detail. The data clarified several important issues. The detachment point in this flow moves downstream. We then attempted to apply the mechanisms learned from the downstream moving detachment point to a case of upstream moving detachment point, e.g. separation on the cylinder or the airfoil. The results are promising.

Unsteady Separation With Downstream Moving Detachment Point

Moore pointed out that the flow reversal will not occur in a situation with the downstream moving detachment point (Fig. 1).

![SEPARATION POINT](image)

**Fig. 1. Separation with downstream moving detachment point (Moore 1958).**

This advantageous characteristic allows us to investigate the velocity in great detail by using hot-wire probes. An interesting unsteady separation occurred in the impinging jet case. When the coherent structure in the jet shear layer, the primary vortex, approaches the wall, a counter-rotating secondary vortex inside the viscous region is induced by the perturbations in the inviscid region Fig. 2. The ejection of the secondary vortex takes the boundary abruptly thicker. After the passing of the vortex the boundary becomes reattached.

From the ensemble averaged velocity measurements, vorticity contours can be constructed. The features of unsteady separation (the ejection of vorticity, thickening of the boundary layer and the way criterion were all observed). With the help of the velocity field, the surface pressure structure of the secondary vortex associated with the unsteady separation can be identified by a sharp rise and peak in the amplitude spectrum peak Fig. 3.

*Associate Professor, Department of Aerospace Engineering.
adverse pressure gradient. A shear layer was created at the viscous and inviscid interface. The vorticity in the unstable shear layer was lumped and formed the secondary vortex. Subsequently, the boundary layer thickened and the flow separated.

Unsteady Separation with Upstream Moving Detachment Point

In the impulsively started cylinder the detachment point moves upstream. A local shear layer was also detected. In the initial stage (Fig. 5-a), several vortices similar to those in the free shear layer were observed near the rear side of the cylinder. Eventually, two large vortices were formed at each side of the stagnation streamline (Fig. 5-b).

Fig. 1. Unsteady separation in an impingement jet.

Fig. 2. Unsteady adverse pressure gradient. A shear layer was created at the viscous and inviscid interface. The vorticity in the unstable shear layer was lumped and formed the secondary vortex. Subsequently, the boundary layer thickened and the flow separated.

Unsteady Separation with Upstream Moving Detachment Point

In the impulsively started cylinder the detachment point moves upstream. A local shear layer was also detected. In the initial stage (Fig. 5-a), several vortices similar to those in the free shear layer were observed near the rear side of the cylinder. Eventually, two large vortices were formed at each side of the stagnation streamline (Fig. 5-b).
Unsteady Separation on the Lifting Surface

In the unsteady airfoil a large hysteresis of lift, drag and moment were found during one cycle of oscillation. Well documented visualization and surface pressure measurements showed that the flow near the leading edge was disturbed by the unsteady separation vortex. Due to practical difficulties, information concerning velocity field around the unsteady vortex is very limited. It is then difficult to obtain a full picture of the initial state of the separation. A detailed look at the individual pressure traces on a plunging airfoil indicated the sharp suction peak similar to that in Fig. 6. It also appears near the separation Fig. 6. An even more interesting feature is that the adverse pressure gradient was detected before the "instance of the unsteady separation Fig. 6."

The visualization on a plunging airfoil had shown that the separation vortex developed from a local shear layer near the surface. Movement similar to that in a free shear layer, - little small vortices and return of these vortices, also have been observed.

Fig. 6. Ensemble averaged pressure traces of a plunging airfoil

\( U_0 = 18.6 \text{ m/sec} \quad x/c = 7.15 \%
\)
\( k = 0.27 \)

Fig. 7. Unsteady adverse pressure gradient on a plunging airfoil

\( k = 0.27 \)

Conclusion

In an unsteady separated flow with a downstream moving detachment point, the separation is developed from a local shear layer produced by an unsteady adverse pressure gradient. In an unsteady airfoil, the mechanism of the separation is not completely understood due to inadequate information of the velocity field. However, the surface pressure measurements indicate that the unsteady pressure that at Still is still in the "area" which produces the separation. The most important implication is that an optical visualization of the pressure gradient on the leading edge of the airfoil is the vital factor of the separation.
process and significantly modify the aerodynamic properties of the unsteady airfoils.

Acknowledgment

The author would like to thank N. Didden and S.H. Chen for their great help during the course of this research.

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