AD-A147 846 A DISCUSSION OF CONING ERRORS EXHIBITED BY INERTIAL NAVIGATION SYSTEMS(U) ROYAL AIRCRAFT ESTABLISHMENT FARNBOROUGH (ENGLAND) D J FLYNN MAR 84 1/1

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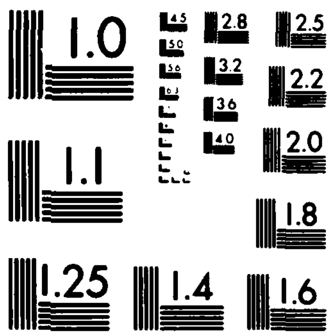

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A DISCUSSION OF CONING ERRORS EXHIBITED BY INERTIAL NAVIGATION SYSTEMS

by

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March 1984

Procurement Executive, Ministry of Defence
Farnborough, Hants

ROYAL AIRCRAFT ESTABLISHMENT

Technical Memorandum Rad-Nav 243

Received for printing 12 March 1984

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SUMMARY

The mechanisms by which so-called 'coning errors' occur in a strapdown inertial navigation system are explained by reference to a single mathematical theorem. Typical values of coning errors are derived and are shown to be significant for a 1 nm/h inertial navigator. Various methods of evaluating and reducing coning errors are discussed.

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1 INTRODUCTION

The term 'coning error' has been coined to cover a class of errors to which a strapdown inertial navigator is subject. There is generally a great deal of confusion and misunderstanding regarding the mechanisms whereby these errors occur. Indeed explanations in the current literature tend to treat only very specific cases. This brief Memorandum therefore seeks to present an explanation of the principal coning error mechanisms. In so doing it is shown that all coning errors may be described by a single mathematical theorem.

Firstly coning errors are classified in section 2 into three distinct types termed "true coning errors - type 1", "true coning errors - type 2" and "pseudo coning errors". Here the causes of each class of coning error are identified and suggestions are given for reducing their effect. Methods for evaluating coning errors are next discussed in section 3 and typical magnitudes of coning errors are calculated (Appendix B). Finally conclusions are drawn in section 4.

2 A CLASSIFICATION OF CONING ERRORS

Coning errors may be classified into two types termed "true coning errors" and "pseudo coning errors". "True coning errors" may in turn be subdivided into two classes termed 'type 1' and 'type 2'.

2.1 True coning errors

For an inertial navigator to exhibit "true coning errors", two conditions must be met. Firstly its inertial measurement unit must perform rigid body coning motion with respect to inertial space. That is to say its attitude oscillates cyclically with differing phase about each of two orthogonal axes (X and Y say) so that the third orthogonal axis (Z) traces out a cone in space (Fig 1a). If for example the motion detected about the X and Y axes is of the form A $\sin(\omega t)$ and A $\cos(\omega t)$ respectively, where A and ω are constants, then the Z axis will execute a simple circular cone in space. In practice the oscillatory motion detected about the X and Y axes may be much more complex resulting in a more complicated cone shape (Fig 1b).

Secondly there must be some deficiency or error in the strapdown system which causes the true attitude of the system to be incorrectly tracked. This second condition may arise by many causes and it is convenient to classify these into two types as follows.

2.1.1 True coning errors - type 1

Here the rigid body coning motion of the inertial navigator stimulates attitude errors such that the system's estimate of orientation executes a coning motion about the true system attitude. It can be shown that when this condition occurs then the cyclic attitude errors will rectify to give a net attitude error drift about the axis of the error cone. (Why this rectification should occur is not at all obvious and is therefore explained in Appendix A.)

Having defined the necessary conditions for "true coning errors - type 1", it is instructive to consider system deficiencies which can cause such errors in strapdown systems.

(i) Gyro quantisation. This will cause the physical coning motion of the system to be incorrectly measured so that the system attitude estimate, on average, lags the true system attitude by upto one quantisation angle. The resulting system attitude estimate will then itself execute a coning motion, although this will in general be a more complicated shape than the physical motion of the system.

Coning errors due to quantisation may be reduced by two methods. Firstly the quantisation of the gyro output could be reduced. This simple solution may not however be realisable in practice. A second alternative would be to eliminate the consistent lag of the quantisation process and consequently the resulting coning motion of the attitude errors. To do this a pulse may be alternatively added and subtracted to the gyro output. However, for this to work in the presence of positive and negative sensed angular rates, the sign of the pulse must be made to take the same sign as the angular rate. This simple algorithm will however slightly increase the quantisation noise, in exchange for the reduction in coning errors.

(ii) Truncation in the attitude algorithm. The numerical integration scheme used in the attitude update algorithm, will only be correct to a certain order of approximation. True coning motion of the system will therefore be incorrectly tracked. It is possible that the resulting attitude errors may themselves exhibit the coning motion about the true system attitude thereby causing true coning errors. Whether this will be the case in practice will depend on the order and type of integration scheme used and each case must therefore be considered individually. In general a clear reduction in coning errors will be effected by merely increasing the order of the integration scheme. However, faster update rates could also be used to reduce the effect of truncation.

It should also be noted that different types of integration scheme using the same order of approximation will yield different errors for a given type of coning motion. Therefore, if truncation errors cause significant coning errors, then an alternative integration scheme of the same order may give a significant improvement for specific types of coning motion.

(iii) Non-commutativity of attitude angles. This again can cause coning errors. In essence the sampled gyro outputs correspond to integrated angular rate about their sensitive axes. This sampled information is however insufficient for the system to accurately update its estimate of attitude. To see this consider a simple sequence of rotations as follows. A 90 deg rotation about the X axis followed by a 90 deg rotation about the Y axis will yield an entirely different attitude change if the sequence of rotations is reversed. The same gyro outputs would however be obtained if samples are only taken at the start and end of the rotation sequence. This shows that a continuous measurement and sampling of attitude changes is required to precisely track an attitude change.

One cannot of course continuously sample the gyros in a digital system, therefore coning motion in particular will not be correctly tracked. If the resulting attitude errors themselves execute a coning motion about the true system attitude, then true coning errors will once again occur.

The problem may be alleviated by increasing the sampling rate of the gyros and amending the attitude update algorithm to use the additional information. In this way one need not increase the overall iteration rate of the attitude algorithm and thus avoid the corresponding computer workload penalties. If the attitude update rate is however increased, this will also serve to reduce truncation errors. Indeed it may be possible in this case to actually reduce the order of integration scheme and still preserve the overall accuracy of the attitude algorithm.

(iv) Limited bandwidth of the attitude update algorithm. This is a different effect from item (iii) above although both are related. Consider, for example, a strapdown computer which samples the gyros at say 100 Hz and updates its estimate of attitude at say 50 Hz. It follows that if motion is present at say 400 Hz, then the attitude algorithm will be totally unable to track the motion. Because the system cannot detect such high frequency coning motion, the term 'undetected coning' is applied in such cases. There will of course also be a grey area at lower frequencies around 15-50 Hz where an attitude algorithm (sampling gyro outputs at 100 Hz say) will attempt to track the motion but will introduce appreciable errors.

The presence of coning motion wholly or partly outside the algorithm bandwidth may therefore cause coning errors. These may be reduced by a variety of methods. Increasing the order of integration scheme will only increase the bandwidth to a limited extent. A better approach would be to increase gyro sampling rates and the integration scheme to make use of this additional information. One could, of course, merely increase the attitude update rate, but this would be a less efficient solution in terms of computer workload. Another alternative would be to use anti-vibration mounts to isolate the system from coning motion of the host vehicle. It should be recognised however that this approach will increase the uncertainty of the system attitude with respect to the host vehicle.

It is worth mentioning here the effect of dither if dithered ring laser gyroscopes are used. By dithering the devices at 200-400 Hz the entire inertial measurement unit will vibrate at dither frequency and may itself execute a coning motion. Whether the gyros sense this motion or not, an attitude update algorithm operating at say 200 Hz will be unable to track it. Therefore attitude errors, equal in magnitude to the coning motion, will be exhibited by the system which will give a smoothed attitude output. Since the errors themselves execute a coning motion then rectification will occur and a gyro drift type effect will be apparent. (Whilst this explanation of undetected coning errors is perfectly valid, the mechanism is perhaps harder to visualise since the coning motion will not be detected by the attitude update algorithm, yet the errors still rectify. An explanation of this effect from a slightly different viewpoint is given in Appendix A.)

In practice such coning errors due to gyro dither will be undistinguishable from true gyro drifts and will therefore be identified in system calibration. If however the inertial measurement unit can vibrate in several distinct coning modes (possibly changing with orientation of the system) then the system will appear to exhibit step changes of gyro drifts which may be attitude dependent. Since these effects can be appreciable (see Appendix B) this phenomenon must be a fundamental consideration when designing the inertial measurement unit and choosing sensors sampling rates for a strapdown inertial navigator.

If however such high frequency coning errors are found to exist once a strapdown has been designed and made, then the following may be considered as possible solutions.

(a) The 'Q' of the sensor mounting block could be altered to reduce resonance around the dither frequency. This could be effected by redesign of the sensor mounting block, by changing the 'Q' of the antivibration mounts, or by introducing a damping mechanism. It should however be noted that such redesigns of completed hardware may prove to be costly and time consuming.

(b) Sensor sampling could be increased to around 10 kHz. The extra information available could then be used to track the motion in a modified attitude update algorithm. Care should however be exercised in this case since the effects of gyro quantisation could become significant. A possible solution to this would be to install a dedicated microprocessor in each gyro which optimally prefilters the sensor outputs at high rate, and which presents the filtered outputs to the strapdown processor.

(c) A third solution would be to cause the inertial measurement cluster to vibrate in only one mode. In this way the coning error would become steady and therefore amenable to calibration. The locking together of the three gyro dither frequencies could be usefully considered in this case.

Section 2.3 below also explains an alternative mechanism whereby gyro dither can cause coning errors.

(v) Computer errors. Finite wordlength and limited accuracy in executing calculations in the attitude update algorithm will inevitably cause errors when the system attempts to track a coning motion. If such coning errors occur, then increasing the wordlength would reduce their effect.

2.1.2 True coning errors - type 2

If the host vehicle executes a coning motion about orthogonal axes X and Y say, so that the Z axis executes a cone in space, and such that at the end of each cycle the attitude returns to a particular value, then one would at first sight expect that there would be no net rotation sensed about the Z axis. Appendix A however shows that this is not the case. Instead a net angular velocity ω is sensed about this axis. This is a real effect and should be correctly detected by the strapdown system so that the attitude estimate can be accurately updated. System errors which arise in the direct measurement of ω constitutes the second type of coning error. (This contrasts with coning errors 'type 1' where the rigid body coning motion of the system merely stimulates cyclic attitude errors which themselves rectify. In 'type 2' coning errors, the physical coning motion of the system rectifies to produce a real angular rate which is incorrectly sensed.)

Such coning errors are principally caused by the following.

(i) Gyro scale-factor error about the coning axis. The presence of this error will cause ω to be incorrectly measured giving a system error which resembles a gyro drift. This coning-induced gyro drift will of course change with the magnitude, frequency and direction of the coning motion. This can be reduced by better calibration of the gyro scale factors. Whilst the effect will be small for systems employing ring laser gyros (RLGs) it will be more significant for DTG based strapdown navigations since the dry tuned gyros in general exhibit significantly greater scale factor errors than the RLG.

(ii) Gyro orthogonality errors. The presence of such errors would cause components of ω to be detected about the X and Y axes and gyro drift type system errors would again result. (Note: this effect can in fact be used to measure gyro orthogonality errors - see section 3.)

Whilst it is recognised that gyro scale factor and orthogonality errors will always cause system errors in a strapdown inertial navigator, their effect is particularly embarrassing when associated with rigid body coning since the motion (and hence the resulting attitude drift) will not be fully predictable but may nevertheless be sustained over long periods of time.

2.2 Pseudo coning errors

If the inertial navigator does not experience coning motion, but the system itself introduces errors which cause the attitude estimate to execute a coning motion about the true attitude, then these errors will again rectify to give a drift about the coning axis. Since no physical coning motion is present, the term 'pseudo coning' is given to this effect. (In contrast, true coning errors as detailed above are stimulated by rigid body coning motion.)

One particular mechanism whereby pseudo coning errors can occur arises from the dithering of ring laser gyros. If this causes flexure of the gyro mounting block or flexure of the gyro sensitive axes with respect to their mounting faces, then cyclic errors in the gyro outputs can occur resulting in pseudo coning errors. (These are not termed "coning errors - type 1" since the inertial sensor cluster is not executing rigid body coning motion.) If a system appears to exhibit such pseudo coning errors then the only suggested remedy is the stiffening of the sensor mounting block or of the gyro itself. It is not considered possible to reduce the coning errors by algorithm redesign (see section 2.1(iv) above) since one would not be able to distinguish between true motion of the sensor block and the deformations.

Pseudo coning errors may also be generated by deficiencies in the attitude update algorithm of the inertial navigator. In this case each algorithm must be considered on its individual merits.

3 METHODS OF EVALUATING CONING ERRORS

There are three basic methods for evaluating coning errors in a strapdown inertial navigator.

(i) Computer simulation. This is an invaluable method for investigating the susceptibility of the strapdown attitude update algorithm to coning errors.

Firstly a routine for simulating the gyro outputs in the presence of coning motion must be written. Careful note must be taken of Appendix A to do this. Next the attitude update routine under test must be written and incorporated into the simulation. This routine will act on the simulated gyro outputs to give an estimate of updated attitude. Finally a method is needed to derive the error in this output.

To calculate the resulting attitude errors one first needs an attitude reference. It may be possible to calculate this analytically, however great care must be exercised in this case since errors in interpreting the simulated motion may easily be made (see Appendix A). Also errors may be made in calculating the sensor outputs which correspond to a particular motion. Since there are many pitfalls in doing this an alternative approach is suggested where a second attitude update algorithm is incorporated into the simulation.

This second algorithm would act as a real-world model and would therefore be made to have a performance vastly superior to the algorithm under test. This would be accomplished by:

faster attitude update rates;
more frequent simulated sensor sampling;
zero sensor quantisation in the sensor signals;
greater wordlength and calculation precision;
using a higher order of integration scheme.

Certainly errors will still be present in this real-world model, however they should be small in comparison to those associated with the algorithm under test.

(ii) Subjecting the strapdown inertial navigator to known coning motions. To do this a machine is required to generate the required coning motion. This may be a programmable two- or three-axis rotation table. Alternatively a device called a 'rocking table' is used at RAE. This fixture consists of a large hemispherical air bearing on which equipment under test can be mounted. The bearing can be driven with sinusoidal motion about three axes and by selecting appropriate frequencies for each axis and fixing the relative phase, a variety of coning motions may be generated.

A diagnostic evaluation of the resulting system errors is now much more complex than case (i) above where only the attitude algorithm was excited. Certainly one can gain an empirical measure of system degradation due to coning motion although it is important that the motion which is performed is realistic in magnitude, type and frequency. In the case of aircraft applications, Refs 1 to 4 may be used to give typical levels of coning motion.

(iii) Direct measurement of gyro dither effects. It is shown in section 2 and Appendix B that the dither of ring laser gyroscopes can cause appreciable coning errors when these devices are used in an inertial navigator. It was shown in particular that the coning errors will occur if the bandwidth of the attitude update algorithm is below the coning frequency and if one of the following also occur.

(a) the mounting block of the inertial sensors flexes at around the dither frequency;

(b) the gyros themselves flex so that their sensitive axes move cyclically;

(c) the entire sensor cluster executes a coning motion at or around the dither frequency.

It is also shown that the resulting coning errors are significantly modified according to the stability or otherwise of (a) to (c) above.

Since the resulting coning errors can be easily calculated (see Appendix A) provided the characteristics of the coning motion are known, it follows that the only problem is the measurement of such coning motions. One approach to this would be to machine or bond a mirror on the mounting block or gyros and to observe the pattern made by a reflected light beam. This would enable coning motions to be readily identified. In addition changes of shape in the pattern would indicate instability of the coning motion. The use of a strobed light source would also be useful in identifying changes in coning modes.

(iv) Mathematical analysis. This method can easily be used to evaluate coning errors resulting from gyro scale factor and gyro orthogonality errors (type 2 coning errors). However it is very difficult to accurately estimate type 1 coning errors. The use of computer simulation (method (a) above) is recommended as a better approach in this case. If however it is decided to use mathematical analysis, then Ref 6 will provide useful information on the relationship between various attitude integration schemes and the precision obtained.

4 CONCLUSIONS

A mathematical theorem has been used to classify and to account for the principal types of coning error to which a strapdown inertial navigator may be subject. In addition it has been used to quantify the likely magnitudes of coning errors for a typical system.

It was found in particular that coning effects induced by gyro dither can substantially degrade velocity performance of a 1 nm/h inertial navigator. It was also shown that large amplitude low frequency coning motion can also cause significant velocity errors if certain gyro orthogonality errors of the order of 6 seconds of arc are present.

Various ways of assessing the effect of coning errors have been suggested and a range of possible methods for reducing their effect have been outlined.

Appendix A

MATHEMATICAL TREATMENT OF CONING ERRORS

This Appendix seeks to account mathematically why system attitude errors which execute a coning motion about the true system attitude will necessarily rectify to produce an attitude error rate about the error cone axis (type 1 coning errors - section 2.1). Additionally it is shown how physical coning motion gives rise to a real rectified angular rate about the coning axis. (The incorrect measurement of this rate gives rise to type 2 coning errors - section 2.2.) An understanding of these rectification processes is fundamental to a true understanding of how the various types of coning error occur and both may be explained by the use of a single mathematical theorem described below.

The theorem was first reported in Ref 5 and, in the context in which it was presented, relates to the motion of a rigid body which executes a cyclic motion such that an axis fixed in it executes a cone in space. (Note: In the case of type 1 coning errors we are interested in system attitude errors which exhibit coning motion with respect to the true system attitude rather than physical coning motion of the system. However the theorem is easier to grasp when related to physical motion so it will be presented first in this context.)

Theorem. Suppose an orthogonal triad of axes X, Y and Z are fixed in a rigid body as shown in Fig 2a. Suppose further that the attitude of the body moves with respect to inertial space. In so doing, the unit vector z which is aligned with the Z axis will trace out a curve Γ on the unit sphere centered at 0, the intersection of X, Y and Z (Fig 2b). Let us further suppose that Γ is a closed curve so that in the course of its motion, the Z axis returns to its original direction. When this happens, the axes X and Y will in general occupy different directions X', Y' than at the start. The difference between the two axes sets X, Y, Z and X', Y', Z' can be completely characterised by a single rotation about the Z axis through an angle ψ. Suppose $\omega_Z(t)$ is the angular velocity of the body about the instantaneous Z axis at a time t whilst Z is moving along Γ. At first sight one might think that by integrating $\omega_Z(t)$ over the duration of the motion the value ψ would result. This is not however the case due essentially to the non-commutativity of the angles which represent the attitude of the body (see section 2). This non-commutativity ensures that the angular motion about the X and Y axes rectifies so that an additional angular displacement occurs about the Z axis. The complete expression for ψ is then given by:

$$\psi = \oint_{\Gamma} \omega_Z dt + A_{\Gamma} + 2\pi n \qquad \text{(A-1)}$$

where A_{Γ} is the area enclosed by Γ on the unit sphere and n is an integer. For small oscillations about Z, n will take the value zero and may be neglected. A proof of this interesting result is beyond the scope of this work but may be found in Ref 5.

The following points are worth noting:

(i) If the body executes completely periodic motion so that when Z returns to its original direction X and Y also resume their original direction, then ψ will be zero and equation (A-1) will, on rearranging, reduce to:

$$\oint_{\Gamma} \omega_Z dt = - A_{\Gamma} \tag{A-2a}$$

This implies that if a body executes coning motion whereby it returns periodically to its original orientation, then a net angular displacement

$$- A_{\Gamma} \tag{A-2b}$$

will be detected about the coning axis. This will give a corresponding angular rate

$$- \frac{A_{\Gamma}}{T} \tag{A-2c}$$

for each cycle lasting time T .

(ii) Secondly the coning motion need not correspond to a circular cone, but may be of arbitrary complexity.

(iii) The sign of the area A_{Γ} is given by the direction in which Γ is traced. Consequently, if Γ is a symmetric figure-of-eight (Fig 3) then the components of A_{Γ} corresponding to each segment will cancel and expression (A-1) will reduce to

$$\psi = \oint_{\Gamma} \omega_Z dt$$

(iv) If Γ does not enclose any area then A_{Γ} will again be zero and equation (A-2) will again hold. Thus rectification will not occur unless Γ encloses a non-zero area.

(v) Finally if the body executes coning motion so that $\omega_Z = 0$, then at the end of each cycle the body will in fact arrive at a different attitude. The difference will be a net rotation ψ about the coning axis where ψ is given by

$$\psi = A_{\Gamma}$$

(It should be noted that this is a real effect and must be clearly understood when simulating coning motion on a computer.)

The above points may now be used in explaining how the various types of coning error occur. True coning errors, type 1 are considered first.

A.1 The theorem applied to true coning errors - type 1

Suppose that the system estimate of attitude executes a coning motion about the true attitude. This can be regarded as equivalent to a rigid body executing a coning motion about some external reference. In this case the above theorem may be immediately applied. When this is done, the quantities used now take on slightly different meanings.

A_Γ is now the area traced out on the unit sphere by the attitude error cone.

The angular rate ω_Z is now an angular error rate which occurs about the Z axis.

The angle ψ is now the net angle between the initial and final orientation of the error axes for one coning cycle.

Thus for each cycle in which the system attitude estimate executes a cone about the true attitude, a net angular drift ψ will accumulate about the coning axis where ψ is given by

$$\psi = \oint_\Gamma \omega_Z dt + A_\Gamma \qquad \text{(A-3)}$$

Since ω_Z is an angular error rate about the Z axis we would expect its integral to contribute to the error drift about Z . The first term is therefore entirely expected. The second term however expresses the rectification effect. For, in the case that there is no error rate ω_Z about the Z axis, equation (A-3) will reduce to

$$\psi = A_\Gamma$$

showing that the cyclic coning motion of the attitude errors has still produced a net (rectified) error ψ about the Z axis over one cycle.

It is therefore seen that the cyclic coning errors will necessarily rectify and the magnitude of the rectification is equal to the area A_Γ enclosed on the unit sphere by the error coning motion curve Γ . By analogy with the physical case considered first, it is seen that the shape of the coning motion is not important but rather the area A_Γ which it subtends on the unit sphere. Further, the sense of the coning motion determines the sign of the rectified error rate. Therefore coning errors which, for example, exhibit a symmetrical figure-of-eight motion would not cause rectified errors since the contributions from the two segments would cancel.

A.2 The theorem applied to true coning errors - type 2

The first point which was noted after the theorem was stated showed how physical coning motion of a given axis, causes a net rectified drift about that axis. Thus, even when completely periodic motion occurs (so that the axes return periodically to a precise orientation), a net angular rate is experienced about the coning axis for each complete cycle. The incorrect measurement of this rectified drift due to sensor misalignments or scale factor errors then causes the so called "type 2 coning errors".

A.3 The theorem applied to undetected coning errors

Finally it is instructive to consider how undetected coning causes a rectified error rate (undetected coning errors). Suppose for example that a strapdown system executes a coning motion which is outside the bandwidth of the attitude update algorithm. Suppose further that the coning motion is completely periodic so that the system returns to a given attitude each coning cycle. Equations (A-2a-c) (relating to coning motion of a rigid body rather than coning in attitude errors) then show that there is a real drift, ω_N , about the coning axis which is associated with the coning motion. The strapdown system will detect this real drift but will not track the coning motion. The system will therefore think it is rotating about the coning axis whereas the system returns to a given attitude each cycle. Therefore rectified errors will result.

Appendix B

TYPICAL MAGNITUDES OF CONING ERRORS FOR A STRAPDOWN INERTIAL NAVIGATOR

In this Appendix the results of Appendix A are used to calculate typical magnitudes of coning errors for a strapdown inertial navigator for the following particular cases.

B.1 Case 1

Suppose the inertial navigator uses ring laser gyroscopes which dither at 400 Hz and that the bandwidth of the attitude update algorithm is less than 400 Hz. Suppose further that the dither stimulates one of the following:

(i) rigid body circular coning motion of the entire sensor cluster with 2 seconds of arc half-angle and 400 Hz frequency;

(ii) cyclic sensor errors due to flexure of the gyros or of the mounting block. In this case the cyclic errors are assumed to be equivalent to a 400 Hz coning motion with a 2 seconds of arc half-angle as in case (i) above.

Then according to section 2 and Appendix A, the resulting angular error about the coning axis will be equal to the area swept out by the coning motion on the unit sphere. Multiplying this error by the coning frequency then yields a coning error equivalent to a gyro drift of 0.02 deg/h. If the coning motion is not stable then this drift cannot be calibrated and would therefore yield a velocity uncertainty of the order of 4 ft/s and a position uncertainty of about 1 nm/h.

B.2 Case 2

Suppose the inertial navigator experiences rigid body coning motion at 0.2 Hz and with 2 deg half-angle. Suppose also that a gyro on the sensor cluster has an orthogonality error of 6 seconds of arc with respect to the coning axis. Then "true coning errors - type 2" will occur.

To quantify the magnitude of the coning errors, it is necessary to calculate the rectified angular rate which is experienced about the coning axis. Equation (A-3) is used and yields a rate of 158 deg/h. The 6 seconds of arc gyro orthogonality error will then cause an erroneous gyro drift of about 0.005 deg/h. This yields a velocity uncertainty of 1 ft/s and a position uncertainty of about 0.25 nm/h.

LIST OF SYMBOLS

A	a constant
A_Γ	an area on the unit sphere enclosed by the curve
n	an integer
T	the time for one coning cycle
X,Y,Z	an orthogonal triad of axes fixed in a body
$\underline{\hat{z}}$	the unit vector along the Z axis
Γ	a curve traced out on the unit sphere by the Z axis
ω	an angular velocity
ω_N	the net angular velocity for one cycle
ψ	an angular displacement about the Z axis over one coning cycle

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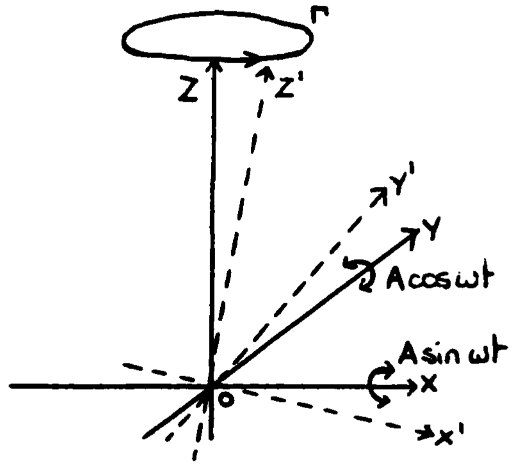

Fig 1a Circular coning - motion of axes

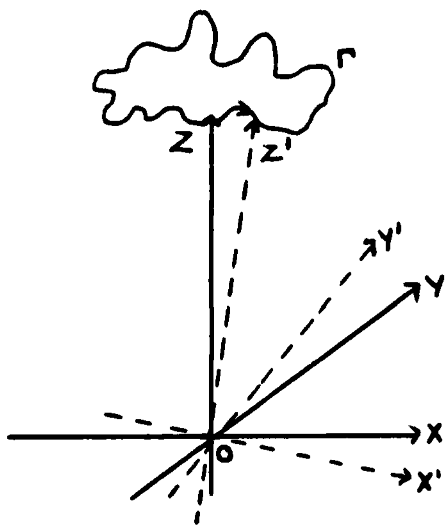

Fig 1b Irregular coning - motion of axes

Fig 1 Types of coning motion

Fig 2a-c

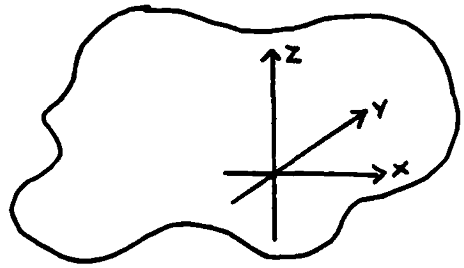

Fig 2a Orthogonal axes X, Y, Z fixed in a rigid body

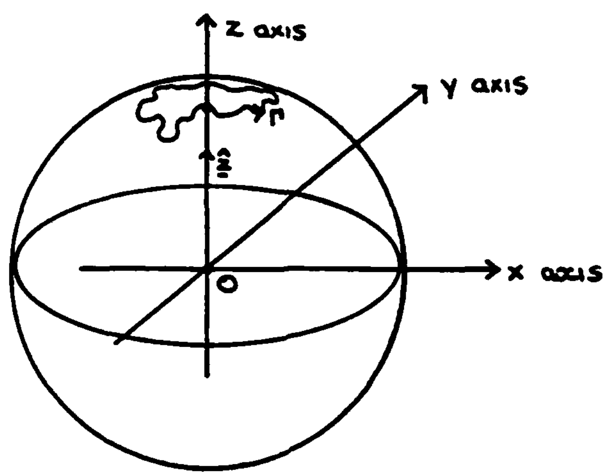

Fig 2b The locus Γ of $\hat{\underline{z}}$ traced on the unit sphere for one coning cycle

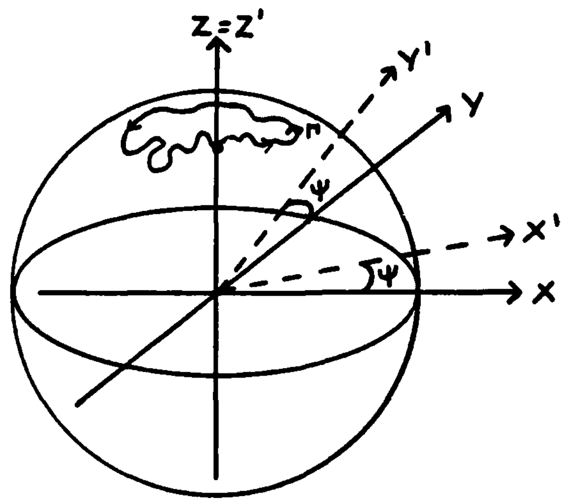

Fig 2c The position of the axes before and after one complete coning angle

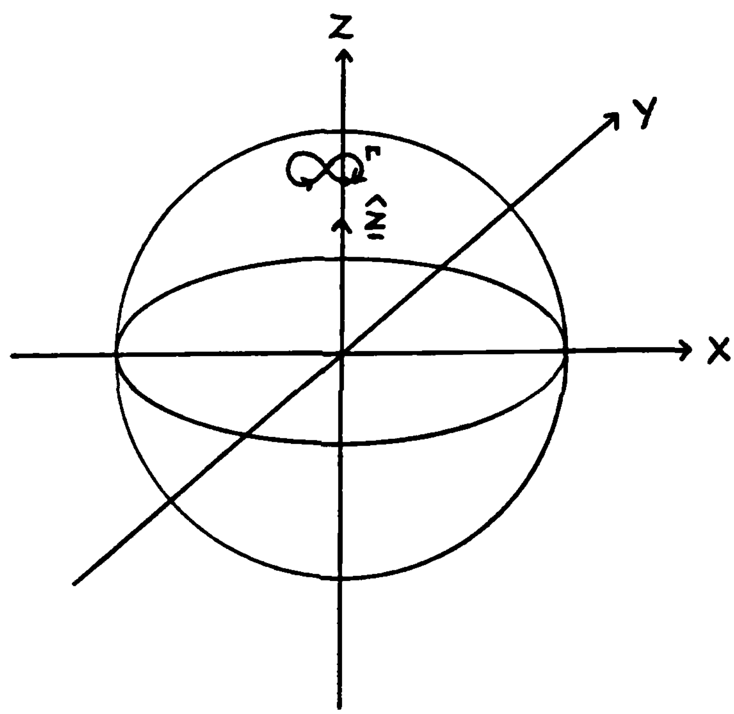

Fig 3 Coning motion where the Z axis executes a symmetric figure of eight

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[redacted] A147846 (U) FIELD/GROUP [redacted]

UNCLASSIFIED TITLE

A DISCUSSION OF CONING ERRORS EXHIBITED BY INERTIAL NAVIGATION SYSTEMS.

ABSTRACT

(U) THE MECHANISMS BY WHICH SO CALLED 'CONING ERRORS' OCCUR IN A STRAPDOWN INERTIAL NAVIGATION SYSTEM ARE EXPLAINE DBY REFERENCE TO A SINGLE MATHEMATICAL THEOREM. TYPICAL VALUES OF CONING ERRORS ARE DERIVED AND AE SHOWN TO BE SIGNIFICANT FOR A 1 NM/H INERTIAL NAVIGATOR. VARIOUS METHODS OF EVALUATING AND REDUCING CONING ERRORS ARE DISCUSSED. ORIGNATOR-SUPPLIED KEYWORDS INCLUDE: DITHER; RING LASER GYROSCOPE; AND VIBRATION.

POSTING TERMS ASSIGNED

CONING ERRORS
  USE CONICAL BODIES
      ERRORS

RING LASER GYROSCOPE
  USE GYROSCOPES)— CO
      RING LASERS)— CO

STRAPDOWN INERTIAL NAVIGATION SYSTEM
  USE INERTIAL NAVIGATION)— CO
      STRAPPED DOWN SYSTEMS)— CO

VIBRATION
  USE VIBRATION)— CO

INERTIAL NAVIGATION SYSTEMS
  USE INERTIAL NAVIGATION

SINGLE MATHEMATICAL THEOREM
  USE MATHEMATICS
      THEOREMS

VALUES OF CONING ERRORS
  USE CONICAL BODIES
      ERRORS)— CO
      VALUE

GREAT BRITAIN)– CO

PHRASES NOT FOUND DURING LEXICAL DICTIONARY MATCH PROCESS

Coning errors)– CO

Dither)– CO

REPORT DOCUMENTATION PAGE

Overall security classification of this page

UNCLASSIFIED

As far as possible this page should contain only unclassified information. If it is necessary to enter classified information, the box above must be marked to indicate the classification, e.g. Restricted, Confidential or Secret.

1. DRIC Reference (to be added by DRIC)	2. Originator's Reference	3. Agency Reference	4. Report Security Classification/Marking
	RAE TM Rad-Nav 243	N/A	UNCLASSIFIED

5. DRIC Code for Originator	6. Originator (Corporate Author) Name and Location
7673000W	Royal Aircraft Establishment, Farnborough, Hants, UK

5a. Sponsoring Agency's Code	6a. Sponsoring Agency (Contract Authority) Name and Location
N/A	N/A

7. Title A discussion of coning errors exhibited by inertial navigation systems

7a. (For Translations) Title in Foreign Language

7b. (For Conference Papers) Title, Place and Date of Conference

8. Author 1. Surname, Initials	9a. Author 2	9b. Authors 3, 4	10. Date	Pages	Refs.
Flynn, D.J.			March 1984	19	6

11. Contract Number	12. Period	13. Project	14. Other Reference Nos.
N/A	N/A	14AA04	

15. Distribution statement

(a) Controlled by – Head of Radio and Navigation Department, RAE

(b) Special limitations (if any) –

16. Descriptors (Keywords) (Descriptors marked * are selected from TEST)

Strapdown. Inertial navigation. Coning. Dither. Ring laser gyroscope. Vibration.

17. Abstract

The mechanisms by which so called 'coning errors' occur in a strapdown inertial navigation system are explained by reference to a single mathematical theorem. Typical values of coning errors are derived and are shown to be significant for a 1 nm/h inertial navigator. Various methods of evaluating and reducing coning errors are discussed.

END

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12-84

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