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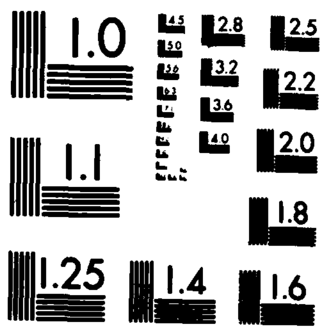
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Final Report

Contract N00014-80-0542

DEVELOPMENT OF A MIXED SCANNING INTERACTIVE SYSTEM FOR DECISION SUPPORT

for the period 06/01/80 - 07/31/83

Submitted to:

Office of Naval Research
800 N. Quincy Street
Arlington, Virginia 22217

Attention: Dr. Willard Vaughn

Submitted by:

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Report No. UVA/525358/SE85/101

July 1984

SCHOOL OF ENGINEERING AND APPLIED SCIENCE

DEPARTMENT OF SYSTEMS ENGINEERING

UNIVERSITY OF VIRGINIA

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This research has resulted in the development of an interactive planning and decision support process that allows the decisionmaker, rather than a paradigm, to iteratively select the mix of parameter value precision and alternative ranking specificity in multiple criteria decision situations. This process permits the decisionmaker to choose a behaviorally meaningful, rational approach to nondominated alternative selection that incorporates parameter imprecision. The parameters that can be imprecisely described are: probabilities, alternative scores on lowest level attributes, and attribute

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tradeoff weights. Development has placed emphasis on the behavioral relevance of the support process. This final report contains, in addition to a summary description of the research, reprints of papers published under ONR sponsorship that provide further analytical and behavioral discussions related to this process.

Final Report

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MOTIVATION FOR THE RESEARCH

It has been observed that the process of choosing among multiattributed alternatives often involves a search for nondominated alternatives, alternatives which are not worse than any other alternative on any attribute and which are better than each other alternative on at least one attribute. In most decision situations, however, there is more than one nondominated alternative, at least initially. In such decision situations, the decisionmaker typically "adjusts" the structure and parameter values of the decision situation such as to identify a single nondominated, or most preferred alternative. This adjustment may involve rational activities, such as aggregation of attributes and compensatory tradeoffs through determination of judgmental weights. It may also involve various rules which may be quite flawed, such as lexicographic ordering in which the best alternative on the most important attribute is selected, or such as sequential pairwise comparison of alternatives using a preference relation that is a function of the two alternatives being compared.

A variety of wholistic, heuristic, or holistic judgmental activities will typically be involved in a search for a dominance structure among the alternatives. Especially when there are a large number of alternative courses of action under consideration, will the decision process typically involve mixed scanning, in which some noncompensatory rule is first used to eliminate grossly inappropriate alternatives. This is then followed by one or more compensatory information evaluation operations that results in a suitable dominance structure that enables final selection.

We have developed a knowledge based system to interactively aid planning and decision support processes through encouragement of search for an alternative dominance structure that is behaviorally realistic and rational, from both a substantive and procedural viewpoint. The support system is called ARIADNE which is an acronym for Alternative Ranking Interactive Aid (based on) DomiNance (structural information) Elicitation. The support system enables use of various integrated forms of wholistic, heuristic, and holistic reasoning in an aided search for dominance information among identified alternatives. It is believed to be flexible enough to closely match diverse decision situations and environments in order to support varying cognitive skills and decision styles; thereby, enabling planners and decision makers to adapt its use to their own cognitive skills, decision styles, and knowledge.

Most of our efforts have concerned choice making situations under certainty and under risk for the single decision epoch case. This formulation allows consideration of a variety of imprecisely known parameters such as attribute tradeoff weights, outcome state values on lowest level attributes, event outcome probabilities, and various combinations of these. Parameter needs are determined from the structure of the decision situation as elicited from the decisionmaker during the formulation and analysis steps of the decision support process.

The decision situation structural model may represent decisions under risk or under certainty. The attribute tree representing the features of decision outcome states may be structured and/or parameterized in a top-down or bottom-up fashion through use of ARIADNE. A single level structure or a multiple level hierarchical structure of attributes may be used with the choice of these being at the discretion of the decisionmaker. Multiple decision node situations may be approaches through a goal directed type decision structuring approach in which the growth of the structure of alternative decisions and event outcomes is guided by sensitivity like computations obtained through use of the ARIADNE algorithms.

Parameters are elicited from the decisionmaker in the form of equalities and ratio inequality bounds. A variety of mathematical programming approaches and graph theory have been used to generate interactive displays of preference digraphs. The mathematical programming approaches are used to determine dominance structures for alternative prioritization that are based on parameter information elicited from the decisionmaker. At present, only a linear programming approach yields a dominance structure on the alternatives, rather than bounds on such a structure, in computational times that are consistent with real time interactive decision aiding.

The purpose of the graph theory algorithms is to enable construction of a domination digraph, or dominance structural model, of preferences that are based upon a dominance reachability matrix determined by the linear programming algorithms from the decision situation structural model and parameters elicited from the decisionmaker. These preference digraphs or dominance structural models encourage either selection of a preferred alternative, or further iteration using the aggregated preference information for feedback learning.

An inverse aiding feature has been incorporated into the decision support system. In this approach, the decisionmaker makes a wholistic prioritization among alternatives. This prioritization may be across some, or all, identified alternatives; and at the top level of the hierarchy of attributes, or at some intermediate level. If we assume that numerical bounds on the attribute scores for those attributes subordinate to and included within the attribute at which alternatives are prioritized, then bounds on attribute weights may be determined that are consistent with the wholistic prioritization by using a linear programming approach. If weights are specified, then it is possible to determine bounds on alternative scores on those attributes subordinate to the attribute at which prioritization was made through use of linear programming algorithms.

As alternatives are identified and prioritized, updates on these bounds are made available. The results obtained from using the inverse aiding feature are comparable in concept to those obtained from the regression analysis based Social Judgement Theory, which results in weight identification only, except that results in the form of bounds on, or ranges of, weights are available with a very few alternative prioritizations. For a large number of prioritizations, the inverse aiding approach will become cumbersome computationally compared to the regression based approach where additional information may be processed in a sequential fashion. Implementation of the combination of inverse and direct aiding, to enhance decisionmaker specification of imprecise values weights and probabilities, will enhance the usefulness of ARIADNE through encouragement to the decisionmaker to become more aware of relevant alternative courses of action and to identify new alternatives on the basis of feedback learning of the impacts of alternatives upon issues and objectives in a behaviorally relevant way that, hopefully, encourages learning.

There have been many realistic paradigms of the process of judgment and choice. We believe that the dominance process model described here it not inconsistent with the primary features and intensions of these. Our purpose, however, is to develop a conceptual design for a prescriptive approach to judgment and choice that will aid in the search for better decisions. We recognize that a truly rational approach to prescriptive decisionmaking must be cognizant of the process of decisionmaking, that is to say process rationality, or it will not be possible to evolve substantively rational support systems.

It is important that an appropriate decision support system be capable of assisting the decisionmaker through encouragement of full information acquisition, including that which may disconfirm strongly held beliefs, and in the analysis and interpretation of this information such as to avoid a variety of cognitive biases and poor information processing heuristics that may lead to flawed judgment and choice. Several desiderata follow from this and these have been incorporated in ARIADNE.

We allow for top-down or bottom-down structuring of the attributes of outcomes or impacts of decisions. The "tree" or "hierarchy" of attributes may be structured to the depth believed appropriate by the decisionmaker. We encourage indentification of alternative courses of action, additional attributes of decision outcomes, and revisions to previously obtained elicitations, at any point in the decision support process as awareness of the decision situation and its structure grows through use of the support system. We do not force a person to quantify parameters to the extent that this becomes overly stressful, or behaviorally and physically irrelevant in view of inherent uncertainties or imprecision associated with the knowledge of parameters characterizing the decision situation structural model.

We allow for revision in the elicited structure of the decision situation and for the identification of new options as awareness of the decision situation grows. Also, we do not require the decisionmaker to quantify parameters beyond the level felt appropriate for the situation at hand. ARIADNE allows parameter imprecision to satisfy this quantification relevancy requirement. We encourage the decisionmaker to specify numerical ranges for facts and values. Thus we allow, for example, expressions for alternative (A) scores on attributes (i) in a form such as $0.2 \leq v_i(A) \leq 0.5$. Weights associated with attribute i might be expressed in the form $0.2 \leq w_i \leq 0.4$ and the probability of event (i) might be stated in the form $0.3 \leq P_i(A) \leq 0.45$. We allow ordinal representations in the linear forms $v_i(A) \leq v_i(B) \leq v_i(C)$, $2w_i \leq w_j \leq w_k$, $P_j(A) \leq P_i(A) \leq 3P_k(A)$, or in similar forms. Quantification of imprecision in the form of numerical bounds on parameters is thus constrained such that it will always lead to what we call behaviorally consistent information sets. Sometimes totally ordinal information may need further quantification in order to make the precision and rigidity of the mathematics correspond to the intensions of the decisionmaker in making an ordinal specification. This is generally not needed to obtain solutions but, rather, to obtain parametric models that are faithful to the understandings of the decisionmaker. For example that ordinal alternative score inequalities $v_i(A) \leq v_i(B) \leq v_i(C)$ are satisfied by the relations $0 \leq v_i(A) \leq 1-2\varepsilon$, $w \leq v_i(B) \leq 1-\varepsilon$, $2w \leq v_i(C) \leq 1$ for small positive ε and w which in the limit become zero. It will generally not be the case that the decisionmaker would express this much imprecision, and would wish to see it more fully quantified.

Considerable effort was devoted to an operational evaluation of ARIADNE. The results of this evaluation, as well as the detailed results of the various positions of this three year research contract are contained in the appendix to this final report which consists of reprints of papers published under the subject contract.

Appendix A

List of Papers Published under Contract #N00014-80-C-0542

1. White, C. C., "Structured Policy Results for Single Stage Decisionmaking under Uncertainty," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-10, No. 12, December 1980, pp. 891-894.

2. DeWispelare, A. R., and Sage, A. P., "On the Application of Multiple Criteria Decision Making to a Problem in Defense Systems Acquisition," Int. Journal on Systems Science, Vol. 11, No. 10, 1980, pp. 1213-1240.

3. DeWispelare, A. R., Sage, A. P., and White, C. C., "A Multi-Criterion Planning Aid for Defense Systems Acquisition with Application to Electric Warfare Retrofit," 9th Annual DOD/FAI Acquisition Research Symposium, Annapolis, Maryland, June 1980, pp. 15-22.

4. White, C. C., and Sage, A. P., "A Multiple Objective Optimization Based Approach to Choice Making," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-10, No. 6, June 1980, pp. 315-326. Reprinted as a chapter in Palmer, J. D., and Salks, R. (eds.), The World of Large Scale Systems, IEEE Press, 1982, pp. 335-346.

5. Sage, A. P., "A Methodology for System Design," Proceedings IEEE Conference on Systems, Man, and Cybernetics, Boston, Massachusetts, October 1980, pp. 272-277.

6. Sage, A. P., "Behavioral and Organizational Considerations in the Design of Information Systems and Processes for Planning and Decision Support," in Proceedings Japan Institute for Systems Research, Vol. 1, No. 5, 1981, pp. 5.65-5.265. Also in IEEE Transactions on Systems, Man, and Cybernetics, Vol. 11, No. 9, September 1981, pp. 640-678.

7. Sage, A. P., and White, C. C., "Second Order Stochastic Dominance for Multiple Criteria Evaluation and Decision Support," Proceedings, International Conference on Cybernetics and Society, Atlanta, Georgia, October 1981, pp. 572-576.

8. Sage, A. P., "A Methodological Framework for Systemic Design and Evaluation of Planning and Decision Support Systems," Computers and Electrical Engineering, Vol. 8, No. 2, June 1981, pp. 87-102.

9. Sage, A. P., "Hierarchical Inference in Large Scale Systems," Proceedings International Federation of Automatic Control VIII Triennial World Congress, Kyoto, Japan, August 1981, pp.

10. Sage, A. P., "On Sensitivity Analysis in Systems for Planning and Decision Support," Journal of the Franklin Institute, Vol. 312, No. 314, September 1981, pp. 265-291.

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12. Sage, A. P., "On Human Information Processing and Inference Analysis as Large Scale Systems Problems," Proceedings of the 1982 Automatic Control Conference, June 1982, pp. 893-898.

13. Sage, A. P., "On Human System Identification, " Proceedings 1982 Inter national Federation of Automatic Control Identification and System Parameter Estimation Symposium, Washington, D. C., June 1982, pp. 592-597.

14. White, C. C., Sage, A. P., and Dozono, S., "An Interactive Approach to Alternative Ranking Involving Inverse Decision Aiding," Proceedings 1982 International Federation of Automatic Control Indentification and System Parameter Estimation, Washington, D. C., June 1982, pp. 604-607.

15. Sage, A. P., and White, C. C., "A Top Down Approach to Imprecise Interactive Attibute Weight Structure and Alternative Score Assesment, " Proceedings 1982 Systems, Man, and Cybernetics Annual Conference, Seattle, Washington, D. C., October, 1982, pp. 663-667.

16. Sage, A. P., and White, E. B., "Decision and Information Structures in Regret Models of Choice Under Uncertainty, " Proceedings 1982 Systems, Man, and Cybernetics Annual Conference, Seattle, Washington, October 1982.

17. White, C. C., and Sage, A. P., "Imprecise Importance Weight Assessment for Multilevel Objectives Hierarchies, " Proceedings 1982 IEEE Large Scale Systems Symposium, Virginia Beach, VA, October 1982, pp. 298-301.

18. Sage, A. P., and Lagomasino, A., "Knowledge Representation and Interpretation in Decision Support Systems, " Proceedings 1982 Systems, Man, and Cybernetics Annual Conference, Seattle, Washington, October 1982, pp. 658-662.

19. Sage, A. P., and White, C. C., "Behavioral Issues in a Knowledge Based Interactive System for Decision Support, " Proceedings IEEE Conference on Decision and Control, Orlando, Florida, December 1982, pp. 1049-1052.

20. Sage, A. P., and White, E. B., "Decision and Information Structures in Regret Models of Judgment and Choice, " IEEE Transactions on Systems, Man, and Cybernetics, Vol. 13, No. 2, March/April 1983, pp. 136-145.

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25. White, C. C., and El Deib, H. K., "Multistage Decisionmaking with Imprecise Utilities," in P. Hansen (ed.), Essays and Surveys on Multiple Criteria Decision Making, Springer Verlag, Verlin, Heidelberg, New York, 1983.

26. Sage, A. P., "Behavioral and Organizational Models for Human Decisionmaking," Policy and Information, Vol. 7, No. 2, 1983, pp. 1-17.

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28. Sage, A. P., and White, C. C., "ARIADNE: A Knowledge Based Interactive System for Planning and Decision Support," IEEE Transactions on Systems, Man, and Cybernetics, Vol. 14, No. 1, January/February 1984, pp. 35-47.

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30. White, C. C., and El Deib, H. K., "Parameter Imprecision in Finite State, Finite Action Dynamic Programs," to be published in Operations Research.

31. White, C. C., "Sequential Decisionmaking Under Uncertain Future Preferences," to be published in Operations Research.

32. White, C. C., and Dozono, S., "A Generalized Model of Sequential Decisionmaking Under Risk," to be published in the European Journal of Operational Research.

Appendix B

REPRINTS OF PAPERS PUBLISHED UNDER ONR CONTRACT #N00014-80-C-0542

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[1] J. N. Warfield, *Societal Systems; Planning, Policy, and Complexity.* New York: Wiley-Interscience, 1976.
[2] A. P. Sage, *Methodology for Large-Scale Systems.* New York: McGraw-Hill, 1977, ch. 4.
[3] F. Harary, *Graph Theory.* Reading, MA: Addison-Wesley, 1969, ch. 16.
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Structured Policy Results for Single Stage Decisionmaking under Uncertainty

CHELSEA C. WHITE, III, MEMBER, IEEE

***Abstract*—A state-dependent single-stage decision analysis problem is examined where the set of underlying states of nature and the set of available actions are both partially ordered. Conditions on the preference structure are presented which guarantee that the optimal policy is monotonically nondecreasing in the density vector over the state set. This intuitively desirable functional form is also shown to be inherited by the nondominated policies for the vector criterion case.**

I. Introduction

The view that a wide variety of decisionmaking situations can be adequately modeled as single-stage decisionmaking problems under uncertainty has served as justification for the development of several decision aids based on a decision tree comprised of a single decision node [1], [3]–[5]. Results presented in [1], [3]–[5] have demonstrated an awareness of the usefulness and intuitiveness of optimal policy structure for decision problems modeled by single stage decision trees. These efforts, however, have not utilized established and more general results found primarily in the operations research literature for determining conditions sufficient for the existence of optimal policies having attractive functional forms.

In this correspondence, we exploit results found in [6] in order to determine constraints on the preference structure which guarantee that the optimal policy has an intuitively desirable functional form for an important class of decision analysis problems. These results, plus those found in [2], are then used to show that, for the vector criterion extension, the search for nondominated policies can be restricted to the set of all policies possessing this intuitively desirable functional form. We now present two examples which have motivated this research.

Example 1 (Military Decisionmaking [1], [3])

Assume the state of the enemy's intent can be any of the following:

1) engage in routine surveillance,
2) fully prepare to attack, stopping short of attack, or
3) commit a hostile action.

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The options assumed available to the commanding officer are

1) continue routine operations (appropriate for state 1),
2) divert effort from other concerns to prepare for attack (appropriate for state 2), or
3) divert effort from other concerns and attack (appropriate for state 3).

We assume that significant disbenefits may accrue if the appropriate option is not instituted. Let x_i be the probability that the state of enemy intent is i. It seems intuitively reasonable that the smaller x_3 becomes and the larger x_1 gets, the smaller the option number selected should be.

Example 2 (Patient/Physician Interaction [4], [5])

Assume a patient can be in one of three different states of health:

1) well,
2) has the disease, or
3) has a severe form of the disease.

Three therapies are assumed available to the physician:

1) do nothing (appropriate for the well state of health),
2) drug regimen (appropriate for the "disease" state of health), or
3) surgery (appropriate for the "disease with complications" state of health).

We assume that if the appropriate therapy is not selected, disbenefits such as risk of complications may result. Once again, it seems reasonable that the lower the probability of being in state of health 3 and the higher the probability of being in state of health 1, the smaller the therapy number should be.

This correspondence is organized as follows. Section II defines the scalar criterion decision problem. A precise definition of a monotonically nondecreasing policy is presented in Section III. Conditions are given which guarantee that the optimal policy has this intuitively desirable functional structure. Section IV considers the vector criterion extension of the problem formulated in Section II, and provides two nondominated structured policy results that appear to have potential use in group decisionmaking situations. Section V summarizes the results determined and discusses a behaviorally relevant issue that suggests a potential impact of these results on utility assessment.

II. Problem Statement

Let S be the (finite) set of all states that the system can assume. We assume that S is partially ordered by the relation $<_S$. For example, in the context of Example 1, $s <_S s'$ might have the meaning that state of enemy intent s is less hostile than state of enemy intent s'; thus (intent 1) $<_S$ (intent 2) and (intent 2) $<_S$ (intent 3).

Let A be the (finite) set of all actions available to the decisionmaker. Similarly, assume that A is partially ordered by the relation $<_A$. Again, in the context of Example 1, $a <_A a'$ might have the interpretation that option a is less provoking than option a', and hence (option 1) $<_A$ (option 2) and (option 2) $<_A$ (option 3).

Uncertainty is described by a probability density vector, $x = \{x_s\}_{s \in S}$, over the state space; i.e., x_s is the probability that the state of the system is $s \in S$. Note that $x \in X$, where $X = \{x = \{x_s\}: x_s > 0 \text{ for } s \in S \text{ and } \sum_{s \in S} x_s = 1\}$.

We remark that $<_S$ induces a partial order $<_X$ on X in the following way. A subset $K \subset S$ is said to be *increasing* if $s \in K$ and $s <_S s'$ implies that $s' \in K$. Thus if $S = \{1, \cdots, \bar{s}\}$, then increasing subsets of S are of the form $\{t, t+1, \cdots, \bar{s}\}$. We say $x <_X x'$ if and only if $\sum_{s \in K} x_s < \sum_{s \in K} x'_s$ for all increasing subsets K. Observe that for the case where S is totally ordered, $x <_X x'$ if and only if the cumulative distribution function associated with x' bounds the cumulative distribution function associated with x from above. With reference to Example 1, note that $x <_X x'$ is equivalent to $x_3 < x_3'$ and $x_2 + x_3 < x_2' + x_3'$, and thus x represents an intuitively less hostile description of state uncertainty than does x'.

Assume that $U: X \times A \to R$ is a given quantitative measure of preference. That is, $U(x, a) > U(x', a')$ if and only if the density vector x and action a are more preferred to the density vector x' and action a'. For example, $U(x, a) = \Sigma_{s \in S} x_s u(s, a)$, where $u(s, a)$ is the utility of being in state s and applying action a and where preference is assumed to be described by expected utility.

The objective of the decision problem modeled as above is to determine an action for each $x \in X$ which maximizes $U(x, a)$. That is, we wish to determine an action selection rule, or *policy*, $\delta: X \to A$, which attains the maximum in $\max_{a \in A} U(x, a)$ on X.

III. Results—Scalar Criterion Case

The intent of this section is to present conditions on U which imply that a maximizing policy δ exists which has the following property:

$$x <_X x' \quad \text{implies} \quad \delta(x) <_A \delta(x').$$

We will refer to such a policy as an *isotone* (monotonically nondecreasing) policy. Note that in Examples 1 and 2 it was argued that an optimal policy should intuitively possess this isotone property. We now present conditions on U which guarantee that an optimal policy can be found which is isotone.

Theorem (Topkis): Assume U is such that if $x <_X x'$ and $a <_A a'$, then

$$U(x, a) + U(x', a') > U(x, a') + U(x', a).$$

Then there exists an optimal policy which is isotone.

The above property on U is often referred to as the *supermodularity* property [6]. This property suggests that on average the "poorly matched" pairs (x, a') and (x', a) are less preferable to the relatively well matched pairs (x, a) and (x', a'). For more discussion, see [6].

Proof: Assume that for $x \in X$, $a <_A a'$ and $U(x, a') - U(x, a) > 0$. If $x <_X x'$, then the supermodularity property implies $U(x', a') - U(x', a) > U(x, a') - U(x, a) > 0$, thus guaranteeing the existence of an optimal policy which is isotone. A generalization of this result is presented in [6]. Q.E.D.

The following result gives conditions which guarantee that an isotone policy exists for the case where U is linear in x, e.g., U is an expected utility function.

Corollary 1: Assume that $U(x, a) = \Sigma_{s \in S} x_s u(s, a)$ and that $u: S \times A \to R$ is such that if $s <_S s'$ and $a <_A a'$, then

$$u(s, a) + u(s', a') > u(s, a') + u(s', a).$$

Then there exists an optimal policy which is isotone. Proof of this result requires the following Lemma.

Lemma: Let $f: S \to R$ be isotone on S; i.e., assume $s <_S s'$ implies $f(s) < f(s')$. Then $x <_X x'$ implies that $\Sigma_s x_s f(s) < \Sigma_s x_s' f(s)$.

Proof: Note that since f is nondecreasing, it can be expressed as $f(s) = \Sigma_K \alpha_K I_K(s)$, where $\alpha_K > 0$ for all $K \neq S$, K is increasing, and $I_K(s) = 1(0)$ if $s \in K (s \notin K)$. Then $\Sigma_s x_s f(s) = \Sigma_s x_s \Sigma_K \alpha_K I_K(s) = \Sigma_K \alpha_K \Sigma_s x_s I_K(s) = \Sigma_K \alpha_K \Sigma_{s \in K} x_s < \Sigma_K \alpha_K \Sigma_{s \in K} x_s' = \Sigma_s x_s' f(s)$. Q.E.D.

Proof of Corollary 1: We now show that the supermodularity of u on $S \times A$ implies the supermodularity of U on $X \times A$. The result then follows from the Theorem.

Note that the supermodularity of u implies that $u(\cdot, a') - u(\cdot, a): S \to R$ is nondecreasing on S if $a <_A a'$. It then follows from the Lemma and the linearity of U in x that

$$\sum_s x_s u(s, a') - \sum_s x_s u(s, a) < \sum_s x_s' u(s, a') - \sum_s x_s' u(s, a),$$

which implies that U is supermodular. Q.E.D.

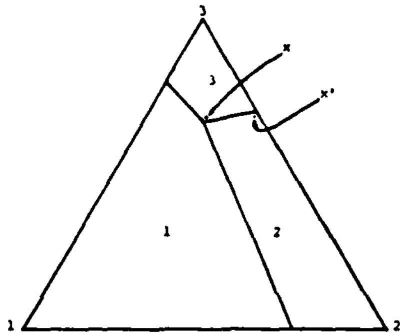

Fig. 1. Probability triangle and optimal policy for Example 3.

Corollary 1 implies that if the utility function assessed is not supermodular, then the optimal policy may not be isotone. We now present an example where an isotone optimal policy appears desirable, the utility structure possesses several agreeable characteristics (one of which is not supermodularity), but the optimal policy is not isotone.

Example 3

Assume that $U(x, a) = \Sigma_s x_s u(s, a)$ for Example 1 and that $u(s, a)$ has the following assessed values:

$$\begin{array}{lll} u(1,1) = 100 & u(1,2) = 30 & u(1,3) = 5 \\ u(2,1) = 25 & u(2,2) = 50 & u(2,3) = 15 \\ u(3,1) = 0 & u(3,2) = 10 & u(3,3) = 25. \end{array}$$

Note that $u(s, a)$ has several desirable functional characteristics:

1) $u(i, j)$ is antitone (monotonically nonincreasing) in $|i - j|$ (the more that we underreact or overreact, the less desirable is our response);
2) $u(i, i) > u(i', i')$ for $i < i'$ (a more peaceful world is more desirable);
3) $u(s, a') > u(s', a)$ for $s < s'$ and $a < a'$ (it is better to overreact than to underreact);
4) $u(2, 1) > u(3, 3)$ and $u(1, 2) > u(3, 3)$ (mild forms of overreaction and underreaction are better than war).

Observe, however, that u is not supermodular since neither $u(\cdot, 2) - u(\cdot, 1)$ nor $u(\cdot, 3) - u(\cdot, 2)$ is nondecreasing. Fig. 1 presents the probability triangle with appropriately marked regions where the various options are optimal.

We remark that for this example the optimal policy is not isotone. To show this, let $x = (0.150, 0.170, 0.680)$ and $x' = (0.010, 0.305, 0.685)$. Observe that x' represents a more hostile description of enemy intent than does x in that, although x_3' is only marginally larger than x_3, $x_2' + x_3'$ is substantially larger than $x_2 + x_3$, and thus $x <_X x'$. It is easily shown, however, that the optimal action for x is option 3, and the optimal action for x' is option 2. A seemingly reasonable utility function has therefore induced an optimal policy with an intuitively undesirable (perhaps untrustworthy) characteristic. Note the optimal policy associated with the weather triangle on [3, p. 27] for a similar contradictory result.

IV. Results—Vector Criterion Case

We now assume that the preference measure function U is vector valued; i.e., assume $U: X \times A \to R^M$, $U(x, a) = \{U_1(x, a), \cdots, U_M(x, a)\}$. The intent of using a vector criterion is that the scalar elements of U represent quantitative measures of noncommensurate preferences. A motivating example is now presented.

Example 4

Assume the utility functions for two different military decisionmakers have been assessed for the decisionmaking problem presented in Example 1, and let u_m be the utility function of the mth decisionmaker, $m = 1,2$. The objective of the analyst is to negotiate a single group policy; however, the two utility functions generate two different optimal policies.

The intent of this section is to provide guidance to the analyst in his or her efforts to determine a negotiated policy. A natural starting point is to restrict the negotiations to the set of all *nondominated* policies, which we now define.

We say that a policy δ is *dominated* if and only if there exists a policy δ' which satisfies the following property: $U_m(x, \delta(x)) \leq U_m(x, \delta'(x))$ for all $x \in X$ for each $m = 1, \cdots, M$ and for some x and m, $U_m(x, \delta(x)) < U_m(x, \delta'(x))$. A *nondominated* policy is not dominated. Observe that if a policy is dominated, there exists another policy which is at least as good as the dominated policy for *all* scalar elements of the preference measure function. Thus, in the context of Example 4, it seems quite natural for the analyst and the decisionmakers to agree that only nondominated policies should be considered in seeking a single negotiated policy.

Assume throughout the remainder of this section that $U(x, a) = \sum_{s \in S} x_s u(s, a)$, where $u(s, a) = \{u_1(s, a), \cdots, u_M(s, a)\}$. It is demonstrated in [2] that if a policy δ causes the maximum to be attained on X for the criterion

$$\max_{a \in A} \sum_{s \in S} x_s \sum_{m=1}^{M} u_m(s, a)\alpha_m \tag{1}$$

for some $\alpha = \{\alpha_m\}$ such that $\alpha_m > 0$ for all m, then δ is a nondominated policy. Observe that it is sufficient to consider only those $\alpha \in \mathcal{Q}$, where $\mathcal{Q} = \{\alpha \in R^M\colon \alpha_m > 0$ for all m and $\sum_m \alpha_m = 1\}$. Determination of all (nonrandom) nondominated policy requires consideration of all $\alpha \in \mathcal{Q}$. We now can state a structured policy result for nondominated policies.

Corollary 2: Assume $u_m\colon S \times A \to R$ is supermodular for all $m = 1, \cdots, M$. Then, search for the set of all nondominated policies can be restricted to the set of isotone policies.

Corollary 2 states that in the context of Example 4, if both decisionmakers have utility structures which imply the existence of optimal isotone policies and if they have agreed to limit their negotiations for seeking a group policy to the set of nondominated policies, then they can restrict their attention to isotone policies. Such a result is perceived as intuitively desirable; if the optimal policy for each decisionmaker is isotone, then the group policy should also be isotone.

Proof of Corollary 2: The proof results from the fact that the sum of nonnegatively weighted supermodular functions is supermodular and from application of the Theorem. Q.E.D.

The fact that the criterion in (1) is isotone in $U(x, a)$ (in fact linear in α) implies the following interesting (and trivially proved) structured policy result.

Corollary 3: Assume that $f\colon R^M \to R$ is isotone and that $x \in X$, $a^* \in A$ is such that $U_m(x, a^*) \geq U_m(x, a)$ for all $a \in A$, for each $m = 1, \cdots, M$. Then, $f[U(x, a^*)] \geq f[U(x, a)]$ for all $a \in A$.

Corollary 3 states in the context of Example 4 that if all the decisionmakers agree on an action for a given point in X, then it is not restrictive to assume that the group policy should also select that action for the given point. We remark that the function f in Corollary 3 is often called a *social choice function*, a function that trades off the measures of the various noncommensurate objectives. Corollaries 2 and 3 are illustrated in the following example.

Example 5

With reference to Example 4, assume the utility functions for decisionmakers 1 and 2 are given in Table I. We seek the set of all nondominated policies in order to focus the determination of a group policy. Figs. 2 and 3 present the optimal policies for decisionmakers 1 and 2, respectively. Corollary 2 implies that since the utility structures given in Table I are both supermodular, search for a group policy can be restricted to the set of isotone policies. Corollary 3 implies that the decisionmakers can restrict their negotiations to determining the action associated

TABLE I
UTILITY FUNCTIONS FOR EXAMPLE 5

$u_1(1, 1) = 100$	$u_1(1, 2) = 40$	$u_1(1, 3) = 10$
$u_1(2, 1) = 40$	$u_1(2, 2) = 50$	$u_1(2, 3) = 30$
$u_1(3, 1) = 0$	$u_1(3, 2) = 20$	$u_1(3, 3) = 40$
$u_2(1, 1) = 100$	$u_2(1, 2) = 50$	$u_2(1, 3) = 10$
$u_2(2, 1) = 60$	$u_2(2, 2) = 70$	$u_2(2, 3) = 30$
$u_2(3, 1) = 0$	$u_2(3, 2) = 40$	$u_2(3, 3) = 50$

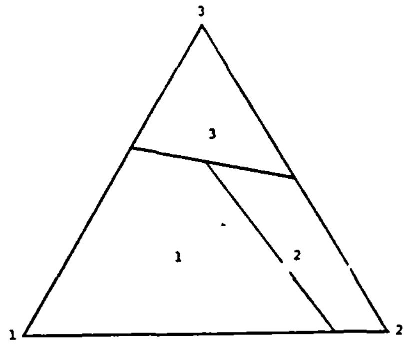

Fig. 2. Optimal policy for DM1 for Example 5.

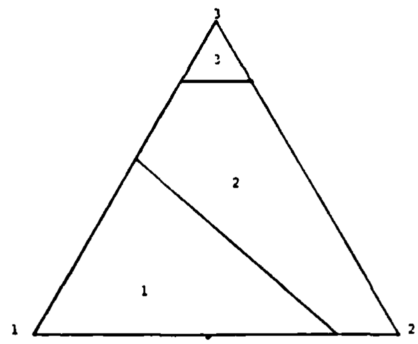

Fig. 3. Optimal policy for DM2 for Example 5.

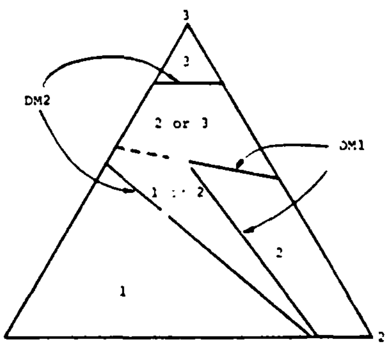

Fig. 4. Characterization of nondominated policies for Example 5.

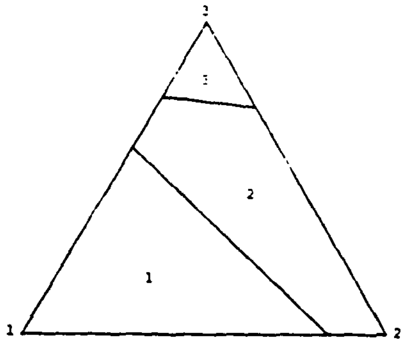

Fig. 5. Nondominated policy for Example 5, $\alpha = 0.25$.

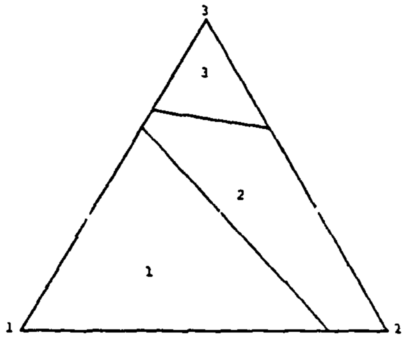

Fig. 6. Nondominated policy for Example 5, $\alpha = 0.50$.

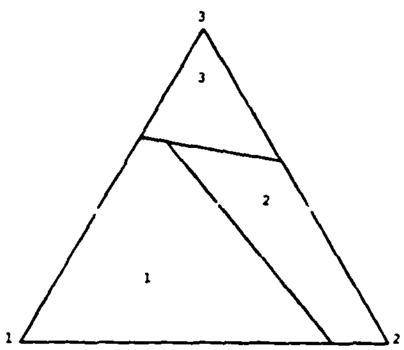

Fig. 7. Nondominated policy for Example 5, $\alpha = 0.75$.

with the group policy in the areas marked "1 or 2" and "2 or 3" in Fig. 4; the areas marked "1," "2," and "3" designate areas of the probability triangle where both decisionmakers agree as to the best action. The linear nature of the criterion given in (1) indicates that the lines separating the various regions of the probability triangle for the nondominated policies will be linear combinations of the lines separating the various regions of the probability triangle associated with each decisionmaker. Three example nondominated policies are displayed in Figs. 5–7 for various values of $\alpha = \alpha_2(\alpha_1 = 1 - \alpha)$.

V. Conclusion

This correspondence has considered a state-dependent single-stage decision analysis problem with partially ordered state and action spaces. The partial order on the state space was used to induce a partial order on the space of all probability density vectors over the state space. It was shown that if the preference structure is supermodular, then there exists an optimal policy which is isotone on the probability density space. For the vector criterion case, we showed that if each scalar element of the criterion is supermodular, then it is sufficient to search only the isotone policies in order to determine nondominated policies.

We believe that the results contained in this correspondence are of importance for the following behaviorally relevant reasons. A (statistically insignificant) number of subjects were asked to consider the statement of Example 1. All agreed that an optimal policy should be isotone. The utility structures assessed, however, almost invariably produced nonisotone policies. For these cases, when confronted with the fact that their utility structure generated a nonisotone policy, each subject felt that isotonicity was still a characteristic that the optimal policy should possess, and hence the utility structure assessed must represent an inaccurate quantification of his preferences. These facts generate the following hypothesis: the isotonicity of the optimal policy is not (implicitly) captured by any of the commonly applied utility assessment procedures. Given that this hypothesis is true, it would therefore seem appropriate for the analyst to affect the assessment procedure so that the optimal policy would be isotone, e.g., explicitly require that the assessed utility structure be supermodular.

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On the Choice of a Military Transport Aircraft Fleet via Worth Assessment Procedure: A Case Study

KEVIN Y. K. NG

Abstract—A model is developed to guide the military planners on the choice of a new military transport aircraft fleet for Canada. It discusses the fleet problem in light of the satisfaction of military requirements, the maximization of industrial benefits, and the minimization of operating costs. The aircraft capability of the fleet is obtained by means of simulation. Worth assessment procedure is then employed to establish a preference relationship.

Introduction

Commencing sometime in the 1980's Canada will need a new transport aircraft to replace its present strategic/tactical fleet. It is expected that the aircraft fleet will be required to carry out

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On the application of multiple criteria decision making to a problem in defence systems acquisition

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Multiple criteria decision theory (MCDT) approaches to choice making are receiving increased attention due to the increasing importance society places on incorporating the non-commensurate and conflicting objectives of a situation into the choice making process. Process algorithms for multiple objective optimization theory (MOOT) and multiple attribute utility theory (MAUT) motivated a combined approach which utilizes, in an efficient manner, the complementary aspects of both processes. An appropriate application for the multiple criteria approach is a specific military equipment acquisition involving aircraft retrofit. The retrofit of a particular aircraft with equipment designed for a mission which the aircraft was not originally designed to fly typically requires a large systems effort. Specifically, the retrofit of an aircraft with sophisticated electronic warfare (EW) equipment has historically involved inefficiencies and inadequacies including schedule and budgetary overruns and a lack of initially specified final product performance. Development of a useful combined MOOT/MAUT process seems a logical choice to ameliorate the difficulties of current electronic warfare aircraft retrofit design (EWARD) processes. This paper generates a set of criteria for evaluation of alternative retrofit systems in the defence systems acquisition cycle and develops an efficient framework for EWARD through extension of a MCDT approach for this application.

1. Introduction

The retrofit of a particular aircraft, with equipment designed for a mission the aircraft was not originally intended to fly, is a complex and time consuming process. When U.S. Air Force requirements for a special purpose aircraft (electronic warfare, reconnaissance, etc.) are developed, there is generally a concerted effort, for economic reasons and reasons of time to completion of effort, to modify an existing airframe as opposed to designing an entirely new one. Difficulty arises concerning how to fit a wide variety of equipment into an airframe and also satisfy all concerned parties with respect to cost, performance, and schedule (USOMB 1976; USAF 1976 b). Previous efforts at Electronic Warfare Aircraft Retrofit Design (EWARD) have met with limited success (Peterson *et al.* 1975; Cook 1977). The basic problem is that the retrofit aircraft is often not what the users originally asked for or need. Instead, the retrofit generally results in an aircraft which often does not sufficiently ameliorate deficiencies which led to the design requirement. A combination of budgetary, political and technical factors often leads to system development delays. This often results in a system being developed in a later time frame than the one in which it was needed and the one in which design requirements were specified.

Current directives (USOMB 1976, USDOD 1977 a, USAF 1977 a, USAF 1976 a) stress the incorporation of the systems oriented approach and evaluation

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criteria concerning performance, cost and schedule. As stated in DOD Directive 5000.1: "System development shall be continuously evaluated against these requirements (programs and equipment which exhibit timely development and high performance at a minimum cost) with the same rigor as that applied to technical requirements. Practical trade-offs shall be made between system capability, cost and schedule." There are many obstacles to implementing the spirit of these current directives as will be described in § 2 of this paper.

The purpose of this effort is to illustrate how multiple criterion decision theory (MCDT) can be applied to the specific application area of an initial phase of the DOD Equipment Acquisition Cycle for an electronic warfare aircraft retrofit design (EWARD). A combined multiple objective optimization theory/multiple attribute utility theory (MOOT/MAUT) approach is applied to EWARD to investigate

(1) If a multiple criteria decision theory (MCDT) approach can improve the EWARD process.
(2) If application of a combined approach using both MOOT and MAUT has merit.
(3) If an adequate set of criteria can be generated to judge the goodness of alternative designs early in the EWARD process.

The retrofitting of a special purpose EW aircraft is a large-scale system problem because of the political, military, economic, and technical overtones. EWARD requirements are therefore appropriate candidates for an approach using systems engineering methodology and MCDT within this methodology. The first three issue formulation steps of problem definition, value system design, and system synthesis are accomplished as described in § 3. Then a combined MOOT/MAUT approach is used to perform the impact analysis and the interpretation of impact steps as described in § 4. The solution obtained from this approach to EWARD was validated on an EW aircraft using appropriate data and advisors who acted as respondents and experts in various elicitation tasks associated with the methodology.

2. The electronic warfare aircraft retrofit decision situation

The retrofitting of equipment to satisfy a particular need is not a new concept in military or civilian history. The size and complexity of current retrofits efforts in the EWARD situation in the U.S.A. and the associated political, economic, military and technical impacts make this a large-scale system problem. The economic concerns are felt from the emphasis of the Congress and upper echelon military policy makers who must consider budgetary constraints and alternative program trade-offs. The size of the problem is significant when one considers that a fleet of fully equipped EW aircraft can cost ten times the price of the original aircraft. The political factors are also considerable when the governmental policy makers consider the ramifications of putting certain equipment on the aircraft which impact on U.S. security, NATO agreements, 'SALT' talks, foreign weapons sales, etc. The technological concerns, while easier to quantify, are nevertheless substantial when one considers the problem of fitting sophisticated electronic equipment into an airframe designed primarily to carry ordnance. The problem of size, weight, volume, antennas, power type, air crew requirements, etc. must be considered (Cook 1977, Peterson *et al.* 1975, USAF 1977 a). Optimization techniques which consider only the technical aspects of the retrofit design problem have met with only limited success (Peterson *et al.* 1975).

To give an appropriate perspective, from which the difficulty in the retrofit design of an EW aircraft is apparent, the prescribed retrofit procedure will be presented along with a set of impediments to enactment of this process. There are five phases of an EW retrofit system program as viewed by government program managers. The five phases of Conceptual, Validation, Development, Production, and Deployment contain various funding decision points. The stakeholders are amalgamated into three groups. Group G-1 (Operations and Intelligence) is composed of the system users ; using commands, Strategic Air Command (SAC), Tactical Air Command (TAC), etc., and the intelligence community. Group G-2 (Government policy) is made up of Headquarters U.S. Air Force (Hq. USAF), Congress, and the excutive branch. Group G-3 (Technical development and assessment) is made up of the industrial contractors, and in-house government development, contract monitoring and user interfacing subgroups ; Air Force Systems Command (AFSC), and Air Force Logistics Command (AFLC). Decision makers and advisors from these three Groups, who are involved in EW equipment acquisition, took part in this effort which is intended only to evaluate a methodological approach.

2.1. *Specified EW retrofit procedure*

The USAF is guided in the procurement of military equipment by various regulations and directives (e.g. USDOD 1977 a, USAF 1969 b). The following discussion describes the prescribed way that an EW retrofit is accomplished in the U.S. Air Force. The process starts with the identification by the using commands in group G-1, or the intelligence community, of a deficiency or need. This deficiency can be a previously recognized weakness which now can be corrected through successful efforts of government laboratories or industrial contractors through the acquisition of a new system. This deficiency or need is presented by the using command to Hq. USAF in the form of a statement of need (SON), and this is where group G-2 becomes involved in the effort. If the initial estimate of system research, development, test, and evaluation (RDT & E) exceeds $75 million or $300 million in production, the program is designated a major systems acquisition. As a major systems acquisition, the Defense Systems Acquisition Review Council (DSARC) review program is required. The mission element need statement (MENS) is next generated. This must identify the mission need in terms of the task to be performed, assessment of projected enemy threat, and existing DOD capability (USDOD 1977 b). Hq. USAF reviews the MENS and forwards it to the Secretary of the Air Force (SAF) who approves it and sends it to the Secretary of Defense (SECDEF) for final approval, or redirects it for appropriate modification or termination. If the need is judged as legitimate and current by the SECDEF the program is initiated (milestone 0) by authorization of funds for the Conceptual Phase.

In the Conceptual Phase, funding is made available to a System Program Office (SPO) cadre in group G-3 to define the acquisition problem, identify program objectives and goals, and alternative candidate systems. The SPO also develops models to evaluate operational considerations, acquisition approaches and associated risk factors. Using cost and performance trade-offs, candidate systems are evaluated to identify one or more alternatives for entry into the Validation Phase. Next, development of a Program Management Plan (PMP) is undertaken as the summary of the previous efforts. The PMP is used as the basic document defining pertinent aspects of the retrofit system. The PMP is used to prepare the Program Management Directive (PMD) which summarizes the previous efforts in the Conceptual Phase, and presents a plan for proceeding into the Validation Phase. Hq. USAF uses the PMD to generate the Decision Coordinating Paper (DCP) as input to the Air Force Systems Acquisition Review Council (AFSARC). AFSARC makes recommendations on the program and forwards these to the SAF. If the DCP is approved, it is passed to the DSARC (milestone I) for action. Following recommendations by DSARC, the SECDEF is tasked with final decision on the program. If approval is granted, funding authorizes proceedings into the Validation Phase. The Conceptual Phase is purely a ' paper ' effort with no funding authorized for hardware.

When the Validation Phase is authorized by the SECDEF, a SPO (G-3) is tasked with generating the basis from which one or more contractors are selected to go into the Development Phase. Validation is achieved through either a contract definition (paper design) or a prototype (hardware demonstration) approach. In the ' contract definition ' approach, usually two (or more) contractors are allowed to compete with each other in an attempt to further define and refine the system. A Request for Proposal (RFP) is issued which initiates the paper study. The results of this phase are system specifications and a statement of work. A source selection team, including representatives from G-1, G-2 and G-3, selects the most attractive contractors from the competing group. A RFP is issued and funding negotiations for the Development Phase are completed with the selected contractors. In the ' prototype ' approach, a Development Concept Paper (DCP) from the Service Secretary (G-2) initiates the process. A formal RFP is distributed to industry, and a Source Selection Team usually chooses one or more contractors to continue as a result of the submitted proposals. The selected contractors fabricate a hardware version of the system under development. This hardware system is evaluated analytically in a demonstration or ' fly-off ' exercise. During this evaluation, a RFP is prepared for Full-Scale Development, and the most satisfactory competitor is selected for further development. In either contract definition or prototype approaches, a PMP is prepared next, followed by the DCP and PMD, and a DSARC board meets for milestone II (G-1, G-2, G3) to judge the worthiness of the program to proceed. If the program is judged essential and proceeding satisfactorily, the SECDEF acts on the program. If the program is approved, a PMD (G-2) is sent to the SPO and funding is approved as authorization to proceed to the Full Scale Development Phase. Other alternative actions to proceeding into Development are to return to more validation, or cancellation.

The Full-Scale Development Phase provides the expanded engineering design, fabrication, testing, evaluation, and support planning for the selected system. The ' user ' and ' supporting commands ' participate in the

Development Test and Evaluation (DT & E) and Initial Operational Test and Evaluation (IOT & E). The contractor negotiates for production during the testing process, and configuration audits (FCA and PCA) are accomplished subsequent to finalizing the system configuration. After this, any change in the system is rigidly controlled and must follow the formal Engineering Change Proposal (ECP) route. The results of the Development Phase are presented to DSARC at milestone III for review. If approval is granted and Office of the Secretary of Defense (OSD) funding procured, the program enters production.

In the Production Phase, the system is produced by the contractors and logistic support is procured. This by far is the most costly and time-consuming phase up to this point. The completed system is turned over to the user in the Deployment Phase by the Systems Manager (SM) in Logistics Command (AFLC). There the system is utilized and maintained until its retirement.

2.2. *EW program complications*

The process just described for EW retrofit of an aircraft is seldom followed exactly because of a number of complex factors pertinent to electronic warfare. There are seven basic reasons for EW retrofit difficulties.

(1) *Electronic warfare is a highly technological, expensive, and specialized business.* EW equipment requires extensive dedicated research and development capabilities that only a limited number of industrial contractors have established. The risks in developing and retrofitting these sophisticated and specialized systems are high, and the spin-offs to commercial application are severely limited.

(2) *There is insufficient communication between all stakeholders at all phases of the system cycle.* There is a lack of effective interchange of information between groups G-1, G-2 and G-3 in the Conceptual Phase of system development. An exception to this lack of communication occasionally exists beween the upper level DMs in groups G-1 and G-2 when politically sensitive equipment is involved. The general lack of communication makes any kind of long range planning for the system retrofit very difficult. This lack of communication prevents the cost and performance people from coming to early agreement which generally means time delays at later phases in the system cycle.

(3) *The decision making structure is multilevel and semidefined.* The decision makers (DMs) and their advisors in G-1 and G-3 groups are defined but arranged in multilevels which makes it difficult for amalgamation of objectives at these various levels. The DMs and their advisors in G-2 group are also arranged in multilevels but are not clearly defined. This means that certain gerents can participate in varying degrees in their decision making role depending on factors such as the political atmosphere. This fragmentation makes it particularly difficult to account for some DMs interactions.

(4) *Government policy makers do not operate in sufficient isolation from private industry.* The government policy advisors in G-1 and G-2 groups perpetuate a long-standing amenitable relationship between themselves and EW industries. While this relationship can be beneficial to the government in certain aspects of contract negotiations, it can cause difficulties such as the fact that system deficiencies (and their amelioration) are often pointed out by the system builder or contractor instead of the intelligence community.

(5) *Long range government policy is difficult to forecast.* The complex issues that affect foreign policy coupled with a bureaucracy that administers it, makes it particularly difficult to estimate accurately what the U.S. foreign policy will be for other than very short planning horizons.

(6) *The current funding directives* (*USOMB Circular* 109) *encourage* (*and occasionally specify*) *dual-contractor development procedure for newly designed equipment.* This is done as a way of ensuring commonality in technology, and preventing a sole source supplier of replacement parts. Unfortunately this practice of carrying two contractors throughout the program also tends to cause funding and scheduling problems.

(7) *The contractor and retrofit program are often given flexibility with respect to cost and schedule commitments.* The primary reason a program gets limited in scope, indefinitely delayed, or cancelled is that it has been surpassed on the priority list (and another program took its funds). The logical reasons for the above actions (lack of performance in the system, cost and schedule overruns) are not considered as prominently.

These factors presented above make the normal EW retrofit procedure of § 2.1 difficult to implement. These items point out the need for a comprehensive approach to the EWARD such as supplied by MOOT or MAUT approaches from systems engineering. These multiple criteria approaches allow the incorporation of a set of salient attributes in a way that allows one to address the requirements by individually considering factors which are affected by the impediments discussed previously. This flexibility is of significant value in a large-scale effort like EWARD. In EWARD, the need exists for an adequate set of criteria which can be utilized in the evaluation of alternatives. The development and subsequent incorporation of these criteria into the difficulties cited produce a cost-effective product that will meet the needs of the users.

3. Structure of the EWARD decision situation

In order to set the stage for the application of techniques from both MOOT and MAUT, an issue formulation effort covering the problem definition, value system design, and system synthesis steps was performed in order to identify and relate the factors of the U.S. Air Force EWARD structure. Sections 1 and 2 have just described pertinent stakeholders. The G-1 group (Operations and Intelligence) is reponsible to point out deficiencies and coordinate requirements so the retrofitted system is operationally satisfactory. This group includes the eventual users of a retrofitted system. The G-2 group (Government policy) is the group that coordinates the systems impact on defence capability and foreign policy. This group also constrains the program with respect to budgetary considerations. The G-3 group (Technical development and assessment) defines the system configuration, and carries out and manages the research, development and production of the retrofit system. While other stakeholders are involved in EWARD, their interactions are extremely difficult if not impossible to define, assess or forecast.

The general needs of an EWARD are defined in the SON and MENS documents which delineate currently existing deficiencies. Historically, specific needs of the new system are described as threats to be countered in terms of requirements and EW techniques. The aircraft which will be retrofitted is often an obvious choice because of the performance requirements, operational situation, and current aircraft inventory supply. The number of these special purpose aircraft required for the specific mission is often specified and considered fixed in the early phases of the DOD equipment acquisition cycle. *Therefore, the aircraft upon which the retrofit will be applied as well* as the fleet size is hypothesized as determined, and its selection is not to be considered in this pre-analysis phase.

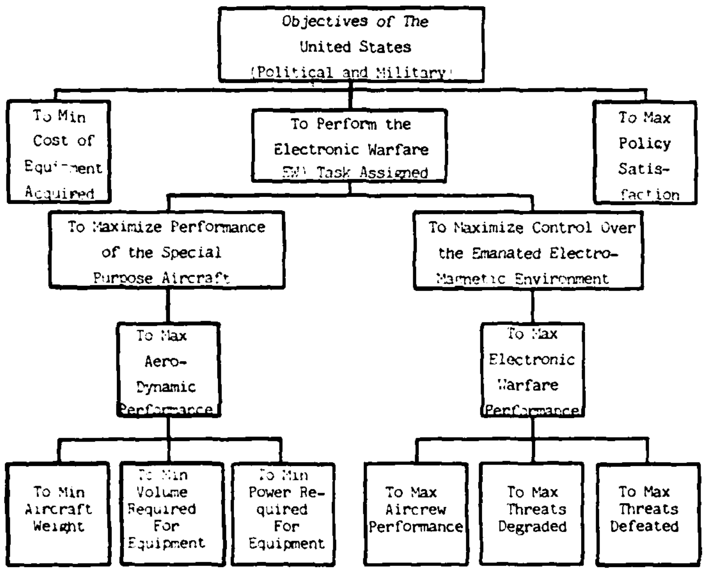


Figure 1. Hierarchy of objectives of the EW selection process.

The possible options to ameliorate current deficiencies are listed in the Conceptual and Validation Phases in terms of passive procedures, transmitting and receiving system techniques, and active and passive expendables disbursement. The major constraints for the retrofit system are size, weight, power, and cooling restrictions (determined by the aircraft itself), funds available (a function of the deficiency priority and other factors), and time until deployment (a function of bureaucratic scheduling, and research and development—R & D status).

The main objectives of the EW task which were constructed as a result of interaction with the stakeholders are shown in Fig. 1. It is this latter set of objectives that were utilized in our solution of the EWARD. The measures, criteria, or attributes by which attainment of the objectives is discerned are shown in Fig. 2. This set of attributes was used in the evaluation process for the alternatives in this EWARD effort.

While lowest level criteria or attribute measures may seem self-explanatory, a brief description is provided to cover some of the aspects of the EWARD.

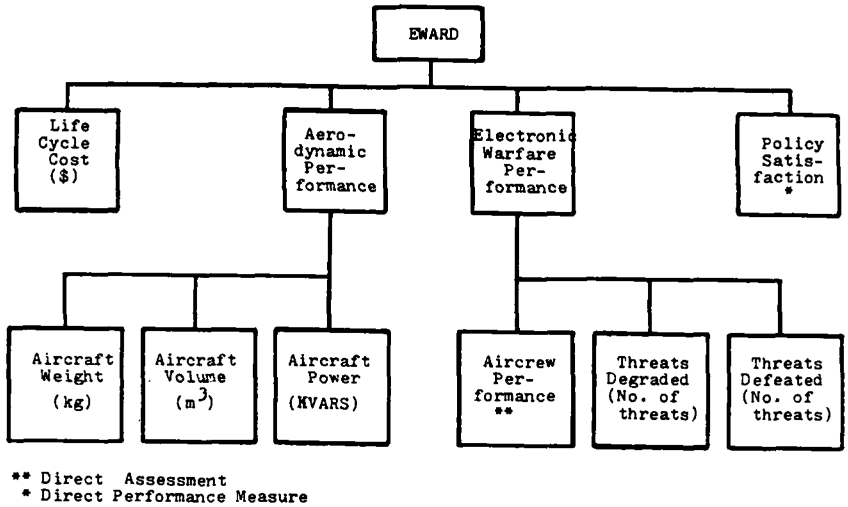


Figure 2. Attribute template for EWARD.

(*a*) The aircraft weight attribute is a measure of the added weight (kg) due to the total EW system plus a penalty figure (also in kg) which accounts for any modifications to the fuselage caused by the EW system which would increase drag such as external antennas, external components, external wing pods, air induction cowlings for cooling, etc. The drag caused to the aircraft fuselage by a system is converted to equivalent penalty weight as a way of accurately quantifying the impact of a specific system on the aircraft performance without double counting with respect to another attribute.

(*b*) The volume attribute (m^3) is a measure of internal volume occupied by the EW system. The available volume for EW equipment is usually limited because of competition from other needed avionics equipment.

(*c*) The power attribute (kvars) is a measure of the peak electrical power required to operate the EW equipment (including auxiliary cooling).

(*d*) The air crew performance attribute (direct performance measure) measures the degree of dedicated service that the EW system demands out of the crew for successful operation (some manual/automatic systems may require more crew operations than is physically possible). Values of this attribute can be obtained by simulation of the cybernetic candidate system. A normalized scale (limits of 0·0 to 1·0) was used to score the various alternative systems. The minimum score of 0·0 represents the case where the air crew was unable to perform the tasks required by an alternative system in a simulated combat environment. The maximum score of 1·0 represents the case where the air crew was able to perform all tasks required for completely successful operation of the alternative system in a simulated combat environment.

(*e*) The threats affected attribute (number of threats degraded) is a measure of the ability of the EW system to affect individual threat types in a dense environment (currently operational and forecast threats validated by a threat group).

(*f*) The threats defeated attribute (number of threats defeated) measures the number of specific threat types defeated which are made inoperable by the EW system.

(*g*) The cost attribute (dollars) is a measure of the life cycle costs of the EW system to include R & D, production, and maintenance of the expected life of the system. This total life cycle cost approach is applied with increasing frequency to DOD equipment acquisition programs.

(*h*) The policy satisfaction attribute (direct effectiveness measure of directives, policies, and requirements satisfied) is a measure of the degree to which the EW retrofitted aircraft satisfies forecast government policies, particularly of the G-2 group, for the production decision point. Policy satisfaction includes concepts influencing candidate EW system political attractiveness at a point in time in the future (the production decision point). This then requires one to forecast the political mood and subsequent government policy. Some of the elements which are included in this factor are: attitude toward U.S. defence posture by the administration and congress, need for foreign arms sales, treaty and alliance commitments, budgetary priorities and amounts of federal spending, military lobby, time to production of EW system, availability of manufacturers to meet the requirement for dual source suppliers of critical parts of a system, employment and unemployment effects on certain contractors, granting of subcontracts of a system for the sake of keeping a base of companies involved in defence oriented work, etc. None of the DMs interviewed were able to quantify all of the above elements, so an attempt at aggregation of the elements met with some success. The DMs were able to express the fact that as the time until production increased, the probability of accurately forecasting the political policies which would need to be satisfied decreased exponentially.

All DMs agreed after discussing that this factor needs inclusion even if it is very difficult to quantify accurately. This concensus among DMs and advisors for including policy satisfaction as one of the objectives justifies the use of a MCDT approach such as MOOT or MAUT. A normalized scale (limits 0·0 to 1·0) was used to score the alternatives with respect to policy satisfaction. The minimum score of 0·0 represents the case where it is estimated that an alternative will not satisfy any of the policies in effect at the production decision point. The maximum score of 1·0 represents the case where it is estimated that an alternative will satisfy all policies in effect at the production decision point. The DMs and advisors estimated policy satisfaction scores for the candidate systems. A maximum time for initiation of production was established at eight years so that the values could be obtained for various candidate systems with certain characteristics.

It is noted that no comprehensive set of attributes (as complete as the set just presented) is now used in the initial design stages of the current EWARD efforts. The DMs and advisors interviewed in this effort agreed that the above set of attributes covers the salient considerations of an EW retrofit program, and that the goodness of a specific system could be adequately evaluated using these attributes.

The alternative policies that achieve the objectives with respect to an EW task are the selection and retrofit of EW equipment into the designated aircraft. A specific action is the selection and retrofit of a specific EW system with the designated number and type of the primary components (including associated airborne and ground equipment) as indicators of this activity (e.g. system α has 16 transmitters, 3 receivers, 1 processor and 4 expendables). Each separate system designated has characteristics measurable by the lower level attributes mentioned earlier (e.g. systems α could have weight β, life cycle cost Γ, degrade Ω threats, etc.). The attribute values and characteristics of each specific system in competitition for selection can have deterministic and probabilistic values (e.g. R & D may not yet be completed and the weight, cost, and number of threats covered are not known with certainty, but utilizing data and estimates, distributions covering the stochastic elements were obtained).

A hypothetical deficiency designation and set of possible EW systems to ameliorate this vulnerability will now be described. A hypothetical situation is selected because the security classification pertaining to past and ongoing EW systems would prevent the publication of these results in unclassified texts thereby prohibiting a general usefulness which is the purpose of this research. While no information is compromised, the hypothetical situation characteristics are relevant to actual EW systems so that DMs and advisors could realistically take part in the EWARD effort (in the opinion of the DMs and advisors interviewed, the hypothetical situation mimicked reality to a high degree). The hypothetical situation was also used to produce a general solution procedure that may be situation specific, but not equipment specific (i.e., the modelling done was intended to be general in nature—applicable to

Lower Level Attribute Limits

Attribute	Greatest Level	Lowest Level
X_1 Aerodynamic performance		
X_{1a} EW system weight	4.5×10^3 kg	5.0×10^2 kg
X_{1b} EW system volume	4.10 m^3	1.5 m^3
X_{1c} EW system power	105.0 KVA	45.0 KVA
X_2 EW system life cycle cost	2.0×10^9 (\$)	1.0×10^8 (\$)
X_3 Electronic warfare performance		
X_{3a} Aircrew effectiveness	1.0 (normalized scale)	0.0
X_{3b} Number of threat types degraded	30.0	5.0
X_{3c} Number of threat types defeated	30.0	0.0
X_4 Degree of policy satisfaction	1.0 (normalized scale)	0.0

Table 1. Lower level attribute limits.

Alternative Retrofit System
Configurations and Characteristics

Alternative	Descriptive Phase
1 (a_1)	Compromise 1 (average)
2 (a_2)	High cost, high reliability
3 (a_3)	High electronic warfare performance
4 (a_4)	High aircraft performance
5 (a_5)	Low electronic warfare performance
6 (a_6)	Low aircraft performance
7 (a_7)	Low cost
8 (a_8)	Compromise 2 (average)

Table 2. Alternative retrofit system configuration and characteristics.

any EW system of the present and near future—and not intended to concentrate only on modelling specific pieces of equipment for a single solution).

Assume the deficiency statements (MENS and SON) designated N' primary threats and N'' secondary threats (specific active—radar, laser, etc., and passive —infra-red, electro-optical, etc.) which are sources of intelligence for enemy fire control systems and armaments (airborne interceptor, anti-aircraft artillery, surface-to-air missiles, etc.). A set of EW system components both developed and proposed to counter these N threats ($N = N' + N''$) is available from governmental and industrial sources. For our hypothetical situation, the attribute limits for the possible system components are shown in Table 1.

A set of alternative proposed systems was assembled which are representative and typical of the spectrum of choices which confront the analysts and policy makers in the Conceptual Phase of EWARD. This list of alternative EW system configurations shown in Table 2 is hypothetically generated as a response of the RFP in the initial stage of the Conceptual Phase. These alternatives are described by a dominant characteristic. These alternative configurations, with expected attribute levels and associated normal probability density functions, were constructed from USAF supplied in-house and contractor empirical data, and supplemented by expert opinion when incomplete data was available. For example, the opinions of DMs and advisors from G-1, G-2 and G-3 were used to estimate levels of attainment for the alternatives and accompanying probability density functions, since little or no data was available for this task. Means and standard deviation for their attribute levels are described by Table 3. Normal density functions are utilized later in the impact assessment and decision making steps.

In order to establish a basis upon which to choose an appropriate MCDT approach, the attributes were investigated with respect to preferential independence. Selected DMs and advisors from each group were interviewed to examine how these gerents trade-off levels of the attributes. Using standard assessment techniques, second order preference independence was explored (Keeney and Raiffa 1976, Keefer 1978, Keeney 1974). The major attributes ($\mathbf{X}_1$, X_2, $\mathbf{X}_3$, X_4) were found to be preferentially independent (PI) and the

Alternative Configurations

Alternative	x_{1a}	x_{1b}	x_{1c}	σ_{1a}	σ_{1b}	σ_{1c}	$x_2(10^9)$	$\sigma_2(10^8)$	x_{3a}	x_{3b}	x_{3c}	σ_{3a}	σ_{3b}	σ_{3c}	x_4	σ_4
*1E	2800.	2.20	79.0				1.20		0.49	16.0	11.0				0.70	
*1D	2905.	2.68	84.1	62.5	0.24	2.55	1.29	0.45	0.37	12.0	9.0	.040	4.0	1.0	0.66	0.020
*1I	2695.	1.7	73.9				1.11		0.57	20.0	13.0				0.74	
2E	2500.	2.10	84.0				1.95		0.81	28.0	18.0				0.34	
2D	2580.	2.51	87.5	40.0	0.21	1.75	2.00	0.25	0.79	26.0	16.0	.010	1.0	1.0	0.26	0.040
2I	2420.	1.69	80.5				1.90		0.83	30.0	20.0				0.42	
3E	3900.	4.00	103.0				1.85		0.96	28.0	25.0				0.42	
3D	4055.	4.10	105.0	77.5	0.05	1.00	1.95	0.50	0.95	27.0	24.0	.005	0.5	0.5	0.36	0.030
3I	3745.	3.90	101.0				1.75		0.97	29.0	26.0				0.48	
4E	810.	1.60	53.0				0.90		0.64	12.0	6.0				0.44	
4D	920.	1.50	57.2	55.0	0.05	2.10	1.10	1.00	0.54	7.0	3.0	.050	2.5	1.5	0.37	0.035
4I	500.	1.70	48.8				0.70		0.74	17.0	9.0				0.51	
5E	1500.	2.90	46.0				0.40		0.52	10.0	3.0				0.39	
5D	1615.	3.36	47.0	57.5	0.23	0.50	0.60	1.00	0.43	5.0	1.0	.045	2.5	1.0	0.33	0.030
5I	1385.	2.44	45.0				0.20		0.61	15.0	5.0				0.45	
6E	4400.	3.70	101.0				1.30		0.72	18.0	12.0				0.45	
6D	4485.	3.92	103.5	42.5	0.11	1.25	1.45	0.75	0.69	14.0	10.0	.015	2.0	1.0	0.38	0.035
6I	4315.	3.48	98.5				1.15		0.75	22.0	14.0				0.52	
7E	1200.	3.10	61.0				0.30		0.43	12.0	4.0				0.65	
7D	1295.	3.60	66.8	47.5	0.25	2.90	0.40	0.50	0.32	6.0	1.0	.055	3.0	1.5	0.70	0.025
7I	1105.	2.60	55.2				0.20		0.54	18.0	7.0				0.60	
8E	2400.	2.10	77.0				1.30		0.74	18.0	13.0				0.68	
8D	2508.	2.41	81.4	54.0	0.16	2.20	1.42	0.60	0.68	15.0	12.0	.030	3.0	0.5	0.62	0.030
8I	2392.	1.79	72.6				1.18		0.80	21.0	14.0				0.74	

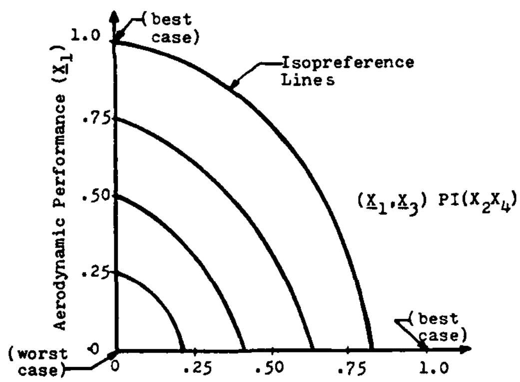
*E = Expected Configuration

*D = Degraded Configuration

*I = Improved Configuration

Attribute and Accompanying Standard Deviations

Table 3. Alternative configurations.



($\underline{X}_1$ and $\underline{X}_3$ were consistently traded off according to the curves above for various joint levels of X_2 and X_4).

Figure 3. Preference independence of attributes.

components of $\mathbf{X}_1$ and $\mathbf{X}_3$ were also found to be PI. An example of the PI assessment for Group 3 is shown in Fig. 3. The set $(\bar{X}_i, \bar{X}_j)$ is preferentially independent of X_{ij} (set levels of all other attributes except X_i and X_j) if preferences for consequences differing only in the value of X_i and X_j do not depend on the fixed value of X_{ij}.

The appropriate point in the defence system equipment program phases for the effort reported here to occur is between Conceptual Phase authorization and Program Management Plan preparation. Our results should be used to aid in judging the contractor competition or sole source performance, and also used as inputs to the final RFP after contractor selection before production. The information needed to perform the portion of the evaluation effort on EWARD as here is identified in the Conceptual Phase.

4. A multicriterion evaluation process

Factors in EWARD such as the lack of an adequate scalar performance measure, the high level of complexity, and the difficulties cited which hamper an efficient acquisition process, make this situation a likely candidate for a comprehensive multiple criteria approach like the combined MOOT/MAUT multicriterion process utilized here.

The EWARD situation contains certain characteristics such as a set of attributes which the DMs and advisors have established as preferentially independent and an indication by the DMs and advisors that they would prefer to reduce the number of alternatives before comparing and ranking the remaining alternatives. The DMs and advisors traditionally attempt to eliminate some alternatives before examining the remaining in more depth, and were comfortable working in this mode. These characteristics in EWARD suggest that a combined MOOT/MAUT process is appropriate for modelling and resolving this decision situation. The MOOT/MAUT process as applied to EWARD will follow the basic algorithm outlined in DeWispelare and Sage (1980). The main steps are (*a*) conduct a pre-analysis or issue formulation step, (*b*) eliminate the inferior alternatives through an elimination by aspects exercises (Tversky 1972) and multiple objective optimization theory (MOOT) to eliminate dominated solutions and form a non-dominated solution set (NDSS), (*c*) elicit the preference structure of the DMs to subsequently develop a scalar choice function which ranks the remaining alternatives to identify the optimal policy using multi-attribute utility theory (MAUT), (*d*) conduct a sensitivity analysis and validation exercise, (*e*) prepare an action plan.

In response to the development of a MENS, retrofit system configurations from the contractors become available which represent the alternative actions. All of the proposed alternative systems are feasible alternatives, and no preliminary scanning is required in EWARD at this point in the effort to eliminate the unacceptable alternatives. Because a large amount of engineering design is required for interfacing a complete retrofit system, information is available at this point (Conceptual Phase) which basically establishes the impacts of each alternative action (implementing an alternative retrofit system). Therefore, no tuning or refining of the alternative systems is required in the Conceptual Phase unless none of the alternative actions are viable solutions to the deficiencies cited in the MENS. The criteria established in the pre-analysis phase was used as the basis for developing the attributes, shown in Fig. 2, used in this effort.

The identification of alternatives which are superior with respect to the attributes developed in the issue formulation step is undertaken through formation of the NDSS. In order to utilize the data concerning the attributes, which include risk information, stochastic dominance was used to indicate which of the alternatives was dominated. The eight alternatives were compared to each other with respect to the stochastic dominance of attribute values to see if any alternatives were dominated with respect to all attributes. The cumulative probability distribution (CDF) for attribute values for each of the alternatives with respect to system weight (x_{1a}) is shown in Fig. 4. When the area

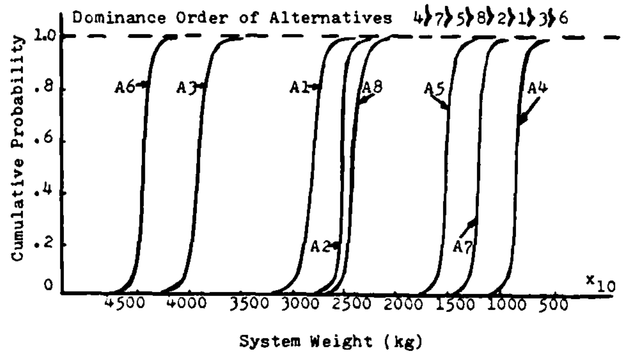


Figure 4. Distribution of system weight for nine EWARD alternatives.

under the CDF for an alternative (A_i) is less than the area under the CDF of another alternative (A_j) (for a specified attribute), A_i is said to dominate A_j in the sense of stochastic dominance for that attribute. A_i is said to dominate A_j if stochastic dominance is present for all attributes for A_i with respect to A_j. Stochastic dominance relationships were established for all alternatives on a pairwise basis for all attributes with the results that only one alternative system, alternative no. 6, is dominated. The large NDSS was expected since there are a number of conflicting objectives. In the interest of reducing the number of alternatives in the NDSS further, to bring the NDSS membership to a number of acceptable by the DMs, an elimination by aspects exercise was conducted. Each group was asked to come up with a set of realistic, from their specific vantage point, minimum acceptable attainment levels that the alternatives would have to pass to be considered in the non-dominated solution set. These levels are shown for all three groups in Table 4. In this elimination by aspects exercise, the analyst must ensure that all attainment levels of the attributes which are exceeded by an alternative reflect an essential shortcoming in the EWARD. This elimination by aspects eliminated three alternatives (alternatives 2, 3 and 5) each of whose attribute values violated many minimum attainment levels of the stakeholder groups. There were then four alternatives comprising the NDSS, alternatives 1, 4, 7 and 8.

In order to select the best alternative system from the reduced NDSS, and to facilitate using risk concepts in the decision process, a MAUT technique was utilized for the ranking of alternative systems decision making phase. The characteristics of the situation such as an available set of attributes, the availability of the directly assessed preference structure of the DMs and advisors, and the availability of probabilistic information concerning the

Required Attainment Levels

Attributes	Group 1	Group 2	Group 3
X_{1a}(kg)	≤ 3000.00	≤ 4500.00	≤ 3500.00
X_{1b}(m^3)	≤ 3.20	≤ 4.10	≤ 3.60
X_{1c}(KVA)	≤ 90.00	≤ 105.00	≤ 90.00
X_2 (\$x10^9)	≤ 1.50	≤ 1.40	≤ 1.70
X_{3a} *	≥ 0.50	≥ 0.00	≥ 0.45
X_{3b}(threats)	≥ 16.00	≥ 10.00	≥ 9.00
X_{3c}(threats)	≥ 10.00	≥ 2.00	≥ 5.00
X_4 *	≥ 0.33	≥ 0.50	≥ 0.40

* Normalized Scale

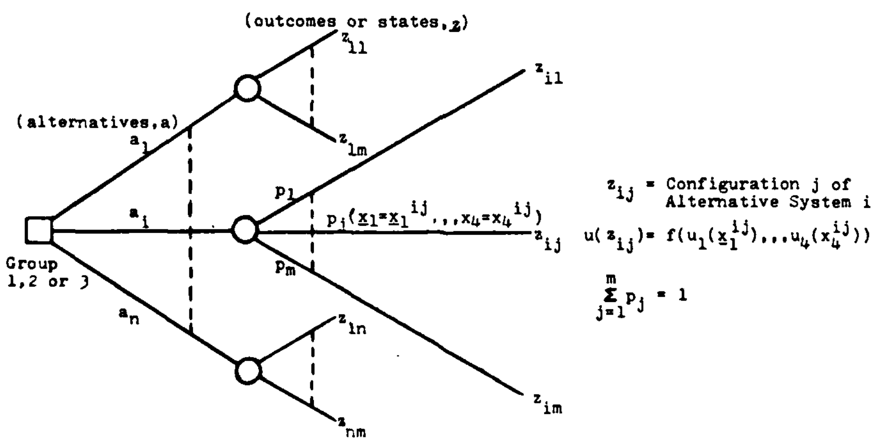
≤ Maximum Attainment Level

≥ Minimum Attainment Level

Table 4. Required attainment levels.

attribute values for the alternatives, suggest the use of multi-attribute utility theory (MAUT).

Multi-attribute utility theory (MAUT) has been a very popular technique for use in decision situations with multiple attributes. This requires the identification of subjective or objective probability for uncertain consequences, and elicitation of DMs preferences or utility over these attributes (Keeney and Raiffa 1976, Keeney and Wood 1977, Brown *et al.* 1974, Barclay *et al.* 1977, Huber and Johnson 1977, Sage 1977, Keeney and Kirkwood 1975). The added



Initial ranking of Alternatives (decisions) is accomplished by averaging out the single stage decision tree: i.e. $\underset{a}{\text{Max}}\, E(u(z,a)) = \underset{a_i}{\text{Max}} (\sum_j p_j(a_i)u(z_{ij}(a_i)))$

Figure 5. Multi-attribute decision analysis model.

complication of multiple DMs and advisors at various levels suggests the consideration of group amalgamation of preferences (Banker and Gupta 1978, Keeney and Kirkwood 1975, Nakayama *et al.* 1979).

The basic decision model is illustrated in Fig. 5. The decision space is made up of individual actions of the reduced NDSS

$$a_i \equiv \text{incorporating set } i \text{ (alternative } i\text{) of equipment as the retrofit system,} \quad i = 1, 2, \ldots, n$$

Thus, set i includes a combination of EW equipment with associated attribute values for X_i, $i = 1, 2, 3, 4$ as discussed in § 3. For example, selecting a_1, implementing alternative 1, causes the retrofit of a system with components which collectively give the attribute levels z_{11}(i.e. $\mathbf{x}_1^{z_{11}}, x_2^{z_{11}}, \mathbf{x}_3^{z_{11}}, x_4^{z_{11}}$) with probability p_1 where p_1 is the combined probability of $\mathbf{x}_1 = \mathbf{x}_1^{z_{11}}$, $x_2 = x_2^{z_{11}}$, $\mathbf{x}_3 = \mathbf{x}_3^{z_{11}}$, $x_4 = x_4^{z_{11}}$. The probability of the system configuration under alternative 1 being at other collective attribute levels or output states is p_i where $i = 1, 2, \ldots, m$ and $\sum_{i=1}^{m} p_i = 1$. These probabilities express the likelihood of the retrofit system giving the estimated values of the attributes in actual operation on the aircraft. In order to incorporate the risk of system implementation into the MAUT format, output configurations other than the expected configuration were constructed from the data and combined with probabilities of realization from Monte Carlo simulations. In order to keep the problem in a framework that current EWARD stakeholders are familiar with, only three resulting configurations were used for each alternative configuration. The characteristics of these three configurations (a degraded, expected, and improved version) of each alternative system is shown in Table 4. Because of the normal density function forms for all candidate systems, three standard probabilities were used for the three approximate outputs of each candidate system ($p = 0{\cdot}63$ for the expected configuration state, $p = 0{\cdot}16$ for the improved configuration state, and $p = 0{\cdot}16$ for the degraded configuration state). Figure 7 illustrates the rationale behind this approximation. This discretization of the probabilistic data means that $m = 3$ and $n = 8$ for the EWARD formulation of MAUT posed in Fig. 5.

This MAUT technique was utilized for specific application in the Conceptual Phase of EWARD so as to be able to incorporate available information from all groups at an early phase, and to specify the retrofit configuration and evaluate the resulting system. Care was taken to separate the subjective probability encoding and utility assessments so as to minimize any interaction confounding effects such as bias and double weighting of certain events and outputs.

The first items addressed were the functional relationships between attributes and the form of the multi-attribute utility function which would measure an aggregated felicity for each alternative retrofit system. Since preferential independence among attributes was established in the pre-analysis phase, first order utility independence (UI) was next examined. X_i is UI of $\bar{X}_i$ (all attributes other than X_i) if preferences for risky choices (lotteries) over X_i with the value of $\bar{X}_i$ held fixed do not depend on the fixed value of $\bar{X}_i$. Using standard MAUT assessment techniques (Keeney and Raiffa 1976), it was found that $\mathbf{X}_1$ was utility independent ; and because ($\mathbf{X}_1$, X_i) is PI, $i = 2, 3, 4$, using Theorem 6.2 of Keeney and Raiffa (1976) it can be concluded that the major attributes are mutually UI (MUI). The components of each of $\mathbf{X}_1$ and $\mathbf{X}_3$ were also found to be MUI. Next, the attributes were examined with respect to Fishburn Marginality (Fishburn 1967, Winterfeldt and Fischer 1973). It was soon pointed out by the interviewees or respondents that additive independence did not hold. Therefore invoking the fact that the attributes are MUI and Theorem 6.1 of Keeney and Raiffa (1976), we conclude that a multiplicative form of multiattribute utility function is appropriate for this application. The multiplicative utility function has the general form

$$1 + Ku = \prod_i (1 + KK_i u_i) \tag{1}$$

where u is the combined utility function, u_i is the ith constituent utility function, K_i is a scaling constant for the ith utility function, and K is the scaling constant for the combined utility function. Specifically, the utility functions for attribute $\mathbf{X}_1$ and $\mathbf{X}_3$ respectively are

$$1 + k_1 u_1 = (1 + k_1 k_{1a} u_{1a}(x_{1a}))(1 + k_1 k_{1b} u_{1b}(x_{1b}))(1 + k_1 k_{1c} u_{1c}(x_{1c})) \tag{2}$$

and

$$1 + k_3 u_3 = (1 + k_3 k_{3a} u_{3a}(x_{3a}))(1 + k_3 k_{3b} u_{3b}(x_{3b}))(1 + k_3 k_{3c} u_{3c}(x_{3c})) \tag{3}$$

where u_{mn} is the lower level attribute utility function, u_j is the higher level utility function, and the k's are scaling constants. The aggregate utility function for each group is

$$1 + KU_i = (1 + KK_1 u_1)(1 + KK_2 u_2)(1 + KK_3 u_3)(1 + KK_4 u_4) \tag{4}$$

where U_i is the combined utility function for each group, $i = 1, 2, 3$ and the K's are scaling constants. The multiplicative form adds complexity of analysis compared to the additive form, but also supplies the required non-compensatory inter-attribute relations necessary in this problem (DeWispelare and Sage 1979).

The utility independence condition, which was verified for all attributes, allows the assessment of eight single dimension utility functions. The utility function for each attribute was needed to incorporate the DMs and advisor' preferences and attitude toward risk. Because of the many gerents at various levels in EWARD, a refinement form of social choice function was used to bring the individual utilities of the DMs and advisors together. Using a modified form of the multiple independent entity/MCDM process of Banker and Gupta (1978) intended for decentralized DMs, the utility functions were extracted for each attribute from the lower level advisors and DMs in each group (G-1, G-2 and G-3). These utility functions were then presented to intermediate and eventually high level DMs in each group for refinement. These refined utility functions were then shown to each level of DM until concensus was reached on the final form for each group (G-1, G-2 and G-3). This way of incorporating the group utility into a single function seemed to work well in this application because it followed the basic organizational 'chain of command' hierarchial decision making process of the government (lower level advisors counsel higher level DMs and higher level DMs feedback policy which focuses this advice thereby causing rapid convergence to a single social choice or welfare function). The refined group utility functions for the attributes are given in Table 5. An example of the group utility curves for attribute x_{3b}, number of threats degraded, is shown in Fig. 6. The majority of the utility curves exhibit a preference structure which is risk averse as illustrated in Fig. 6. The risk averse tendencies by the DMs and advisors were expected in a project which involves government officials (Levy 1974).

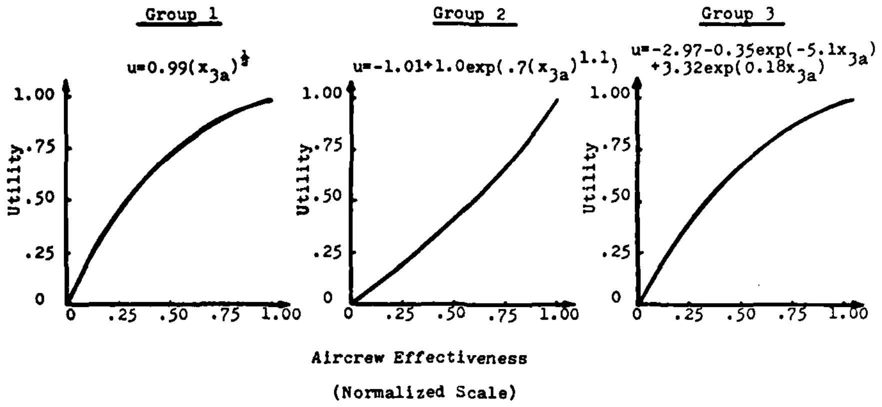


Figure 6. Group utility functions for attribute x_{3a}.

Group Utility Functions For Each Attribute

	Group 1	Group 2	Group 3
$u_{1a}(x_{1a})$	$1.52-0.44\exp(3.1E-4x_{1a})$; $x_{1a} \leq 3500$ $-0.97+2.4\exp(-2.0E-4x_{1a})$; $x_{1a} \geq 3500$	$1.13-2.5E-4\, x_{1a}$	$1.08-0.06\exp(6.5E-4x_{1a})$
$u_{1b}(x_{1b})$	$1.26-0.10\exp(0.61x_{1b})$	$1.65-0.39\exp(0.35x_{1b})$	$1.41-0.19\exp(0.49x_{1b})$
$u_{1c}(x_{1c})$	$1.37-0.15\exp(0.02\, x_{1c})$	$1.28-0.09\exp(0.02\, x_{1c})$	$1.19-0.05\exp(0.03x_{1c})$
$u_2(x_2)$	$1.13-0.11 \cdot \exp(1.15\, E-9\, x_2)$	$1.01-0.01\exp(2.1\, E-9x_2)$	$1.18-0.16\exp(1.0\, E-9\, x_2)$
$u_{3a}(x_{3a})$	$0.99(x_{3a})^{\frac{1}{2}}$	$-1.01+1.0\exp(0.7(x_{3a})^{1.1})$	$-2.97-0.35\exp(-5.1x_{3a})$ $+3.32\exp(0.18x_{3a})$
$u_{3b}(x_{3b})$	$4.92-5.15\exp(-0.01\, x_{3b})$	$2.53-2.80\exp(-0.02x_{3b})$	$1.18-1.72\exp(-0.08x_{3b})$
$u_{3c}(x_{3c})$	$1.52-1.53\exp(-0.04\, x_{3c})$	$1.59-1.59\exp(-0.02\, x_{3c})$; $x_{3c} \leq 20$ $-0.52+0.60\exp(0.03(x_{3c})$; $x_{3c} \geq 20$	$1.01-1.01\exp(-0.13x_{3c})$
$u_4(x_4)$	$1.0x_4$	$1.01-1.01\exp(-5.1x_4)$	$-1.0+1.01\exp(0.69x_4)$

Table 5. Group utility functions for each attribute.

Since we have established the multiplicative form of combined multi-attribute utility function, the task of evaluating the scaling constants for this form were approached next. Using suggested assessment techniques (Keeney and Raiffa 1976) with the interviewees, the group utility functions, Table 5, and the attribute ranges, Table 1, the scaling constants were evaluated for each group and the results are shown in Table 6 for the multiplicative utility functions. Each group's relative strengths of the preferences for the attributes is evident in this table. For instance, the values of the combined utility function scaling coefficients show that Group 2 prefers a high level policy satisfaction (K_4) and low cost (K_2) over performance (K_1 and K_3) while Group 3 values aerodynamic (K_1) and electronic warfare performance (K_3) over political (K_4) and financial (K_2) considerations.

Group Utility Functions

Top Level Utility Functions

$$1+KU_i = (1+KK_1u_1)(1+KK_2u_2)(1+KK_3u_3)(1+KK_4u_4)$$

where U_i = combined group utility function, $i = 1,2,3$

u_j = higher level attribute constituent utility function $j = 1,2,3,4$

Scaling Coefficients	Group 1	Group 2	Group 3
K	-0.76	-0.97	-0.98
K_1	0.51	0.30	0.69
K_2	0.33	0.75	0.46
K_3	0.49	0.32	0.92
K_4	0.21	0.80	0.45

Lowest Level Attribute Utility Functions

$$1+k_1u_1 = (1+k_1k_{1a}u_{1a}(x_{1a}))(1+k_1k_{1b}u_{1b}(x_{1b}))(1+k_1k_{1c}u_{1c}(x_{1c}))$$

$$u_2 = u_2(x_2)$$

$$1+k_3u_3 = (1+k_3k_{3a}u_{3a}(x_{3a}))(1+k_3k_{3b}u_{3b}(x_{3b}))(1+k_3k_{3c}(x_{3c}))$$

$$u_4 = u_4(x_4)$$

where u_{mn} = lower level attribute utility function; $m = 1,3$ $n = a,b,c$

Scaling Coefficients	Group 1	Group 2	Group 3
k_1	-0.60	-0.29	0.14
k_{1a}	0.48	0.40	0.38
k_{1b}	0.59	0.37	0.26
k_{1c}	0.24	0.35	0.31
k_3	-0.40	5.04	-0.97
k_{3a}	0.51	0.12	0.42
k_{3b}	0.29	0.25	0.48
k_{3c}	0.38	0.13	0.95

Table 6. Group utility functions.

Each group's (G-1, G-2, G-3) utility for the set of alternatives described in Tables 2 and 3 were calculated using the multiplicative form and scaling constants listed in Table 6 for the attribute template of Fig. 2. The constituent utilities for the expected values of the candidate systems for all three groups are listed in Table 7. The combined group felicity for each alternative EW retrofit system (expected version, improved version, and degraded version) in the reduced XDSS is shown in Table 8. The ranking of alternatives step

Utility Values For The Expected Alternative Configurations

Alternatives	$u^*_{1a_1}$	$u^*_{1a_2}$	$u^*_{1a_3}$	$u^*_{1b_1}$	$u^*_{1b_2}$	$u^*_{1b_3}$	$u^*_{1c_1}$	$u^*_{1c_2}$	$u^*_{1c_3}$	$u^{**}_{A_1}$	$u^{**}_{A_2}$	$u^{**}_{A_3}$	u'_{2_1}	u'_{2_2}	u'_{2_3}
1	.47	.43	.72	.87	.81	.85	.58	.61	.64	.75	.65	.70	.68	.82	.65
4	.95	.93	.98	.98	.97	.99	.91	.93	.94	.98	.97	.86	.81	.91	.78
7	.88	.83	.95	.58	.49	.54	.83	.85	.87	.80	.76	.77	.97	.98	.96
8	.59	.53	.80	.89	.84	.87	.60	.63	.66	.80	.65	.70	.62	.78	.59

Alternatives	$u^*_{3a_1}$	$u^*_{3a_2}$	$u^*_{3a_3}$	$u^*_{3b_1}$	$u^*_{3b_2}$	$u^*_{3b_3}$	$u^*_{3c_1}$	$u^*_{3c_2}$	$u^*_{3c_3}$	$u^*_{C_1}$	$u^*_{C_2}$	$u^*_{C_3}$	u'_{4_1}	u'_{4_2}	u'_{4_3}
1	.69	.38	.63	.47	.52	.65	.47	.36	.83	.60	.28	.90	.70	.98	.62
4	.79	.53	.75	.28	.32	.47	.27	.20	.54	.50	.17	.72	.44	.90	.37
7	.65	.32	.58	.28	.32	.47	.18	.14	.39	.41	.12	.55	.65	.97	.57
8	.87	.65	.82	.54	.57	.73	.54	.41	.83	.73	.43	.95	.68	.97	.60

* u_{ij_k} = utility of component j (j = a,b,c) of attribute $\underline{X}_i$ (i = 1,3) for Group k (k = 1,2,3)

** u_{m_n} = combined utility of attribute X_m (m = A,C,A for $\underline{X}_1$ and C for $\underline{X}_3$)for Group n (n = 1,2,3)

' u_{s_t} = utility of attribute X_s (s = 2,4) for Group 5 (t = 1,2,3)

Table 7. Utility values for the expected alternative configurations.

Combined Utility For The Various Configurations Of Alternatives

Alternatives	u_{1E}	u_{2E}	u_{3E}	u_{1D}	u_{2D}	u_{3D}	u_{1I}	u_{2I}	u_{3I}
1	0.78	0.96	0.96	0.76	0.94	0.94	0.80	0.98	0.97
4	0.83	0.97	0.93	0.82	0.95	0.89	0.86	0.98	0.94
7	0.78	0.97	0.90	0.77	0.90	0.89	0.81	0.94	0.90
8	0.82	0.95	0.97	0.80	0.93	0.94	0.84	0.97	0.98

u_{ij} = utility for configuration j (j = E(Expected, I(Improved), D(Degraded)) of Group i (i = 1,2,3)

Table 8. Combined utility for the various configurations of alternatives.

was next accomplished by maximizing the expected individual group utilities using the decision tree of Fig. 6. The resulting group utilities for each alternative were generated by averaging out the single stage MAUT formulation of Fig. 6 for each alternative using the relation

$$E[u(z, a_i)] = \sum_j p_j(a_i)u[z_{ij}(a_i)] \tag{5}$$

where $p_j(a_i)$ is the probability of outcome z_{ij}, and $u[z_{ij}(a_i)]$ is the multi-attribute utility of outcome z_{ij}, and $E[u(z, a_i)]$ represents the expected utility or score for alternative a_i. The utility data of Table 8 and the probability data discussed earlier and illustrated in Fig. 7 were substituted into this expected utility formulation. Table 9 shows the output of these calculations and the resulting preference rankings for the individual groups with respect to the alternatives. The closeness of the utility scores for each group is explained by the fact that even though utilities were elicited for the full range of each attribute, the alternatives in the NDSS had attribute values that were in a much narrower range.

Group Preference Rankings of Alternatives

Alternatives	u_1	Ranking	u_2	Ranking	u_3	Ranking
1	0.781	4	0.965	2	0.958	2
4	0.833	1	0.968	1	0.923	3
7	0.784	3	0.959	3	0.906	4
8	0.821	2	0.951	4	0.965	1

u = total utility for given alternative of Group k (k=1,2,3)

Table 9. Group preference rankings of alternatives.

These rankings show that the G-1 (operations, intelligence) prefer the alternatives which aid aircraft performance and are average or compromise systems. This was expected since many of the participants in G-1 have air crew experience (pilots, navigators, weapons operators, etc.). G-2 preferred the compromise and low cost alternatives. This also was expected since budgetary constraints and political agreements are driving forces for the government policy makers. It is noted that there is a correlation in the upper levels of alternative choices of G-1 and G-2 which is possibly precipitated by the close interaction between the DMs and advisors in G-1 and G-2 discussed in § 2. G-3 (program managers, contractors, analysts, etc.) preferred the high EW and aircraft performance, high cost, and compromise alternatives. These choices were anticipated since engineers, contractors, and analysts in G-3 are basically performance oriented and consider cost secondarily. Alternatives 4 (high aircraft performance) and 8 (best compromise) were highly preferred by all groups. Because no single alternative was ranked as most preferred by all groups, the final modelling step was the combination of the three separate group (G-1, G-2 and G-3) multi-attribute utility functions into a single social choice function so that final ranking can be performed for the EWARD presented. Because it can be seen from Table 9 that there are conflicting ranking for alternatives, a means was required to obtain combined group consensus. Because there are three groups and more than three alternatives, there is no guaranteed way of obtaining consensus (through popular techniques like simple majority rule, simple additive weighting, etc.) without violating Arrow's axioms in the impossibility theorem (Arrow 1963). This is because interpersonal comparison of utilities is required. Nevertheless, concensus is required, and two techniques which can aggregate the group utilities are the cardinal social welfare function discussed by Keeney and Kirkwood (1975) and the Extended Contributive Rule method (ECR) discussed by Nakayama *et al.* (1979).

The Extended Contributive Rule method (ECR) was used to aggregate individual groups' preferences into a single choice function (Nakayama *et al.* 1979). ECR amalgamates the preferences of the DMs in a way that incorporates directly the degree of confidence of all the individual groups in their own preferences and in other groups' preferences, and the intensity of each preference. ECR was particularly applicable to EWARD since it considers preferences between two alternatives at a time and therefore given an output which is readily transformed into a directed graph indicating preference relationships (Sage 1977, White and Sage 1980) for display purposes. The ECR method of combining group preferences was chosen over the simple linear additive and multiplicative forms of combining group utility in this application because of two reasons :

(1) ECR specifies a way of making interpersonal comparison of utility. Each group ranks the importance of the opinion of all groups so that weights can be established for each group's utility. This concept of intergroup weighing makes intuitive sense in a governmental setting where the various groups realize their relative position with respect to the political structure which must eventually authorize funding for the EWARD production.
(2) ECR allows a preference threshold to be incorporated into the ranking of alternatives step. This thresholding feature affords the DM the opportunity to not only establish specific preference relations between alternatives, but also reveal the strengths of these preference relations. These pairwise preference relations can then be easily displayed on a digraph.

The ECR algorithm is now described. 'r a set of alternatives $A=(A_1, \ldots, A_n)$, and group utility functions $u_i=(u_1, u_2, u_3)$, let w_{yz} be the weight which group y imposes on the utility of group z. That is, w_{yz} represents group z influences. We define the quantity

$$\bar{A}=\sum_{i=1}^{3} w_i \,.\, \Delta u_{jk}{}^{i}+\alpha\left(\sum_{i=1}^{3} w_i \,.\, \min\,(0, \Delta u_{jk}{}^{i})-\beta\right) \tag{6}$$

where

$$w_i=\left(\sum_{y=1}^{3} w_{yi}\right)/3, \quad \alpha \geqslant 0, \quad \beta \geqslant 0 \quad \text{and} \quad \Delta u_{jk}{}^{i}=u_i(A_j)-u_i(A_k)$$

For two alternatives A_j, $A_k \in A$; A_j is preferred to A_k ($A_j \succ \alpha A_k$), at an opinion level α if and only if $\bar{A}>0$. All possible combinations of alternatives taken two at a time are categorized with respect to a preference existing or not (binary relation). The parameter α indicates the weighing which is given to opposite preference opinions. It is a way of requiring a level of coincidence (all or a majority of opinions agree in order to establish a preference relationship) of individual group opinions (i.e. if $\alpha=0$, no consideration for opposite opinions is given (no agreement of opinion is required) and the ECR takes the form of a linear additive SWF ; if α is large, then complete unanimity of opinion is required to establish preference relations as an opposing preference is heavily weighed). The parameter β is used to indicate the intensity (strong, weak, etc.) of the preference relations. This parameter allows the DM to differentiate the strong from the weak preference relations. It is in effect a threshold which can be set to allow only preference relations above this value to be recognized. The parameters α and β are initially set at large values and the pairwise preference relations are determined. These preference relations are transformed into a digraph indicating preferences. If the digraph arrangement of alternatives does not have vertical structure, then the threshold β is decreased in small steps. This has the effect of establishing the weaker pairwise preference relations which were previously cancelled due to the large threshold. Ultimately a vertical digraph is established. If $\beta = 0$ and a vertical digraph is still not evident, then the coincidence of opinion parameter, α, is decreased and the algorithm repeated until a vertical or near vertical digraph results.

The weights shown in Table 10 were elicited from each group to establish the intergroup comparison of utilities. The ECR algorithm was evaluated using information from Tables 9 and 10 along with various values of α and β to establish the preferences shown in Fig. 7.

Intergroup Weighing Of Utilities

	w_{i1}	w_{i2}	w_{i3}	
Group 1	0.38	0.41	0.21	$\sum_j w_{ij} = 1$ (normalized)
Group 2	0.25	0.50	0.25	
Group 3	0.25	0.25	0.50	
average w_j ($w_j = \sum_i w_{ij}/3$)	0.29	0.39	0.32	

- w_{ij} is the weight assigned to the utility of Group j by Group i
- w_j is the resulting average weight of Group j

Table 10. Intergroup weighting of utilities.

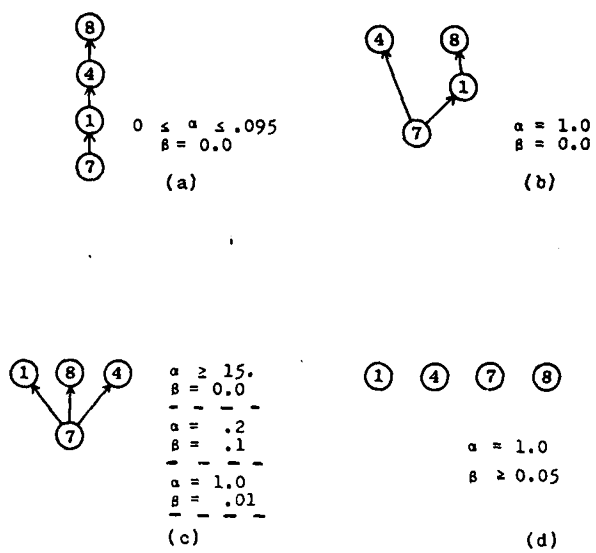


Figure 7. Group ranking of alternatives using ECR.

Figure 7 shows that there are three alternatives which rank consistently higher than the other alternatives. These alternatives (1, 4 and 8) exhibit medium or compromise values of attributes of cost and EW performance along with high aircraft performance. For α (the parameter which weighs opposite preference opinions heavily) in the range of $0 \leqslant \alpha \leqslant 0 \cdot 095$ and $\beta = 0$ (with $\alpha = 0$, the ECR social choice function assumes the form of a linear additive SWF) alternative 8 dominates (Fig. 9 (*a*)). As α is increased to larger values, the alternatives aligned themselves in two groups of a preferred set (1, 4 and 8) and a dominated set (7) as illustrated in Fig. 7 (*b*) and 7 (*c*). As a threshold value was brought in (β increases from 0), the ranking of Figure 7 (*a*) started to decompose until the strengths of all preference relations are overcome and complete unaniminity would be required to establish a preference relation (Fig. 7 (*d*)). When these results were presented to the advisors and DMs in all groups, there was concensus that alternative 8 was the most desirable and its specifications would be used as the standard in the RFP.

A sensitivity analysis was conducted to examine the robustness of the final ranking of alternative systems, and to point out critical areas in this approach to EWARD.

As a check of the DMs consistency in the MOOT/MAUT process, the utility based ranking of alternatives was made just prior to aggregating the group utilities into a joint SCF. The group utility was calculated for all alternatives not in the NDSS. The combined group utilities for these alternatives were calculated and compared to those in the NDSS. The resulting rankings showed that alternatives 2, 3, 5 and 6 are indeed dominated in all groups. This indicates that the DMs were consistent in the value scoring of alternatives, elimination by aspects, and utility elicitation tasks.

The significance of the accuracy of the input data on the ranking of the alternatives was tested by varying the average values of Table 3. Varying the values of all the attributes scores by 5% for each alternative, first in a beneficial and then detrimental direction, did not affect the groups preference ordering of the top three alternatives. Varying all attribute scores by 5% in a beneficial direction for the second best alternative, caused this alternative (no. 4) to become optimal. Changing the life cycle cost attribute weight by 6% caused a decision switch from alternative 8 to alternative 4. These results should alert analyst and decision maker to the realization that alternative system parameter accuracy is important in that alternative 4 is almost as good as alternative 8. This is particularly significant also when considering that the data for the policy satisfaction attribute was obtained primarily from subjective estimates.

Sensitivity to variation in the individual group functions weighting coefficients was greater than that due to variation of alternative scores but 5% changes in weights did not produce any consequential change in rankings.

A check on the effects of the variation of the individual group's stated minimum acceptable attainment level was accomplished next since these attainment levels were the basis for an elimination by aspect exercise. It is noted that because all attainment levels of each individual group had to be justified to the other groups, only reasonable and fairly conservative [illegible]ment levels were supplied by the groups. While variation by 20% in several attainment levels (policy satisfaction, threats degraded, etc.) caused variation in the membership of alternatives in the NDSS, these affected alternatives (5 and 7) were marginal and not prominently ranked in the final analysis.

Lastly, the robustness of the selection of the optimum alternative system was examined with respect to the weighing of each other's opinion by the individual groups. In the ECR group utility aggregation algorithm, the intergroup opinion weights were varied up to 10% with only minor variation in the final alternative rankings (alternative system 4 and alternative system 1 changed places in the rankings). Alternative 8 was still the optimum choice even with this variation in the intergroup comparison of utilities.

5. Validation, acceptability, and implementation considerations

The DMs and advisors who took part in this exercise expressed satisfaction in the MOOT/MAUT approach to EWARD. Therefore, this technique was used in a validation exercise to see how it would have done on an actual system which is operational now. The data from a recent EW retrofit system was processed to fit the modelling approach of the combined MOOT/MAUT method presented in this paper. Four alternative systems were reduced in the three groups (G-1, G-2, G-3) to a set of non-dominated alternatives from which the groups selected a 'best' alternative. The alternative which was selected was a modification of the system which was actually retrofit on the aircraft. All groups agreed that the system selected by the MOOT/MAUT approach was superior, and was selected in a more efficient manner compared to the system which was eventually retrofit. The DMs expressed their opinions for the discrepancy and these can be summed up *in two points*:

(1) There was no comprehensive set of criteria, such as in Fig. 2, which could be used to judge system goodness early in the acquisition life cycle in the actual retrofit.
(2) There was a lack of communication between pertinent stakeholders in the actual retrofit which was significantly ameliorated by the suggested technique. Therefore, it is concluded that this exercise validates the appropriateness and efficacy of the use of MCDT (specifically the MOOT/MAUT approach) in the early phases of the DOD Equipment Acquisition Life Cycle.

Twenty-one DMs and advisors from various stakeholder groups took part in the elicitation part of this effort. Interviews with these participants consisted of fact finding and preference elicitation sessions. The average time spent with each participant was 2·4 hours, and the maximum time for any individual was 18 hours. The majority of participants (nineteen) expressed satisfaction and acceptance of this approach to EWARD, and indicated they would be in favour of using this technique in future efforts. The majority of participants gave the opinion that the two main benefits of this approach to EWARD are:

(1) The final product is an acceptable, cost effective system.
(2) There is a definite savings in time accrued through this approach (the participants estimated that up to one years time (50%) in the Conceptual Phase could be saved utilizing this technique) compared to the present design procedure.

As mentioned in §§ 2 and 3, the application of the MOOT/MAUT approach to EWARD should be in the Conceptual Phase of the Defence Systems Acquisition Cycle before the PMP is prepared. The application at this point in the acquisition cycle would allow for a maximum of benefits (determine the system configuration and get all pertinent groups communicating) without the need for legislation, or any changes of the current regulations. The current regulations do not generally specify application of particular techniques, but they allow for the application of desirable techniques. The analysts required could be drawn from the staffs in all three groups (User Commands, Hq. USAF and/or DOD, and Systems/Logistics Commands), and formed into a dedicated team for the EWARD (versus working at the retrofit problem in fragmented groups as they do currently) without the requirement for additional personnel. This team would ideally be responsible to the DSZRC/SECDEF to allow the team to operate with the cooperation of all groups but immune from the influence of any one group.

6. Summary

In this effort, a decision situation involving Electronic Warfare Aircraft Retrofit Design (EWARD) was modelled and resolved using the combined MOOT/MAUT approach. We began our efforts with an overview of the DOD Equipment Acquisition Cycle as set down in various regulations and directives. Several considerations in the electronic warfare community were described because of difficulties they cause in the acquisition procedure. The primary stakeholder groups were identified along with their interaction in the equipment retrofit situation. The decision making procedure was applied to the Conceptual Phase of the DOD Equipment Acquisition Cycle where it was intended to produce two results :

(1) A salient set of design criteria (candidate system descriptors) which can be used in judging the goodness of a proposed EW alternative design in the early phases of the EW equipment acquisition.
(2) A decision making procedure for candidate system selection as a basis for system selection and the RFP.

The EWARD situation was modelled around attributes adequate for judging the EW retrofit system and a set of realistically based alternatives for a retrofit design. These alternatives were supplied with attribute levels and the associated risk for realization of the alternatives. Using the MCDT approach, the combined MOOT/MAUT process was used to solve the EWARD situation. A pre-analysis phase was used to structure the decision situation. The optimization step consisted of a multi-dimensional elimination by stochastic dominance exercise since the impacts of implementing the various alternative systems are provided by the competing contractors. The resulting NDSS was further reduced in the number of alternative systems when the individual groups supplied minimum acceptable attainment figures for the attributes as an input to an elimination by aspects exercise. The MAUT technique of multi-attribute decision analysis was used to model and rank the alternatives in the NDSS by developing a cardinal utility SCF for each group. These group utility functions were amalgamated into an ultimate SCF which produced a final ranking of the NDSS and identification of alternative 8 as the optimum system configuration.

Group consensus corroborated this alternative selection. A certain amount of iteration was required in all steps of the MOOT/MAUT process in order to converge to an acceptable policy choice.

A subsequent sensitivity analysis of the final ranking of the alternatives, with alternative 8 as the identified optimal configuration, produced the following observations: the DMs were consistent with respect to value scoring and utility function scoring of the attribute as a basis for forming the NDSS and then ranking the resulting alternatives; the final ranking of alternatives was somewhat sensitive to the accuracy of the alternatives systems impact data, and the stated minimum acceptable attainment levels of the individual groups; the final ranking was sensitive to the accuracy of the weighting constants in the individual group utility functions.

A validation exercise was then carried out using data from a recent EW retrofit system. The groups selected a system using the approach which all DMs and advisors stated was the superior system of the alternatives available.

We have illustrated the use of a MCDT approach in the early phases of a particular equipment acquisition effort. The approach can produce a solution, based on values and data provided by users, which contains risk/uncertainty elements. The validation effort demonstrated that such an approach forces DMs and their advisors to consider the salient attributes early and to communicate, or at least add opinions and inputs to the situation, with other stakeholder groups. The technique aids the decision maker in evaluating the most desirable alternative. The MOOT/MAUT approach involves the DMs in a fashion that should give the DMs confidence in the results obtained.

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Sensitivity Analysis in Systems for Planning and Decision Support†*

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ABSTRACT: *This paper surveys contemporary research involving error and sensitivity analysis approaches useful for the design of aids for planning and decision support. Discussed are structural sensitivity considerations as well as the effects of errors, for both single and multi-attribute cases, in estimation or elicitation of probabilities and utilities. One of the major uses for sensitivity analysis type results is in bounded prioritization of alternatives using ordinal information. This use of sensitivity analysis is discussed and illustrated with examples.*

I. Introduction

A contemporary effort of much interest is the design of evaluation and choice making aids for planning and decision support processes. These adjuvants are sometimes called management information systems, although we feel that the terms decision support system, or planning and decision support system, are more appropriate. A central purpose in use of these systems is not just presentation of information representing facts and values, but the aggregation of this information to aid in evaluation and choicemaking. Research in this area involves many disciplines and perspectives; and thus we have a large scale systems problem. There are a number of sources of error in the design of decision analysis algorithms for planning and decision support systems. We discuss several of these in this paper, namely: errors in the structure of the decision situation, errors in the elicitation of probabilities, and errors in the assessment of single and multiple attribute utility functions. Decisionmakers sometimes find it very difficult to provide precise (cardinal) estimates of weights and find it much less stressful to provide ordinal values. As shown in Section IV, sensitivity results can often then be used to infer priorities. We conclude our survey and presentation with a discussion of some contemporary research needs in this area.

II. Sensitivity to Probability Estimation Errors

In decision analysis problems under risk, it is necessary to obtain an objective or subjective estimate of the probability that various outcome states

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will result from decision alternatives. Once a recommended decision has been established, it is often useful to determine the magnitude of the changes in the state probabilities required for the recommended decision to become less desirable than another decision alternative. This magnitude, coupled with some knowledge of the quality of the state probability estimates, can be used to determine how confident we are in the optimality of the recommended decision.

We consider the three outcome state case first; then we generalize these results to the n outcome state case. For convenience, we assume that the probability associated with each outcome state is independent of the action alternatives. With alternative a^i we associate an outcome utility vector

$$(u^i)^T = [u^i_1\ u^i_2 u^i_3] \tag{1}$$

and we write for the outcome state probability

$$p^T = [p_1\ p_2\ p_3]. \tag{2}$$

The expected utility of alternative a^i is then

$$EU(a^i) = p^T u^i = p_1 u^i_1 + p_2 u^i_2 + p_3 u^i_3. \tag{3}$$

Now suppose that the probabilities, p_i, are perturbed. Since we must maintain

$$p_1 + p_2 + p_3 = 1, \quad p_i \geq 0 \tag{4}$$

we must have

$$\Delta p_1 + \Delta p_2 + \Delta p_3 = 0. \tag{5}$$

By substituting (4) into (3), we see that the equation for constant expected utility is that of a straight line

$$EU(a^i) = u^i_3 + p_1(u^i_1 - u^i_3) + p_2(u^i_2 - u^i_3) \tag{6}$$

in two dimensions, p_1 and p_2. The difference in expected utility for alternative i and j is some amount Δ^{ij} given by

$$\Delta^{ij} = EU(a^i) - EU(a^j) = p_1(u^i_1 - u^j_1) + p_2(u^i_2 - u^j_2) + p_3(u^i_3 - u^j_3). \tag{7}$$

As long as $\Delta^{ij} \geq 0$ we know that alternative a^i is preferred to alternative a^j. The relation $\Delta^{ij} = 0$ graphs as a straight line in the p_1, p_2 plane if we make use of (4) to eliminate p_3. Doing this, however, distorts the planes of interest somewhat. Fortunately, the three dimensional space for p_1, p_2, p_3 becomes a plane when we associate the constraint of (4) with this space.

Figure (1) indicates a typical probability triangle. It is straightforward to

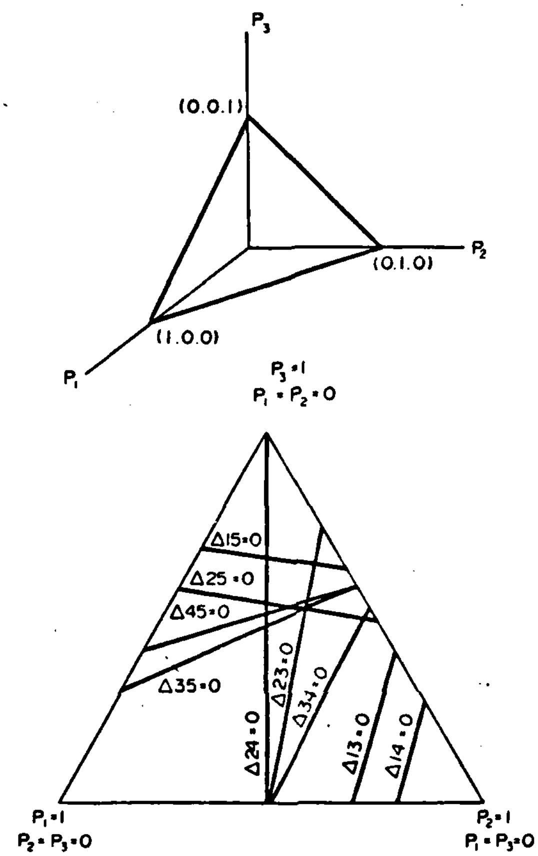

FIG. 1. Probability triangle and planar projection with decision switch points from Table I.

show that graphs of $\Delta^{ij} = 0$ are straight lines in Fig. 1. For example, the graphs of $\Delta^{ij} = 0$ for the problem of Table I are shown in Fig. 1.

TABLE I
Illustrative utilities for a simple example

	Outcome states		
Alternative	x_1	x_2	x_3
a^1	1.0	0.7	0.5
a^2	0.9	0.6	0.3
a^3	0.6	0.9	0.2
a^4	0.7	0.8	0.3
a^5	0.0	0.0	1.0

Of interest in this figure is the fact that alternative 1 dominates alternative 2 in the utility of each outcome state is greater for alternative 1 than for alternative 2. Further, it is not possible for alternative 4 to be the best alternative and the optimum decision regions are as shown in Fig. 2.

For more than three outcome states, the graphical approach suggested here is infeasible. Isaacs (12) and Fishburn *et al.* (7) describe a general approach that is applicable to the n dimensional case. This approach allows determination of that "second best" alternative which could become the best alternative due to a minimum overall variation in probability.

There are, in general, a variety of possible ways in which probability estimates can be incomplete. Among these are the following (6, 12):

(1) The decision maker provides an estimate of the p_i.

(2) The decision maker provides a probability density function, $g(p_i)$, for the p_i.

(3) There exists no information about the p_i.

(4) There exists an ordinal measure, or ordering of the p_i. For convenience, and without loss of generality, we may assume that $p_1 \geq p_2 \geq \cdots \geq p_n$.

(5) There exists bounded internal measures such that each p_i is bounded, such as $\alpha_i \leq p_i \leq \alpha_i + \epsilon_i$ with $\alpha_i \geq 0$, $\epsilon_i \geq 0$.

(6) There exists a set of inequalities relating the P_i, such as

$$\sum_{j=1}^{k+1} p_{j+1} \geq p_j \geq \sum_{j=1}^{k} p_{j+1}.$$

(7) Some of the p_i are known whereas others are related by inequalities of the forms given in (4), (5) or (6).

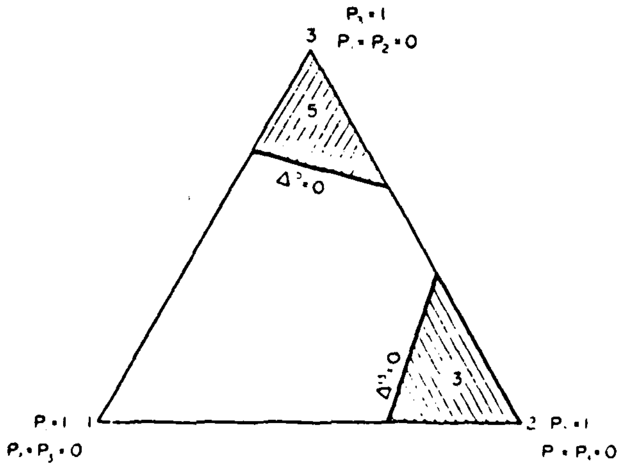

FIG. 2. Optimum decision regions as a function of probabilities.

There exists several ways in which the, possibly partial, information concerning probabilities may be processed to assist in evaluation of the alternative courses of action. Among these are:

(a) The estimate of the p_i, if provided, may be used to obtain the subjective expected utility of each alternative a^i from the relation $\sum_{j=1}^{n} p_j u_j^i$. A sensitivity analysis similar to that of our Section 3 may be used to determine the possibility of a decision switch due to probability elicitation errors.

(b) The expected probability of each event outcome may be computed from $\hat{p}_j = \int_{-\infty}^{\infty} p_j g(p_j)\,dp_j$ and the $\hat{p}_j$ used in place of the p_j. Estimates of subjective expected utility are obtained as in (a) above.

(c) Various dominance relations may be obtained from the ordinal bounds and bounded interval measures provided by the decision maker.

(d) Various minimum changes for a decision switch may be obtained.

(e) Regions in which various alternatives are best may be determined and displayed for the decision maker.

Often, various forms of stochastic dominance **(27, 28)** can be used to eliminate alternatives from consideration even when there exists little or no information concerning event outcome probabilities. For case 3, in which there exists no probability information, alternative j will dominate alternative i if $u^j \geq u^i$, with the inequality holding for at least one component of the utility vector. In component form this becomes $u_k^j \geq u_k^i$, $k = 1, 2, \ldots, n$. For example, we easily see that alternative 1 dominates alternative 2 for the alternatives and utilities illustrated in Table I. This may be written $a^1 > a^2$.

Further, it is often possible to identify an alternative which may not be dominated but which is inferior to or dominated by a mixed strategy consisting of a mass probability $F^T = [F^1, F^2, \ldots, F^m]$ on the set of primary alternatives $a^T = [a^1, a^2, \ldots, a^m]$. If the decision maker adopts mixed strategy F, then alternative a^i is elected with probability F^i. For the example posed by the data Table I we see that $0.5u_j^1 + 0.5u_j^3 \geq u_j^4$, $j = 1, 2, 3$. Thus alternative 4 is dominated by the mixed strategy $F^T = [0.5, 0, 0.5, 0, 0]$. Consequently we may delete alternative 4 from further consideration for the particular case when mixed strategies can be considered and when there is no information available concerning the probabilities of event outcomes. It is dominated by the mixed strategy of choosing alternative 1 with probability 0.5 and alternative 3 with probability 0.5.

In the general case, with no assessment of probabilities, alternative a^i is dominated by a mixed strategy F, assuming the probability vector $\mathbf{p}$ is the same for all alternatives, if

$$\sum_{j=1} F^j u_k^j \geq u_k^i \quad \text{for} \quad k = 1, 2, \ldots, n. \tag{8}$$

If there exists an ordinal ranking of probabilities, as in case 4, then we can easily show that option alternative a^i is dominated by a mixed strategy if there exists a mixed strategy F such that

$$\sum_{j \neq i} F^j\left(\sum_{k=1}^{j} u_k^j\right) \geq \sum_{k=1}^{j} u_k^i \quad \text{for} \quad j = 1, 2, \ldots, m. \tag{9}$$

Unfortunately there does not appear to be any method that is general and simple to use to determine appropriate mixed strategies. Often, also, the appropriateness of mixed strategies must be questioned for many applications.

For the case where there exists bounded interval measures in the form of case 5, then the primary strategy alternative a^i dominates alternative a^j if

$$\Theta + \left[\sum_{k=1}^{n} \alpha_k (u_k^i - u_k^j)\right] \geq 0 \tag{10}$$

where Θ is the minimum value of the objective function for the linear programming problem

$$\min \sum_{k=1}^{n} x_k (u_k^i - u_k^j) \tag{11}$$

subject to $0 \leq x_k \leq \epsilon_k$ and

$$\sum_{k=1}^{n} x_k = 1 - \sum_{k=1}^{n} \alpha_k. \tag{12}$$

There appear to be no general formulae for cases 6 and 7 in which there exists sets of inequalities governing the p_i. For any given set of inequalities, we may write equations involving expected utilities and then equate coefficients. A paper by Barron (1) provides two detailed examples of computations involving these inequalities. Additional details concerning sensitivity of decisions to probability estimation errors may be found in (**6, 7, 23, 28, 29**).

III. Sensitivity to Variations in Utility—Single Attribute Utility Functions for Decisions Under Uncertainty

In this section, we examine sensitivity relations to changes in utility functions. This section will concern the single attribute case. We will extend the results in this section to the scalar multi-attribute case in the next section. The vector multi-attribute case is considered in (**28**). When considering utility function changes, it is convenient but not at all necessary to consider that the same output value or utility function is common to all alternatives. We, therefore, represent different alternatives by different probability density or mass functions.

For the continuous state case, we consider a utility function $u(x)$ and alternative a^i defined by the associated probability density function $f^i(x)$. For the finite state case we consider a utility function $u(x_i)$, $i = 1, 2, \ldots, n$ and alternatives a^j defined by an associated probability mass vector function p^j

which has n components, $(p^i)^T = [p^i(x_1), p^i(x_2), \ldots, p^i(x_n)] = [p_1^i, p_2^i, \ldots, p_n^i]$.

There are a variety of ways in which utility estimates can be stated, possibly incompletely. Among these are the following:

(1) The decision maker may provide a complete estimate of the value function $v(x)$ and utility function $u(x)$.

(2) The utility function $u(x)$ or value function may be completely unspecified.

(3) Only ordinal preferences are specified.

(4) The value function may be specified, but not the risk aversion coefficient. In this case, the utility function is unspecified.

If the value and utility function are completely specified, the expected utility of each alternative may be computed and a sensitivity analysis similar to that of Section IV conducted to determine the potential of a likely decision switch due to utility and value function elicitation errors. If the value function is completely unspecified, then the decision maker is inchoate and there is virtually nothing that can be done to aid the decision maker except through procedures that will enhance the value coherence of the decision maker.

Much information concerning alternative option preferences can be obtained from just an ordinal ranking of preference for outcome states. In the sequel we will assume that the decision maker can always express an ordinal preference for the value of event outcomes of the form $v(x_1) \leq v(x_2) \leq \cdots \leq v(x_n)$.

When ordinal preferences among event outcomes can be elicited, and when probabilistic estimates of occurence of these states can be obtained, then concepts of stochastic dominance can be used to determine bounds on alternative preferences. And if values but not utilities are specified, then it will often be possible to specify risk aversion coefficients which bound alternative preferences. We will illustrate each of these claims by means of a simple example. Prior to doing this, however, it is desirable to establish some fundamental concepts concerning stochastic dominance (29).

We say that alternative a^i is preferred to alternative a^j whenever the expected utility of alternative a^i is greater than that for alternative a^j. In symbols, we have if $a^i > a^j$

$$\int_{-\infty}^{\infty} f_i(x)u(x)\,\mathrm{d}x > \int_{-\infty}^{\infty} f_j(x)u(x)\,\mathrm{d}x \tag{13}$$

where $f_i(x)$ is the probability density function for the event outcome, x, associated with alternative a^i; and $u(x)$ is the utility of the outcome states. We wish to provide for a value function $v(x)$ and will restrict this, for convenience to the interval $[0, 1]$. The value function is not necessarily a monotone function of the outcome states, x. The utility function, however, should be isotone in v. Thus it is convenient to rewrite (13) as $a^i \geq a^j$ if

$$\int_0^1 f_i(v)u(v)\,\mathrm{d}v \geq \int_0^1 f_j(v)u(v)\,\mathrm{d}v \tag{14}$$

where we realize that the utility function is written as $u[v(x)]$ but where we delete the x symbol for convenience.

Stochastic dominance concepts are based upon the imposition of a series of increasing constraints upon the form of the utility function $u(v)$. The most trivial assumption is that utility is a monotone increasing function of increasing value. Thus we require $\mathrm{d}v(v)/\mathrm{d}v = u'(v) \geq 0$. We now integrate (14) by parts. Since we have

$$\int_0^1 f_i(v)u(v)\,\mathrm{d}v = P_i(v)u(v)\,|_0^1 - \int_0^1 P_i(v)u'(v)\,\mathrm{d}v = 1 - \int_0^1 P_i(v)u'(v)\,\mathrm{d}v$$

we obtain for (14)

$$\int_0^1 [P_i(v) - P_j(v)]u'(v)\,\mathrm{d}v \leq 0. \tag{15}$$

Without specification of $u(v)$ but with specification that $u'(v) \geq 0$, we see that the inequality of (15) will be satisfied if and only if

$$P_i(v) \leq P_j(v), \quad \forall v \in [0, 1]. \tag{16}$$

With the inequality holding for at least one i. This is the requirement for *first order stochastic dominance.* When the inequality of (16) holds we say that a^i dominates a^j by first order stochastic dominance or $a^i >_1 a^j$.

We can rewrite the expression for the probability mass function

$$P_i(v_k) = \int_0^{v_k} f_i(v)\,\mathrm{d}v$$

in terms of discrete state probabilities $p_i(v_l)$, $l = 1, 2, \ldots, k$ as

$$P_i(v_k) = \sum_{l=1}^{k} p_i(v_l).$$

If, in addition to requiring monotonicity of the utility function, we also require risk aversion; then we impose the further requirement that $\mathrm{d}^2u(v)/\mathrm{d}v^2 = u''(v) \leq 0$. We integrate (15) by parts to obtain

$$u'(1)\Gamma(1) - \int_0^1 \Gamma(v)u''(v)\,\mathrm{d}v \leq 0$$

where

$$\Gamma(v) = \int_0^v [P_i(\alpha) - P_j(\alpha)]\,\mathrm{d}\alpha.$$

To satisfy this requirement for $u'(1) \geq 0$ and $u''(v) \leq 0$, we must require

$\Gamma(v) \leq 0$ in the interval 0 to 1. Our requirement for what is called *second order stochastic dominance* becomes, therefore,

$$\int_0^v [P_i(\alpha) - P_j(\alpha)]\,\mathrm{d}\alpha \leq 0, \quad \forall v \in [0, 1]. \tag{17}$$

For the finite state case we replace (17) with

$$\sum_{l=1}^{k} [P_i(v_l) - P_j(v_l)][v_{l+1} - v_l] \leq 0, \quad \forall k \in [1, n] \tag{18}$$

with the inequality holding for at least one k.

An excellent discussion of stochastic dominance concepts is presented in the chapter by Fishburn and Vickson in (29).

In the general case we can write the requirement for kth order stochastic dominance of alternative a^i over a^j, $a^i >_2 a^j$, as

$$\Gamma_k(v) \leq 0, \quad \forall v \in [0, 1] \tag{19}$$

where

$$\Gamma_k(v) = \int_0^v \Gamma_{k-1}(\alpha)\,\mathrm{d}\alpha$$

$$\Gamma_1(v) = P_i(v) - P_j(v).$$

Satisfaction of (19) will insure, for increasing k, various increasing requirements on risk aversion of the form $u'(v) \geq 0$, $u''(v) \leq 0, \ldots, (-1)^k$, $u^k(v) \leq 0$.

A particularly interesting case occurs for $k = \infty$. The utility curve for infinite risk aversion is given by $u(v) = 0$, $v = 0$ and $u(v) = 1$, $\forall v \in (0, 1]$. Generally, it will be relatively easy to determine requirements for infinite order stochastic dominance.† Since the strength of the dominance relation increases with increasing order, we can use this concept to advantage, especially in the multi-attribute case. Note that we will be able to determine first order dominance, and infinite order dominance, for many discrete state problems with only ordinal event outcome preference information. Value preference bounds may be determined from ordinal preference information from higher order stochastic domination concepts. This requires the solution of linear programs, much the same as those of (10) to (12) (27–29).

Mean variance dominance, or expected value dominance, is a concept that has often been used, especially in capital budgeting efforts. Alternative a^i dominates alternative a^j in terms of expectation or mean value domination if

†We can also compute, with relative ease, the expected utility for the modified infinite risk aversion case where $u(v) = 0$, $\forall v \in [0, v_1]$ and $u(v) = 1$, $\forall v \in (v_1, 1]$.

it has a greater expected value such that

$$\bar{a}_i = \int_0^1 v f_i(v)\,\mathrm{d}v \geq \int_0^1 v f_i(v)\,\mathrm{d}v = \bar{a}_j. \tag{20}$$

Alternative a^i dominates alternative a^j in terms of variance if the variance associated with alternative i is less than that associated with alternative j, or

$$\sigma_i^2 = \int_0^1 v^2 f_i(v)\,\mathrm{d}v - a_i^2 \leq \int_0^1 v^2 f_j(v)\,\mathrm{d}v - a_j^2 = \sigma_j^2. \tag{21}$$

When an alternative dominates another alternative in both expectation and variance, it is said to dominate it in an EV domination sense. Generally, domination in either expectation or variance is not necessarily meaningful. And even EV domination is a less valuable concept than the various stochastic dominance concepts, in that one can easily configure problems for which a non preferred alternative, that is one either stochastically dominated by another alternative or with a lower expected utility than another alternative, may dominate another alternative in an EV domination sense (28). Thus the EV domination concept must be used with caution.

As an example of sensitivity calculations which use stochastic domination concepts, we consider the problem posed by Table II.

TABLE II.
Probability of occurrence of various event outcome states

Alternative	State x_1 $v(x_1) = 0$	x_2 $v(x_2) = 0.25$	x_3 $v(x_3) = 0.5$	x_4 $v(x_4) = 0.75$	x_5 $v(x_5) = 1.0$
a^1	0.25	0.25	0.25	0.25	0
a^2	0	0.25	0.25	0.25	0.25
a^3	0.15	0.15	0.20	0.25	0.25
a^4	0.20	0.80	0	0	0
a^5	0.50	0	0	0	0.50
a^6	0	0	1.0	0	0

Also shown are cardinal values, which are used in some calculations.

For first order stochastic dominance we have, from (16), the first order stochastic dominance reachability matrix and digraph (21) shown in Fig. 3. Note that we need only use the ordinal preferences information among state values to determine this domination relationship.

We must know cardinal values in order to use second and higher order domination relations, however. Bounds on these values can be utilized, and we can consider multivariate, multi-attribute outcomes, as discussed in (27).

	a^1	a^2	a^3	a^4	a^5	a^6
a^1	1	0	0	0	0	0
a^2	1	1	1	1	0	0
a^3	1	0	1	1	0	0
a^4	0	0	0	1	0	0
a^5	0	0	0	0	1	0
a^6	0	0	0	1	0	1

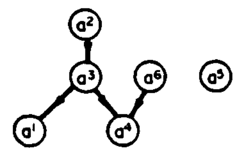

FIG. 3. Reachability matrix and minimum edge digraph for first order stochastic domination.

(28) and (29). Unfortunately, this requires resolution of a number of linear programs and this can be computationally rather unattractive. If we use the cardinal values specified in Table II, it is a simple matter to use (17) or (18) to obtain the reachability matrix and minimum edge digraph shown in Fig. 4. Note that first order stochastic dominance ensures second order stochastic dominance.† Thus, there is no need to determine dominance for the dominated relations of Fig. 3. From Fig. 3 we see that the only possible dominance relations which we need check are

$$a^1 \text{ vs } a^4, a^5, a^6$$

$$a^2 \text{vs } a^5, a^6$$

$$a^3 \text{vs } a^5, a^6$$

$$a^4 \text{vs } a^5$$

$$a^5 \text{vs } a^6.$$

	a^1	a^2	a^3	a^4	a^5	a^6
a^1	1	0	0	0	0	0
a^2	1	1	1	1	1	0
a^3	1	0	1	1	1	0
a^4	0	0	0	1	0	0
a^5	0	0	0	0	1	0
a^6	1	0	0	1	1	1

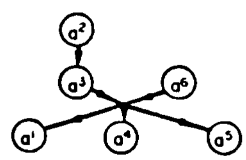

FIG. 4. Reachability matrix and minimum edge digraph for second order stochastic dominance.

†Stochastic dominance of order n guarantees stochastic dominance of order greater than n.

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From these candidate relations we determine that

$$a^6 >_2 a^1, \quad a^2 >_2 a^5, \quad a^3 >_2 a^5, \quad a^6 >_2 a^5$$

and these are, of course, shown in Fig. 4.

For infinite order stochastic dominance we easily see, from Table II, that

$$a^2 \sim a^6 >_\infty a^3 >_\infty a^4 >_\infty a^1 >_\infty a^5$$

and this dominance pattern is illustrated in Fig. 5. This dominance digraph indicates that the best two alternatives are a^2 and a^6; just as did the second order stochastic dominance effort. But, infinite order stochastic dominance results, effectively, in a maximization of the probability that the alternative selected will result in other than the minimum return. It can be a rather pessimistic criterion, therefore. It may well turn out that alternative a^3 is preferred to alternative a^6 for a more realistic utility function.

We next examine EV domination and easily establish the results shown in Fig. 6. From this figure we indeed see that domination based on expectation only is a poor indicator to use for choicemaking. Although the EV dominance digraph shown in Fig. 6 is slightly different from that obtained using the second order stochastic domination results, it does indicate that the best three alternatives are a^2, a^3, a^6.

To determine the final choice alternative we might assume a standard exponential relationship, expressing constant risk aversion r, to relate utility and value. Here we will use

$$u(v) = \frac{1 - e^{-rv}}{1 - e^{-r}}.$$

The expected utilities for alternatives a^2, a^3, a^6 are easily determined for various r as shown in Fig. 7. There is no way, of course, that a^3 can be

	a^1	a^2	a^3	a^4	a^5	a^6
a^1	1	0	0	0	1	0
a^2	1	1	1	1	1	0
a^3	1	0	1	1	1	0
a^4	1	0	0	1	1	0
a^5	0	0	0	0	1	0
a^6	1	0	1	1	1	1

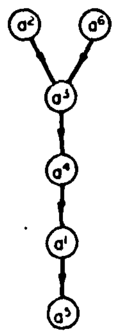
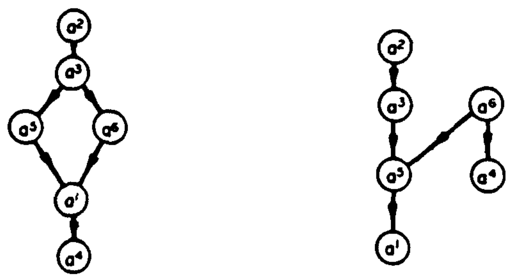


FIG. 5. Reachability matrix and minimum edge digraph for infinite order stochastic dominance.

	Mean	Variance
a^1	0.375	0.7813
a^2	0.625	0.7813
a^3	0.575	0.11938
a^4	0.2	0.01
a^5	0.5	0.25
a^6	0.5	0

(a) Mean and variance values for example



(b) Domination based on expectation only

(c) Domination based on expectation and variance

FIG. 6. EV dominance results.

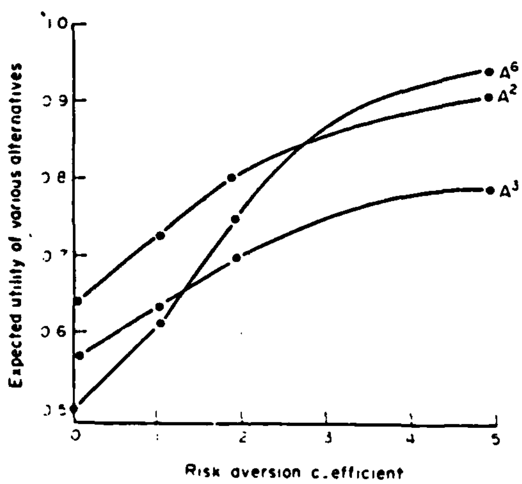


FIG. 7. Sensitivity effect of risk aversion coefficient in determining best alternatives.

the preferred alternative since it is dominated by a^2. a^6 can be either the most preferred, the second most preferred, or the third most preferred alternative depending upon the amount of risk aversion. Fig. 7 indicates transition points where the various optimum decision alternatives change.

IV. Sensitivity in Scalar Multiple Attribute Utility Analysis

Much contemporary emphasis has been placed on the evaluation of alternatives using multiple attribute utility theory. The numerical utility that results from use of multi-attribute utility theory depends upon the structure of the multiple attribute utility function, the scaling coefficients within this structure, and the individual single attribute utility functions which are aggregated to determine the multiple attribute utility functions.

Fishburn (8, 9) has considered approximations of scalar multi-attribute utility functions in which $u(x)$ is a continuous real valued scalar cardinal utility function and $v(x)$ is an approximation for $u(x)$. A number of approximations to $u(x)$ of the general form

$$v(x) = v(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{k} f_{1_j}(x_1) f_{2_j}(x_2) \ldots f_{n_j}(x_n)$$

are considered. A distance metric in the form of the uniform norm

$$D(u, v) = \sup |u(x) - v(x)|$$

is minimized. Among the results of these efforts, the following are especially significant:

(1) An additive utility function $u(x)$ can be approximated to arbitrary closeness by the multiplicative form

$$v(x) = \prod_{i=1}^{n} v(x_i)$$

(2) If $u(x)$ is multiplicative, then there is a lower bound on the distance between the actual utility and an additive approximation.

One of the interesting conclusions of this effort is that simple additive approximations may function as well as the more complicated multiplicative approximations for cases in which the true utility function, $u(x)$, is neither multiplicative nor additive.

Much effort has been devoted to parameter sensitivity in deterministic additive models. Among the useful results obtained from these studies is the indication that differential weighting of attributes is often not necessary and that equal weighting may perform essentially as well. We caution that differential weighting is needed, however, if alternatives are of nearly equal value. Leung (14) provides a survey of much of this work with a number of references to contemporary literature.

Among the more useful of sensitivity type results that can be established for multi-attribute decisionmaking under certainty are various types of dominance results for the case where attribute scores, in the range 0 to 1, can be established for each alternative. In the most unspecified case nothing is known about the n weights for an assumed linear multi-attribute utility function except that

$$w_i \geq 0, \quad i = 1, 2, \ldots, n \tag{22}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{23}$$

$$u^j = u(a^j) = w^T u^j = \sum_{i=1}^{n} w_i u_i(a^j) = \sum_{i=1}^{n} w_i u_i^j. \tag{24}$$

If it turns out that $u_i^j \geq u_i^k \ \forall i$, with the inequality holding for at least one i, then we easily see that alternative j must have, regardless of the weights, a greater utility, $u(a^j)$, than alternative k. We say that a^j dominates a^k. If we have, for example,

$$u^1 = 0.2w_1 + 0.2w_2$$
$$u^2 = 0.5w_1 + 0.2w_2$$
$$u^3 = 0.3w_1 + 0.2w_2$$
$$u^4 = 0.5w_1 + 0w_2$$

we see that $a^2 \succcurlyeq a^1$, $a^2 \succcurlyeq a^3$, and $a^2 \succcurlyeq a^4$ for any w_1, w_2 subject to (22) and (23). Figure 8 illustrates these four utilities and the associated dominance relations. We see from this figure that there is no dominance of alternative 3 by alternative 1; nor is there dominance of alternative 3 by alternative 4. Yet we see that there are no values of the weights such that a^3 is preferred to a^1

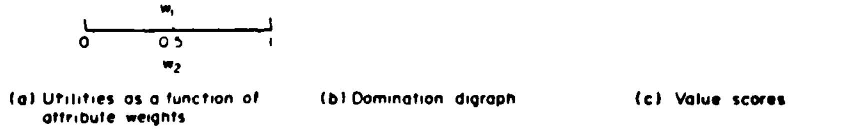


FIG. 8. Preference relations for a simple example.

and a^4. We may establish this fact by noting that the requirements for $a^3 \succcurlyeq a^1$, which is $w_1 \geq w_2$, and $a^3 \succcurlyeq a^4$, which is $w_2 \geq 2w_1$, are inconsistent.

In the general case where utilities are defined by (24) and the constraints of (22) and (23) hold, we can show that alternative k will be dominated by a mixed strategy of the $(m-1)$ remaining alternatives if there is a non zero (positive) solution J to the linear programming problem

$$J = \min \sum_{\substack{i=1 \\ i \neq k}}^{m} d_i^k, \quad w_j \geq 0, \quad j = 1, 2, \ldots, n \tag{25}$$

$$\sum_{i=1}^{n} w_i = 1, \quad d_j^k \geq 0, \quad j = 1, 2, \ldots, k-1, k+1, \ldots, m \tag{26}$$

$$\sum_{i=1}^{n} w_i(u_i^k - u_i^j) + d_j^k \geq 0, \quad j = 1, 2, \ldots, k-1, k+1, \ldots, m. \tag{27}$$

When we compare alternative 3 with a mixed strategy of alternatives 1 and 4, for the alternatives of Fig. 8, we see that the foregoing relations become

$$\begin{gathered} J = \min (d_1^3 + d_4^3) \\ w_1 \geq 0 \\ w_2 \geq 0 \\ w_1 + w_2 = 1 \\ d_1^3 \geq 0 \\ d_4^3 \geq 0 \\ 0.1w_1 - 0.1w_2 + d_1^3 \geq 0 \\ -0.2w_1 + 0.1w_2 + d_4^3 \geq 0. \end{gathered}$$

The solution to this linear programming problem is

$$d_1^3 = 0.1 - 0.2w_1 \ \forall \ w_1 \leq 0.5$$

$$d_4^3 = 0.3w_1 - 0.1 \ \forall \ w_1 \geq 0.33.$$

We see that there is a non zero J over the entire range of w_1 and thus note again that alternative 3 is dominated by a mixed strategy of alternatives 1 and 4.

In many cases it will be possible for the decisionmaker to express ordinal weights, or bounds, on the weights w_i in the form

$$w_1 \geq w_2 \geq \cdots w_n \geq 0. \tag{28}$$

Suppose also it turns out that

$$\begin{aligned}
u_1^j &= u_1^k + \epsilon_1 \\
u_2^j &= u_2^k + \epsilon_2 - \epsilon_1 \\
u_3^j &= u_3^k + \epsilon_3 - \epsilon_2 \\
&\vdots \\
u_i^j &= u_i^k + \epsilon_i - \epsilon_{i-1} \\
&\vdots \\
u_n^j &= u_n^k + \epsilon_n - \epsilon_{n-1}
\end{aligned} \tag{29}$$

where $\epsilon_i \geq 0\ \forall i$. Then we have for the utility of alternative j

$$u^j = \sum_{i=1}^{n} w_i u_i^j = \sum_{i=1}^{n} w_i (u_i^k + \epsilon_i - \epsilon_{i-1}) \tag{30}$$

where $\epsilon_0 = 0$. We can write, by changing the summation index,

$$\sum_{i=1}^{n} w_i \epsilon_{i-1} = \sum_{i=0}^{n-1} w_{i+1} \epsilon_i$$

such that (29) becomes, using (24),

$$u^j = u^k + \sum_{i=1}^{n-1} \epsilon_i (w_i - w_{i+1}) + \epsilon_n w_n.$$

Since we know that $\epsilon_i \geq 0$ and $w_i \geq w_{i+1} \geq 0$, we have established the fact that if the equalities of (29) hold, we must have $a^j > a^i$ regardless of the value of the weights. Equation (29) is not in the form most suited for actual use and can easily be rewritten in terms of the cumulative difference inequality

$$\sum_{i=1}^{m} u_i^j \geq \sum_{i=1}^{m} u_i^k, \quad m = 1, 2, \ldots, n \tag{31}$$

which is in a form quite suitable for actual use.

For the four alternatives and utilities considered earlier in this section we still have $a^2 \geqslant a^1$, $a^2 \geqslant a^3$ and $a^2 \geqslant a^4$. We also have $a^4 \geqslant a^3$, $a^3 \geqslant a^1$, and the implied transitive preference, $a^4 \geqslant a^1$ as well. In this particular case, specifying an ordinal scale for the attribute weights has completely ordered the (dominance) preference relationship. As indicated in Fig. 9, the imposition of the constraint $w_1 \geq w_2$ simply eliminates the right half of the space in Fig. 8 where $w_2 \geq 0.5$.

Even though alternative k may not be dominated by alternative j, it may or may not always have a lower utility than the mixed strategy of alternative i or

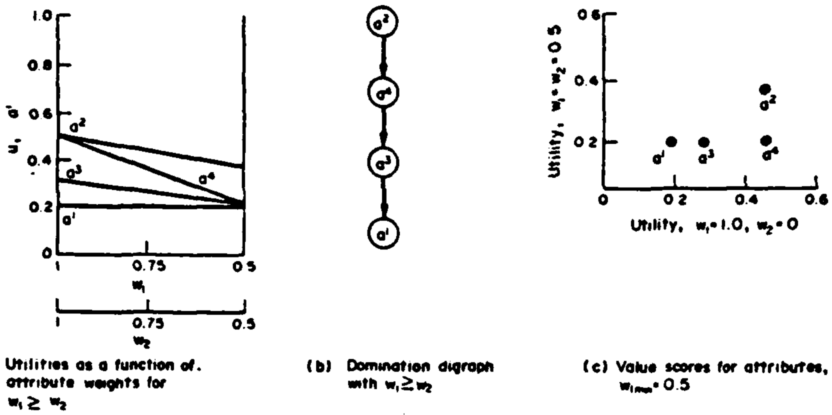


FIG. 9. Preference relations for a simple example, $w_1 \geq w_2$.

alternative *j*. For example in

$$u^5 = 0.6w_1 + 0.4w_2$$
$$u^6 = 0.3w_1 + 0.9w_2$$
$$u^7 = 0.7w_1 + 0.2w_2$$

we see that alternative 5 is not dominated by a mixture of alternatives 5 and 7 if $\omega_1 \geq \omega_2$.† However, in order to have $a^5 \geqslant a^6$ and $a^5 \geqslant a^7$ we must have

$$w_1 \geq w_2$$
$$0.6w_1 + 0.4w_2 \geq 0.3w_1 + 0.9w_2$$
$$0.6w_1 + 0.4w_2 \geq 0.7w_1 + 0.2w_2$$

or

$$1.67w_2 \leq w_1 \leq 2w_2$$

or, since $w_2 = 1 - w_1$,

$$5/8 \leq w_1 \leq 2/3.$$

Thus, dominance of a^5 over a^6 and a^7 is not guaranteed except over a small range of w_1. On the other hand if we have

$$u^8 = 0.8w_1 + 0.1w_2$$

†If, $w_2 > w_1$ then alternative 6 dominates alternative 5 as can be shown.

then we will have $a^5 > a^6$ and $a^5 > a^8$ only if

$$1.67 w_2 \leq w_1 \leq 1.5 w_2$$

and this is inconsistent. Thus alternative 5 cannot be the alternative with the highest utility in that the utility of alternative 6 or alternative 8 is always greater than this. Figure 10 illustrates preference relationships among alternative 5, 6, 7 and 8 as obtained here.

For the general case where the attribute weights are ordered as in (28), it turns out that alternative k will be dominated by a mixed strategy of the $(m-1)$ remaining alternatives if there is a non zero solution J to the linear programming problem

$$J = \min \sum_{\substack{i=1 \\ i \neq k}}^{m} d_i^k, \quad w_1 \geq w_2 \geq \cdots \geq w_n \geq 0$$

$$\sum_{i=1}^{n} w_i = 1, \quad d_j^k > 0, \quad j = 1, 2, \ldots, k-1, k+1, \ldots, m \tag{32}$$

$$\sum_{i=1}^{n} w_i (u_i^k - u_i^j) + d_j^k \geq 0, \quad j = 1, 2, \ldots, k-1, k+1, \ldots, m.$$

There have been a number of sensitivity studies of multi-attribute utility in the psychological literature (3–5, 24, 30) and the effect of errors, including cognitive bias induced errors, upon risk and uncertainty estimation (22). A major goal of this psychological research is, a theory of errors. This will allow

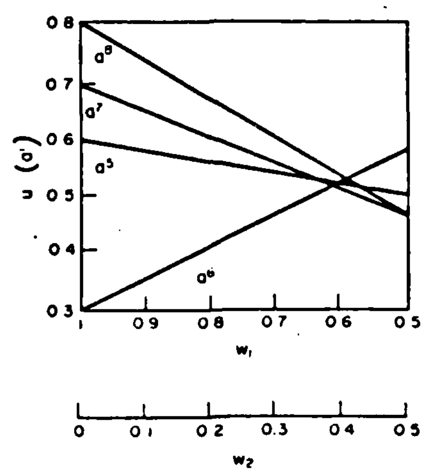


FIG. 10. Utilities as functions of attribute weights. Note that a mixed strategy of alternatives 8 and 6 may always be better than alternative 5 or 7.

determination of the effects of poor structuring of decision situation models and poor elicitations of values and uncertainties and the aggregation of these into decision rules. A hoped for achievement of all this research is a theory that explains and clarifies descriptive and prescriptive approaches to substantive and procedural judgment and decision processes such as to enable the design of more efficacious systems for planning and decision support. In this section we have indicated how sensitivity results can be used to this end, and how sensitivity analysis can be used to reduce, often considerably, the needed number of precise weights. This should generally reduce the potential cognitive stress involved in decision analysis efforts. Some rather general results concerning partially identified parameters and associated sensitivity analysis for planning and decision support are given in (**28**).

V. Sensitivity and the Structure of Decision Situation Models

Sensitivity analysis results can be used to guide the structuring of decision situations. This has been the thrust of recent efforts by Leal *et al.* (**13, 16**); Merkhoffer *et al.* (**15**); and Rajala and Sage (**17–20**). Of interest also is the related work of Chen and Jarboe (**2**); Haruna and Komoda (**10**); and Howard *et al.* (**11**).

Use of sensitivity measures to guide the structuring of decision trees, and related structures such as fault trees, requires the availability of preference or utility measurements for event outcomes and uncertainty measures over events. There are four concepts of value in structuring decision trees using sensitivity concepts:

(1) sensitivity differential of a node,
(2) relative sensitivity differential of a node,
(3) expected value of resolving residual uncertainty and
(4) decision sensitivity to outcome variable uncertainty resolution.

The sensitivity differential of a node, Δ, is the change that must occur in the value of that node in order to cause a change in the currently best initial decision. In Fig. 11 for example, a decrease in the value,† or utility, $v(x, d)$ at node B of more than 0.95 units will cause the best initial decision to switch from a^1 to a^3. It would require a decrease of -0.125 units in the value at node E, for example, for this to happen. A recursive relation

$$\Delta(i) = \begin{cases} \dfrac{\Delta(i-1)}{p_i}, & \text{node } i \text{ is an event node} \\ \Delta(i-1) + v_{i-1}(x) - v_i(x), & \text{node } i \text{ is a decision node} \end{cases}$$

may be determined (**13**). Here $\Delta(i)$ is the sensitivity differential associated with node i and $\Delta(i-1)$ is the sensitivity differential of the preceeding node. $v_i(x)$ and $v_{i-1}(x)$ refer to the expected values at nodes i and $i-1$.

†We use the value symbol for convenience. All of the discussion in this section applies to utilities as well as to values.

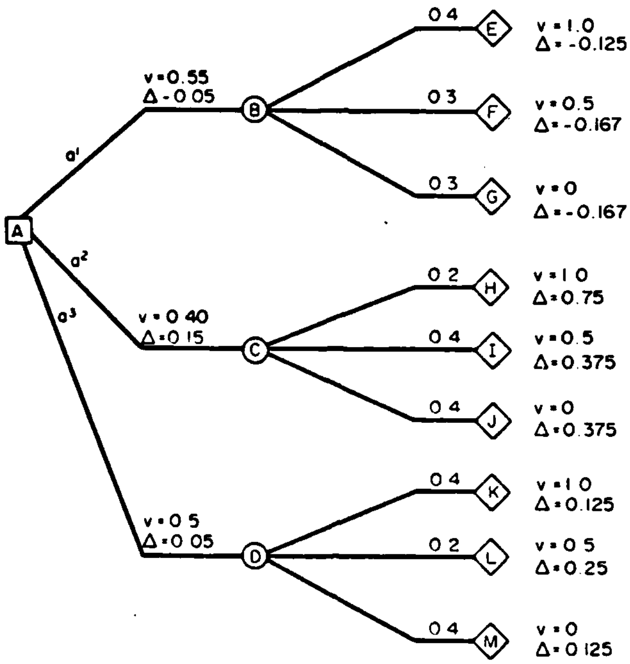


FIG. 11. Decision tree with expected (rollback) values and sensitivity differentials.

The relative sensitivity differential of node i is given by

$$s_r(i) = \frac{\sigma_v(i)}{\Delta(i)}$$

where $\sigma_v(i)$ is the anticipated change in the value at node i which may result from further refinement. Unfortunately there is no general way to determine $\sigma_v(i)$ except by elicitations from the decision maker. It is reasonable that $\sigma_v(i)$ is monotone increasing with increases in $v_i(x)$ since greater inaccuracies typically are associated with larger values.

The expected value of resolving residual uncertainty (EVRRU) may be easily computed (15). We assume that, in Fig. 12, the current best decision is $[a^1, a^4]$. The following results are obtained:

(1) The EVRRU is zero if we consider a node, such as node 1, in which the node is along a path leading from the current best *initial* decision, a^1, but not the best current decision $[a^1, a^4]$.

(2) The EVRRU is zero if we consider a node such as node 2, in which the node is along a path leading from the current best initial decision, a^1, but where

$$p_i[v(x, a^1, a^4) - v(x, a^1, a^5)] \leq v(x, a^1) - v(x, a^2).$$

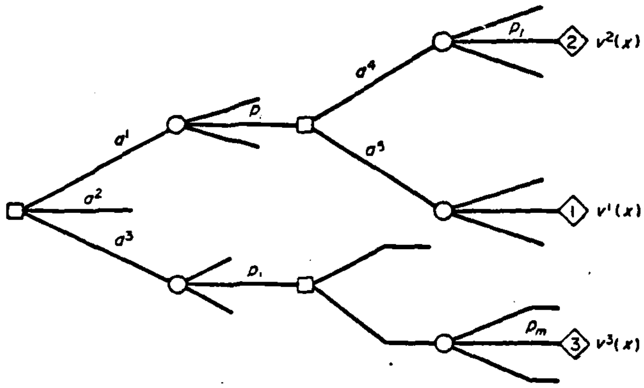


Fig. 12. Prototypical decision tree for computation of the expected value of resolving residual uncertainty.

(3) For case 2, the EVRRU is given by

$$\text{EVRRU} = p_i p_j f[v^2(x) + \Delta - m_*^2]$$

where

$$f = \text{prob}\,[v^{*2}(x) \le v^2(x) + \Delta]$$
$$m_*^2 = E[v^{*2}(x) | v^{*2}(x) \le v(x) + \Delta]$$

if the inequality in case 2 is reversed.

(4) If we consider node 3 which is not along the path leading from the best initial decision, then we have

$$\text{EVRRU} = -p_L p_m g[v^3(x) + \Delta - m^*]$$

where

$$g = \text{prob}\,[v^{*3}(x) \ge v^3(x) + \Delta]$$
$$m^* = E[v^{*3}(x) | v^{*3}(x) > v(x) + \Delta].$$

Rajala and Sage (18) have developed a 9-step tree expansion structuring procedure based upon these sensitivity relations. Steps in this procedure are:

(1) Identify the initial decision alternatives and represent them by branches emanating from the first decision node.

(2) Identify state variables of importance in determining the value of each alternative.

(3) Elicit a value or utility function.

(4) Encode provisional probability distributions on each state variable for each alternative course of action.

(5) Using the value model compute probability distributions, f and g, on the rollback value at the terminal nodes and each interior node using the state variable distributions.

(6) Determine the next appropriate node for expansion. Either the EVRRU and/or the relative sensitivity, $s_r(i)$, is appropriate as an aid in this determination. If either, or both, of these are below some threshold for all remaining modes we stop expansion of the tree. Otherwise we go to Step 7.

(7) The sensitivity of the current best initial decision to uncertainty resolution in each state variable is determined. Multi-attribute utility functions are especially appropriate to aid in this task.

(8) We verify that the best course of action may be affected by incorporating factors determined in Steps 6 and 7 into the decision model. Often this can be accomplished by determining whether the event probability required for a decision switch converges to an amount less than or equal to the amount elicited from the decisionmaker. If it does we go to Step 9. If it does not we return to Step 7 and repeat this step until we are convinced that no switch in the decision is feasible.

(9) Incorporate relevant features in the decision tree to obtain a new and improved representation of the decision situation. We return to Step 4 and continue the process until the EVRUU and/or $s_r(i)$ are sufficiently low at all remaining nodes such that we conclude that no further improvement in the decision situation structural model is feasible.

We shall limit our discussion concerning the value of information measures to a primary decision situation that involves only a single decision d whose uncertain outcomes are represented by the discrete state variables $(x_1, \ldots, x_n)$. The uncertainty on this state vector x is encoded in the distribution $f(x)$ and the value of the outcome is measured by the value model $v(x, d)$.

The purpose behind considering the possibility of acquiring additional information, given this complete model of the primary decision situation, is to determine the worth of eliminating remaining uncertainty on the state variables. An important quantity which establishes an upper bound to the amount that the decisionmaker should pay to eliminate all uncertainty on a state vector is the value of perfect information.

If a clairvoyant, an individual capable of indicating the exact outcome of an uncertain quantity, were to report to the decisionmaker that a particular state vector x' would occur, then the decisionmakers could select the appropriate course of action d_* that gives the maximum value, denoted by $v(x', d_*)$. However, since the decisionmaker does not know what x' the clairvoyant would report, the value that would be realized from a particular outcome must be weighted by the probability that the outcome will be reported. The probability that is assigned is just prior probability, $f(x')$. To the decisionmaker, the expected value of the decision situation with perfect information is

$$E[v(x, d_*)] = \sum_{i} f(x')v(x', d_*).$$

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The well known expected value of perfect information, EVPI, which is a measure of the upper bound the decisionmaker should be willing to pay to resolve all uncertainty on a state vector x, is the difference between the quantity determined in the equation above and the expected value of the optimal course of action based only on information encoded from the decisionmaker's prior knowledge and experience, that is

$$\mathrm{EVPI} = E[v(x, d_*)] - \max_d E[v(x, d)]$$

$$= \sum_i f(x^i) v(x^k, d_*) - \max_d \sum_i f(x^i) v(x^k, d).$$

The magnitude of EVPI can assist the analyst in determining the level of effort to be directed toward identifying and organizing information gathering decisions into the decision model. A lower magnitude of EVPI generally warrants a lower level of structuring activity. These methods provide a basis which allows the analyst to prompt the decisionmaker into identifying information gathering alternatives, with EVPI providing additional incentive to the decisionmaker. The expected value of information from these methods is, of course, bounded above by EVPI, assuming no structural modelling errors.

The information z obtained about the state vector x from an information gathering alternative causes the decisionmaker's experience and knowledge to change, and its effect may be completely accounted for by a revision in the probability distribution on x. The revised distribution on x is determined from Bayes' rule

$$f(x|z) = \frac{f(z|x) f(x)}{f(z)}$$

where the probability functions are appropriately defined. The expected value of information, EVI, is the difference between the optimal course of action with additional information and the optimal course of action without additional information. It is computed as

$$\mathrm{EVI} = E_z[\max_{d(z, c_z)} E\{v[x, d(z, c_z)]\}] - \max_d E[v(x, d)]$$

$$= \sum_z \left[\max_{d(z, c_z)} \sum_x f(x|z) v[x, d(z, c_z)]\right] f(z) - \max_d \sum_x f(x) v(x, d)$$

where $d(z, c_z)$ indicates the primary decision d is based on information z obtained at cost c_z. If EVI is positive, then the expected value of the decision situation will increase when the secondary decision to gather additional information is made. If EVI is negative, the expected value will decrease, reflecting that information costs more than it is worth. If $c_z = 0$, then EVI cannot be negative since it is never harmful mathematically to obtain additional information.

Rajala and Sage (17–20) present several examples of these procedures. These approaches are especially capable for use as structuring tools to determine parsimonious decision trees which are reflective of the decision-maker's perception of the decision situation. Also, the approach provides a general method to use in "pruning" already structured trees. Of particular interest, in this connection, would be the ability to deal with multi parametric sensitivity issues (25).

VI. Conclusions

This paper has presented a discussion of contemporary efforts involving error and sensitivity analysis of decision analysis algorithms for evaluation and choicemaking associated with planning and decision support. We have examined sensitivity to probability estimation errors, sensitivity to utility elicitation errors, sensitivity to variations in the structure and parameters of multi-attribute utility functions, and sensitivity to variations in the structure as well as the probability and utility parameters. This structural sensitivity to expansion of the decision tree is especially useful in that it provides an aid in the determination of parsimonious models of decision situations.

Our recent research has also concerned a mixed scanning based planning and decision support system which involves a vector multiple attribute utility function (26, 27). In this approach, which is believed behaviorally relevant, we intentionally avoid elicitation of all possible parameters and only elicit those which can be shown to be most beneficial in increasing the domination pattern among alternatives. Sensitivity results of the type obtained in this paper have been found useful in guiding the partial aggregation of values. This results in a planning and decision support system in which the parameter weight elicitation procedure is guided by an interaction process involving the judgement and desires of the decision maker and suggestions concerning the efficiency of value aggregation that are determined by sensitivity analysis approaches.

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SECOND ORDER STOCHASTIC DOMINANCE FOR MULTIPLE CRITERIA EVALUATION AND DECISION SUPPORT

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ABSTRACT

In this paper, we examine a single-stage, multiobjective decisionmaking problem under uncertainty. The decisionmaker can select any one of a finite number of alternatives that are assumed to have been identified. After any alternative is chosen, one of a finite number of outcomes will result. With each outcome is associated a number of criteria or attributes. Cardinal preference relations across the set of possible outcomes have been elicited from the decisionmaker for each attribute criteria. No attempt to tradeoff criteria is necessarily assumed. The probabilistic relationship between each alternative and each outcome is presumed to be known. The decisionmaker is risk averse. Our objective is to determine the smallest subset of alternatives that is guaranteed to contain the most preferred alternative on the basis of this assumption. The achievement of this objective enhances evaluation prioritization and decisionmaking since alternative selection is generally easier if made from a subset of the alternative set rather than from the entire alternative set.

We present an approach which achieves this objective and which has computational times amenable to interactive decision aiding. We make use of a result, due to Fishburn and Vickson, which states that the feasibility of a certain collection of linear equalities and inequalities represents a necessary and sufficient condition for one alternative to be weakly preferred to another with respect to the second order stochastic dominance (SSD) relation. The approach presented here uses transitivity and upper and lower bounds on this relation in order to reduce the number of concomitant linear programs necessary for solution. The lower bound is provided by the first order stochastic dominance relation; the upper bound is given by a relation that is equivalent to the second order stochastic dominance relation when certain independence conditions hold. An example illustrates results of using this approach.

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I. INTRODUCTION

Many well-known approaches for multiobjective, single-stage decision aiding under uncertainty [1-3] have, in part, the following tasks associated with them:

1) Determine the utility function of the decisionmaker (DM).

2) Calculate the (subjective) expected utility for each alternative. (We assume throughout that the outcome probabilities as a function of alternative are given. These may be objective or subjective probabilities which obey the sum to one property that have been elicited from the DM or otherwise determined).

3) Select the alternative having the largest (subjective) expected utility.

A potential difficulty with implementing this approach in practice is that complete utility function assessment may be a stressful task that requires a substantial amount of time and effort. Additionally, utility assessment can require cognitive perspectives not within the previous experience of the DM, and this can be a source of frustration which may well lead to results of lower quality than potentially achievable. A potentially useful tactic for reducing these difficulties is to use a less than complete description of a multiple attribute utility function; one which infers the maximum amount possible from ordinal preference information. The various stochastic dominance procedures [4-6] provide a useful approach towards achieving this goal.

Less than a complete description of the utility function, however, almost invariably produces less than a total ordering on the alternative set, and a weaker, partial ordering on the alternative set usually cannot identify the most preferred alternative. A partial order can, however, identify the nondominated set, a set of alternatives that is guaranteed, under mild conditions, to contain the most preferred alternative. The identification of the nondominated set is often quite adequate for decision aiding [7]. The number of alternatives in the nondominated set is dependent on the amount of partial preference information known [8,9]. When a scalar utility function representing a complete multiattribute assessment has been identified, there is but one alternative in the nondominated set.

The apparent fact that the nondominated set is often a sufficiently informative aid for evaluation, prioritization and decisionmaking has motivated the development of a decision aiding procedure that allows the mix and specificity of tasks associated with utility of value function identification to be adaptively determined by the DM [8,9]. Adaptive determination of this mix requires that the time necessary to determine the impact of initial or additional preference information on alternative order specificity be small, due to the substantial constraints that are often placed on the available time of most DM's such that the procedure is potentially interactive. One of the decision aiding procedures presented in [9] is based on second order stochastic dominance (SSD) and hence on only cardinal preference information across outcomes for each attribute, very incomplete knowledge concerning tradeoff weights among noncommensurate attributes, and the often behaviorally relevant assumption that the DM is risk averse. A straightforward implementation of this SSD-based decision aiding procedure requires the formulation and solution of P(P-1) linear programs for a problem having P alternatives. Our current computational experience indicates that such an implementation almost invariably requires computation times that are unacceptably large, even for small problems, for interactive decision aiding. The intent of this paper is to propose a set of procedures that may significantly reduce the computational time and effort necessary to order the alternative set using SSD, thus enhancing the potential of the SSD-based approach as an interactive decision aiding procedure.

This paper is organized as follows. The problem is formulated and preliminary results and definitions are given in Section II. In Section III, we state and investigate conditions that may often reduce the time required by the approach taken in [9] to order the alternative set using SSD. Conclusions are presented in the final section.

II. PROBLEM FORMULATION AND PRELIMINARIES

We assume that the DM can select for implementation any one of P predetermined alternatives from the alternative set $\Pi = \{\pi^1, .., \pi^P\}$. After an alternative is implemented, any one of M possible outcomes will occur. There are N objectives under consideration. Let v_n^m be the predetermined value score of the m^{th} outcome with respect to the n^{th} objective. The real number v_n^m is isotone (monotonically nondecreasing) in preference with respect to the n^{th} objective; that is, outcome m' is (weakly) preferred to outcome m with respect to objective n if and only if $v_n^{m'} \geq v_n^m$. Let $V = \{v^m, m=1, ..., M\}$, the set of all value score vectors, where $v^m = \{v_1^m, ..., v_N^m\}$. This requirement for cardinal preference information across alternatives for each attribute can be relaxed and ordinal measurement scales, such as the ratio scale used by Saaty [10, pp. 53-64], used instead. This simplifies the elicitation task and does not introduce a significant possibility for error, in most instances. Sensitivity analysis techniques [6] could be used, in any specific case, to determine possible error effects of ordinal measurement scales.

The probability that outcome m will result if alternative π^P is selected is $\pi^P(v^m)$. This model of the decision situation in which the outcome states are the same and have the same values for all alternatives is convenient here, but not at all necessary. We could also model the decision situation as one in which the probability of a given outcome is the same for all alternatives and where the utility or value of the outcome state differs across alternatives. Alternately we could model the decision situation as one in which both the probability of a given outcome and the value of the outcome are functions of alternatives. All three representations may be shown to be mathematically equivalent. The one we use is, we believe, more compelling behaviorally and mathematically.

We assume that there exists a presumably unassessed utility function u: $V \to R$ which reflects the DM's preferences in that outcome m' is (weakly) preferred to outcome m with all objectives under consideration if and only if $u(v^{m'}) \geq u(v^m)$. Alternatives are compared on the basis of (subjective) expected utility: alternative π' is (weakly) preferred to alternative π if and only if $E(u, \pi') \geq E(u, \pi)$, where

$$E(u, \pi) = \sum_{v \varepsilon V} u(v)\pi(v).$$

We make only the following three assumptions about the DM:

1) The DM prefers outcome m' to outcome m when only objective n is considered. Let this statement be true for all n = 1, .., N. Then, the DM prefers outcome m' to outcome m when considering all objectives simultaneously (consistency of ordinal preferences).

2) The DM prefers the expected consequence of a lottery to that lottery (risk aversion).

3) The DM can express cardinal value preferences across outcome states for each attribute. As previously noted, this assumption can be relaxed.

These three assumptions imply that the DM's scalar utility function is isotone and concave, respectively, and thus is a member of the set U_2 = {u: u is isotone and concave}. The objective is to provide the DM with a set of alternatives which is guaranteed to contain the most preferred alternative under the assumption that all that is known about the DM's scalar utility function is that it is a member of U_2.

We say that alternative π' is (weakly) preferred to alternative π with respect to SSD, i.e. $\pi' R_2 \pi$, if and only if $E(u, \pi') \geq E(u, \pi)$ for all $u \in U_2$. Thus, if the DM is consistent and risk averse, $\pi' R_2 \pi$ implies that π' can be expected to be at least as good an alternative choice as π. It is well-known [8] that the most preferred alternative is a member of the nondominated set $\{\pi \in \Pi$: there does not exist a $\pi' \in \Pi$ such that $\pi' R_2$ and not $\pi R_2 \pi'\}$. Determining the nondominated set of Π with respect to R_2, or more generally determining the set $P_2 \subseteq \Pi \times \Pi$, where $(\pi', \pi) \in P_2$ if and only if $\pi' R_2 \pi$, helps to restrict the search for the most preferred alternative, thus presumably aiding decisionmaking. (Note that π is nondominated if there is no $\pi' \in \Pi$ such that $(\pi', \pi) \in P_2$ and $(\pi, \pi') \notin P_2$; thus, knowledge of P_2 can be used to determine the nondominated set of Π with respect to R_2.)

The following necessary and sufficient conditions are a slightly generalized version of a result presented in Fishburn and Vickson [5], and suggest an approach for determining P_2:

THEOREM 1: $\pi' R_2 \pi$ if and only if there exists a feasible solution to the set of linear equalities and inequalities:

(i) $\quad d_{ij} \geq 0$ for all i, j = 1, ..., M

(ii) $\quad \sum_{j=1}^{M} d_{ij} = 1$ for all i = 1, ..., M such that $\pi'(v^i) \neq 0$

(iii) $\quad \pi(v^j) = \sum_{i=1}^{M} \pi'(v^i) d_{ij}$ for all j = 1, ..., M

(iv) $\quad \sum_{i=1}^{M} d_{ij} v_n^j \leq v_n^i$ for all i = 1, ..., M such that $\pi'(v^i) \neq 0$ and all n = 1, ..., N.

A probabilistic interpretation of $\{d_{ij}\}$ is given by Fishburn and Vickson, [5] in terms of a preference separation between π and π'. Thus the d_{ij} represent domination strength between alternatives on the various value dimensions.

A brute-force application of Theorem 1 for determining P_2 requires the formulation and solution of P(P-1) linear programs, each having up to M(M + N + 2) decision variables, 2M of which are artificial variables, and up to M(N + 2) side constraints. Considerable experience with the process of formulating and solving these linear programs indicates that they may often require computer time that is unacceptably large for interactive decision aiding, even with relatively small problems and a relatively fast computer. For example, an P=6, M=5, and N=4 problem required roughly 1.5 minutes of CPU time on a CDC-6400. Our computations were done interactively in a time sharing mode. Turn around time can was experienced to be between 5 and 15 minutes, depending on the system load, for the problem. This length of time seems excessive for interactive decision aiding even when using a rather fast computer. The motivation to use microprocessors in planning and decision support is strong; consequently there is much motivation to seek an approach to determining nondominated sets that are associated with reduced computational complexity. In the next section, we present bounds on SSD and associated theory and techniques that yield a strong second order dominance result which will reduce, sometimes substantially, the computational burden associated with standard approaches to calculating SSD relations.

III. MAIN RESULTS

In the previous section, a procedure for determining P_2 was suggested that involved a straightforward application of Theorem 1. In this section, we present several results that will typically reduce, sometimes substantially, the computational times associated with this procedure. These results are presented and proved following three preliminary definitions.

1. The alternative π' is said to be (weakly) preferred to π with respect to first order stochastic dominance (FSD), $\pi' R_1 \pi$, if and only if $E(u, \pi') \geq E(u, \pi)$ for all $u \in U_1$ = {u: u is isotone}.

2. The alternative π' is said to be (weakly) preferred to π with respect to strong-SSD ($\bar{S}SD$), i.e. $\pi' \bar{R}_2 \pi$, if and only if, for each n = 1, ..., N, there exists a feasible solution to the set of linear equalities and inequalities (i), (ii), (iii), and

(iv) $\quad \sum_{i=1}^{M} d_{ij} v_n^j \leq v_n^i$ for all i = 1,..., M such that $\pi'(v^i) \neq 0$

3. An arbitrary relation R_b on Π is said to be stronger than (more precisely, at least as strong as) an arbitrary relation R_a on Π, i.e. R_a

$\subseteq R_b$, if and only if $\pi' R_a\ \pi$ implies $\pi'\ R_b\ \pi$, for any pair $\pi', \pi \in \Pi$.

We remark that $\pi' \bar{R}_2\ \pi$ is equivalent to N separate checks for univariate SSD, for which there exists a computationally simple procedure [5, Section 2.14]. We also note that under certain independence conditions, presented in Theorem 2.11 of [5], that $R_2 = \bar{R}_2$, or $R_2 \subseteq \bar{R}_2$ and $\bar{R}_2 \subseteq R_2$. We now present our main result, the proof of which is in the Appendix.

THEOREM 2: R_1, R_2, and $\bar{R}_2$ are transitive, and $R_1 \subseteq R_2 \subseteq \bar{R}_2$.

The impact of this result is due to the fact that it can be used to reduce, often drastically, the number of linear programs that require solution in order to construct the set P_2. Transitivity implies that it is not necessary to check whether or not $\pi''\ R\ \pi$, if $\pi''\ R\ \pi'$ and $\pi'\ R\ \pi$, for arbitrary transitive relation R. The relation $R_1 \subseteq R_2 \subseteq \bar{R}_2$ implies that, once P_1 and $\bar{P}_2$ are known where we define P_1 and $\bar{P}_2$ similarly to P_2, the only pairs that require the linear program check in order to complete determination of P_2 are those pairs in $\bar{P}_2$ which are not in P_1 and which have not already been added to P_2 by the above transitivity argument. Theorem 2, therefore, suggests the following four step procedure for determining P_2:

(1) Determine P_1. Relatively simple computational procedures for determining P_1 are presented in [9].

(2) Determine $\bar{P}_2$. Relatively simple computational procedures for determining $\bar{P}_2$ are suggested in [5]. The algorithms of Theorem 1 may also be used for this purpose but are more complex than needed.

(3) Evaluate all pairs in P_1 which are not in $\bar{P}_2$, using Theorem 1.

(4) Construct P_2 by adding the appropriate pairs from Step 3 to P_1.

The above procedure prompts three comments. First, the transitivity of all relations can be useful in reducing the number of linear program formulations and solutions, a fact not explicitly mentioned above. Second, it is inconsequential, in terms of substantive results, whether Step 1 is performed prior to Step 2. Whichever of the first two steps is performed first can, however, impact on the time required to perform the second step. This impact is due to the facts that if $(\pi', \pi) \in P_1$, then $(\pi', \pi) \in \bar{P}_2$ and if $(\pi', \pi) \notin \bar{P}_2$, then $(\pi', \pi) \notin P_1$, since $P_1 \subseteq \bar{P}_2$. It should be possible to construct an algorithm to determine an approach which yields maximum transitive inference. But this algorithm might well require greater execution time than it would save. Third, in checking for strong-SSD, it is necessary to examine only down to the first objective that fails to satisfy the univariate SSD criterion (if one exists).

We now present an example illustrating the above procedure.

EXAMPLE: Consider Example 3 in [8]. In that example problem, there were six available alternatives, five possible outcomes, and four objectives under consideration prior to the objective aggregation procedure, i.e. P=6, M=5, and N=4. Table 1 presents the assumed data.

Table 1. Data for the Example

		Outcome Number				
		m=1	m=2	m=3	m=4	m=5
Objective Number	n=1	10	5	5	0	5
	n=2	10	0	0	0	0
	n=3	3	3	10	0	3
	n=4	5	5	5	0	10

(a) Value Scores for Each Outcome and Objective

	Outcome Number				
	m=1	m=2	m=3	m=4	m=5
p=1	0.6	0.1	0.2	0.1	0.0
p=2	0.7	0.0	0.1	0.2	0.0
p=3	0.3	0.1	0.0	0.4	0.2
p=4	0.3	0.0	0.1	0.1	0.5
p=5	0.1	0.1	0.0	0.1	0.7
p=6	0.0	0.1	0.1	0.0	0.8

(b) Outcome Probabilities for Each Alternative

Results in [8] indicate that $P_1 = \{(4,3)\}$. Calculations based on procedures suggested in Section 2.14 of [5] or our Theorem 1 show that $\bar{P}_2 = \{(1,3), (2,3), (4,3)\}$. Thus, it is only necessary to check the pairs (1,3) and (2,3) in order to determine P_2. Solution of the two associated linear programs indicates that $(1,3) \in P_2$ and $(2,3) \notin P_2$, and hence $P_2 = P_1\ \ (1,3) = \{(1,3), (4,3)\}$. Note that $P_2 \neq \bar{P}_2$. This result is in agreement with a result found in [9], which was determined from the formulation and solution of P(P-1) = 6(6-1) = 30 linear programs.

The first three objectives were linearly aggregated in Example 3 [8] with weights 0.1, 0.1, and 0.8, respectively. As a result, $P_1 = \{(1,2), (4,3), (6,5)\}$. Calculations show that $\bar{P}_2 = \{(1,2), (1,3), (2,3), (4,3), (6,5), (6,3), (5,3)\}$ where it is noted that (1,3) and (6,3) are members of $\bar{P}_2$ by transitivity. Thus, the only pairs that actually require examination by the procedure suggested in Theorem 1 are: (1,3), (2,3), (6,3), (5,3). We observe that if $(2,3) \varepsilon P_2$ and $(5,3) \varepsilon P_2$, then it is not necessary to check if $(1,3) \varepsilon P_2$ and $(6,3) \varepsilon P_2$, respectively, because of the transitivity of P_2. Solution of the associated linear programs show that (1,3) and (6,3) are members of P_2. We have therefore determined that $P_2 = \bar{P}_2$ by formulating and solving only two linear programs. This result is in agreement with a result found in [9], which again was determined from the formulation and solution of 30 linear programs.

IV. CONCLUSIONS

This paper has investigated procedures for making SSD a viable concept for interactive decision aiding. Our primary contribution toward achieving this objective has been the identification of a partial order that acts as an upper bound on the SSD partial order. Our present level of experience indicates that this upper bound, the FSD partial order lower bound, the transitivity of all three partial orders, and necessary and sufficient conditions due to Fishburn and Vickson [5] can often be used to obtain a significant reduction in the computational demands associated with a straightforward application of our Theorem 1 in determining P_2.

The four step procedure for determining P_2 proposed here, however, may not always reduce computational time. Although straightforward application of Theorem 1 requires formulating and computing at least as many linear programs as required by the four step procedure, it does not require the determination of P_1, $\bar{P}_2$ or transitivity checks. If $P_1 = \phi$ and $\bar{P}_2 = \Pi \times \Pi$, it is clear that an application of Theorem 1 that allows for transitivity checks will be computationally quicker and therefore superior to the four step procedure. We have found, however, that the number of pairs in $\bar{P}_2$ which are not in P_2 have usually been a small fraction of P(P-1) and that the time necessary to calculate P_1 and $\bar{P}_2$ has typically been significantly smaller than the time necessary to formulate and calculate the additional linear programs. Considerable computational experience to date indicates the merits of both approaches for calculating P_2.

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APPENDIX: Proof of Theorem 2.

It is shown in [5] that R_1 and R_2 are transitive and that $R_1 \subseteq R_2$. A simple argument, based on the fact that the univariate SSD relation is transitive, proves that $\bar{R}_2$ is transitive. In order to prove $R_2 \subseteq \bar{R}_2$, define D_0 as the set of all $\{d_{ij}\}$ such that (i), (ii), and (iii) hold and D_n as the set of all $\{d_{ij}\}$ such that (iv)' holds. Note that:

(a) $\pi' R_2 \pi$ if and only if

$$D_0 \cup \left[\bigcap_{n=1}^{N} D_n \right] \neq \phi$$

(b) $\pi' \bar{R}_2 \pi$ if and only if

$$D_0 \cap D_n \neq \phi \text{ for all } n = 1, \ldots, N.$$

Use of the fact that

$$D_0 \cap \left[\bigcap_{n=1}^{N} D_n \right] = \bigcap_{n=1}^{N} \left[D_0 \cap D_n \right]$$

easily implies $R_2 \subseteq \bar{R}_2$. □

Behavioral and Organizational Considerations in the Design of Information Systems and Processes for Planning and Decision Support

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Abstract—Determinants of performance of systems and processes for planning and decision support are discussed. This paper is directed at people who design such systems and processes, who use such systems and processes, and who manage organizations in which these may be used. The literature cited is associated with several areas including psychology, organizational behavior and design, information science, management science, computer science, and related disciplines. Performance determinants and design requirements for systems and processes for planning and decision support are especially stressed. A number of areas where additional research appears needed are mentioned, and some recommendations and interpretations are given concerning both contemporary efforts and needed future efforts.

I. Introduction

That there is much interest in planning and decisionmaking efforts to determine effective public and private sector policies is made evident by the number of recent texts and case studies devoted to these topics [2], [4], [13], [18], [20], [21], [44], [45], [48], [51], [80], [84]–[86], [89], [104], [105], [108], [134], [135], [139], [141], [150], [178], [179], [198], [212], [219]–[222], [237], [243], [283], [293], [318]–[320], [334], [359], [361], [363], [377], [394], [397], [398], [400], [412]. These in part, concern the numerous complexities associated with practical implementation of the results of systemic efforts for planning and decision support. Advances in digital computer technology coupled with advances in systems science, systems methodology and design, and systems management suggest extension of the information analysis and display capability provided by management information systems to include interpretation and aggregation of information and values such as to result in decision support systems (DSS) or planning and decision support systems. There is a growing literature in this area [5], [36], [39]–[41], [76], [86], [110], [133], [138], [224], [226], [227], [239], [240], [258], [309], [350], [356], [366] and this indicates much contemporary interest and activity.

There are a number of requirements for design success with respect to systems for planning and decision support. These involve a considerable number of disciplines. The result of not making appropriate use of pertinent contributions from a number of disciplines in the design of systems for planning and decision support is likely to be a system or process that is deficient in one or more important ways. The purpose of this effort is to discuss, from a systems engineering perspective, some of the many requirements for design success in this area.

It is possible to disaggregate planning and decisionmaking processes into a number of steps. In essence, they are purposeful futuristic efforts which involve the entire systems engineering process [301]–[305], [307], [308] and can, therefore, be described by any of a number of frameworks for systems engineering such as the three-or the seven-step framework which involves:

1) Formulation of the issue.
   a) problem definition (determination of needs, constraints, alterables)
   b) value system design (determination of objectives and objectives measures)
   c) system synthesis (identification of possible decisions or action alternatives and measures of the accomplishment of these);
2) Analysis of the issue.
   d) systems analysis and modeling (determination of the structure of the decision situation, the impacts of identified decisions or action alternatives and the sensitivity of these to possible change in conditions)
   e) optimization or refinement of alternatives (adjustment of parameters or activities such that each identified decision is the best possible in accordance with the value system);
3) Interpretation of the issue
   f) evaluation and decisionmaking (each possible decision alternative is evaluated, prioritized, and one or more alternatives are selected for implementation action)
   g) planning for action (commitment of resources are made and implementation is accomplished).

Janis and Mann [177] have identified a four-stage model of the decisionmaking process. Fig. 1 presents a slightly modified version of this decision process model. We note that it contains the same essential steps involved in the systems engineering process. Of particular interest are the questions asked at each step of the process. We will elaborate upon this model and other models of the decisionmaking process in our efforts to follow.

Comprehensive efforts involving decisionmaking will be complex because of the many disciplines and areas involved as well as because of the subject matter itself. Probably the formal study of decisionmaking first began with the rational economic man concepts of the 18th century mathematicians Cramer and Bernoulli who explained the St. Petersburg paradox. Since then there have been many workers from a large number of disciplines who have been concerned with various types of decisionmaking studies and the provision of assistance to enhance the understanding of rationale for plans and decisions as well as improvements in the efficiency, effectiveness, and equity of the resource allocations that constitute planning and decisionmaking.

Contemporary choicemaking issues in the public and private sector are complex, contain much uncertainty, and require inputs from many sectors for full understanding and resolution. Many writers have indicated bounded rationality limits in decisionmaking that would appear to make provision of information system

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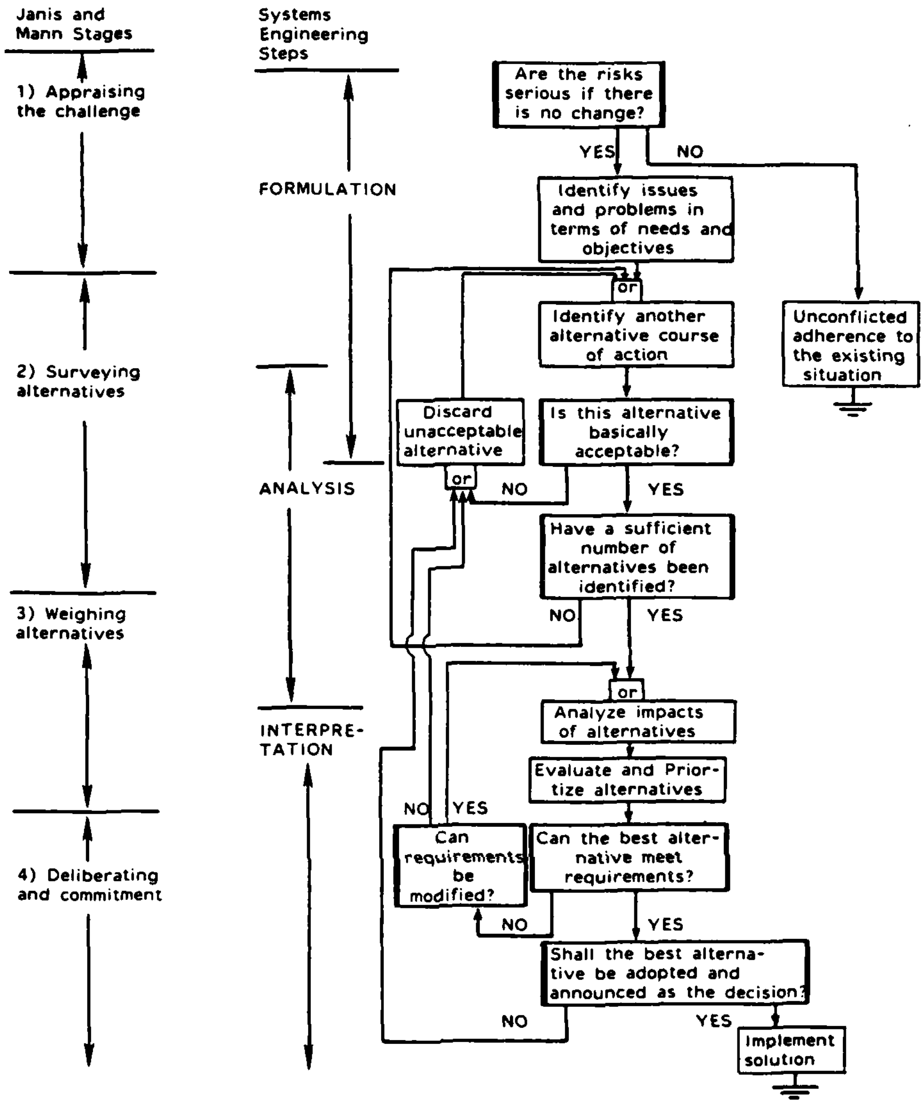

Fig. 1. Systems engineering interpretation of the decision process model of Janis and Mann.

adjuvants for choicemaking normatively very desirable. Such planning and decision support systems could, in principle, provide decisionmakers with rapid access to the information and knowledge needed to enhance decision quality. Unfortunately this promise of enhanced decision quality has not always been realized in practice. There are, doubtlessly, a number of causitive factors inhibiting the potential benefits possible from information systems for planning and decision support. Principal among these factors which, at present, pose fundamental limits to information system success appear to be

1) the need to insure substantive or input–output rationality, such that evaluations of plans and decisions are veridical;
2) the need to insure process rationality, such that the information system accommodates the capabilities of, and the constraints placed upon, the user;
3) the need to understand and cope with human cognitive limitations as they affect the formulation, analysis, and interpretation of decision situations and alternatives; and
4) the need to understand and integrate the normative or prescriptive components with the descriptive components of decision situations in order to evolve realistic adjuvants for the formulation, analysis, and interpretation of decision options.

This paper presents a survey, status report, and integration and interpretation of research from a diversity of areas that supports the design of information systems capable of coping with the needs and fundamental limits to improved judgment. We discuss and describe

1) the cognitive styles of decisionmakers;
2) individual human information processing in decision situations and biases in the acquisition, analysis, and interpretation of information;
3) decision rules for individual decision situations;
4) contingency task structural models of decision situations; and
5) decisionmaking frameworks, organizational settings, and information processing in group and organizational decision situations.

In a very real sense the structural models section, Section V, is the principal portion of this effort. It contains the basic decisionmaking paradigm, including action selection. The contingency task structure, which comprises the issue at hand, the environment into which the issue is imbedded, the decisionmaker, and decisionmaker experiential familiarity with the issue and environment, is the determinant of cognitive style and performance objectives for the particular task at hand. These, in turn, influence selection of an information processing approach and selection of a decision rule.

The literature in this area is enormous. But there is the need for efforts to integrate it from the perspective of systems engineering design of information systems for planning and decision support. There are a number of recent surveys available that discuss one or a limited number of the topics important for the design of planning and decision support systems. These include the surveys of Barron [27], Benbasat and Taylor [35], Bettman [37], Craik [67], Dunnette [87], Einhorn, Kleinmuntz, and Kleinmuntz [95]; Einhorn and Hogarth [98], Ericsson and Simon [99], Hammond, McClelland, and Mumpower [142]; Hammond [143], Hogarth [159], Hogarth and Makridakis [161], Johnson and Huber [179], Kassin [190], Keen [193], Libby and Fishburn [214], Libby and Lewis [215], Mintzberg [248], Nisbett and Ross [264], Nutt [266], Robey and Taggart [292], Sage and White [304], Schneider and Shiffrin [313], Slovic and Lichtenstein [345], Slovic, Fishhoff, and Lichtenstein [348]; Svenson [365], and Zmud [415]. This work attempts a selective integration of this voluminous literature and extensions and interpretations of it from the perspective of ultimate potential usefulness for the design of information systems for planning and decision support. Generally, references are provided only to published literature of the last half decade with limited references to earlier seminal literature and reports. This was felt desirable in order to limit the reference list to an almost manageable size. Despite our attempt to make this report comprehensive, it doubtlessly fails to incorporate the important contributions of a number of authors. And there are doubtlessly unintentional misattributions and misinterpretations as well. For this apologies are offered and forgiveness requested.

II. Cognitive Styles

It is becoming increasingly clear that it is necessary to incorporate not only problem characteristics, but also problem solver or decisionmaker characteristics, into the design of information systems for planning and decision support. A deficiency in some past designs has been the neglect of the human decisionmakers' role and characteristics and their effects. Essentially all available evidence suggests that problem characteristics and user characteristics influence the planning and choice strategies adopted by the decisionmaker. This section discusses a number of cognitive style models from these perspectives.

Mason and Mitroff [238] have suggested that each person possesses a particular specific psychological cognitive style or "personality" and that each personality type utilizes information in different ways. In their research on (MIS) management information system design, they claim that an information system consists of a person of a certain psychological type who faces a problem in some organizational context for which needed evidence to arrive at a solution is made available through some mode of presentation.

There are five essential variables in the information system characterization of Mason and Mitroff. Each of these are disaggregated into subelements. Mason and Mitroff characterize the *psychological-type variable* according to the Jungian stereotypology. In this typology, people differ according to their preference for information acquisition and analysis and the preferred approach to information evaluation and interpretation. At extremes in the information acquisition dimension are sensing-oriented or sensation types who prefer detailed well-structured problems and who like precise routine tasks, and intuitive-oriented type people, who dislike precise routine structured tasks and perceive issues wholistically. At extremes in the information evaluation dimension are feeling-oriented people, who rely on emotions, situational ethics, and personal values in making decisions; and thinking-oriented individuals, who rely on impersonal logical arguments in reaching decisions.

Mason and Mitroff characterize the *problem variable* into structured and unstructured problems. These may be further divided into decisions under certainty, decisions under risk, and decisions under uncertainty. The *organizational context variable* is characterized as strategic planning, management control, and operational control. The *method-of-evidence-generation variable* involves five types of inquiry systems: the data-based Lockean inquiry system, the model-based Leibnitzian inquiry system, the multiple model-based Kantian inquiry system, the conflicting model-based Hegelian inquiry system, and the learning system based Singerian–Churchmanian inquiry system [254]. A fifth variable, *mode of presentation*, includes personalistic modes of presentation such as one-on-one contact as in drama and art and impersonalistic modes such as abstract analytical models and company reports. These latter four variables do not formally relate to cognitive styles, and some further comment on them is contained in other portions of our effort. A number of works by Mason and Mitroff and their colleagues discuss various aspects of this categorization [252]–[254]. Of interest in this regard is a work by Kilmann [200] which suggests the design of organizations with the Jungian personality characteristics of individuals.

Among the many other studies which have emphasized the need to incorporate decisionmaker characteristics into information system design is that of Doktor and Hamilton [78]. They studied the influence of cognitive style on the acceptance of management science recommendations and found a strong correlation between the decisionmaker's cognitive style and willingness to accept these recommendations. They found that differences in acceptance rates were due not only to differences in cognitive style but also to differences in this subject population. From this and many other investigations [34], [74], [77], [79], [101], [151], [166], [174], [229], [252], [253], [263], [267], [268], [311], [330], [369], it appears that appropriate consideration of the human behavioral variable of cognitive style is very necessary for successful design of decision support systems.

A number of studies such as those by Taylor [369], Craik [67], Payne [272], Schneider and Shiffrin [313], [327] and Simon [342], indicate, as we will discuss in later sections, that human decisionmakers attempt to bring order into their information processing activities when confronted with excess information or the lack of sufficient information. Many early studies assumed that static fixed patterns of dealing with information were "preferred" by the decisionmaker for the process of experiencing the world, and these were referred to as "cognitive style." Some early studies view cognitive style as a mode of functioning that is static and pervasive throughout a person's perceptive and intellectual activities. A number of intellectual processes are subsumed within the term cognitive style. These concern the way in which information is acquired or formulated, analyzed, and interpreted. Thus, cognitive style includes such human activities as information filtering and pattern recognition.

Zmud has indicated [414], [415] that those individual differences which influence information system success most strongly involve cognitive style, personality, and demographic/situational variables. Cognitive style refers to the process behavior that individuals exhibit in the formulation or acquisition, analysis, and interpretation of information or data of presumed value for decisionmaking. Doubtlessly cognitive style is somewhat influenced by such personality variables as dogmatism, introversion, extroversion, and tolerance for ambiguity. However, little appears known concerning these influences. Gough discusses personality and personality assessment in his chapter [87]; but it is rare to find, with some notable exceptions [249]–[251], [330], [332], [352], discussions of personality effects upon decisionmaking behavior in cognition studies. The demographic/situational variables involve personal characteristics such as intellectual ability, education, experience with and knowledge of specific contingency tasks, age, and the like. An important situational variable is the level of stress encountered by the decisionmaker in a specific problem situation. The level of stress, which results in the adoption of a coping pattern, influences the decisionmaker's ability in acquisition and processing of the information necessary for decisionmaking. The subject of stress will be dealt with in some detail in Section V. Many variables are especially important for an information processing model of cognitive behavior. Some will be discussed in Section III. Our efforts in this section will be devoted primarily, therefore, to cognitive style concepts, especially the role of personality variables in the adoption of cognitive style.

There are a number of cognitive style models in addition to that of Mason and Mitroff. Bariff and Lusk [24], for example, have discussed three cognitive style characteristics relevant to information system design: cognitive complexity, field dependent/independent, and systematic/heuristic. The cognitive complexity characteristic involves three structural characteristics of thinking and perception: differentiation, the number of dimensions sought or extracted and assimilated from data discrimination; the fineness of the articulation process in which stimuli are assigned to the same or different categories; and integration, the number and completeness of interconnections among rules for combining information.

Benbasat and Taylor [35] note that much cognitive complexity research deals with interpersonal perception and has limited value for modeling activities of managers in processing information and making decisions. Mischel is especially perceptive in discussing the potential hazards of attributions and enduring categorizations of people into fixed slots on the basis of a few behavioral signs in his study of the interface between cognition and personality [251]. The assumptions that static characterizations are sufficiently informative to enable behavior predictions in specific settings are strongly challenged. An evaluation of the uses and limitations of static trait characterization of individuals is presented and the strong interacting role of context is emphasized. Mischel is especially concerned with "cognitive economics," that is to say the recognition that people are easily overloaded with an abundance of information and that simplified methods of acquisition and processing of information are, therefore, used. He is especially concerned also with growth of self-knowledge and rules for self-regulation with maturation, topics to be discussed in Section V. We concur with these views in that we believe that it is the individual's experience with the task at hand that is the primary determinant of cognitive style. Further we believe that it is an individual's information processing capacity under various levels of stress and in different contingency task structures that determines, in part, the quality of decisionmaking. These factors depend strongly upon experience. Thus we support the information processing view of Simon [337]–[344] that few characteristics of the human information processing system are invariant over the decisionmaker and the task. These characteristics are generally experiential and evolve over time in a dynamic fashion. They are not static and can not be treated as static and task invariant for a given individual.

In the Bariff and Lusk cognitive style model [24], individuals may be categorized according to whether they are tightly bound by external referrents in structuring cognitions, in which case they are called field dependent or low analytic; or whether they can make use of internal referrents as well as external referrents in structuring cognitions, in which case they are high analytic or field independent. In a field-dependent mode, perception is dominated by the overall organization of the field. There is limited ability to perceive discrete parts of a field, especially as distinct from a specific organized background. Field independent people have more analytical and structuring abilities in comparison to field dependent people in that they can disaggregate a whole into its component parts.

The systematic–heuristic categorization of Bariff and Lusk describes cognitive styles associated with people who either search information for causal relationships that promote algorithmic solutions, or who search information by trial and error hypothesis testing. Systematic individuals utilize abstract logical models and processes in their cognition efforts. Heuristic individuals utilize common sense, past experience, and intuitive "feel." Systematic individuals would be able to cope with well-structured problems without difficulty and would approach unstructured problems by attempting to seek underlying structural relations; whereas heuristic individuals would attempt to cope with unstructured problems without a conscious effort to seek structural identification.

Of particular importance with respect to cognitive styles are relationships between the environmental complexity of the contingency task structure and information processing characteristics. A number of authors have attempted experiments based on the hypothesis that the conceptual structure of the individual determines information processing characteristics. Conceptual structure is typically measured on a dimension of abstract versus concrete. Abstract individuals would be capable of using integratively more complex conceptual processes than concrete type individuals. Abstractness may be characterized by the ability to differentiate a greater variety of information and to discriminate and integrate information in complex ways. Abstract individuals would, therefore, be expected to base actions on more information and to develop more complex strategies for information evaluation than concrete individuals. This is somewhat similar to Piaget's account of evolving cognitive development,[1] in that the "formal" thinker is capable of abstract thought whereas the "concrete" thinker relies more on preceptual experience as a basis for thought and problem solution. While the work of Piaget appears to assume that cognitive capacity evolves over time, some research involving personality and cognitive style assumes that an individual's cognitive style is not task dependent and not subject to change as a function of contingency variables such as experience.

Among other efforts, Driver and Mock [83] developed decision-style theory, a set of four decision styles based upon the heuristic–analytic characterization of Huysman [172] to relate conceptual structure of decisionmakers to both the amount of information they tend to use and the degree of focusing that they exhibit in the use of information. A heuristic person will use intuition, past experience, concrete thought, and a wholistic approach to reach decisions. An analytic person will utilize abstract logical models and will search for causal relationships and underlying structure to evolve rationale for decisionmaking. The four decision styles are determined by the degree of focus in the use of information and the amount of information desired. A decisive person is one who wishes to see the minimum possible amount of information and who will likely identify a single workable decision. Decision speed obtained from short summary, often verbal reports, is a characteristic of the decisive person. A flexible person is one who utilizes minimum information but who will identify a number of potentially acceptable decisions. A hierarchic person is one who utilizes much information, often obtained in a thorough way from long involved precise reports to identify a single acceptable decision. An integrative person utilizes much information to identify a number of potentially acceptable decisions.

Vasarhelyi [389] has also examined the analytic–heuristic dimension. His experimental results indicate generally that analytic type people tend to use computers and other analytic tools more in planning than do heuristic types. Heuristic types use less information than the analytic types and are more concerned with the lack of flexibility in computers than analytic types. However, his study of correlations among various style-measuring instruments indicates that these are relatively low.

Driver and Mock also suggested a fifth style which they referred to as the complex style, which is characterized by a wide search and analysis of information. It is a mixture of the integra-

[1] See Section V of this paper.

TABLE I
FOUR MODELS OF COGNITIVE STYLE

BARIFF AND LUSK [24]	DRIVER AND MOCK [83]
COGNITIVE COMPLEXITY · DIFFERENTIATION · DISCRIMINATION · INTEGRATION	DEGREE OF FOCUS IN USE OF INFORMATION · MULTIPLE SOLUTIONS IDENTIFIED · ONE SOLUTION IDENTIFIED
FIELD INDEPENDENT/DEPENDENT SYSTEMATIC/HEURISTIC	AMOUNT OF INFORMATION USED · MAXIMUM · MINIMUM
MCKEENEY AND KEEN [242]	**MASON AND MITROFF [238]**
INFORMATION ACQUISITION · RECEPTIVE · PRECEPTIVE	INFORMATION ACQUISITION · INTUITIVE · SENSING
INFORMATION EVALUATION AND INTERPRETATION · SYSTEMATIC · INTUITIVE	INFORMATION EVALUATION AND INTERPRETATION · THINKING · FEELING

tive and hierarchic types. Zmud [415] has performed some experimental studies of this decision style theory. His findings indicate that perceptual differences can indeed be observed for specific cognitive styles and among subjects with different educational and experiential backgrounds. However, his results also indicate that there is no apparent relationship between cognitive style perceptions and actual cognitive behavior despite consistent differences in perceptions of cognitive styles.

McKenney and Keen [242] have done extensive work on cognitive style measurements. These have become, in part, the basis for several definitive efforts [192], [193], [258] in decision support system design. They conceptualize cognitive style in two dimensions: information acquisition and information processing and evaluation. The information acquisition mode consists of receptive and preceptive behavior, both at the opposite extremes of a continuum. They claim that preceptive decisionmakers use concepts, or precepts, to filter data, to focus on patterns of information, and to look for deviations from or conformities with their expectations. Receptive people tend to focus on detail rather than patterns and derive implications from data by direct observation of it, rather than by fitting it to their own precepts.

With respect to information processing and evaluation, McKenney and Keen measured individuals on a scale, with the systematic thinker at one extreme and the intuitive thinker at the other extreme. They have shown, using a battery of pencil and paper tests, that systematic thinkers approach a problem by structuring it in terms of some method which would lead to a solution, whereas intuitive thinkers use trial and error, intuition, and previous experience to obtain solutions.

We have examined four cognitive style characterizations in this section. Table I summarizes the models of cognitive style that result from these efforts. We note the considerable similarity among these four constructs. There have been a number of studies of the measuring instruments involved in classifying people according to these cognitive styles. Many, such as the study by Vasarhelyi [389] mentioned previously, have found rather low correlations among test instruments. Zmud [413], [415] has indicated low correlation also among test scores on different instruments indicating cognitive styles. Chervany, Senn, and Dickson [57], [76] have expressed much concern and pessimism concerning the validity of much of the contemporary research in this area. They comment that the study of individual personality differences as predictors of human behavior and performance have been basically unsuccessful in that it has not been possible to predict performance on the basis of personality characteristics. Their comment and the comment of others that the characteristics of the task in which the individual involved is a prime determinant of human behavior, appears unassailable. We will provide and discuss additional evidence supporting a dynamic cognitive style characterization that will incorporate the contingency task structure and the decisionmaker's task experience in several other sections of this paper. In particular we emphasize the strong need for consideration of the structure and the content of planning and decision situations in order to evolve contextually meaningful support.

III. INFORMATION PROCESSING

Problem solving, judgment, and decisionmaking imply both thought and action. Hence decisionmaking can be defined as the processes of thought and action involving an irrevocable allocation of resources that culminates in choice behavior. In making a decision, more often than not, the decisionmaker is dealing with environments characterized by risks, hazards, uncertainty, complexity, changes over time, and conflict. Further, the quality of a decision depends upon how well the decisionmaker is able to acquire information, to analyze information, and to evaluate and interpret information such as to discriminate between relevant and irrelevant bits of data. Decision quality also depends upon how well the decisionmaker is able to cope with stress, which is invariably encountered in important decision circumstances. Effective management of these factors enables strategies by which the decisionmaker may arrive at a good problem solution, decision, or judgment.

A number of studies such as those by Barron [28], Bettman [37], Chorba and New [58], Delaney and Wallsten [73], Feather [107], Howell and Fleishman [165], O. Huber, [168], G. Huber [170], [171]; Ives, Hamilton, and Davis [174]; Libby and Lewis [215], Lucas and Nielson [228], MacCrimmon and Taylor [232], Montgomery and Svenson [256], Moskowitz, Schaefer, and Borcherding [259]; Payne [275], Simon [342], Tushman and Nadler [379], Tuggle and Gerwin [380], Wallsten [395], [396]; and Wright [402]–[406] discuss the vital role of human information processing in decisionmaking. Most contemporary researchers regard information processing as a crucial task for effective decisionmaking and state that the type of decision problem, the nature of the decision environment, and the current state of the decisionmaker combine to determine decision style and decision strategy for a specific task. The term information processing refers to the processing of verbal reports as well as quantitative data since verbal reports are data [99].

An information processing theory of problem solving, judgment, and decisionmaking is based on the assumption that individuals have an input mechanism for acquisition of information, an output mechanism for interpretation and choicemaking, internal processes for filtering and other analysis efforts associated with information, and memories for long- and short-term storage of information. There are a large number of ways of representing

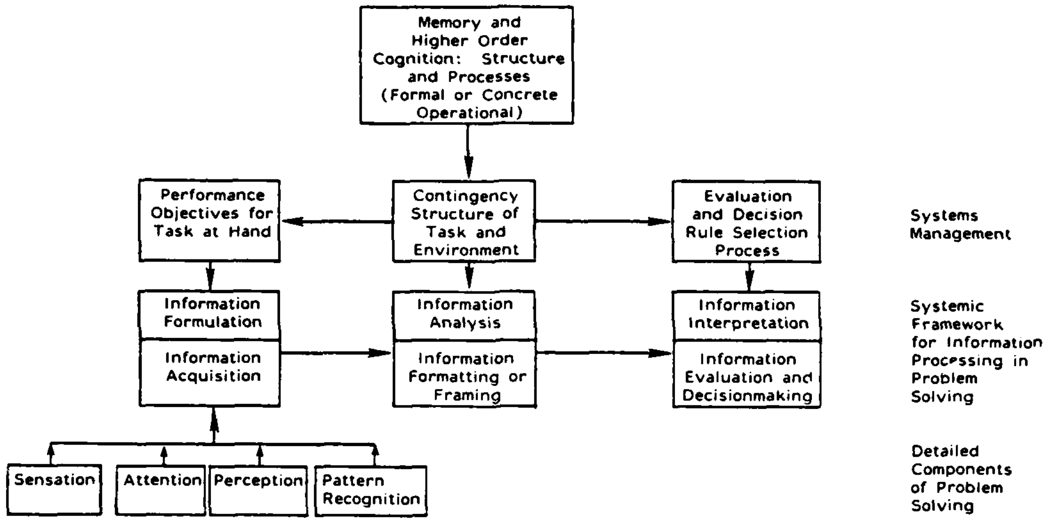

Fig. 2. A systems engineering conceptual model of human information processing.

human information processing. Many of these are described in texts in cognitive psychology such as Anderson [7], Posner [281], or Solso [354], and in works in consumer choice such as Bettman [37]. Much of the work in this area owes a great deal to Simon [334]–[344] who has developed information processing theories in psychology and in artificial intelligence.

Fig. 2 presents a conceptual model of a systems engineering framework [308] for human information processing. There are doubtlessly a number of components missing from this model. It does not show, for example, the essentially iterative nature of the process. Nevertheless we feel that it provides a useful point of departure and a structure for our efforts to follow.

The key functions, which determine how a specific problem or decision situation is cognized, depend upon an interaction of the memory and higher-order cognition of the problem solver with the environment through the contingency task structure. We will be very concerned with development of a conceptual model of higher-order cognition and the contingency task structure in Section V. It is appropriate to remark here that the various information analysis and interpretation processes of thinking, task performance objective identification, evaluation, and decision rule identification, are called "higher order" cognition. This is not because they are somehow more important than the so called "lower order" cognition efforts of information acquisition involving formulation: sensation, attention, perception, and pattern recognition; but because they occur later in time in the overall information processing effort.

It is important to note that information processing and decisionmaking efforts intimately involve memory. Memory [102] influences human judgment in a number of ways. It will influence the perception of the contingency task structure associated with an issue as well as the decision rules used for evaluation of alternatives. Two characteristics of human memory are of special importance for our efforts here. First, information will be encoded in more or less efficient and effective ways in terms of human abilities for recall. The coding process is dependent, also, upon the interpretation attached to information and this strongly influences event recall, perceptions, and associated cognitive biases. The literature concerned with memory and its components, and their relations and interaction with human perceptual experience and behavior is vast and speculative in nature. There have been many studies, both physiological and psychological, concerned with the identification of the memory "engram," which is hypothesized to be the fundamental unit of memory. We need not be especially concerned in this effort with the various physiological structures and processes associated with human memory; or with various related behavior therapies [109]; however the essentials are reviewed below briefly. A useful brief survey of the literature on memory is presented by Thomassen and Kempen in chapter 3, vol. II [244], by Fox [128], and by Radcliff [284].

Human memory constitutes two major components, short-term memory and long-term memory. Short-term memory plays a key role in immediate recall of actively rehearsed limited information [7], [354]. Unless conscious effort is put forth in recalling information from short-term memory, this cannot be done after a lapse of 30 to 60 s from initial presentation. Models of a working short-term memory involve a number of mechanisms, such as an articulatory rehearsal loop that has the capacity to retain short verbal sequences. This is just one mechanism by which short-term retention is possible. There are a number of other sensory registers. It is important to note that short-term memory is an integrated network of many mechanisms, and is associated, in use, with a number of skilled processes.

Shiffrin and Schneider [313], [327] incorporate concepts of attention, memory, and perceptual learning in their theory of short-term retention. They hypothesize short-term storage, the function of which is active control of thinking, reasoning, and general memory processes. According to Shiffrin & Schneider, short-term storage is an activated subset of long-term storage. Transfer of information from short-term storage to long-term storage is dependent on attentional limitations, interference from strong external and internal stimuli, extent of analysis of information, and formation of associations in long-term storage. There have been many studies involving concepts such as retrieval processes, memory trace identification, encoding processes, and recognition which we will not discuss as they appear of secondary importance to the goals of this particular effort. While five to seven unconnected items is believed to be the maximum amount of information that can be retained in short-term memory, long-term memory may contain a virtually limitless amount of information.

Thus we see an enormous difference between human abilities and computer abilities. Because of its large long-term memory and ability for quick search and recall, a human mind easily reasons wholistically. Wholistic reasoning, such as reasoning by analogy, is not at all easy, at this time at least, for a computer. Significant unaided computational effort would be difficult for a human since computation must be done in short-term memory. There exists the possibility that information stored in long-term memory is flawed because of cognitive-biases introduced by processing in short-term memory. A principal task of computer-aided support must be to augment human capabilities in need of augmentation, while not diminishing abilities in those areas in which human abilities exceed those of the computer [81].

Our effort in the remainder of this section will be devoted to a description of the various processes which support information acquisition and information analysis. We will also discuss some of the cognitive biases that can result from "poor" information acquisition and information analysis. Information interpretation, which leads to alternative evaluation and decisionmaking, is an important and somewhat distinct part of the overall information processing model. It will be discussed in the next four sections from several perspectives.

The types of operations involved in information acquisition are sensation, attention, perception, and pattern recognition. Doubtlessly there are other valid ways of categorizing these operations [7], [37], [67], [100], [137], [148], [175], [281], [297], [298], [313], [327], [333] but the taxonomy used here is sufficient for our purposes. In sensation, information is acquired through the five major sense modalities, which are environmentally activated, in response to a specific array of stimulus energies. In a specific decisionmaking situation, the decisionmaker filters out bits of data believed to be irrelevant. The filtering process is based upon task characteristics, experience, motivation, as well as other features and demands of the specific decisionmaking situation. If such a filtering mechanism were not to exist, the decisionmaker would often encounter information overload which generally results in saturation and the inability to process sufficient information for the task at hand. Short-term and long-term memory components play key roles in the information acquisition process as the decisionmaker proceeds with efforts that culminate in choice. A response system couples the memory system to the sensory system and the environment. Thus it controls or activates the sensory modalities on the basis of the actions taken. Through the response system we close the information flow feedback loop. Bower, in volume 1 of Estes [100], has summarized principal components of the flow system. A model of the principal components of information flow might consist of: the response system, the sensory system, the memory system, and the central processor. The central processor coordinates memorizing, thinking, evaluation of information, and final decisionmaking.

Ultimately involved in retention processes is the notion of attention [7]. In order for information to be transferred from short-term memory to long-term memory, constant conscious attention, in terms of rehearsal, is required. Information entering short-term memory that is not attended to, through specific conscious processes, is lost. Processing of information demands attending to relevant bits of incoming data and transfer of the data into long-term memory for future retrieval for making a decision. Interferences of various types may interrupt attention and thus hinder transfer and retention of relevant stimuli into long-term memory.

Inherent in the processing of information acquisition, is the process of pattern recognition. This process generally involves two phases: extraction and identification. A given stimulus is "coded" in terms of its features. These extracted features of the object or stimulus describe the stimulus. The term "features" implies such characteristics as angles, lines, or edges. A stimulus may be received through any of the sense modalities. The meaning that this conveys to the decisionmaker, or the manner in which the decisionmaker perceives the stimulus, is dependent upon the patterns extracted from the stimulus. In the identification phase, the sensory—perceptual system classifies the stimulus object. The way in which this is often assumed to occur is by a weighted matching of the current feature list against a likely set of prototypes in long-term memory [7], [313], [327], [354] with the input being classified according to the name of the best matching prototype. The quality or extent of the sensory information extracted determines the accuracy of identification.

Thus pattern recognition processes involve memory and the other three components of information acquisition: sensation, or the initial experience of stimulation from the sensory modalities; attention, or the concentration of cognitive effort on sensory stimuli; and perception, or the use of higher-order cognition to interpret sensory stimuli.

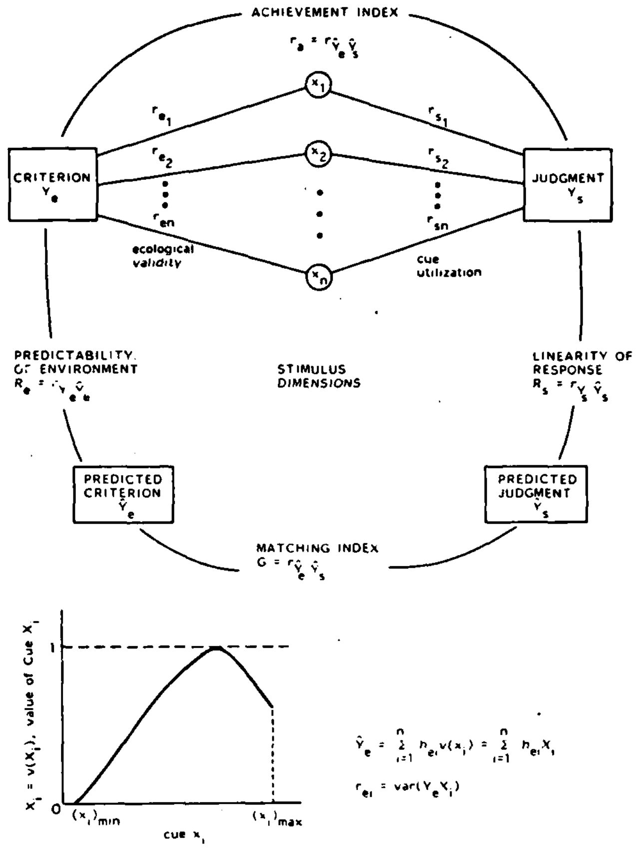

Fig. 3. The Brunswick lens model and its relation to Hammond's social judgment theory.

We have just described what might be regarded as a component or physiological model of information processing. In these stimulus response approaches, behavior is seen as being initiated by the onset of stimuli. A seeming deficiency in approaches of this sort is that there is little consideration of how information bits are aggregated to influence choice and how the decisionmaker goes about the process of information formulation or acquisition, analysis, and interpretation.

A lens model developed by Brunswick and his students is a notable exception to this. The Brunswick lens model is the basis for the policy capture or social judgement theory approach of Hammond and his colleagues [140]–[143]. The lens model, displayed in Fig. 3, assumes that people are guided by rational programs in their attempt to adapt to the environment. There is a criterion value Y_e, and the subjects response, judgment, or inference Y_s. The left side of Fig. 3 represents ecological cue validities which are the correlations r_{ei} between the cues and the criterion value. On the right or organismic side of Fig. 3, a subject will base a response, judgment, or inference Y_s, on the perceived ecological structure. By calculating the correlations r_{si}, that exist between the cues and the response or criterion evaluation, learning concerning the response system can be obtained.

We note that the value of the environmental criterion Y_e and the subject inference Y_s are directly comparable if linear combinations of the cues are assumed. We have, for n cues,

$$Y_e = \sum_{i=1}^{n} h_{ei} x_i + \nu_e, \qquad \hat{Y}_e = \sum_{i=1}^{n} h_{ei} x_i$$

$$Y_s = \sum_{i=1}^{n} h_{si} x_i + \nu_s, \qquad \hat{Y}_s = \sum_{i=1}^{n} h_{si} x_i$$

where h_{ei} and h_{si} are optimum regression weights for the independent cues x_i which provide measures of the importance weights of the cues, ν_e and ν_s are error terms due to inadequacy of the linear model, Y_e and Y_s are the true criterion value and subject response, and $\hat{Y}_e$ and $\hat{Y}_s$ are the predicted criterion value and subject response based on the observed cues. The many works of Hammond and his associates [2], [21], [45], [54], [140]-[143], [186], [187], [261], [290], [294], [295], [404] concerning social judgment theory make use of this lens model. The approach has been shown to be useful in a variety of areas such as policy formulation, negotiation, and conflict resolution. Recent efforts by Hoffman, Earle, and Slovic [154] have shown that the computer displays of social judgment theory: which show both task characteristics, in terms of cue values and corresponding criterion values; and response characteristics, in terms of individual cue values and associated subject responses and judgments; provide a very effective feedback mechanism which might enable people to effectively learn much about complex functional relationships and tasks. There are a number of studies of regression analysis approaches to determination of parameters for decision rules [260], [290]. Use of regression analysis is central to social judgment theory. Recent applications of the approach [261] have involved using simulation models to generate responses which are valuated by the decisionmaker.

Questions concerning the cognitive style used by the decisionmaking are, we believe, very important. Information analysis and information interpretation may be accomplished in a concrete operational mode of thought or in a formal operational mode. We will describe the essential features of these two higher-level cognition processes in Section V. The concrete operational thought process, which is typically applied in familiar situations which people perceive to be well structured, may involve efforts such as reasoning by analogy, or affect, or standard operating procedures. The formal operational thought process, typically applied in situations with which the problem solver is unfamiliar and inexperienced, may involve explicit use of quantitative or qualitative analytical thought.

In either of these modes or "styles" of thought or cognition, information acquisition, analysis, and interpretation may be quite flawed. Many recent studies emphasize the strong need for modeling problem solving behavior in a descriptive, or positive sense in order to detect possible flaws in information processing. Our discussions thus far in this section have been concerned with physiological models in which people have input and output mechanisms, a memory for information storage and retrieval, and a central processor for coordination and control. Here, we wish especially to underscore the need not only for physiological or stimulus-response models but especially for process tracing [72], [95–98], [255] models of information formulation, analysis, and interpretation as well as associated decisionmaking. Knowledge of the actual unaided process of problem solving or descriptive process tracing should serve as a useful guide to the design of information systems that avoid, or at least ameliorate the effects of, cognitive heuristics and biases. This involves requirements for a knowledge of the ways in which people apply strategies in order to reach judgments.

A large number of contemporary studies in cognitive psychology indicate that the attempts of people, including experts, to apply various intuitive strategies in order to acquire and analyze information for purposes such as prediction, forecasting, and planning are often flawed. Many studies have been conducted to describe and explain the way information is acquired and analyzed and the results of faulty acquisition and analysis. Generally the descriptive behavior of subjects in tasks involving information acquisition and analysis is compared to the normative results that would prevail if people followed an "optimal" procedure. There have been a number of recent discussions of cognitive biases from several perspectives [61], [62], [98], [142], [154], [156], [160], [161], [185], [234], [263], [304], [309], [346–349], [351], [352], [385], [386], [406–408]. The recent texts by Nisbett and Ross [264] and Hogarth [159] concerning the strategies and biases associated with judgment and choice are especially noteworthy. Among the cognitive biases that have been identified are several which affect information formulation or acquisition, information analysis, and interpretation. Among these biases, which are not independent, are the following.

1) Adjustment and Anchoring [345], [383]—Often a person finds that difficulty in problem solving is due not to the lack of data and information, but rather to the existence of excess data and information. In such situations, the person often resorts to heuristics which may reduce the mental efforts required to arrive at a solution. In using the anchoring and adjustment heuristic when confronted with a large amount of data, the person selects a particular datum, such as the mean, as an initial or starting point, or anchor, and then adjusts that value improperly in order to incorporate the rest of the data such as to result in flawed information analysis.

2) Availability [383], [385]—The decisionmaker uses only easily available information and ignores not easily available sources of significant information. An event is believed to occur frequently, that is with high probability, if it is easy to recall similar events.

3) Base Rate [25], [291], [386]—The likelihood of occurrence of two events is often compared by contrasting the number of times the two events occur and ignoring the rate of occurrence of each event. This bias often occurs when the decisionmaker has concrete experience with one event but only statistical or abstract information on the other. Generally abstract information will be ignored at the expense of concrete information. A base rate determined primarily from concrete information may be called a causal base rate whereas that determined from abstract information is an incidental base rate. When information updates occur, this individuating information often is given much more weight than it deserves. It is much easier for individuating information to override incidental base rates than causal base rates.

4) Conservatism [210], [259], [345]—The failure to revise estimates as much as they should be revised, based on receipt of new significant information, is known as conservatism. This is related to data saturation and regression effects biases.

5) Data Presentation Context [161]—The impact of summarized data, for example, may be much greater than that of the same data presented in detailed, nonsummarized form. Also different scales may be used to considerably change the impact of the same data.

6) Data Saturation—People often reach premature conclusions on the basis of too small a sample of information while ignoring the rest of the data that is received later on, or stopping acquisition of data prematurely.

7) Desire for Self-Fulfilling Prophecies—The decisionmaker values a certain outcome, interpretation, or conclusion and acquires and analyzes only information that supports this conclusion. This is another form of selective perception.

8) Ease of Recall [205], [382], [383]—Data which can easily be recalled or assessed will affect perception of the likelihood of similar events occurring again. People typically weigh easily recalled data more in decisionmaking than those data which cannot easily be recalled.

9) Expectations [161], [235]—People often remember and attach higher validity to information which confirms their previously held beliefs and expectations than they do to disconfirming information. Thus the presence of large amounts of information makes it easier for one to selectively ignore disconfirming information such as to reach any conclusion and thereby prove anything that one desires to prove.

10) Fact-Value Confusion—Strongly held values may often be regarded and presented as facts. That type of information is sought which confirms or lends credibility to one's views and values. Information which contradicts one's views or values is ignored. This is related to wishful thinking in that both are forms of selective perception.

11) Fundamental Attribution Error (Success/Failure Error) [263], [264]—The decisionmaker associates success with personal inherent ability and associates failure with poor luck in chance events.

This is related to availability and representativeness.

12) Gamblers Fallacy—The decisionmaker falsely assumes that unexpected occurrence of a "run" of some events enhances the probability of occurrence of an event that has not occurred.

13) Habit—Familiarity with a particular rule for solving a problem may result in reutilization of the same procedure and selection of the same alternative when confronted with a similar type of problem and similar information. We choose an alternative because it has previously been acceptable for a perceived similar purpose or because of superstition.

14) Hindsight [112–114], [116]—People are often unable to think objectively if they receive information that an outcome has occurred and they are told to ignore this information. With hindsight, outcomes that have occurred seem to have been inevitable. We see relationships much more easily in hindsight than in foresight and find it easy to change our predictions after the fact to correspond to what we know has occurred.

15) Illusion of Control [209], [210]—A good outcome in a chance situation may well have resulted from a poor decision. The decisionmaker may assume a feeling of control over events that is not reasonable.

16) Illusion of Correlation [115], [383]—A mistaken belief that two events covary when they do not covary is known as the illusion of correlation.

17) Law of Small Numbers (see Kahneman and Tversky [235])—People are insufficiently sensitive to quality of evidence. They often express greater confidence in predictions based on small samples of data with nondisconfirming evidence than in much larger samples with minor disconfirming evidence. Sample size and reliability often have little influence on confidence.

18) Order Effects [161], [184]—The order in which information is presented affects information retention in memory. Typically the first piece of information presented (primacy effect) and the last presented (recency effect) assume undue importance in the mind of the decisionmaker.

19) Outcome Irrelevant Learning System [96], [97]—Use of an inferior processing or decision rule can lead to poor results and the decisionmaker can believe that these are good because of inability to evaluate the impacts of the choices not selected and the hypotheses not tested.

20) Overconfidence [114], [183], [216]—People generally ascribe more credibility to data than is warranted and hence overestimate the probability of success merely due to the presence of an abundance of data. The greater the amount of data, the more confident the person is in the accuracy of the data.

21) Redundancy—The more redundancy in the data, the more confidence people often have in their predictions, although this overconfidence is usually unwarranted.

22) Reference Effect [30], [383]—People normally perceive and evaluate stimuli in accordance with their present and past experiential level for the stimuli. They sense a reference level in accordance with past experience. Thus reactions to stimuli, such as a comment from an associate, are interpreted favorably or unfavorably in accordance with our previous expectations and experiences. A reference point defines an operating point in the space of outcomes. Changes in perceptions due to changes in the reference point are called reference effects. These changes may not be based upon proper, statistically relevant computations.

23) Regression Effects [183], [383]—The largest observed values of observations are used without regressing towards the mean to consider the effects of noisy measurements. In effect this ignores uncertainties.

24) Representativeness [382], [383]—When making inference from data too much weight is given to results of small samples. As sample size is increased, the results of small samples are taken to be representative of the larger population. The "laws" of representativeness differ considerably from the laws of probability and violations of the conjunction rule $P(A \cap B) \leq P(A)$ are often observed.

25) Selective Perceptions [161]—People often seek only information that confirms their views and values. They disregard or ignore disconfirming evidence. Issues are structured on the basis of personal experience and wishful thinking. There are many illustrations of selective perception. One is "reading between the lines" such as, for example, to deny antecedent statements and, as a consequence, accept "if you don't promote me, I won't perform well" as following inferentially from "I will perform well if you promote me."

26) Spurious Cues [161]—Often cues appear only by occurrence of a low probability event but they are accepted by the decisionmaker as commonly occurring.

27) Wishful Thinking—The preference of the decisionmaker for particular outcomes and particular decisions can lead the decisionmaker to choose an alternative that the decisionmaker would like to have associated with a desirable outcome. This implies a confounding of facts and values and is a form of selective perception.

Doubtlessly there are other information acquisition, analysis, and interpretation biases that we have not identified here. Any categorization into acquisition, analysis, and interpretation bias is somewhat arbitrary since iteration and feedback will often, in practice, not allow this separation. Also, many of the identified biases overlap in meaning and, therefore, are related to others. Some further discussion of cognitive biases will be presented in our discussion of the situation framing phase of prospect theory in Section III. Certainty, reflection, and isolation effects are three results of these biases that have particular prominence in prospect theory.

Of particular interest are circumstances under which these biases occur, their effects on activities such as decisionmaking, issue resolution, planning, and forecasting and assessment; and appropriate styles which might result in debiasing or amelioration of the effects of cognitive bias.

Many of the cognitive biases that have been found to exist have been found in the unfamiliar surroundings of the experimental laboratory, and generalization of this work to real world situations is a contemporary research area of much interest. However most of the laboratory experiments have concerned very simple if unfamiliar tasks. A number of studies have compared unaided expert performance with simple quantitative models for judgment and decisionmaking, such as those by Brehmer [47], Cohen [62], Dawes [70], [71], Goldsmith [132], Kleinmuntz and Kleinmuntz [204], and by several authors in Wallstein's recent definitive work concerning cognitive processes in choice and decision behavior [396]. While there is controversy [62], [263], [349], most studies have shown that simple quantitative models perform better in human judgment and decisionmaking tasks, including information processing, than wholistic expert performance in similar tasks. This would appear to have major implications and to sound major caveats for such areas as "expert forecasting." This caution is strongly emphasized in the works of Hogarth and Makridakis [161], Makridakis and Wheelright [235], and Armstrong [14]–[16]. This is a caution noted in but a few [18] of the contemporary works on forecasting and assessment.

There are a number of prescriptions which might be given to encourage avoidance of possible cognitive biases and to debias those that do occur [96], [98], [161], [184], [235], [355], [386]. Some suggestions to avoid cognitive bias follow.

1) Sample information from a broad data base and be especially careful to include data bases which might contain disconfirming information.

2) Include sample size, confidence intervals, and other measures of information validity in addition to mean values.

3) Encourage use of models and quantitative aids to improve upon information analysis through proper aggregation of acquired information.

4) Avoid the hindsight bias by providing access to information at critical past times.

5) Encourage decisionmakers to distinguish good and bad decisions from good and bad outcomes in order to avoid various forms of selective perception such as, for example, the illusion of control.

6) Encourage effective learning from experience. Encourage understanding of the decision situation and methods and rules used in practice to process information and make decisions such as to avoid outcome irrelevant learning systems.

7) Use structured frameworks based on logical reasoning [255], [376] in order to avoid confusing facts and values and wishful thinking and to assist in processing information updates.

8) Both qualitative and quantitative data should be collected, and all data should be regarded with "appropriate" emphasis. None of the data should be overweighted or underweighted in accordance with personal views, beliefs, or values only.

9) People should be reminded, from time to time, concerning what type or size of sample from which data are being gathered, so as to avoid the representativeness bias.

10) Information should be presented in several orderings so as to avoid recency and primacy order effects, and the data presentation context and data saturation biases.

Kahneman and Tversky [235] discuss a systemic procedure to enhance debiasing of information processing activities. A definitive discussion of debiasing methods for hindsight and overconfidence is presented by Fischhoff [185]. Lichtenstein and Fischhoff present a number of helpful guidelines to assist in training for calibration [217]. Clearly more efforts along these lines are needed. Studies to determine the extent to which learning feedback acquired through use of methods such as social judgment theory contributes to debiasing would be especially rewarding. This is especially the case since confidence in unaided judgment is learned and maintained through feedback even when there is very little or no justification for this confidence [94]. Typically outcomes which follow from decisions based on negative judgments are not observed. Reinforcements of self-fulfilling prophecy type judgments through positive outcome feedback only occur in spite of, rather than due to, judgment validity.

Research integrating the methods whereby people integrate or aggregate information and attribute causes [8]–[12], [142], [143], [186], [190], [199], [321], [364] with methods for the identification and amelioration of cognitive biases would be of interest and of much potential use also.

In a sense, the results of this section are disturbing in that they tend to support the "intellectual cripple" hypothesis of Slovic ([142], pg. 14) and imply that humans may well be little more than masters of the art of self deception. On the other hand there is strong evidence that humans are very strongly motivated to understand, to cope with, and to improve themselves and the environment in which they function. While there are a number of fundamental limitations to systemic efforts to assist in bettering the quality of human judgment, choice, and decisions [282], [307], there are also a number of desirable activities [16], [305], [385]. These can assist in increasing the relevance of systemic approaches such as those which result in information processing adjuvants for policy analysis, forecasting, planning, and other judgment and decision tasks in which information acquisition, analysis, and interpretation play a needed and vital role.

IV. Decision Rules

In order to select an alternative plan or course of action for ultimate implementation, the decisionmaker applies one or more decision rules which enable comparison prioritization, and ultimately, selection of a single policy alternative from among a set of choice alternatives. The purpose of a decision rule is to specify the most preferred alternative generally from a partial or total ordering or prioritization of alternatives. To utilize a decision rule we must have a set of alternatives, a set of objectives to be accomplished by the alternatives, a knowledge of the impacts of the alternatives, evaluation of these impacts, and associated preference information. Decision rules may be explicit or implicit in terms of the way in which they are used in the decision process.

We can assume, without loss of generality, that each single policy alternative may represent a complex portfolio of individual alternatives and that the set of choice alternatives contains mutually exclusive components. This formulation can always be accomplished but may result in a very large set of policy alternatives since n individual alternatives can be combined into 2^n possible portfolios of alternatives. Failure to consider a combination of alternatives may result in significant errors in decisionmaking unless each of the individual alternatives represents one component of a portfolio of all possible combinations of individual alternatives, or unless the individual alternatives are independent or mutually exclusive.

It is assumed at the interpretation step of the decision process that formulation and analysis have been accomplished such that there exists a decision situation structural model and the results of exercising the model. Thus objectives, relevant constraints, some bounds on the issue, possible policy alternatives, impacts of policy alternatives, etc. are assumed known. The choice of a decision rule will depend, in large measure, upon the decision situation structural model as reflected in the contingency task structure. We will discuss dynamic models for contingency task structures in our next section.

The above discussion may appear representative primarily of the judgment and decision process associated with the formal operational thought model that we will elaborate upon in our next section. For purposes of clarity of exposition, we have presented an oversimplified view of how decision rules are used to aggregate information and evaluate alternatives. The sequence we have described implies comparison and evaluation of alternatives only after we have first accomplished formulation and analysis of the issue under consideration. As we have noted throughout our discussion, decisionmakers typically compare and evaluate alternatives while they are in the process of decision situation formulation and analysis. These partial comparisons and evaluations lead to searches for additional policy alternatives, additional analysis, etc. As we have also noted, the entire decision process typically occurs in a parallel-simultaneous-iterative fashion rather than an exclusively sequential series of steps in which formulation is followed by analysis which is followed by interpretation.

Individuals and decision environments vary so greatly that there are a great number of decision rules that will be needed to describe actual decision situations. Schoemaker [315] is among a number of authors [121], [255], [364], [365], [372] who have attempted classification schemes to allow categorization of various descriptive decision rule models. His first level categorization separates decision rules into holistic and nonholistic categories. In a holistic decision rule each alternative or portfolio of alternatives is evaluated and assigned a value or utility. After all alternatives have been evaluated, they are compared and alternative A is said to be preferred to alternative B if its evaluation has given it a greater utility such that $U(A) > U(B)$. In nonholistic decision rules individual alternatives or portfolios of alternatives are generally compared with one another in a sequential elimination process. This comparison may be against some standard across a few attributes within alternative pairs or across alternatives, with alternative attributes being compared one at a time.

Each of these categories appears to imply disaggregation into components of the event outcomes likely to follow from decisions. Our section on contingency task structure models will propose a dynamic evolving cognitive style model which admits of expert situational understanding that involves reasoning by analogy, intuitive affect, and other forms of nonverbal almost unconscious perception. We elect to call this type of reasoning wholistic and add a third category to the classification scheme of Schoemaker.

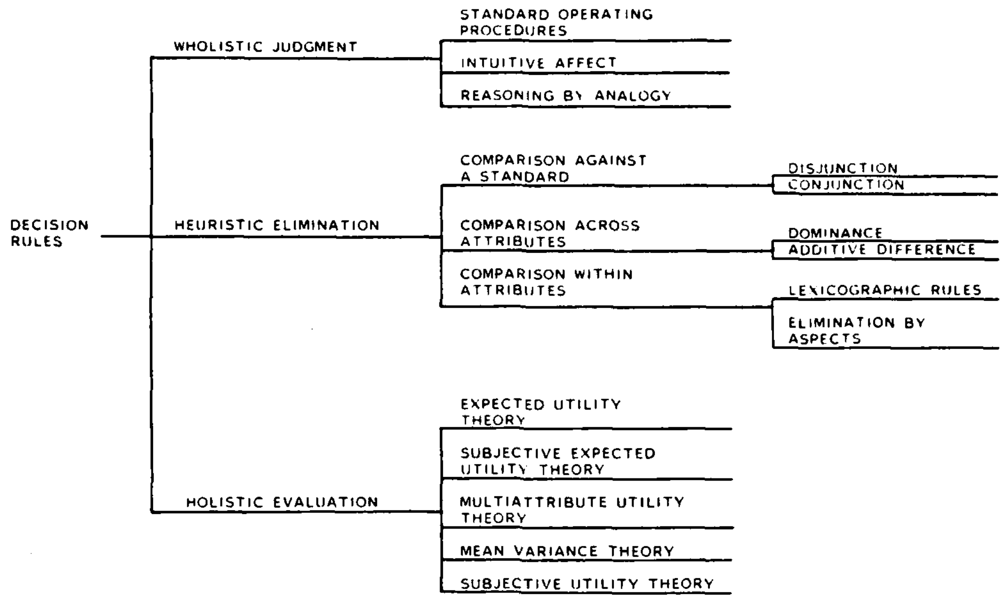

Fig. 4. Hierarchical structure of decision rules.

Consequently we envision three first level general categories of decision rules: holistic, heuristic, and wholistic. In a *holistic* decision rule there is an attempt to consider all aspects of a decision situation in evaluating choices by means of disaggregation of various choice components. In a *heuristic* decision rule, detailed complicated comparisons are not used. Rather, simplified approximations to holistic decision rules are used. In a *wholistic* decision rule, the evaluation and choice of alternatives is based upon use of previous experience, hopefully true expertise, with respect to similar decision situations. The selection of an alternative is based upon its perceived or presumed worth as a whole and without detailed conscious consideration of the individual aspects of each alternative. It is possible to define a number of decision rules and categorize them. The first level categories we have defined are not mutually exclusive. A number of decision rules doubtlessly can be categorized into more than one of these first-level decision categories. Figure 4 illustrates a possible inclusion structure for the decision rules we will describe here.

Expected Utility Theory: Our first decision rule is based on expected utility theory and is doubtlessly the most familiar decision rule to engineers. This rule derives from a "rational actor"[2] decision model [3], [4], [89], [103], [121], [134], [169], [192], [222], [256], [265], [285], [315], [359], [397] which is more fully discussed in Section VI.

The rational actor model is a normative model. Von Neuman and Morgenstern, who introduced the axioms of the model of rational man, stated the purpose of their work as

> to find mathematically complete principles which define "rational behavior" . . . a set of rules for each participant which tell him how to behave in every situation which may conceivably arise.

The idea of rationality originated in the economics literature where microeconomic models of the consumer and the firm assumed complete information and rationality. The rational person is assumed to have identified a set of well-defined objectives and goals and is assumed to be able to express preferences between different states of affairs according to the degree of satisfaction of attaining these objectives and goals. A rational person has identified available alternative courses of action and the possible consequences of each alternative. The rational person makes a consistent choice of alternative actions in order to maximize the expected degree of satisfaction associated with attaining identified objectives and goals.

A number of elements are assumed to exist in the rational actor model:

1) a set of policy alternatives A;
2) the set of possible consequences of choice or future states of nature or decision outcomes called S;
3) a utility function $U(s)$ that is defined for all elements s of S;
4) information as to which outcomes will occur if a particular policy alternative a in A is chosen; and
5) information as to the probability of occurrence of any particular outcome if an alternative $a \in A$ is chosen. $P_a(s)$ is the probability that $s \in S$ will occur if $a \in A$ is chosen.

There are a number of ways in which the axioms associated with the rational actor model may be stated. Each statement of the axioms allows proof of the fact that cardinal utility functions will exist and be unique only up to positive linear transformations. Further, the evaluation of expected utility allows choicemaking and prioritization of alternatives in accordance with the expected utility of each alternative. There are a number of textbook accounts of expected utility theory to which the interested reader of this review may turn for alternative sets of axioms and detailed accounts of the use of expected utility theory [51], [163], [196], [222], [285], [302], [315]. MacCrimmon and Larson interrelate the major axiom systems in expected utility theory [3] in a noteworthy contribution to understanding of the several systems that lead to (essentially) the same results for the rational actor model. A very readable introductory treatment of expected utility theory, relating descriptive psychological concerns with normative concerns, is presented by Vlek [244].

The rational actor model is often accepted as a normative model of how decisions should be made, at least in a substantive or "as if" fashion. It is often observed that the model is not an accurate description of either the substance or the process of actual unaided choicemaking behavior. Some of these observers use empirical evidence of the deviation of actual decisionmakers from either substantive rationality or process rationality. These observations are doubtlessly correct. The rational actor model is,

[2] Technological or economic rationality would be a more appropriate term.

however, invaluable in that it can be often used as reference for comparison of actual behavior with ideal "aided" or normative behavior. Further, it provides a benchmark against which to compare simplified heuristics. Our efforts and discussions in this section concern primarily substantive behavior although we recognize the great difficulty, in practice, of separating substance from process.

Simon and his colleagues introduced the concept of bounded rationality and developed a satisficing model for individual choicemaking. It is worth noting that boundedly rational actors are basically rational subject to constraints on the formulation, analysis, and interpretation of information, and the substitution of achievement of a target level of return, or aspiration level, for selection of the best alternative. Typically people satisfice by adaptive adjustment [72] of aspirations such that, in repetitive decision situations, optimizing behavior is approached [270].

There is absolutely nothing in the formulation of the rational actor model which requires identification of *all* objectives, *all* possible alternatives, *all* possible impacts of alternatives, etc. The rational actor model is perfectly capable of being used to allow prioritization and selection of the best alternative by evaluating some impacts and with knowledge of some objectives, from among an incomplete set. It in no sense necessarily requires completeness in everything and the associated complexity that this would require. Actual decisionmaking behavior may not, however, even be boundedly rational but may employ such poor heuristics as to result in inferior choicemaking even to the extent of selecting inferior choices from among those in a bounded set.

There have been a number of experimental studies and field studies of the appropriateness of the expected utility model [3], [111], [117], [119], [125], [184]–[186], [236], [336]–[341], [385] as a descriptive model of substantive unaided behavior. Among the surveys which comment upon the experimental and field studies are [27], [98], [206], [348], [372]. Schoemaker [315] provides a very readable brief survey of some of this literature. While the evidence is mixed, most studies indicate that the expected utility decision rule simply does not function well in a *descriptive substantive* sense.

In its simplest form the expected utility of alternative a_i is computed from

$$E\{U(a_i)\} = \sum_{j=1}^{n} p[s_j(a_i)]U[s_j(a_i)] \tag{1}$$

where the $s_j(a_i)$, $j = 1, 2, \ldots, n$, are the states which may result from alternative a_i and the $p[s_j(a_i)]$ are the associated probabilities. In the expected utility formulation the $p[s_j(a_i)] = p_j(a_i) = p_j$ are assumed to be objective probabilities and, of course, $\sum_{j=1}^{n} p_j = 1$. Generally these probabilities are not alternative invariant although notationally they are sometimes written as if they were independent of alternatives. The $U[s_j(a_i)]$ are the utilities or values [296] of the decisionmaker for the various outcome states. Johnson and Huber [179] survey a number of procedures that can be used to elicit utility functions. Most of the textbooks cited earlier also contain discussion of utility assessment procedures.

Subjective Expected Utility: Often it occurs that objective probabilities are, for any of a variety of reasons, unavailable in a given situation. The subjective expected utility model is obtained when subjective probabilities $f(p_j)$ are substituted for the p_j in (1). The $f(p_j)$ are generally elicited such that $\sum_{j=1}^{n} f(p_j) = 1$ and so the subjective probabilities behave in a way consistent with the laws of probability. There are a number of discussions concerning probability elicitation [31], [223], [257], [355] that present appropriate procedures to enable determination of subjective probabilities from individuals and groups. Conventional approaches to elicitation of utility in expected utility theory may confound strength of preference felt for alternative event outcomes and attitude toward risk. Also the elicitation procedure can become cumbersome. Recent research has formally separated these factors [33] and shows much promise in enhancing understanding of attitude towards risk. In this approach the utility concept is devoid of risk. It takes on a meaning more like that in conventional microeconomics where it measures strength of preference for certain outcomes only. This research [33] could provide additional linkages and understanding between the expected utility and subjective expected utility concepts by providing for incorporation of risk aversion effects in a relatively simple way. A related approach to incorporation of risk aversion is described by Howard and decision analysts at the Stanford Research Institute [164] who have been responsible for a number of major application studies in this area. There have been a number of related approaches [65], [66], [121] and the subjects of risk and uncertainty are of much contemporary interest [6], [136], [153], [304].

A number of studies have indicated that the relation between subjective and objective probabilities is nonlinear and situation dependent. It is usually indicated that people often underestimate high probabilities and overestimate low ones. More recent research has indicated that this appears true only for favorable outcomes. Just the opposite appears true when the outcome is unfavorable. This appears to be a form of wishful thinking for low probability events and "everything bad happens to me" for high probabilities. What we will call subjective utility theory attempts to incorporate situation dependent nonlinearities that may exist between subjective and objective probabilities.

Multiattribute Outcomes: Often decision situations are sufficiently complex that it is difficult to evaluate, in a wholistic fashion, the utility of each outcome. Often it is possible to disaggregate the features on which utility is based into a number of components called attributes. An attribute tree is a hierarchical structure which, when quantified through elicitation of values of the outcomes on the lowest level attributes and relative weights of the attributes, can be used to determine the utility of event outcomes. The types of multiattribute utility models used have varied from very simple unit weight linear models to rather complex multiplicative models [106]. Dawes [71] documents the robust beauty of linear models of the form

$$U(s_i) = \sum_{j=1}^{m} h_j u_j(s_i), \qquad \sum_{j=1}^{m} h_j = 1 \tag{2}$$

where there are assumed to be m attributes, h_j is the weight of the jth attribute and $u_j(s_i)$ the value score on the jth attribute of outcome s_i. In much of the work in this area decisions under certainty are considered such that there is a one to one correspondence between alternative a_i and outcome s_i. Under decision-under-certainty conditions we can let $s_i = a_i$ in (2).

Multiattribute models have been very successfully used to predict the decision behavior in field settings or many professional groups. Hammond [140]–[142] and his colleagues have, as discussed in Section III, developed an approach known as social judgment theory in which the "policy" of the decisionmaker, equivalent in this circumstance to the weights h_j, are identified from wholistic prioritization of decision outcomes through use of regression analysis techniques. Ward Edwards and his colleagues [186], [301] and elsewhere, elicit weights from decisionmakers for the model of (2) in a useful straightforward procedure called simple multiple attribute ranking technique (SMART) that has seen a number of realistic applications. Results of the surveys of Armstrong [14], [15]; Fischer [111], Slovic and Lichtenstein [345], Slovic, Fischhoff, and Lichtenstein [348]; Shanteau [324], and others indicate that simple linear models [64] are very potent predictors of reliable judgment, especially under conditions of certainty in that one can replicate the substantive judgment of decisionmakers. This is the case even though the simple linear model may not do a very good job of modeling the decision process. "Boot strapping" is the name given to the task of substituting a decision rule for the decisionmaker. The studies in the cited references show that the elimination of human judgment error made possible by boot strapping enables it to be superior to unaided human judgment. One can even misspecify weights and ignore attribute dependencies and still find that weighted linear models do quite well [71].

The fact that the weighted linear rule may be so good is a rather mixed blessing. In circumstances in which there is no requirement for knowledge of the underlying decision process, the substantive predictive ability of the linear additive model may make it quite useful. Situations such as evaluating credit card applicants or applicants for admissions to colleges are repetitive judgment and decision situations which fit into this category. Use of simple formal linear model may well, in situations such as these, lead to a more efficient as well as more effective and equitable selection process than one based on unaided human intuition [70], [71], (Dawes in Shweder [332]). In unstructured or semistructured nonrepetitive decision situations it is much less clear that a decision rule that is not guaranteed to be faithful to the underlying decision process will be nearly as valuable as one that is in terms of enabling decisionmakers to make better decisions. Fischhoff, Goitein, and Shipira [119] provide a number of perceptive comments concerning this and the consequent need for a theory of errors to explicate the effects of poor decision situation structural models and parameters within the structure. A hoped-for achievement is a sensitivity-based analysis of deviations from optimality to determine, among other things, the role of experience in decisionmaking and those components and principles of decisionmaking which can be usefully and meaningfully learned from experience [47], [94]–[97], [115], [116].

Multiattribute utility models based on the expected utility theory of von Neumann and Morgenstern are considerably more complex than those of behavioral decision theory. Often there are efforts to determine existence of various attribute independence conditions such as to validate use of a linear model of the form of (2) or a multiplicative model of the form

$$1 + HU(s_i) = \prod_{j=1}^{m} \left[1 + h_j H u_j(s_i)\right], \qquad \sum_{j=1}^{n} h_j = 1. \quad (3)$$

The foremost proponents of this approach are Keeney and Raiffa [196]. There are many contributions to this area and variations of the basic approach [23], [29], [75], [93], [127], [231], [277], [278], [300], [301], [302], [358], [399]. It is proposed exclusively as a normative approach and has been successfully used for a variety of applications including proposal evaluation [245], [310], siting power plants [197]; and budgeting and planning [52], [191].

Mean Variance—There are a number of models and associated decision rules based upon mean-variance (EV) models. Markowitz's portfolio theory, which is well summarized in Libby and Fishburn [214] and Baron [26], is based in part on the assumption of a quadratic utility function

$$U(s) = \alpha + \beta s + \gamma s^2 \quad (4)$$

where the same states are assumed invariant over all alternatives such that we have a quadratic programming problem in prioritizing alternatives where

$$E\{U(a_i)\} = \sum_{j=1}^{n} p_j(a_i) U(s_j)$$
$$= \alpha + \beta E\{a_i\} + \gamma E\{a_i^2\}$$
$$= \alpha + \beta\mu_i + \gamma(\sigma_i^2 + \mu_i^2).$$

Coombs [65], [66], [185] has also been concerned with portfolio theory and assumes an optimum risk level in the form of a single peaked risk preference function for every expected value level. Gambles of equal expected value are judged on the basis of lower variance in the Markowitz' portfolio theory and on the basis of deviation from optimum risk level in Coombs' portfolio theory. Stochastic dominance concepts [124] are especially useful in dealing with problems in the mean-variance models of portfolio theory. Unfortunately as has been shown by a number of authors [124], the results from using mean-variance portfolio theory are not necessarily consistent with results obtained from expected utility theory. For example, if the outcomes of decision a_1 are $10 with probability 0.5 and $20 with probability 0.5 and the outcome of decision a_2 is $10 with probability 1.0; then the EV rule ($\mu_{a1} = \$15$, $\sigma_{a2} = \$5$) ($\mu_{a2} = \$10$, $\sigma_{a2} = 0$) is indeterminate in that there is no pareto superior or dominance alternative in an EV sense. Yet any reasonable person would prefer alternative a_1 to alternative a_2.

Fishburn [123] has considered a variation of the mean-variance model which involves concepts based upon target level of return or aspiration level or reference level to define the risk of an alternative. The "risk" of alternative a is determined from the probability of receiving a return not to exceed x, denoted $F(x)$, by

$$R(a) = \int_{-\infty}^{t} (t - x)^{\alpha}\, dF(x) = \int_{-\infty}^{t} (t - x)^{\alpha} p(x)\, dx$$

where t is the target return, α is a nonnegative parameter that is used to indicate relative importance of deviations below target return. For $0 \leq \alpha < 1$ the decisionmakers primary concern is failure to achieve the target with little regard to the size of the deviation. For $\alpha > 1$ the decisionmaker is very concerned with sizeable deviations from target and relatively unconcerned with small deviations. In the former case the decisionmaker is risk seeking for losses and has a utility function that is convex for losses. In the latter case the decisionmaker is risk averse for losses and has a utility function that is concave for losses.

In this model the mean return from an alternative and its risk are the two attributes determining preference. This model thus appears much similar to the standard EV model in that $a_1 > a_2$ if and only if $\mu(a_1) \geq \mu(a_2)$ and $R(a_1) \leq R(a_2)$ with at least one inequality being valid. In the example just considered, the mean values are as given previously and the risks are

$$R(a_1) = \begin{cases} 0, & t \leq 10 \\ 0.5(t - 10)^{\alpha}, & 10 \leq t \leq 20 \\ 0.5(t - 10)^{\alpha} + 0.5(t - 20)^{\alpha}, & 20 \leq t \end{cases}$$

$$R(a_2) = \begin{cases} 0, & t \leq 10 \\ (t - 10)^{\alpha}, & 10 \leq t \end{cases}$$

Thus we see that the risk is the same, that is zero, if $t \leq 10$ and so we prefer a_1. The risk associated with a_1 is one half that associated with a_2 if the target return is between $10 and $20. The risk associated with a_1 is less than that associated with a_2 if $t \geq 20$. And so, since $\mu(a_1) > \mu(a_2)$, we prefer a_1 regardless of the target return. Generally, as in this case, Fishburn's below-target model will resolve ambiguities associated with the standard mean-variance model. The decisionmaker is free to specify α and t. Thus this represents a rather useful dominance type decision rule. Extensions of this rule to the case of multiattribute and multiple objective preferences would have considerable value.

Subjective Utility Theory: A number of researchers have proposed holistic decision rules based on the observation that people, in unaided situations, do not typically perceive (objective) probabilities such that the fundamental probability property $\sum_{i=1}^{n} p_i = 1$ is satisfied. There presently exists several decision situation models based upon a subjective utility theory in which probabilities do not sum to one. Among these are certainty equivalence theory due to Handa [144], subjectively weighted utility theory due to Karmarkar [188], [189]; and prospect theory due to Tversky and Kahneman [184], [385]. There have been several additional studies involving prospect theory including those of Thaler [371], and Hershey and Schoemaker [152], [153]. Some of the foundations for these subjective utility theory efforts can be found in the early work of Allais [3] who was among the first to note that the normative expected utility approach of von Neumann and Morgenstern, and the subjective expected utility modifications, did not necessarily describe actual descriptive choice behavior. We believe that these studies are especially relevant to information system design and so summarize relevant features from these effects here.

In certainty equivalence theory five axioms are assumed. We will use the term prospect or prospect (s, P) to mean the opportunity to obtain outcome s with probability P. Simply stated, these are as follows.

1) Preferences are governed only by utilities and outcomes. One is indifferent between a nonsimple prospect and an actuarially identical simple prospect with a single event node.
2) Complete ordering of prospects is possible and transitivity of prospects exists.
3) Continuity exists such that if $(s_1, P_1) \succcurlyeq (s_2, P_2), \succcurlyeq (s_3, P_3)$ then there exists an α such that $(s_2, P_2) \sim (\alpha s_1, P_1) + (s_3 - \alpha s_3, P_3)$.
4) Independence exists such that if $(s_i, P_i) \sim (x_i, 1) \forall i$, then $(\underline{s}, \underline{P}) \sim (\Sigma x_i, 1)$ where $\underline{s}$ and $\underline{P}$ represent vectors of outcomes and probabilities s_i and $\tilde{P}_i$.
5) Enhanced prospects are preferred if and only if a basic prospect is preferred. Thus $(\beta s_1, P_1) \succcurlyeq (\beta s_2, P_2) \forall \beta \geq 0$ if and only if $(s_1, P_1) \succcurlyeq (s_2, P_2)$.

These axioms are sufficient to insure that the subjective utility function of alternative a_i, $CE(a_i) = CE[s(a_i), P(a_i)] = U(\underline{s}', \underline{P}')$, is linear in s_i and of the form

$$U(\underline{s}', \underline{p}') = \sum_{j=1}^{n} s_j' w(P_j') = \underline{w}^T(\underline{P}')\underline{s}'. \tag{5}$$

Axioms 1, 4, and 5 incorporate the major changes from the von Neumann–Morgenstern axioms. It appears unduly restrictive to require that the utility function be linear in the outcome and this is reason enough to warrant the development of a more robust theory.

Fishburn [125], however, has shown that certainty equivalence theory must reduce to the expected value model, $U(\underline{s}, \underline{P}) = \underline{P}^T\underline{s}$, $\Sigma_{j=1}^n w(P_j) = 1$. This occurs because of the requirement that one must be indifferent between a nonsimple prospect and an actuarially equivalent simple prospect. To insure this for the two outcome case, for the general actuarially equivalent two outcome prospects of Fig. 5, requires that $w(P) + w(1 - P) = 1$.[3] This certainty must be viewed as another limitation of this certainty equivalence theory and indicates the considerable care that must be exercised in modifying the basic utility theory axioms.

The subjective weighted utility model yields for the SWU of alternative a_i

$$SWU(a_i) = \sum_{j=1}^{n} w[P_j(a_i)]U[s_j(a_i)] \tag{6}$$

where the subjective weighted probabilities are

$$w[P_i(a)] = \frac{f[P_i(a)]}{\sum_{j=1}^{n} f[P_i(a)]}. \tag{7}$$

Although a variety of probability weighting functions are possible, Karmarkar [188], [189] proposes use of a log normal function

$$\ln\left(\frac{f}{1-f}\right) = a\ln\left(\frac{P}{1-P}\right) \tag{8}$$

[3]For the n outcome case we would have $\Sigma_{j=1}^n w(P_j) = 1$ and we see that the only general $w(P_j)$ that will insure this is $w(P_j) = P_j$.

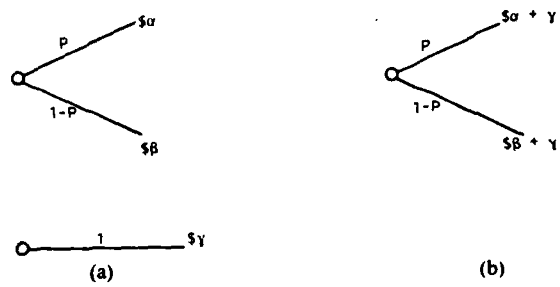

Fig. 5. Two actuarially equivalent prospects.

or

$$f(P) = \frac{P^\alpha}{P^\alpha + (1-P)^\alpha} \tag{9}$$

where $0 \leq \alpha \leq 1$. This transformation of probabilities is such that large probabilities are understated and small probabilities overstated. Karmarkar emphasizes that the probability weighting function does not represent a probability perception phenomenon but represents a bias in the way in which (objective) probabilities are descriptively incorporated into the evaluation, prioritization, and choicemaking process. In this model, the final weighted probabilities do sum to one in accordance with the conventional subjective expected utility theory. However, the expression for any normalized weight $w[P_i(a)]$ is actually a function of the value of all other probabilities as seen in (7). The effects of this confounding of influence remain to be investigated.

The considerably more sophisticated prospect theory of Tversky and Kahneman [184], [385], contains a number of modifications to expected utility theory. Prospect theory consists of an editing phase involving framing of contingencies, alternatives, and outcomes, followed by an evaluation phase. These modify subjective expected utility theory such as to enhance unaided descriptive realism of the theory

1) In the editing phase, the decision situation is recast into a number of simpler situations in order to make the evaluation task simpler for the choicemaker. The tasks in editing are very much dependent on the contingency situation at hand and offer possibilities for coding, combining, segregating, cancelling, and detection of dominance.
2) Value functions are devoid of risk attitude and are unique only up to positive ratio transformations.
3) Outcomes are expressed as positive or negative deviations from a reference or nominal outcome which is assigned a value of zero. Thus value changes represent changes in asset position. Positive and negative values are treated differently with the typical value function being an S-shaped curve that is convex below the reference point and concave above it. Displeasure with loss is typically greater than pleasure associated with the same gain.
4) Probability weights $w[P_i(a)]$ reflect an uncertain outcome contribution to the attractiveness of a prospect. As in SWU theory, high probabilities are underweighted and low ones overweighted. The following are among the properties of the probability weighting function.
   a) True at extremes $w(0) = 0$, $w(1) = 1$
   b) Subadditive at low P, $w(\alpha P) > \alpha w(P), 0 < \alpha$
   c) Overweighted for small p, $w(p) > p$, $p \ll 1$
   d) Underweighted for large P, $w(P) < P$, $P \gg 0$
   e) Subcertain $w(p) + w(1-p) < 1$
   f) Subproportional $w(\alpha P)/w(P) \leq w(\alpha\beta P)/w(\beta P)$, $0 < \alpha, \beta, \leq 1$.
5) The value of a prospect $(\underline{s}, \underline{P}) = (s_1, P_1) + (s_2, P_2)$ is given by
   a) $V(\underline{s}, \underline{P}) = v(s_2) + w(P_1)[v(s_1) - v(s_2)]$ (10)

for strictly positive prospects in which $P_1 + P_2 = 1$ and $s_1 > s_2 > 0$, or strictly negative prospects in which $P_1 + P_2 = 1, s_1 < s_2 < 0$.

b) $$V(s, P) = w(P_1)v(S_1) + w(P_2)v(s_2) \tag{11}$$

for regular prospects which are prospects that are neither strictly positive nor strictly negative in that either $P_1 + P_2 \neq 1$ and/or $v(s_1)$ and $v(s_2)$ are of opposite sign.

In no sense is prospect theory posed as a normative theory of how people should make decisions. The editing or framing of contingencies, alternative acts, and outcomes is similar to the formulation step of the systems process. It is in this forming phase that the contingency task structure and decision situation model are, in effect, formed. For example, in a population of one million people where black lung disease might kill two thousand people, possible forms are

Form 1—Alternative a_1 will save 500 people whereas if alternative a_2 is adopted there is a 0.25 probability of saving 2000 people and a 0.75 probability of not saving anyone.

Form 2—Alternative a_3 will result in death of 1500 people whereas alternative a_4 will result in a 0.25 probability that no one will die and a 0.75 probability that 2000 people will die.

These two forms are really the same, yet many people will interpret them differently. The editing or forming phase of prospect theory allows different interpretations and thus makes provision for different evaluation of results in terms of alternative formulations of the same issue.

Prospect theory is especially able to cope with *certainty effects* in which people overweight outcomes considered certain compared with those considered only highly probable; *reflection effects* in which preferences are reversed when two positively valued outcomes are replaced by two negatively valued outcomes; and *isolation effects* in which people disregard common outcome components shared by outcomes and focus only on components that distinguish alternatives. Kahneman and Tversky have established an axiomatic basis for prospect theory [184] for the two outcome case.

In a recent study involving prospect theory, Hershey and Schoemaker [152] question the generality of the reflection hypothesis of prospect theory which states that asymmetric preferences are found when comparing gain prospects with loss prospects. They introduce four types of reflectivity depending upon whether subjects choose positive prospect (s_1, P_1) or the noninferior prospect (s_2, P_2), and whether they choose negative prospect $(-s_1, P_1)$ or $(-s_2, P_2)$. Across-subject and within-subject reflectivity are examined in terms of whether subjects do or do not choose and do or do not switch from safe to risky prospects. They conclude that predictions of prospect theory concerning reflectivity depend upon the size of probabilities. For P large enough to insure underweighting of probabilities, it appears that the reflectivity hypothesis is quite valid. For smaller values of P, reflectivity is neither predicted nor excluded from the results of Hershey and Schoemaker.

In another study Hershey and Schoemaker [153] examine preferences for basic insurance-loss lotteries and show that risk taking is prevalent in the domain of losses. They suggest a utility function which is concave for low losses and convex for larger ones. They indicate a context effect in which various insurance formulations lead to more risk averse behavior than for statistically equivalent gambling formulations. Their conclusion that probabilities and outcomes may be of less guidance in influencing decision behavior as uncertainties concerning their magnitude increase, strengthens conjectures concerning the influence of context and perceptions of decision situation structural models upon decision results.

Thaler [371] examines a number of the tenets of prospect theory with generally very positive confirming results. Additional comments concerning the seminal prospect theory appear in a previous survey in these transactions [304] including the observation that a number of the results of prospect theory, which are seemingly at variance with expected utility theory, can be accommodated successfully using multiple attribute utility theory. Extensions of prospect theory to include multiple attribute preferences, large number of outcomes, sequential multistage decisionmaking, risk aversion coefficients, and subjective probability effects would do much to enable this significant development to be of even greater usefulness in explaining complex positive or descriptive decision behavior. This might well be of much normative use as well.

Heuristic Decision Rules

A number of decision rules do not involve comparisons in a true holistic fashion. Rather they involve comparisons of one alternative with another, generally within a restricted alternative set and attribute set. Within the heuristic class of decision rules we may distinguish those which compare alternatives against some standard by means of conjunctive or disjunctive comparisons, those which compare alternatives across attributes, and those which make comparisons within attributes. Generally, these rules can result, when improperly applied, in intransitive choices [289]. We will consider several rules from each subcategory. First we will discuss two noncompensatory rules [90] that are often used when there is an overabundance of data present.

Disjunctive—A disjunctive decision rule is one in which the decisionmaker identifies minimally acceptable value standards for each relevant attribute. Alternatives which pass the critical standard on one or more attributes are retained. Alternatives which fall below the critical standards on all attributes are eliminated. A single alternative is accepted when the critical standards are set such that all but one alternative fail to exceed any of the critical standards on any attributes. Unlike multiattribute utility theory (MAUT) rules, where poor performance on one attribute can be made up by good performance on other attributes such that the rule is compensatory, a disjunctive decision rule is noncompensatory. A compensatory approximation to a disjunctive decision rule for attributes s_i is

$$U = \sum_{i=1}^{m} \frac{1}{\left(1 + \frac{s_i}{c_i}\right)n_i}, \quad n_i \gg 1 \tag{12}$$

where m represents the number of attributes and c_i is the critical value on the ith attribute. If U is greater than one, the alternative in question is retained.

Conjunctive—A conjunctive decision rule is one in which minimally acceptable value standards for each relevant attribute are identified. Alternatives are acceptable if they exceed all minimum standards. They are rejected if they fail to exceed any minimum standard. The critical values for disjunctive and conjunctive rules are generally different. A compensatory approximation to the noncompensatory conjunctive decision rule is

$$U = \prod_{i=1}^{m} \frac{1}{\left(1 + \frac{c_i}{s_i}\right)n_i}, \quad n_i \gg 1. \tag{13}$$

An alternative is retained if the corresponding utility U is above a threshold which is set just slightly below 1. These approximations for the disjunctive and conjunctive rules become noncompensatory as n_i approaches infinity.

By iterating through the conjunctive acceptance and disjunctive rejection rule several times with adjustable critical values or aspiration levels, these rules become, in effect, forms of satisficing rules.

Dominance models and additive difference models are two examples of models which lead to decision rules involving comparison across some, but not necessarily all, attributes. No minimum standard of performance on attributes, that is to say minimum aspects, are identified.

Dominance—A dominance decision rule is one which chooses alternative a_1 over a_2 if a_1 is better than a_2 on at least one aspect and not worse than a_2 on any other aspect. An aspect is the score of a specific attention on a specific attribute. There are a number of applications of dominance theory, including stochastic dominance, to decisionmaking situations [33], [54], [75], [124], [358], [399].

Additive difference—In an additive difference rule [382]–[385], a binary choice is made between alternatives a_1 and a_2. Differences are considered between values for a_1 and a_2 on each relevant attribute. Differences of the form $U_i(a_1) - U_i(a_2)$ are computed. Each of the differences is weighted in proportion to the importance of the differences between alternatives on the various attributes. The resulting weight is $f_i[U_i(a_1) - U_i(a_2)]$. Alternative 1 is preferred to alternative 2 only if

$$\sum_{i=1}^{n} f_i[U_i(a_1) - U_i(a_2)] > 0.$$

This is a compensatory rule and can be used to compare any number of alternatives merely by retaining the winner in each comparison [272]. Only if the functions f_i are linear will the additive difference rule necessarily lead to transitive choices.

A third important subcategorization involves comparison within attributes. There are a variety of lexicographic procedures [121] and the elimination by aspects rule [381], [382] which explicitly involve comparison of alternatives on one, or at most a few, attributes.

Lexicographic Decision Rule: This rule prescribes a choice of the alternative which is most attractive on the most important attribute. If two aspects on this attribute are equally attractive, the decision will be based upon the most attractive aspect on the attribute next in order of importance, etc.

Minimum Difference Lexicographic Rule: This rule is much like the lexicographic rule, with the additional assumption that for each attribute there is a minimum acceptable difference Δ_i of alternative scores. Thus, only differences greater than Δ_i between the attractiveness values of two alternatives may determine a decision. If the difference on the most important attribute is less than Δ_i, then the attribute next in the lexicographic order is considered. The lexicographic semiorder rule is a special case of this decision rule where Δ_i is defined only for the most important attribute. For all other attributes $\Delta_i = 0$. This procedure may easily be extended to cases where the Δ_i are defined for the two most important aspects. This rule is often used in situations where information about attributes are missing as a result of imperfect discrimination among alternatives on a given attribute or of unreliability of available information. In general this rule leads to intransitive choices when there are more than two alternatives. It may even lead to agenda dependent results for the case where there are only three alternatives. One should be especially careful to examine relations used for ordering alternatives to attempt to detect use of heuristics such as this, especially if concepts such as transitivity are used, perhaps inferentially, to determine partial orderings. This suggests the need for special care when attempting to use transitivity concepts to infer ordinal preferences. The resulting failure to seek disconfirming information may well create structural preference illusions.

Einhorn [96], [97] first uses the term "outcome irrelevant learning structure" to describe processes which used deficient heuristics, and which then reinforces poor choices through experiences involving feedback and lack of discomfirming evidence. These outcome irrelevant learning structures (OILS) may result either from unaided judgment processes or from poorly conceived or possibly well conceived but improperly utilized, and therefore irrelevant, systemic methods or processes.

The Maximizing Number of Attributes in Greater Attractiveness Rule: This rule prescribes a choice of the alternative that has the greater number of favorable attributes. Specifically the rule requires that the aspect of a decision alternative must be classified for each attribute as better, equal, or worse than the attractiveness of the other alternative on that attribute. The preferred alternative will be that which has the greatest number of favorable classifications.

Elimination by Aspects: In this rule [288], [381] attributes are assumed to have different importance weights. An attribute is selected with which to compare alternatives with a probability that is proportional to its weight. Alternatives which do not have attribute scores above some aspiration or critical level are eliminated. A second attribute is selected with probability proportional to its weight and evaluation by elimination continues. The elimination by aspects model is thus seen to be a lexicographic rule in which decision-forming attributes are picked according to a probabilistic mechanism.

Wholistic Decision Rules

It is not possible to provide anywhere near a complete listing or discussion of the many possible wholistic decision rules. Three of these wholistic judgment processes occur perhaps more frequently than others: standard operating procedures, intuitive affect, and reasoning by analogy.

Standard Operating Procedures: Standard operating procedures may result from the application of holistic or heuristic procedures, or other wholistic judgment approaches. A standard operating procedure is essentially what the name implies, a set of experience-based guides to behavior which are typically used without resort to the underlying rationale which led to the procedure. Often standard operating procedures are formulated by one person or group and then implemented by another person or group. Sometimes they involve habit or folk custom, such as "drink white wine with fish."

Often user's guides and operating manuals are written in attempts to standardize operating procedures for performance. The greatest value of these procedures is as a checklist, reminder, or options, profile of attributes to look for, judgments to make or activities to select or perform. A fundamental often occurring difficulty is that an expert may be able to use a checklist or profile of options as a guide to performance based upon the ability of the expert to quickly recognize the features inherent in the situation. Lack of training and experience will often make it not possible for the novice to utilize this capacity for task need recognition. Klein and Weitzenfeld [202], [230] pose that the lack of training and experience inherent in the novice, the associated lack of ability to recognize contextual relations and analogous situations, and the inability of guides to be able to teach this ability are all fundamental impediments to the use of many standard operating procedure type guides to judgment and performance.

Intuitive Affect

A person who makes judgments based on intuitive affect typically takes in information by looking at the "whole" of a situation rather than by disaggregating the situation into its component parts and acquiring data on the parts. Valuation is typically based on an attempt to determine whether alternatives are pleasant or unpleasant, likeable or unlikeable, good or bad for individuals. It stressed the uniqueness of personalistic value judgments. Zajonc [411] presents a very useful discussion of affect or feeling as postcognitive activity.

Reasoning by Analogy

Many philosophers of science claim reasoning by analogy [55], [130], [360] is the basis of hypothesis generation. It is fundamentally different than deductive inference or inductive inference-based reasoning. In analogic reasoning we use analogies, prototypes, or other paradigms with which we are familiar to guide us in new tasks. These exemplars encourage recognition in a present situation in terms of experientially based knowledge.

Doubtlessly analogic reasoning, as well as reasoning by intuitive affect and standard operating procedures, are each heavily influenced by the contingency structure of the task at hand and the environment. These are the judgment processes used by many in reaching decisions. We will comment further in Sections V and VI upon wholistic judgment and its role [81], [82], [98], [116]–[119], [202], [203] in choicemaking.

In this section we have examined a number of decision rules. We have discussed holistic, heuristic, and wholistic rules. The holistic models or rules are generally substantive and not necessarily process models. They may be prescriptive or descriptive in intent and use. The heuristic and wholistic models are more process oriented than the heuristic models. In unaided situations people generally do not have the cognitive stamina to utilize the holistic rules or may not sense a need for them even if they could utilize them. A variety of contemporary research [273]–[275] has presented the strongest of evidence that choice of decision rules is very task dependent and actual choices may vary appreciably across different interpretations of the same decision situations. Preference reversals have even been noted with translation of gambles and target return, reference point, or aspiration level effects. Phenomena such as these have recently been studied [274] and shown to be potentially explainable by a descriptive model of risky choice due to Fishburn [123] and by prospect theory.

We note that people use different decision rules and models at different phases of a decision process as a function of a number of influencing variables, such as education, experience, motivation, familiarity with the environment, and above all, stress. Etzioni [103], [104] has proposed a mixed scanning model of decisionmaking that forms the basis for some current research in information systems for planning and decision support [75], [76], [245], [399]. There are a number of contemporary efforts and approaches that support the design of systemic aids that will be more responsive to decisionmaker requirements. Especially important in this regard are the efforts of Einhorn, Kleinmuntz, and Kleinmuntz [95]; Hogarth and Makridakis [161], Huber [168], Jungerman [181], Kleinmutz and Kleinmutz [204], Lad [208], Libby [213], Montgomery and Svenson [256], Payne [271]–[275], Rouse [299], Svenson [364], [365], Thorngate [372], Toda [373]–[375], Tversky and Sattagh [384], Tweney *et al.* [387], Vlek [392], [393]; Wallsten [395], [396]. Efforts which concern the integration of descriptive and prescriptive components of decisionmaking [142], [307], [322], [323]; efforts which concern determination of cognitive choice models in realistic settings [22], [88], [157], [158], [314], [325]; efforts which involve formulation and structuring of decision situations [1], [247], [248], [255], [265], [286], [287], [300], [301], [302], [353], [397]; and efforts which involve the cognitive effort involved in decisionmaking [180], [328], may offer much promise as well.

V. Contingency Task Structure Models

The designer of information systems for planning and decision support must be concerned both with normative models of decision and choice processes and with descriptive models of how people perform, and can perform, in given situations. Thus our discussions of information processing and decision or evaluation rule selection in the previous two sections take on particular meaning in that they comment on the wide variety of possible behaviors. We will be especially concerned, in this section, with describing cognitive processes as they are influenced by the contingency structural elements of task, environment and the human problem solver's experience with these. There have been a limited number of efforts to describe these such as those by Allais and Hagen [3], Beach and Mitchell [32], Borgida and Nisbett [42], Broadbent [49], Bunn [53], Carrol [56], Dreyfus and Dreyfus [82], Einhorn [96], [97], Einhorn and Hogarth [98], Harsanyi [147], Hauser [149], Howell and Fleishman [165], Huber [168], Janis and Mann [176], [177], Jungerman [182], Klein [202], [203], Kleinmutz and Kleinmutz [204], Kunruether and Schoemaker [207], MacKinnon and Wearing [233], Montgomery and Svenson [256], Payne [272], [275], Sage [308], Simon [338], [340], [341], [343], Soelberg [353], and Wallsten [395], [396]. This is an area in which additional research could pay major dividends in ultimately increasing the effectiveness of information systems in coping with the contingency task structure variables in planning and decision support.

The contingency task structure model we first describe is related to Piaget's theory of intellectual development [43], [126], [131], [205], [262], [362]. After a description of this model [308] we indicate implications for information system design and the relationship of this model to models that have been proposed by others.

Insights into the nature of cognitive development and insights into a conceptual model of cognitive activity is contained in the works of Piaget, the founder of "genetic epistemology." According to Piaget, there are four stages of intellectual development:

1) sensory motor
2) preoperational
3) concrete operational
4) formal operational.

The last two of these are of particular importance to our efforts here. In the writings of Piaget, intellectual development is seen as a function of four variables:

1) maturation
2) experience
3) education
4) self regulation—a process of mental struggle with discomforting information until identification of a satisfactory mental construction allows intellectual growth or learning.

In Piaget's model of intellectual development concrete operational thinkers can deal logically with empirical data, manipulate symbols, and organize facts towards the solution of well structured and personally familiar problems. Formal operational thinkers can cope in this fashion also. A major difference, however, is that those concrete thinkers who are not also capable of formal thought lack the capacity to reason hypothetically and to consider the effect of different variables or possibilities outside of personal experience. Concrete operational thinkers, for instance, will often have difficulty in responding "true" or "false" to the statement, "six is not equal to three plus four." As another example: "A card has a number on one side and a letter on the other; test the hypothesis that a card with a vowel on one side will have an even number on the other side." Concrete operational thinkers will have difficulty selecting cards for bottom side examination if the top sides of four cards are a, b, 2, 3. However, failure to pick the cards with "a" and 3 on top may not indicate inability as a formal operational thinker but, rather, failure to properly diagnose the task and determine the need for formal operational thought.

We wish to develop a model of higher order cognitive processing that describes the mature adult decisionmaker. Such a decisionmaker will typically be capable of both formal and concrete operational thought. As we will argue, selection of a formal or concrete cognitive process will depend upon the decisionmaker's diagnosis of need with respect to a particular task. That need will

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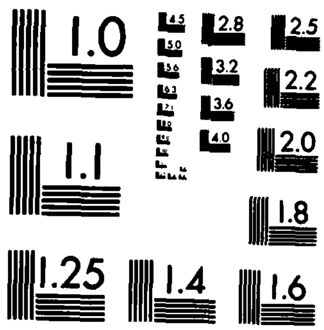
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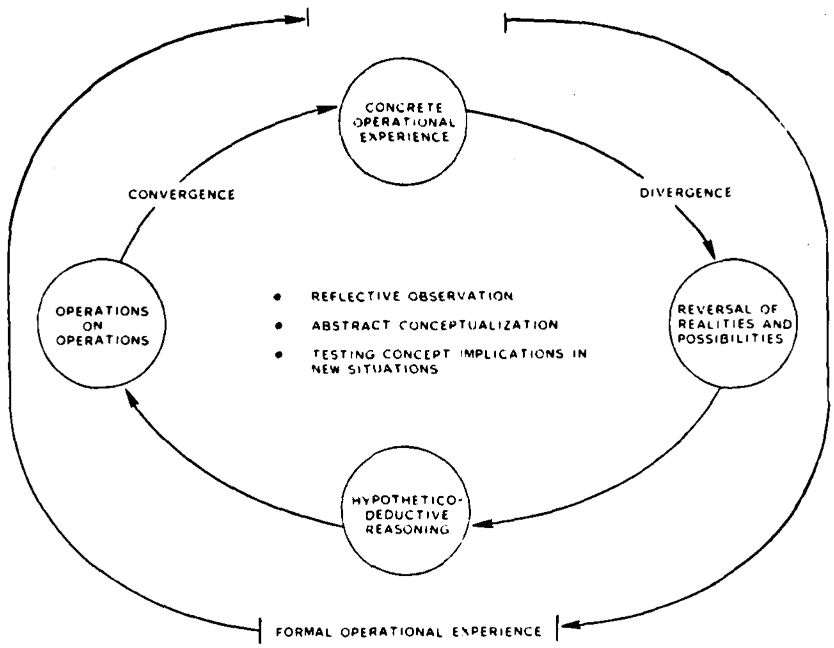

Fig. 6. Learning through formal operational experiences.

depend upon a decisionmaker's maturity, experience, and education with respect to a particular problem. Each of these influence cognitive strain or stress, a subject that will be discussed later in this section. Ordinarily, a decisionmaker will prefer a concrete operational thought process and will make use of a formal operational thought process only when concrete operational thought is perceived inappropriate. In general, a concrete operational thought process involves less stress and may well involve repetitive and previously learned behavioral patterns. Familiarity and experience, with the issue at hand or with issues perceived to be similar or analogous, play a vital role in concrete operational thought. In novel situations, which are unstructured and where new learning is required, formal operational thought is typically more appropriate than concrete operational thought.

We see, in the foregoing discussion, the dominant role of the contingency task structure in guiding problem solving efforts. In concrete operational thought, people use concepts which

1) are drawn directly from their personal experiences,
2) involve elementary classification and generalization concerning tangible and familiar objects,
3) involve direct cause and effect relationships, typically in simple two-variable situations;
4) can be taught or understood by analogy, algorithms, affect, standard operating policy, or recipe; and which
5) are "closed" in the sense of not demanding exploration of possibilities outside the known environment of the person and stated data.

In formal operational thought, people use concepts which may

1) be imagined, hypothetical, based on alternative scenarios, and/or which may be contrary to fact;
2) be "open ended" in the sense of requiring speculation about unstated possibilities,
3) require deductive reasoning using unverified and perhaps flawed hypotheses,
4) require definition by means of other concepts or abstractions that may have little or no obvious correlation with contemporary reality, and which may
5) require the identification and structuring of intermediate concepts not initially specified.

Formal operational thought involves three principal stages:

1) reversal of realities and possibilities
2) hypothetico-deductive reasoning
3) operations on operations

as shown in Fig. 6. These are accomplished through reflective observation, abstract conceptualization, and the testing of the resulting concept implications in new situations. It is in this way that the divergence produced by discomforting new experiences allows the learning of new developments and concepts to be "stored" in memory as part of one's concrete operational experiences.

A number of the cognitive style investigations discussed in Section II have concluded that "abstract" decisionmakers are more information oriented and would typically process much information in complex decision environments. "Concrete" decisionmakers, on the other hand, could be expected to reach an information overloaded state at lower levels of environmental complexity; hence they would tend to process less information than would the abstract decisionmaker. Some models of cognitive style are based on the assumption that "concrete" decisionmakers need more information to arrive at a decision than do "abstract" decisionmakers, suggesting that "concrete" decisionmakers do not give existing information its full worth and more are prone to fits of skepticism than "abstract" decisionmakers. At first glance, cognitive style models such as the one suggested here appear to run in parallel to Piaget's concepts of concrete operational and formal operational thought. But there are very important and very significant differences. These are explicable through the contingency task structure and concept of task, environment, and decisionmaker; a concept that appears, with some notable exceptions, missing in much of the existing cognitive style research cited in Section II.

The concrete operational thinker does not necessarily have limited abilities to process or integrate information, and the "formal" operational thinker is not necessarily capable of "abstract" thought in the specific contingency situation at hand. The formal thinker is neither necessarily able to process information which encompasses more complexity nor better able to cope with uncertainty and disjointedness in the decision environment than is one who uses concrete operational thought in a given decision situation. Our contingency task structural model for the mature, perhaps expert, adult decisionmaker is one in which the decisionmaker may use formal or concrete operational thought based primarily on diagnosis of the contingency structure of the decision situation, and the stress that is perceived to be associated with the decision situation. This election of a formal or concrete operational mode of thought may be appropriate or inappropriate.

Systemic process design must be responsive to the observation that there are two fundamentally different thought or cognition processes. These are often associated with different halves of the brain [38], [67], [120], [246–248], [254], [410]. One type of thought process is described by the adjectives verbal, logical, sequenced, thinking, and analytical; whereas the second is described as nonverbal, intuitive, wholistic, feeling, and heuristic. The verbal process is typically viewed as superior in engineering and natural science. But this viewpoint on the nature of thought appears wrong and should be discouraged as positively harmful. For the two processes are complementary and compatible. They are not competitive and incompatible in any meaningful way. One thought process may be deficient, in fact, if it is not supported by the other. The nonverbal supports the verbal by suggesting ideas, alternatives, etc. The verbal supports the nonverbal by expressing, structuring, analyzing, and validating the creative ideas that occur in the nonverbal process. An appropriate planning and decision support process must provide for verbal and nonverbal support. An appropriate planning and decision support process must be tolerant and supportive of a decisionmaker's cognitive (thought) processes. These will typically vary across individuals and within the same individual as a function of the environment, the individual's previous experience with the environment, and those associated factors which introduce varying amounts of stress. Thus a contingency task structural view of individuals and organizations in decision situations is needed as contrasted with a stereotypical view in which individuals are assumed to process fixed, static, and unchanging cognitive characteristics which are uninfluenced by environmental considerations.

This view will encourage us not only to consider the evolution of future events over time as inherently probabilistic, but also to consider value change over time. It is especially important that we consider values as containing noncommensurate, ambiguous, and uncertain components, rather than as being absolute, consistent, precise, and exogenous with respect to choice [236], [238].

Typically we learn from experience and adopt various decision rules in the form of cognitive heuristics based upon this experience. The strength of belief that we have in the usefulness of heuristics is often based on reinforcement through feedback. Einhorn [96], [97] has described several supporting illustrations of this. As we have indicated in Section IV. The use of various types of lexicographic semiorders often lead to intransitive choices which are often not recognized as intransitive. We often define issues by content rather than structure and convince ourselves to like what we get from a decision. As a consequence we find it hard to separate decisions from outcomes in retrospective evaluations of our judgments. Much of this is probably due to changing our attitudes and our perceptions in a very selective way without being aware of the change and to changing our forecasts, retrospectively, to correspond to events that have occurred without recognizing this change [117]–[119]. Thus we adopt a hindsight or "knew it all along" bias influenced by a variety of highly selective perceptions of reality.

We are most likely to have coherent value preferences and are able to develop and utilize appropriate evaluation heuristics in well-structured situations with which we are familiar. Learning by trial and error and development of judgment based on either reasoning by analogy, standard operating procedures, or organizational rules typically results from these "concrete operational" situations and experiences. Long standing use of these "rules" results in purely affective judgment and decision responses. In a familiar and simple world, a "concrete operational" world, these judgment guides and judgment heuristics might well be, and in fact often are, quite acceptable. In a changing and uncertain environment, an environment that is different from the one with which we are familiar, we may well err considerably by using these concrete operational world appropriate judgment heuristics. If we do not have a developed set of coherent values relative to a changing environment, we may respond affectively with the first alternative option that comes to mind. We may well adopt postdecision behavior such as to support and maintain a chosen response and employ cognitive biases and cognitive heuristics to justify this potentially ill-chosen response. This results in an affective response appropriate for a "concrete operational" situation when an analytical response, appropriate for a "formal operational" situation, is needed. In the Janis and Mann [177] terminology we adopt a coping pattern based on unconflicted adherence or unconflicted change whereas vigilance is called for.

A serious problem in practice is that we get used to very simple heuristics that are appropriate for "concrete operational" situations in a familiar world and we continue to use them in "formal operational" situations in an unfamiliar world in which they may be very inappropriate. A typical heuristic is incrementalism: "Go ahead and crowd one more beast into the commons." Such a heuristic may be appropriate in the familiar situation our forbears encountered in a new unexplored continent. But the "social traps" produced by such judgmental heuristics in a now crowded environment may be inappropriate. There are numerous contemporary issues to support this assertion.

Styles or modes of information processing, which includes information acquisition and information analysis, are of much importance in the design of information systems for interpretation of the impacts of proposed policy. Information acquisition refers to the perceptual process by which the mind organizes the verbal and visual stimuli that it encounters. As indicated in Section II, McKenney and Keen [242] discuss two modes of information acquisition, a preceptive mode and a receptive mode. We essentially utilize these modes for our model of information acquisition and analysis.

a) In preceptive acquisition and analysis, individuals bring existing experiential concepts and precepts to bear to filter data. They focus on structural relations between items and look for deviations from their expectations. They then use formal precepts as cues for acquisition, analysis, and associated structuring of data.
b) In receptive acquisition and analysis, individuals focus on contextual detail rather than presumed structural relationships. They infer structure and impacts from direct and detailed examination of information, generally including potentially discomfirming information, rather than from fitting it to their precepts.

There is nothing inherently good or bad in either mode of information acquisition, analysis, and associated structuring. The same individual may use different modes as a function of contingency task structure. Most people will have preferences for one mode or the other in a particular situation, depending upon their diagnosis of the contingency task structure and perceived needs to accomplish effective information interpretation and associated decisionmaking. It is our hypothesis that cognitive biases often arise, or are initiated, by use of a situationally incorrect mode of information acquisition and structuring. To use preceptive acquisition when receptive acquisition is more appropriate would appear to invite one or more of the many biases associated with selective perception. To use receptive acquisition when preceptive acquisition is appropriate would appear to introduce much stress associated with the low likelihood of being able to resolve an issue in the time available.

Information evaluation and interpretation refers to the decision rule portion of the problem solution. We advocate a model

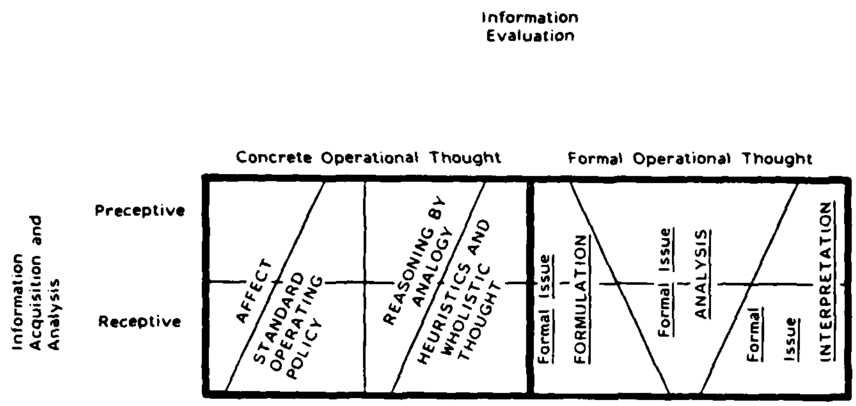

Fig. 7. Conceptualization of problem solving styles.

based on the use of the Piaget theory of concrete and formal operational thinking as a useful precept for information evaluation and interpretation. These thought process models may be summarized as follows.

a) In concrete operational thought, individuals approach problems either through intuitive affect, analogic reasoning, or through following a standard operating policy or organizational processes, or some related process.
b) In formal operational thought, individuals approach problems through structuring in terms of imbedding realities into possibility scenarios, hypothetico-deductive reasoning, and interpretation in terms of operations on operations.

Fig. 7 presents our conceptualization of information acquisition, analysis, and interpretation or problem solving styles. This figure does not illustrate, however, the fundamentally dynamic nature of this process model. Fig. 6 has presented some of the dynamic learning experiences which link the concrete operational and formal operational thought processes. Again we argue that no style is inherently appropriate or inappropriate. Appropriateness of a particular style, as has been mentioned before, is very much task, environment, and experience dependent. That most decisionmakers function as concrete operational thinkers is doubtlessly correct. A principal task of a well-designed information system is to assist in aiding the decisionmaker to detect the appropriate style for a given task, environment, and decisionmaker experience level. Another task is to enhance transfer of formal operational experiences to concrete operational experiences, such as through conceptualization and evolution of appropriate heuristics, wholistic thought, analogous reasoning guides, standard operating procedures, other forms of affective thought, and perhaps even precognitive responses. We posit that both types of information acquisition and analysis may occur with either concrete or formal thought although the appropriate balance of receptive and preceptive acquisition and analysis will vary from situation to situation.

Our discussions have indicated the strong environmental dependence of the formulation, analysis, and interpretation steps necessary for problem resolution. These steps are necessary steps in the resolution of any issue using systemic means, regardless of the "style" adopted for problem solution. Environments, organizations, and technologies are three dominant concerns of systems engineering in general and for the design of systems for planning and decision support in particular. It is the interaction of the environment with an organization and a technology that results in a management technology. Systems management is the term we use to denote the interaction of human judgment with methodological concerns [305]–[308]. Systems management denotes, therefore, concerns at the cognitive process level that involve the contingency task structure and its role in influencing the selection of performance objectives and decision rules for evaluation of options associated with issue resolution. There are many influences which act on the contingency task structure. Fig. 2 indicates, conceptually, how the contingency task structure, and the environment which influences it, acts to specify and direct problem solving efforts through selection of performance objectives and associated information processing and decision rule paradigms.

It is our belief that the dynamic cognitive style models of Figs. 6 and 7 can be used as guides to illustrate both those modes of information acquisition and information evaluation that should be used and that will be used on a given issue. We stress that the particular cognitive style most appropriate for a given issue will depend upon the decisionmakers familiarity with a given issue, the issue itself, and the environment into which the issue is imbedded. Thus a receptive or preceptive information acquisition style will be appropriate in a formal operational setting if the issue at hand is an unfamiliar and unstructured one. The appropriate balance between preceptive and receptive information acquisition will be dependent upon the type of issue and the experience or familiarity the decisionmaker has with possible information sources and their likely reliability. It will, of course, also be influenced by the "personal" style of the decisionmaker and the type, if any, of interaction with the systems analyst as well as upon other characteristics of the decision situation. We accept the view that systems methodologies, especially as implemented through use of human judgment to form a systemic process, are highly value dependent. Different systems methodologies allow one to define issues in different ways and are responsive in differing amounts to value concerns, such as equity. Some methodologies explicitly encourage for example, detection of the use of deficient heuristics and encourage correction. The "transparency" and communicability of a decision process, for example, is very much a function of the methodologies used in process aiding for the formulation of issues, the analysis of alternatives, and associated interpretation efforts. This value dependence of systems methodologies is, therefore, an important aspect of information system design and is related to performance objectives for the task at hand.

There have been a number of studies which focus upon the critical importance of task description and the decisionmaker's interaction with the task through the environment. Dawes [70], [71] stresses the critical interaction among the mind and the task, and integrated models of the mind and the task requirements. He discusses the "even numbered-vowel" experiment described earlier in this section as does Anderson [7]. Anderson indicates that the failure, and a majority of educated adults do fail, to correctly resolve this task is due to difficulties in applying the *modus tollens* concept of conditional deductive reasoning, a concept which requires thinking about what is not the case. Anderson also discusses a slight variation of this task, which is generally the same, and in which almost all subjects performed correctly. The task involved looking at four pictures of ordinary letter envelopes with the possibility of a stamp on them and picking the letters which should be turned over to test the hypothesis: if a letter is sealed, it has a 18¢ stamp on it. The critical difference between the two tasks is the fact that most people have experiences similar to the second task. It is relatively familiar compared to the first task, concerning which people do not have significant experience.

We should be rather cautious however in the apparently reasonable inference that we learn *correctly* from experience. A number of important studies by Brehmer [46], [47] have shown that by no means do people always improve their judgment and decisionmaking ability on the basis of increased experience. Biases, such as the tendency to use confirming evidence to the neglect of disconfirming evidence, are the key culprits. Brehmer [47] indicates how these biases can be understood in terms of available information. He concludes that truth is not manifest. It needs to be inferred in order to extract from experience information components that will truly lead to better judgments and decisions. The recent definitive discussion of judgment and choice processes by Einhorn and Hogarth [98] emphasizes the importance and the interdependence of attention, memory, cognitive representations, learning, conflict, and feedback. It provides much valuable perspective concerning the importance of these topics for judgment and decisionmaking.

Carroll [56] is much concerned also with understanding decision behavior, especially through the process tracing techniques that have been emphasized by Payne [272]–[275]. Carroll proposes that the decisionmaker might better be portrayed as possessing a rich store of knowledge organized around a variety of evoked schemas, those complex units of organized knowledge which guide the acquisition and use of case information, rather than exclusively considering the decisionmaker as exhaustively following the prescriptions of normative models. Many of the chapters in the recently edited works of Estes [100], Hamilton [137], Howell [167], Howell and Fleishman [165], Schweder [332], and Wallsten [396] discuss issues related to cognitive factors in judgment processes, including task descriptions for scripts, those stereotypical sequences of actions and event schemas which often are of much use in explaining judgment.

Studies of information support for U.S. Air Force command and communication systems accomplished by Klein [202], [203] express a number of concerns regarding artificial intelligence and information processing approaches for decision aiding. These reservations concern potential inabilities of humans to disaggregate situations into components and to analyze these discrete components. He indicates that the proficient performance of experts may well be based more on reasoning by analogy than by representations in terms of step by step descriptions capable of (discrete) digital computer processing. Further, expert proficient performers may not follow explicit conscious rules. Requiring them to do so may reduce performance quality, and they will be unable to accurately describe the rules that they do follow. Klein views expertise as arising from perceptual abilities including recognitional capacity in terms of analogous situations, sensitivity to environmental context in the sense of appreciation of the significance of subtle variations, and sensitivity to intentional context by viewing the relevance and importance of task components as a whole by anticipating what has to occur to achieve a goal rather than just what will occur at the next time instant or step. He presents a comparison-guided model of proficient decisionmaking. In this model [203]

1) a current decision situation is perceived in terms of objectives;
2) the decisionmakers' experience allows recognition of a comparison situation;
3) similarities and differences between the comparison situation and the current situation are noted;
4) this application suggests options, including evaluation of options and selection of a preferred option based on what worked in the comparison option; and
5) the way the objectives and the decision are perceived, possible further adjustments of options, generation of new options, and combination of options, follow from this.

Klein strongly encourages development of decision aids to support the recognitional capacity of the expert; aids that will assist the expert in recognizing new situations in terms of analogous comparison cases and in using these to define options or alternatives. The adjuvant would also keep track of options, assist in generation of new ones, and perform computations to assess the impacts of various options. It certainly appears that this is a needed and necessary role for information systems adjuvants for planning and decision support. But it must be remembered that not all users of such a system will be proficient and expert in all of the tasks they are to perform. We suggest the need also for provisions for formal operational thought type processes for those contingency task situations that have not been sufficiently cognized such that appropriate use of concrete operational thought necessarily leads to efficient and effective performance.

Dreyfus and Dreyfus [82] also argue that experienced and expert human decisionmakers solve new problems primarily by seeing similarities to previously experienced situations in them. They argue strongly that since similarity based processes actually used by experienced and expert humans lead to better performance than formal approaches practiced by beginners, decisionmaking based on proven expertise should not be replaced by formal models. They pose a model which contains five developmental stages through which a person passes in acquiring a skill such as to become a proficient expert. Their basic tenet is that people demand less and less on abstract principles and more and more on concrete experience as they become proficient. Their five stages and suggested instruction at each stage are as follows.

1) *Novice*—Decompose the task environment into context-free nonsituational features which the beginner can recognize without experience. Give the beginner *rules* for determining action and provide monitoring and feedback to improve rule following.

2) *Competence*—Encourage aspect recognition not by calling attention to recurrent sets of features, but rather by singling out perspicuous examples. Encourage recognition of dangerous aspects and knowledge of *guidelines* to correct these conditions. Equal importance weights are typically associated with aspects at this stage.

3) *Proficiency*—This comes with increased practice that exposes one to a variety of whole situations. Aspects appear more or less important depending upon relevance to goal achievement. Contextual identification is now possible and memorized principles, called *maxims*, are used to determine action.

4) *Expertise*—The repertoire of experienced situations is now vast, such that the occurrence of a specific situation triggers an *intuitively* appropriate action.

5) *Mastery*—The expert is *absorbed* and no longer needs to devote constant attention to performance. There is no need for self monitoring of performance and energy is devoted only to identifying the appropriate perspectives and appropriate alternative actions.

Dreyfus and Dreyfus associate the development of these five skill categories with successive transformation of four mental functions. Fig. 8 [82] indicates how these transformations occur with increased stages of proficiency. While developed primarily for training this model contains much of importance with respect to information system design to support planning and decisionmaking as well. A key issue in this table would appear to be the development of concrete situational experience which first occurs when a person is able to recognize aspects. There seems to exist some complimentarity between our model of the cognitive judgment and decision process and that of Dreyfus and Dreyfus. The concrete operational thought of experienced decisionmakers

	MENTAL FUNCTION			
SKILL LEVEL	RECOLLECTION OF SIMILAR FEATURES	RECOGNITION OF ASPECTS	DECISION PARADIGM	AWARENESS OF TASK
NOVICE	NON SITUATIONAL	DECOMPOSED		
COMPETENT			ANALYTICAL	MONITORING
PROFICIENT	SITUATIONAL			
EXPERT		WHOLISTIC	INTUITIVE	
MASTER				ABSORBED

Fig. 8. Dreyfus' judgment and decision process model.

would appear to be much the same as the thought of the expert and the master. Of course in all of these models, "expert" is a relative term, with the environment and the contingency task structure of a specific situation needed to determine whether a decisionmaker is familiar and experienced with it. Some differences in the models are doubtlessly present as well. Some of these depend upon precisely what is meant by "processing information." Our definition is rather broad and certainly not restricted to quantitative processing. Generally information processing, in our view, includes the formulation or acquisition, analysis, and interpretation of data of value for decisionmaking. This can be accomplished holistically, heuristically, or wholistically.

Very important concerns exist, in our view, with respect to possible cognitive bias and value incoherencies in the concrete operational decisionmaking of experts or masters. Questions related to the effects of changing environments upon the judgment and decision quality of masters and novices alike are very important in all of these models. For intuitive experience may not be a good guide for judgments and decisions in uncertain, unfamiliar, and/or rapidly changing environments. But quantitative or qualitative analysis-based efforts may well not be very good either due to changed decision situation and contingency task structural models. In our view it is possible to become a "master;" but unfortunately possible to become a master of the art of self-deception as well as of a specific task. The external behavior of the two "masters" may well be the same; situational, wholistic, intuitive, and absorbed. What was an appropriate style for one "master" may well be inappropriate for another.

Behavior in familiar but uncertain environments is of much interest. Studies of failure, situations in which experts and masters fail or misdiagnose their degree of expertise or mastery, could yield exceptionally useful results and would also serve to incorporate and integrate much of the experimental work involving biases, poor heuristics, and value coherences into a more real decision situation. We hypothesize that the dynamic models of decision styles presented in this section will be useful vehicles to these ends.

Judgment and decisionmaking efforts are often characterized by intense emotion, stress, and conflict; especially when there are significant consequences likely to follow from decisions. As the decisionmaker becomes aware of various risks and uncertainties that may be associated with a course of action, this stress becomes all the more acute. Janis and Mann [176], [177] have developed a conflict model of decisionmaking. Conflict here refers to "simultaneous and opposing tendencies within the individual to accept and reject a given course of action." Symptoms of conflicts may be hesitation, feelings of uncertainty, vacillation, and acute emotional stress with an unpleasant feeling of distress being, typically, the most prevalent of all characteristics associated with decisionmaking [49]. The major elements associated with the conflict model are the concept of vigilant information processing, the distinction between hot and cold cognitions, and several coping patterns associated with judgments.

Cold cognitions are those made in a calm detached environmental state. The changes in utility possible due to different decisions are small and easy to determine. Hot cognitions are those associated with vital issues and concerns, and are associated with a high level of stress. Whether a cognition is, or should be, hot or cold is dependent upon the task at hand and the experiential familiarity and expertness of the decisionmaker with respect to the task. The symptoms of stress include feelings of apprehensiveness, a desire to escape from the distressing choice dilemma, and self-blame for having allowed oneself to get into a predicament where one is forced to choose between unsatisfactory alternatives. Janis and Mann [177] state that "psychological stress" is used as a generic term to designate unpleasant emotional states evoked by threatening environmental events or stimuli. They define a "stressful" event as "any change in the environment that typically induces a high degree of unpleasant emotion, such as anxiety, guilt, or shame, and which affects normal patterns of information processing." Janis and Mann describe five functional relationships between psychological stress and decision conflict.

1) The degree of stress generated by decision conflict is a function of those objectives which the decisionmaker expects to remain unsatisfied after implementing a decision.
2) Often a person encounters new threats or opportunities that motivate consideration of a new course of action. The degree of decision stress is a function of the degree of commitment to adhere to the present course of action.
3) When decision conflict is severe because all identified alternatives pose serious risks, failure to identify a better decision than the least objectionable one will lead to defensive avoidance.
4) In severe decision conflict when the decisionmaker anticipates having insufficient time to identify an adequate alternative that will avoid serious losses, the level of stress remains extremely high. The likelihood that the dominant pattern of response will be hypervigilance, or panic, increases.
5) A moderate degree of stress, which results when there is sufficient time to identify acceptable alternatives, in response to a challenging situation, induces a vigilant effort to carefully scrutinize all identified alternative courses of action and to select a good decision.

Based upon these five functional relation propositions, Janis and Mann present five coping patterns which a decisionmaker would use as a function of the level of stress: unconflicted adherence or inertia, unconflicted change to a new course of action, defensive avoidance, hypervigilance or panic, and vigilance. These five coping patterns, in conjunction with the five functional relation propositions of psychological stress, were used

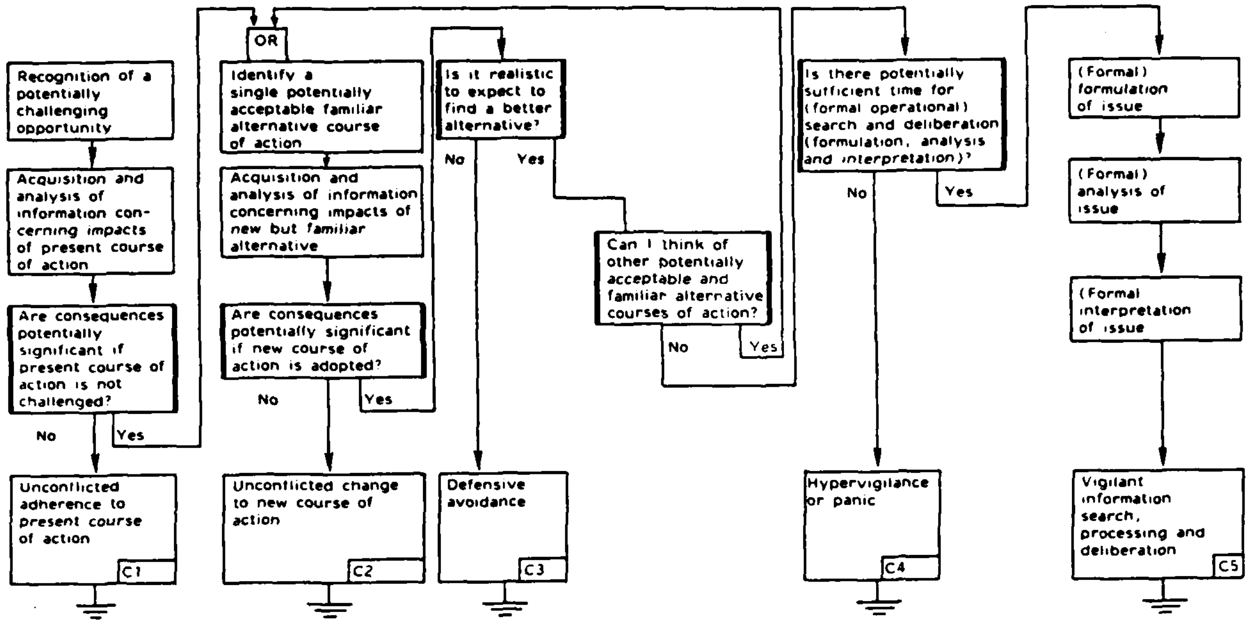

Fig. 9. Interpretation of the Janis and Mann [177] conflict model of decisionmaking.

by Janis and Mann to devise their conflict model of decisionmaking. This model postulates that each pattern of decision stress for coping is associated with a characteristic mode of information processing. It is this mode of information processing which governs the type and amount of information the decisionmaker will prefer. Fig. 9 presents an interpretation of this conflict model of decisionmaking in terms of the systems engineering contingency models discussed in this section. This model points to a number of markedly different tendencies which become dominant under particular conditions of stress. These include open-mindedness, indifference, active evasion of discomfirming information, failure to assimilate new information, and all of the other cognitive information processing biases identified in Section III. Table II summarizes information processing preferences and decision styles generated by this conflict mode. The table depicts the striking complexity entailed by the vigilant information processing pattern in comparison to the other coping patterns. The vigilance pattern is characterized by seven key steps which require somewhat prolonged deliberation. The other four coping patterns require that only a few key steps be addressed. Selection of a coping pattern may be made properly or unwisely, just as selection of a decision style may be proper or improper. The seven steps of vigilant information processing appear quite equivalent to the steps of systems engineering.

Janis and Mann [177] combine the hypotheses they present concerning the four stages of the decisionmaking (which we discuss in Section I), the five functional relation propositions of psychological stress, and the five stress coping patterns. Also they present a decision balance sheet, an adaptation of the moral algebra of Benjamin Franklin [177], on which to construct a profile of the identified options together with various cost and benefit attributes of possible decision outcomes. They have shown that decision regret reduction and increased adherence to the adopted decision results from use of this balance sheet. Strategies for challenging outworn decisions and improving decisions quality are also developed in this seminal work.

It would be of considerable interest to indicate the typical interactions between this model of Janis and Mann, which would be an expanded version of Fig. 1 and the other three contingency task structure models of decision style that we have discussed in this section. We believe each of these models to be appropriate and to portray different relevant features of task evaluation, information processing preference, and decision rule selection in terms of contingency elements associated with the environment and the decisionmaker's prior experiences.

There are, of course, other models of the planning and decisionmaking process. Within the field of artificial intelligence there exists a growing important body of literature concerning models of cognitive processes in planning and decisionmaking [150], [398]. The work of the Hayes-Roths [150] is definitive in this regard. It presents a model of the independent actions of a large number of cognitive specialists who make tentative decisions for incorporation into a tentative plan. Different specialists influence different portions of a plan and reguard decisions concerning the plan on a common data structure called a "blackboard." This blackboard allows specialists to retrieve prior plans and decisions and to combine earlier decisions with present decisions, thereby potentially generating new decisions. A process description of how knowledge generated during issue resolution is structured, stored, and used is available.

The basis assumptions underlying the planning model are: that decisions occur at several different levels of abstraction, that present decisions will constrain subsequent decisions, and that people can adopt alternative and appropriate strategies for planning. This important research suggests areas in which present and prospective planners need training by potentially allowing determination of differences in the information processing and judgment strategies of people as a function of task and environment.

VI. Decisionmaking Frameworks and Organizational Settings

We have already discussed such topics as decisionmaking rules, cognitive styles, information processing and contingency task structural models. Each of these represents a necessary component in the description of components of the decisionmaking process. While these components are all necessary for understanding of the decision process, they are not sufficient. In particular the nature of the decisionmaking process is very much influenced by the topics to be discussed in this section: various types of reasoning, the degree of approximation to various conceptual models of decisionmaking, the degree of centralization of the decision process, and the effects of these factors upon information acquisition. All of these factors are typically related and all are part of the contingency task structure.

Characterizations of Rationality

Diesing [77] is among several writers such as Steinbruner [359], who have defined several forms or types of rationality. Diesing defines five forms of rationality.

1) *Technical Rationality*—This results from efficient achievement of a single goal. A technically rational organization is one in which all of the activities of the organization are efficiently organized to achieve the goal of the organization. Technological progress requires an increase in the efficiency of the productive process and the existence of social conditions that make this increased efficiency possible. Diesing notes that a technological innovation that deals only with more efficient means to a single end will often have rather limited influence if the impacts of the technology and resulting attributes are morally and psychologically isolated from one another.

2) *Economic Rationality*—This results from maximum achievement of a plurality of goals. There are four characteristics needed for existence of an economy. Two of these relate to allocation: plurality of alternative ends, common means to the ends, scarcity of resources; and availability of a value system and associated measurements. Two characteristics relate to exchange: plurality of economic units, and a different prioritization of values among these units. Diesing claims that maximum goal achievement or economic rationality is possible if

a) the ends (goals) of the economic units are comparable and measurable on a single scale;
b) there are no limits on the assignability and use of the means;
c) economic units are integrated enough to engage in rational allocation and exchange; and
d) information about the supply–demand relationships for the various units is available and known to all.

Consequently economizing includes both evaluation and selection of various ends and means. Clearly it is desirable that conditions a)–d) hold but there exist many approaches to maximization under constraints that may be used to yield optimum resource allocation under constraints. Economic progress is equivalent to an increase in productivity per labor hour and, consequently, increased productivity can only result from economic and technical change. Economic progress will typically spread rapidly throughout a culture because it allows more and more ends to become both alternatives to each other and means to other ends as well. Generally the rational actor model we have discussed before is equivalent to Diesing's model of economic rationality.

3) *Social Rationality*—A social system is an organization of cultural roles such as expectations, obligations, and ideals. A social system is said to be integrated when the various associated activities fit well, support, confirm, enrich, and reinforce one another. Social integration is more than mechanical efficiency and consistency due to the mutual support, enrichment, confirmation, and reinforcement requirement. This integration makes action possible by

a) channeling emotional energy and preventing it from being diffused and lost;
b) eliminating conflict which could block action;
c) providing those supporting factors which strengthen action and which allow action to be carried through to completion; and
d) making actions more meaningful by allowing them to be related to past and future actions.

An integrated social system is a rational social system that enhances the meaningful and successful completion of actions. Successfully completed actions are not necessarily either efficient or effective as integration promotes survivability of the system and not necessarily the people within it. In extreme cases of inefficiency or ineffectiveness, people may leave the system and establish another one. Four characteristics of a *rational* social organization, as described by Diesing, are

a) internally consistent roles that can be carried out by the society without great strain;
b) harmonious roles that fit together without conflict among roles;
c) smoothly evolving roles such that there exists continuity and stability with no sharp impulsive changes in roles over time; and
d) roles compatible with the nonsocial (i.e., geographic, technoeconomic, temporal, and physiological) environment.

As it develops and becomes more integrated, a social system develops a value system that reinforces, through feedback, the structure of, and roles within, the social system. Well-integrated socially-rational systems typically resist change and avoid risk in our interpretation of Diesing. One might argue, of course, that a well-integrated social system *should* be adaptive to change and that failure to do so will subject it to a greater long term risk than if it were organically adaptive to change. This is, perhaps, the difference between a descriptive view and a normative view of a well-integrated social system.

4) *Legal Rationality*—A legally rational system is a system of rules which are complex, consistent, precise, and detailed enough to be capable of unambiguous application. Some of these rules may apply impartially to all people, while others may apply differently to different classes of people. A "legally rational" system is rational because, and if, it is effective in preventing disputes. It does this by providing a framework which defines and supports performance of economic and social rules. This framework also provides a procedure for settlement of those disputes which occur.

5) *Political Rationality*—This is the rationality of decisionmaking structures. A decisionmaking structure is composed of a set of discussion relationships and a set of beliefs and values that are imbedded into a set of recognized roles. These roles have been assigned to individuals such as to enable actions within the context of previous actions and commitments. Politically rational decision structures are based upon three guiding imperatives, according to Diesing:

a) maintenance of independence of the group despite all pressures for dependence;
b) actions to structure the political group such that pressures are balanced and moderate; and
c) preparation for future pressures which act to increase the stability and political rationality of the decision structure by providing unification and broadening of participation.

These forms of rationality are certainly related. Technical rationality is necessary for, and a part of, economic rationality. The primary characteristic which follows from rational economic behavior is a detachment or neutrality of intrinsically valueless commodities. These are useful only as means to ends such that scalar optimization may be used to select the commodity bundle of alternate means. Particularism and loyalty are the primary characteristics of social rationality such that obligations evolve from particular social relations with individuals and groups rather than general, universalist-detached relations which are applicable to all. Ascription, in which actions towards people evolve from particular relations rather than as a response to achievement, is another characteristic of social rationality. Thus we see that the characteristics of economic rationality *may* contrast sharply with those of social rationality. But this, we believe, is not necessarily the case. For, as Diesing indicates, neither form of rationality can exist without some form of the other. Economic rationality theories are based on the assumption that social integration is a reality; such that there exist communication and valuation capabilities and no goal conflicts or factionalism. In a similar way social rationality assumes that societies' economic resource allocation problems are solved.

Social and political rationality are related in the sense that both are primarily concerned with internal structural concerns involving process and procedure; that is, the structure of interpersonal relations, or the accumulation of power, or the direction of pressure. Economic and legal rationality are primarily concerned with the substantive behavior as contrasted with procedural and internal structural concerns. We have argued strongly in previous sections that substantive and procedural rationality [206], [336] are each necessary considerations in information system design.

Decision Frameworks

We have presented a detailed synopsis of the perceptive work of Diesing [77] concerned with five different forms of rationality. Additional forms of rationality [50], perhaps based upon the ten interacting societal sectors noted [304], [307] could doubtlessly be developed. It would be of interest to determine the extent to which these additional forms of rationality would be subsets of and independent of the five forms of Diesing.

March [236] discusses bounded rationality, limited rationality, contextual rationality, game rationality, and process rationality. A study of relations of these forms of rationality both to the rationality forms of Diesing and decision frameworks, which we will now discuss, could lead to useful insights and more relevant systemic process designs.

The organizational science literature contains much discussion relative to the development of conceptual models for decisionmaking based upon various rationality conceptualizations. Among these are the (economic) rational actor model, the satisficing or bounded rationality model, the bureaucratic politics, incremental, or "muddling through" model, the organizational processes model, and the garbage can model. These are related to the five types of rationality described by Diesing in relatively obvious ways that follow directly from a description of these decision frameworks.

1) *The Rational Actor Model*: The decisionmaker becomes aware of a problem, studies it, carefully weighs alternative means to a solution and makes a choice or decision based on an objective set of values. This is comparable to technical and economic rationality as described by Diesing. At first glance the rational actor model appears to contain much of value and to be especially well matched to the detached neutrality, calculative orientation, and avoidance of favoritism associated with the achievement-oriented entrepreneurial western society. In rational planning or decisionmaking

a) the decisionmaker is confronted with an issue that can be meaningfully isolated from other issues;
b) objectives are identified, structured, and weighted according to their importance in achieving need satisfaction on various aspects;
c) possible activities to resolve needs are identified;
d) the impacts of action alternatives are determined;
e) the utility of each alternative is evaluated in terms of its impacts upon needs; and
f) the utilities of all alternatives are compared and the policy with the highest utility is selected for action implementation.

These are essentially equivalent to the vigilant information processing steps of Janis and Mann [177].

Unfortunately, there are several substantive requirements for successful *complete* rational decisionmaking that will not generally be met in practice. These include

a) comprehensive identification of *all* needs, constraints, and alterables relevant to planning and decisionmaking is, of course, not possible;
b) determination and clarification of *all* relevant objectives is, of course, not possible;
c) determination and minimization of costs and maximization of effectiveness will not necessarily lead to the "best" results because of a) and b);
d) detached neutrality and a calculative orientation rather than arbitrariness, conflict, and coercion are not always possible;
e) a unified process that will cope with interdependent decisions will often be very complex;
f) sufficient time to use the method will often not be available;
g) sufficient information to enable use of the method will often be difficult and expensive to obtain; and
h) sufficient cognitive capacity to use the method will often not exist.

It has long been recognized by systems engineers and management scientists that the attempt to use a normatively optimum process will result in less than optimum results because of these modeling inaccuracies, cognitive limitations, and solution time constraints. Thus the presence of the realities of a)–h) will, because of a combination of resource and intellectual constraints, lead to selection of an alternative that is best only within constraints posed by the model actually used. We may also observe that an economically rational decision would only be appropriate when the decision situation structural model is such that an economically rational process is possible and desirable, and that the intellectual and resource conditions extant make substantive use of the rational actor model feasible.

Simon [337], [339], [340], [343] was perhaps the first to observe that unaided decisionmakers may not be able to make complete substantive, that is "as if," use of the model possible. The concepts of bounded rationality and satisficing represent much more realistic substantive models of actual decision rules and practices. We have described a variety of satisficing heuristic rules in Section IV. Unless very carefully developed and applied, these rules may result in very inferior decisions; decisions which are reinforced through feedback and repetition such as to result in experiences that are, by no means, the best teacher.

Of possibly even greater importance to information system design is the fact that completely economically rational processes may be neither desirable nor possible. Social, political, or legal rationality concerns may well prevail. And one of the other decision frameworks we describe here may well be more appropriate if these concerns are dominant over economic rationality concerns.

2) *The Satisficing or Bounded Rationality Model*: The decisionmaker looks for a course of action that is basically good enough to meet a minimum set of requirements. The goal is to "not shake the system" or "play it safe" by making decisions primarily on the basis of short-term acceptability rather than seeking a long-term optimum.

Simon introduced the concept of satisficing or bounded rationality as an effort to

> replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capabilities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist

He suggested that decisionmakers compensate for their limited abilities by constructing a simplified representation of the problem and then behaving rationally within the constraints imposed by this model. The need for this rests in the fact that many decisionmakers satisfice by finding either optimum solutions in a simplified world or satisfactory solutions in a more realistic world. As Simon says, "neither approach dominates the other [341]".

Satisficing is actually searching for a "good enough" choice. Simon suggested that the threshold for satisfaction, or aspiration level, may change according to the ease or difficulty of search. If many alternatives can be found the conclusion is reached that the aspiration level is too low and needs to be increased. The converse is true if no satisfactory alternatives can be found. This may lead to a unique solution through iteration.

The principle of bounded rationality and the resulting satisficing model suggests that simple heuristics may well be adequate for complex problem solving situations. Satisficing strategies may be excellent for repetitive problems. They may also lead to premature choices that result in unforeseen disastrous consequences; consequences which could have been foreseen by more careful analysis. The heuristic decision rules described in Section IV are all versions of satisficing strategies. A recent paper by Thorngate [372] provides useful descriptions of ways in which heuristic decision rules may be used and abused. Development of efficient and effective decision heuristics is a contemporary need for the analysis of decision behavior [56], [59], [60], the modeling of organizational and individual decisions [292], [365] as well as for the design of normative systems to aid decisionmaking [316]. We believe that to be effective as well as efficient, heuristics will have to be developed in a very cautious way with due considerations for the many implications of the contingency task structure of a decision situation [326].

3) *The Bureaucratic Politics, Incrementalism, or "Muddling Through" Model:* After problems arise which require a change of policy, policymakers consider only a very narrow range of alternatives differing to a small degree from the existing policy. One alternative is selected and tried with unforseen consequences left to be discovered and treated by subsequent incremental policies. This is the incremental view.

In 1959 Lindblom postulated the approach called incrementalism, or muddling through [218]–[221], to cope with perceived limitations in the economically rational approach. Marginal values of change only are considered—and these for only a few dimensions of value whereas the rational approach calls for exhaustive analysis of each identified alternative along all identified dimensions of value. A number of authors have shown incrementalism to be the typical, common, and currently practiced process of groups in pluralistic societies. Coalitions of special interest groups make cumulative decisions and arrive at a workable compromise through a give and take process that Lindblom calls "partisan mutual adjustment." He indicates that ideological and other value differences do not influence marginal decisions as much as major changes and that, in fact, considering marginal values subject to practical constraints will lead to agreement on marginal programs. Further, incrementalism can result in agreement on decisions and plans even by those who are in fundamental disagreement on values. However incrementalism appears based on keeping the masses marginally content and thus may not be able to do much to help the greatly underprivileged and unrepresented. It is, of course, a combination of Diesing's social and political rationality. Boulding has compared incrementalism to "staggering through history like a drunk putting one disjointed incremented foot after another." Yet there have been a number of studies, such as Allison's study of the Cuban missile crisis [4], Steinbruner's case studies [359], and others [44], [108], [135], [400] which indicate this to be an often used approach in practice.

It is important to note [218] that Lindblom rejects (economic) comprehensive rationality even as a normative model and indicates that systems analysis will often lead to ill-considered, often accidental incompleteness. He indicates the following inevitable limitations to analysis.

a) It is fallible, never rises to infallibility, and can be poorly informed, superficial, biased, or mendacious.
b) It cannot wholly resolve conflicts of value and interests.
c) Sustained analysis may be too slow and too costly compared with realistic needs.
d) Issue formulation questions call for acts of choice or will and suggests that analysis must allow room for politics.

A perceived more practical model process for decisionmaki[ng] than the rational actor model is, therefore, called for. The mod[el] is descriptive and is an extreme version of the bounded rationality model. Alternative models have been proposed [317].

The main features of the model proposed by Lindblom are th[e] following.

1) Ends and means are viewed as not distinct. Consequentl[y] means-ends analysis is viewed as often inappropriate.
2) Identification of values and goals is not distinct from the analysis of alternative actions. Rather, the two processes are confounded.
3) The test for a good policy is, typically, that various decisionmakers, or analysts, agree on a policy as appropriate without necessarily agreeing that it is the most appropriate means to an end.
4) Analysis is drastically limited, important policy options are neglected, and important outcomes are not considered.
5) By proceeding incrementally and comparing the results of each new policy with the old, decisionmakers reduce or eliminate reliance on theory.
6) There is a greater preoccupation with ills to be remedied rather than positive goals to be sought.

In a very readable recent work concerning "muddling through [221]," Lindblom classified incremental analysis at three levels: simple, disjointed, and strategic. Incremental analysis is, as we have indicated, a good description of political decisionmaking and is sometimes referred to as the political process model.

4) *The Organizational Processes Model*: Plans and decisions are the result of interpretation of standard operating procedures. Improvements are obtained by careful identification of existing standard operating procedures and associated organizational structures and determination of improvements in these.

The organizational process model, originally due to Cyert and March [68], functions by relying on standard operating procedures which constitute the memory or intelligence bank of the organization. Only if the standard operating procedures fail will the organization attempt to develop new standard procedures.

The organizational processes model may be viewed as an extension of the concept of bounded rationality to choicemaking in organizations. It is clearly an application of reasoning and rationality as discovery and application of rules to cases. It may be viewed as a hybrid of economic and legal rationality. It typically involves concrete operational thought as we have indicated in Section V. The main concepts of the behavioral theory of the firm, which is suggested as a descriptive model of actual choicemaking in organizations are as follows.

1) Quasiresolution of conflict: Major problems are disaggregated and each subproblem is attacked locally by a department. An acceptable conflict resolution between the efforts of different departments is reached through sequential attention to departmental goals.
2) Uncertainty avoidance is achieved
   a) by reacting to external feedback,
   b) by emphasizing short-term choices, and
   c) by advocating negotiated futures.
3) Problem search in which
   a) search is stimulated by encountering issues,
   b) a form of "satisficing" is used as a decision rule, and
   c) search in the neighborhood of the status quo only is attempted and only incremental solutions are considered.
4) Organization learning: Organizations adapt on the basis of experience.

The organizational process model may be viewed as suggesting that decisions at time t may be forecasted with almost complete certainty from knowledge of decisions at time $t - T$ where T is the planning or forecasting period. Standard operating procedures or "programs," and education motivation and experience or "programming" of management are the critical determinants of behavior for the organizational process model.

5) *The Garbage Can Model:* This relatively new model [63] views organizational decisionmaking as resulting from four variables: problems, solutions, choice opportunities, and people. Decisions result from the interaction of solutions looking for problems, problems looking for solutions, decision opportunities, and participants in the problem solving process. The model allows for these variables being selected more or less at random from a garbage can. Doubtlessly this is a realistic descriptive model. It is especially able to cope with ambiguity of intention, understanding, history, and organization. Further, it provides much support for a feedback model of organizational choice in which preferences and cognitions of individuals affect their behavior, behavior of individuals affect organizational choices, organizational choices affect environmental activities and responses, and environmental activities and responses affect the preferences and cognition of individuals [237]. An extensive and definitive discussion of ambiguity and choice in organization, with emphasis upon the garbage can model, is contained in [237].

All five of the models or frameworks for decisionmaking have both desirable and undesirable characteristics. Conclusions may be drawn from these models and the fact that any of them may be relevant in specific circumstances. If we accept the facts that

1) decisionmakers use a variety of methods to select among alternatives for action implementation;
2) these methods are frequently suboptimal; and
3) most decisionmakers desire to enhance their decisionmaking efficiency and effectiveness;

then we must conclude that there is much motivation and need for research and ultimate design and development of planning and decision support systems. But these five models make it very clear that improved planning and decisionmaking efficiency and effectiveness and aids to this end can only be accomplished if we understand human decisionmaking as it is as well as how it might be and allow for incorporation of this understanding in systemic process adjuvants. One of the requirements imposed on these adjuvants will be relevance to the individual and group decisionmaking structure [181], [237], [286], [303], [401]. Another requirement is relevance to the information requirements of the decisionmaker. We discuss both of these in this section of our survey and interpretation.

There have been many studies of group decisionmaking. These include the fundamental theoretical studies of Arrow [17] and others which show that, under a very mild set of realistic axioms, there is no assuredly successful and meaningful way in which ordinal preference functions of individuals may be combined into a preference function for society [17], [196], [279], [302]. Conflicting values [378] are the major culprit preventing this combination. This has a number of implications which suggest much caution in using ordinal preference voting systems and any systemic approach based only on ordinal, possibly wholistic, or heuristic preferences among alternatives. Among other possible debilitating occurrences are agenda dependent results which can, of course, be due to other effects [280]. There have been a number of studies of group decisions and social and organizational interactions such as those by Bacharagh [19], Davis [69], Ebert and Mitchell [89], Einhorn, Hogarth, and Klempner [92], Holloman and Hendrick [162], Janis and Mann [176], [177], Leavitt [211], Mintzberg [248], Penrod and Hastie [276], Schein [312], Shumway, *et al.* [329], Simon [341], Vinokur and Burnstein [390], [391] and in the edited work of Hooker, Leach, and McClennen [163]. Several systemic methods have been proposed for forming and aggregrating group opinions as described in the works of Hogarth [155], Huber [169], Hylland and Zeckhauser [173], Rohrbaugh [295], Van de Ven and Delbecq [388]. An excellent survey of voting methods and associated paradoxes is presented by Fishburn [122] and by Plott [279].

Very definitive studies of the interpersonal comparison of utilities have been conducted by Harsanyi [145]–[147]. He argues convincingly that we make interpersonal utility comparisons all the time whenever we make any allocation of resources to those to whom we feel the allocation will do the most good. The prescription against such comparisons is one of two key restrictions which lead to the Arrow impossibility theorem. By using cardinal utilities such that it becomes possible to determine preferences among utility differences (i.e. whether $u(a) - u(b) > u(b) - u(c)$), and interpersonal comparison of utilities, Harsanyi shows that Arrow's impossibility theorem becomes a possibility theorem. This is a major point in that it is generally not possible for a group to express meaningful transitive ordinal preferences for three or more alternatives even though all individuals in the group have individually meaningful transitive ordinal preferences.

Harsanyi is concerned primarily with organizational design [147]: how to design social decisionmaking units so as to maximize attainment of social objectives or value criteria. He shows that rational morality is based on maximization of the average (cardinal) utility level for all individuals in society. The utilitarian criterion is applied first to moral rules and then these moral rules are used to direct individual choices. Thus each utilitarian agent chooses a strategy to maximize social utility under the assumption that all other agents will follow the same strategy. Harsanyi recognizes a potential difficulty [147] with this particular utilitarian theory of morality in that it is open to dangerous political abuses as well as the numerous problems associated with information acquisition and analysis in a large centralized system. He posits a difference between moral rationality and game-theoretic rationality. He argues the unavoidable use of interpersonal cardinal utility comparisons in moral rationality and the inadmissibility of such comparisons in game theory. Much of Harsanyi's efforts concern game situations [146] in which outcomes depend on mutual interactions between morally rational individuals, each attempting to better their own interests. We will not attempt to explore here the very interesting subjects of bargaining, conflict, resolution, and negotiation and the use of systems for planning and decision support to these ends [21], [45], [269].

Harsanyi's concept of utilitarianism has occasionally been criticized for making inadequate provision for equity, or equivalently for social group equality. John Rawls, a philosopher, has presented a theory of justice [291] which involves a difference principle in which decisions are made under uncertainty rather than under risk. This difference principle advocates selection of the alternative choice which is the best for the worst-off member of society and is, therefore, the direct social analog of the maximum principle for the problem of individual decisions under certainty. Rawls uses a "veil of ignorance" concept in which individuals must determine equitable distribution of societies' resources before they know their position in society. His argument is essentially that people will select a resource allocation rule that maximizes the utility of the worst-off member of society. Discussions of some of the potential difficulties associated with Rawls' "social contract" justice theory are presented by Ellsworth and Gauthier [163].

Other useful interpretations of cardinal utility and interpersonal utility comparisons have been made by Keeney and Kirkwood [194] and Keeney [195]. Their axioms allow development of a multiplicative group utility function in contrast to the additive utility function of Harsanyi. It is possible to more directly deal with equity considerations in a multiplicative group utility model than in an additive model. Papers by Bodily, Brock, and Keeney [201] contain insightful discussions concerning group and individual utilities of a multiattribute nature. Ulvila and Snider [201] illustrate use of multiattribute utility models in negotiations.

We are particularly interested, here, in describing decisionmaking efforts in hierarchical organizations [241]. This leads naturally to a study of information processing in organizations

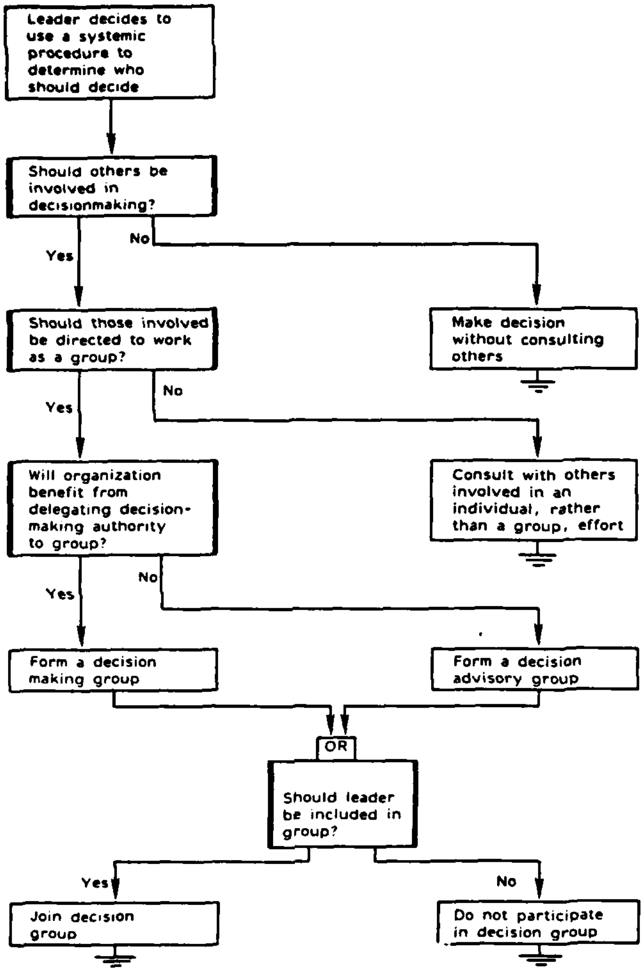

Fig. 10. Delta chart on how to decide who should decide (after [169]).

and a description of how decisionmakers may determine information needs. While there have been a number of studies of group decisionmaking roles and organizational behavior [357], [370], our efforts will be based primarily on those of Vroom and Yetton [394] and Huber [169].

Huber, Vroom, and Yetton have indicated a number of potential advantages and disadvantages to group participation in decisionmaking. Since a group has more information and knowledge potentially available to it than any individual in the group, it *should* be capable of making a better decision than an individual. Group decisions are often more easily implemented than individual decisions since participation will generally increase decision acceptance as well as understanding of the decision. Also group participation increased the skills and information that members may need in making future organizational decisions. On the other hand there are disadvantages to groups. They consume more time in decisionmaking than individuals. The decisions may not fully support higher organizational goals. Group participation may lead to unrealistic anticipations of involvement in future decisions and resentment towards subsequent individual decisions in which they have not participated. Finally, there is no guarantee that the group will converge on a decision alternative.

Huber asks four primary questions the answers to which determine guidelines for selection of a particular form of group decisionmaking. The delta chart of Fig. 10 indicates how the responses to these questions determines an appropriate form of group decisionmaking. There are a number of subsidiary questions concerned with each of the primary questions. For example, we may determine whether or not to involve others by posing questions involving decision quality, understanding, and acceptance, personnel development, and relationships, and time required.

Vroom and Yetton have been much concerned with leadership and decisionmaking [394]. Their primary concern is with effective decision behaviors. They develop a number of clearly articulated normative models of leadership style for individual and group decisions. These should be of use to those attempting to structure normative or prescriptive models of the leadership style portion of decision situations which are capable of operational implementation. We will not illustrate these here since they essentially involve generalizations of Fig. 10. It is the apparent goal of Vroom and Yetton to move beyond generalities such as the leadership style theory X-theory Y [211], [394]. They desire to come to grips with and explicitly use leadership behavior and situational variables to enhance organizational effectiveness.

Much of our discussion in this section has concerned the evaluation component of various decisionmaking frameworks and organizational settings. Effective planning and decision support is based not only upon evaluation, but upon information acquisition and processing as well. We have emphasized this in our discussions thus far in terms of individual information processing behavior; but have not yet given explicit consideration to information processing behavior in organizations.

Keen [193] acknowledges four causes of inertia relative to organizational information systems. He indicates that information is only a small component, human information processing is experiential and relies on simplification, organizational change is incremental and evolutionary with large changes being avoided, and that data is a political resource affecting particular groups as well as an intellectual commodity. Each of these suggests the importance of a knowledge of the way in which information is processed by organizations.

Of particular interest among studies concerning information processing in organizations are the works of Baron [28], Ebert and Mitchell [89], Fick and Sprague [110], Gerwin and Tuggle [129], Howell and Fleishman [165], Huber [169]–[171], Keen [193], Libby and Lewis [215], Lucas [225]–[228], O'Reilly [268], Shumway *et al.* [329], Simon [342], Starbuck and Nystrom [357], Taggart and Tharp [367], Tushman and Nadler [379], Tuggle and Gerwin [380], Wright [406], and Zedeck [409].

The purpose of systems for planning and decision support is to provide timely, relevant, and accurate information to system users such as to enhance human judgment, and decisionmaking efficiency and effectiveness concerning resource allocations that affect issues under consideration. To enhance efficiency and effectiveness available resources must be allocated and coordinated in space, across a hierarchy of decisionmakers; and in time, as new information arrives and the environmental situation extant changes. Associated information acquisition, analysis, and evaluation and interpretation must, as a consequence, often be distributed both in space and in time. This must be accomplished selectively in space and time since different decisionmakers have different information needs. In addition, it will be physically impossible and often behaviorally undesirable to supply all relevant information to each decisionmaker in the hope that it will be effectively cognized and utilized. Further, differences in education, motivation, experiences with the environmental situation extant, and stress will influence cognitive information processing style. Consequently a central task in the design of effective information systems is that of selection and choice of appropriate information system architecture to enhance selective information processing in order to provide each user of the system with the most appropriate information at the most appropriate time. Thus questions of information selection, information aggregation in space and in time, and the contingency task structure become of major importance.

It is desirable that an appropriately designed system, and the associated process, be capable of the following.

1) Assisting in the evaluation of alternative plans and courses of action that involve formal operational thought processes.
2) Assisting in the transfer of formal operational situations to concrete operational situations.
3) Assisting in evaluation of alternative plans and courses of action that involve concrete operational thought processes.
4) Assisting in the avoidance of information processing biases and poor judgmental heuristics.
5) Assisting in the proper aggregation of information cues from multiple distributed sources.
6) Assisting in the use of a variety of judgmental heuristics appropriate for given operational environments as natural extensions of a decisionmaker's normal cognitive style.
7) Assisting, to the extent possible, in the determination of whether a formal or concrete style of cognition is most appropriate in a given situation.
8) Assisting decisionmakers who need to use formal operational thought and those whose expertise allows appropriate and effective use of concrete operational thought to function together in a symbiotic and mutually supportive way.

Clearly there is a space–time and an organizational dependence associated with these desired capabilities. Among the many concerns that dictate needs and requirements for automated support systems is the fact that decisionmakers must typically make more judgments and associated decisions in a given period of time than they can comfortably make. This creates a stressful situation which can lead, as has been noted, to the use of poor information processing and judgmental heuristics, especially since judgments and decisions are typically based on forecasts of the future and, therefore, inherently involve uncertainty.

There are formidable needs and issues to be resolved that are associated with the design of information processing and judgment aiding support systems. These relate to questions concerning appropriate functions for the decisionmaker and staff to perform. They concern the type of information which should be available and how this information should be acquired, analyzed, stored, aggregated, and presented such that it can be used most effectively in a variety of potential operational environments. They concern design of information systems with strong space-time-environmental dependencies. They concern design of information systems that can effectively "train" people to adapt and use appropriate concrete operational heuristics in those environments in which inexperience dictates initial use of formal operational thought. They concern design and use of information systems that support environmentally experienced decisionmakers in the use of a variety of effective concrete operational heuristics. And because of their use by multiple decisionmakers, these tasks must be accomplished in a parallel architectural fashion.

Huber [170]–[171] and Tushman and Nadler [379] have developed a number of propositions, based on their own research and upon the research of others, reflecting various aspects of information processing in organizations. There are a number of fundamental propositions developed by Tushman and Nadler which relate to the development of a model of an organization as an information processing system. These fundamental propositions include [379] the following.

FP1: Tasks of organizations and their subunits vary in uncertainty and risk variables.
FP2: As uncertainty and risk increase so also does the need for information and increased information processing capability.
FP3: Capacities and capabilities in information processing will vary as a function of organization structure.
FP4: Organizational effectiveness increases as the match between information processing requirements and information processing capacity increases.
FP5: Effectiveness of organizational units will depend upon their ability to adapt their internal structures over time to meet the changed information processing requirements that will be associated with environmental changes.

In an effort to enhance efficiency, organizational information processing typically requires selective routing of messages and summarization of messages. Huber [171] identifies six variables associated with the routing of messages. Six propositions relative to message routing are identified and associated with these variables. Three propositions are associated with delay in messages, eight with organizational message modification, and four with message summarization. Table II presents an interpretation of the impacts of the variables associated with organizational information processing and the probabilities of routing, delay, modification, and summarization of messages. It is possible to infer a few impacts not discussed in this noteworthy work of Huber. Most of these simply relate to the observation that if something happens to decrease the probability of sending a message unmodified then the probability of the message being delayed and/or modified is increased.

Identification of other variables which influence information processing by organizations would represent a desirable activity. To determine how these information processing variables are influenced by the information processing biases of individuals discussed in Section III would seem especially desirable in terms of the likely usefulness of the results and the need for an expanded theory of group information processing biases. There

TABLE II
CROSS IMPACT MATRIX BETWEEN VARIABLES AFFECTING ORGANIZATIONAL INFORMATION AND ASSOCIATED ACTIVITIES

	A) PROBABILITY OF MESSAGE ROUTING (IN UNDISTORTED, UNSUMMARIZED FORM)	B) PROBABILITY OF MESSAGE DELAY	C) PROBABILITY OR EXTENT OF MESSAGE MODIFICATION	D) PROBABILITY OR EXTENT OF MESSAGE SUMMARIZATION
1. INCREASES IN ECONOMIC AND OTHER COSTS OF A TRANSMISSION SENDING	−	⊖	−	−
2. INCREASES IN WORKLOAD OF SENDING UNIT	−	+	+	−
3. PERCEIVED RELEVANCE OF MESSAGE TO SENDING UNIT	+	⊖	⊖	+
4. DECREASES IN PERCEIVED GOAL ATTAINMENT, STATUS OR POWER OF THE SENDING UNIT RESULTING FROM ROUTING	−	⊕	⊕	
5. INCREASES IN PERCEIVED GOAL ATTAINMENT, STATUS OR POWER RESULTING FROM MODIFICATION	⊖		+	
6. PERCEIVED GOAL ATTAINMENT, STATUS, OR POWER OF THE SENDING UNIT	+	⊖	⊖	
7. FREQUENCY OF PAST COMMUNICATION OF SIMILAR MESSAGES	+	⊖	⊖	
8. PERCEIVED TIMELESS OF MESSAGE FOR THE RECEIVING UNIT		−		
9. NUMBER OF ACTIVE COMMUNICATION LINKS IN THE CHAIN BETWEEN RECEIVER AND SENDER	⊖	+	+	+
10. DECREASE IN STRESS OF THE RECEIVER PERCEIVED BY THE SENDER TO RESULT FROM MODIFICATION	⊖		+	
11. AMOUNT OF DISCRETION ALLOWED ALTERING OR CHOOSING THE MESSAGE FORMAT	⊖		+	
12. INCREASED INDIFFERENCE BETWEEN ACTUAL MESSAGE CONTENT AND TRANSMITTER'S DESIRED CONTENT	⊖	⊕	+	
13. INCREASED IN PERCEIVED AMBIGUITY OF DATA ON WHICH MESSAGE IS BASED	⊖	⊕	+	
14. INCREASES IN SAVINGS DUE TO SUMMARIZATION				+

+ = ENHANCING IMPACT SEEN BY [171]
− = INHIBITING IMPACT SEEN BY [171]
⊕ = INFERRED ENHANCING IMPACT
⊖ = INFERRED INHIBITING IMPACT

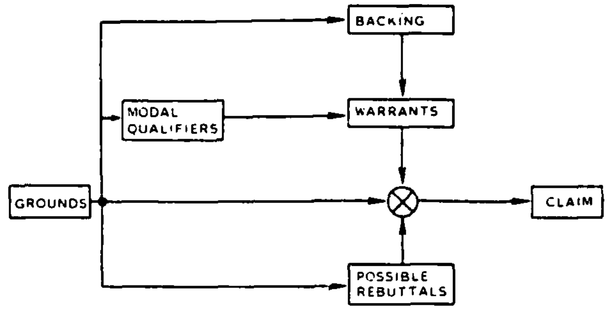

Fig. 11. A possible structure for information processing based upon the six elements of logical reasoning.

appears to have been only limited results obtained in the area of cognitive information processing biases and use of inferior heuristics on the part of groups. Thus many of the areas discussed in Sections III and IV could be extended to groups.

Especially noteworthy concerning results that have been obtained in this area are the groupthink studies of Janis and Mann [177]. Groupthink is a collective pattern of defensive avoidance, a concurrence-seeking tendency of highly cohesive groups. When groupthink occurs, people develop rationalizations to support selectively perceived illusions or wishful thinking about issues at hand and typically participate collectively in development of a defensive avoidance pattern. In groupthink a group collectively falls victim to one or more of the cognitive biases described in Section III.

Among the conditions which lead to groupthink are high cohesiveness, insulation, lack of use of systemic procedures for search and appraisal, highly directive leadership, and a contingency task situation which leads to high stress. Among the symptoms of groupthink cited by Janis and Mann [177] are an illusion of invulnerability, collective rationalization, belief in inherent group morality, excessive pressure against dissenting views, self-censorship, illusions of unanimity, and members who shield the group from disconfirming information. They cite a large number of case studies involving groupthink; cases where incrementalism and bureaucratic politics were the dominant decisionmaking framework typically adopted. Nine prescriptions are offered to avoid groupthink.

1) The group leadership should be noncommitted to particular alternative courses of action.
2) The group leader should encourage critical evaluation.
3) "Devil's advocates" should be included in the group.
4) Subgroups should be formed, allowed to function independently, and then meet with other subgroups to express generated ideas and resolve differences.
5) A variety of alternative scenarios of potential opponents intentions should be developed.
6) Second-opinion meetings should be held to allow full expression of doubts and rethinking of the issue.
7) Experts with opposite viewpoints to the majority view should be encouraged to present challenging views.
8) A small "policy" subgroup should always discuss subgroup deliberation with the larger group to attempt to obtain disconfirming feedback.
9) Independent policy planning and evaluation groups should be formed.

The suggestions offered in Section III to avoid cognitive bias and to ameliorate the effects of those that do occur appear capable of application to groups as well as to individuals. Explicit study of group and organizational bias that would compliment and extend existing studies [16], [19], [129], [155], [173], [257], [276], [279], [280], [388], [390], [391], [394] of group and organizational decisionmaking should yield results that are valuable for the design of planning and decision support systems.

A major difficulty in cognitive information processing seems to be failure to identify and use an appropriate structure that allows appropriate weighting of observed data. Investigation of the effects of various structured-information processing, decision aiding protocols upon the acquisition, analysis, and interpretation of information and its integration with judgment and decisionmaking activities would appear to be a contemporary need in information system design. There are six elements found in explicit argument [376].

1) *claims* or hypotheses,
2) *grounds* or foundations to support the claims,
3) *warrants* or justification for the grounds or foundations,
4) *backing* or the general body of information that is presupposed by the warrant,

TABLE III
INTERPRETATION OF THE JANIS AND MANN [177] COPING PATTERNS

Janis and Mann Coping Pattern	Likely information processing characteristics	Level of interest and importance attached to information & decision task	Stress level	Information evaluation style	Potential coping errors	Characteristics of possible errors	Characteristics of proper selection of coping pattern
Unconflicted adherence or unconflicted change	Indifference to information acquisition and analysis	Low	Low, calm demeanor	Concrete operational	Lack of interest in information processing has caused a generally unstructured and unfamiliar issue to be diagnosed as a familiar well structured, or an inconsequential, one.	An unconcerned tranquil journey to disaster.	efficient and effective use of past experience to select an appropriate decision
Defensive avoidance	May vary from indifference to information, to highly selective processing of information to avoiding disconfirming information, thereby encouraging "wishful thinking".	Low to medium	Highly variable from low to high	Generally flawed and concrete operational. Inattention, delay, and refusal to evaluate and act.	Decisions are postponed, procrastinated, or avoided by shifting responsibility to others and ignoring essentially all information.	No decision is, in reality, a decision. One of the possible consequences of no decision will occur.	This is never a proper coping pattern; only "blind luck" will result in a good outcome.
					A form of wishful thinking is used to evolve a decision. Highly selective perception is used to bolster the decision	*Often value labilities* and inchoateness will result in accepting an hypothesis that the decisionmaker believes is deserving to be *true as a self fulfilling prophecy.*	
Hypervigilance	Very indiscriminate and erratic search	Very high	Very high	Flawed and formal operational	Failure in acquisition and analysis of information resulting from information overload due to panic.	Highly variable from snap judgments and acceptance of overly simplified and flawed decision rules to freezing such that valuable time, in which a good decision could be made, is lost	This s never a pro er coping pattern, only "blind luck" will result in a good outcome.
Vigilance	Openminded discriminating information acquisition and analysis. Typically this involves: a) thorough widescope canvas of potential alternatives b) identification of all *relevant objectives* c) thorough impact analysis d) intensive search for new information e) proper cognizing of disconfirming as well as confirming information f) sensitivity analysis to parameter changes g) detailed implementation provisions and contingency planning	High	Moderate	Formal operational	The issue is inconsequential or well within the experiential background of the decisionmaker.	Wasted time and possibly an inferior decision to one that could have been obtained from affect or or analogous reasoning or other concrete operational thought.	openminded discriminating search and deliberation involving an unstructured selection to select an appropriate decision

5) *modal qualifiers* or circumstances, contingencies, or restrictions which will have to exist in order that the warrant truly supports the grounds.
6) *possible rebuttals* or circumstances, contingencies, or restrictions which, if they exist, will refute or diminish the force of the warrant.

A simplified block diagram of the interaction among these elements is given in Fig. 11. The information processing "structure," consisting in part of the decisionmakers' view of possible and probable action courses and the "decision situation model," is specified by elements 3–6. Element 2, the "grounds," comprises the situational data pertaining to the operational conditions extant. The claim, element 1, is the empirical statement which is supported by other elements in this information processing structure. Toulmin shows through examples that the six elements for logical argument and reasoning can be used as a model for rational reasoning in a number of areas including law, science, the arts, management, and ethics. This structured information processing model is also sufficiently general to accommodate analytical hierarchical inference [165], [306]. Thus it may well provide a structured framework for information processing that can accommodate a variety of information processing styles and approaches ranging from the purely qualitative and affective, to reasoning by analogy which may be a blend of qualitative and quantitative, to quantitatively based filtering and detection algorithms. This could provide enhanced understanding of the implications of chance, cause, and reason [55] in human information processing.

Use of a structured information processing format may reduce the tendency for message distortion due to the exacerbating variables presented in Table III, perhaps to a considerable extent. Mitroff and Mason [255] have presented some suggestions concerning use of structured logical reasoning to cope with ill-structured policy problems and the often occurring divergence between opposing formulations and perceptions of large-scale issues.

Information summarization is needed in information systems for a variety of reasons. Procedures to condense and organize information into a form that can be managed and used in an efficient manner are, therefore, important. The structured information processing model suggested here may well provide organizational support for message aggregation and integration that will accommodate and encourage effective information summarization. We postulate that this framework may accommodate both receptive and preceptive styles of processing and summarization of information, that it will also accommodate nonnumerical and numerical information; and thus hopefully enable rapid conversion from one to the other as needed or desired for different situations.

In this section we have examined a number of frameworks for decisionmaking. Our particular interest is in the description of these frameworks in a way compatible with and supportive of the effective design of systems and processes to aid groups in planning and decisionmaking. We describe a number of "rational" ways in which groups make decisions and pay particular attention to information processing needs in group decisionmaking. Use of a structured protocol for information processing in systems for planning and decision support is suggested as a generic suggestion of potential ways to detect and correct possible cognitive biases that affect many judgment tasks.

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A Multiple Objective Optimization-Based Approach to Choicemaking

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Abstract—An approach for choicemaking under certainty, based in part upon multiple objective optimization, is presented in this paper. With this approach clearly inferior alternatives are eliminated and subsets of alternatives are determined which are guaranteed to contain the most preferred alternatives through use of an induced partial order on the set of alternatives. Use of such a relation to order the alternative set partially is reminiscent of multiobjective optimization theory (MOOT). Our MOOT based approach is believed behaviorally relevant in that it is interactive and can accommodate a variety of cognitive styles and levels of on-line decisionmaker preference feedback. It does not require elicitation of all criterion or attribute weights as does the multiple attribute utility theory (MAUT) approach. MAUT is compatible with and can be used to complement and potentially enhance the effectiveness of the approach for value elicitation for choicemaking. Application of the MOOT based approach is illustrated by several examples.

I. Introduction

THE FOLLOWING problem statement is central to many different choicemaking problems and may be compatible with several decisionmaker cognitive styles.

P: select, and rank to the extent possible, the N most preferred alternatives from X, the finite set of all alternatives under consideration.

The motivation for problem P is due to the fact that decisionmakers and policymakers, in seeking the most preferred alternative, typically will first request a modest number, perhaps between three and five, of highly preferred alternatives from their analysts. They then will explore further the rationale behind the selection and ranking of these alternatives. The goal of the analysts, therefore, is to provide the N most preferred alternatives and (generally) an explanation of the rationale behind their selection and ranking in such a manner that disbenefits of planning and decisionmaking such as the time, cost, and stress of choicemaking are kept to a tolerably low level. We will assume throughout that the set of alternatives X has previously been identified and perhaps refined with the aid of an elimination by attributes procedure which eliminates clearly undesirable alternatives (Tversky [10]).

A standard approach to solving problem P is multiattribute utility theory (MAUT) (Keeney and Raiffa [4], Edwards [1]), which can be described as follows.

1) Assess a real-valued (utility) function u, defined on the alternative set $X, u: X \to R$, which satisfies the property that the utility of alternative x is greater than the utility of alternative x' if and only if alternative x is preferred to alternative x'; i.e., $u(x) > u(x')$ if and only if $x \succ x'$. Mathematically, the converse is also true. That is $x \succ x'$ if and only if $u(x) > u(x')$. In practice we elicit utility or value functions and from this infer preferences. Behaviorally the concept of preference must preceed that of utility.
2) Compute $u(x)$ for all $x \in X$ and rank order the computed utilities.
3) Identify the alternatives that generate the N largest utility values.

There are conditions,[1] e.g., mutual utility independence, which imply that utility functions having special functional forms, e.g., the multilinear form, adequately model a decisionmaker's preferences (Keeney and Raiffa [4]). Such specialized functional forms significantly reduce the amount of time and effort required to complete the assessment of a multiattribute utility function. However, verification of sufficient conditions to insure existence of these functional forms can be a time and effort consuming task. Also, if such sufficient conditions do not hold, utility functions with functional forms based on these conditions may not represent good quantitative models of decisionmaker preference. Finally, the cognitive style of many decisionmakers may make elicitation of the complete set of attribute weights needed for MAUT very difficult. Thus, although MAUT provides a solution to problem P, accurate execution of step 1) can be a time consuming and stressful activity in practice, especially if there are many attributes under consideration and the decisionmaker is not comfortable with the highly structured elicitation procedure required to determine attribute weights.

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[1] Many authors distinguish between preference (value) functions and preferential independence conditions for the riskless case and utility functions and utility independence conditions for the risky case. We use the word utility to represent both of these.

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In this paper we propose a behaviorally attractive complementary and alternative approach to solving problem P. This new approach only attempts, a least initially, to order the alternative set partially. This is suggestive of a search for the Pareto-optimal, noninferior, or nondominated set (or frontier) in multiobjective optimization theory (MOOT). We therefore refer to the choicemaking approach to be presented and analyzed in this paper as the *MOOT based* approach. There have been a number of previous studies concerning multiple objective optimization theory (Haimes [3], MacCrimmon [6], MacCrimmon and Siu [5], Starr and Zeleny [9], and Zeleny [12]). However, our approach is fundamentally much more an evaluation, refinement, or ranking approach than an optimization approach.

The primary advantage of at least initially assessing only a partial order on the alternative set is that the often difficult and time consuming effort of trading off noncommensurate objectives, or attributes, may be partially avoided. Avoidance or delay in making trade-offs will eliminate or at least postpone explication of those values and goals that the decisionmaker may wish to leave initially implicit. The disadvantage of a partial order is that the resulting nondominated set of alternatives will contain, in general, more than one alternative, thereby requiring further effort for final choice selection. The reason for determining the partial order, however, is that if the nondominated set is sufficiently small, the decisionmaker may be able to select the most preferred alternative from the nondominated set without any significant further external assessment. Equivalently, the partial order can be thought of as eliminating dominated alternatives from further consideration. We remark that since, for example, the second most preferred alternative may not be in the original nondominated set, determination of only the original nondominated set is not guaranteed to solve problem P.

The MOOT based approach to solving problem P is presented in Section II. The key assumption made is that the decisionmaker can select the most preferred alternative from any given set of nondominated alternatives. This assumption is referred to as the *complete preference feedback* assumption. Three procedures are offered for enhancing the likelihood that the complete preference feedback assumption will hold.

An algorithm for solving problem P for the complete preference feedback case and an example are presented in Section III. Our development of this algorithm, presented in Section IV, is based on use of a directed graph representation of the partial order on the set of alternatives X induced by a vector of objective functions.

In Section IV we modify the statement of problem P and the associated analysis to accommodate the possibility that complete preference feedback may not be available. An algorithm and several examples are presented for this *partial preference feedback* case.

A rather complete hypothetical example is presented in Section V to illustrate several of the notions developed earlier.

II. The MOOT Based Choicemaking Approach—Complete Preference Feedback

In this section we present, discuss, and analyze the MOOT based approach to solving problem P. We make a complete preference feedback assumption: that the decisionmaker can choose the most preferred alternative from any given nondominated set. Procedures are suggested for improving the likelihood that this assumption will hold. This section begins with several preliminary definitions and assumptions.

We assume that M presumably noncommensurate attributes have been identified and that for each attribute m, a real-valued objective function on $X, v_m: X \rightarrow R$, has been assessed.[2] For the mth attribute, v_m is equivalent to a total order, $\prec_m$, on X; i.e., x' is preferred to x with respect to the mth attribute ($x \prec_m x'$) if and only if $v_m(x) < v_m(x')$.

Although the MOOT based approach to choicemaking presented in this paper requires M objective functions to be assessed, compared to only one utility function to be assessed for the MAUT approach, assessment of the complete set of lowest level objectives $\{v_i\}$ is often significantly easier than complete assessment of u. Typically, assessment of a complete scalar utility function will require assessment of M lowest level component utility functions v_i or u_i, $i = 1, 2, \cdots, M$, and trading off all of these utility functions by assessing at least $M-1$ utility or attribute weights. Assessment of $\{v_i\}$ for the MOOT based approach does not require trading off the M lowest level noncommensurate objectives, an activity that utility function assessment cannot avoid. For example, few would disagree that 1) maximizing economic benefit and 2) minimizing environmental pollution are two important objectives in the management of many industrial innovation situations. It is unlikely that determining objective measure functions for these two objectives would be unduly time consuming, stressful, or controversial. Trading off achievement levels for these two objectives, however, may be agonizing.[3] To the extent possible, it may be desirable to avoid, or at least postpone, making such trade-offs, especially if they are not really necessary, as may often be the case.

Let the vector function $v: X \rightarrow R^M$ be defined as $v(x) = \{v_1(x), \cdots, v_M(x)\}$. Note that v is equivalent to a partial order on X; i.e., x' is preferred to x ($x \prec x'$) if and only if $v(x) < v(x')$. Let $v(X) = \{v(x) : x \in X\}$, and assume without significant loss of generality that $v: X \rightarrow v(X)$ is one-to-one and onto. The M-vector element $w \in v(X)$ and its corresponding alternative are said to be *nondominated* if there does not exist another element $w' \in v(X)$ such that $w < w'$ and $w \neq w'$.

The MOOT based choicemaking approach for solving problem P consists of two basic steps.

[2] For notational convenience we will use v_i to represent attributes and objectives for the MOOT based approach and u_i to represent attributes for the MAUT based approach.

[3] Generally, practical determination of objective measure functions requires preference independence among objectives, a requirement common to both MAUT and our MOOT based approach.

1′ Select the most preferred element in $v(X)$ from the nondominated set of $v(X)$, i.e., the set of all nondominated elements in $v(X)$.
2′ If the number of alternatives determined thus far is less than N, say N', then redefine X as X with these N' alternatives removed and go to 1); otherwise stop.

We now discuss several important issues raised by each of these two steps.

A) Step 1′

Two fundamental assumptions must hold in order that Step 1′ can be successfully accomplished:

a) the most preferred element in $v(X)$ can be found in the nondominated set of $v(X)$;
b) the decisionmaker can select the most preferred element in the nondominated set of $v(X)$. This is the complete preference feedback assumption.

We remark that it is particularly desirable for the first assumption to hold since, in general, the fewer the alternatives the decisionmaker has from which to choose, the easier it is to select the most preferred alternatives. We will show that the first assumption holds under a weak and behaviorally relevant hypothesis, which will be presented after a needed definition.

Definition: A scalar *social choice function* is a function $V: v(X) \rightarrow R$ which when composed with the objectives function v produces the utility function u; i.e., V is such that $u(x) = V[v(x)]$ for all $x \in X$.

The scalar social choice function trades off the objectives functions to produce a real-valued measure of preference. In our approach, we wish to avoid a complete explicit assessment of V as much and for as long as possible. To verify that it is sufficient to examine only elements in the nondominated set of $v(X)$ in searching for the most preferred element in $v(X)$, however, we must assume its existence.

Lemma 1: Assume that the social choice function V is isotone (monotonically nondecreasing). Then, there is a vector element w in the nondominated set of $v(X)$ such that $V(w) > V(w')$ for all $w' \in v(X)$.

Proof: Assume that there is a $\hat{w} \in v(X)$ which is not in the nondominated set of $v(X)$ but which has the property: $V(\hat{w}) > V(\tilde{w})$ for all $\tilde{w}$ in the nondominated set of $v(X)$. This property implies that $\hat{w} > \tilde{w}$ for all $\tilde{w}$ in the nondominated set of $v(X)$; hence, $\hat{w}$ is a member of the nondominated set of $v(X)$, which is a contradiction.

In light of Lemma 1, we therefore assume throughout the remainder of this paper that the social choice function is isotone in the value function in order to preserve the isotonicity of the utility function u with respect to the partial ordering induced by v on X.

We remark that if mutual utility independence (Keeney and Raiffa [4]) holds, then the multiplicative form of risk preference or utility under risk

$$1 + Ku(x) = \prod_{m=1}^{M} \left[1 + Kk_m u_m(x) \right]$$

represents a special case of an isotone social choice function.

The primary difficulty in successfully accomplishing Step 1′ is that the decisionmaker may not be able to easily select the most preferred element in the nondominated set of $v(X)$, i.e., the complete preference feedback assumption may not be valid, in general. The ease with which this selection can be made depends on the specific decision situation, the efficiency of the attribute or objective structuring procedure, the number of attributes, and the cognitive style of the decisionmaker and decision situation associated task characteristics.

In some cases the nondominated set of $v(X)$ may be sufficiently small and/or its alternative elements may be sufficiently distinguishable, and/or the dimension of the objectives vector v may be sufficiently small for the decisionmaker to select the most preferred alternative without further preference elicitation. For example, a recently developed medical diagnostic decision aid, which uses a specialized and simplified version of the MOOT based approach, typically provides a nondominated diagnostic test set which is sufficiently small to cause a physician little difficulty in selecting a test from the nondominated set without further explicit preference elicitation (White *et al.* [11]). If this is not the case and the decisionmaker finds it difficult to select a most preferred alternative from the nondominated set of $v(X)$, two approaches, not requiring complete preference elicitation, can be considered.

a) Decrease the size of the nondominated set until it is small enough for the decisionmaker to be able to select a most preferred alternative.
b) Modify problem statement P and the desired end result of the analysis in order to accommodate the fact that complete preference feedback is not available.

The second approach will be considered in detail in Section IV and will not be discussed further in this section. We devote the remainder of this section to the first approach.

There appears to be three potentially implementable, general procedures to accomplish the first approach:

1) trading off objectives such as to reduce, in effect, the number of objectives by moving up the objectives tree,
2) determining bounds on the objective trade-off weights,
3) utilizing prior elicited knowledge concerning the functional form of the social choice function.

The first procedure results in trading off some of the objectives. This is equivalent to redefinition of some lower level objectives in terms of higher level objectives and, in this way, "moving up" the objectives tree. Following several preliminary definitions, we will show that, under proper assumptions, trading off some of the objectives is a generally desirable activity, especially where there are many lowest level objectives. Trading off some of the objectives:

a) "strengthens" the induced partial order
b) does not add elements to the set of nondominated elements and often results in a reduction in the number of elements in the nondominated set.

Let $\tilde{v}: X \to R^N, N \leq M$, be defined as $\tilde{v}(x) = f[v(x)]$ for all $x \in X$, where $f: R^M \to R^N$. Let $\tilde{\prec}$ be the partial order on X induced by $\tilde{v}$. Define $\bar{X}$ and $\tilde{X}$ as the nondominated sets in X generated by the respective partial orders $\prec$ and $\tilde{\prec}$.

We remark that the function f can represent a social choice function (if it is isotone and if $N=1$) or a function that trades off less than all of the objectives functions. For example, let $f(w_1, \cdots, w_M) = \{w_1, \cdots, w_{M-2}, g(w_{M-1}, w_M)\} \in R^{M-1}$, where the function $g: R^2 \to R^1$ represents a (presumably isotone) "local" social choice function of the two objectives functions w_{M-1} and w_M. The function f can also delete some of the objectives from consideration (let $f(w_1, \cdots, w_M) = \{w_1, \cdots, w_N\}$, where $N < M$). See (Fishburn [2]) for a related discussion which involves lexicographic ordering. Deletion of objectives, however, appears best accomplished as part of the prescreening and prescanning of alternatives, or alternatives refinement step, which would precede the use of the MOOT based procedure.

Lemma 2:

a) If f is isotone, then for any pair $x, x' \in X$, the fact that $x \prec x'$ implies $x \tilde{\prec} x'$.
b) If f is strictly isotone, then $\tilde{X} \subset \bar{X}$.

Proof: We note that $x \prec x'$ if and only if $v(x) \leq v(x')$ which implies $\tilde{v}(x) = f[v(x)] \leq f[v(x')] = \tilde{v}(x')$ if and only if $x \tilde{\prec} x'$. Thus part a) is shown to be correct.

To verify the correctness of part b) we will prove that $\bar{X}^c \subset \tilde{X}^c$, where the superscript c stands for complementation with respect to X. Let $x \in \bar{X}^c$. Then there exists an $x' \in \bar{X}$ such that $v(x) \leq v(x')$ and $v(x) \neq v(x')$. The strict isotonicity of f preserves this property under composition.[4] Hence, $\tilde{v}(x) = f[v(x)] \leq f[v(x')] = \tilde{v}(x')$ and $\tilde{v}(x) \neq \tilde{v}(x')$, which implies $x \notin \tilde{X}$.

Part a) of Lemma 2 states that if alternative x' is preferred to alternative x before a trade-off is made, then this preference will also hold after the trade-off is made. Clearly the converse may not be true; for example, let $w = (1,2)$, $w' = (2,1)$, and $f(w) = w_1$. Thus, a trade-off will always "strengthen" (more properly, not weaken) a partial ordering on X (and $\tilde{\prec}$ is said to be stronger than $\prec$) in that there will be at least as many pairs of alternatives having elements that can be related by $\tilde{\prec}$ as could be related by $\prec$.

Part b) of Lemma 2 states that, under the assumption that the trade-off function is strictly isotone, the set of nondominated alternatives resulting from a trade-off will never contain more alternatives than before the trade-off was made. This is desirable since a decisionmaker would not want to spend the time and effort involved in trade-off elicitations if this were to result in an increased number of alternatives to be considered. We remark, however, that strict isotonicity is required. For example, if $f(w) = 1$ for all $w \in v(X)$, then $\tilde{X} = X$.

The procedure of making trade-offs may be accomplished by the use of an attribute or objectives measure template, or tree, the structure of which is generally obtained as part of the impact assessment and analysis phase of a choicemaking effort. As has been demonstrated by Lemma 2, moving up the objectives template by increasing the number of objectives traded off will not increase the size of the nondominated set. It is therefore beneficial to perform as many attribute or objectives trade-offs initially as is comfortably possible (i.e., make the number of objectives M as small as is comfortably possible). Trading off all of the objectives (through use of an assessed social choice function which may be an approximation based on ordinal preference data or a function based on a complete cardinal preference assessment) is mathematically, though not necessarily behaviorally, equivalent to assessing a utility function and would produce a nondominated set consisting of only the most preferred alternative.

The second procedure to decrease the number of elements in the nondominated set of $v(X)$ involves determining bounds on the objective trade-off weights. This procedure would impose at least a partial ordering on the alternatives within a nondominated set. For example, consider the following $M=2$ case. The isotonicity of a social choice function guarantees that if an equipreference curve goes through the point w in Fig. 1, then the remainder of the curve will be in the shaded area. Therefore, the isotonicity of the social choice function does not provide sufficient information to determine how the points w and w' are related by preference. If, however, it could be determined that the equipreference curve would remain in the shaded region depicted in Fig. 2, then we would be assured that w is preferred to w'. For example, if the decisionmaker can state that the weight of objective 2 can never exceed α times the weight of objective 1 and that $\alpha > (w_1 - w_1')/(w_2' - w_2)$, then we can surely assume that w is preferred to w'.

The third procedure for reducing the number of elements in the nondominated set of $v(X)$ requires utilizing what information there exists regarding the functional form of the social choice function V. For example, assume V is linear in the objectives functions; hence the equipreference curves are lines having negative slope as in Fig. 3. Consider three alternatives x_1, x_2, and x_3, each producing objective vectors, $v(x_i)$, of (0,1), (1,0), and (0.4,0.4), respectively. Clearly, all three alternatives are nondominated; however, it is easy to demonstrate that the linearity of V implies that alternative x_3 can not be preferred to both x_1 and x_2.

Step 2'

Once an alternative has been determined as most preferred in Step 1', Step 2' requires that this alternative be

[4] f composed with v is the function $f[v(\cdot)]$.

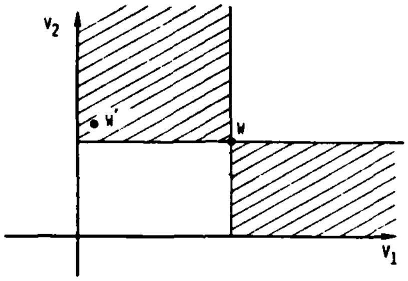

Fig. 1. Bounds on equipreference curves.

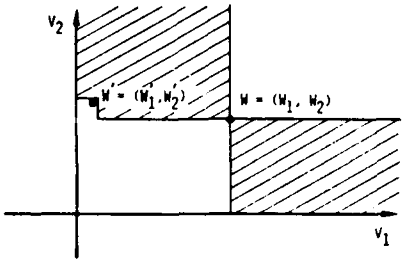

Fig. 2. Reduced region for equipreference curves.

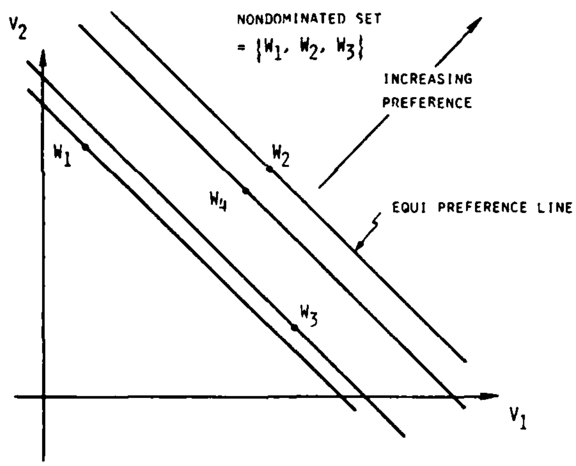

Fig. 3. Dominated and nondominated alternatives.

removed from further consideration. Even though the most preferred alternative must be in the nondominated set, the second most preferred alternative may not be. Fig. 3 illustrates this fact, which is very important in practice for successful use of multiple objective based approaches.

III. Algorithm for the Complete Preference Feedback Case

Following some preliminaries, we present an algorithm in this section for determining the N most preferred alternatives and their ranking from a finite set of alternatives. This algorithm assumes that the decisionmaker can always identify the most preferred element from any given nondominated set.

The algorithm for the complete preference feedback case as well as one other algorithm to be presented in Section IV, will be based on a directed graph, or digraph,

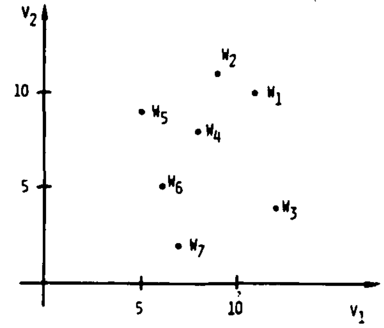

Fig. 4. Elements of $v(X)$ for the example.

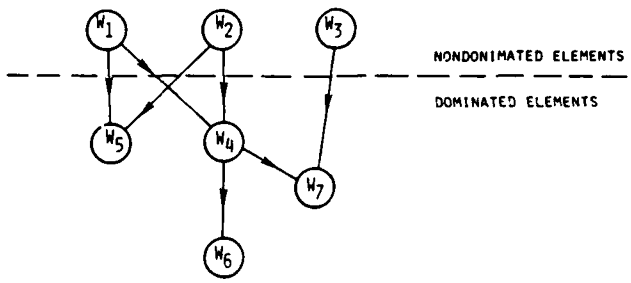

Fig. 5. Initial digraph for the example: contextual relation "dominates."

constructed using the contextual relation "dominates." Details of the construction of such digraphs are given in [8] and elsewhere. We now present an example in order to illustrate the relationship between a set $v(X)$ and its associated digraph.

Example: 1: Assume $M = 2$ and that $v(X)$ is composed of the following points, where $w_i = \{w_{i1}, w_{i2}\}$:

$$w_1 = (11, 10)$$
$$w_2 = (9, 11)$$
$$w_3 = (12, 4)$$
$$w_4 = (8, 8)$$
$$w_5 = (5, 9)$$
$$w_6 = (6, 5)$$
$$w_7 = (7, 2).$$

These points are displayed in Fig. 4; the associated digraph is presented in Fig. 5.

Consider the following algorithm.

0) Set $n = 1$.
1) Determine the most preferred element in the top level set (or nondominated set) of the digraph. This is equivalent to determining the nth most preferred alternative.
2) Remove the selected element from the digraph. Move up to the top level all second level elements which were predecessors of *only* the removed element.
3) If $n = N$ stop; if not, increase n by one and go to step 1).

We illustrate the above algorithm with the following example.

Example: 2: Assume that $X=\{x_1,\cdots,x_7\}$ and $w_i=v(x_i)$, $i=1,\cdots,7$, as given in Example 1 and depicted in Fig. 4 and 5. We note from Fig. 5 that search for the most preferred alternative can be restricted to those represented by $\{w_1,w_2,w_3\}$ (or equivalently alternatives $\{x_1,x_2,x_3\}$); consequently, the decisionmaker can ignore the elements w_i, $i=4,\cdots,7$, in selecting the most preferred alternative. Assume that the decisionmaker selects x_1 as the most preferred alternative. We then eliminate w_1 to produce Fig. 6. Observe that although elements w_4 and w_5 are no longer dominated by w_1 (which is now removed), they remain dominated by w_2, and hence no new elements move into the nondominated set. A search for the second most preferred element can therefore be restricted to the set $\{w_2,w_3\}$. Assume the decisionmaker selects w_2 as the second most preferred score. Removal of w_2 from Fig. 6 now allows w_4 and w_5 to enter the nondominated set, as shown in Fig. 7. A search for the third most preferred element in $v(X)$ can therefore be restricted to the set of attribute or objective scores $\{w_3,w_4,w_5\}$ and the alternatives x_3, x_4, and x_5 that they represent.

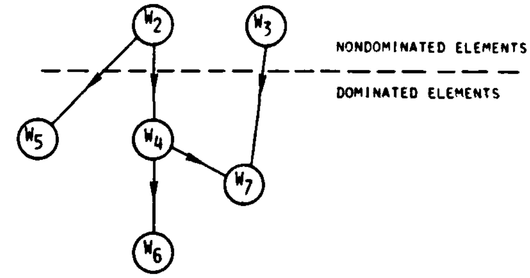

Fig. 6. Digraph with w_1 eliminated.

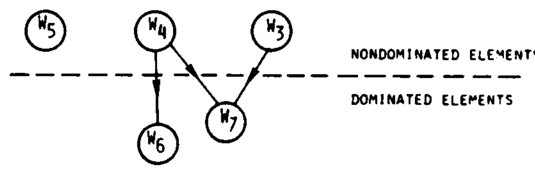

Fig. 7. Digraph with w_1 and w_2 eliminated.

IV. The Partial Preference Feedback Case

As discussed in Section II, the complete preference feedback assumption may not always be satisfied without making difficult trade-offs to insure a sufficiently small dimension for the objectives vector and sufficiently small size of the nondominated set. In order to be able to accommodate the possibility that complete preference feedback may not be available, we modify the statement of problem P as follows.

P': determine $\mathcal{X}_N$, a collection of ordered subsets of X, where

1) each element in $\mathcal{X}_N$ contains N alternatives,
2) each element in $\mathcal{X}_N$ is a candidate for the ordered set of the N most preferred alternatives based on:
   a) the digraph induced by the vector function v
   b) preferences additional to those already expressed in determining the vector-valued function v.

We remark that although the information contained in $\mathcal{X}_N$ is of obvious value for decisionmaking, $\mathcal{X}_N$ may not be the most desirable form in which to present this information. A desirable aggregated form might be $X_N\subset X$, where X_N is the subset that is guaranteed to contain the N most preferred alternatives. The subset X_N is easily obtained by taking the union of subsets in $\mathcal{X}_N$. Observe that in general $\mathcal{X}_N$ cannot be reconstructed from X_N and the digraph because of the additional preference information supplied by the decisionmaker. It is true, however, that $\mathcal{X}_N$ is a subset of the set of all combinations of alternatives in X_N which are compatible with the original digraph.

We observe that any subset of X having cardinality n (i.e., containing n elements) may not be a member of $\mathcal{X}_n$. For example, $\{x_1,x_2,x_7\}$ cannot be a member of $\mathcal{X}_3$ for the set of alternatives considered in Examples 1 and 2 because it is incompatible with the digraph displayed in Fig. 5; i.e., the digraph indicates that x_7 cannot be the third most preferred alternative.

Lemma 2b) demonstrated, for the complete preference feedback case, that trading off objectives would not cause additional alternatives to be included in the nondominated set. We now prove an analogous result for the partial preference feedback case, following several preliminary definitions. Let $\tilde{\mathcal{X}}_N$ be the collection of ordered subsets of X which is defined as $\mathcal{X}_N$ is defined, except $\tilde{\mathcal{X}}_N$ is determined based on a partial order $\tilde{\prec}$ that is stronger than $\prec$.[5] Similarly, we define $\tilde{X}_N\subset X$ as the union of all subsets of X in $\tilde{\mathcal{X}}_N$.

Corollary: Assume that the function f defined in Section II is strictly isotone. Then, $\tilde{\mathcal{X}}_N\subset\mathcal{X}_N$ and $\tilde{X}_N\subset X_N$.

Proof: Clearly, $\tilde{X}_N\subset X_N$ if $\tilde{\mathcal{X}}_N\subset\mathcal{X}_N$. Lemma 2b) implies that $\tilde{\mathcal{X}}_1\subset\mathcal{X}_1$; assume $\tilde{\mathcal{X}}_{n-1}\subset\mathcal{X}_{n-1}$. It is easily shown that Lemma 2b) then implies $\tilde{\mathcal{X}}_n\subset\mathcal{X}_n$, and the result follows by induction.

An algorithm for the determination of $\mathcal{X}_N$ is as follows.

0) Set $\mathcal{X}_0=\varnothing$ and $n=1$.
1) Assume $\mathcal{X}_{n-1}=\{X^1_{n-1},\cdots,X^K_{n-1}\}$ and $\mathcal{X}_n=\varnothing$; set $k=1$.
2) Construct a digraph from the alternatives in $X\sim X^k_{n-1}$.[6]
3) Assume there are L nondominated alternatives in the digraph constructed in step 2). Set $l=1$.
4) Let $Y=\{X^k_{n-1},x\}$, where $x\in X$ is the lth nondominated alternative in the digraph constructed in step 2).
5) Determine whether or not the set $Y\subset X$ should be included in $\mathcal{X}_n$ and include it or exclude it as appropriate. This determination should result from

[5] Refer to Section II for further explication of these sets and subsets.

[6] $A\sim B=A\cap B^c$; that is, $A\sim B$ is the set of all points in A which are not also contained in B.

either direct interaction with the decisionmaker or from preference information, additional to the information contained in the function v, obtained from the decisionmaker.

6) If $l > L$, GO TO 7). If $l < L$, let $l = l+1$ and GO TO 4).

7) If $k > K$, GO TO 8). If $k < K$, let $k = k+1$ and GO TO 2).

8) If $n > N$, STOP. If $n < N$, let $n = n+1$ and GO TO 1).

We can now demonstrate that problem P' generalizes problem P for the case of complete preference feedback. Observe that complete preference feedback would select $\mathcal{X}_1$ to be composed of one subset. This subset would contain only one alternative, the most preferred alternative. We assume $\mathcal{X}_{n-1}$ contains only one ordered subset of X which contains the $n-1$ most preferred alternatives. Then, a quick iteration of the algorithm indicates that step 5) determines the nth most preferred alternative which is included with the subset of $\mathcal{X}_{n-1}$ to result in the single subset contained in $\mathcal{X}_n$ for this complete preference feedback case.

The following four examples illustrate the MOOT based choicemaking algorithm for the partial preference feedback case and several forms of the partial preference information that may be available from the decisionmaker.

Example 3: Consider the set of alternatives and their objective measures which were examined in Examples 1 and 2. We will assume, however, that the decisionmaker can supply no preference feedback beyond the preference information already contained in the function v. This assumption is the opposite extreme of the complete preference feedback case and will be referred to as the *no preference feedback* case. Note that step 5) in the above partial preference feedback algorithm is essentially removed in this case. Thus,

$$\mathcal{X}_1 = \{\{x_1\},\{x_2\},\{x_3\}\},$$

$$\mathcal{X}_2 = \{\{x_1,x_2\},\{x_1,x_3\},\{x_2,x_1\},\{x_2,x_3\}\{x_3,x_1\},\{x_3,x_2\}\},$$

$$\mathcal{X}_3 = \{\{x_1,x_2,x_3\},\{x_1,x_2,x_4\},\{x_1,x_2,x_5\},\{x_1,x_3,x_2\},\{x_2,x_1,x_3\},\{x_2,x_1,x_4\},\{x_2,x_1,x_5\},\{x_2,x_3,x_1\},\{x_3,x_1,x_2\},\{x_3,x_2,x_1\}\}$$

as can easily be seen by inspection of Fig. 5. We note that

$$X_1 = \{x_1,x_2,x_3\}$$

$$X_2 = \{x_1,x_2,x_3\}$$

$$X_3 = \{x_1,x_2,x_3,x_4,x_5\}.$$

It is of interest to observe that if the first three most preferred alternatives are sought, then alternatives x_6 and x_7 can always be excluded from consideration, even if no preference feedback is available from the decisionmaker.

We remark that for the no preference feedback case $\mathcal{X}_N$ can be reconstructed exactly from X_N and the original digraph.

Example 4: Assume that in examining the results of Example 3, the decisionmaker decides that x_4 cannot be a member of X_3. Then, $\mathcal{X}_3$ becomes

$$\mathcal{X}_3 = \{\{x_1,x_2,x_3\},\{x_1,x_2,x_5\},\{x_1,x_3,x_2\},\{x_2,x_1,x_3\},\{x_2,x_1,x_5\},\{x_2,x_3,x_1\},\{x_3,x_1,x_2\},\{x_3,x_2,x_1\}\}.$$

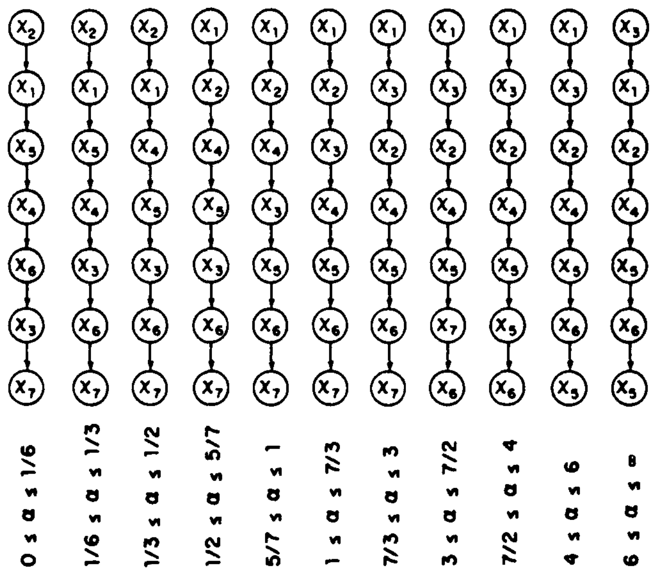

Fig. 8. Preferences and domination relations for linear social choice function.

Example 5: Assume that the decisionmaker cannot exclude any of the original three nondominated alternatives as the most preferred alternative, given only the information provided in Examples 1 and 2. That is, $\mathcal{X}_1 = \{\{x_1\},\{x_2\},\{x_3\}\}$. For the $n=2$ case, however, we assume that the decisionmaker feels that both x_1 and x_2 cannot be more preferred than x_3, or equivalently that x_3 must either be preferred to x_1 or to x_2. Therefore,

$$\mathcal{X}_2 = \{\{x_1,x_3\},\{x_2,x_3\},\{x_3,x_1\},\{x_3,x_2\}\},$$

which is identical to the $\mathcal{X}_2$ determined in Example 3 with the sets $\{x_1,x_2\}$ and $\{x_2,x_1\}$ excluded. If no further preferences can be expressed, it then follows that

$$\mathcal{X}_3 = \{\{x_1,x_3,x_2\},\{x_2,x_3,x_1\},\{x_3,x_1,x_2\},\{x_3,x_2,x_1\}\}.$$

Thus, $X_1 = X_2 = X_3 = \{x_1,x_2,x_3\}$. We now observe that alternatives x_i, $i = 4,\cdots,7$, can be excluded from consideration in determining the three most preferred alternatives. x_6 and x_7 were already excludable; the information that both alternatives x_1 and x_2 cannot be more preferred than x_3 allows us to exclude x_4 and x_5 from consideration as candidates for the three most preferred alternatives. Note that this exclusion has been accomplished without detailed elicitation of weights.

Example 6: Again consider the data presented in Examples 1 and 2. Assume preference information from the decisionmaker has implied that the three most preferred alternatives are x_1, x_2, and x_4 and that the social choice function is linear. Let $v(x) = \{v_1(x), v_2(x)\}$ and assume that the social choice function is $V[v(x)] = \alpha v_1(x) + v_2(x), \alpha > 0$. We can easily obtain the preference or domination digraph of Fig. 8. Since we have assessed that $\mathcal{X}_3 = \{\{x_1,x_2,x_4\}\}$, we see from Fig. 8 that $\frac{1}{2} < \alpha < 1$. It also follows without further preference information that $\mathfrak{X}_4 = \{\{x_1, x_2, x_4, x_3\}, \{x_1, x_2, x_4, x_5\}\}$. Note that $\{x_1, x_2, x_4, x_6\} \notin \mathfrak{X}_4$ due to the linearity of the social choice function even though x_6 is in the nondominated set of X with alternatives x_1, x_2, and x_4 removed. Thus we see that complementary use of the proposed MOOT based approach with an ordinal linear social choice function has allowed bounding of the linear social choice function and elimination of infeasible alternatives without detailed efforts to elicit all attribute weights. Of course we, indirectly, use many of the attractive and useful features of MAUT in this procedure. However we selectively use portions of the attribute or objectives template and our MOOT based approach to bound and limit criterion or attribute weight elicitation. Here we are able to eliminate x_6 from further consideration and, as a byproduct, determine bounds on a linear social choice function trading off $v_1(x)$ and $v_2(x)$.

V. An Example Application of the MOOT Based Approach

We now present a hypothetical example of an interactive decision support process which utilizes the MOOT based choicemaking approach. It is assumed that the template displayed in Fig. 9 represents the attribute structure of the problem. For example, if the problem under consideration is to purchase a new automobile, the attributes in Fig. 9 might have the following meaning:

A) safety
B) initial cost
C) fuel economy
D) scheduled maintenance expenses
E) unscheduled maintenance expenses
F) resale value
G) attractiveness
H) trunk and passenger compartment capacity
I) desirability
C)–E) operating cost
B)–F) cost.

Note that the template indicates that "cost" is composed of "initial cost," "operating cost," and "resale value."

Assume that six (6) hypothetical alternatives are under consideration, x_1 through x_6. Table I presents alternative scores for each alternative and for each lowest level attribute. These scores would be assessed from experts familiar with the particular attribute and alternative. They correspond to the $w_i = v(x_i)$ function form concepts used in our previous examples.

The objectives measure value for alternative x_1 is at least as great as the objectives measure value for x_5 for every attribute, and therefore x_1 dominates x_5. Similarly, x_2 dominates x_6. Fig. 10 presents the associated digraph. If the intent is to select a single alternative eventually, then both x_5 and x_6 could be deleted from further consideration at this point.[7] If the ultimate intent is to select more than just the best alternative, then all alternatives require further consideration. We also observe that if the two most preferred alternatives are to be selected and $x_1(x_2)$ is considered the most preferred alternative, then $x_6(x_5)$ can be deleted from further consideration.

Fig. 10 may contain a sufficient amount of information for choicemaking; however, if this is not the case, which is very likely since there are eight lowest level attributes, some trade-off assessment becomes necessary. In order to strengthen the partial order on the set of alternatives, we move up the objectives template by performing our first trade-off. The attributes which we assume are first traded

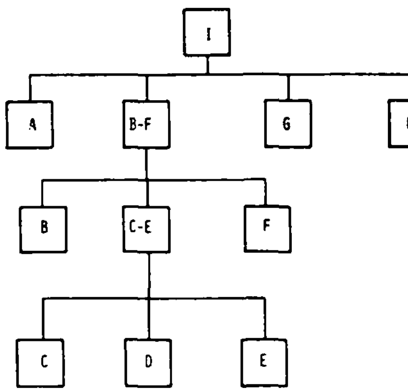

Fig. 9. Attribute structure.

TABLE I
Alternative Scores for Lower Level Attributes or Criteria

ALTERNATIVES		ALTERNATIVE SCORES FOR ATTRIBUTES							
		A	B	C	D	E	F	G	H
	x_1	70	100	65	40	80	10	100	60
	x_2	100	40	70	30	100	100	10	100
	x_3	60	35	70	35	10	10	40	50
	x_4	50	0	100	100	0	90	10	100
	x_5	65	40	0	40	75	0	30	55
	x_6	0	35	60	0	90	40	0	0

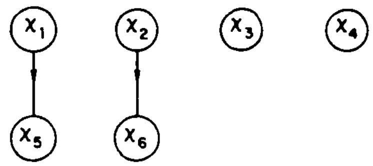

Fig. 10. Domination digraph for Table I.

[7] Unless there exists the possibility that x_1 may be "unavailable," in which case x_5 should be retained, or the possibility that x_2 may be "unavailable" in which case x_6 should be retained. Henceforth we will routinely assume potential availability of all alternatives.

TABLE II
TRADE-OFF WEIGHTS FOR ATTRIBUTES C, D, AND E

	ATTRIBUTES C	D	E
BEST ALTERNATIVES	x_4	x_4	x_2
WORST ALTERNATIVES	x_5	x_6	x_4
WEIGHTS	.20	.35	.45

TABLE III
ALTERNATIVE SCORES FOR ALL OBJECTIVE MEASURE VALUES: FIRST TRADE-OFF CASE

		ALTERNATIVE SCORES FOR ATTRIBUTES					
		A	B	C-E	F	G	H
ALTERNATIVES	x_1	70	100	63	10	100	60
	x_2	100	40	69.5	100	10	100
	x_3	60	35	30.75	10	40	50
	x_4	50	0	55	90	10	100
	x_5	65	40	47.75	0	30	55
	x_6	0	35	52.5	40	0	0

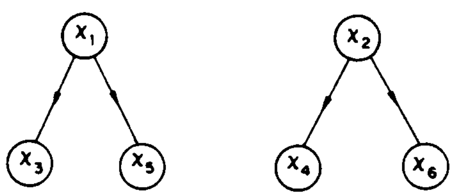

Fig. 11. Domination digraph for Table III.

TABLE IV
TRADE-OFF WEIGHTS FOR ATTRIBUTES B, C–E, AND F

	ATTRIBUTES B	C-E	F
BEST ALTERNATIVES	x_1	x_2	x_2
WORST ALTERNATIVES	x_4	x_3	x_5
WEIGHTS	.40	.10	.50

TABLE V
ALTERNATIVE SCORES FOR OBJECTIVE MEASURE VALUES: SECOND TRADE-OFF CASE

		ALTERNATIVE SCORES			
		A	B-F	G	H
ALTERNATIVES	x_1	70	51.3	100	60
	x_2	100	72.95	10	100
	x_3	60	22.075	40	50
	x_4	50	50.5	10	100
	x_5	65	20.775	30	55
	x_6	0	39.25	0	0

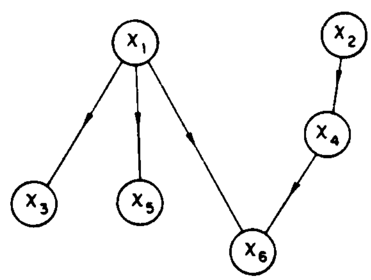

Fig. 12. Domination digraph for Table V.

off are C, D, and E. We note that with respect to each of these three attributes, the best and the worst alternatives are given in Table II. Assume that the decisionmaker provides the following information: the linear weights for comparing the difference between the best alternative and the worst alternative are 0.20, 0.35, and 0.45 for the three attributes C, D, and E, respectively. Table III gives the resulting objectives measure values. We now note that x_1 dominates x_3 and x_5, and x_2 dominates x_4 and x_6. Thus we have the digraph of Fig. 11. Note that if the intent were to determine a single best alternative, it would be sufficient at this point to eliminate alternatives x_3, x_4, x_5, and x_6 from further consideration. A comparison of Figs. 10 and 11 indicates the impact of strengthening the partial order on the set of alternatives by trading off the three attributes C, D, and E. If selecting more than the best alternative were the objectives of the choicemaking effort, once again all alternatives would have to be considered further. We observe also that if the two most preferred alternatives were being sought and $x_1(x_2)$ were chosen as most preferred, then x_4 and x_6 (x_3 and x_4) could be eliminated from further consideration.

Fig. 11 may convey enough information to the decisionmaker for alternative selection. If not, as may well be the case since there are now six lowest level objectives, further trade-off assessment is required. In order to further strengthen the partial order on the set of alternatives, we assume that the decisionmaker has decided to move up the template by trading off the attributes B, C–E, and F. The best and worst alternatives for each of these attributes are given in Table IV.

Assume that the decisionmaker provides the following trade-off information: linear weights for comparing the difference between the best alternative and the worst alternative are 0.40, 0.10, and 0.50 for the attributes B, C–E, and F, respectively. Table V provides the resulting objectives measure values. We note that x_1 continues to dominate x_3 and x_5 but now also dominates x_6. Once again, x_2 dominates x_4 and x_6. Also, x_4 dominates x_6. The digraph of these new relationships is presented in Fig. 12.

TABLE VI
TRADE-OFF WEIGHTS FOR ATTRIBUTES G AND H

	ATTRIBUTES	
	G	H
BEST ALTERNATIVES	x_1	x_2, x_4
WORST ALTERNATIVES	x_6	x_6
WEIGHTS	.10	.90

TABLE VII
ALTERNATIVE SCORES FOR OBJECTIVE MEASURE VALUES: THIRD TRADE-OFF CASE

		ALTERNATIVE SCORES		
		A	B-F	G-H
ALTERNATIVES	x_1	70	51.3	64
	x_2	100	72.95	91
	x_3	60	22.075	49
	x_4	50	50.5	91
	x_5	65	20.775	52.5
	x_6	0	39.25	0

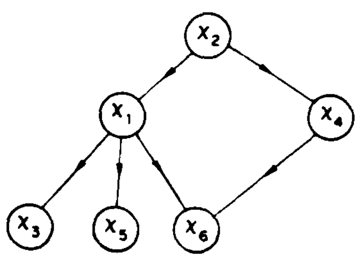

Fig. 13. Domination digraph for Table VI.

From this digraph we observe that x_6 need not be considered further if the objective is to seek the best and second best alternatives. We also note that if the decisionmaker could select x_2 as the most preferred alternative without further assessment, then x_3 and x_5 each would join x_6 as being at best the third most preferred alternative. If x_1 is the most preferred, then x_2, x_3, and x_5 would compete for second best, x_4 could be no better than third best, and x_6 could not be better than the fourth most preferred.

If the second trade-off has not provided the decisionmaker with enough information to make the desired selections, another trade-off assessment is required. We therefore move up the template by trading off at least some of the second level attributes. Assume the decisionmaker elects to trade off the attributes G and H. Table VI lists the best and worst alternatives and their associated incremental weights for each of these two attributes. Table VII lists the resulting objectives measure values, and Fig. 13 presents the associated digraph. It is now clear that x_2 is the most preferred alternative and x_1 and x_4 would com-

TABLE VIII
TRADE-OFF WEIGHTS FOR ATTRIBUTES A, G-F, AND G-H

	ALTERNATIVES		
	A	B-F	G-H
BEST ALTERNATIVES	x_2	x_2	x_2, x_4
WORST ALTERNATIVES	x_6	x_5	x_6
WEIGHTS	.40	.50	.10

TABLE IX
ALTERNATIVE SCORES FOR OBJECTIVE MEASURES VALUES: FINAL CASE

		ALTERNATIVE SCORES
		A-H
ALTERNATIVES	x_1	60.05
	x_2	85.575
	x_3	39.9375
	x_4	54.35
	x_5	41.6375
	x_6	19.625

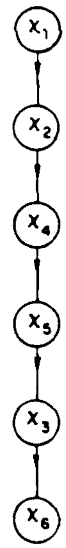

Fig. 14. Domination digraph for Table IX.

TABLE X
FINAL WEIGHTS FOR LOWEST LEVEL ATTRIBUTES

A	0.4000
B	0.2000
C	0.0100
D	0.0175
E	0.0225
F	0.2500
G	0.0100
H	0.0900

TABLE XI
ALTERNATIVE SCORES FOR OBJECTIVE MEASURE VALUES: THIRD TRADE-OFF CASE WITH VARIED WEIGHTS

ALTERNATIVE	ALTERNATIVE SCORES		
	A	B-F	GH
x_1	70	51.3	74
x_2	100	72.95	68.5
x_3	60	22.075	46.5
x_4	50	50.5	68.5
x_5	65	20.775	46.25
x_6	0	39.25	0

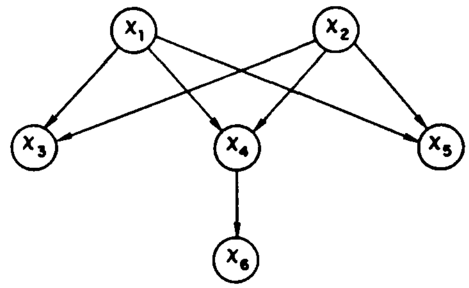

Fig. 15. Domination digraph for Table XI.

TABLE XII
POSSIBILITY SUBSETS FOR N BEST ALTERNATIVES, $N = 1, 2$

TABLE	FIGURE	ORDERED SUBSET CANDIDATES FOR N BEST ALTERNATIVES	SUBSET GUARANTEED TO CONTAIN N MOST PREFERRED ALTERNATIVES
1	9	$X_1 = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$ $X_2 = \{\{x_1,x_2\}, \{x_1,x_3\}, \{x_1,x_4\}, \{x_1,x_5\}, \{x_2,x_1\}, \{x_2,x_3\}, \{x_2,x_4\}, \{x_2,x_6\}, \{x_3,x_1\}, \{x_3,x_2\}, \{x_3,x_4\}, \{x_4,x_1\}, \{x_4,x_2\}, \{x_4,x_3\}\}$	$X_1 = \{x_1,x_2,x_3,x_4\}$ $X_2 = \{x_1,x_2,x_3,x_4,x_5,x_6\}$
3	10	$X_1 = \{\{x_1\}, \{x_2\}\}$ $X_2 = \{\{x_1,x_2\}, \{x_1,x_3\}, \{x_1,x_5\}, \{x_2,x_1\}, \{x_2,x_4\}, \{x_2,x_6\}\}$	$X_1 = \{x_1,x_2\}$ $X_2 = \{x_1,x_2,x_3,x_4,x_5,x_6\}$
5	11	$X_1 = \{\{x_1\}, \{x_2\}\}$ $X_2 = \{\{x_1,x_2\}, \{x_1,x_3\}, \{x_1,x_5\}, \{x_2,x_1\}, \{x_2,x_4\}\}$	$X_1 = \{x_1,x_2\}$ $X_2 = \{x_1,x_2,x_3,x_4,x_5\}$
6	12	$X_1 = \{\{x_2\}\}$ $X_2 = \{\{x_2,x_1\}, \{x_2,x_4\}\}$	$X_1 = \{x_2\}$ $X_2 = \{x_1,x_2,x_4\}$
9	13	$X_1 = \{\{x_2\}\}$ $X_2 = \{\{x_2,x_1\}\}$	$X_1 = \{x_2\}$ $X_2 = \{x_1,x_2\}$

pete for second most preferred. If x_4 were chosen to be second best, then x_1 would be the third most preferred alternative, with x_3, x_5, and x_6 competing for fourth best. If, however, x_1 were chosen as second best, x_3, x_4, and x_5 would compete for third best, and x_6 would be at best the fourth most preferred. We now have moved up the attribute tree or template such that there are only three lowest level attributes. They correspond to safety, costs, and attractiveness in terms of beauty and spaciousness. At higher levels in the attribute template, trade-offs become more difficult to make as the "noncommensurateness" of the attributes increases as we move up the tree. Choicemaking at this point, or prior to this point, seems a strong possibility. Interactive computer graphics to present the results of figures and tables such as these to the decisionmaker will be most desirable as aids in choicemaking. It will be very difficult to do this for many objectives which is another reason that appears to support the potential attractiveness of our proposed approach.

In order to order the alternatives totally, if such a total ordering were necessary, trading off all three of the remaining attributes would be required. All hypothetically assessed trade-off information and associated results are provided in Tables VIII and IX and in Fig. 14; Table X lists the final weights for the initial attributes.

We expect that the decisionmaker frequently will not require full trade-off assessment for choicemaking and hence will elect to reach a decision using the decisionmaking support process before the final assessment. Regardless of the stage at which final evaluation is made, however, the decisionmaker is likely to want to know the impact of trade-off weight variation and initial objective measure value variation on the digraph structure. For example, let the weight for attribute G in Table VI be changed from 0.10 to 0.35, thus changing the weight for attribute H to 0.65. The new table of objectives measure values are presented in Table XI and its associated digraph in Fig. 15. This sort of sensitivity analysis should always precede final alternative selection especially where there are any questions concerning subjectivities of either the alternative scores or criterion (attribute) weights.

Table XII presents the $\mathcal{X}_n$ and X_n, $n=1,2$, for this example for each level of trade-off considered. We note that the number of elements in both $\mathcal{X}_n$ and X_n decrease as the number of trade-offs increase, which is in agreement with the corollary presented in Section IV.

VI. Conclusion

We have presented an approach to choicemaking which may substantially reduce the amount of trade-off information required from the decisionmaker relative to the MAUT approach. The approach presented may be used to complement the MAUT approach and as an alternative to it. The most general assumption considered relative to the amount of preference information that should be elicited from the decisionmaker was the partial preference feedback assumption. Two special cases of the partial preference feedback assumption were considered. The complete preference feedback case was considered in detail, and three procedures were presented to enhance the likelihood that this assumption would hold. The no preference feedback case was examined by means of an example. We remark that the partial preference feedback case is, to borrow a phrase from the optimum systems control literature, a closed-loop procedure involving decisionmaker preferences which augment those preferences elicited in construction of the vector of objectives functions. By analogy, the no preference feedback case is an open-loop procedure.

Future research and application of the MOOT based approach to choicemaking will include risk and uncertainty considerations and will indicate behaviorally relevant procedures for implementation of the approach to form a planning and decision support process for aided choicemaking. Contrasts, comparisons, and complementarity with the well-known MAUT approach will, of course, be of interest.

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ON HUMAN INFORMATION PROCESSING AND INFERENCE ANALYSIS AS LARGE SCALE SYSTEMS PROBLEMS

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This paper presents a survey of human information processing and associated inference analysis in decentralized organizations as a large scale systems problem. One purpose of aids for inference analysis is to assist humans in using techniques which allow them to make better use of information. The use of hierarchical systems concepts with structured logical reasoning protocols is discussed and illustrated in this overview paper.

INTRODUCTION

In processing information, it is generally assumed that observed data is directly related to a hypothesis under consideration. Further, it is assumed that the impact of data on the probability of a hypothesis being correct can be computed in a straightforward fashion by use of Bayes' rule. This process is known as inference. It may operate in an inductive or deductive manner. Typically inferences are structured in a deductive fashion and data are processed in an inductive manner. There are at least two complexities which may, in practice, make this difficult to accomplish:

1. The complexity and knowledge requirements associated with many information processing tasks may be so great that no single individual may be able to supply the required information.

2. Multi-stage information processing tasks, that is tasks where data are not directly and immediately related to the hypothesis under test, are complex. Rather involved computations may be required.

Hierarchically decomposing multiple stage inferential information processing tasks into a number of single stage inference tasks, and then aggregating the results of these single stage inferences, may resolve each of these two potential difficulties. It may allow experts, with an appropriate knowledge base, to participate in portions of the inference task in a logically consistent manner that is also appropriate in terms of the experts knowledge base. Computations in a hierarchical framework will generally be easier than computations in an undecomposed format. There is experimental evidence to indicate [1, 2] that hierarchical decomposition of human inference tasks reduces the tendency of humans to selectively process more favored data while virtually ignoring other data. The result of this is cognitive information processing bias [3, 4] that may result in a very flawed judgment and choice.

This paper discusses, at a conceptual level, relations between frameworks for information processing and hierarchical information processing. We indicate, in a preliminary way, how structured frameworks of decision situations influence information needs and information processing within a hierarchical inference format.

2. Hierarchical Inference Analysis

In a very large number of contemporary areas of interest, there are issues involving the possible occurrence of uncertain-to-occur events. Often we can obtain more veridical results in determining probabilities and likelihoods for event outcomes if they are conditioned upon the occurrence of other events and/or decisions. This approach forms the basis for cross impact analysis, hierarchical inference analysis, and other related approaches in which probability structures or probability diagrams have proven to be of considerable value [5].

In these approaches, it is assumed that observed information can be directly related to postulated hypotheses and that the impact of observed information on the probability of a given hypothesis being true, or occurring, evolves sequentially according to Bayes rule. The complexity of many contemporary large scale issues is such that the amount and type of cognitive skills and technical knowledge, required to express all appropriate probabilities or likelihood ratios which infer or link all information elements to postulated hypotheses, is beyond the unaided capability of any single individual, and often even the aided capability. One possible approach to ameliorate this situation is to disaggregate the complex issue into

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a hierarchical structure such as the generic structured framework suggested by Figure 1.

In the hierarchical approach to information structuring and associated inference, a number of intermediate elements are identified. These elements typically represent activity states or activity indicators which are relevant to the postulated hypotheses. These activity states or activity indicators are presumed to be meaningful representations of a portion of the issue. They correspond to the six elements in Figure 1. Probabilities which relate observed information to these intermediate elements are assessed or elicited, as well as the probabilities which relate these intermediate elements to the postulated hypotheses. Also included are propositions and implications from logical reasoning such that it becomes possible to display credibility and plausibility indices that encourage detection of individual inconsistencies as well as group differences with respect to recognition and enhanced communication of differences in message interpretation and the rationale for this.

The rationale behind this approach is behaviorally and organizationally compelling. Parties at interest to a given issue may be expert in diverse portions of the complete issue and a single individual, or group, will probably not have sufficient experience and knowledge to relate lowest level information to highest level hypotheses. If a complex issue is hierarchically decomposed, it may be possible to utilize the abilities and knowledge of various experts in an efficient and logically consistent fashion. Presumably, organizations are structured in a hierarchical manner to take advantage of opportunities such as these.

There is no unique structure for a hierarchical model of a given issue. These models are necessarily subjective contingency structures in that they can only represent a conceptual model of a particular issue, and the way in which a particular issue is disaggregated. The influence of various cognitive styles of the individual or group constructing the model and the influence of various constraints, such as environmental constraints, are generally the strongest determinants affecting the choice of a particular hierarchical model. Still the generic structure of Figure 1 appears fully appropriate to capture the subleties required by many effective information processing tasks. It would appear appropriate as well for providing a logical framework for discussing differing plausible interpretations of assumptions and intents.

A generic prototypical hierarchical inference structure will consist of five different types of elements:

1. H_i, i = 1,2,..., a set of mutually exclusive hypotheses which describe the issue under consideration, often in terms of claims.

2. A_i, i = 1,2,..., a set of identified inferred activities, or possible surrogate activities, which represent observed cues relative to the issue or diagnostic of the hypotheses under consideration. Each activity category, A_i, may be further disaggregated into a set of mutually exclusive exhaustive subactivity states A^j_i, j = 1,2,... .

3. I_i, i = 1,2,..., activity indicators which impact upon the hypotheses throgh various activities. Any activity indicator may consist of a number of mutually exclusive exhaustive subactivity indicator state variables, I^j_i, j = 1,2,..., which are presumed to be diagnostic of the related impact activity state variable, A_i.

4. E_i, i = 1,2,... events, or outcome states that influence the indicators. The events are not guaranteed to have occurred; rather there are information reports on occurrence, or nonoccurrence of events.

5. V_i, i = 1,2,..., data or information variables which may impact directly on hypotheses, activities, activity indicators, or events.

Data or information variables or elements may connect with, or impact, directly to hypotheses, an activity, or activity indicator variables or elements. Activity indicator states must connect with or directly impact activity states. Activity states may connect to activity indicator states or hypotheses. By definition, the hypotheses are at the top level of the inference hierarchy and correspond to the "claim" in Figure 1. Since an activity state may be just a surrogate for a hierarchy, we can, in effect, allow hypotheses to appear at other than the top level of the structure. Formally, however, hypotheses will be at the top of the structure. Figure 2 represents a prototypical hierarchical inference structure. Hypotheses are at the top level of these structures. The second level contains various activities, A_i, which directly influence the hypotheses. The next level contains indicators which impact directly on activities that are at the second level. The fourth level contains the events, which influence various activity indicators. At the fifth and final level of these structures, we have various information variables or observables. Figure 2 represents the inference structure thus described. The procedure to develop an hierarchical inference tree involves determination of an important influential or causal structure. There are a variety of structuring procedures that can be used for this purpose [6, 7]. In general, the inference tree can have many more than 5 levels.

Computations based upon hierarchical inference are conceptualized in a relatively simple way. The posterior probability that a particular hypothesis is true given an observed datum is, where P (/) denotes conditional probability,

$$P\ (H/V) = \frac{P\ (H)}{P\ (V)}\ P\ (V/H) \qquad (1)$$

To make this calculation we need to know the prior probability that the hypothesis will be correct and the prior probability that the data will be observed. Further we need to know the conditional probability that the data will be observed conditioned upon the hypothesis being valid.

When data influence various events, E_j, then we can calculate the conditional probability of a hypothesis being correct from

$$P\ (H/V) = \frac{P(H)}{P(V)} \sum_j P\ (V/E_j,H)P(E_j/H) \qquad (2)$$

Now we need to know the probabilities that the datum will occur conditioned upon the events and the hypothesis. This sort of expansion can be continued such that we can evaluate the probability of a hypothesis conditioned upon many events, activities, activity indicators, and data. Generally, the calculations are extraordinarily complex unless independence, $P(V/E_j,H) = P(V/E_j)$, exists.

We need to obtain P(H/V) for use in a decision analysis effort. However, estimates of the quantities on the right hand side of Exs. (1) and (2) are normally much more easily obtained. These are readily available from a hierarchical inference structure.

We may assess probabilities for a hierarchical inference structure in either deductive or indictive form. In a deductive structure, the hypotheses at the top of the structure are "source" elements for a single source digraph tree which begins at the various hypotheses. In an inductive structure the hypotheses at the top of the structure are "sink" elements for a single sink digraph tree which terminates at the various hypotheses at the top of the tree. The hierarchical inference structure of Figure 2 may be replicated several times over to form the structure of Figure 1.

3. Hierarchical Inference as a Large Scale System Issue

It is possible to examine and evaluate the potential increase in potential predictive accuracy made possible by replacing simple hierarchical inference models in which information samples are assumed to be received independently, by one in which information samples are assumed dependent. There are two basic types of information sample dependencies; enhancing and inhibiting. In the enhancing dependency case, the probability in favor of a hypothesis being true if two correlated information samples are received is greater than what it would be if the two samples were independent. The opposite is true in the case of inhibiting dependency information. If either enhancing or inhibiting information is present, i.e., if dependent information samples are observed, then an extra activity state variable can be introduced as a surrogate to formally make the information samples independent. The use of this intervening activity state variable will allow observed information to be diagnostic of this surrogate state which is, in turn, diagnostic [8] of the fundamental hypotheses under consideration. Figure 3 displays some possible numerical results to support this.

There is a considerable elicitation and assessment effort saving if information samples are independent since it is not necessary to elicit or estimate conditionally dependent probabilities or likelihoods. There are at least two major implications of this for inference analysis:

a) Careful structuring of decision situation and inference structures to result in a minimum of information dependencies may be very rewarding. A study of the tradeoffs between structural and information dependency complexities is, therefore, an important research topic.

b) Aggregating strongly correlated information samples prior to elicitation or estimation of probabilities or likelihoods may result in reduced dependencies and therefore reduced overall effort. Again, there are some interesting tradeoffs involved in this although they will be more subjective and difficult to quantify than in case a.

It is possible, also, to examine and investigate the effects of nonstationary environments upon hierarchical inference. In most hierarchical inference models, it is assumed that the marginal or prior probability of hypothesis H_i being true, $P(H_i)$, is independent of information being received, or time. This is, of course, not necessarily true. A Kalman-filter-like structure [9,10] may be used to accommodate the estimator part of a combined estimation-detection type scheme for processing inferential information. Although much has been written about combined estimation and detection for optimal control type problems there appears to have been few if any efforts to relate this to typical decision analysis and inference analysis type problems. A difficulty in accomplishing this is that structural changes are typically incorporated in a deductive fashion whereas parametric information updates are processed in an inductive fashion in the Kalman filter.

Of course, a simple probability function for hypothesis change can be used to adjust the prior probabilities after each information update. This change probability would be a function of information received and time. Clearly the approach is suboptimal but it may be effective in that the approach is simple, relatively easy to understand, and probably does not introduce major errors. Thus, it should result in enhanced analysis ability with respect to modeling and comprehension of decision situations in nonstationary environments. A simple and appropriate, but suboptimal, approach is almost always better than a more correct complex, difficult to comprehend, and improperly used approach, especially in situations in which process concerns, which involve human and behavioral factors, are important.

Also, it is possible to examine and evaluate the effects of various stopping rules and newly generated or identified hypotheses. In many information purchasing studies, a decisionmaker may make a decision after receipt of any given sample of information. "Optimal stopping" is a term used to refer to sufficiently well structured inference situations where the costs, diagnosticity, utilities and payoffs, are well known. One can precompute and select a best time to make a decision. Several realities complicate the picture, however. Information gathering and processing behavior may be quite different after a decision has been made than before the decision was made. More information of a given quality may be required in order to influence a decisionmaker to change a decision once it has been made, i.e., after commitment to a decision, than that initially required to make the decision. Prior to decision commitment, information is processed to increase the likelihood of making a good decision. Postdecisional commitment information is often used, or misused, to justify the already committed to course of action. Clearly, inferences are made postdecision as well as predicision, and a study of postdecision inference analysis should lead to rather interesting and useful results.

Hypothesis evaluation and inferences are made possible because of a very precise and specific problem formulation and structuring in which uncertainties are represented by a (finite) set of exhaustive and mutually exclusive hypotheses. This exclusivity requirement precludes identifying and expanding the hypothesis space as the inference analysis effort progresses in time. In realistic situations, the receipt of additional information will often result in suggestions for and identification of new hypotheses as well as in updates to hypothesis probabilities.

There are a number of interesting implications to this. One and only one hypothesis from a mutually exclusive well-posed set can be true. The conventional structure of an inference analysis problem does not provide for the possibility that more than one of the hypothesis is true, or that none of the initially posted hypotheses may be true, or the one that is true may not be identified until later. Structurally, we may add additional hypotheses: or for that matter activities, activity indicators, events and/or information sources. The structural changes due to bordering an additional inference element onto an existing inference tree result in some interesting and apparently unexplored problems in structural sensitivity analysis. To investigate the effect of inference structure changes due to sequential hypothesis generation may lead to interesting conclusions concerning the "value of structure and information" that may compliment the many existing studies concerning (only) the value of information. This will be especially the case when it becomes possible to determine a useful problem formulation to allow joint consideration of optimum stopping, postdecision inferences, and pre and postdecision sequential hypothesis generation.

4. Summary

This paper has described contemporary efforts in hierarchical inference as a large scale systems problem. A number of suggestions have been made for efforts which add considerable realism to, and which enhance the utility of, formal hierarchical inference approaches in enhancing human information processing.

Many recent studies in cognitive science indicate that people are flawed information processors due to their failure to [12]:

1. Seek disconfirming information
2. Analyze disconfirming information
3. Identify alternative hypotheses
4. Consider whether evidence supporting a favored hypothesis supports alternative hypotheses just as well; or perhaps even better.

The goal of adjuvants to human information processing is to eliminate these flaws through appropriate detection of biases and flaws, and through appropriate debiasing procedures.

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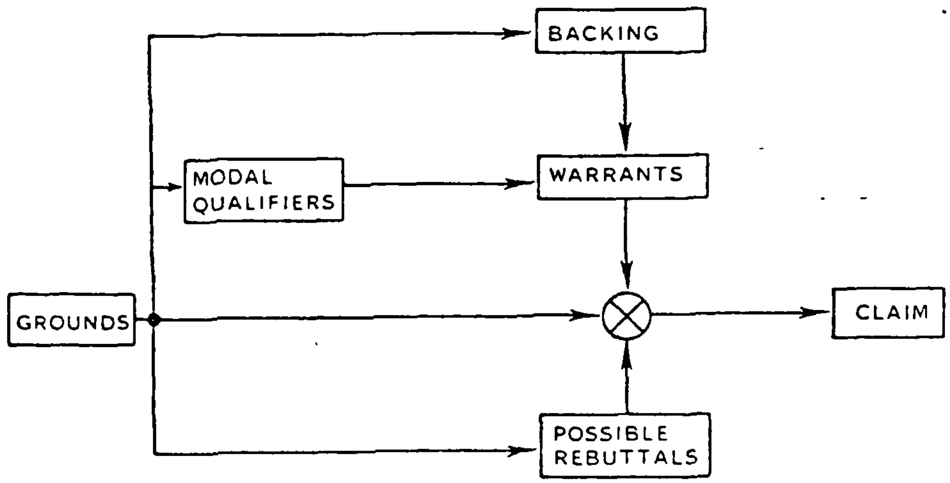


Fig. 1. A Possible Structure for Information Processing Based Upon the Six Elements of Logical Reasoning

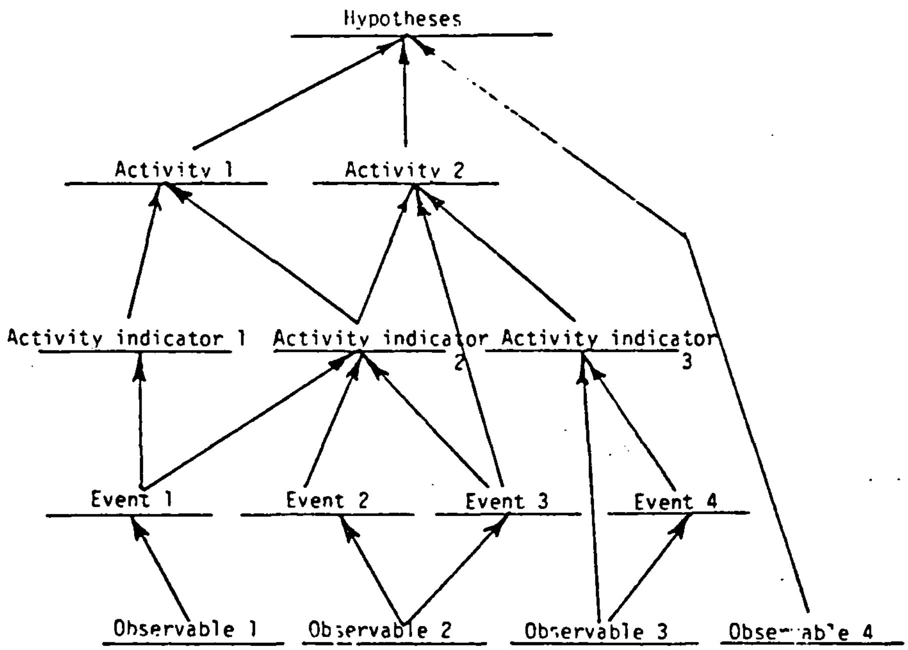
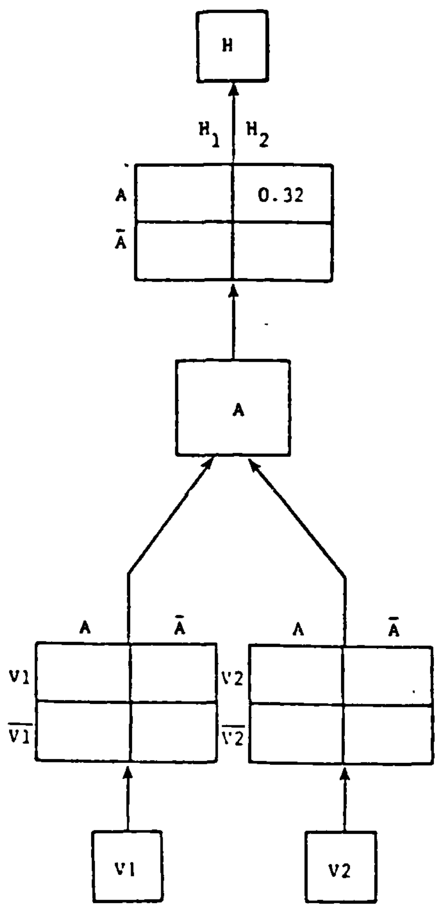


Fig. 2. Prototypical Hierarchical Inference Structure



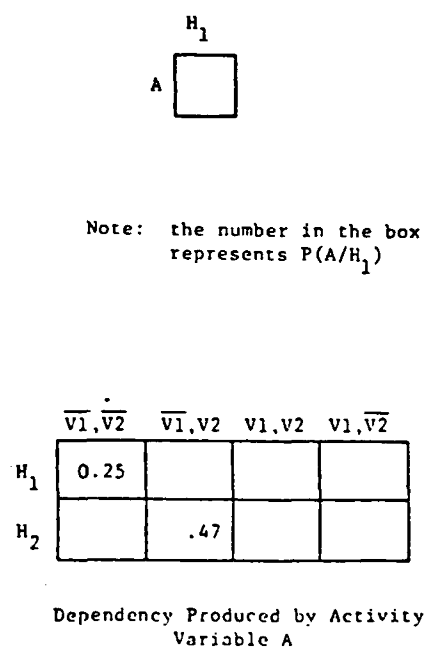


Fig. 3. Illustration Showing how Redundancy or Dependency can Result from Independent Data and Introduction of a State Variable. Continued use of Bayes' Rule enables determination of $P(H,/V1,V2)$ etc.

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ON HUMAN SYSTEM IDENTIFICATION

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Abstract. This paper discuss needs for and potential accomplishments that could result from, development of a theory of human system identification.

NEEDS FOR HUMAN SYSTEM IDENTIFICATION

Much recent research has concerned cognitive heuristics and biases [1] that degrade information processing [2]; human inference; and associated human judgment, choice and decisionmaking [3,4,5]. It is very important that planning and decision support processes deter use of these biases as well as seek to overcome the many other fundamental limitations of systems engineering and related areas [6]. To do this requires the design of planning and decision support systems and processes [5,7] that are adaptive to user requirements, problem situations, and the experiential familiarity of the user with these elements that, taken together, constitute the contingency task structure. Design of an appropriate planning and decision support system requires an integration of the descriptive and the prescriptive. It requires that we be able to not only detect, but that we also correct for shortcomings in human information processing such as to support efficient and effective management of information. It requires, in effect, system identification of the human-organization-computer process that involves knowledge organization, information processing, and associated judgment and choice.

The purpose of Information Systems for planning and decision support is to provide timely, relevant, and accurate information such as to enhance human judgment, and decisionmaking efficiency and effectiveness, concerning planning and the associated resource allocations. Not only should these systems, and associated processes, aid and support the decisionmaker in making efficient and effective decisions; they should also enhance confidence both in present decisions and in the ability to make more efficient and more effective decisions in the future.

For maximum efficiency and effectiveness, available resources must be allocated and coordinated both in space, through a hierarchy of decisionmakers, and in time, as new information arrives and the environmental situation extant changes. Associated information acquisition, analysis, and evaluation must, as a consequence, be distributed both in space and in time. Information acquisition, analysis and evaluation must be accomplished selectively in space and time since different decisionmakers have different information needs. In addition, it will be physically impossible and behaviorally undesirable to supply all relevant information to each decisionmaker in the hope that it will be effectively cognized and utilized. Further, differences in education, motivation, experiences with the environmental situation extant and stress will influence cognitive information processing style. Consequently, a central task in the design of systems is that of selection and choice of appropriate information system architecture to enhance selective information acquisition, analysis, and interpretation in order to provide each user of the system with the most appropriate information at that appropriate time. Thus questions of information selection, information aggregation in space and in time, and the contingency task structure, which is a function of the environment and the decisionmakers, become of major importance.

As is true with systems engineering efforts generally, the structural components of a planning and decision support system consist of adjuvants for:

a) issue formulation

b) issue analysis

c) issue interpretation

The issue formulation adjuvant will enable decisionmakers to acquire, process, and evaluate information in order to perceive the current state of the environment; to compare that perception with a desired state; and to identify possible action alternatives which might cause the environmental state extant to change such as to be more in conformity with the desired environmental state.

The issue analysis adjuvant will enable determination or analysis of the impacts of the proposed action alternatives in terms of environmental state changes. Finally, the interpretation adjuvant will allow valuation, in accordance with a value system, of the identified action alternatives in terms of their impacts upon the environment such as to enable selection of one or more of the proposed action alternatives for deployment or implementation.

A variety of questions and concerns make design and application of the aforementioned adjuvants a non routine task. Perhaps central among these questions and concerns is the fact that decisionmakers may differ considerably in their education, motivation, and prior experience with particular operational environment conditions extant. Whether a formal operational style or concrete operational style of cognition is appropriate for a particular contingency task, and whether or not this will be used, is strongly dependent upon these factors and the type of stress that they produce [8].

Automated aids must, consequently, be flexible in the sense of being capable of adaptation to a variety of information processing and judgmental styles that are difficult to specify a priori. Further, they must be parsimonious with respect to overloading neither the data base management subsystem which will degrade efficiency, effectiveness and reliability of information retrieval from computer memory; nor the human information subsystem, which will result in an increase in stress and the likelihood of use of cognitive information processing biases and poor judgmental heuristics.

Expert decisionmaking is typically done in a concrete operational mode of cognition, and involves use of one or more of a variety of typically wholistic judgmental efforts. These decision styles are doubtlessly appropriate for, and potentially capable of excellent results in environments when the decisionmakers diagnosis of situationally caused stress, and when other components of the contingency task structure, are appropriate for these forms of concrete operational behavior. In unstructured situations and in unfamiliar environments, a formal operational mode of cognition is generally appropriate. One task of an automated system is to assist decisionmakers in acquiring the experience and situational familiarity appropriate for cognizing the formal operational thought process into a situation where concrete operational thought is efficient, effective, and otherwise appropriate. Concrete operational thought typically involves use of forward processed judgmental heuristics based upon a preceptive mode of information processing. It is the preferred cognitive style when and if it is "fully appropriate" for information processing and judgment. Inferior cognition and/or poorly perceived concrete environmental situations may, however, result in a combination of information processing biases, poor judgmental heuristics, and value incoherencies which may result in extremely poor use of information and poor aggregation of facts and values, perhaps accomplished intuitively and wholistically, to form judgments [1,4]. Extraordinarily poor concrete operational thought and associated judgment may well be the result of these maladies which may be very effectively reinforced through feedback to create an Outcome Irrelevant Learning System [9]. A major result of many recent studies in behavioral decisionmaking is that by no means do we necessarily learn well from experience [10].

Thus it is desirable that appropriately designed automated aids, or adjuvants, for planning and decision support systems be capable of:

- assisting in the evaluation of alternative plans and course of action that involve formal operational thought processes,
- assisting in the transfer of formal operational situations to concrete operational situations,
- assisting in evaluation of alternative plans and courses of action that involve concrete operational thought processes,
- assisting in the avoidance of information processing biases and poor judgmental heuristics,
- assisting users to combine cues received from multiple source in an appropriate fashion,
- assisting in the use of a variety of judgmental heuristics appropriate for given operational environments as natural extensions of a decisionmaker's normal cognitive style,
- assisting, to the extent possible, in the determination of whether a formal or concrete style of cognition is most appropriate in a given situation,
- assisting decisionmakers who need to use formal operational thought, and those whose expertise allows appropriate and effect use of concrete operational thought, to function together in a symbiotic and mutually supportive way,
- assisting decisionmakers in maximizing the many beneficial aspects of experiential learning.

Clearly there is a space-time dependency associated with these desired capabilities.

Also among the many concerns that dictate needs and requirements for automated systems is the fact that decisionmakers must typically make more judgments and associated decisions in a given period of time than they can comfortably make. This creates a stressful situation which can lead, as has been noted, to the use of poor information processing and judgmental heuristics, especially since judgments and decisions are typically based on forecasts of the future and, as a consequence, involve uncertainties and risks.

The basic requirement for an aiding procedure is to enhance the initial acquisition of information, and the structuring and analysis of the information received from multiple sources. Objectives associated with improvement in the situation extant and possible courses of action will also be identified. Typically there will be uncertainty with respect to this information and the events likely to follow from various alternative actions. Additional information may reduce this uncertainty but can only be obtained at various costs. Determination of the value of additional information will enable determination of whether it is worthwhile to acquire and analyze it. When new information is obtained, it is aggregated in with existing information. Additional information is obtained if it is believed to be desirable, that is to say cost effective, and is processed in a similar manner. This information processing sequence terminates when the decisionmaker determines that it would be non cost effective to obtain additional information [11]. This can occur because of one or more of a number of considerations and constraints: temporal, human and other resource limitations, etc. When information acquisition and analysis finally terminates, information evaluation and interpretation is accomplished and a judgment or decision is made. There have been a number of studies of human information processing [5] which generally indicate unaided information processing and associated judgment to often be flawed.

Not only is there a need to solve problems over a particular planning horizon, but it is often necessary to update the resulting solutions "periodically" as better information is obtained. Thus it is necessary to cope not only with planning horizons, but also to update solutions, or recommended alternative courses of actions, at various planning periods. These difficulties are further confounded with space and multiple decisionmaker, and related organizational, issues; with the cognitive style, experience, and contingency task structure related determinants of human information processing and judgmental mechanisms; and with potential information processing bias and judgment heuristics.

Thus we see that there are indeed formidable needs and issues to be resolved that are associated with the design of information processing and judgment aiding support systems. These relate to questions concerning appropriate functions for humans to perform. They concern the type of information which should be available and how this information should be acquired, analyzed, evaluated, summarized, stored, aggregated and presented such that it can be used most effectively in a variety of potential operational environments. They concern design of information systems with strong space-time-environmental dependencies. They concern design of information systems that can effectively "train" users to adapt and use appropriate concrete operational heuristics in those environments in which inexperience dictates initial use of the more inefficient and time consuming, but potentially more effective and reliable, formal operational thought. They concern design and use of information systems that support environmentally experienced users in the use of a variety of effective concrete operational heuristics. And because of use of the system by multiple decisionmakers, these tasks must be accomplished in a parallel architectural fashion.

ORGANIZATIONAL INFORMATION PROCESSING

Keen [12] acknowledges four causes of inertia relative to organizational information systems. He indicates that: information is only a small component; human information processing is experiential and relies on simplification; organizational change is incremental and evolutionary with large changes being avoided; and that data is a political resource affecting particular groups as well as an intellectual commodity. Each of these suggests the importance of a knowledge of the way in which information is processed by organizations. Only with a knowledge of descriptive process components of information processing can we design useful prescriptive aids.

Of particular interest among studies concerning information processing in organizations is a large body of literature concerning how individuals integrate or aggregate information and attribute causes [3]; how various ways of presenting information and how various cognitive structures influence behavior [13]; how decision makers determine information requirements and associated analysis techniques [14]; and how the actual process, as opposed to the substance, of judgment and decisionmaking evolves in particular situations.

Huber [15] and Tushman and Nadler [16] have developed a number of propositions, based on their own research and upon the research of others, reflecting various aspects of information processing and associated decisionmaking in organizatic... [.7]. In an effort to enhance efficiency, organizational information processing typically requires selective routing of messages and summarization of messages. Huber [15] identifies six variables associated with the routing of messages. Six propositions relative to message routing are identified and associated with these variables. Three propositions are associated with delay in messages, eight with organizational message modification, and four with message summarization. Identification of other variables which influence information processing by organizations would represent a desirable activity. To determine how these information processing variables are influenced by the information processing biases of individuals would seem especially desirable in terms of the likely usefulness of the results and the need for an expanded theory of group information processing biases. There appears to have been only limited results obtained in the area of cognitive information processing biases and use of inferior decision heuristics on the part of groups.

Especially noteworthy concerning results that have been obtained in this area are the groupthink studies of Janis and Mann [18]. Groupthink is a collective pattern of defensive avoidance, a concurrence seeking tendency of highly cohesive groups. When groupthink occurs, people develop rationalizations to support selectively perceived illusions or wishful thinking about issues at hand and collectively participate in development and use of a defensive avoidance pattern. In groupthink, a group collectively falls victim to one or more of the many cognitive biases.

Among the conditions which lead to groupthink are: high cohesiveness, insul-tion, lack of use of systemic procedures for search and appraisal, highly directive leadership, and a contingency task situation which leads to high stress. Among symptoms of groupthink are: an illusion of invulnerability, collective rationalization, belief in inherent group morality, excessive pressure against dissenting views, self censorship, illusions of unanimity, and members who shield the group from disconfirming information. Nine prescriptions are offered to avoid groupthink by Janis and Mann[18].

A major difficulty in human information processing seems to be failure to identify and use an appropriate structure that allows appropriate weighting of observed data. It is both the structure and the content within the structure that determines the essence of a decision situation. Investigation of the effects of various structured information processing/decision aiding frameworks and protocols upon the acquisition, analysis, and interpretation of information, and its integration with judgment and decision making activities, would appear to be a contemporary need in information system design. There are six elements found in explicit argument [19]:

1. claims or hypotheses
2. grounds or foundations to support the claims
3. warrants or justification for the grounds or foundations
4. backing or the general body of information that is presupposed by the warrant
5. modal qualifiers or circumstances contingencies or restrictions which will have to exist in order that the warrant truly supports the grounds
6. possible rebuttals or circumstances, contingencies, or restrictions which, if they exist, will refute or diminish the force of the warrant would appear to be the elements of interest for development of structured protocols.

A simplified block diagram of the interaction among these elements is given in Fig. 1. The information processing "structure," consisting in part of the decisionmakers view of possible and probable action courses and the "decision situation model," is specified by elements 3-6. Element 2, the "grounds," comprises the situational data pertaining to the operational conditions extant. The claim, element 1, is the empirical statement which is supported by other elements in this information processing structure. This structured information processing model is also sufficiently general to accommodate analytical hierarchical inference. Thus it may well provide a structured framework for information processing that can accommodate a variety of information processing styles and approaches and be appropriate for a variety of operational environments and contingency task structures. It would appear to so structure the information processing framework that areas where additional information is needed can be identified. Thus, this structured framework represents a communicable plausible way of capturing, as well as explaining, a rather complete view of the belief system of a decisionmaker with respect to a given issue.

Use of a structured information processing format may reduce the tendency for message distortion, perhaps to a considerable extent. Mitroff and Mason [20], have presented some suggestions concerning use of structured logical reasoning to cope with ill structured policy problems and the often occurring divergence between opposing formulations and perceptions of large scale issues. These appear especially relevant to the design of knowledge based information systems for planning and decision support. Potentially very useful results are obtained by using the structure and logic of rational argument to develop a basis for detection of inconsistencies and bias. These models may be used to detect inferences, and their likely consequences, that are the result of an inconsistent set of premises. It should be possible to do this in such a way as to make explicit those inconsistencies which are due to different perceptions of an issue. This aids in understanding the viewpoints of others as well as providing a basis for determining, perhaps by assigning a quantitative index to, the credibility of verbal reports and quantitative information.

Information processing biases that occur during formulation, analysis and interpretation of information are influenced by a number of factors; with stress and the contingency task structure being among the dominant influences. Also, there are a number of feedback mechanisms involved which influence these biases [3,9]. The often occurring propensity to not seek disconfirming information, and consequent reliance upon positive feedback only, leads to unwarranted confidence in judgment and various maladies as, for example, groupthink. Also, attention to content in messages at the expense of structure may result in the logical error of denying the antecedent statement when there may be no information whatever that supports this inference. There are at least four related consequences of human information processing limitations: selective perception of information in accordance with prior expectations; improper aggregation of information cues received from multiple sources; improper use of feedback; and limited and faulty memory and recall. To avoid these limitations, it would appear necessary to identify and analyze potentially disconfirmation evidence, and to identify and test alternative hypotheses, using all available information. This must, of course, be accomplished within an appropriate framework.

Information formulation biases are generally due to the use of agenda dependent "editing" or "framing" rules. Organizations doubtlessly utilize somewhat similar editing and framing rules in their transmission of messages. A number of these have been suggested by Tversky and Kahneman [21,22] and others. It is on the basis of scenario descriptions and summaries that issues, and prior statistics for information processing, are determined. One of the fundamental claims of prospect theory, and related theories, is that these editing or framing rules lead to agenda dependent information formulation and related information acquisition. Knowledge of editing rules used in a given situation will generally allow determination of a set of debiasing procedures. It is quite possible that various structured procedures and protocols, based upon a set of required specifications to determine information formulation needs for reasoning models could serve as useful adjuvants to the effective determination of "proper" editing and summarization rules which avoid various error formulation biases. In effect, the structured framework would become a normative framing and editing guide for decision situations.

SUMMARY

We have examined some of the needs for human system identification as they relate to the design of knowledge based systems for planning and decision support.

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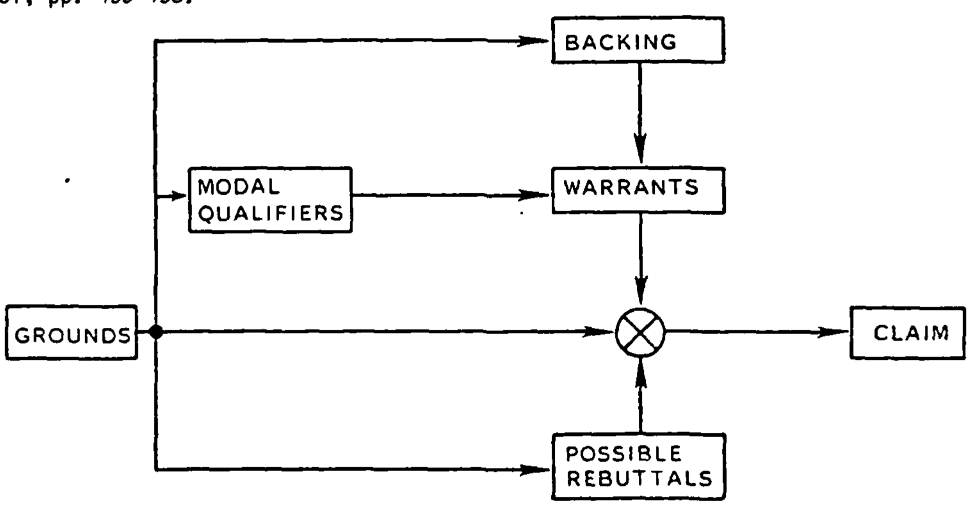
Fig. 1. A logic Structure for Rational Argument



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AN INTERACTIVE APPROACH TO ALTERNATIVE RANKING INVOLVING INVERSE DECISION AIDING

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Abstract. We present an iterative and interactive approach to aid a decisionmaker in selecting an alternative from a finite set of alternatives under conditions of outcome uncertainty. This approach allows parameters, i.e., probabilities and utilities, to be described imprecisely. The decisionmaker can: determine the impact of imprecise parameter values on alternative ranking, determine the implications of a wholistically determined alternative ranking on parameter values, and elect to more precisely describe the values of the parameters. The intent of iteratively performing these three tasks is to determine descriptions of parameter values and an alternative ranking that are acceptable, consistent, and support the selection of the most preferred alternative.

INTRODUCTION

In this paper, we consider the issue of helping a decisionmaker select a single alternative from a finite set of alternatives under conditions of outcome uncertainty. A normative approach is taken which is based on the maximization of expected utility. Three key characteristics of the approach are:

1. probabilities and utilities may be imprecisely described as being members of given sets,
2. the ultimate goal of the analysis is to provide only enough alternative ranking information in order to support the decisionmaker's desire to select the most preferred alternative,
3. the decisionmaker is allowed to iteratively provide information about the probability and utility values, examine the impact of this information on alternative ranking and adjust and augment the parameter information as needed in order to produce a desired alternative ranking.

These three characteristics distinguish our approach from multiattribute decision analysis (DA), as described, for example in Keeney and Raiffa (1976). The DA paradigm typically requires precise probability and utility initially and then examines the impact of parameter imprecision on optimality via a standard post-optimality analysis. As indicated by the first characteristic, we allow parameters to be imprecisely described initially, thus essentially eliminating the need for post-optimality analysis.

The result of a DA exercise is a total ordering of the alternatives. Such alternative ranking specificity may be more than is necessary in order to support the decisionmaker's selection of a most preferred alternative. For example, physicians diagnosing a common ambulatory complaint found it quite satisfactory to have 10 to 12 diagnostic tests eliminated from a total number of 14 prior to test selection (White, C.C., E.C. Wilson and A.C. Weaver,1982). As indicated by the second characteristic, we do not necessarily require the assessment of information sufficient to determine a total order on the alternatives, which tends to reduce our data needs relative to DA.

The third characteristic suggests two features of our approach:

1. the decisionmaker, rather than a paradigm iteratively selects how much alternative ranking specificity is necessary for alternative selection.
2. the decisionmaker is allowed to (a) go back and adjust information regarding probability and utility values if those values produce a counterintuitive alternative ranking and/or (b) readjust his wholistic view of how the alternatives should be ranked on the basis of the impact of parameter values on alternative ranking.

The decisionmaker therefore has substantial control over how the decision aiding process evolves, a fact which presumably enhances decision aid acceptability. This and other behavioral and organizational considerations important in the design of decision support systems are discussed in (Sage, 1981).

We begin by formulating the decision aiding problem and specifying the types of information assumed to be available for its resolution. We state our model of parameter imprecision and present a quadratic program which determines an alternative ranking based on this parameter imprecision. Our approach to decision aiding is then presented. It can involve the determination of a description of parameter values based on a wholistic alternative ranking, which we call the inverse decision aiding problem and which we at least partially analyze. These results are illustrated by examples. Conclusions of our results are then presented.

PROBLEM FORMULATION

A single alternative is to be chosen from a finite set of alternatives A. Once an alternative is selected, a single consequence, from a finite set of consequences C, will result. The problem objective is to select the most preferred alternative from A.

SOURCES OF INFORMATION

We consider two possible sources of information useful in selecting the most preferred alternative: direct information and indirect information. Each type of information is described as a relation on A. R_D represents the presumably transitive relation determined by direct information and R_I represents the relation determined by indirect information. The pair $(a',a)\epsilon R_D$ if and only if the decisionmaker is willing to wholistically state a preference for alternative a' relative to alternative a.

PROBABILITIES AND UTILITIES

The determination of R_I requires more structure. Let p(a,c) represent the probability of consequence c occurring, given that alternative a was selected. We assume $p(a,c) \geq 0$ for all $a\epsilon A$ and $c\epsilon C$ and that $\Sigma_{c\epsilon C}p(a,c)=1$ for all $a\epsilon A$. Let u(a,c) represent the utility of selecting alternative a and receiving consequence c. We assume that $\min_{a\epsilon A} u(a,c)=0$ and $\max_{a\epsilon A} u(a,c)=1$ for all $c\epsilon C$.

PARAMETER IMPRECISION

Expected utility will serve as the basis for determining R_I; however, we will assume that the probabilities and utilities may not be precisely known. The justification of this assumption is as follows. Facts, represented by probabilities, may not be known precisely due to large confidence intervals, if the probabilities are statistically determined, or due to lack of consensus or confident expertise, if the probabilities are subjectively determined. Values, represented by utilities, may not be known precisely because the decisionmaker may not want to explicitly reveal preferences for a variety of reasons, e.g., legal, political, personal. Lack of time necessary for parameter assessment also represents a possible barrier to precise parameter value determination.

We describe parameter imprecision as follows. Let $\#C$ represent the total number of possible consequences, i.e., card $(C)=\#C$, u(a)=row $[u(a,1),\ldots, u(a,\#C)]$, and p(a)=col $[p(a,1),\ldots, p(a,\#C)]$. Assume for each pair $(a',a)\epsilon A\times A$, $a'\neq a$, there is a set U(a',a) and a set P(a',a). U(a',a) represents constraints on u(a') and u(a), i.e., $[u(a'), u(a)]\epsilon U(a',a)$, and P(a',a) represents constraints on p(a') and p(a), i.e., $[p(a'), p(a)]\epsilon P(a',a)$. The following example illustrates the usefulness of these descriptions of parameter imprecision.

EXAMPLE 1. A physician must decide whether to operate (OP) or to treat with drugs (T), i.e., A={OP,T}. Two consequences are possible, bad (B) and good (G), i.e., C={B,G}. If the consequence is good, it is better to have intervened as little as possible; however, if the consequence is bad, more intervention would be perceived as better. Thus, $0=u(T,B)\leq u(OP,B)\leq u(OP,G)\leq u(T,G)=1$, and U(T,OB) represents the set of all utility values that can satisfy these (linear) inequalities. The probability of a good consequence is enhanced by the operation. Therefore, $p(OP,B)\leq p(T,B)$, and P(T,OB) represents the set of all probability values that can satisfy this (also linear) inequality.

DEFINITION AND DETERMINATION OF R_I

We can now define R_I: $(a',a)\epsilon R_I$ if and only if $u(a')p(a')\geq u(a)p(a)$ for all $[u(a'),u(a)]\epsilon U(a',a)$ and $[p(a'), p(a)]\epsilon P(a',a)$. We observe that if U(a',a) and P(a',a) represent linear constraints, then $(a',a)\epsilon R_I$ if and only if the optimal value of the criterion of the following quadratic program is nonnegative:

$$\text{minimize: } u(a')p(a')-u(a)p(a)$$

$$\text{subject to: } [u(a'),u(a)]\epsilon U(a',a)$$
$$[p(a'),p(a)]\epsilon P(a',a).$$

We observe that if either probabilities or utilities are known precisely, the quadratic programs become linear programs. Unfortunately, Kuhn-Tucker conditions for these quadratic programs are only necessary, an easily shown fact, and hence their use will produce only an upper bound $\bar{R}_I$ on R_I, i.e., $R_I \subseteq \bar{R}_I$.

Use of this quadratic programming procedure is illustrated in the following example.

EXAMPLE 2: Consider the case where $A=\{1,2,3,4\}$, $C=\{B,G\}$, $p_i=p(i,B)$, and

$$p_4 \geq p_3 \geq p_2 \geq p_1,\ p_4 \leq 0.1$$

$$0 = u(4,B) \leq u(3,B) \leq u(2,B) \leq u(1,B)$$

$$u(1,B) \leq u(1,G)$$

$$u(1,G) \leq u(2,G) \leq u(3,G) \leq u(4,G) = 1$$

$$u(3,B) = 0.1,\ u(2,B) = 0.2$$

$$0.8 \leq u(3,G) \leq 0.9$$

$$0.5 \leq u(2,G) \leq 0.6.$$

This example represents an extension of Example 1 for the case where there are four available alternatives and where certain additional restrictions on the parameters are given as above. Presumably, we have been able to assess an upper bound on p_4, exact values for u(3,B) and u(2,B), and bounds on u(3,G) and u(2,G). Use of standard quadratic programming software indicates that $\hat{R}_I = \{(4,2), (4,3)\}$. Routine algebraic analysis confirms that $R_I = \hat{R}_I$; that is, alternative 4 dominates alternatives 2 and 3 and alternatives 1 and 4 are nondominated. Hence, alternatives 1 and 4 are possible contenders for the most preferred alternative.

AN ITERATIVE PROCEDURE FOR DECISION AIDING

For the case where R_I represents the basis for alternative selection, the following iterative procedure has been suggested for decision aiding (White and Sage, 1980; White, Sage and Scherer, 1981):

IP1:

1. Assess U(a',a) and P(a',a) for all $(a',a)\in A\times A$, $a'\neq a$.
2. Determine R_I.
3. If an alternative can be selected from the nondominated set generated by R_I, then stop. If not, assess more precise parameter information, update U(a',a) and P(a',a) for all (a',a), and go to Step 2.

Generally, but not always, more precise parameter information produces an R_I with a smaller nondominated set, which presumably enhances alternative selection. Thus, the decisionmaker provides parameter value information, examines the resulting nondominated set, and then provides additional parameter value information if the nondominated set is too large for alternative set.

The availability of R_D suggests the following modification of the above iterative procedure:

IP2:

1. Assess R_D and parameter value information, and determine R_I.
2. Combine R_D and R_I to produce a new relation, R, on A.
3. If an alternative can be selected on the basis of R, then stop. If not, assess more wholistic and parameter value information, redetermine R_D and R_I, and go to Step 2.

We remark that by encouraging the decisionmaker to revise wholistic preferences among alternatives, i.e. to revise R_D, we are subjecting the associated knowledge base to a learning effect, which in general should improve the quality of human judgment. It is not, at present, clear what protocols to use in order to best accomplish Step 2. We feel, however, that potentially valuable components of Step 2 are comparing R_D to R_I, examining the implications of R_D in terms of parameter value information, and possibly revising the parameter value information and/or R_D due to learning effects. Such a procedure clearly puts the decisionmaker in more control of the decisionmaking process than does IP1. Support for the claim that such an occurrence enhances potential user acceptability can be found in Adelman and others (1982) and Beach (1975). We now examine a key substep of Step 2 in IP2: the determination of parameter value information that generates a given relation on the alternatives.

INVERSE DECISION AIDING

Assume that we wish to determine conditions on the parameters that would produce a given relation R, which we refer to as the inverse decision aiding problem. Clearly, u(a) and p(a), $a\in A$, must be such that if $(a',a)\in R$, then $u(a')p(a') \geq u(a)p(a)$. One of our motivations in examining the inverse decision problem is to find elements (pairs of alternatives) in R_D that generate acceptable constraints on the parameters additional to those constraints assessed from the decisionmaker. Additional constraints tend to make the parameter values more precise, which in turn tends to make the nondominated set associated with R_I smaller and hence more helpful in the alternative selection process. The procedure that we earlier proposed for determining R_I was based on the assumption that the constraints on the parameters were linear. We therefore prefer that a given relation R would produce linear constraints on the parameters. Additionally, it may be easier for a decisionmaker to understand the factual and/or preferential meaning of linear, rather than nonlinear, constraints on the parameters.

If lowest level attribute scores and probabilities are known precisely for problems involving multiple objectives and an additive utility function, then linear inequalities can be determined on the tradeoff weights. That is, if $p(a)$ is precisely known for all $a \in A$, $u(a) = \sum_n \lambda_n u_n(a) = \lambda \bar{u}(a)$ for all $a \in A$, and the appropriately defined matrix $\bar{u}(a)$ is precisely known for all $a \in A$, then $(a',a) \in R$ implies $\lambda[\bar{u}(a')p(a') - p(a)] \geq 0$. Linear inequalities can also result under conditions involving some knowledge of parameter values, as we illustrate in the following example.

EXAMPLE 3: Assume $A = \{1,2,3,4\}$, $C = \{B,G\}$, and $R = \{(4,3), (4,2)\}$. We wish to determine what conditions on the parameters must be satisfied in order for $R \subset R_I$. First, consider the assumption that $(4,3) \in R$. This assumption holds if

$$p_3 u(3,B)+(1-p_3)u(3,G) \leq p_4 u(4,B)+(1-p_4)u(4,G),$$

a nonlinear relationship. Assume the decisionmaker agrees that $u(4,B) = 0$ and $u(4,G) = 1$. Then, algebraic manipulation implies that

$$p_4 \leq p_3[u(3,G)-u(3,B)] + [1-u(3,G)].$$

If the decisionmaker is further willing to put a lower bound on $u(3,G) - u(3,B)$ and an upper bound on $u(3,G)$, say 0.7 and 0.9 respectively, then we know that $p_4 \leq 0.7p_3 + 0.1$ implies $(4,3) \in R_I$. Similarly, if $u(2,G)-u(2,B)$ has a lower bound of 0.3 and $u(2,G)$ has an upper bound of 0.6, then $p_4 \leq 0.3p_2 + 0.4$ implies $(4,2) \in R_I$. Thus, if $u(4,B) = 0$, $u(4,G) = 1$, $0.7 \leq u(3,G) - u(3,B)$, $0.9 \geq u(3,G)$, $0.3 \leq u(2,G) - \bar{u}(2,B)$, $0.6 \geq u(2,G)$, and $p_4 \leq 0.3 \min (p_2,p_3) + 0.1$, then $\{(4,3), (4,2)\} \subset R_I$. These conditions are weaker than those imposed in Example 2. It is also easy to show that more precise parameter value information leads to a relation on A containing more alternative pairs. Thus, the above conditions imply $\{(4,3), (4,2)\} = R_I$.

CONCLUSIONS

We have presented and discussed a decision aiding procedure that allows for probabilities and utilities to be imprecisely expressed and permits the constraints on their values to be iteratively modified in order to produce a desirable ranking on the alternative set. The decisionmaker can see how his perceptions of facts and values affect alternative ranking and, importantly, what facts and values are required to support perceptions of a reasonable alternative ranking. Alternating between these two modes of thought will produce a set of probabilities and utilities, perhaps only imprecisely described, and a relation on the alternatives that are both justifiable and consistent. Future efforts will be directed toward further refining IP2 and developing a general perspective on the concept of inverse decision aiding. We anticipate that this concept will compliment social judgment theory (Hammond, McClelland and Mumpower, 1980), a procedure for identifying precise tradeoff weights based on an application of regression analysis to wholistic preferences among alternatives. An outstanding unresolved issue is how to deal with the situation where a set of probabilities and utilities and a relation on the alternatives exist that are believable to, perhaps even advocated by, the decisionmaker yet are not consistent.

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A TOP DOWN APPROACH TO IMPRECISE INTERACTIVE ATTRIBUTE WEIGHT STRUCTURE AND ALTERNATIVE SCORE ASSESSMENT*

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ABSTRACT

This paper describes a sensitivity based interactive top down procedure that will allow identification of the structure of an attribute tree, elicitation of attribute weights and alternative scores on lowest level attributes in such a way as to ultimately result in evaluation of alternative courses of action.

INTRODUCTION

For a variety of reasons a decisionmaker may be unwilling or unable to provide the precise information that is necessary to assess utilities and identify preferences. Generally, this complicates the search for a dominance structure that enables selection of a single nondominant alternative, from a mutually exclusive set, for implementation action (6-8). In order to cope with this possibility, we have developed a procedure for alternative evaluation that allows for elicitation of a decision situation structural model and identification of imprecise parameters within this structure (8-12). These imprecise parameters may represent event outcome probabilities, alternative performance scores on attributes, and tradeoff weights across attributes.

In this paper, we present a procedure for enhancing the assessment of a top down structure, and parameter information within this structure, for a hierarchical tree of attributes. We assume that any probabilities associated with event outcomes are precisely known. For ease in presentation we will illustrate our results only for the certain case. However our results apply equally well to the risky case.

The interactive decision support problem formulation is presented in the next section. The third section develops the approach to top down aiding and presents an illustrative example. Conclusions are presented in the final section.

PROBLEM FORMULATION

We consider a normatively based model of decision making under risk. We assume that there exists A alternative courses of action a_i, $i=1,2,\ldots,A$. When an alternative a_i is selected, one of C consequences, $x_j(a_i)$, $i=1,2,\ldots,C$ occurs. Consequence x_j follows from a_i with probability $p(x_j|a_i) = p_j(a_i)$. The utility, or bliss, associated with consequence $x_j(a_i)$ is denoted $U(x_j,a_i)$. We assume that the conditions for additive independence are satisfied such that the substantive criterion of choice that any sensible person should follow is to evaluate the subjective expected utility (SEU) of alternative a_i given by

$$EU(a_i) = \sum_{j=1}^{C} U(x_j,a_i)p(x_j|a_i) \tag{1}$$

for $i=1,2,\ldots,A$, and select the alternative with the largest SEU. Often the decisionmaker will find it difficult to specify the single scalar utility associated with implementing a_i and receiving consequence x_j. When the additive utility conditions hold, we can write the scalar total utility of implementing option a_i and receiving consequence $x_j(a_i)$ in terms of N components

$$U(x_j,a_i) = \sum_{k=1}^{N} c_k u_k(x_j,a_i) \tag{2}$$

where c_k is the tradeoff weight associated with the k^{th} attribute. Generally it is helpful to define outcome states x_j that are functionally independent of alternative a_i. Then the utility function will be, functionally, alternative independent. It is particularly desirable to accomplish this modeling since this will insure that the importance weights, the c_k, will not be alternative dependent. In this representation we have a common set of outcome states, x_j, for all alternatives. We are then assured that the weights, c_k, are alternative independent. If

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this modeling is not accomplished, assessment of utilities and weights can become a very tedious, although certainly not impossible, task.

With this assumption, our expected utility expression becomes in scalar notation and in somewhat more convenient vector-matrix form,

$$EU(a_i) = \sum_{k=1}^{N} \sum_{j=1}^{C} \rho_k u_{kj} p_j(a_i) = \rho^T U p(a_i) \tag{3}$$

We assume that the conditional probabilities $p_j(a_i) = p(x_j|a_i)$ are all known precisely. The utility components, u_{kj}, may be precisely known or may be imprecisely known with the imprecision expressed by inequalities

$$\alpha_j \le u_{kj} \le \beta_j$$

where α_j and β_j may be numbers or linear functions of the u_{km}, for $m=1,2,\ldots,C$. It is very important to note that our linear utility inequalities must be expressed within attributes. Thus u_{kj} cannot be expressed as a function of u_{nm} for $k \ne n$. This is behaviorally realistic. It would be difficult to imagine utility assessment across attributes.

In a similar way we assume that the attribute tradeoff weights are expressable by linear inequalities of the form $D\rho + e \ge 0$, subject of course to $\sum_{k=1}^{N} \rho_k = 1$, $\rho_k \ge 0$. Generally it is desirable that we obtain fixed numerical limits for probabilities and utilities in the form $\alpha_j \le u_{kj} \le \beta_j$ and $\varepsilon_k \le \rho_k \le \eta_k$. Linear equalities such as $\rho_1 \le \rho_2 \le \rho_3$ and $u_{k1} \le u_{k2} \le u_{k3}$ are quite acceptable mathematically but purely ordinal specifications such as these may well convey such great imprecision that it is not possible to establish any meaningful dominance pattern.

We can now write the minimum and maximum values of $EU(a_j)$. We obtain

$$\underline{EU}(a_i) = \min_{\rho,U} EU(a_i) = \min_{\rho} \sum_{k=1}^{N} \rho_k \underline{V}_k(a_i) \tag{4}$$

$$\text{where} \quad \underline{V}_k(a_i) = \min_{u_{kj}} \sum_{j=1}^{C} u_{kj} p_j(a_i) \tag{5}$$

for $k=1,2,\ldots,N$. We can minimize the expression for V_k term by term only because of the assumption that we express set inequalities in u_{kj} only within attributes and not across attributes. In a similar way we have

$$\overline{EU}(a_i) = \max_{\rho,U} EU(a_i) = \max_{\rho} \sum_{k=1}^{N} \rho_k \overline{V}_k(a_i) \tag{6}$$

$$\text{where} \quad \overline{V}_k(a_i) = \max_{u_{kj}} \sum_{j=1}^{C} u_{kj} p_j(a_i) \tag{7}$$

Despite the possible utility and weight parameter imprecision we can say with certainty that alternative a_i is preferred to alternative a_m ($a_i \succ a_m$) if and only if

$$\min_{\rho,U} [EU(a_i) - EU(a_m)] \ge 0$$

Using the definition of SEU in Eq. (3) in the foregoing results in

$$\min_{\rho} \sum_{k=1}^{N} \rho_k V_k(a_i, a_m) \ge 0 \tag{8}$$

$$\text{where} \quad V_k(a_i, a_m) = \min_{u_{kj}} \sum_{j=1}^{C} u_{kj}[p_j(a_i) - p_j(a_m)] \tag{9}$$

It is a simple matter to show (4) that

$$\min_{\rho,U}[EU(a_i) - EU(a_m)] \ge \min_{\rho,U} EU(a_i) - \max_{\rho,U} EU(a_m)$$

It might be tempting to believe that we could determine whether $a_i \succ a_m$ by checking to see whether $\underline{EU}(a_i) \ge EU(a_m)$ where $\underline{EU}(a_i) = \min_{\rho,U} EU(a_i)$ and $\overline{EU}(a_i) = \max_{\rho,U} EU(a_i)$. If $\underline{EU}(a_i) \ge \overline{EU}(a_m)$ we have sufficient, but not necessary conditions for $a_i \succ a_m$. Using this relation will guarantee conservatism. If we determine that $\underline{EU}(a_i) \ge \overline{EU}(a_m)$ we know for sure that $a_i \succ a_m$, but the converse is not necessarily true. We point this out here since we will generally wish to use $\overline{EU}(a_i)$ and $\underline{EU}(a_i)$ as displays to the decisionmaker to assist in the attribute tree structuring effort. It will be a very useful guide, but only a guide. Large differences between $\overline{EU}(a_i)$ and $\underline{EU}(a_i)$ at any attribute suggest that it is advantageous to disaggregate the tree from that attribute node downward in an attempt to get greater parametric precision.

The certain outcome case is a special subset of the results presented here thus far. Alternative a_i leads to outcome x_i with certainty and C=A. We use $p_j(a_i) = \delta_{j,i}(a_i)$ where $\delta_{j,i}(a_i)$ is the Kronecker delta function. Equations (5), (7) and (9) become

$$\underline{V}_k(a_i) = \min_{U_k(a_i)} u_k(a_i) = \underline{u}_k(a_i)$$

$$\overline{V}_k(a_i) = \max_{u_k(a_i)} u_k(a_i) = \overline{u}_k(a_i)$$

$$V_k(a_i, a_m) = \min_{u_k(a_i), u_k(a_m)} [u_k(a_i) - u_k(a_m)]$$

where $u_k(a_i) = u_{kj} p_j(a_i) \Big|_{j=i}$
When the inequalities on $u_k(a_i)$ are a set of numbers only we obtain

$$V_k(a_i, a_m) = \underline{V}_k(a_i) - \overline{V}_k(a_m)$$

but we caution that this still results in the inequality

$$\min_{\rho, u(a_i), u(a_m)} \rho^T[u(a_i) - u(a_m)] = \min_{\rho} \rho^T V_k(a_i, a_m) \geq$$

$$\min_{\rho} \sum_{k=1}^{N} \rho_k \underline{u}_k(a_i) - \max_{\rho} \sum_{k=1}^{N} \rho_k u_k(a_m)$$

rather than an equality between these expressions.

THE ATTRIBUTE TREE STRUCTURING PROCEDURE

In some approaches to attribute tree structuring it is assumed that all attributes have been identified and related to one another through the contextual relation "is included in". While this approach certainly works, there is no way prior to constructing the structure and parameterizing it to know that the tree is too deep in the sense of more attributes being included than are really needed to meaningfully characterize the utility function. The approach described here is a sensitivity based approach (2,4,5) in which sensitivity is interpreted by means of the parameter precision which the decisionmaker expresses. This is used to guide the attribute tree structuring such that a more or less minimal but sufficiently complex tree structure results.

Initially we assume that the decisionmaker cannot select wholistically from among A attributes. Two or more attributes next to the top of the tree are identified, such as, for example, cost and effectiveness. The decisionmaker expresses an initial belief in the range of costs and effectiveness to be expected from identified alternatives, and a range of importance weights expressing perceptions concerning tradeoffs among these attributes. It may well be that this is sufficient to identify a single non-dominant alternative. More often than not, this will not be possible. Calculations of the differences between maximum and minimum performance for each alternative on each attribute suggest the direction of tree expansion most likely to yield significant changes in alternative preference specificity. Even when a single non dominant alternative has been found, it is often advisable to extend the tree one level lower and then aggregate up the tree or do a conventional MAUT assessment (1.3) as a check on the veracity of the structuring and parameterization effort.

The prescribed approach is perhaps best illustrated by means of an example.

A SIMPLE EXAMPLE

Suppose that the decisionmaker perceives the attribute tree shown in Figure 1. We do not assume that the entire tree is known prior to the elicitation effort but merely pose this now to avoid sketching several partial trees.

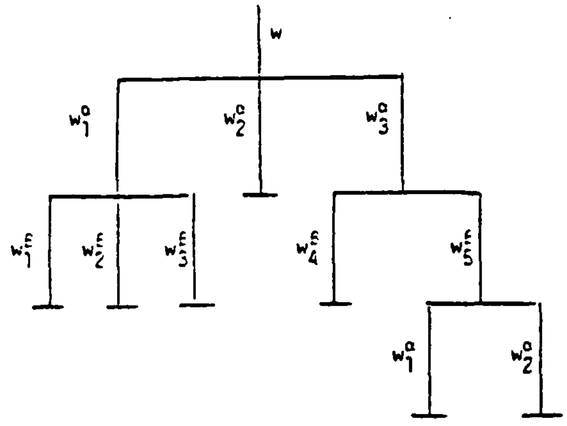

Figure 1 - Attribute Tree for Example

Suppose there are 4 non dominant alternatives initially identified. The decisionmaker elects to do a top down evaluation. Three attributes are initially identified. The decisionmaker initially identifies the following scores on the three first level attributes, where a – b indicates that the score may vary from a to b.

Attribute Scores

Alternative	$u_1^\alpha(a_i)$	$u_2^\alpha(a_i)$	$u_3^\alpha(a_i)$
a_1	0.3 → 0.6	0	1
a_2	1	1	0
a_3	0.4 → 0.9	0.2 → 0.3	0.3 → 0.6
a_4	0	0.6 → 0.7	0.1 → 0.8

We note that there is little imprecision over scores on attribute w_2^α. But, there is considerable imprecision over the other top level attributes. If the decisionmaker cannot identify a unique best and unique worst alternative on each attribute, the attributes are generally in need of redefinition. If this is infeasible, surrogate best and worst alternatives should be introduced. In terms of the difference between best and worst scores on each of these three attributes, suppose that the decisionmaker initially identifies ranges for the swing weights as $w_2^\alpha = 0.3$ and $w_3^\alpha \leq w_1^\alpha \leq 3w_3^\alpha$. Then we ask if the interpretation $0.35 \leq w_1^\alpha \leq .467$, $w_2^\alpha = 0.3$, and $0.233 \leq w_3^\alpha \leq 0.35$ is a reasonable set of bounds. The decisionmaker agrees and then we compute, using Eqs. (4) through (7), the minimum and maximum ranges for the composite scalar utility

Alternative	$U(a_i)$ min	max
a_1	0.3731 →	0.56
a_2	0.65 –	.767
a_3	0.305 →	.65
a_4	0.2033 –	.50

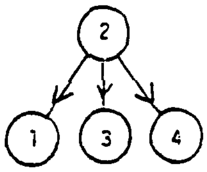

Figure 2 - Domination Digraph After Disaggregating One Level Down the Attribute Tree

These sufficient conditions enable us to draw the domination digraph of Figure 2. It appears that alternative 2 is the single nondominant digraph. However small changes could easily result in alternative 3 becoming (also) nondominant. So we may wish to go further down the tree.

Suppose that the decisionmaker wishes to prioritize the alternatives and so elects to go further down the tree. We have shown that, except in pathological cases, aggregating up the tree strengthens the parameter specificity order. Thus disaggregating down the tree might seem counterproductive. In a top down approach, however, we necessarily may have little confidence in initial weights and utility scores elicited at the top of the tree but are able to refine these to have greater precision as we move down the tree.

We might, for example disaggregate the components of the utility $u_1^c(a)$ to get

Attribute Scores

Alternative	$u_1^c(a_i)$	$u_2^c(a_i)$	$u_3^c(a_i)$
a_1	.6	1	.2
a_2	1	0	1
a_3	.7	.4	0
a_4	0	.6	.3

The decisionmaker might also say that $w_1^c = 0.5$, $w_2^c = 0.2$, and $w_3^c = 0.3$. Now we are able to calculate the utility score at attribute u_1^α. We obtain

Alternative	$u_1^\alpha(a_i)$
a_1	0.56
a_2	0.80
a_3	0.43
a_4	0.21

Generally, these agree with the results obtained earlier. And the value scores are precise now. But we had best be careful in using the old weights (w_1^c, w_2^c, and w_3^c)to find the total utility. They were based on anchoring value scores at 0 and 1. We can change these scores for $u_1^\alpha(a_i)$ to the interval $0 \to 1$ by use of $y = ax + b$ with $a = 1.6949$ and $b = -.3559$. Then we get

Alternative	$u_1^\alpha(a)$
a_1	.6034
a_2	1.0
a_3	.3729
a_4	0

Now, perhaps the DM suggests the same weights as before. Then we can calculate

Alternative	$u(a_i)$ min	max
a_1	0.5148 →	0.5612
a_2	0.65 →	0.767
a_3	0.2955 →	0.4305
a_4	0.2033 →	0.50

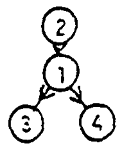

Figure 3 - Domination Digraph

and can now select the two best altetnatives as shown in the domination digraph of Figure 3.

Next the DM perhaps suggests that disaggregation of the components of $u_3^c(a)$ is desirable. Suppose that we obtain

Alternative	$u_4^{\varepsilon}(a_i)$	$u_5^{\varepsilon}(a_i)$
a_1	1	1
a_2	0	0.1 → 0.4
a_3	0.3	0.5 → 0.8
a_4	0.2 → 0.7	0

...ose that we unitially obtain $w_4^{\varepsilon} > 3w_5^{\varepsilon}$ and we ...final agreement that $0 \leq w_5^{\varepsilon} \leq 0.25$ and ... $\leq w_4^{\varepsilon} \leq 1.0$. These are, of course required ...this inequality. Next we calculate

Alternative	$u_3^{\alpha}(a_i)$
a_1	1
a_2	0 → 0.1
a_3	0.3 → 0.425
a_4	0.15 → 0.70

...have lost our anchor again. We might wish to ...ansform the maximum value such that 1 remains at ...and the 0.1 minimum score shifts to 0. We use ... = ax + b with a = 1.111 and b = -.1111 and the ...ive attribute score matrix becomes

Alternative	$u_3^{\alpha}(a)$
a_1	1.0
a_2	0
a_3	.3 → .3611
a_4	.15 → .667

...ain we see that we have reduced the imprecision. ...ce more the DM is asked whether or not it is ...esirable to change the weights w_i^{α}. Perhaps the ...swer is no. Then, we can calculate the scalar ...ility score

Alternative	$u(a_i)$ min	max
a_1	0.5148 →	0.5612
a_2	0.65 →	0.767
a_3	0.296 →	.444
a_4	0.2150 →	.444

...e cannot get any greater alternative specificity ...om this expression compared to that of Figure 3. ...wever, we are now much more confident that alter...native two is better than one which is better than ...three and four, than we were earlier since there is ...much more separation between $\underline{U}(a_2)$ and $\overline{U}(a_1)$ than previously as well as greater separation between $\underline{U}(a_1)$ and $\overline{U}(a_3)$ or $\overline{U}(a_4)$.

Of course we could expand the attribute tree still farther but the effort thus far seems to strongly suggest that alternative 2 is our most preferred alternative. At this point we might wish to use the structure that we have elicited, and redo the parametric elicitation effort as a check on our approach. We would use Eqs. (6) and (7) which yield necessary and sufficient conditions for $a_i \succ a_m$.

SUMMARY

We have used a sensitivity approach, based on parameter imprecision and a top down approach, to elicit the minimally relevant structure for an attribute tree and to prioritize a set of alternatives.

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DECISION AND INFORMATION STRUCTURES IN REGRET MODELS OF CHOICE UNDER UNCERTAINTY

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ABSTRACT

This paper first discusses the use of the preference relation "selection of A and rejection of B is preferable to selection of B and rejection of A." This contextual relation yields descriptive models of choice under uncertainty that explain many observed violations of classical single attribute utility theory. It is shown that successful use of this relation, for descriptive purposes, requires careful attention to elicitation of realistic decision situation structural models and information observability conditions within this model. Reservations with respect to use of the regret model for normative purposes, due to potential nontransitivities of the contextual relation, are discussed.

1. INTRODUCTION

There is much evidence [6,9,10,17-21] available that many people systematically violate the tenets of subjective expected utility theory in actual, or descriptive, choice situations. Many human information processing and judgmental flaws could be cited. Among the many detailed studies of these information processing biases and flawed cognitive heuristics that may lead to poor judgment with reference to much contemporary literature are [10,18-21]. Prospect theory [9,21] represents perhaps the most significant study of systematic descriptive violations of the normative expected utility theory. Our purpose in this paper is to discuss the recently revived interest and extensions to regret theory [1-3,13] especially as they relate to descriptive and normative decision behavior. We are especially concerned with extensions of regret theory to incorporate decision situation framing and information availability perspectives.

2. THE FRAMING OF DECISION SITUATIONS AND ASSOCIATED EFFECTS ON CHOICE

Violations of consistency and coherence in choice may often be traced to cognitive limitations which govern the perceptions of decision situations, the processing of information, and the evaluation of options. A strong determinant of the frame or structure adopted for a given decision is the decisionmaker's experiential familiarity with the issue under consideration. Because of imperfections in human cognition, changes of perspective often reverse the relative apparent desirability of achieving various objectives and, consequently, the relative desirability of options that lead to objective attainment. Similarly, variations in the framing of alternatives, contingencies, and outcomes may result in systematic reversal of preferences. The order in which information is presented (primacy or recency effects) often unduly influences ones initial estimate (anchor) of a situation. It is essential that problems be framed in a very careful fashion such that mental models, or any other models that the decisionmaker may choose for the decision situations, are truly representative of the essential features of the situation at hand in order to avoid, or at least minimize, possibilities for cognitive bias that may be stimulated by an "improper" frame.

Consider, for example, the decision situation illustrated in Figure 1. Alternative B should be chosen over Alternative A, assuming that our decision rule is to select the alternative with the greatest expected return. Generally, the value of money is not a linear function of the amount of money, however, due to the satiation effect. Also we have an attitude toward risk that further shapes the value function into a utility function. These separate and distinct issues of preferences and risk aversion will be discussed later. Further, negative outcomes are valued in a different fashion than positive outcomes of the same amount. Value functions are often convex for gains and concave for losses [9].

Let us now examine the various urn representations shown in Figure 2 from which the decision situation graphically illustrated in Figure 1 could have been obtained; let us also speculate on how the decisionmaker might react in each case. There are four choice situations illustrated. In each case, the decisionmaker is able to view the ball drawn from the urn. In some cases, additional information is available.

Suppose that we must choose between the options described in choice situation 1. If we choose option A and a white or shaded ball is drawn, then we obtain nothing. However, it does not appear that we would feel badly about not having chosen option B since, in this latter case, we would surely have lost money. We might well feel quite good at choosing option A and not choosing urn B. Thus, we see that the value felt from a decision outcome may be a combination of value for what we did obtain, as well as regret or joy for what we could have obtained had we chosen the other option. If after having selected option A, a black ball is drawn, we might or might not have regret associated with not having selected option B where we would have won $500. It is possible that we would just feel good about having played it safe and won $100. Alternatively, there could be regret at not having chosen option B. It appears that many people would express post-decision regret at selecting option A and obtaining a black ball. Therefore, the need to include regret as an attribute of the descriptive choice situation becomes more apparent.

Suppose that we choose option B. If a white or shaded ball is drawn, it is very possible that we would feel badly about having been greedy and chosen option B over the safer bet of selecting option A. Thus, there exists regret associated with this choice. Here, again, one experiences post-decisional regret. If a black ball is drawn, we would be quite happy with having selected option B in the

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first place. Again, we see a post-decision effect, bliss or sadness, that is modified from the pre-decision situation. Examination of choice situations 2, 3 and 4 show that these are not at all equivalent, even though they are represented by the same decision tree.

3. THE NEED FOR AVAILABLE INFORMATION SENSITIVE THEORIES OF REGRET

Bell [1-3] was perhaps the first to advocate the inclusion of regret as a second attribute to more fully capture the decisionmaker's values. It has often been suggested that regret be used as a surrogate for value and that an option alternative be selected which minimizes regret. But regret, in the definitive work of Bell, is measured against some ideal best and worst outcomes and these same anchors are used to measure regret for all outcomes. Bell's effort, and related independently obtained results due to Loomes and Sugden [13], consider value and regret as simultaneously present attributes of decision outcomes. Two-option situations are considered and regret is a differential concept that is measured within the outcome states across the options that could have led to these states.

This method appears applicable in two outcome win/lose situations where the selection of a winner in one choice would result in post-decision error-free identification of a win or loss situation had the other option been selected. There are, of course, many examples of decision situations where one may obtain, post-decision, full knowledge about what would have happened under the option not selected. There are, however, other decision situations where complete post-decision knowledge about what would have happened under the option not selected is not available. And there are many decision situations which involve more than two alternative courses of action. This effort extends regret theory to incorporate these considerations.

Consider, for example, the problem represented in Figure 3 where option A gives \$1,000 with probability 0.9 and \$0 otherwise. Option B gives \$750 for sure. The choice of option B over lottery A may be attributed to a certainty effect; we prefer a certain \$750 to a 0.9 chance of winning \$1,000 and a 0.1 chance of winning \$0. In the full information framing situation, the decisionmaker views the problem as shown in Figure 7. In the ordered pair (a,b), the first attribute "a" represents what we actually gain or lose and the second attribute "b" represents the assets foregone by not having selected the other alternative. We assume a linear multi-attribute utility function form such that for n outcomes of a decison alternative A, we have

$$EU(A) = \sum p_i(A) [u_i(A) + f_i(A)]. \qquad (1)$$

Here $p_i(A)$ is the probability of obtaining outcome i given that option A is selected. $u_i(A)$ is the utility associated with obtaining outcome i from selection of option A. $f_i(A)$ is the regret associated with obtaining outcome i from option A and not obtaining "some other outcome." Clearly, we need to be careful in anchoring this other outcome. In order to have option B preferred to option A, $B \succ A$, we must have, where we let $u(1000) = 1$ and $u(0) = 0$,

$$\begin{aligned} p[1 + f(M-C)] + (1-p)f(-C) &< u(C) \\ &+ pf(C-M) + (1-p)f(C) \end{aligned} \qquad (2)$$

The second attribute in the ordered pair is referred to as the foregone assets under the total information assumption since it is just the outcome in the other lottery. It is very important to note that value here is a cardinal value function measured with some presumed anchor whereas regret is a differential cardinal measure anchored on another outcome that "could have been."

This procedure suffices for the special case where the problem being evaluated may be represented by the one-urn, two-outcome full information model shown in Figure 4. In this case, if one wins or loses after having selected one option, then it can be determined whether one would have won or lost had the other option been selected. However, suppose that Figure 3 was intended to be representative of the two urn problem shown in Figure 5. There are only two cases associated with the model of Figure 5 where regret enters the decision situation as a second attribute. These involve option A. If we select option B and the certain return, there is no way of knowing whether we would have won or lost had the other urn been selected.

A question of significant interest is whether or not post decision knowledge of the outcomes increases or decreases the overall utility of the certain outcome alternative B. The utility of $\bar{B}$, with no post-decision information about the outcomes obtained from alternative $\bar{A}$ available, is u(C). With post-decision information (regret) available, it is

$$EU(B) = u(C) + pf(C-M) + (1-p)f(C) \qquad (3)$$

The answer to our question seemingly lies in the regret function. Post-decision outcome information increases the utility of alternative $\bar{B}$, such that $\bar{B} \succ B$, if

$$pf(C-M) + (1-p)f(C) < 0 \qquad (4)$$

and decreases the utility of alternative $\bar{B}$, such that $B \succ \bar{B}$, if

$$pf(C-M) + (1-p)f(C) > 0 \qquad (5)$$

The alternatives A and $\bar{A}$ should have the same expected utility here for these pairwise preference comparison examples. We will have alternative $\bar{A}$ preferred to $\bar{B}$, $\bar{A} \succ \bar{B}$, if

$$p[1 + f(M-C)] + (1-p)f(-C) > u(C) \qquad (6)$$

where we let $f(0) = 0$ for convenience. We see that there will exist preference reversals, $B \succ A \sim \bar{A} \succ \bar{B}$, if Eqns. (2) and (6) are satisfied. If this occurs, Eqn. (5) is satisfied. In a similar way we obtain preference reversals, $\bar{B} \succ \bar{A} \sim A \succ B$, if neither Eqn. (2) nor Eqn. (6) is satisfied. If this occurs, Eqn. (4) is satisfied. Thus we see that, under appropriate circumstances, we can obtain preference reversals by changing the information set available to the decisionmaker. In most cases we expect that greater post decision outcome information will increase the utility of the uncertain outcome alternative, assuming it is favorable, and then we expect Eqn. (4) to hold.

An interesting modification to the urn model decision situations can be made by inserting k no-win white balls in each urn for the decision situations depicted in Figures 4 and 5. We preserve the regret that is associated with choice situation 7 but the certainty regret effect associated with choice situation 8 seems to vanish. There is now no certain way of knowing what would have occurred under outcome $\bar{B}$ if we select option $\bar{A}$. Of course, if k is small we have a relatively good idea of what would occur, but there is no way in which we can know for sure. Suppose, for example, that we let M = \$1,000, C = \$750, m = 45, and n = 5. It seems not at all unreasonable that we prefer choice A to choice B in choice situation 7 and $\bar{B}$ $\bar{A}$ in choice situation 8. This might be especially the case if we represent a group, and if we might encounter group criticism should we select an option that yields a result that is less than that which is known to have been obtained under the other option.

Now suppose we add k = 950 white balls to each urn. We then obtain the decision situation structural models of Figure 6. Our preferences should remain the same regardless of k if we assume that conventional utility theory is fully applicable. The sure thing principal of Savage and the strong independence axiom of Samuelson each require this. These early seminal results in decision analysis are based on the assumption that pre-decision and post-decision regret information is the same.

It is very likely, however, that we will prefer the more risky options A and $\bar{A}$ since there is a 33% greater return with only slightly greater risk. At first glance, this might appear to contradict the results that follow from use of Eqn. (6). But Eqn. (6) is formally not applicable as addition of the k white balls to the urn has modified the information content present. Now we no longer know that option $\bar{B}$ will produce a winner, and the decision situation model is now that of Figure 6a and not that of Figure 5.

The significance of this observation is that a knowledge of information patterns in the decision situation model is essential; and that the notion of "regret" is not at all independent of risk levels, especially when there are post-decision information uncertainties. If k = 1, for example, it is quite obvious that the regret model of Figure 8 would be more applicable than that of Figure 5a. However, when there is full post-decision outcome information, then it would appear that regret is not strongly dependent upon the risks involved. With k = 950 such that Figure 5b results for the full post-decision outcome information case, and with k = 0 for which we obtain Figure 4, the regret terms are essentially the same.

There have been a number of attempts to illustrate the nonrationality of choices which violate one or more of the classic axioms of decision theory. We shall briefly examine a simple situation which is illustrative of these here. Our purpose in doing this is to demonstrate the need to carefully construct decision situation structural models, especially with respect to information flow patterns, including the nature of any "regret" that is associated with the decision situation. In the "full information" case illustrated in Figure 4 many people will prefer B = (C,1) to A = (M,P) but will prefer $\bar{A}$ = (M,ap) to $\bar{B}$ = (C,a), where M > C and $0 \le a, p \le 1$. There are at least two decision matrices which may be claimed to describe this decision outcome situation:

(a) Probability of Outcome

Option	a^2p	$ap(1-a)$	$(1-ap)a$
$\bar{A}$	M	M	0
$\bar{B}$	C	0	C

(b) Probability of Outcome

Option	ap	$a(1-p)$	$1-a$
$\bar{A}$	M	0	0
$\bar{B}$	C	C	0

In each of the above matrices we have A = $\bar{A}$ (a = 0) and B = $\bar{B}$ (a = 0). The question which immediately arises is: what does "full information" infer here? The answer is that there is no answer to the question as posed. We simply must know about the decision situation structural model or frame used to represent the task at hand. We may represent these two "full information" decision situations by the urn models and decision trees of Figure 7. If we view value and regret as the components of utility, the decision criteria for these two frames are quite different and given by $\bar{A} \rangle \bar{B}$ iff

$$p - v(c) + ap[f(M-C) - f(C-M)] + p(1-a)[f(M) - f(-M)] + (1-ap)[f(-C) - f(C)] > 0$$

for the situation model of case (a) and

$$p - v(c) + p[f(M-C) - f(C-M)] + (1-p)[f(-C) - f(C)] > 0$$

for the situation model of case (b). It is this latter situation model that we have considered in our previous discussions. For the situation model of Figure 7(b) we go from options $\bar{A}$ and $\bar{B}$ to options A and B by changing a from some small number (typically) to 1. This simply means that we remove shaded balls from the urn. In each case, we obtain the standard utility expressions if we remove the regret terms. Also, the certainty effect is absent in the model of Figure 7(b).

By no means does this discussion suggest that the situation model of Figure 7(b) is more realistic than that of Figure 7(a). A judgment of this sort must necessarily depend upon the task at hand. If we consider the specific situation where C = $5,000, M = $4,500, p = 0.98 and a = 0.05 then changing a to 1 results in quite different frames of the decision situation as indicated in Figure 10, which is computed with n = 1000. To use either of these models requires some illustration of the physical situation involved!

Thus, it is not fully meaningful to speak of value and regret associated with prospects A = (M,p) and B = (C,1). There is no formal difficulty in using the value concept since it is presumably anchored on some ideal best and ideal worst possibilities. But the regret concept is based upon opportunity foregone by not selecting the other option and the question immediately arises concerning available information about what we would have obtained under the option not selected. The conclusion that we must exercize considerable care in obtaining the decision situation structural model is inescapable.

4. THE POTENTIAL FOR NONTRANSITIVE BEHAVIOR USING PAIRWISE REGRET COMPARISONS

It is well known that sets of pairwise preference comparisons are often nontransitive. This may well occur using the regret concepts presented here since the regret associated with selecting an alternative must necessarily be associated with the alternative not selected. Thus, it is not meaningful to speak of *the expected utility of alternative A* when regret associated with not selecting alternative B is involved. We could use

$$EU(A,B) = \sum_{i=1}^{N} p_i(A)\,[u_i(A) + f_i(A,B)]$$

where the $u_i(A)$ are the, perhaps multi-attributed, components of the utility of the i^{th} outcome of option A: the $f_i(A,B)$ are the regrets, which are negative for true regret and positive for rejoicing, associated with obtaining the i^{th} outcome from alternative A rather than the possibly known outcome associated with rejecting alternative B; and the $p_i(A)$ are the probabilities associated with obtaining the i^{th} outcome state following choice of alternative A. It is very convenient but not strictly necessary that this probability be the same across the N outcome states for options A and B. We will say that $A \rangle B$ iff EU(A,B) > EU(B,A). From this, we easily see that there is no reason to infer that if $A \rangle B$ and $B \rangle C$, we must necessarily have $A \rangle C$. Figure 8 illustrates a three-choice situation with preference intransitivities that occur because of the different information sets available in the three pairwise preference comparisons. This would suggest much caution in the use of any prioritization approach that is based on pairwise preference comparisons and assumed, but nonverified, transitivity among preference relations as it becomes extraordinarily easy to produce agenda dependent results.

5. CONCLUSIONS AND SUMMARY

In this paper, we have examined the recently introduced concept of pairwise comparison regret. As expected, the regret concept does not necessarily lead to transitive preference comparisons. We have demonstrated the strong need to incorporate decision process descriptions in framing of regret situations.

Several illustrative examples indicate that the framing of decision output states and the information available concerning the outputs, resulting from decision options not selected, greatly influences the regret calculations. The central conclusion from this is that careful decision situation structuring must be associated with the regret concept if the results obtained are to have descriptive value.

Even more interesting and potentially useful questions concern the value of the regret concept in a normative sense. Clearly, many people experience notions of regret and hesitate rejecting alternatives because of some desirable properties of the rejected alternatives that will then be foregone. We must be careful however to note that the "preference" relation implied, through use of regret, by $A \rangle B$ is not just "A is preferred to B" but "selecting A and rejecting B is preferable to selecting B and rejecting A." Thus the two preference relations $A \rangle B$ and $B \rangle C$ do not provide sufficient information to infer $A \rangle C$. The contextual relation is not uniquely defined here. It should really be written to infer the true contextuality of the situation at hand. What we have really shown is that $A \rangle_{AB} B$ and $B \rangle_{BC} C$ say about $\rangle_{AC}$. It might well be a serious mistake to use approaches which infer that it does if we allow for pairwise comparisons of this sort.

We agree, strongly, with the comment of Kahneman and Tversky [9] that those departures from subjective expected utility theory that prospect theory (and regret theory) described are such that they "must lead to normatively unacceptable consequences." A well conceived decision support process should assist the decisionmaker in realizing the potential errors involved. For these reasons, among others, we strongly encourage elicitation processes that encourage comparison within attributes and across outcome states for alternatives. It would seem that pairwise preference comparison of alternatives is fraught with difficulties and, unless great care is exercized, can lead to potentially misleading results. At the very least, we must be fully aware of the detailed potentially context dependent definition of the pairwise relation, $\rangle$, and that this relation is strictly speaking, incomparable in the expressions $A \rangle_{AB} B$ and $B \rangle_{BC} C$ when these expressions imply pairwise comparison with no ideal alternatives to serve as a reference point, anchor, or basis for the comparison. In other words, we must be careful to insure that the relation $\rangle$ is defined in a fashion such that it is context independent across alternatives if we are to use it in the manner indicated here.

This cannot, however, be used as an invocation against using the regret concept, or the more general prospect theory, to describe behavior. For descriptive purposes these approaches have much to recommend them. Understanding of descriptive reality is a necessary first step towards a meaningful normative process. In this sense, among others, these approaches have considerable value. And, of course, it may well turn out that the regret approach, with suitable information framing, will not lead to preference intransitivities. In this case, assuming validity of the value and regret elicitations [8,12], the approach would seem to have much potential normative, as well as descriptive, appeal.

A number of extensions to our efforts suggest themselves. Two that seem particularly cogent concern examination of regret elicitation approaches and a closer comparison contrast and integration of the regret concept with prospect theory [7,9,21], other axiomatic approaches [14-16], and the recently introduced very promising concept of relative risk aversion [4,11].

6. ACKNOWLEDGMENTS

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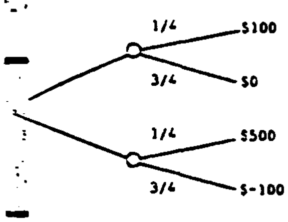


2. Decision tree for a simple example.

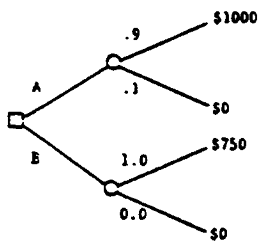


Fig.3. Gen. probl. which ill. the certainty effect.

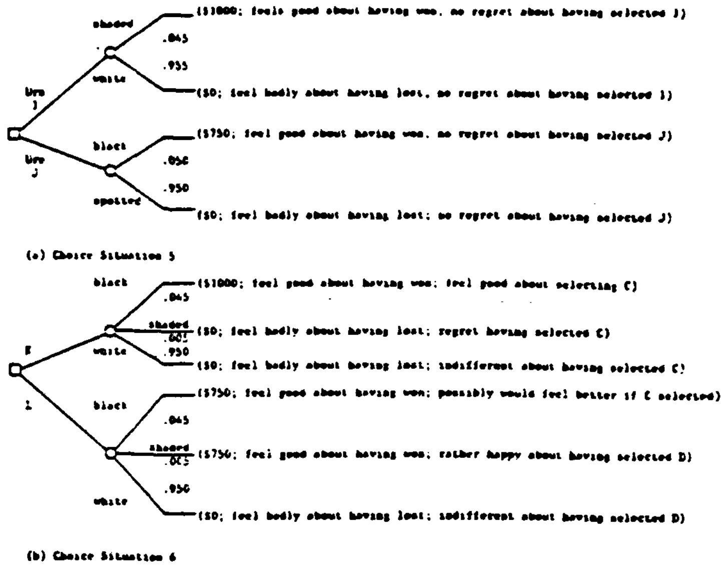


Figure 6. Illustrations of regret as a post decision outcome attribute.

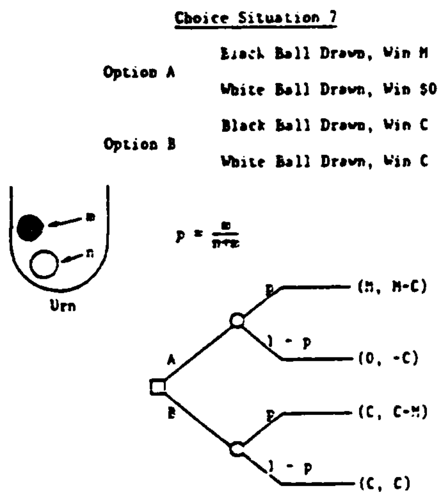


illustration of the certainty effect

Fig. 4. Regret model of a dec. sit. represented by the urn model shown -

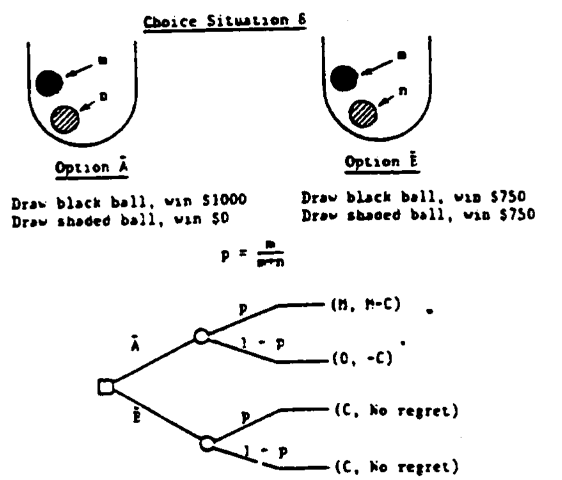
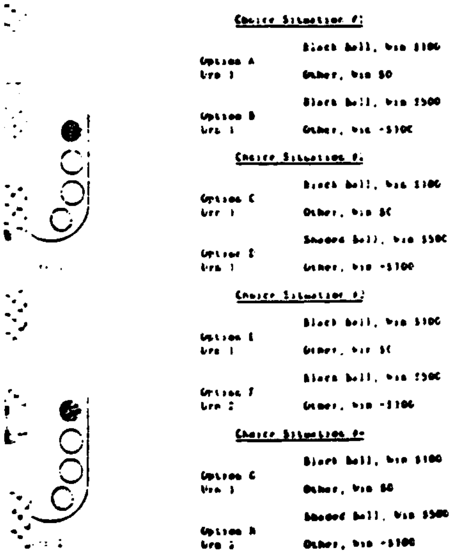


Fig. 5. Two-urn model with certainty effect.



2. Choice situations that have a common decision tree.

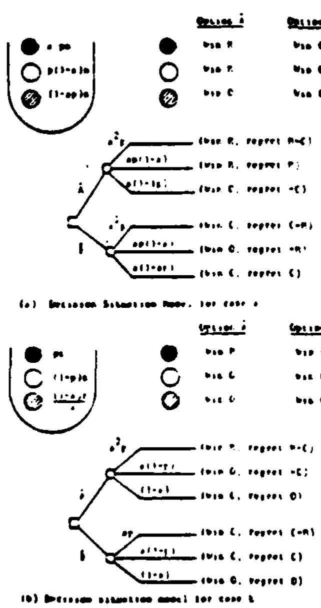


Fig.7. Two different inf. flow models which yield the same prospects.

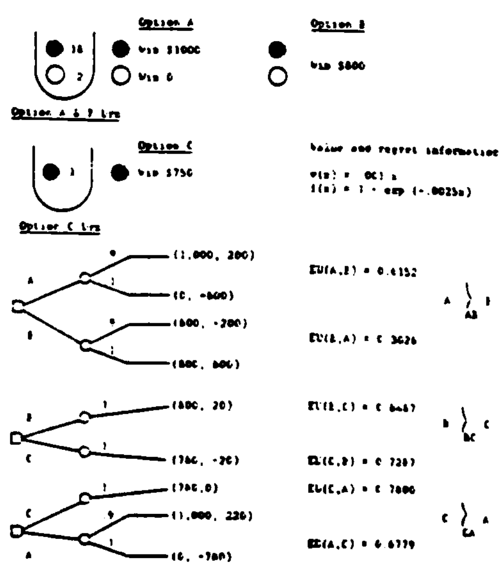


Figure 8. Illustration of nontransitivities resulting from pairwise comparisons using regret.

IMPRECISE IMPORTANCE WEIGHT ASSESSMENT FOR MULTILEVEL OBJECTIVES HIERARCHIES*

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We present a procedure for importance weight assessment for a single stage decisionmaking model under uncertainty that allows imprecision in the utility values and in the importance weights. An assessment approach for dealing with nonunique best and/or worst alternatives for a given attribute is suggested. An example illustrates the procedure and how the procedure could be implemented using currently existing software.

INTRODUCTION

A decisionmaker may not be able and/or willing to provide information necessary to precisely *identify* his or her preferences, expressed in terms of a utility function. In order to deal with this possibility and with the possibility that probabilities also may be imprecisely known, Sarin (1977a, 1977b), Fishburn (1965), White and Sage (1980), and White, et al. (1982a, 1982b) have developed procedures for alternative evaluation, prioritization, and ranking based on imprecisely known parameter values (e.g., probability, importance weight, lowest level utility scores) for single stage decisionmaking models.

In this paper we present a procedure for enhancing the assessment of possibly imprecise importance weight information for the case where lowest level utility scores in a multilevel objective hierarchy may be imprecise and where probabilities are precisely known. We also indicate how currently available software, which has been named ARIADNE for Alternative Ranking Interactive Aid based on DomiNance information Elicitation, can be used to provide information useful for this assessment. The problem considered is formulated in Section 2, and the assessment procedure is described in Section 3. An illustrative example is presented in Section 4. Conclusions are given in the final section.

PROBLEM FORMULATION:

We now present a normatively based and behaviorally relevant model of single-stage decisionmaking under uncertainty. Let A be the number of available alternatives. Once an alternative is selected, one of C consequences will occur. If alternative a is selected, then consequence c will occur with probability $p_c(a)$. If alternative a is selected and consequence c occurs, then a utility of $u_{ic}(a)$ is accrued with respect to attribute i, $i = 1,\ldots,I$. We assume that the attributes are additive independent (Keeney and Raiffa, 1976; p. 295). Thus the expected utility of alternative a can be expressed as

$$\sum_{i=1}^{I} \rho_i \sum_{c=1}^{C} u_{ic}(a)p_c(a) = \rho u(a)p(a),$$

where $\rho_i \geq 0$ is the importance weight associated with attribute i, $\sum_{i=1}^{I} \rho_i = 1$, $\max\{u_{ic}(a)$: for all a and c$\} = 1$, and $\min\{u_{ic}(a)$: for all a and c$\} = 0$.

Assume that all of the probabilities $p_c(a)$ are known precisely, $u_i = \{u_{ic}(a)$: for all a and c$\}$ is known to be a member of the set $U_i, i=1,\ldots,I$, and $\rho = \{\rho_i$, for all i$\}$ is known to be a member of the set P. Let R be the relation on the alternatives defined as follows: $(a',a)\epsilon R$ if and only if the expected utility of alternative a' is at least as great as the expected utility of alternative a for all possible values of $u_i, i=1,\ldots,I$, and ρ. Thus, $(a',a)\epsilon R$ if and only if

$$\min \rho[u(a')p(a')-u(a)p(a)] \qquad (1)$$

is nonnegative, where the minimum is with respect to all $u_i \epsilon U_i, i=1,\ldots,I$, and $\rho \epsilon P$. It is shown in White, et al. (1982) that the determination of (1) can be computed by the following two-level set of

* This research supported by ONR Contract N00014-80-C-0542.

linear programs when U_i, i=1,...I, and P are polyhedra:

$$\min_{\rho \epsilon P} \sum_i \rho_i v_i(a',a)$$

where

$$v_i(a',a) = \min_{u_i \epsilon U_i} \sum_c [u_{ic}(a')p_c(a') - u_{ic}(a)p_c(a)].$$

Note that complete determination of R can require the solution of A (A-1) and A (A-1)I linear programs, respectively. Use of the easily proved fact that R is transitive may possibly reduce the number of linear programs required to determine R. We remark that the forms of the U_i and P may indicate that there exists simplified procedures for solving the above linear programs. Simplified procedures, e.g. procedures for solving linear programs with bounded variables (Bradley et al., 1977, p. 78), may be of considerable importance if quick response is necessary and/or a small computer environment is required.

Determination of the $v_i(a',a)$ may require the solution of a mixed integer program if the best and/or worst alternative/consequence pairs with respect to an attribute are not uniquely identifiable. To illustrate this fact, assume C=1 (decisionmaking under certainty), A=5, $0 \leq u(a) \leq 1$ for all a, and

$$u(1) = 1 \text{ and/or } u(2) = 1$$

$$u(4) < u(3) < u(1)$$
$$u(5) \leq u(3) \leq u(2)$$

$$u(4) = 0 \text{ and/or } u(5) = 0.$$

Such inequalities perhaps indicate an ordinal ranking of the alternatives where alternatives 1 and 2 are ranked "good", alternative 3 is ranked "fair" and alternatives 4 and 5 are ranked "poor". Note that our model insures that $\max_a\{u(a)\}=1$ and $\min_a\{u(a)\}=0$, a common convention (see Keeney and Raiffa, 1976, p. 119 and 231). The above inequalities would then have the following interpretation:

$$1 \geq u(1) \geq 1-\delta_1$$
$$1 \geq u(2) \geq 1-\delta_2$$
$$u(1) \geq u(3) \geq u(4)$$
$$u(2) \geq u(3) \geq u(5)$$
$$\delta_4 \geq u(4) \geq 0$$
$$\delta_5 \geq u(5) \geq 0$$
$$0 \leq \delta_1 + \delta_2 \leq 1$$
$$0 \leq \delta_4 + \delta_5 \leq 1$$

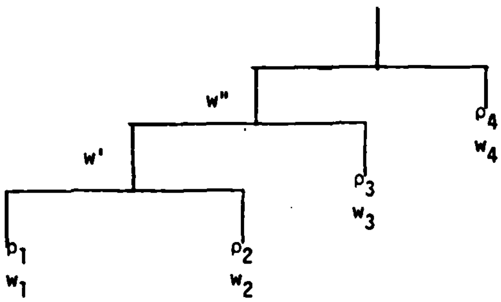
$$\delta_i \epsilon \{0,1\},\ i=1,2,4,5.$$

We will observe shortly that if for a given attribute the best and worst alternatives are not uniquely identified, then importance weight assessment can become complicated, sometimes significantly. Thus, in most circumstances it will be beneficial to refine knowledge of the utility functions through further assessment and/or to possibly restructure various portions of the decisionmaking problem in order to avoid as much as possible, assessment difficulties associated with nonunique best and/or worst alternatives for a given attribute. Although the utility functions assessed at the terminal branches of the objectives hierarchy may uniquely identify best and worst alternatives for each attribute, higher level importance weight assessment may still involve nonunique best and/or worst alternatives. This fact will be illustrated by the example presented in Section 4.

ASSESSMENT OF P:

We will assume that constraints on the $u_i=\{u_{ic}(a)$, for all a and c$\}$ are known. These constraints might be based on assessment procedures discussed, for example, in Farquhar (1982, p. 22) and/or the binary variable description just presented.

Assessment of the importance weights are typically based on information contained in (1) the utilities for each attribute and (2) an objectives hierarchy. An objectives hierarchy is a graphical depiction of the functional relationship of the objectives. For example, consider the objectives hierarchy in Figure 1. If ρ_1 and ρ_2 are associated with



$$w_2 + w_2 = 1$$
$$w' + w_3 = 1$$
$$w'' + w_4 = 1$$
$$\rho_1 + \rho_2 + \rho_3 + \rho_4 = 1$$

FIGURE 1: An Example of an Objectives Hierarchy.

initial cost and maintenance cost, respectively, then w' would be associated with a notion of cost, say overall cost, that represents a combination of initial cost and maintenance cost.

We remark that importance weights can assume two basic forms which we will call the ρ form and the w form. These forms are illustrated in Figure 1 for the objectives hierarchy presented there. Note that ρ_i is the importance weight for the i^{th} terminal attribute. The w form of importance weights assumes that the importance weights are normalized at each level of the objectives hierarchy, as indicated in Figure 1. The relationship between the ρ and w forms is straightforward to determine. For example, for the objectives hierarchy depicted in Figure 1,

$$w_i = \rho_i/(\rho_3+\rho_2) \quad , \quad i=1,2,$$

$$w' = (\rho_1+\rho_2)/(\rho_1+\rho_2+\rho_3)$$

$$w_3 = \rho_3/(\rho_1+\rho_2+\rho_3)$$

$$w'' = \rho_3+\rho_2+\rho_3$$

$$w_4 = \rho_4$$

$$\rho_i = w_i w' w'' \quad , \quad i=1,2,$$

$$\rho_3 = w_3 w''$$

$$\rho_4 = w_4 \quad .$$

Note that

$$w_i \; w_j = \rho_i/\rho_j \quad , \quad i,j\epsilon\{1,2\},$$

$$w' \; w_3 = (\rho_1+\rho_2)/\rho_3$$

$$w''|w_4 = (\rho_1+\rho_2+\rho_3)/\rho_4 \quad .$$

Also, observe that

$$w_i \leq b \qquad \rho_i \leq b(\rho_1+\rho_2) \quad , \quad i=1,2,$$

$$w' \leq b \qquad \rho_1+\rho_2 \leq b(\rho_1+\rho_2+\rho_3)$$

$$w_3 \leq b \qquad \rho_3 \leq b(\rho_1+\rho_2+\rho_3)$$

$$w'' \leq b \qquad \rho_1+\rho_2+\rho_3 \leq b$$

$$w^4 \leq b \qquad \rho_4 \leq b \quad .$$

Thus, assessment in terms of the w form can easily be described in terms of the ρ form. Description of the importance weights in the ρ form is preferred from a computational perspective because it is most directly applicable to the aforementioned linear and/or mixed integer programming procedures. However, it is often useful to use the w form for assessment.

A standard approach for precise assessment of the importance weights when no uncertainty exists and when the $u_i(a)$ (since the consequence is certain, it is without loss of generality that we drop the dependence of $u_{ic}(a)$ on c) are known precisely is as follows. For each i, identify the most and least preferred alternatives, a_i^* and a_{i*}, respectively; thus, $u_i(a_i^*)=1$ and $u_i(a_{i*})=0$, for all i. The importance of $u_i(a_i^*)-u_i(a_{i*})$ relative to $u_j(a_j^*)-u_j(a_{j*})$ then represents the relative value of ρ_i to ρ_j (or of w_i to w_j, if it is preferred to deal with the w form of importance weights). For example, again referring to Figure 1, assume ρ_1 and ρ_2 are associated with initial cost and maintenance cost, respectively. If the difference between best and worst initial costs is considered to be twice as important as the difference between best and worst maintenance costs, then $\rho_1 = 2\rho_2$ and hence $w_1=2/3$ and $w_2=1/3$. Due to the additive form of the utility function, we can then determine the utility values for the attribute "overall cost" by calculating $u(a)=w_1u_1(a)+w_2u_2(a)$. We have thus "moved up" the hierarchy. The best and worst alternatives with respect to "overall cost" and attribute 3, whatever it might represent, can then be combined to produce utility scores for the upper level attribute associated with w". Once w" and w_4 are assessed, a total order on the alternatives can be determined and the most preferred alternative identified.

We now present a generalization of the above importance weight assessment procedure that allows utility function and importance weight imprecision. Let S_i^* (S_{i*}) be the set of all possible most (least) preferred alternative/consequence pairs with respect to attribute i. These sets can be obtained directly from the assessed $\{u_{ic}(a)$, for all a and c$\}$. Assume that we wish to assess the relationship between w_i and w_j, i=j, where i and/or j may be other than lowest level objectives. Select $(a_i^*,c_i^*)\epsilon S_i^*$, $(a_{i*},c_{i*})\epsilon S_{i*}$, $(a_j^*c_j^*)\epsilon S_j^*$, $(a_{j*},c_{j*})\epsilon S_{j*}$. On the assumption that (a_j^*,c_j^*) $((a_{j*},c_{j*}))$ is the most (least) preferred alternative/consequence pair with respect to attribute i and that (a_i^*,c_i^*) $((a_{j*},c_{j*}))$ is the most (least) preferred alternative/consequence pair with respect to attribute j, we assess bounds on the ratio w_i/w_j, $\alpha \leq w_i/w_j \leq \beta$. Assume such bounds are assessed for each possible combination of pairs in S_i^*, S_{i*}, S_j^*, and S_{j*}. Let α_* (β^*) be the lowest such lower (largest such upper) bound on w_i/w_j. We then assume that $\alpha_* \leq w_i/w_j \leq \beta^*$. Let attributes i and j be the lower level attributes associated with the higher level attribute k. Then,

$$u_{kc}(a) = w_i\tilde{u}_{ic}(a)+w_j\tilde{u}_{jc}(a), \; w_i+w_j = 1,$$

where $\tilde{u}_{ic}(a)$ has been normalized as follows:

$$\tilde{u}_{ic}(a) = [u_{ic}(a)-u_{ic_{i*}}(a)]/[u_{ic_i^*}(a_i^*)-u_{ic_{i*}}(a)]$$

with $\tilde{u}_{jc}(a)$ analogously defined.

AN ILLUSTPATIVE EXAMPLE:

We now present an example to illustrate the above assessment procedure. We will also describe how currently available software, ARIADNE, can be used to solve this example. For simplicity, assume C=1, i.e. the decisionmaking under certainty case. Let the $u_1(a)$ satisfy:

$$u_1(1)=0, \; u_1(2)=1, \; 0.3 \leq u_1(3) \leq 0.5, \; 0.2 \leq u_1(4) \leq 0.3$$

$u_2(1)=0, u_2(2)=1,\ 0.2\le u_2(3)\le 0.5,\ 0\le u_2(4)\le 0.4$

$0.3\le u_3(1)\le 1,\ 0.8\le u_3(2)\le 1,\ u_3(3)=1,\ u_3(4)=0$

$u_4(1)=1, 0.7\le u_4(2)\le 0.75,\ u_4(3)=0,\ 0.4\le u_4(4)\le 0.75.$

On the basis of this information and no importance weight information, $R = \phi$, i.e. none of the alternatives is preferred to any of the other alternatives. This fact can be determined using ARIADNE by introducing the above equalities and inequalities on the $u_i(a)$. We remark that ARIADNE can also consider bounds on ratios of the utility scores as long as comparison is not made across attributes; e.g., $b_* \le u_{ic}(a)/u_{ic'}(a') \le b^*$.

Let Figure 1 represent the objectives hierarchy for this example. Clearly, $S_i^* = \{2\}$ and $S_{i*} = \{1\}$ for $i=1,2$. Assume an assessment provides $0.95\le w_1/w_2\le 1.05$, or equivalently, $0.95\le \rho_1/\rho_2\le 1.05$. This information does not affect R; hence R remains null. We can use ARIADNE to determine this fact by allowing it to consider in addition to the utility scores the following importance weight information: $0.95\le \rho_1/\rho_2\le 1.05$.

Let $u'(a) = w_1u_1(a)+w_2u_2(a)$, the utility function associated with w'. It is easily shown that $u'(1)=0, u'(2)=1,\ 0.2487\le u'(3)\le 0.5$, and $0.0974\le u'(4)\le 0.3513$. We note that $S^{*'} = \{2\}$, $S'_* = \{1\}$, $S_3^* = \{3\}$, and $S_{3*} = \{4\}$. ARIADNE can be used to provide $S^{*'}$ and S'_* (S_3^* and S_{3*}) by letting it consider in addition to the utility score information, the following importance weight information: $0.95\le \rho_1/\rho_2\le 1.05$, $\rho_3=\rho_4=\rho_5=0 (\rho_3=1)$. Assume an assessment provides $10w_3\le w'$, or equivalently $10\rho_3\le \rho_1+\rho_2$. Analysis then indicates that $R=\{(2,3)\}$, i.e. alternative 2 is at least as preferred as alternative 3. ARIADNE can be used to determine this alternative relation, given as input the utility score information and the following importance weight information: $0.95\le \rho_1/\rho_2\le 1.05$ and $10\rho_3\le \rho_1+\rho_2$. Thus, alternative 1, 2, or 4 may be the most preferred.

If the above alternative ranking information is not sufficient for alternative selection, we consider the utility function $u''(a) = w'u'(a)+w_3u_3(a)$, which is associated with the importance weight w''. Calculations show that $0\le u(1)\le 0.0909$, $0.9818\le u''(2)\le 1$, $0.2487\le u''(3)\le 0.5455$, and $0.0885\le u''(4)\le 0.3513$. Thus, $S^{*''} = \{2\}$, $S''_* = \{1,4\}$, $S_4^* = \{1\}$, and $S_{4*} = \{3\}$. We can use ARIADNE to determine $S^{*''}$ and S''_* (S_4^* and S_{4*}) by providing ARIADNE with the utility score information and the following importance weight information: $0.95\le \rho_1/\rho_2\le 1.05$, $10\rho_3\le \rho_1+\rho_2$, and $\rho_4=0$ $(\rho_4=1)$. Suppose an assessment provides $10w_4\le w''$, or equivalently $10\rho_4\le \rho_1+\rho_2+\rho_3$. Then, $R = \{(2,3), (2,4), (3,1)\}$ and thus alternative 2 is the most preferred alternative. We determine this information using ARIADNE by providing as input the utility score information and the following importance weight information: $0.95\le \rho_1/\rho_2\le 1.05$, $10\rho_3\le \rho_1+\rho_2$, and $10\rho_4\le \rho_1+\rho_2+\rho_3$.

CONCLUSIONS:

In this paper, we have presented a procedure for importance weight assessment which allows imprecision in the utility values and in the importance weights. We have noted that a characteristic of this procedure is that best and/or worst alternatives for a given attribute may be nonunique. An assessment approach for dealing with this characteristic was suggested. An example was used to illustrate the procedure and how the procedure could be implemented using a currently existing software system, ARIADNE. Of course, assessment of importance weights given precise utility scores does not necessarily have to involve best and worst alternatives for each attribute. Other approaches to importance weight assessment given imprecise utility scores are under development.

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Decision and Information Structures in Regret Models of Judgment and Choice

ANDREW P. SAGE, FELLOW, IEEE, AND ELBERT B. WHITE

Abstract—The use of the preference relation "selection of A and rejection of B is preferable to selection of B and rejection of A" is discussed. This contextual relation yields descriptive models of choice under uncertainty that explain many observed violations of classical single attribute utility theory. It is shown that successful use of this relation, for descriptive purposes, requires careful attention to elicitation of realistic decision situation structural models and pre- and post-decision information availability conditions within these models. Reservations with respect to use of the regret model for normative purposes, due to potential nontransitivities of the contextual relation, are discussed.

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I. Introduction

THERE IS much evidence [6], [9], [10], [17]–[21] available that many people systematically violate the tenets of subjective expected utility theory in actual, or descriptive, choice situations. Many human information processing and judgmental flaws could be cited. Among the many detailed studies of these information processing biases and flawed cognitive heuristics that may lead to poor judgment with reference to much contemporary literature are [10], [18]–[21]. Prospect theory [9], [21] represents perhaps the most significant study of systematic descriptive violations of the normative expected utility theory. Our purpose in this paper is to discuss the recently revived interest and extensions to regret theory [1]–[3], [13] especially as they relate to descriptive and normative decision behavior. We are especially concerned with extensions of regret theory to incorporate decision situation framing and information availability perspectives.

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We will first discuss the framing of decision situations. Next, we will incorporate notions of decision situation framing within regret theory, as it now exists. We will be especially concerned with information availability. A discussion of potentially nontransitive behavior [5] that may result from the use of the pairwise comparisons implied by regret theory is followed by our conclusions and suggestions for extensions of this effort.

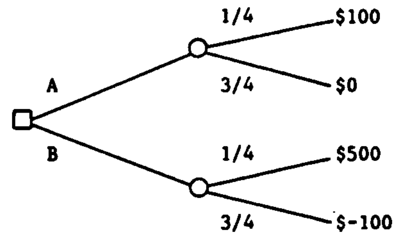

Fig. 1. Decision tree for a simple example.

II. The Framing of Decision Situations and Associated Effects on Choice

Violations of consistency and coherence in choice may often be traced to cognitive limitations which govern the perceptions of decision situations, the processing of information, and the evaluation of options. A strong determinant of the frame or structure adopted for a given decision is the decisionmaker's experiential familiarity with the issue under consideration. Because of imperfections in human cognition, changes of perspective often reverse the relative apparent desirability of achieving various objectives and, consequently, the relative desirability of options that lead to objective attainment. Similarly, variations in the framing of alternatives, contingencies, and outcomes may result in systematic reversal of preferences. The order in which information is presented (primacy or recency effects) often unduly influences ones initial estimate (anchor) of a situation. It is essential that problems be framed in a very careful fashion such that mental models, or any other models that the decisionmaker may choose for the decision situations, are truly representative of the essential features of the situation at hand in order to avoid, or at least minimize, possibilities for cognitive bias that may be stimulated by an "improper" frame.

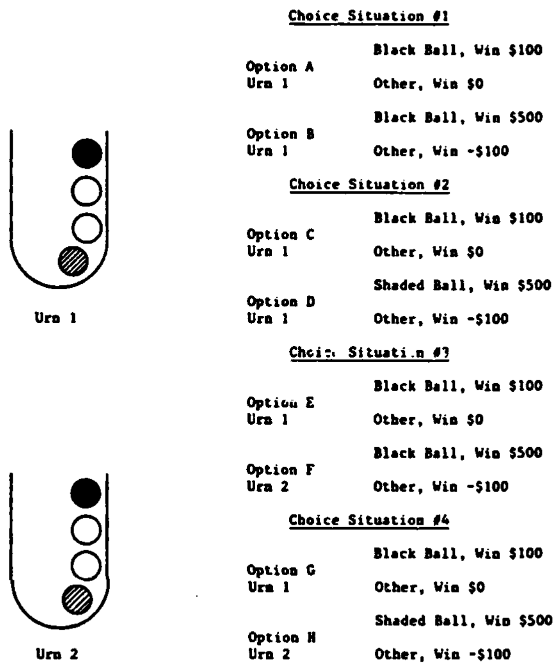

Fig. 2. Choice situations that have a common decision tree.

Consider, for example, the decision situation illustrated in Fig. 1. Alternative *B* should be chosen over Alternative *A*, assuming that our decision rule is to select the alternative with the greatest expected return. Generally, the value of money is not a linear function of the amount of money, however, due to the satiation effect. Also we have an attitude toward risk that further shapes that value function into a utility function. These separate and distinct issues of preferences and risk aversion will be discussed later. Further, negative outcomes are valued in a different fashion than positive outcomes of the same amount. Value functions are often convex for gains and concave for losses [9].

Let us now examine the various urn representations shown in Fig. 2 from which the decision situation graphically illustrated in Fig. 1 could have been obtained; let us also speculate on how the decisionmaker might react in each case. There are four choice situations illustrated. In each case, the decisionmaker is able to view the ball drawn from the urn. In some cases, additional information is available.

Problem 1: Suppose that we must choose between the options described in choice situation 1. If we choose option *A* and a white or shaded ball is drawn, then we obtain nothing. However, it does not appear that we would feel badly about not having chosen option *B* since, in this latter case, we would surely have lost money. We might well feel quite good at choosing option *A* and not choosing option *B*. Thus, we see that the value felt from a decision outcome may be a combination of value for what we did obtain, as well as regret or joy for what we could have obtained had we chosen the other option. If after having selected option *A*, a black ball is drawn, we might or might not have regret associated with not having selected option *B* where we would have won $500. It is possible that we would just feel good about having played it safe and won $100. Alternatively, there could be regret at not having chosen option *B*. It appears that many people would express post-decision regret at selecting option *A* and obtaining a black ball. Therefore, the need to include regret as an attribute of the descriptive choice situation becomes more apparent.

Suppose that we choose option *B*. If a white or shaded ball is drawn, it is very possible that we would feel badly about having been greedy and chosen option *B* over the safer bet of selecting option *A*. Thus, there exists regret associated with this choice. Here, again, one experiences post-decisional regret. If a black ball is drawn, we would be quite happy with having selected option *B* in the first

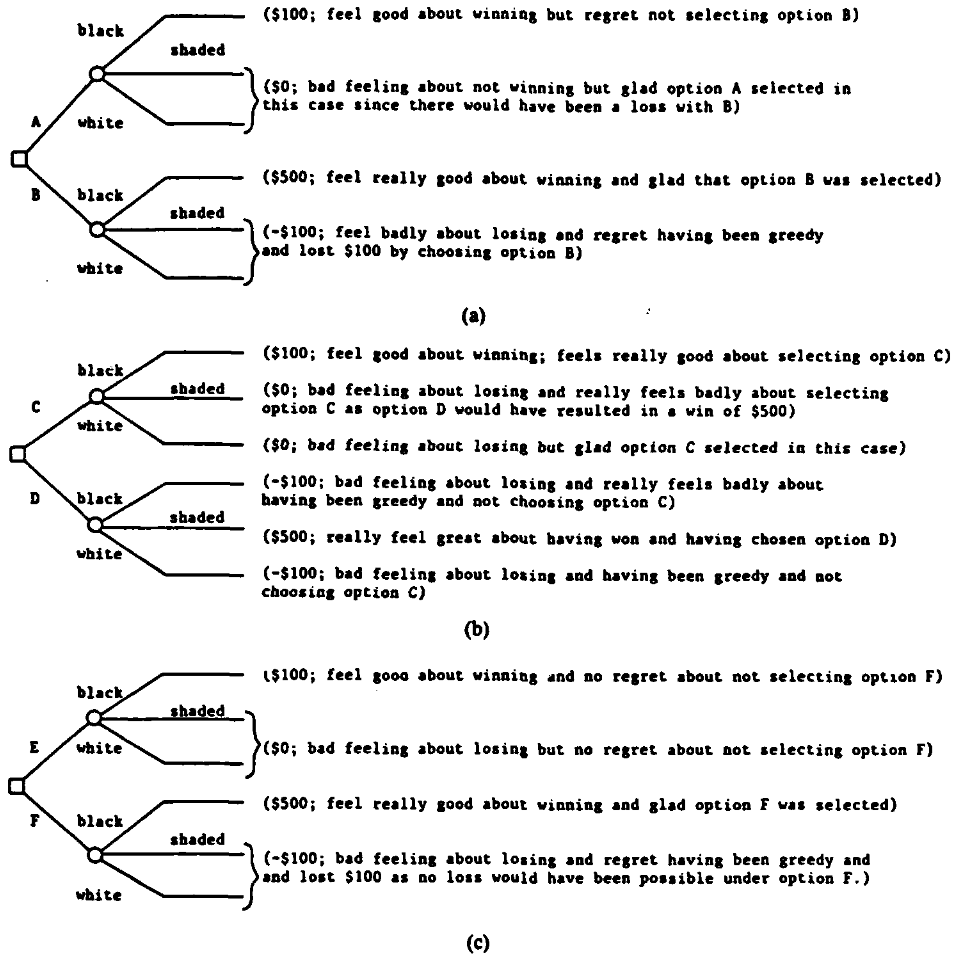

Fig. 3. Decision tree with outcomes characterized by happiness outcome and post-decision regret. (a) Problem 1. (We are allowed to see outcome results for options not selected.) (b) Problem 2. (We are allowed to see outcome results from options not selected.) (c) Problem 3. (We are not allowed to see outcome results from options not selected.)

place. Again, we see a post-decision effect, bliss or sadness, that is modified from the pre-decision situation.

Problem 2: Suppose that we must choose between the options described in choice situation 2 of Fig. 2. If a black ball is drawn, we should really feel good about having selected option C instead of option D. One hundred dollars would have been lost had option D been selected. If a shaded ball is drawn, no money is lost, but $500 would have been won had option D been selected. Here, we may feel quite badly about not having selected D in the first place as we know, post-decision, that we would have won if we had selected option D. If a white ball is drawn, we are likely rather glad that option D was not selected, since there would have been a loss of $100 with this option. Suppose that we choose option D. If a white ball is drawn, we would most likely feel badly about having been greedy in selecting option D instead of option C. If a black ball is drawn, we would probably feel badly about not having selected option C. Finally, if a shaded ball is drawn, then we would probably feel good over having both drawn the shaded ball and selected option D.

Problem 3: Assume that we are only informed that we have won or lost according to the color of the ball drawn. This may be represented by the two-urn model for choice situation number 3 or 4 involving options E and F for situation 3, or G and H for situation 4. Suppose that we choose option E. If a white ball or a shaded ball is drawn, we are simply informed of the loss. Since there is no way of determining what would have been the outcome had the other option been selected, we have no basis for regretting not having chosen the other option. If a black ball is drawn, we are informed that we have won. Since there is no way of determining what would have occurred had the other option been selected, we can only be happy about having won. We surely should not assume that we would have obtained a black ball if we selected option F in choice situation 3 where we would be happier, or in option H where we would be very sad. Suppose that we do choose option F. If a white or shaded ball is drawn, we would only feel badly about having lost. There should be no regret. If a black ball is drawn, we would be happy with having gone for the "big win" and won.

In our discussion, we assumed that the decisionmaker had explicit information available concerning the outcomes from the option not selected in choice situations 1 and 2, the one-urn problems. We assumed that no information was available concerning the outcomes from the option not selected in choice situations 3 and 4, the two-urn problems. Even though there are, indeed, subtle differences that are perceived to exist between the choice situations, they are sometimes modeled in the same way in the decision tree representation shown in Fig. 1, which is a good representation of the decision tree portion of the decision situation. However it is not a good representation of the attributes of decision outcomes. For this, we need an attribute tree, or a more complete labeling of the outcomes in terms of their attributes, as shown in the decision tree of Fig. 3. Even in the very simple situations, we see that we have identified two attributes associated with the outcomes. These are the amount of money, won or lost, and the regret, or rejoicing, associated with what could have been obtained from the decision not selected.

In the single-urn model, we generally have post-decision regret or post-decision conflict associated with outcomes. In the two-urn model, we do not generally have post-decision regret since we do not know the outcome for the option not selected. The central issue in this is that the decision situation model should not be the same for all of these choice situations. The outcomes and the associated values are not the same, and our feelings towards them are not the same since the available information is different in each case. Post outcome information about outcomes from decisions not selected is both context dependent and different from predecision information. Thus, we need to examine models which allow inclusion of the available pre- and post-decision information on choice situations.

In each of the problems just described, the probabilities of winning were the same; however, the amount to win varied. Now consider the decision situations represented in Fig. 4 where options *I* and *K* result in a 0.045 chance of winning $1000 and options *J* and *L* result in a 0.05 chance of winning $750. The decision options represented in choice situation 6 are such that one knows exactly what would have been won or lost for both options once an outcome has been observed. The lotteries represented in choice situation 5 are such that there is no means of determining what would have been won or lost had the other option been selected. There are many other urn representations which result in different post-decision information available to the decisionmaker that will have the same decision tree representation, even though the pre-decision information is the same.

Fig. 5 presents the information that the decisionmaker has available (post-decision) under the framings represented in choice situations 5 and 6. Again, we see that the attributes of the choice situations may be quite different. We wish to explore the implications of this here. The primary questions that we wish to resolve in this paper are as follows.

1) Is there a difference between choice situations 1, 2, 3, and 4? Should there be a difference with respect to judgment concerning the most preferred alternatives in each situation?

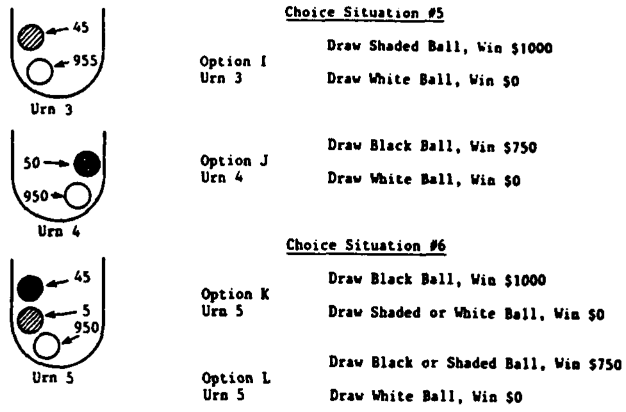

Fig. 4. Urn model representation of two different choice situations.

2) Is there, and should there, be a difference between choice situations 5 and 6?
3) What constructs are available to explain, in descriptive and normative senses, any differences in these?
4) Are there possibilities for nontransitivities or other anomalous results from using these constructs?

III. The Need for Available Information Sensitive Theories of Regret

Bell [1]–[3] was perhaps the first to advocate the inclusion of regret as a second attribute to more fully capture the decisionmaker's values. It has often been suggested that regret be used as a surrogate for value and that an option alternative be selected which minimizes regret. But regret, in the definitive work of Bell, is measured against some ideal best and worst outcomes, associated with the specific choice situation, and these same anchors are used to measure regret for all outcomes. Bell's effort, and related independently obtained results due to Loomes and Sugden [13], consider value and regret as simultaneously present attributes of decision outcomes. Two-option situations are considered, and regret is a differential concept that is measured within the outcome states across the options that could have led to these states, as in Bell's work.

This method appears applicable in two outcome win/lose situations where the selection of a winner in one choice would result in post-decision error-free identification of a win or loss situation had the other option been selected. There are, of course, many examples of decision situations where one may obtain, post-decision, full knowledge about what would have happened under the option not selected. There are, however, other decision situations where complete post-decision knowledge about what would have happened under the option not selected is not available. Also there are many decision situations which involve more than two alternative courses of action. This effort extends regret theory to incorporate these considerations.

Consider, for example, the problem represented in Fig. 6 where option *A* gives $1000 with probability 0.9 and $0 otherwise. Option B gives $750 for sure. The choice of option *B* over lottery *A* may be attributed to a certainty

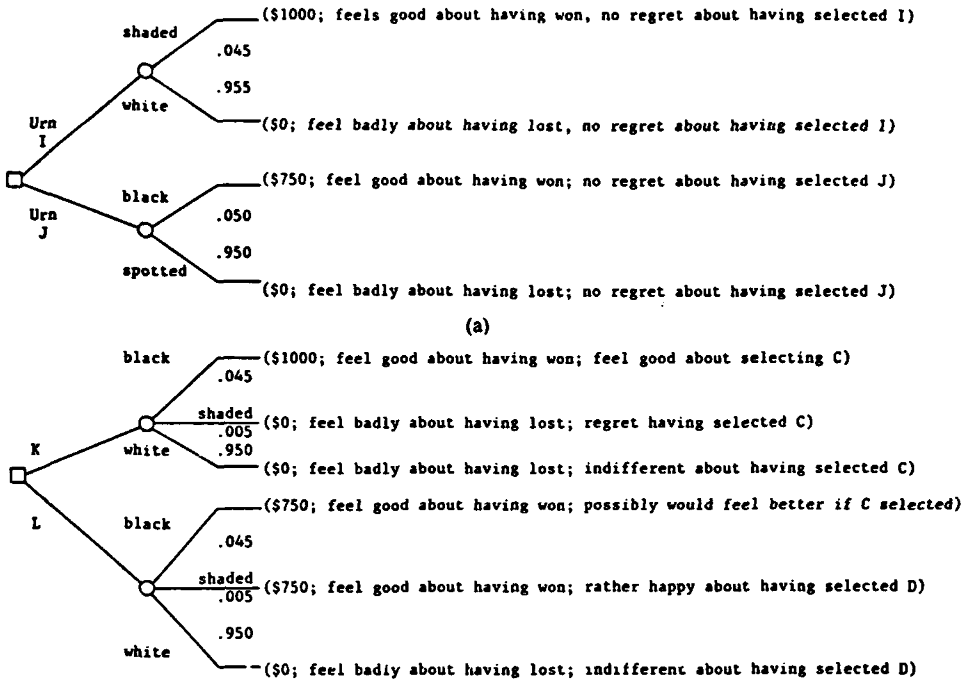

Fig. 5. Illustrations of regret as post-decision outcome attribute. (a) Choice situation 5. (b) Choice situation 6.

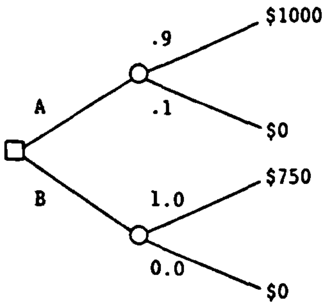

Fig. 6. General problem which illustrates certainty effect.

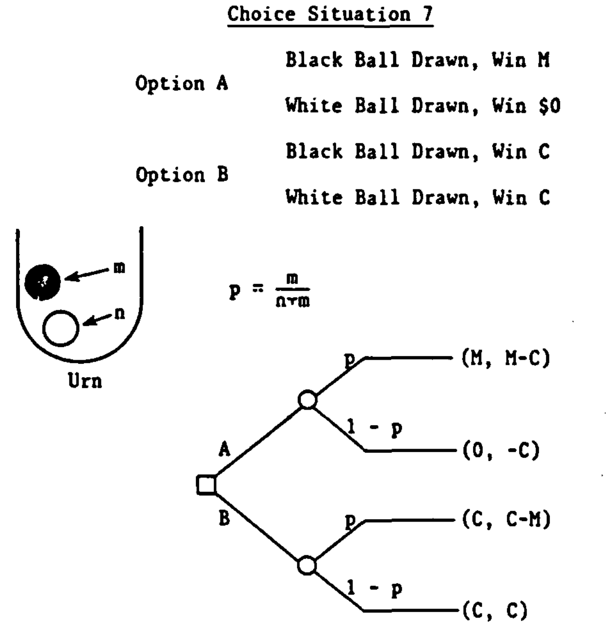

Fig. 7. Regret model of a decision situation represented by urn model shown—illustration of certainty effect.

effect; we prefer a certain $750 to a 0.9 chance of winning $1000 and a 0.1 chance of winning $0. In the full information framing situation, the decsionmaker views the problem as shown in Fig. 7. In the ordered pair $\langle a, b\rangle$, the first attribute a represents what we actually gain or lose and the second attribute b represents the assets foregone by not having selected the other alternative. We assume a linear multiattribute utility function form such that for n outcomes of a decision alternative A, we have

$$EU(A) = \sum p_i(A)\left[v_i(A) + f_i(A)\right]. \qquad (1)$$

Here $p_i(A)$ is the probability of obtaining outcome i given that option A is selected. $v_i(A)$ is the value associated with obtaining outcome i from selection of option A. $f_i(A)$ is the regret associated with obtaining outcome i from option A and *not* obtaining "some other outcome." Clearly, we need to be careful in anchoring this other outcome. In order to have option B preferred to option A, $B \succ A$, we must have, where we let $u(1000) = 1$ and $u(0) = 0$, and now use $f(\alpha)$ to represent the regret associated with obtaining the outcome α,

$$p[1 + f(M - C)] + (1 - p)f(-C)i < v(C) + pf(C - M) + (1 - p)f(C). \qquad (2)$$

The second attribute in the ordered pair is referred to as the foregone assets since it is just the difference between the outcome actually realized and the outcome in the other lottery. It is very important to note that value here is a

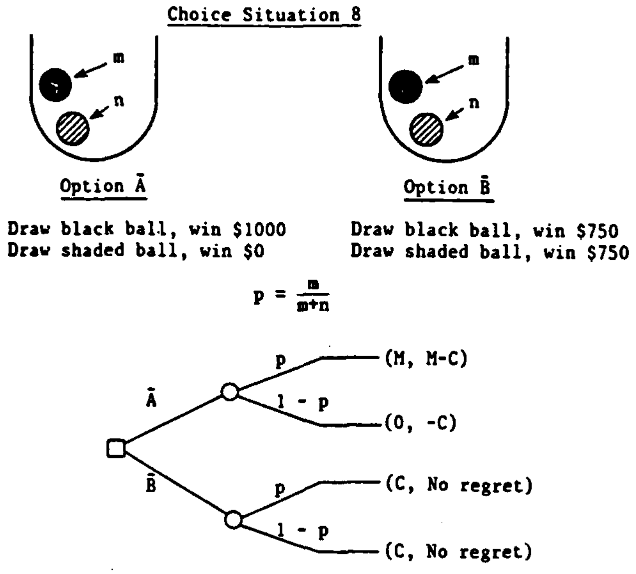

Fig. 8. Two-urn model with certainty effect.

cardinal value function measured with some presumed anchor whereas regret is a differential cardinal measure anchored on another outcome that "could have been."

This procedure suffices for the special case where the problem being evaluated may be represented by the one-urn, two-outcome full information model shown in Fig. 7. In this case, if one wins or loses after having selected one option, then it can be determined whether one would have won or lost had the other option been selected. However, suppose that Fig. 6 was intended to be representative of the two-urn problem shown in Fig. 8. There are only two cases associated with the model of Fig. 8 where regret enters the decision situation as a second attribute. These involve option $\bar{A}$. If we select option $\bar{B}$ and the certain return, there is no way of knowing whether we would have won or lost had the other urn been selected.

A question of significant interest is whether or not post-decision knowledge of the outcomes increases or decreases the overall utility of the certain outcome alternative B. The value of $\bar{B}$, with no post-decision information about the outcomes obtained from alternative $\bar{A}$ available, is $v(C)$. With post-decision information (regret information) available, it is (from Fig. 7)

$$EU(B) = v(C) + pf(C - M) + (1 - p)f(C) \quad (3)$$

The answer to our question seemingly lies in the regret function. Post-decision outcome information will increase the utility of alternative $\bar{B}$, relative to that of alternative B, such that $\bar{B} \succ B$, if $EU(\bar{B}) - EU(B) > 0$, or if

$$pf(C - M) + (1 - p)f(C) < 0 \quad (4)$$

and will decrease the utility of alternative $\bar{B}$, relative to that of alternative B, such that $B \succ \bar{B}$, if

$$pf(C - M) + (1 - p)f(C) > 0. \quad (5)$$

The alternatives A and $\bar{A}$ should have the same expected utility here for these pairwise preference comparison examples, since we know in each case what outcome occurs from the option not selected. We will have alternative $\bar{A}$ preferred to $\bar{B}$, $\bar{A} \succ \bar{B}$, if

$$p[1 + f(M - C)] + (1 - p)f(-C) > v(C), \quad (6)$$

where we let $f(0) = 0$ for convenience. We see that there will exist preference reversals, $B \succ A \sim \bar{A} \succ \bar{B}$, if (2) and (6) are satisfied. If this occurs, (5) is satisfied. In a similar way we obtain preference reversals, $\bar{B} \succ \bar{A} \sim A \succ B$, if neither (2) nor (6) is satisfied. If this occurs, (4) is satisfied. Thus we see that, under appropriate circumstances, preference reversals may be made to occur by changing the information set available to the decisionmaker. Changing the probabilities of obtaining the outcomes may also result in preference reversals. In most cases we expect that greater post-decision outcome information will increase the utility of the certain outcome alternative, assuming it is favorable, relative to the uncertain outcome alternative, and then we expect (5) to hold.

An interesting modification to the urn-model decision situations can be made by inserting k no-win white balls in each urn for the decision situations depicted in Figs. 7 and 8. The resulting situations are just those depicted in Figs. 4 and 5 for a specific set of numbers for k, m, and n. We preserve the regret that is associated with choice situation 7 but the certainty effect associated with choice situation 8 seems to vanish. There is now no certain way of knowing what would have occurred under outcome $\bar{B}$ if we select option $\bar{A}$. Of course, if k is small we have a relatively good idea of what would occur, but there is no way in which we can know for sure. Suppose, for example, that we let $M = \$1000$, $C = \$750$, $m = 45$, and $n = 5$. It seems not at all unreasonable that we prefer choice A to choice B in choice situation 7 and $\bar{B} \succ \bar{A}$ in choice situation 8.

Now suppose we add $k = 950$ white balls to each urn. We then obtain the decision situation structural models of Fig. 5. Our preferences should remain the same regardless of k if we assume that conventional utility theory is fully applicable. The sure thing principle of Savage and the strong independence axiom of Samuelson each require this. These early seminal results in decision analysis are based on the assumption that pre-decision and post-decision regret information is the same.

It is very likely, however, that we will prefer the more risky options A and $\bar{A}$ since there is a 33 percent greater return with only slightly greater risk. At first glance, this might appear to contradict the results that follow from use of (6). However (6) is formally not applicable as addition of the k white balls to the urn has modified the information content present. Now we no longer know that option $\bar{B}$ will produce a winner, and the decision situation model is now that of Fig. 5(a) and not that of Fig. 8.

The significance of this observation is that a knowledge of information patterns in the decision situation model is essential; and that the notion of "regret" is not at all independent of risk levels, especially when there are post-decision information uncertainties. If $k = 1$, for example, it is quite obvious that the regret model of Fig. 8 would be more applicable than that of Fig. 5(a). However, when there is full post-decision outcome information, then it would appear that regret is not strongly dependent upon the risks involved. With $k = 950$ such that Fig. 5(b) results for the full post-decision outcome information case, and with $k = 0$ for which we obtain Fig. 7, the regret terms are essentially the same.

There have been a number of attempts to illustrate the nonrationality of choices which violate one or more of the classic axioms of decision theory. We shall briefly examine a simple situation which is illustrative of these here. Our purpose in doing this is to demonstrate the need to carefully construct decision situation structural models, especially with respect to information flow patterns, including the nature of any "regret" that is associated with the decision situation. In the "full information" case illustrated in Fig. 7 many people will prefer $B = (C, 1)$ to $A = (M, p)$ but will prefer $\bar{A} = (M, ap)$ to $\bar{B} = (C, a)$, where $M > C$ and $0 \leqslant a, p \leqslant 1$. There are at least two decision matrices which may be claimed to describe this decision outcome situation:

(a) Probability of Outcome

Option	a^2p	$ap(1-a)$	$(1-ap)a$	$(1-a)(1-ap)$
$\bar{A}$	M	M	0	0
$\bar{B}$	C	0	C	0

(b) Probability of Outcome

Option	ap	$a(1-p)$	$1-a$
$\bar{A}$	M	0	0
$\bar{B}$	C	C	0

In each of the above matrices we have $A = \bar{A}(a = 1)$ and $B = \bar{B}(a = 1)$. The question which immediately arises is: what does "full information" infer here? The answer is that there is no answer to the question as posed. We simply must know about the decision situation structural model or frame used to represent the task at hand. We may represent these two "full information" decision situations by the urn models and decision trees of Fig. 9. If we view value and regret, and probability, as the components of utility, the decision criteria for these two frames are quite different and given by $\bar{A} \succ \bar{B}$ if we have

$$p - v(c) + ap[f(M - C) - f(C - M)] + p(1 - a) \cdot [f(M) - f(-M)] + (1 - ap)[f(-C) - f(C)] > 0$$

for the situation model of case a and

$$p - v(c) + p[f(M - C) - f(C - M)] + (1 - p)[f(-C) - f(C)] > 0$$

for the situation model of case b. Here we let $v(0) = 0$ and $v(M) = 1$. It is this latter situation model that we have considered in our previous discussions. For the situation model of Fig. 9(b) we go from options $\bar{A}$ and $\bar{B}$ to options A and B by changing a from some small number (typically) to 1. This simply means that we remove shaded balls from the urn in Fig. 9(b). In each case, we obtain the standard utility expressions if we remove the regret terms. Also, the certainty effect is absent in the model of Fig. 9(b).

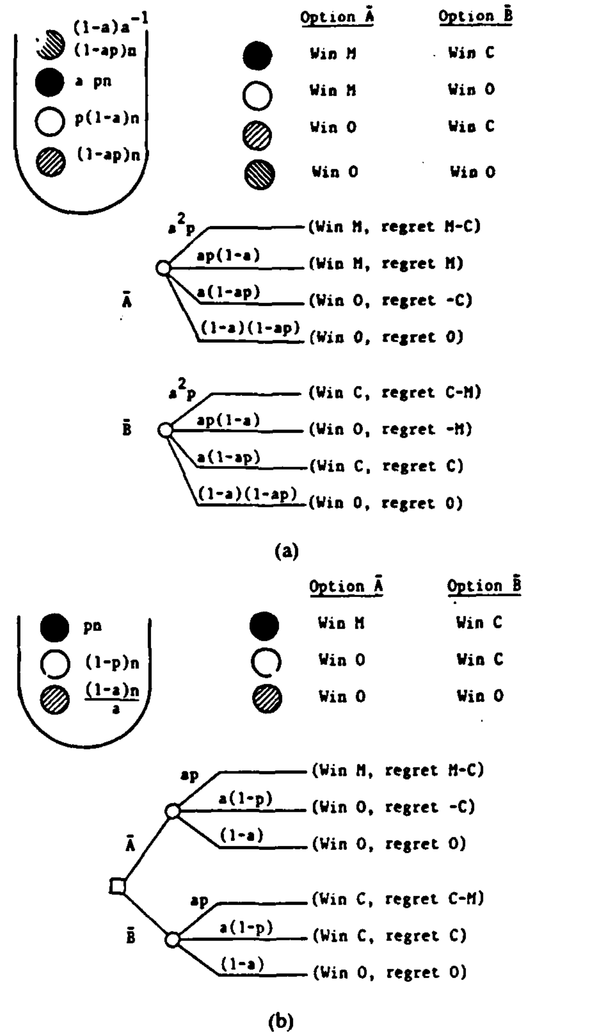

Fig. 9. Two different information flow models which yield same prospects. (a) Decision situation model for case a. (b) Decision situation for case b.

By no means does this discussion suggest that the situation model of Fig. 9(b) is more realistic than that of Fig. 9(a). A judgment of this sort must necessarily depend upon the task at hand. If we consider the specific situation where $C = \$5000$, $M = \$4500$, $p = 0.98$ and $a = 0.05$ then changing a to 1 results in quite different frames of the decision situation as indicated in Fig. 10, which is computed with $n = 1000$. To use either of these models requires some illustration of the physical situation involved!

Thus, it is not fully meaningful to speak of value and regret associated with prospects $A = (M, p)$ and $B = (C, 1)$. There is no formal difficulty in using the value concept since it is presumably anchored on some ideal best and ideal worst possibilities. However the regret concept is based upon opportunity foregone by not selecting the other option and the question immediately arises concerning available information about what we would have obtained under the option not selected. The conclusion that we must exercise considerable care in obtaining the decision situation structural model is inescapable.

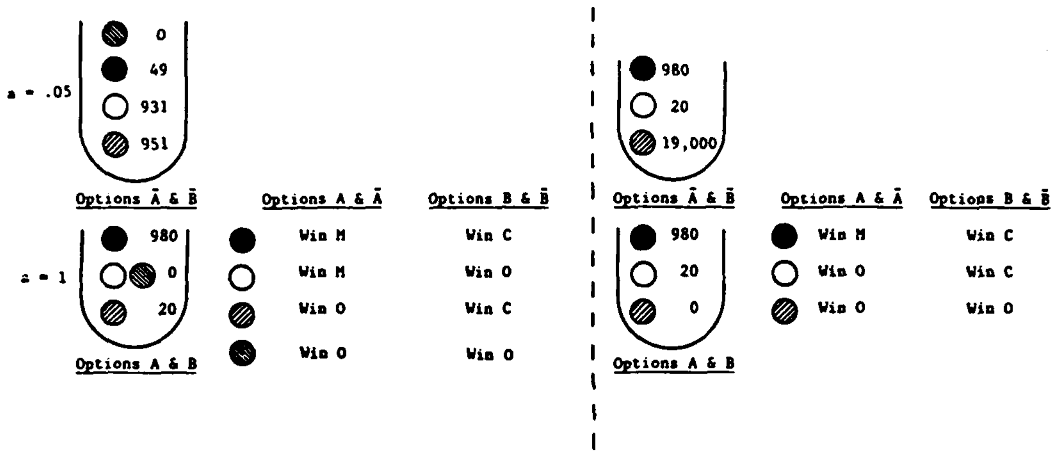

Fig. 10. Illustration of urn model changes for two different decision frames. (a) Case a. (b) Case b.

IV. The Potential for Nontransitive Behavior Using Pairwise Regret Comparisons

It is well-known that sets of pairwise preference comparisons are often nontransitive. This may well occur using the regret concepts presented here since the regret associated with selecting an alternative must necessarily be associated with the alternative not selected. Thus it is not meaningful to speak of the expected utility of alternative A when regret associated with not selecting alternative B is involved. We should use for (1)

$$EU(A, B) = \sum_{i=1}^{N} p_i(A)[v_i(A) + f_i(A, B)],$$

where the $v_i(A)$ are the, perhaps multiattributed, components of the utility of the ith outcome of option A; the $f_i(A, B)$ are the regrets, which are negative for true regret and positive for rejoicing, associated with obtaining the ith outcome from alternative A rather than the possibly known outcome associated with rejecting alternative B; and the $p_i(A)$ are the probabilities associated with obtaining the ith outcome state following choice of alternative A. It is very convenient but not strictly necessary that this probability be the same across the N outcome states for options A and B. When these probabilities are not the same we must generally reformulate the situation model in a meaningful way such that we obtain $p_i(A) = p_i(B)$. We will say that $A \succ B$ if $EU(A, B) > EU(B, A)$. From this, we easily see that there is no reason to infer that if $A \succ B$ and $B \succ C$, we must necessarily have $A \succ C$. Fig. 11 illustrates a three-choice situation with preference intransitivities that occur because of the different information sets available in the three pairwise preference comparisons. This would suggest much caution in the use of any prioritization approach that is based on pairwise preference comparisons and assumed, but nonverified, transitivity among preference relations as it becomes extraordinarily easy to produce agenda dependent results.

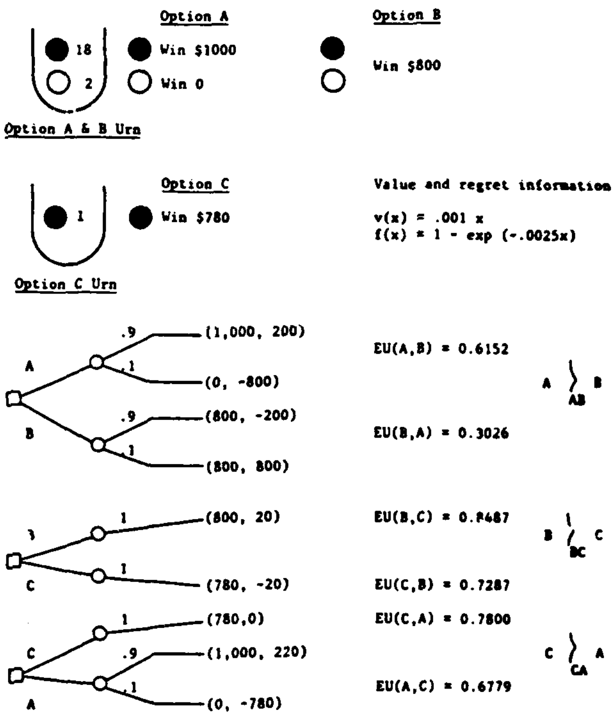

Fig. 11. Illustration of nontransitivities resulting from pairwise comparisons using regret.

V. Conclusion

In this paper, we have examined the recently introduced concept of pairwise comparison regret. As expected, the regret concept does not necessarily lead to transitive preference comparisons. We have demonstrated the strong need to incorporate decision process descriptions in framing of regret situations. Several illustrative examples indicate that the framing of decision output states and the information available concerning the outputs, resulting from decision options not selected, greatly influences the regret calculations. The central conclusion from this is that careful decision situation structuring must be associated with the regret concept if the results obtained are to have descriptive value.

Even more interesting and potentially useful questions concern usefulness of the regret concepts in a *normative* sense. Clearly, many people experience notions of regret and hesitate rejecting alternatives because of some desirable properties of the rejected alternatives that will then be foregone. We must be careful however to note that the "preference" relation implied, through use of regret, by $A \succ B$ is not just "A is preferred to B" but "selecting A and rejecting B is preferable to selecting B and rejecting A." Thus the two preference relations $A \succ B$ and $B \succ C$ do not provide sufficient information to infer $A \succ C$. The contextual relation $\succ$ is not uniquely defined here. It should really be written to infer the true contextuality of the situation at hand. What we have really shown is that $A \succ_{AB} B$ and $B \succ_{BC} C$ do not necessarily allow us, when comparing A and C, to say anything about $\succ_{AC}$. It might well be a serious mistake to use approaches which infer that it does if we allow for pairwise comparisons of this sort.

We agree, strongly, with the comment of Kahneman and Tversky [9] that those departures from subjective expected utility theory that prospect theory (and regret theory as well) describes "must lead to normatively unacceptable consequences." A well-conceived decision support process should assist the decisionmaker in understanding the potential decision process errors involved in the use of flawed judgment heuristics. For these reasons, among others, we strongly encourage elicitation processes that encourage comparison within attributes and across outcome states for alternatives. It would seem that pairwise preference comparison of alternatives is fraught with difficulties and, unless great care is exercized, can lead to potentially misleading results. At the very least, we must be fully aware of the potentially context and associated framing dependent definition of the pairwise relation, $\succ$, and that this relation is, strictly speaking, incomparable in the expressions $A \succ_{AB} B$ and $B \succ_{BC} C$ when these expressions imply pairwise comparison with no ideal alternatives to serve as a reference point, anchor, or basis for the comparison. In other words, we must be careful to insure that the relation is defined in a fashion such that it is context independent across alternatives if we are to use it in the manner indicated here, for multiple pairwise comparisons across alternatives.

This cannot, however, be used as an invocation against using the regret concept, or the more general prospect theory, to describe behavior. For descriptive purposes these approaches have much to recommend them. Understanding of descriptive reality is a necessary first step towards a meaningful normative process. In this sense, among others, these approaches have considerable value. And, of course, it may well turn out that the regret approach, with suitable information framing, will not lead to preference nontransitivities. In this case, assuming validity of the value and regret elicitations [8], [12], the approach would seem to have much potential normative, as well as descriptive, appeal.

A number of extensions to our efforts suggest themselves. Two that seem particularly cogent concern examination of regret elicitation approaches and a closer comparison contrast and integration of the regret concept with prospect theory [7], [9], [21], other axiomatic approaches [14], especially the insightful generalized expected utility analysis of Machina [15], [16] in which a nonlinear value functional is proposed which depends upon event outcome probability as an attribute and where only a local linear utility function satisfies the savage substitution axiom, and the recently introduced very promising concept of relative risk aversion due to Dyer, Sarin, and Krzysztofowicz [4], [11].

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Multiple objective evaluation and prioritization under risk with partial preference information†

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Much current interest surrounds the design and development of interactive multi-criterion decision support processes. We consider a finite alternative course of action, a single-stage, decision situation model under risk with multiple attributes, criteria or objectives. In this approach, a decision-maker is able to select interactively and iteratively a mixture of complete judgement (e.g. intuitive affect), heuristic elimination (e.g. elimination by aspects), and holistic assessment (e.g. expected utility theory) of partial preference information to enable aggregation of some, but not necessarily all, criteria or attributes. We consider three dominance relations on the alternative set: expected-value score dominance, first-order stochastic dominance, and second-order stochastic dominance. Two important issues are resolved here. The first of these involves establishment of the impact of various forms of partial objective aggregation information, in the form of attribute weights, upon dominance relations for the alternative option set. The second concerns the computational tractability of the second-order dominance relations. In resolving the first issue, we show that under suitable conditions that will almost always prevail, additional attribute aggregation information implies greater alternative discrimination specificity for the three dominance relations considered. A fourth dominance relation is defined that represents a computationally attractive upper bound on the numerically demanding second-order stochastic dominance relation. The ultimate goal of the proposed procedure is rational and behaviourally relevant assistance in the search for a dominance structure.

1. Scope and purpose

In this paper, we consider two behaviourally relevant issues that arise from the application of a recently developed decision aiding approach to an important class of multiple objective decision-making problems under risk. A key characteristic of the decision aiding approach is that the decision-maker may iteratively and interactively select the mix of effort devoted to (1) assessing the relative importance of some, but not necessarily all, objectives, and (2) completely selecting the most preferred alternative on the basis of a dominance structure. The two issues, posed as questions, are: (1) does more of the former type of effort reduce the requirements for effort of the latter type: and (2) is the application of the decision aiding approach to this class of problems amenable to interactive computing? In resolving the first issue, we show that under mild and generally prevalent conditions, more information assessed regarding the relative importance of the objectives or attributes tends to reduce the number of candidates for the most preferred alternative. With respect to the resolution of the second issue, we present an approximation that is substantially more computationally attractive than an important method for determining the second-order stochastic dominance relation and associated preferences among the alternatives. These results,

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we feel, significantly enhance the potential computational efficiency and effectiveness, and hence acceptability, of interactive multicriterion decision aiding approaches.

2. Introduction

Many decision situations can be characterized as choice problems that involve selection of a preferred alternative, or sometimes selection of the N most preferred alternatives, from among a finite number of alternatives in a single decision node multiple-outcome model of decision making under risk. Crawford *et al.* (1978), Keefer (1978) and Pliskin *et al.* (1980) are among those who have considered applications of a decision situation model of this type.

Our research considers the following general approach for aiding single-stage finite-alternative multicriteria decision-making under risk. We assume that the structure of the problem has been identified, and that probabilities and value scores for each objective criterion or attribute have been assessed. However, attribute aggregation information, in the form of attribute weights, has not been assessed. We assume that the assessment of weights to enable attribute aggregation information, the central focus of this paper, is carried out in a three-step iterative procedure.

(1) We use various algorithmic procedures to partially order the alternatives into a dominance structure on the basis of currently available aggregation information.
(2) If this partial order provides enough information so that an alternative can be selected completely without more specific alternative ranking information, then stop.
(3) If complete selection cannot be made, then assess further objectives aggregation information and return to Step 1.

We envisage three forms of decision rules to enable judgement here. In a holistic rule, the whole is disaggregated into its parts, values for the parts are determined and aggregated to form a single-scale value measure. In heuristic elimination, alternatives perceived as unacceptable are eliminated by use of various conjunctive or disjunctive scanning approaches, etc. In wholistic judgement, alternative selection is made on the basis of reasoning by analogy, intuitive affect, or other forms of judgement that consider the whole of a situation without disaggregating criteria associated with outcomes into various criteria. See Sage (1981) for a discussion of various decision rules associated with these three judgement types.

Three basic dominance procedures for ordering alternatives into a dominance structure are considered here.

(1) Expected value score dominance (EVSD).
(2) First order stochastic dominance (FOSD).
(3) Second order stochastic dominance (SOSD).

We remark that utilization of the iterative approach for decision aiding suggested here allows the decision-maker (DM), rather than a paradigm, to determine the mix of holistic judgement or heuristic elimination (Step 2), and assessment of objectives aggregation information (Step 3). Support to the claim that this iterative procedure enhances the flexibility and the behavioural acceptability of decision support systems (DSS) can be found in White and Sage (1980) and White *et al.* (1982). Single-stage alternative selection under certainty is discussed in White and Sage (1980). The design, implementation and partial evaluation of a microprocessor-based DSS based on a single-pass version of the above iterative procedure is reported in White *et al.* (1982). The intent of the DSS discussed by White *et al.* is to provide a physician with a set of non-dominated diagnostic tests, determined on the basis of cost and expected information flow, in order to aid in the diagnosis of a common medical complaint. The set of non-dominated diagnostic tests usually contains no more than four tests from an alternative set containing fourteen tests. Physicists who have used the DSS have found that such a small initial non-dominated set usually represents sufficient support for alternative selection, which in part justifies the restriction of the above iterative procedure to a single pass in this decision-making environment. An evaluation of the aid's capability has indicated that its use has a significant beneficial impact on the cost and the frequency of major diagnostic errors. Perhaps more interestingly, an evaluation of the aid's acceptability in an operational (i.e. clinical) environment has indicated a promising level of user acceptability, relative to the level of user acceptability usually experienced by other DSSs (Adelman *et al.* 1981, Sage and White 1980).

We assume that the objectives aggregation information assessed represents a global description of the amount of influence that two or more lower (not necessarily lowest) level objectives have on a higher (not necessarily highest) level objective. In contrast, a complete scalar multiattribute utility assessment represents a global description of the amount of influence that all lowest objectives have on the highest level objective (Keeney and Raiffa, 1976). Several iterations of the aforementioned three-step procedure may therefore be required in order to specify fully the DMs utility function. The suggested procedure does not, however, necessarily require a complete specification of the utility function to enable evaluation and prioritization of the alternatives. This may reduce the time and stress associated with complete preference information assessment, sometimes considerably. If desired, the approach may be continued so that ultimately, a completely specified scalar multiattribute utility function is obtained such that the dominance structure is that of totally ordered alternatives.

A similarity exists between this decision support (evaluation) approach and various interactive multiple-objective optimization approaches (e.g. Geoffrion *et al.* 1972, Hwang *et al.* 1980, Oppenheimer 1978, Musselman and Talavage 1980, Zionts and Wallenius 1976) in that both interactively and iteratively attempt to eliminate clearly inferior alternatives from further consideration. However, our problem definition differs fundamentally from the problem formulations on which the interactive multiple-objective optimization procedures are based. Most of the multiple-objective optimization formulations, such as those cited, are deterministic and assume a non-denumerable set of alternatives and structural conditions which allow for the direct application of various mathematical programming procedures. Furthermore, the interactive multiple-objective optimization procedures are fundamentally impact assessment and optimization procedures which most appropriately refine parameters within a system, rather than evaluate and prioritize different policy alternatives. Within the systems engineering framework (Sage 1982), these are analysis and optimization procedures, whereas we present an evaluation and interpretation procedure for decision support.

Two important issues may be posed as questions. These naturally arise from our approach to decision aiding. Their resolution is the primary concern of this paper. The two questions are as follows.

(1) Under what conditions will additional objectives or criteria aggregation information imply a reduction in the number of alternatives requiring holistic or heuristic judgement ?
(2) Can the various algorithmic tasks associated with this approach be carried out in times compatible with interactive computing ?

This first issue arises from our perception that most DMs will expect that more effort devoted to the assessment of objectives aggregation information will reduce the effort required for holistic judgement or heuristic elimination by reducing the number of alternatives that are candidates for the most preferred alternative, and conversely. In fact, this expectation may not always be realized. We provide conditions in this paper which guarantee that additional objectives aggregation information implies increased alternative discrimination specificity.

The second issue has become of concern due to the fact that the SOSD relation often requires significant computational effort. In order to reduce this effort, we take advantage of the facts that the FOSD relation is a lower bound on the SOSD relation and that there is a computationally attractive upper bound on the SOSD relation. We call this upper bound the strong SOSD relation (SSOSD). All of these relations are transitive.

This paper is organized as follows. Several preliminary definitions and remarks are presented in the next section. The EVSD, FOSD, SOSD and SSOSD relations are defined in §§ 3–6. In §§ 3–5 we present conditions which guarantee that additional objectives aggregation information implies increased alternative discrimination specificity. These results resolve the first issue, at least partially. In § 6, procedures are developed for reducing the computational effort associated with the SOSD relation, thus at least partially resolving the second issue concerning computational attractiveness.

3. Preliminary definitions and remarks

We assume that the DM can select for implementation any one of P predetermined alternatives from the finite alternative set $\Pi = \{\pi^1, \dots, \pi^P\}$. Implementation of an alternative will result in the occurrence of any one of M possible outcomes. There are N objectives associated with each outcome. Let $v_n{}^m$ be the alternative independent, predetermined *value score* of the mth outcome with respect to the nth objective. The real number $v_n{}^m$ is isotone (monotonically non-decreasing) in preference with respect to the nth objective; that is, outcome m' is (weakly) preferred to outcome m with respect to objective n, if and only if $v_n{}^{m'} \geq v_n{}^m$. We define the value score vector associated with the mth outcome as $v^m = \{v_1{}^m, \ldots, v_N{}^m\}$ and let $V = \{v^m, m = 1, \ldots, M\}$ be the *collection of all value score vectors*. The *probability* that outcome m will result if alternative π^μ is selected is the scalar $\pi^\mu(v^m)$. We note that π^μ is equivalently considered to be a probability mass vector having mth scalar component $\pi^\mu(v^m)$; i.e. $\pi^\mu = \{\pi^\mu(v^m)\}_{m=1}{}^M$. We remark that alternative dependent value scores can be considered in this framework by properly expanding the outcome set and appropriately redefining the outcome probabilities.

Each of the four relations, EVSD, FOSD, SOSD and SSOSD, will be described as a subset, say R, of ordered pairs $\Pi \times \Pi$. The relation $(\pi', \pi) \in R$ will be given the interpretation that alternative π' is (weakly) preferred to alternative π (with respect to R). The set R of ordered pairs represents a valuable aid in selecting an alternative. The non-dominated set of R is guaranteed to contain the most preferred alternative. (Alternative $\pi \in \Pi$ is said to be dominated if there is a $\pi' \in \Pi$ such that $(\pi', \pi) \in R$ and $(\pi, \pi') \notin R$. The set of all alternatives in Π that are not dominated is called the non-dominated set of R.) Hence, search for the most preferred alternative can be confined to the non-dominated set. (However, the second most preferred alternative may not be in the non-dominated set. This may cause concern if the objective of decision aiding is to select the K most preferred alternatives (White and Sage 1980).) More specifically, if there is a $\pi' \in \Pi$ such that $(\pi', \pi) \in R$ for all $\pi \in \Pi$, then π' is an optimal alternative. Additionally, if $(\pi, \pi') \notin R$ for all π except for the case where $\pi = \pi'$, then π' is the unique optimal alternative.

Throughout, the function $f: \mathbb{R}^N \to \mathbb{R}^L$ will represent the objective or attribute aggregation function, a functional representation of the DMs preferences, regarding value score aggregation. For example, we might let $L = N - 2 \geqslant 1$, $f_l(v) = v_l (l = 1, \ldots, N-3)$, $f_{N-2}(v_{N-2}, v_{N-1}, v_N)$ represent the value score for the aggregated attribute 'operating expenses', where v_{N-2}, v_{N-1} and v_N represent the value scores of the three attributes 'fuel expenses', 'scheduled maintenance expenses', and 'unscheduled maintenance expenses' that, taken together, comprise the higher order attribute 'operating expenses'.

In the next three sections, we will examine the impact of objectives aggregation functions on the EVSD, FOSD and SOSD relations. Specifically, let R and $\bar{R}$ be any one of these three relations on Π before and after an objectives aggregation, respectively. We would generally hope and expect that $R \subseteq \bar{R}$. This is equivalent to the statement that objectives aggregation has provided at least as much alternative discrimination specificity as there existed before the aggregation. However, $R \subseteq \bar{R}$ does not guarantee that the non-dominated set in Π associated with $\bar{R}$ is smaller than that associated with R. Examples of a non-dominated set becoming larger following the elimination of an attribute are easy to construct but are somewhat pathelogical in practice. In applications, non-dominated sets tend to grow smaller as more preference information is assessed. Thus, from a practical perspective, $R \subseteq \bar{R}$ essentially supports resolution of the first issue addressed in this paper. Our intent, therefore, in the following three sections is to determine what conditions imply that $R \subseteq \bar{R}$ for the EVSD, FOSD and SOSD relations.

4. Expected-value score dominance

Let R_e be the subset of ordered pairs in $\Pi \times \Pi$ defined as follows : $(\pi', \pi) \in R_e$ if and only if the vector relation

$$\sum_{v \in V} v\pi'(v) \geqslant \sum_{v \in V} v\pi(v)$$

is satisfied. (Note that

$$\sum_{v \in V} v\pi(v) = \sum_{m=1}^{M} v^m \pi(v^m)$$

where $v^m\pi(v^m)$ is the N-vector having nth scalar element $v_n{}^m\, \pi(v^m)$.) We refer to R_e as the expected-value score relation on Π, a relation which is clearly transitive. (The relation R is transitive if and only if $(\pi'', \pi') \in R$ and $(\pi', \pi) \in R$ imply $(\pi'', \pi) \in R$.) Note that $(\pi', \pi) \in R_e$ if and only if the expected-value score associated with alternative π' is at least as great as the expected-value score associated with alternative π for every objective. Thus if the value score vector represents Von Neuman Morgenstern utility, then we are calculating expected (vector) utility. Our interest in R_e is due to the fact that it represents the basis for several operational decision aids (Kelly 1978). Let $\bar{R}_e$ be the expected-value score relation after an objectives aggregation ; that is, let R_e be such that $(\pi', \pi) \in \bar{R}_e$ if and only if

$$\sum_{v \in V} f(v)\pi'(v) \geqslant \sum_{v \in V} f(v)\pi(v)$$

The following result presents the relationship between R_e and $\bar{R}_e$.

Proposition 1

If either

(*a*) f is affine (linear on V plus a constant) and isotone, or
(*b*) $M = 2$ and f is isotone,

then $R_e \subseteq \bar{R}_e$.

Under the additional assumption that f is strictly isotone, a straightforward argument shows that the aggregation will not cause the non-dominated set to increase. Linear objectives aggregation represents a very common aggregation procedure for operational decision aids (Kelly 1978), and is the ultimate result of continuing the process of obtaining greater attribute aggregation at each iteration.

Proof

(*a*) Assume $(\pi', \pi) \in R_e$. By the isotonicity of f

$$f[\Sigma_v v\pi'(v)] \geqslant f[\Sigma_v v\pi(v)]$$

That f is affine then implies $(\pi', \pi) \in \bar{R}_e$.

(*b*) Note that for the case where $M = 2$

$$\sum_{v \in V} v\pi'(v) \geqslant \sum_{v \in V} v\pi(v)$$

is equivalent to

$$v^1[\pi'(v^1) - \pi(v^1)] \geqslant v^2[\pi'(v^1) - \pi(v^1)]$$

Assume $\pi'(v^1)-\pi(v^1)>0$, which implies $v^1 \geqslant v^2$ from the foregoing inequality. Note that $v^1 \geqslant v^2$ implies $f(v^1) \geqslant f(v^2)$, which in turn implies

$$f(v^1)[\pi'(v^1)-\pi(v^1)] \geqslant f(v^2)[\pi'(v^1)-\pi(v^1)]$$

which is equivalent to the desired result. The $\pi'(v^1)-\pi(v^1)=0$ case is trivial; the $\pi'(v^1)-\pi(v^1)<0$ case proceeds as above. □

We now present a counter-example to the possibility intuitive claim that, in general, $R_e \subseteq \bar{R}_e$.

Example 1

We consider a specific two-alternative three-outcome case where there are two attributes corresponding to each outcome. Let

$$v^1=\begin{bmatrix}0\\1\end{bmatrix},\quad v^2=\begin{bmatrix}1\\0\end{bmatrix},\quad v^3=\begin{bmatrix}0{\cdot}75\\0{\cdot}75\end{bmatrix}$$

$$\pi'(v^1)=0{\cdot}2,\quad \pi'(v^2)=0{\cdot}2,\quad \pi'(v^3)=0{\cdot}6$$

$$\pi(v^1)=0{\cdot}3,\quad \pi(v^2)=0{\cdot}3,\quad \pi(v^3)=0{\cdot}4$$

We see that

$$\sum_{v\in V} v\pi'(v)=\begin{bmatrix}0{\cdot}65\\0{\cdot}65\end{bmatrix}$$

$$\sum_{v\in V} v\pi(v)=\begin{bmatrix}0{\cdot}60\\0{\cdot}60\end{bmatrix}$$

Thus $(\pi',\pi)\in R_e$; that is, alternative π' dominates π with respect to the EVSD relation. This means we can expect that π' is preferred to π no matter what the relative weights of the two attributes.

Consider the case where the aggregation function is the multilinear case: $f(v^i)=k_1v_1{}^i+k_2v_2{}^i+(1-k_1-k_2)v_1{}^i\,v_2{}^i$ for $i=1$, 2 or 3. A multilinear utility function is a necessary condition for mutual utility independence; see Keeney and Raiffa (1976) for details. Note that $f(v^1)=k_2$, $f(v^2)=k_1$ and $f(v^3)=3(k_1+k_2)/16+9/16$. It follows that the expected values of the two alternatives are

$$\sum_{v\in V} f(v)\pi'(v)=\tfrac{5}{16}(k_1+k_2)+\tfrac{27}{80}$$

$$\sum_{v\in V} f(v)\pi(v)=\tfrac{3}{8}(k_1+k_2)+\tfrac{9}{40}$$

Observe that if $(k_1+k_2)\leqslant 9/5$, then $(\pi',\pi)\in\bar{R}_e$; however, when $(k_1+k_2)\geqslant 9/5$, $(\pi,\pi')\in\bar{R}_e$. Thus when the sum k_1+k_2 exceeds 9.5, objectives aggregation using the multilinear utility function does not imply that $R_e\subseteq\bar{R}_e$. For the particular case where the multilinear function is linear, we have $k_1+k_2=1$ and $(\pi',\pi)\in R_e$.

Example 2

We now present a simple numerical example which illustrates the determination of R_e and $\bar{R}_e$. Assume there are five possible outcomes, four lowest level attributes initially under consideration, and six available alternatives, i.e. $M=5$, $N=4$, $P=6$. Table 1 presents assumed data, and the expected value scores are listed in Table 2 (a). On the basis of the information contained in Table 2 (a), $R_e=\{(1,3),(2,3),(4,3)\}$ and the expected-value dominance digraph for the six alternatives is given by Fig. 1.

Assume $f(v)=Av$, where the first three attributes are aggregated together into a single attribute according to

$$A=\begin{bmatrix}0\cdot1 & 0\cdot1 & 0\cdot8 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Table 3 presents the value scores resulting from aggregating the value scores of Table 1 (a) and these weights. Here, the aggregation function is isotone

		Outcome number, m				
		1	2	3	4	5
(a) Attribute number, n	1	1	0·5	0·5	0	0·5
	2	1	0	0	0	0
	3	0·3	0·3	1	0	0·3
	4	0·5	0·5	0·5	0	1
(b) Alternative number, p	1	0·6	0·1	0·2	0·1	0·0
	2	0·7	0·0	0·1	0·2	0·0
	3	0·3	0·1	0·0	0·4	0·2
	4	0·3	0·0	0·1	0·1	0·5
	5	0·1	0·1	0·0	0·1	0·7
	6	0·0	0·1	0·1	0·0	0·8

Table 1. Data for Example 2. (a) Value scores v_n^m for each outcome. (b) Probabilities for each alternative, $\pi^p(v^m)$.

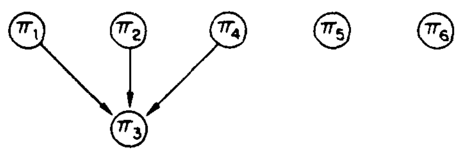

Figure 1. Expected-value dominance digraph for alternatives before aggregation.

		Alternative numbers					
		1	2	3	4	5	6
(*a*) Attribute number	1	0·75	0·75	0·45	0·60	0·50	0·50
	2	0·60	0·70	0·30	0·30	0·10	0
	3	0·41	0·31	0·18	0·34	0·27	0·37
	4	0·45	0·40	0·40	0·70	0·80	0·90
(*b*) Attribute number	1–3	0·463	0·393	0·219	0·362	0·276	0·346
	4	0·450	0·400	0·400	0·700	0·800	0·900

Table 2. Expected-value scores for Example (2): (*a*) before the aggregation, and (*b*) after the aggregation.

		Outcome number. *m*				
		1	2	3	4	5
Attribute number	1–3	0·44	0·29	0·85	0	0·29
	4	0·5	0·5	0·5	0	1

Table 3. Value scores v_n^{kl} for each outcome.

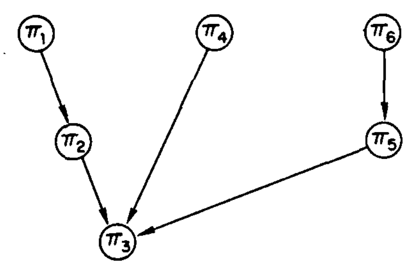

Figure 2. Expected-value dominance digraph for alternatives after aggregation.

and linear and the trade-off weights for the three attributes 1, 2 and 3 or 0·1, 0·1 and 0·8, respectively. Table 2 (*b*) gives the expected-value scores after the aggregation, indicating that $\bar{R}_e = \{(1,2), (2,3), (4,3), (6,5), 5,3)\}$. Note that the aggregation has produced a non-dominated set, the set $\{1, 4, 6\}$, which is strictly smaller than the non-dominated set before the aggregation, $\{1, 2, 4, 5, 6\}$. Figure 2 indicates the expected-value dominance digraph for the six alternatives after aggregation.

5. First-order stochastic dominance

Let R_1 be the subset of ordered pairs in $\Pi \times \Pi$ defined as follows : $(\pi', \pi) \in R_1$ if and only $E(u, \pi') \geq E(u, \pi)$ for all $u \in U_1$, where U_1 is the set of all real-valued functions on V that are isotone and where

$$E(u, \pi) = \sum_{v \in V} u(v)\pi(v)$$

Thus, $E(u, \pi)$ is the expected utility vector associated with utility function u and probability mass function π. We refer to R_1 as the first-order stochastic dominance (FOSD) relation on Π, a relation that is both transitive and anti-symmetric (Fishburn and Vickson 1978). An equivalent and more operationally useful definition of R_1 is as follows (Lehmann 1955) : $(\pi', \pi) \in R_1$ if and only if $\pi'(K) \geq \pi(K)$ for all $K \in \mathscr{K} = \{K \subseteq V :$ if $v \in K$, $v' \in V$ and $v' \geq v$, then $v' \in K\}$ where

$$\pi(K) = \sum_{v \in K} \pi(v)$$

$\mathscr{K}$ is the (finite) collection of all the so-called increasing sets in V.

Let $\bar{\mathscr{K}} = \{K \subseteq V :$ if $v \in K$, $v' \in V$ and $f(v') \geq f(v)$, then $v' \in K\}$, the collection of increasing sets after an objectives aggregation. Define $\bar{R}_1$ exactly as R_1 was defined, except replace $\mathscr{K}$ with $\bar{\mathscr{K}}$. We now present our main result involving objectives aggregation for FOSD.

Proposition 2

If f is isotone, then $R_1 \subseteq \bar{R}_1$.

Proof

A simple argument demonstrates that $\bar{\mathscr{K}} \subseteq \mathscr{K}$ which implies $R_1 \subseteq \bar{R}_1$ directly from the definition. □

We now illustrate Proposition 2 with the following example.

Example 3

Consider the problem presented in Example 2. The data presented in Table 1 (*a*) generate the dominance digraph of outcome state elements in V in Fig. 4. The increasing sets in $\mathscr{K}$ associated with this digraph which are of interest are : $\{1\}$, $\{3\}$, $\{5\}$, $\{1, 3\}$, $\{1, 5\}$, $\{3, 5\}$, $\{1, 3, 5\}$ and $\{1, 2, 3, 5\}$. We delete consideration of $\{1, 2, 3, 4, 5\} \in \mathscr{K}$ and $\phi \in \mathscr{K}$ since $\pi(\{1, 2, 3, 4, 5\}) = 1{\cdot}0$ and $\pi(\phi) = 0{\cdot}0$ for all $\pi \in \Pi$.

We do *not* need the cardinal values given in Table 1 (*a*) to generate the increasing sets shown here. Any ordinal preference relations that imply the

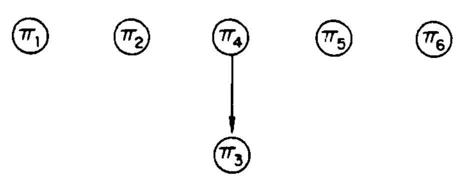

Figure 3. First-order stochastic dominance digraph for alternatives before attribute aggregation.

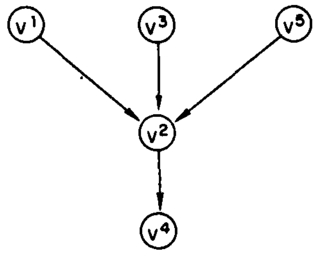

Figure 4. Domination digraph of Example 3 for the value scores associated with Table 1.

Increasing subset	Alternative 1	2	3	4	5	6
{1}	0·6	0·7	0·3	0·3	0·1	0·0
{3}	0·2	0·1	0·0	0·1	0·0	0·1
{5}	0·0	0·0	0·2	0·5	0·7	0·8
{1, 3}	0·8	0·8	0·3	0·4	0·1	0·1
{1, 5}	0·6	0·7	0·5	0·8	0·8	0·8
{3, 5}	0·2	0·1	0·2	0·6	0·7	0·9
{1, 3, 5}	0·8	0·8	0·5	0·9	0·8	0·9
{1, 2, 3, 5}	0·9	0·8	0·6	0·9	0·9	1·0

(*a*)

{3}	0·2	0·1	0·0	0·1	0·0	0·1
{5}	0·0	0·0	0·2	0·5	0·7	0·8
{3, 5}	0·2	0·1	0·2	0·5	0·7	0·8
{1, 3}	0·8	0·8	0·3	0·4	0·1	0·1
{1, 3, 5}	0·8	0·8	0·5	0·9	0·8	0·9
{1, 2, 3, 5}	0·9	0·8	0·6	0·9	0·9	0·0

(*b*)

Table 4. Probabilities of the increasing sets for each alternative, i.e. $P(v \in K | \pi)$ for each $K \in \mathscr{K}$ and $\pi \in \Pi$: (*a*) before and (*b*) after the value score aggregation in Example 3.

outcome state domination digraph of Fig. 1 will suffice. We do use these values for aggregation later in this example, however.

The probabilities associated with each of the above sets for each alternative are displayed in Table 4 (*a*). We note that the only alternative pair (π_i, π_j) for which $\pi_i(K) \geqslant \pi_j(K)$ for all $K \in \mathscr{K}$ is the pair $i = 4$, $j = 3$. Thus, it follows that $R_1 = \{(4, 3)\}$, and hence for this example $R_1 \subseteq R_c$. Figure 3 illustrates the FOSD relation among alternatives before an attribute tradeoff. There will generally be 'less dominance' associated with FOSD than EVSD.

As in Example 2, let f be linear and assume the value scores for attributes 1, 2 and 3 are traded-off with weights 0·1, 0·1 and 0·8, respectively. Again, this results in the value score matrix given in Table 3. The resulting dominance digraph of output state elements in V is given in Fig. 5. This digraph has the following increasing sets of interest: $\{3\}$, $\{5\}$, $\{3, 5\}$, $\{1, 3\}$, $\{1, 3, 5\}$ and $\{1, 2, 3, 5\}$. Notice that the total number of increasing sets has been reduced by the aggregation. Table 4 (b) presents the probabilities associated with these increasing sets. It then follows that $\bar{R}_1 = \{(1, 2), (4, 3), (6, 5)\}$, which provides more preference information with respect to the alternatives than does R_1. Again we see that there is less preference information given by FOSD than by EVSD. Thus $\bar{R}_1 \subseteq \bar{R}_e$. Figure 6 shows the FOSD digraph for the six alternatives of this example.

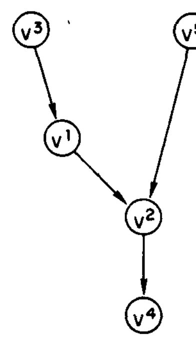

Figure 5. Output state value domination digraph for the value scores associated with Table 3 after the value score aggregation for Example 3.

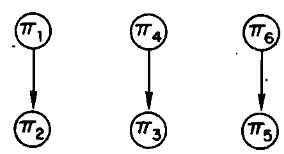

Figure 6. First-order stochastic dominance of alternatives after attribute aggregation.

It is of interest to give conditions that relate the two orders R_e and R_1. Let the function $I_K : V \to \{0, 1\}$ be such that $I_K(v) = 1$ if $v \in K$ and $I_K(v) = 0$ if $v \notin K$. Let $I = \{I_K\}_{K \in \mathscr{K}}$. Thus, $I(v)$ is a γ-vector, if $\mathscr{K}$ contains γ subsets of V, which identifies the increasing sets containing v. It is easy to demonstrate that $(\pi', \pi) \in R_1$ is equivalent to

$$\Sigma_v I(v)\pi'(v) \geqslant \Sigma_v I(v)\pi(v)$$

It is now straightforward to show that the following results hold.

Proposition 3

(a) Assume there is an isotone, affine function α such that $\alpha[I(v)] = v$ for all $v \in V$. Then $R_1 \subseteq R_e$.

(b) Assume there is an isotone, affine function β such that $\beta(v) = I(v)$ for all $v \in V$. Then $R_e \subseteq R_1$.

We now illustrate Proposition 3 with the following example.

Example 4

Consider the data presented in Tables 1 (*a*) and 4 (*a*). With reference to Table 4 (*a*), note that $I(v^1) = \text{col}\{1, 0, 0, 1, 1, 0, 1, 1\}$, in that $1 \in \{1\}$, $1 \notin \{3\}$, $1 \notin \{5\}$, $1 \in \{1, 3\}$, and so forth. The collection of column vectors $\{I(v^1), \ldots, I(v^5)\}$ is then

$$[I(v^1), \ldots, I(v^5)] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

In order to illustrate Proposition 3 (*a*), we seek an isotone, affine function α such that

$$[v^1, \ldots, v^5] = \alpha[I(v^1), \ldots, I(v^5)]$$

where, from Table 1 (*a*)

$$[v^1, \ldots, v^5] = \begin{bmatrix} 1 & 0{\cdot}5 & 0{\cdot}5 & 0 & 0{\cdot}5 \\ 1 & 0 & 0 & 0 & 0 \\ 0{\cdot}3 & 0{\cdot}3 & 1 & 0 & 0{\cdot}3 \\ 0{\cdot}5 & 0{\cdot}5 & 0{\cdot}5 & 0 & 1 \end{bmatrix}$$

Note that since $v^4 = \alpha I(v^4)$, α *is required to be linear, i.e.* α is a 4×8 matrix. We can now omit v^4 and $I(v^4)$ from further consideration. Standard algebraic procedures show that

$$\alpha = \begin{bmatrix} 0{\cdot}5 & 0 & 0 & 0 & 0 & 0 & 0 & 0{\cdot}5 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0{\cdot}7 & 0 & 0 & 0 & 0 & 0 & 0{\cdot}3 \\ 0 & 0 & 0{\cdot}5 & 0 & 0 & 0 & 0 & 0{\cdot}5 \end{bmatrix}$$

is such that $\alpha[I(v)] = v$ for all $v \in V$. We remark that α is not unique and that other α can be found which are linear and satisfy $\alpha[I(v)] = v$ but are not isotone. Clearly α is isotone, and hence from Proposition 3 (*a*), $R_1 \subseteq R_a$, which is verified by the results in Examples 2 and 3. Since the aggregation function used was linear in both Examples 2 and 3, it is straightforward to show that $\bar{R}_1 \subseteq \bar{R}_e$, using Proposition 3 (*a*) and the same α as above.

Straightforward algebraic manipulation shows that there exists a unique 8×4 matrix β such that $\beta v = I(v)$ for all $v \in V$, where

$$\beta = \begin{bmatrix} 0 & 1/10 & 0 & 0 \\ -3/35 & 3/70 & 1/7 & 0 \\ -1/5 & 1/10 & 0 & 1/5 \\ -3/35 & 1/7 & 1/7 & 0 \\ -1/5 & 1/5 & 0 & 1/5 \\ -2/7 & 1/7 & 1/7 & 1/5 \\ -2/7 & 17/70 & 1/7 & 1/5 \\ 1/5 & -1/10 & 0 & 0 \end{bmatrix}$$

Note, however, that β is not isotone and thus it does not necessarily follow from Proposition 3 (*b*) that $R_e \subseteq R_1$. In fact, we observe from Examples 2 and 3 that $R_e \subseteq R_1$ does not hold.

In concluding this section, we present two facts that have potential computational impact for determining R_1. Let $\mathscr{K}_1 = \{K \in \mathscr{K} : K^c \in \mathscr{K}\}$, $\mathscr{K}_2 = \mathscr{K}_1^c$, where $\mathscr{K}_1^c$ is the complement of $\mathscr{K}_1$. It is interesting to note that $\pi'(K) = \pi(K)$ for all $K \in \mathscr{K}_1$ if and only if $(\pi', \pi) \in R_1$. This fact suggests that in trying to determine whether or not $(\pi', \pi) \in R_1$ a simple initial check would be to see if $\pi'(K) = \pi(K)$ for all $K \in \mathscr{K}_1$. We observe that only half of the $\mathscr{K}_1$ sets need to be checked since $\pi'(K) = \pi(K)$ iff $\pi'(K^c) = \pi(K^c)$, thus reducing the computational burden of checking all of the sets in $\mathscr{K}_1$. It can easily be shown that if all elements in the outcome set are non-dominated, i.e. $\mathscr{K} = \mathscr{K}_1$, then all elements in Π are non-dominated ; thus, if all elements in the outcome set are non-dominated, it is unnecessary to determine the relation on the alternative set.

6. Second-order stochastic dominance

Let R_2 be the subset of ordered pairs in $\Pi \times \Pi$ defined as follows : $(\pi', \pi) \in R_2$ if and only if $E(u, \pi') \geqslant E(u, \pi)$ for all $u \in U_2$, where U_2 is the set of all real-valued functions on Γ that are both isotone and concave. We refer to R_2 as the second-order stochastic dominance (SOSD) relation on Π. The concavity of a utility function is the functional representation of global risk aversion, describing the preference structure of a decision-maker who will never invest in an actuarially fair prospect.

Fishburn and Vickson (1978) have presented a slightly less general version of the following necessary and sufficient conditions for $(\pi', \pi) \in R_2$: $(\pi', \pi) \in R_2$

if and only if there exists a feasible solution to the following set of linear equalities and inequalities

(i) $d_{ij} \geq 0$ for all $i, j = 1, \ldots, M$

(ii) $\sum_{j=1}^{M} d_{ij} = 1$ for all $i = 1, \ldots, M$

such that $\pi'(v^i) \neq 0$

(iii) $\pi(v^j) = \sum_{i=1}^{M} \pi'(v^i) d_{ij}$ for all $j = 1, \ldots, M$

(iv) $\sum_{j=1}^{M} d_{ij} v_n{}^j \leq v_n{}^i$ for all $i = 1, \ldots, M$

such that $\pi'(v^i) \neq 0$ and all $n = 1, \ldots, N$, where $v_n{}^j$ is the nth scalar entry of $v^j \in V$; i.e. $v^j = \{v_1{}^j, \ldots, v_N{}^j\}$.

Interpretations of the solution $\{d_{ij}\}$ to the above can be found in Fishburn and Vickson (1978). To proceed, it is desirable to determine conditions under which $R_2 \subseteq \bar{R}_2$. We define $\bar{R}_2$ as R_2 was defined, except replace U_2 with $\bar{U}_2$, where

$$\bar{U}_1 = \{u : u \text{ is isotone on } f(\mathbb{R}^N)\}$$

$$\bar{U}_2 = \{u \in \bar{U}_1 : u \text{ is concave on } f(\mathbb{R}^N)\}$$

We now present the conditions on f which insure $R_2 \subseteq \bar{R}_2$.

Proposition 4

If we assume that objectives aggregation function f satisfies the following conditions

(1) $f_l(v^j) = \sum_{n=1}^{N} v_n{}^j b_{nl}$ where $b_{nl} \geq 0$

for all $n = 1, \ldots, N$ and $l = 1, \ldots, L$

(2) if $v^i \neq v^j$, then $f(v^i) \neq (v^j)$

then $R_2 \subseteq \bar{R}_2$.

Proof

We wish to show that if (iv) holds, then

$$\sum_{j=1}^{M} d_{ij} f_l(v^j) \leq f_l(v^i)$$

for all $i = 1, \ldots, M$ such that $\pi'(v^i) \neq 0$ and all $l = 1, \ldots, L$. Condition (2) implies that the number of v^i such that $\pi'(v^i) \neq 0$ does not change after the aggregation is performed. Condition (1) can be used to show easily that (iv) implies the above inequality. □

We remark that R_2 is both transitive and antisymmetric and that $R_1 \subseteq R_2$ (Fishburn and Vickson 1978). The following example illustrates the latter fact and Proposition 4.

Example 5

Again consider the problem presented in Example 2, using second-order stochastic dominance as the method of generating a partial order on l. It follows from application of the results of conditions (i) to (iv) that $R_2 = \{(1, 3), (4, 3)\}$ and $\bar{R}_2 = \{(1, 2), (2, 3), (4, 3), (6, 5), (5, 3)\}$. Note that by transitivity (1, 3) and (6, 3) are members of $\bar{R}_2$. Note also that it is only coincidental that $\bar{R}_2 = \bar{R}_c$. We observe that the assumption of risk aversion has caused the relationship $R_1 \subseteq R_2$ to be strict both before the aggregation and after the aggregation. Figure 7 illustrates the second-order stochastic dominance before and after the aggregation of the first three attributes, with weights (0·1, 0·1, 0·8) into a single attribute.

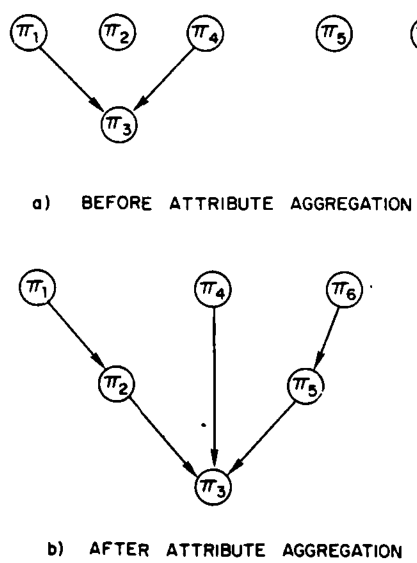

Figure 7. Second-order stochastic digraphs of alternatives for Example 5.

7. Strong second-order stochastic dominance

We observe that determining R_2 requires formulating and determining the feasibility of $P(P-1)$ linear programs, each having up to $M(M+N+2)$ decision variables ($2M$ of which are artificial variables) and up to $M(N+2)$ side constraints. Of course, these computational requirements may be decreased if the transitivity of R_2, and/or the fact that $R_1 \subseteq R_2$ are efficiently utilized. Our experience indicates, however, that for even modestly sized problems where card (R_1)/card $(\Pi \times \Pi)$ is relatively small (which is common), determination of R_2 often generates computer times that are unacceptably large for interactive computing. (Card A is the cardinality of the set A ;

i.e. the number of elements in the set A if A is finite.) For such cases, it would be useful to have an easily computed upper bound on R_2. We now investigate one such upper bound.

Let $R_S{}^n$ be the subset of ordered pairs in $\Pi \times \Pi$ defined as follows: $(\pi', \pi) \in R_S{}^n$ if and only if $E(u, \pi') \geqslant E(u, \pi)$ for all $u \in U_S{}^n$, where $U_S{}^n$ is the set of all real-valued functions on $V^n = \{v^{nm}, m = 1, \ldots, M\}$ that are both isotone and concave. Thus $R_S{}^n$ is equivalent to R_2 when only the nth objective is under consideration, and hence $R_2 \subseteq R_S{}^n$ for all $n = 1, \ldots, N$. Let

$$R_S = \bigcap_{n=1}^{N} R_S{}^n$$

and call R_S the strong second-order stochastic dominance (SSOSD) relation. Since each $R_S{}^n$ is transitive and antisymmetric for all n, then so is R_S. Clearly, $R_2 \subseteq R_S$. We remark that determining whether or not $(\pi', \pi) \in R_S$ is equivalent to at most N separate checks for univariate second-order stochastic dominance, for which there exists a computationally simple procedure (cf. § 2.14 in Fishburn and Vickson (1978)). We describe this procedure as follows. Assume for notational convenience that for objective n, $m' \geqslant m$ implies $v_n{}^{m'} \geqslant v_n{}^m$; i.e. with respect to objective n, outcome $m+1$ is at least as preferred as outcome m, for $m = 1, \ldots, M-1$. Then $(\pi', \pi) \in R_S{}^n$ if and only if

$$\sum_{i=1}^{l} \left[\sum_{j=1}^{i} \pi'(v^j) \right] (v_n{}^{i+1} - v_n{}^i) \leqslant \sum_{i=1}^{l} \left[\sum_{j=1}^{i} \pi(v^j) \right] (v_n{}^{i+1} - v_n{}^i)$$

for all $l = 1, \ldots, M-1$.

We also note that under certain independence conditions (presented, for example, in Theorem 2.11 of Fishburn and Vickson (1978)), $R_2 = R_S$. We now present an example illustrating the computational usefulness of R_S.

Example 6

We again consider the problem presented in Example 2. Recall that $R_1 = \{(4, 3)\}$. Calculations based on procedures suggested in § 2.14 of Fishburn and Vickson (1978) show that $R_S = \{(1, 3), (2, 3), (4, 3)\}$. Thus it is only necessary to check the pairs (1, 3) and (2, 3) in order to determine R_2. Solution of the two associated linear programs indicates that $(1, 3) \in R_2$ and $(2, 3) \notin R_2$, and hence $R_2 = R_1 \cup (1, 3) = \{(1, 3), (4, 3)\}$, which is, of course, in agreement with the results of Example 5.

Calculations show that $\bar{R}_S = \{(1, 2), (2, 3), (4, 3), (6, 5), (5, 3)\}$. Recalling that $\bar{R}_1 = \{(1, 2), (4, 3), (6, 5)\}$, the only pairs that require examination are: (2, 2), (5,3). The pairs (1, 3) and (6, 3) are members of $\bar{R}_S$ by transitivity. We observe that if $(2, 3) \in \bar{R}_2$ and $(6, 3) \in \bar{R}_2$, then it is not necessary to check if $(1, 3) \in \bar{R}_2$ and $(5, 3) \in \bar{R}_2$, respectively, because of the transitivity of $\bar{R}_2$. Solution of the associated linear programs shows that (1, 3) and (6, 3) are members of $\bar{R}_2$. We have therefore determined that $\bar{R}_2 = \bar{R}_S$ by formulating and solving only two linear programs. Dominance digraphs for the SOSD relation are presented in Fig. 7.

8. Conclusions

We have addressed two important issues that arise in the application of stochastic dominance concepts to interactive decision support with a single-stage finite-alternative multicriteria decision-under-risk model. The key characteristics of this approach are that the DM, rather than the paradigm, is allowed to select iteratively and interactively a mix of wholistic judgement, heuristic elimination, and holistic evaluation of partial preference information to enable aggregation of some but not necessarily all criteria or attributes. Conditions on the attribute aggregation function were given which guarantee that preference information supplied by the DM in the form of attribute-value score aggregations is transmitted to the relation on the alternative set in the form of a stronger partial order for three important relations : expected-value stochastic dominance, first-order stochastic dominance, and second-order stochastic dominance. A fourth relation was introduced to reduce the computational burden that is often associated with the determination of the second-order stochastic dominance relation.

With respect to the interactive decision support approach outlined in the Introduction, the behavioural impacts of the results obtained here are as follows.

(1) Under weak conditions, more effort devoted by the DM to the assessment of objectives aggregation information will result in a reduction of effort by the DM required to obtain a rational dominance structure of alternative prioritization that is suitable for wholistic judgement.

(2) There is a computationally attractive upper bound on the SOSD relation which enhances our ability to obtain approximate and useful SOSD results in a timely manner that is consistent with interactive decision support.

It is of interest to observe that the expected-value score relation and the stochastic dominance relations differ fundamentally in their interpretation of a complete-value score aggregation, i.e. for the case where the aggregation function $f: \mathbb{R}^N \rightarrow \mathbb{R}^1$ (the $L=1$ case). In this case, $\bar{R}_e$ is a linear order and $\bar{R}_1$ and $\bar{R}_2$ may only be partial orders. $\bar{R}_e$ essentially treats $f(v)$ as a utility function, i.e. $u(x)=f[v(x)]$, whereas $\bar{R}_1$ and $\bar{R}_2$ assume there exists a still unassessed function U which, when composed with $f(v)$, produces the desired utility function, i.e. $u(x)=U[f[v(x)]]$. A discussion of the usefulness of assuming the existence of such a function U, especially for modelling risk, can be found in Bodily (1980).

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A KNOWLEDGE BASED INTERACTIVE PROCEDURE FOR PLANNING AND DECISION SUPPORT UNDER UNCERTAINTY AND PARAMETER IMPRECISION

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Abstract

We summarize key features of an interactive planning and decision support process for multiple criteria alternative selection situations. Probabilities, utility scores for the lowest level attributes, and attribute tradeoff weights, i.e., the parameters, can be imprecisely described by set inclusion. Within a specified structural model of the decision situation, the process allows the decisionmaker to iteratively select the mix of parameter value precision and alternative ranking specificity. By selecting this mix, the decisionmaker is able to direct the alternative selection process in an interactive manner, using alternative selection strategies based on behaviorally meaningful dominance search strategies. Emphasis is placed on the motivation of the research and the behavioral relevance of the support process.

1. Introduction

The process of choosing among multiattributed alternatives often involves an *initial search for a* dominance structure and ultimate identification of a set of nondominated alternatives. By definition, a nondominant alternative is one which is not worse than any other alternative on any attribute and which is better than each other alternative on at least one attribute. In most decision situations, however, there is *no single alternative* that dominates all other alternatives, at least initially. In such decision situations, the decisionmaker typically "adjusts" the structure of the decision situation, and parameter values within this structure, so as to identify a dominance structure which contains a single nondominant alternative. This search may involve rational activities, such as aggregation of attributes and compensatory tradeoffs through determination of judgmental weights. Alternately, it may involve various rules which may be quite flawed. Examples of such rules are (i) lexicographic ordering, in which the best alternative on the most important attribute is selected, and (ii) sequential pairwise comparison of alternatives using a preference relation that is a function of the two alternatives being compared. In this latter case, nontransitive preferences may easily result due to the fact that the contextual relation used to determine *preferences changes from* binary comparison to binary comparison.

A variety of holistic, heuristic, and wholistic judgmental activities will typically be involved in the search *for a dominance* structure among the alternatives. These take on various forms and mixtures of formal knowledge based, rule based or skill based activities as deemed appropriate for the task at hand [1,2]. Especially when there are a large number of alternative courses of action under consideration, the decision process will typically involve mixed *scanning, where some noncompensatory rule is first used* to eliminate grossly inappropriate alternatives. This is then followed by one or more compensatory information evaluation operations that results in a dominance structure which enables final judgment and alternative selection.

The research discussed here is based upon the hypothesis that people are able to evaluate alternative plans and decisions efficiently and effectively, and with low stress, when there is a clear dominance pattern among alternatives that enables establishment of a sufficiently discriminatory priority structure. Our goal is to provide a knowledge based decision support process that enhances the quality of the dominance structure used for judgment and choice.

The next section will present a summary discussion *of the features and structural* constructs of our decision support system. The following section presents a more detailed discussion of these structural constructs and introduces some of the modes in which the support process can be used. Then we discuss some behavioral issues that relate to the conceptual design of ARIADNE. The list of references contains citations to a number of works which discuss the algorithmic content of the decision support system.

2. Features of the Decision Support System

We now investigate concepts for the design and evaluation of an interactive knowledge based planning and decision support system which combines, or allows combination of, several evaluation rules and contingency structures often used as a basis for evaluation, prioritization, judgment, and choice. We have developed *a knowledge based* system to interactively aid planning and decision support processes through encouragement of search for a dominance structure that is behaviorally realistic and rational, from both a substantive and procedural viewpoint. The support system is called ARIADNE for Alternative Ranking Interactive Aid based on DomiNance structural information Elicitation. The support system enables use of various integrated forms of wholistic, heuristic, and holistic reasoning in an aided search for dominance information among identified alternatives. We believe it to be flexible enough to closely match diverse decision situations and environments in order to support varying cognitive skills and decision styles, thereby enabling planners and decision makers to adapt its use to their own cognitive skills, decision styles, and knowledge.

Our efforts have concerned choice making situations *under certainty and under risk,* primarily for the single decision node case. This formulation allows

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consideration of a variety of imprecisely known parameters such as: attribute tradeoff weights, outcome state values on lowest level attributes, event outcome probabilities, and various combinations of these. Parameter needs are determined from the structure of the decision situa tion, as elicited from the decisionmaker during the formulation and analysis steps of the decision support process. We consider these formulation and analysis steps to be outside the scope of our present software developments but recognize the essential need for them in a complete decision support process.

The decision situation structural model may represent decisions under risk or under certainty. The attribute tree representing the features of decision outcome states may be structured and/or parameterized in a top-down or bottom-up fashion through use of ARIADNE. A single level structure or a multiple level hierarchical structure of attributes may be used with the choice of these being at the discretion of the decisionmaker. Multiple decision node situations may be approached through a goal directed decision structuring approach in which the growth of the structure of alternative decisions and event outcomes is guided by sensitivity-like computations obtained through use of the ARIADNE algorithms [3-5].

Parameters are elicited from the decisionmaker in the form of equalities and inequality bounds. A variety of mathematical programming approaches and graph theory, have been used to generate interactive displays of preference digraphs. These mathematical programming approaches are used to determine dominance structures for alternative prioritization that are based on parameter information elicited from the decisionmaker. At present, only a linear programming approach will yield necessary and sufficient conditions for determination of a priority structure and computational times that are consistent with interactive decision aiding. This requires that we elicit structural parameter information in a slightly restricted form which we denote the "behaviorally consistent information set" (BCIS). Often this BCIS will be in such a form that solution of the generally nonlinear programming problems associated with determination of dominance structures can be replaced by the solution of simple, computationally amenable linear programs with bounded variables. The major simplification associated with eliciting parameter imprecision in a prespecified structural format, however, is in the natural language dialogue needed to establish a model of the decision situation.

The purpose of the graph theory algorithms is to enable construction of a domination digraph, or dominance structural model. This digraph is a pictorial representation of the ordinal preferences as determined from a dominance reachability matrix. This matrix is determined by the linear programming algorithms from the decision situation structural model and parameters elicited from the decisionmaker. These domination digraphs encourage either selection of a preferred alternative, or further iteration using the aggregated preference information for feedback learning.

An inverse aiding feature is currently being incorporated into the decision support system [6,7]. This feature allows the decisionmaker to make wholistic, skill based prioritizations among alternatives. These prioritizations may be across some, or all, identified alternatives, at the top level of the hierarchy of attributes or at some intermediate level. If we elicit numerical bounds on the attribute scores for those attributes which are subordinate to and included within the attribute at which alternatives are prioritized, then bounds on attribute weights, consistent with the wholistic prioritization, may be determined by using a linear programming approach. Alternately, if weights are specified, then it is possible to determine bounds on alternative scores on those attributes subordinate to the attribute at which prioritization was made through use of linear programming algorithms.

As alternatives are identified and prioritized, updates on these bounds are made available. The results obtained from using the inverse aiding feature are, in many ways, comparable to those obtained from the regression analysis based Social Judgement Theory [8]. This approach provides weight identification only, with a "confidence" measurement concerning the validity of weights, cardinal preferences are assumed. Results in the form of bounds on, or ranges of, weights are available with a very few alternative prioritizations in the inverse aiding approach. The prioritizations needed may involve a mixture of cardinal and ordinal preferences. For a large number of prioritizations, the inverse aiding approach may become cumbersome computationally compared to the regression based approach, where additional information may be easily processed in a sequential fashion.

Combination of inverse and direct aiding to enhance decisionmaker specification of imprecise values, weights, and probabilities enhances the usefulness of ARIADNE since it allows for judgments and their explanation, using a combination of formal knowledge based and skill based modes. This enhanced usefulness will also occur through encouragement to the decisionmaker to become more aware of relevant alternative courses of action and to identify new alternatives on the basis of feedback learning of the impacts of alternatives upon issues and objectives in a behaviorally relevant way that, *hopefully*, encourages "double-loop learning" [9].

3. Structure of ARIADNE

A complete set of activities envisioned in using the single stage, or single decision node, version of ARIADNE involves the following set of activities.

Formulation of the Decision Situation

1. Define the problem or issue that requires planning and decisionmaking by identification of its elements in terms of

   (a) Needs, and

   (b) Constraints or bounds on the issue.

2. Identify a value system with which to evaluate alternative courses of action, and identify objectives or attributes of the outcomes of possible decisions or alternative courses of action.

3. Identify possible alternative courses of action, or option generation.

Analysis of the Decision Situation

1. Determine outcome scenarios.

2. Identify decision structural model elements, that is those elements or factors from the conceptual formulation framework which appear pertinent for incorporation into a decision situation structural model.

3. Structure decision model elements:

   (a) Structure decision tree,

   (b) Structure information acquisition and processing tree--which may be part of the basic decision tree, and

   (c) Structure attribute tree or objectives hierarchy.

4. Determine independence conditions among elements of the attribute tree and decision alternatives.

5. Identify potential for the use of deficient information processing heuristics and provide appropriate debiasing procedures.

6. Determine impacts of, or outcomes thay may result from, alternative courses of action.

7. Encode uncertainty elements in the form of event outcome probabilities, or bounds on these, to the extent possible.

8. Identify risk aversion coefficients, if needed, to the extent possible.

9. *Identify preference or value functions, or bounds* on these functions, to the extent possible.

10. Identify attribute weights, or bounds on these functions, to the extent possible.

11. Identify wholistic preferences among alternatives to the extent that this is possible.

12. Identify possible disjunctive and conjuctive aspects, or thresholds for attributes, of identified alternative courses of action.

<u>Evaluation and Interpretation of the Outcome of Alternative Courses of Action</u>

1. Identify a decision aiding protocol, or plan, for evaluation and interpretation of the decision situation.

2. Identify potential for use of deficient judgment heuristics.

3. Use conjunctive and/or disjunctive scanning to eliminate very deficient alternatives and retain alternatives meeting minimum acceptability criteria across attributes.

4. Determine the maximum amount of domination information possible:

   (a) *Display domination digraph.*

   (b) Identify alternative courses of action which could not be among the N most preferred alternatives. Normally these are deleted from further consideration.

   (c) If the decisionmaker can select an alternative for implementation by wholistic judgment, or prioritize the remaining alternative set through heuristic elimination, then go to step 6 of Evaluation and Interpretation.

   (d) If a choice cannot be made, then assess further information about values of imprecisely known parameters by iterating through steps 6-11 of Analysis (B), then return to step 1 of the Evaluation and Interpretation (C). There exists many possibilities for obtaining greater alternative evaluation specificity such as:

   (i) setting higher aspiration levels or aspects,

   (ii) moving up the attribute tree by determination of a subset of attribute tradeoff weights,

   (iii) "tightening" bounds on attribute tradeoff weights,

   (iv) "tightening" bounds on event outcome probabilities, possibly through information processing updates,

   (v) "tightening bounds" on value or preference functions.

5. If the decisionmaker has provided (partial) wholistic preferences as part of the analysis effort, use these with the inverse aiding feature of the aid to determine bounds on attribute weights implied by these preferences such as to provide learning feedback to decisionmaker.

6. Conduct sensitivity analysis. Provide the decisionmaker with an indication of how sensitive the optimal action alternative, or prioritization of alternatives, is with respect to changes in values and information about impacts.

7. Evaluate validity and veracity of the approach. Encourage judgment concerning whether the formulation, analysis, and *interpretation are* sound. If not, encourage appropriate modification to structure and parameters associated with the decision situation, including identification of additional attributes and alternative courses of action. Then, iterate back to an appropriate step and continue.

In our work to date, we assume that the details of issue formulation and analysis are accomplished external to the interactive aid itself. There are a variety of procedures for accomplishing these tasks. [10] Our research assumes that there exists an issue formulation structure and that the impacts of alternatives are known. These are provided through various elicitation activities. We <u>do</u> <u>not</u> envision that the software we develop for interactive <u>interpretation</u>, including evaluation and prioritization, will generally be suitable for use independent of a trained decision analyst. Whether software can be evolved to result in an appropriate "stand alone" aid is very dependent upon the environment and other factors that constitute the *contingency task structure* for a specific situation. In situations which are repetitive and environments which are stable, such as in health care or equipment fault diagnosis situations for example, it seems entirely possible to design useful "stand alone" aids. In most strategic, and in many tactical situations there will not be a stable underlying structure that will easily allow this. The activities involved in issue framing and the identification of a dominance structure appropriate *for decisionmaking are often very* situation dependent.

There are a number of considerations that influence planning and decision support processes. The person using a decision support system should be aware of these considerations if best use of the aiding process is to be obtained. Generally these considera-

AD-A146 895 DEVELOPMENT OF A MIXED SCANNING INTERACTIVE SYSTEM FOR DECISION SUPPORT..(U) TECHNISCHE HOCHSCHULE VIENNA (AUSTRIA) INST OF INORGANIC AND G.. A P SAGE ET AL. 3/4

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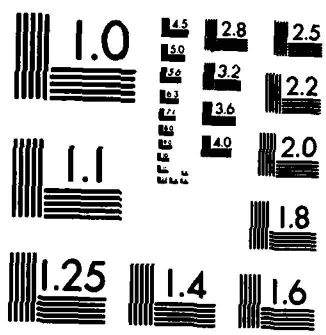
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tions involve the operational environment and the familiarity of the decisionmaker with the environment and task at hand. It is the interaction of these factors that influence:

(1) behavioral characteristics of the decisionmaker,

(2) interaction between decisionmaker and analyst,

(3) choice of computer-based support for decisionmaker analyst interaction

Among the behavioral characteristics of the decisionmaker that influence aiding consideration strongly are the facts that the decisionmaker:

(1) is often impatient with time consuming and stressful assessment procedures;

(2) wants to see some preliminary results promptly if these are needed or wanted;

(3) may lack interest in interacting directly with complex quantitative procedures for decision aiding that do not seem tailored to the specific contingency task structure of the issue at hand; and, as a consequence,

(4) requires a decision aiding approach that adapts to the decisionmaking style appropriate for the decisionmaker in the given contingency task structure.

There are a number of considerations that influence the most desirable interaction between the decisionmaker and the analyst. The interaction must be such that these result:

(a) a list of objectives and an objectives hierarchy;

(b) a list of alternatives; and

(c) a list of outcomes for each alternative.

The extent of the need for the use of these identified lists will vary greatly with the "expertise" of the decisionmaker. A major task of the analyst in the formulation and analysis portion of the aiding effort is to assist the decisionmaker in obtaining these "lists" in a behaviorally relevant and realistic manner.

The analyst must also ensure, to the extent possible, that:

(1) the above lists are reasonably complete;

(2) the lowest level objectives are additively independent;

(3) the alternatives are mutually exclusive, and

(4) the outcomes that follow from each alternative are mutually exclusive and exhaustive.

The nature of the interactive process is such that iterative changes can be made in terms of addition or deletion of alternatives and attributes. Nevertheless, there are significant advantages in attempting to be reasonably complete at the start of the interpretation portion of the process.

The decisionmaker must provide the analyst, following behaviorally realistic elicitation procedures, information regarding:

(1) alternative scores on lowest level attributes,

(2) tradeoff weights,

(3) probabilities, and

(4) relative risk aversion coefficients

or appropriate ratios or bounds on these quantities which represent the precision that the decisionmaker believes appropriate or is capable of providing for the given decision situation.

There are many computer based support considerations which evolve from decisionmaker-analyst interaction considerations. A goal of all decision support system design efforts is to obtain "friendly software", software that is friendly both to the decisionmaker and the analyst. In particular, the analyst must be able to interpret the decisionmaker's structural and parameter information for input to the computer. To do this may require:

(1) redefining the outcome space, such as redefinition of attributes to ensure satisfaction of independence considerations and

(2) describing parameter information in terms of inequalities (or more generally set membership).

The analyst must be able to interpret computer output in a fashion that facilitates the decisionmaker's understanding and decisionmaking abilities. The analyst must be able to assist the decisionmaker in responding to the following question which is central in our interactive knowledge based support system:

> Has sufficient preference and structural information been elicited from and provided to the decisionmaker for alternative selection, or is more information required for identification of a dominance structure that is relevant and appropriate for quality decision support?

If the decisionmaker feels that an alternative can be selected for action implementation at any stage in the interactive aiding effort, the analyst must be able to encourage decisionmaker judgment concerning whether or not the issue formulation, analysis, and interpretation are sound. If the issue formulation, analysis, and/or interpretation are not perceived as sound by the decisionmaker, the analyst must be able to encourage appropriate structural and parameter value modification, typically by means of sensitivity analysis, in order to insure effective, explicable, and valid planning and decision support. If the decisionmaker cannot choose an alternative from among those considered, the analyst must be capable of eliciting further structural and/or parameter information to enhance appropriate selection of alternative courses of action.

One very important feature of a knowledge based system for planning and decision support is encouragement to the decisionmaker for generating new options, outcomes, and attributes at essentially any point in the aiding effort; and ability to properly evaluate these new options. The analyst must be able to cope with this additional information under the assumption that:

(1) the new information is consistent with previously obtained information; or

(2) the new information is not consistent with previously obtained information due to

 (a) structural inconsistencies, or
 (b) parameter inconsistencies.

Thus, the capacity to resolve potential inconsistencies through interaction with the decisionmaker must exist within the planning and decision support process. The indirect, or inverse decision aiding, feature should be of particular value to this end. In a "policy capture" like fashion, this indirect feature will allow identification of bounds on attribute weights in terms of wholistic preferences among some, or all, alternatives. In the direct aiding feature, values, weights, and probabilities are identified and prioritization of alternatives result from this. Combined use of the direct aiding feature with indirect aiding should result in much learning feedback concerning relations among the various modes of judgment.

ARIADNE, as we have noted, does not contain software to assist in the formulation and analysis portion of the planning and decision support effort. It is in these two steps that alternative choices, attributes and decision impacts or outcomes are elicited or identified. Our effort is much more concerned with the interpretation part of a decisionmaking effort; that is to say how information is processed concerning formulation and analysis based quantities such as probabilities, values, weights, ratios, and bounds upon these. We are concerned also with the way in which this information is aggregated, by any of a variety of formal knowledge, rule based, or skill based modes of cognition that result in judgment and choice. We recognize the difficulties in separating the tasks of formulation and analysis from those of interpretation. There are difficulties at the systems management level since the way in which people cognize a problem, as part of the contingency task structure of a particular situation, determines the way in which they will go about resolving it. Thus the performance objectives, information processing style, and decision style that are most appropriate and that are likely to be used for a given task, are very much dependent upon the task itself. When a particular concrete operational or skill based strategy has yielded previous satisfactory results, many people will tend to use that strategy unquestioningly and uncritically in new situations perceived to be similar. This can result in very unsatisfactory judgments and choices in decision situations that have changed and that are not recognized as different from familiar past situations. This may result in premature cessation of search and evaluation of alternatives prior to identification of quality strategies, even for familiar situations. The efforts can be devastating in unfamiliar environments that are not so recognized [11].

The strategies which a decisionmaker will desire to use for interactive interpretation will be strongly dependent upon the way in which the task requirements are initially cognized. This will influence the objectives, attributes, and alternatives generated in the formulation step and the value scores or impacts associated with them in the analysis step. The input information to the interpretation step is just this information. Adequacy of the interactive interpretation step will clearly be dependent upon the "quality" of the information input to it.

Many recent studies [12] have indicated that people often constr... selectively perceived simple deterministic representations of decision situations that make information processing easy and which do not reflect the complexities and uncertainties that are associated with the actual situation. A goal of a decision support system is to encourage wide scope perceptions and associated information processing. The process used to assess probabilities, utilities, and weights will doubtlessly affect the quantities that are elicited. It is possible, for example, that a poor elicitation procedure may, unknowingly or knowingly, create rather than measure values [13]. An advantage to formal support for planning and decisionmaking processes is that it is possible to conduct a search for inconsistent judgment and perhaps even detect flawed information processing heuristics if process tracing is used. When inconsistencies are discovered, it then becomes possible, at least in principle, to examine the judgment process to determine which judgments imply flawed information processing, and/or incoherent or labile values, and/or deficient decision rules. A major ultimate goal, outside the scope of our present study, is to suggest debiasing and other corrective procedures to enhance the quality of human information processing and decision rule selection.

This mixed scanning based planning and decision support system is based upon rational search for a dominance structure which will enable exposure of some of the processes upon which judgment and choice is based. In particular, it enables determination of the precise point in a dominance structure search process when a decisionmaker is able to select a single nondominated alternative. We should be able to do this without resorting to a complete elicitation of precise parameter information and prioritization of all alternatives. The activity of complete precise determination of all parameter information is often stressful and time consuming, may require perspectives outside of the experiential familiarity of the decisionmaker, and allows few results until conclusion of the aiding effort.

The overall process described here appears well suited to accommodating the fact that neither individuals nor groups possess static decision styles capable of being stereotyped and captured by a rigid, inflexible support process. It is specifically recognized that an interactive process is needed that is capable of adaptation to a variety of decision styles that are contingency task structure dependent. System design should reflect the realization that is generally not possible to define a problem or issue fully until one knows potential solutions to the issue. A major cause of this is the fact that information to fully define the issue generally becomes available only as one evaluates potential solutions. Planning and decisionmaking will therefore necessarily be iterative.

4. Behavioral Relevance Issues

Our decision support system design paradigm is based upon a process model of decisionmaking in which a person perceives an issue which may require a change in the existing course of action. On the basis of a framing of the decision situation, one or more alternative courses of action, in addition to the present option which may be continued, are identified. A preliminary screening of the alternatives, using conjunctive and disjunctive scanning, may eliminate all but one alternative course of action. Unconflicted adherence to the present course of action or unconflicted change to a new option may well be the meta strategy for judgment and choice that is adopted if the decisionmaker perceives that the decision situation is a familiar one and that the stakes are not so high that a more thorough search and deliberation is needed [11].

Alternately, if the decision environment is an unfamiliar one, or the stakes associated with judgment and choice are high, a more vigilant form of information acquisition, analysis, and interpretation are called for. This desire for more vigilant information processing leads to a search for a dominance pattern among alternatives, the search for new alternatives that are not dominated by presently identified alternatives, and the elimination from further consideration of alternatives that are dominated. If no single non-dominated alternative is found, adjustments to the dominance structure of alternatives are made through various forms of cognitive activity such as: attribute aggregation, additional information acquisition and analysis, and identification of additional attributes and/or alternatives. This is continued until the structure of needs, objectives, attributes, and alternative action options, and their impacts are such that identification of a single non-dominated alternative results. This "single alternative" may well represent a combination of subalternatives. If there is insufficient time and experience to accomplish these cognitive activities, hypervigilance generally results. The decisionmaker is in a situation where the present course of action is diagnosed as unfortunate and there is a shortage of time and experience that might enable identification and evaluation of an appropriate one.

Given sufficient time and experience, vigilant information processing often results from the aforementioned tasks. Figure 1 presents some salient features of this dominance process model for search, discovery, judgment, and choice.

The mode of judgment and choice that is "proper" depends upon the decisionmaker's situation diagnosis of the contingency task structure. Here, "proper" decision behavior is based upon the assumption that the environment, the task, the experiential familiarity with the task, and the environment that constitutes the contingency task structure are diagnosed correctly. If this is not the case, then the strategies leading to unconflicted change, adherence, or vigilant information processing may be significantly flawed. The role of the contingency task structure in situation diagnosis and in influencing, at a meta level or systems management level, the process of judgment and choice is, therefore, a very important one.

There have been many realistic paradigms of the process of judgment and choice. We believe that the dominance process model described here is not inconsistent with the primary features and intensions of these <u>descriptive</u> models. Our purpose, however, is to develop a conceptual design for a <u>prescriptive</u> approach to judgment and choice that will aid in the search for better decisions. We recognize that a truly rational approach to prescriptive decisionmaking must be cognizant of the process of decisionmaking as it evolves in a descriptive fashion, that is to say process rationality, or it will not be possible to evolve substantively rational support systems.

It is important that an appropriate decision support system be capable of assisting the decisionmaker through encouragement of full information acquisition, including that which may disconfirm strongly held beliefs, and in the analysis and interpretation of this information such as to avoid a variety of cognitive biases and poor information processing heuristics that may lead to flawed judgment and choice [2,12].

A realistic decision support process is necessarily iterative. Several desiderata follow from this:

1. We should allow for top-down or bottom-up structuring of the attributes of outcomes, or impacts of decisions. The "tree" or "hierarchy" of attributes should be structured to the depth believed appropriate by the decisionmaker.

2. Rather than force a decision situation structural model in the form of a tree, we should encourage the decisionmaker to identify a cognitive map of goals, objectives, needs, attributes, alternatives, and impacts that is reflective of the way in which the decisionmaker perceives diagnostic and causal inferences to occur. At some later time this cognitive map may be used to structure a multinode decision tree which is representative of substantive rationality, but not at all necessarily representative of process rationality.

3. We should encourage identification of alternative courses of action, additional attributes of decision outcomes, and revisions to previously obtained elicitations, at any point in the decision support process as awareness of the decision situation and its structure grows through use of the support system.

4. We should not force a person to quantify parameters to the extent that this becomes overly stressful, or behaviorally and physically irrelevant in view of the inherent uncertainties or imprecision that is associated with the knowledge of parameters characterizing the decision situation structural model or their assessment.

These have two primary implications with respect to our interpretation efforts. We allow for revision in the elicited structure of the decision situation and for the identification of new options as awareness of the decision situation grows. Also, we do not require the decisionmaker to quantify parameters beyond the level felt appropriate for the situation at hand. If the decisionmaker feels comfortable in exercising precision with respect to factual outcomes, this is perfectly acceptable and desirable. But parameter imprecision should be allowed if we are to have a realistic support process.

ARIADNE allows parameter imprecision in order to satisfy this quantification relevancy requirement, as do approaches based on fuzzy set theory [16]. We encourage the decisionmaker to specify precise values or numerical ranges for facts and values. Thus we allow, for example, expressions for alternative (A) scores on attributes (i) in the form $0.2 \le v_i(A) \le 0.5$, weights associated with attribute i in the form $0.2 \le w_i \le 0.4$, and probabilities of event (i) resulting from alternative A in the form $0.3 \le P_i(A) \le 0.45$. We allow ordinal representations in the linear forms $v_i(A) \le v_i(B) \le v_i(C)$, $2w_i < w_j < w_k$, $P_j(A) \le P_i(A) \le 3P_k(A)$, or in similar forms. Quantification of imprecision in the form of numerical bounds on parameters always leads to what we call "behaviorally consistent information sets (BCIS)." Sometimes totally ordinal information may need further quantification in order to make the precision and rigidity of the mathematics correspond to the intensions of the decisionmaker in making a purely ordinal specification. This is generally not needed to obtain solutions but, rather, to obtain parametric models that are faithful to the understandings of the decisionmaker. For example that ordinal alternative score inequalities $0 \leq v_i(A) \leq v_i(B) \leq v_i(C) \leq 1$ are satisfied by the relations $0 \leq v_i(A) \leq 1-2t$, $W \leq v_i(B) \leq 1-t$, $2W \leq v_i(C) \leq 1$ for small positive t and W which in the limit become zero. It will generally not be the case that the decisionmaker would express this much imprecision, and would wish to see it more fully quantified to be reflective of (subjective) beliefs. It is, therefore, important that a simple and informative display of value scores, weights, and probabilities be provided to the decisionmaker. This will enhance interactive use of the support system and will enable learning of the impact of these parameters, and associated imprecision, upon decisions.

5. Conclusions

In this paper, we have examined some underlying considerations that have influenced the development of a decision support system that specifically recognizes that imprecise and incomplete knowledge is important to judgment and choice and which allows for its incorporation in the knowledge base of a decision support system. The system allows for judgment and choice at a skill based wholistic level as well as at the formal reasoning based level at which most decision analysis based paridms operate. For detailed discussions of the algorithmic content of ARIADNE, the reader is referred to [6,7,14-26].

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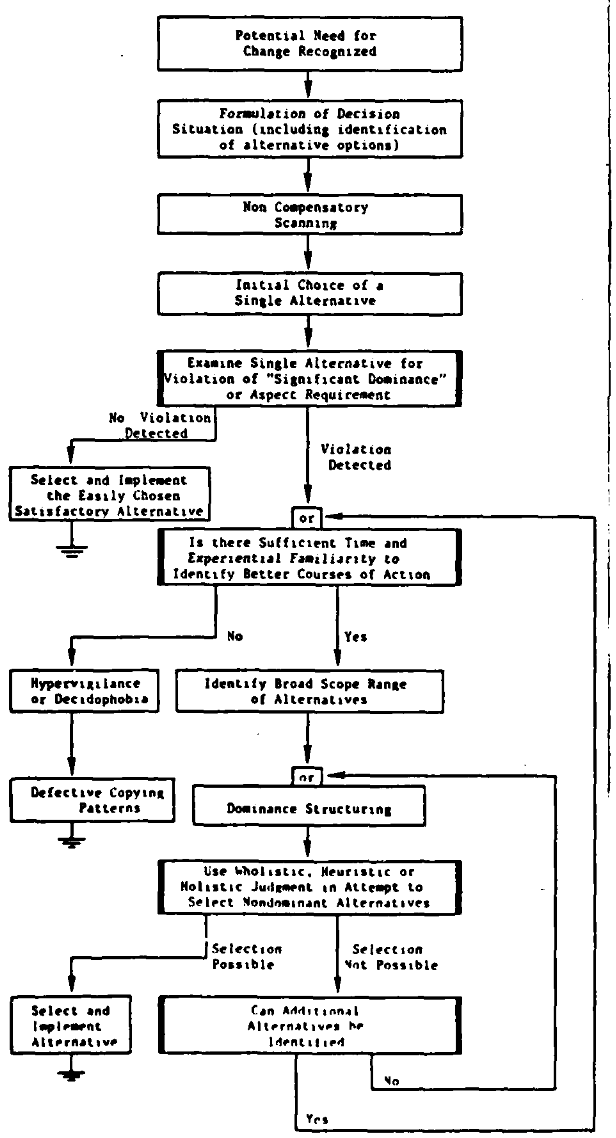

Figure 1. Descriptive Dominance Structural Model of Decision Process.

Command and Control ... July 1983

AN EVALUATION OF ARIADNE

by

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ABSTRACT

In this paper, we present the objectives, operational details, results, and conclusions of an evaluation of a decision aiding procedure ARIADNE. The results of the evaluation indicate that ARIADNE, in comparison to a well-known decision aiding procedure called SMART, (1) has a more flexible model of parameter value description that tends to reduce assessment stress and makes ARIADNE more useful in situations where information precision is poor, (2) allows earlier presentation of initial alternative ranking information and (3) allows the decisionmaker to adjust the mix of alternative ranking specificity and parameter value precision.

I. INTRODUCTION

In this paper, we present the objectives, operational details, results, and conclusions of an evaluation of two decision aiding approaches, ARIADNE and SMART. An indepth description of SMART (Simple, MultiAttribute Rating Technique) can be found in Edwards (1977); Detailed discussions and algorithmic descriptions of ARIADNE (Alternative Ranking Interactive Aid based on DomiNance structure information Elicitation) are presented in the companion paper (Sage and White, 1983). In Section II, we list the hypotheses that were tested during the evaluation. The operational aspects of the evaluation are detailed in Section III. We present the results of the evaluation, and the conclusions that we obtain from these relative to the identified hypotheses, in Section IV.

II. IDENTIFIED HYPOTHESES:

The intent of the evaluation was to test the following hypotheses:

1. Use of the more general model of parameter value description in ARIADNE tends to reduce stress associated with the assessment of alternative values and attribute weights.

2. Use of the more general model of parameter value description in ARIADNE tends to increase confidence in the final alternative selected.

3. The ability to provide additional parameter information, in a form and sequence selected by the decisionmaker, and to observe its impact on alternative ranking in an iterative fashion is a desirable feature of ARIADNE.

4. ARIADNE requires less time for use than does SMART.

5. Decisionmakers do not feel that it is necessary for an aid to produce a single best alternative to assist the decisionmaker in selecting the most preferred alternative.

6. ARIADNE is more useful than SMART in situations where information precision is poor.

7. Problems typically encountered in the subjects' operational environment would be more appropriately examined aided by ARIADNE than aided by SMART.

8. ARIADNE is no more difficult to understand and use than SMART.

We now discuss the procedures for testing these hypotheses.

III. *OPERATIONAL ASPECTS OF EVALUATION*

Eight (8) civilians employed by the United States Army Foreign Science and Technology Center (FSTC) participated as subjects in the evaluation. Each of the subjects had had extensive involvement in technical project evaluation in a military environment and thus had sufficient experience to appreciate the difficulties and operational issues involving proposal evaluation. Proposal evaluation was a subject addressed in the more specialized of the two decisionmaking scenarios examined by the subjects during the evaluation.

The two scenarios developed for the evaluation were proposal evaluation and sports car selection. Each of these scenarios can be obtained from the authors. It was assumed that each scenario involved decisionmaking under certainty. The first scenario was designed to represent a realistic proposal evaluation problem that might occur in a DOD funding agency. Although attributes and some ordinal relations among attribute weights were specified in the RFP to which the proposals were to respond, information presented in the five (5) submitted proposals from which to deduce utility scores and hence tradeoff weights was often vague and/or not available. Also, there was room for judgement in strengthening the ordinal relations among the attribute weights that were provided in the RFP summarized in the scenario. The sports car selection scenario was designed to represent a much more precisely defined alternative selection problem.

Standard procedures were used in order to investigate and compensate for effects due to facilitation style and order with respect to decision aid and scenario.

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The chronology for the evaluation was as follows:

1. General briefing. A briefing was given to the volunteer subjects regarding the purpose of the evaluation and the characteristics of the two aids. Individual evaluation sessions were scheduled, and both scenarios were given to the subjects to read prior to the individual sessions.

2. Individual sessions. Individual evaluation sessions were conducted. Each session for each individual subject involved a subject, facilitator, and computer terminal operator. If ARIADNE was used, assessed information regarding lowest level attribute utility scores and tradeoff weights was allowed to be imprecise and was translated into linear inequalities by the facilitator and/or computer terminal operator. Initially, only utility score information for the alternatives on the identified attributes was assessed. Once this assessment was completed, a domination digraph on the alternatives was computed and displayed to the subject. The subject could also view a score sheet of values from which this digraph was obtained. If this digraph provided sufficient information for alternative selection, then this portion of the session was halted. If not, then further utility score and/or tradeoff weight information was requested and the resulting domination digraph displayed. This information could concern attribute scores and weights not previously obtained or more precise estimates of previously elicited scores and weights. This iterative procedure continued until the subject halted the process.

If SMART was used, all parameters were precisely assessed. Then the total linear order on the alternatives was displayed. If the subject wished, a post optimal sensitivity analysis was performed on whatever single parameter values were of concern. Detailed descriptions of facilitation protocols can be obtained from the authors.

During the examinations of each scenario, the computer terminal operator completed an Analyst Information Sheet, detailing various times and types of requests. After completing both scenarios, the subjects were asked to complete a short questionnaire and return it. Copies of both the Analyst Information Sheet and the Questionnaire can be obtained from the authors.

IV. RESULTS AND CONCLUSIONS:

We now examine each of the hypotheses in the context of data collected during the evaluation.

Hypothesis # 1: Use of the more general model of parameter value description in ARIADNE tends to reduce stress associated with the assessment of alternative values and attribute weights. Relevant Questionnaire questions (Q) and responses (R):

Q1: Being allowed to express parameter values imprecisely using ARIADNE produced more stress than being required to state all parameter values precisely using SMART.

R: (presenting individual subject scores below the line; +5(-5) indicates strong (dis)agreement)

-5				0		+5
1	6	8	7	2	4	5
				3		

indicating a tendency to agree with the hypothesis. There were two relevant comments: S1 (Subject 1). "Liked the flexibility."

S3: "Perhaps more stress initially (with ARIADNE) because I didn't understand the process. But after using it, I would be inclined to say far less stress (with ARIADNE) for complex applications."

Q6: What was the most comfortable way of expressing parameter value information for you?

R: Indicated that two subjects (S2, S5) preferred exact values, two subjects (S1, S8) preferred interval estimates, and four (S3, S4, S6, S7) preferred ranking statements for expressing parameter value information. There was one relevant comment from S3: "for the exercise today, [exact value] would be the answer; however, for actual application in complex areas, [ranking statements] is my answer." These responses cause us to conjecture that if the subjects had been more experienced in expressing parameter values imprecisely (several had experience in expressing value scores and tradeoff weights precisely) then there would have been stronger support for this hypothesis.

Hypothesis # 2: Use of the more general model of parameter value description in ARIADNE tends to increase confidence in the final alternative selected. Relevant Questionnaire questions and responses:

Q2: I felt more confidence in the final alternatives chosen when aided by ARIADNE than in the final alternative produced when aided by SMART.

R:

-5			0			+5
	2	7	4	1	3	
			5		8	
			6			

indicating slight support for the hypothesis. There were no especially relevant written comments.

Q8: I disagreed with the action alternative recommendation obtained using ARIADNE.

R: (S1,...,S7) "no"; S8 "don't remember."

Q14: I disagreed with the action alternative recommendation obtained using SMART.

R: S2 "yes;" (S1, S3, ..., S7) "no;" S8 "don't remember." Thus, responses to questions the latter 2 indicate no recognizable difference in the perceived quality of the decisions made using ARIADNE versus those made using SMART. There were no relevant comments associated with either question.

Q17: Which approach would you prefer to use to make recommendations to others concerning evaluation and prioritization and why?

R: (S1, S3, S7, S8) "ARIADNE;" (S2, S5, S6) "SMART;" S4 "neither." Three relevant comments were made: S4: "Depends on Situation." S6: "Since SMART gives percentages, one can see if two proposals are close and then adjust the rankings by consideration of factors that were not originally considered." S7: "By only indicating preference as opposed to exact values is very helpful; weight max-min display nice feature."

We feel S4's comments tends to explain his response to this question. The comments of S6 and S7 indicate that the max-min display (indicating the maximum and minimum values of expected utility for each alternative) incorporated into ARIADNE was liked by S7 and may not have been requested by S6. We conjecture that had S6 seen this display, his response would have been different.

Q12: Use of ARIADNE encouraged me to carefully weigh the positive and negative consequences of each alternative.

Q16: Use of SMART encouraged me to carefully weigh the positive and negative consequences of each alternative.

Responses to Q12 and Q16 were, respectively:

```
-5                    0                   5
 ____________________________________________
              6       5    2    3    4       1
                           7    8
```

```
-5                    0                   5
 ____________________________________________
        3             1    7    2       5
                                4
                                6
                                8
```

indicating that ARIADNE and SMART provided an approximately equal level of encouragement to the subject for him to carefully weigh the positive and negative consequences of each alternative.

In summary, questionnaire responses indicate that the level of confidence in the output of ARIADNE and SMART appear to be quite similar. Also indicated was that both aids equally encourage the careful weighing of the possible consequences of the alternatives.

Hypothesis # 3. The ability to provide additional parameter information, in a form and sequence selected by the decisionmaker, and to observe its impact on alternative ranking in an iterative fashion is a desirable feature of ARIADNE. Relevant Questionnaire questions and responses:

Q3: Being able to provide additional parameter information and then to observe its impact on alternative ranking in an iterative fashion, was a desirable feature of ARIADNE.

R:

```
-5                    0                   5
 ____________________________________________
                      5        6    2    1    8
                               7    3
                                    4
```

indicating strong support for the hypothesis.

As an indication of how often the feature of ARIADNE being evaluated in this hypothesis test was exercised, the computer terminal operator recorded on the analyst information sheet the number of iterations required in constructing the final domination digraph. The number of iterations for each subject was respectively: 3, 3, 0, 4, 0, 2, 5, and 3.

Thus the subjects developed a reasonable level of experience with this feature of ARIADNE, placing a high level of confidence in the responses to Q3.

Q4: Knowing that a single iteration always produces a best alternative is a desirable feature of SMART.

R:

```
-5                    0                   5
 ____________________________________________
     6              7 1    2         4   5
                      3    8
```

indicating that the subjects found moderately attractive the fact that SMART always produces a best alternative in one iteration.

The responses to Q3 and Q4 indicate that the decisionmaker should be encouraged to be as precise as possible in order to reduce the number of iterations of ARIADNE for final decision selection.

Hypothesis # 4: ARIADNE requires less time for use than does SMART. Relevant questions and responses:

Q5: Use of SMART lead me to a decision more quickly than use of ARIADNE.

R:

```
-5                    0                   5
 ____________________________________________
    6    8            7       1    2         4
                                   3         5
```

indicating that the perceptions of the subjects tend not to support the hypothesis.

Timing data recorded on the analyst information sheets support the perceptions of the subjects with regard to the average total length of time per session. Let VET = value elicitation time, WET = weight elicitation time, TST = total session time. Then, the timing data (in minutes) are as follows:

	1	2	3	4
		APS		
VET	41.75	18.75	19.25	21.25
WET	9.5	10.5	6.0	14.0
TST	57.0	41.25	47.25	51.0
		CCW		
VET	29.5	24.25	28.75	25.0
WET	11.75	9.75	10.0	11.5
TST	57.5	40.75	47.0	51.25
		TOTAL		
VET	35.625	21.5	24.0	23.125
WET	10.625	10.125	8.00	12.75
TST	57.25	41.0	47.125	51.125

where 1 = ARIADNE, 2 = SMART, 4 = proposal evaluation, 3 = sports car selection. Thus, use of ARIADNE required on average 40% more total time than did SMART and the proposal evaluation scenario required on average 8.5% more time to evaluate than did the sports car selection scenario. Both facilitators on average required 49.125 minutes per scenario.

Also recorded on the analyst information sheet was the length of time between the beginning of the session and the beginning of the weight elicitation process. For ARIADNE, this length of time represents an upper bound on the length of time to the presentation of the first digraph. These times for the 8 subjects were: 25, 41, 20, 30, 30, 15, 25, 35 for an average of 27.625 seconds. We remark that SMART required approximately 48% more time to provide initial alternative ranking information to the decisionmaker than did ARIADNE. If we assume that total session times for SMART and the lengths of time between the beginning of the session and the beginning of the weight elicitation process for ARIADNE are realizations of normally distributed random variables with unknown means and unknown variances, then a standard statistical test indicates that these realizations come from two different random variables with a confidence level of greater than 0.95.

In summary, ARIADNE required less time to provide initial alternative selection feedback than did SMART but more time to complete the entire session. Therefore, if the alternative selection situation is such that some alternative ranking information is better than none, ARIADNE would tend to be the preferred aiding procedure.

Hypothesis # 5. Decisionmakers do not feel that it is necessary for an aid to produce a single best alternative to assist the decisionmaker in selecting the most preferred alternative. Relevant Questionnaire question and response:

Q7: It would be necessary for an aid to produce a single most preferred alternative before I would feel that I could select the best alternative.

R:

-5					0					5
1		4	2	8	3	7				
		5								
		6								

indicating strong support for the hypothesis. A recent evaluation of a decision support system, designed to provide a nondominated set of alternatives to the decisionmaker rather than a single most preferred alternative, also has supported this hypothesis (White et al., 1982). Therefore, we conclude with high confidence that decision situations do exist where the decisionmaker does not find it necessary for a decision aiding procedure to identify a single most preferred alternative.

Hypothesis # 6. ARIADNE is more useful than SMART in situations where information precision is poor. Relevant Questionnaire questions and response:

Q9: ARIADNE is more useful than SMART in situations when information precision is poor.

R:

-5					0					5
							2		6	1
							5		7	3
										4
										8

indicating strong support for the hypothesis. There was one relevant comment: S8: "Very much agree that ARIADNE aids one with imprecise data."

Hypothesis # 7. Problems typically encountered in the subject's operational environment would be more appropriately examined aided by ARIADNE than aided by SMART. Relevant Questionnaire questions and responses:

Q10: Typical problems encountered in my operational environment would not be appropriately examined by SMART.

Q13: Typical problems encountered in my operational environment would not be appropriately examined by ARIADNE.

Responses to Q10 and Q13 were, respectively:

-5					0					5
5			2		6	4	3			1
							7			
							8			

-5					0					5
1	8	3		7	2					
				4	5					
					6					

These responses indicate a perceptable preference for ARIADNE over SMART in the subjects' operational environment.

There were no relevant comments to Q10. Relevant comments to Q13 were: S2: "My operational environment does not involve actual decisionmaking with multiple criteria. It does involve analysis of the decisionmaking of other parties/governments." S8: "Much of the data I deal with are abstract."The response of (and further discussions with) S2 indicate a need for an inverse decision aiding procedure that is not currently operational with ARIADNE. The response of S8 to Q10 and Q13 indicate that "abstract" appears to be synonymous with "vague" and "imprecise."

We conclude that ARIADNE appears to be somewhat better suited than SMART for the operational environments of the subjects. We suspect that this is in large part due to the fact that hypothesis # 6 had strong support.

Hypothesis # 8: ARIADNE is no more difficult to understand and use than SMART. Relevant Questionnaire questions and responses:

Q11: SMART is easier to use than is ARIADNE.

Q15: ARIADNE is easier to understand than is SMART.

Responses to Q11 and Q15 were, respectively:

-5					0					5
1		6			5	7	2	4	3	
								8		

-5					0					5
		3	2		1		7			
		4	8		5					

These responses indicate that SMART is both simpler to use and easier to understand than ARIADNE and therefore contradict the validity of the hypothesis.

Relevant comments to Q11 were: S6: "Since one has to be specific in SMART, it is harder to feel comfortable with the exactness of the results." S8: "ARIADNE does require some knowledge of computers and the manipulation of data on them; or at least one that can operate a computer." The S6 response appears to support hypothesis # 2.

The relevant comment to Q15 was: S6: "Since an "interpreter" was used, most interfacing problems were eliminated."

The above data indicate that the evaluation results contradict the veracity of this hypothesis. However, there are two factors which contribute to this contradiction that could possibly be eliminated or mollified. First, many of the subjects had had previous experience with SMART-like scoring and weighting assessment procedures, and none had had any experience with ARIADNE. Second, current facilitation procedures for ARIADNE are more complicated and less established than those for SMART. We conjecture that: 1. a sufficient amount of familiarity with the more general model of parameter value incorporated into ARIADNE and 2. the completion of truly established facilitation procedures for ARIADNE (as exist for

SMART)would cause ARIADNE to be perceived by the user as no more difficult to understand and use than SMART. Training time to achieve such a level of familiarity is likely to be longer for ARIADNE than for SMART due to the relative increased flexibility inherent in ARIADNE.

Other issues and associated questions:

Q18: Was the posterior sensitivity analysis associated with SMART helpful?

Q19: Which decision making scenario was the most appropriate for ARIADNE and why?

Q20: Which decision making scenario was the most appropriate for SMART and why?

Responses to Q18, Q19, and Q20 indicated, respectively, that: 1. the post-optimality sensitivity analysis feature was useful in SMART, 2. the proposal evaluation scenario was most appropriately evaluated using ARIADNE, and 3. the sports car selection scenario was most appropriately evaluated using SMART. Generally, comments to questions # 19 and # 20 indicated that for complex decision selection situations having less quantitative information available, ARIADNE would be preferred to SMART, which provides further support for hypothesis # 6.

Summary, our evaluation lends credence to the following claims:

1. The more flexible model of parameter value description employed by ARIADNE tends to reduce assessment stress and makes ARIADNE more useful than SMART in situations where information precision is poor.

2. The iterative, progressive information requirements associated with ARIADNE is a desirable feature that allows earlier presentation of initial alternative ranking information than does SMART.

3. SMART requires less total time for use than does ARIADNE.

4. Being able to adjust the mix of alternative ranking specificity and parameter value precision is a desirable feature of ARIADNE.

5. ARIADNE may require more training than would be required by SMART for successful use.

6. The level of confidence in the output of ARIADNE and SMART appear to be quite similar.

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5.

INCONSISTENCY RESOLUTION IN MULTIATTRIBUTE DECISIONMAKING AND TRADEOFF WEIGHT DETERMINATION UNDER RISK*

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ABSTRACT

In this paper, we examine the issue of inconsistency and present a new approach to its resolution for a model of single stage, multiattribute decisionmaking and tradeoff weight determination under risk. The model considered, a generalization of a standard decision analysis paradigm (see Keeney and Raiffa, 1976; Sarin, 1977a, b; White et al., 1983), allows the decisionmaker to describe the value or possible values of the tradeoff weights by (1) (possibly imprecise) tradeoff weight assessment and (2) directly expressed preferences among the alternatives. Human decisionmakers, however, occasionally produce noncoherent responses in judgmental tasks. This noncoherence may be compounded by the different perspectives implied by the above two approaches to tradeoff weight determination. As a consequence, the set of all tradeoff weights that can satisfy both the results of the tradeoff weight assessment and the directly expressed preference exercise may be null. The intent of this paper is to present a new approach to resolving this type of inconsistency.

I. PROBLEM FORMULATION AND PRELIMINARIES:

We assume:

A is the finite set of alternatives,

I is the number of additively independent attributes,

C is the number of consequences,

$u_{ic}(a)$ is the utility associated with alternative i of selecting alternative a and receiving consequence c,

$p_c(a)$ is the probability of receiving consequence c, given that alternative a was selected,

w_i is the tradeoff weight associated with attribute i.

Let $w = \mathrm{row}\{w_1,\ldots,w_I\}$, $u(a) = \{u_{ic}(a)\}$, and $p(a) = \mathrm{col}\{p_1(a),\ldots,p_C(a)\}$. Then,

$$wu(a)p(a) = \sum_{i=1}^{I} w_i \sum_{c=1}^{C} u_{ic}(a)p_c(a)$$

represents the expected utility of selecting alternative a. Alternative a' is said to be at least as preferred to alternative a if and only if

$$wu(a')p(a') \geq wu(a)p(a).$$

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Assume p(a) is known precisely for all a A. Let $W \subseteq \bar{W} = \{w \in R^I : w_i \geq 0, \sum_i w_i = 1\}$ represent the set of all possible tradeoff weight vectors, a set presumably derived from information provided by the decisionmaker regarding the relative importance of attributes. The set W is assumed to be described by a set of linear inequalities $\hat{S}_w$.

For each i, i=1,...,I, let U_i represent the set of all possible values of the #AxC matrix $u_i = \{u_{ic}(a)\}_{a,c}$, where #A is the number of elements in A. We assume that the set U_i is derived from information provided by the decisionmaker regarding alternative and consequence rankings for attribute i. Typically, the U_i will be described by sets of linear inequalities. We will assume throughout that $U_i \neq \emptyset$, for all i=1,...,I, where $\emptyset$ is the null set.

Let the information provided directly by the decisionmaker regarding the relative desirability of the alternatives be described by the relation $R \subseteq A \times A$, where (a',a) R if and only if the decisionmaker has stated that alternative a' is at least as preferred as alternative a. We interpret (a',a) R to mean that

$$wu(a')p(a') \geq wu(a)p(a)$$

for <u>all</u> possible values of w, u(a'), and u(a). Thus, (a',a) R implies the following restriction on the tradeoff weights:

$$\sum_i w_i \min_{u_i \in U_i} v_i(a',a) \geq 0,$$

where $v_i(a',a) = [u_i(a')p(a') - u_i(a)p(a)]$, and $u_i(a) = \{u_{ic}(a)\}_c$. We observe that if U_i is described by a finite set of linear inequalities, then

$$\min_{u_i \in U_i} v_i(a',a)$$

is a linear program. Let $\hat{S}_R$ represent the set of all such linear inequalities generated by the ordered pairs in R.

Define $\hat{P}$ as the coarsest partition of W generated by the inequalities in $\hat{S} = \hat{S}_W \cup \hat{S}_R$. For each $P \in \hat{P}$, let $\hat{S}(P) \subseteq \hat{S}$ represent the set of inequalities that are violated by the w P. We say that $\hat{S}$ is <u>consistent</u> (<u>inconsistent</u>) if there exists (does not exist) a P P such that $\hat{S}(P) = \emptyset$. That is, $\hat{S}$ is consistent (inconsistent) if there is (is not) a tradeoff weight in W for which no inequality in $\hat{S}$ is violated. Note that there can be at most one element $P \in \hat{P}$ such that $\hat{S}(P) = \emptyset$. Define the relation on $\hat{P}$ as follows: $P' \succ P$ if and only if $\hat{S}(P') \subseteq \hat{S}(P)$. Thus, if $P' \succ P$, then each tradeoff weight vector in P violates at least as many inequalities in $\hat{S}$ as any of the tradeoff weight vectors in P'. We illuminate these concepts with the following example.

II. *EXAMPLE:*

Consider the following alternative selection problem under certainty (i.e., C = 1). Assume a decisionmaker wishes to purchase an automobile from a group of 4 automobiles ($A = \{a_1,\ldots,a_4\}$). Selection is to be based on 3 attributes (I = 3): safety, cost, and attractiveness. The lowest level value scores are shown in Table 1. In Table 1, 1(0) indicates the most (least) preferred alternative for a specific attribute. The decisionmaker states a preference for automobile a_3 over automobile a_1 ($(a_3,a_1) \in R$) and feels that in the context of the automobiles under consideration, safety is more important than cost and cost is more important than attractiveness; that is, $\hat{S}_W = \{w_1 \geq w_2,\ w_2 \geq w_3 = 1 - w_1 - w_2\}$. Straightforward analysis shows that $\hat{S}_R = \{-(5/2)w_1 + 1 \geq w_2\}$. Thus, $\hat{S} = \{-(5/2)w_1 + 1 \geq w_2,\ w_1 \geq w_2,\ 2w_2 \geq 1 - w_1\}$. The resulting partition of W is given in Figure 1; note that $\hat{P} = \{P_1,\ldots,P_7\}$. The elements in $\hat{S}(P_n)$, n=1,...,7, are presented in Table 2, where Y(N) indicates that the inequality is in (is not in) $\hat{S}(P_n)$. For example, $\hat{S}(P_3) = \{2w_2 \geq 1-w_1\}$. The resulting domination digraph is presented in Figure 2. We note that $\hat{S}$ is inconsistent and that the inconsistency exists due to a conflict between the inequality in $\hat{S}_R$ and the inequalities in $\hat{S}_W$. It is indicated in Table 2 and Figure 2 that the elimination of any one inequality in $\hat{S}$ will produce consistancy.

III. INCONSISTENCY RESOLUTION:

Assume there is no $P \in \hat{P}$ such that $\hat{S}(P) = \emptyset$. Then, there is no tradeoff weight in W that satisfies all of the inequalities in $\hat{S}$. Two objectives appear reasonable:

1. Determine a tradeoff weight that minimizes, in some sense, the extent of violations of the inequalities in $\hat{S}$.

2. Request that the decisionmaker reevaluate preferences associated with the inequalities in $\hat{S}$ with the hope that some of the inequalities can be removed or modified in order to produce a consistent set of inequalities and associated judgments.

For the type of model being considered here, linear programming procedures like those found in Pekelman and Sen (1974) can be useful in helping to achieve the first objective. The difficulty with such procedures is that the criteria may not always produce behaviorally justifiable results. For example, consider the first model presented in Pekelman and Sen (1974):

$$\text{minimize} \sum_{k=1}^{K} y_k$$

$$\text{subject to} \sum_{i=1}^{I} a_{ik} w_i + y_k \geq 0,\ k=1,\ldots,K$$

$$\sum_{i=1}^{I} w_i = 1$$

$$w_i,\ y_k \geq 0$$

where K is the number of inequalities in $\hat{S}$ and $\sum_{i=1}^{I} a_{ik} w_i \geq 0$ is the kth inequality in $\hat{S}$. The intent of solving this linear program is to minimize the sum of the y_k, where y_k represents a measure of the amount of violation of the kth inequality. Note that if the optimal criterion value is zero, then $\hat{S}$ is consistent. Application of the above linear program to the (inconsistent) example presented in Section 2, produces the tradeoff weights $w_1 = w_2 = 2/7$, $w_3 = 3/7$. However, these weights indicate that $w_2 < w_3$, which contradicts the decisionmaker's statement that cost is relatively more important than attractiveness. We see no behavioral justification for contradicting this statement in preference to contradicting any other statement associated with inequalities in $\hat{S}$. Thus, the resulting solutions of such approaches cannot be accepted without question.

The second objective, removal or modification of an inequality in $\hat{S}$ requires that the decisionmaker rethink and perhaps retract a former statement of preference. Clearly, it is behaviorally desirable to minimize such effort. We now indicate a procedure for identifying minimal subsets in $\hat{S}$ which if removed would cause $\hat{S}$ to become consistent.

Let $\hat{P}^{ND} \subseteq \hat{P}$ be the set of all nondominated elements of $\hat{P}$; i.e., $P \in \hat{P}^{ND}$ if and only if there is no $P' \in \hat{P}$ such that $P' \succ P$ and not $P \succ P'$. Observe that $\hat{S} \sim \hat{S}(P)$ (i.e., the set of all the inequalities in $\hat{S}$ except those in $\hat{S}(P)$) is consistent for any $P \in \hat{P}$. Note, however, that if $P \in \hat{P}$ is dominated, there is a $P' \in \hat{P}^{ND}$ such that $\hat{S}(P') \subseteq \hat{S}(P)$. Thus, it is reasonable to investigate procedures that will remove from $\hat{S}$ the inequalities in $\hat{S}(P)$ for any of the P $\hat{P}^{ND}$.

We now present a procedure for identifying the $\hat{S}(P)$ for all $P \in \hat{P}^{ND}$. Let $\hat{D} = \{0,1\}^K$, and let $\hat{D}_1$ be the set of all elements in $\hat{D}$ having exactly 1 0's and (K-1) 1's. For $D = \{d_k, k=1,\ldots,K\} \in \hat{D}$, consider the linear program:

$$\text{minimize} \sum_{k=1}^{K} d_k y_k$$

$$\text{subject to } \sum_{i=1}^{I} w_i = 1,$$

$$w_i,\ y_k \geq 0 \text{ for all } i \text{ and } k,$$

$$\sum_{i=1}^{I} a_{ik} w_i + y_k \geq 0 \text{ for all } k \text{ such that } d_k = 1$$

$$\sum_{i=1}^{I} a_{ik} w_i \leq 0 \quad \text{for all } k \text{ such that } d_k = 0.$$

The sequence D for the above linear program is said to be <u>possibly</u> <u>nondominated</u> if:

1. there exists a feasible solution,

2. the optimal criterion value equals 0,

3. the slack variables for the $d_k=0$ inequalities are all positive in the first tableau.

Observe that if a $D \in \hat{D}$ is possibly nondominated, then there is a $P \in \hat{P}$ such that $\hat{S}(P)$ is comprised of the inequalities

$$\sum_{i=1}^{I} a_{ik} w_k \geq 0$$

for all k such that $d_k=0$. Note that if D and D' are both possibly nondominated, D is associated with $P \in \hat{P}$, D' is associated with $P' \in \hat{P}$, and $D' \leq D$, then $\hat{S}(P) \subseteq \hat{S}(P')$. Thus, in searching for $\hat{S}(P)$ for all $P \in \hat{P}^{ND}$, it is sufficient to search for the set of all possibly nondominated elements in $\hat{D}$ that are not bounded above by any other possibly nondominated element in $\hat{D}$. We now present an algorithm for this search:

0. Set n=0.

1. Let $\hat{D}_n = \{D_1,\ldots,D_M\}$, where $M = \binom{K}{n}$; set m=1.

2. Is there a previously identified D' equivalent to $\hat{S}(P)$ for some $P \in \hat{P}^{ND}$ such that $D_m \leq D'$? If no, go to 3. If yes, go to 4.

3. Is D_m possibly nondominated? If yes, save D_m; it is equivalent to $\hat{S}(P)$ for some $P \in \hat{P}^{ND}$. If no, go to 4.

4. Let m=m+1 if m < M, and go to 2. If m=M, then set n=n+1. If $n \leq K$, then go to 1. If n > K, then stop.

With respect to modifying elements in $\hat{S}$, we have noted previously that more precise assessment of the U_i causes the set of all tradeoff weights satisfying inequalities in $\hat{S}_R$ to enlarge. Thus, more precise assessment of the U_i may cause $\hat{S}$ to become consistent. However, there is a limit as to how much impact more precise assessment of the U_i can have on the consistency of $\hat{S}$. To determine this limit, let $\hat{S}_R$ represent the set of all linear inequalities of the form

$$\sum_i w_i \max_{u_i\ U_i} v_i(a',a) \geq 0,$$

or equivalently,

$$\sum_i w_i \min_{u_i\ U_i} v_i(a,a') \leq 0,$$

for all $(a',a) \in R$. Define $\hat{\hat{S}} = \hat{S}_R \cup \hat{S}_W$. Then if $\hat{\hat{S}}$ is consistent, more precise assessment of the U_i may produce a consistent $\hat{S}$. If $\hat{S}$ is not consistent, then more precise assessment of the U_i alone will not produce a consistent $\hat{S}$. For example, consider the example presented in Section 2. Then, $\hat{S}_R = \{-5w_1 + 2 \geq w_2\}$, which produces the partition shown in Figure 3. The tradeoff weight (1/3, 1/3, 1/3) satisfies all inequalities in $\hat{\hat{S}}$, and hence S is consistent. However, since $\hat{\hat{S}}$ permits only a single tradeoff weight satisfying all inequalities, it is unlikely that only more precise assessment of the U_i will produce a consistent $\hat{S}$.

IV. CONCLUSIONS:

We have examined a model of single stage decisionmaking and tradeoff weight determination under risk and tradeoff weight and utility score imprecision in the context of inconsistency resolution with respect to tradeoff weight determination. A procedure for identifying the minimal sets of inequalities causing inconsistency, and a method for determining if more precise assessment of the U_i could lead to consistency, were given.

The results presented in this paper raise an interesting behavioral issue. If $\hat{S}_W$ and/or $\hat{S}_R$ is inconsistent, then clearly $\hat{S}$ is inconsistent. Inconsistency can also result when a consistent $\hat{S}_W$ is combined with a consistent $\hat{S}_R$; e.g., the example problem in Section 2. An investigation to determine the origins of and behavioral explanations for inconsistency is a topic for future research. Such an investigation could be expected to lead to interactive procedures that would tend to reduce inconsistency.

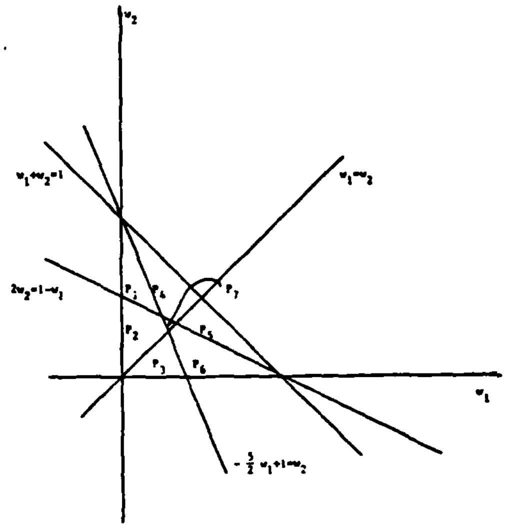

FIGURE 1: $\hat{P}$ for the Example.

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	Safety(i_1)	Cost(i_2)	Attractiveness(i_3)
a_1	1	$0.20 \leq u_2(1) \leq 0.40$ $u_2(1) \leq u_2(3)$	0.3
a_2	0.8	0	1
a_3	0.1	$0.35 \leq u_2(3) \leq 0.50$	0.9
u_4	0	1	0

TABLE 1: Utility Score Information for the Example

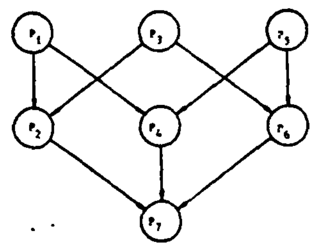

FIGURE 2: Domination Digraph for the Example

n	$-(5/2)w_1+1 \geq w_2$	$w_1 \geq w_2$	$2w_2 \geq 1-w_1$
1	N	Y	N
2	N	Y	Y
3	N	N	Y
4	Y	Y	N
5	Y	N	N
6	Y	N	Y
7	Y	Y	Y

TABLE 2: $\hat{S}(P_n)$, n=1,...,7, for the Example.

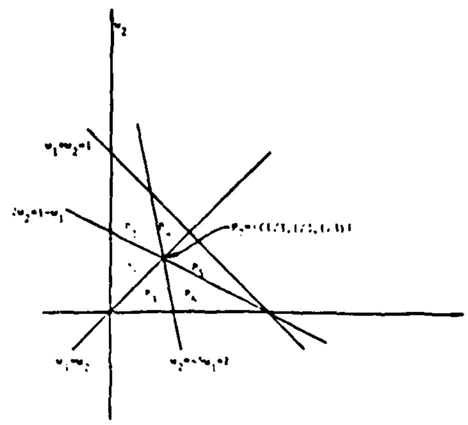

FIGURE 3: Partition Generated by $\hat{S}$

MULTI-STAGE DECISIONMAKING WITH IMPRECISE UTILITIES

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ABSTRACT

We present a dynamic program for determining the set of all possibly optimal strategies for a decision analysis problem with imprecisely known utility function, where imprecision is described by set inclusion. This procedure is based on the assumption that the utility function is affine in an unknown parameter, which for example might be the vector of utility values itself or the vector of importance weights. A simple multiattribute example is presented to illustrate the theory and the computational procedure.

Keywords: Decision analysis, parametric dynamic programming.

INTRODUCTION

The standard decision analysis (DA) problem, as presented in Keeney and Raiffa (1976) for example, assumes that various outcome probabilities and terminal node utilities are known precisely. In reality, such parameters may be only imprecisely known. For example, the decisionmaker (DM) may find precise utility assessment too stressful and/or too time consuming or due to the nature of the problem, the DM may wish to be vague about his preferences.

Fishburn (1965), Sarin (1977a, 1977b), and White et al (1982) have investigated the implications of parameter imprecision in selecting a most preferred alternative for a single-stage DA problem. Their model of parameter imprecision was set inclusion. Cyert and DeGroot (1975) examined a sequential decisionmaking problem with a utility function dependent on unknown, static parameters. A Bayesian estimate of the utility function was updated at each decision epoch and an alternative selected on the basis of this estimate.

In this paper, we examine a multi-stage DA problem having precise probabilities but imprecise terminal utility values. Utility imprecision is described by set inclusion. We assume that knowledge of the set of all possible terminal utility values does not change over the planning horizon; thus the utility function is static and no information arrives over the planning horizon to revise our knowledge of the set of possible utility values.

This paper is organized as follows. The problem is formulated in Section 2, and a solution procedure is derived in Section 3. A numerically illustrative example is presented in Section 4. Conclusions and directions for future research are presented in the last section.

PROBLEM FORMULATION

Assume that the given decision tree has a maximum of K stages. For simplicity, we will assume that all branches of the tree have exactly K stages, which can be achieved by adding the appropriate number of decision nodes with single actions and chance nodes with single outcomes to branches having less than K stages.

Let z_{k+1} be the outcome received after having selected alternative, a_k, $k=0,1,\ldots,K-1$. Define $s_k = \{a_0,z_1,\ldots,a_{k-1},z_k\}$, which we call the state at stage k. Note that s_k uniquely identifies a decision node in the decision tree if $k < K$ and uniquely identifies a terminal node in the decision tree if $k=K$. Assume all probabilities of the form $p(z_{k+1}|s_k,a_k)$, and hence $p(s_{k+1}|s_k,a_k)$, are known. Let S_K be the set of all terminal nodes, and assume $u:S_K\to R$ is the utility function of the problem. The DM chooses an alternative a_k at stage k on the basis of the stage number k and the current decision node s_k; thus, $a_k=\lambda_k(s_k)$, where $\lambda = \{\lambda_k,\ k=0,1,\ldots,K-1\}$ is referred to as a strategy. Let Λ be the set of all such strategies. Our criterion is expected utility, $E_\lambda\{u(s_K)\}$. Assume all that is known about $u=\{u(s_K),\ s_K\in S_K\}$ is that u is affine in an imprecisely known parameter ρ and that $\rho\in P\subseteq R^N$ where P is a given set. That is, $u(s_K) = h^1(s_K) + \sum_{n=1}^N h^2(s_K)_n\rho_n = h^1(s_K) + h^2(s_K)\rho$, where $\rho = \{\rho_n, n=1,\ldots,N\}\in P$. Our objective is to determine the set of all strategies that maximize the expected utility criterion for some allowable parameter value. That is, we seek all $\lambda^*\in\Lambda$ such that for some $\rho\in P$

$$E_{\lambda^*}[h^1(s_K) + h^2(s_K)\rho] \geq E_\lambda[h^1(s_K) + h^2(s_K)\rho]$$

for all $\lambda\in\Lambda$.

We remark that the affine form of the utility function appears to allow for a wide variety of interesting problem formulations. For example, if u is the imprecisely known parameter, then $N = \#S_K$, $h^1(s_K) = 0$, and $h^1(s_K)_n = 1$ if $s_K = n$ (= 0 if $s_K\neq n$). As another example, let N represent the number of objectives in a multicriteria decisionmaking problem having an additive utility function with imprecisely known importance weights. Then, let ρ_n be the importance weight for the n^{th} objective, and define $h^1(s)$ and $h^2(s)$ appropriately.

SOLUTION PROCEDURE

Dynamic programming will be used to achieve the problem objective. Let $f_k(s,\rho)$ be the optimal expected utility to be accrued if at stage k, the current state is s and the value of the imprecisely known parameter is ρ. Then, f_k satisfies the following dynamic programming equation and boundary condition:

$$f_k(s,\rho) = \max_{a\epsilon A_k(s)} \{\sum_{s'} p(s')|s,a)f_{k+1}(s',\rho)\}$$

for $k < K$, where $f_K(s,\rho) = h^1(s) + h^2(s)\rho$ and where $A_k(s)$ represents the set of all available alternatives at stage k, given that the decision node under consideration is node s (Bertsekas, 1976). Let δ be the function that achieves the above maximum, which is a function of stage, state, and parameter value. That is, assume $\delta = \{\delta_k, k=0,1,\ldots,K-1\}$ is such that

$$f_k(s,\rho) = \sum_{s'} p[s'|s,\delta_k(s,\rho)]f_{k+1}(s',\rho)$$

for all k, s, and ρ. Then, δ is an optimal parameter dependent strategy (Bertsekas, 1976). Note that if λ^* is such that $\lambda_k^*(s) = \delta_k(s,\rho)$ for all k and s for some $\rho\epsilon P$, then λ^* represents one of the (parameter independent) strategies which we seek. Thus, our problem objective is attained once δ, and hence f_k, $k=0,\ldots,K$, are determined. We now show that f_k, $k=0,\ldots,K$, has a computationally interesting functional form.

<u>Proposition</u>. For each k and s, $f_k(s,\rho)$ is piecewise affine and convex in ρ on P.

<u>Proof</u>: The Proposition equivalently states that for each k and s, there is a set $A_k(s)$ of pairs (α,γ) such that

$$f_k(s,\rho) = \max \{\alpha+\gamma\rho: (\alpha,\gamma)\epsilon A_k(s)\},$$

where $\alpha\epsilon R$ and $\gamma\epsilon R^N$ for any $(\alpha,\gamma)\epsilon A_k(s)$. Clearly, $f_K(s,\rho)$ is piecewise affine and convex in ρ for each s; $A_K(s) = \{(h^1(s), h^2(s))\}$. Assume for each $(k+1)^{st}$ stage decision node, $f_{k+1}(s,\rho)$ is piecewise affine and convex in ρ. Then, for each $a\epsilon A_k(s)$,

$$\sum_{s'} p(s'|s,a)f_{k+1}(s',\rho) = \sum_{s'} p(s'|s,a)\max\{\alpha+\gamma\rho:(\alpha,\gamma)\epsilon A_{k+1}(s')\} =$$

$$\max\{\sum_{s'} p(s')|s,a)\alpha(s') + \sum_{s'} p(s'|s,a)\gamma(s')\rho\},$$

where the last maximum is taken over all $(\alpha(s'),\gamma(s'))\epsilon A_{k+1}(s')$ for all states s' at stage k+1. Clearly, the right hand side of the above equality string is piecewise affine and convex. The function $f_k(s,\rho)$ is then piecewise affine and convex since the maximum of a finite number of piecewise affine and convex functions is piecewise affine and convex. □

The proof of the Proposition suggests the following computational procedure which is based on a similar procedure found in (*Smallwood and Sondik*, 1973):

0. Define $A_K(s) = \{(h^1(s), h^2(s))\}$; set k=K-1.
1. Define $A_k(s,a)$ as the set of all pairs (α',γ') where

$$\alpha' = \sum_{s'} p(s'|s,a)\alpha(s')$$

$$\gamma' = \sum_{s'} p(s'|s,a)\gamma(s')$$

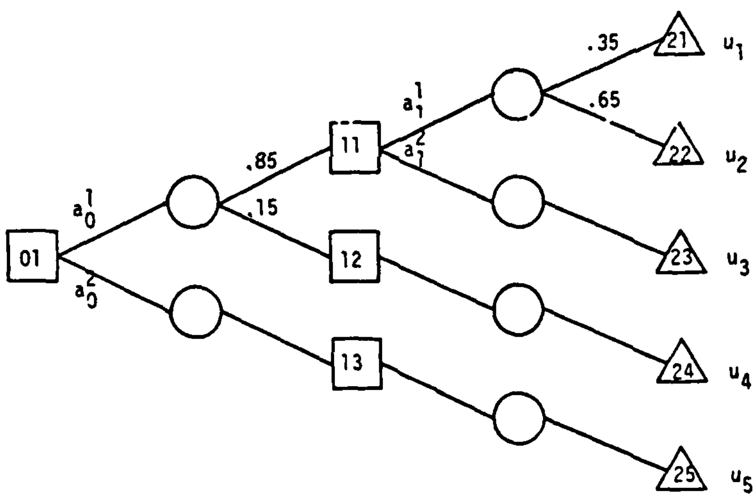
where $(\alpha(s'),\gamma(s'))\varepsilon A_{k+1}(s')$.

2. Define $A_k(s) = \cup_a A_s(s,a)$.
3. If k=0, stop; if not, set k=k-1 and go to Step 1.

We remark that the number of pairs in $A_k(s,a)$ may be reducible by eliminating all pairs (α',γ') in $A_k(s,a)$ that do not achieve the maximum in $\max\{\alpha'+\gamma'\rho: (\alpha',\gamma')\varepsilon A_k(s,a)$ for some value of $\rho\varepsilon P$. An analogous statement can be made regarding the set $A_k(s)$.

EXAMPLE

We now present a simple numerical example to illustrate the theoretical and computational results obtained above. Consider the following decision tree:



Let $u_i = u_{i1}\rho_1 + u_{i2}\rho_2 + u_{i3}\rho_3$, $\rho_n \geq 0$ and $\sum_n \rho_n = 1$, where:

u_{in}:	$n = 1$	2	3
$i = 1$	1	0.7	0.75
2	0.8	0.4	0.85
3	0.2	1	0
4	0.35	0	0.75
5	0	0.3	1

We note that there are three parameter invariant strategies:

$$\lambda^1(01) = a_0^2$$
$$\lambda^2(01) = a_0^1 \quad \text{and } \lambda^2(11) = a_1^1$$
$$\lambda^3(01) = a_0^1 \quad \text{and } \lambda^3(11) = a_1^2$$

The boundary conditions and the first iteration of the dynamic program imply that:

$$f(21,\rho) = \rho_1 + 0.7\rho_2 + 0.75\rho_3$$
$$f(22,\rho) = 0.8\rho_1 + 0.4\rho_2 + 0.85\rho_3$$
$$f(23,\rho) = 0.2\rho_1 + \rho_2$$
$$f(24,\rho) = 0.35\rho_1 + 0.75\rho_3$$
$$f(25,\rho) = + 0.3\rho_2 + \rho_3$$
$$f(11,\rho) = \max\{0.87\rho_1 + 0.505\rho_2 + 0.815\rho_3,\ 0.2\rho_1 + \rho_2\}$$
$$f(12,\rho) = 0.35\rho_1 + 0.75\rho_2$$
$$f(13,\rho) = 0.3\rho_1 + \rho_3$$

We now see that $a_1^1(a_1^2)$ should be selected rather than $a_1^2(a_1^1)$ if $0.87\rho_1 + 0.505\rho_2 + 0.815\rho_3 \geq (\leq)\ 0.2\rho_1 + \rho_2$. For the moment, let us not make any assumptions regarding P. Then,

$$f(01,\rho) = \max\{0.3\rho_1 + \rho_3,\ \max\{0.792\rho_1 + 0.42925\rho_2 + 0.80525\rho_3,\ 0.2225\rho_1 + 0.85\rho_2 + 0.1125\rho_3\}\},$$

where $a_0^2(a_0^1)$ should be selected rather than $a_0^1(a_0^2)$ if $0.3\rho_1 + \rho_3 \geq (\leq)$ $\max\{0.792\rho_1 + 0.4925\rho_2 + 0.80525\rho_3,\ 0.2225\rho_1 + 0.85\rho_2 + 0.1125\rho_3\}$. We note that these inequalities divide the set $\{\rho\epsilon R^3\colon \rho_n \geq 0,\ \sum_n \rho_n = 1\}$ into three regions, each of which represents the set of all points where one of the three strategies is optimal. For example, if all that is known about ρ is that $\rho_1 \geq \rho_2 \geq \rho_3$ and $\rho_1 \geq 0.6$, then

λ^2 is optimal. As another example, if $\rho_3 \geq 0.85$, then λ^1 is optimal. We remark that if $\rho_1 \geq 0.6$, then it would have been sufficient to let $f(11,\rho) = 0.87\rho_1 + 0.505\rho_2 + 0.815\rho_3$.

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

We have presented a numerically efficient procedure for determining the set of all possibly optimal parameter invariant strategies for a decision analysis problem with imprecisely known utility functions. Future research directions include extending these results to other sequential decisionmaking problems for both the finite and infinite planning horizon cases.

ACKNOWLEDGEMENTS

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Behavioral and Organizational Models for Human Decisionmaking

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ABSTRACT

This paper presents a discussion and interpretation of critical factors relevant to the design of support systems to enhance the quality of organizational decisionmaking. The roles of information, feedback, and individual and organizational learning in determining choice, and the organizational objectives that lead to choice and that are responsive to choice, are emphasized.

1. INTRODUCTION

Information-based technologies are major potential aids to organizational decisionmaking. Sound design and implementation of information and knowledge based support to organizational decisionmaking require a knowledge of the ways in which organizations can acquire and process information, the ways in which organizations adapt to their internal and external environment, the ways in which organizations cope with conflict, the ways in which organizational preferences result from decisions as well as being determined by them, and the ways in which organizations learn and fail to learn. We discuss each of these in this paper from the perspective of decision support system design.

2. ORGANIZATIONAL REALITIES

There are a variety of definitions of an organization:

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- a system of consciously coordinated activities of two or more people [6]
- social units deliberately constructed to seek specific goals [13]
- collectives that have been established on a relatively continuous basis in an environment, with relatively fixed boundaries, a normative order, authority ranks, communication systems, and an incentive system designed to enable participants to engage in activities in general pursuit of a common set of goals [15]
- a set of individuals (with bounded rationality) engaged in the decisionmaking process [30]

Organizations can be viewed from a closed system perspective which views an organization as an instrument designed to enable pursuit of well defined specified objectives. In this view an organization will be concerned primarily with four objectives: efficiency, effectiveness, flexibility or adaptability to external environmental influences, and job satisfaction [16]. Four organizational means or activities follow from this: complexity and specialization, centralization or hierarchy of authority, formalization or standardization of jobs, and stratification of employment levels [16]. In this view, everything is functional and tuned such that all resource inputs are optimum and the associated responses fit into a well-defined master plan.

March and Simon [29], among others, discuss the inherent shortcomings associated with this closed system model of humans as machines. Not only is the human as machine view inappropriate but there are pitfalls associated with viewing environmental influences as "noise."

In the open systems view of an organization, concern is not only with objectives but with appropriate responses to a number of internal and external influences. Weick [50] describes organizational activities of enactment, selection, and retention which assist in the processing of ambiguous information that results from an organization's interactions with ecological changes in the environment. The overall result of this process is the minimization of information equivocality such that the organization is able to understand its environment, recognize problems, diagnose their causes, identify policies to potentially resolve problems, evaluate efficacy of these policies, and select a priority order for problem resolution. Figure 1 presents a partial interpretation of Weick's social theory of organizing.

The result of the enactment activities of the organization is the enacted environment of the organization. This enacted environment contains an external part, which represents the activities of the organization in product markets, and an internal part which is the result of organizing people into a structure to achieve organizational goals. Each of these environments is subject to uncontrollable ecological influences due to economic, social, and other changes. Selection activities allow perception framing, editing and interpretation of the effects of the organization's actions upon the external and internal environments such as to enable selection of a set of relationships believed of importance. Retention activities allow admission, rejection, and modification of the set of selected knowledge in accordance with existing retained knowledge and integration

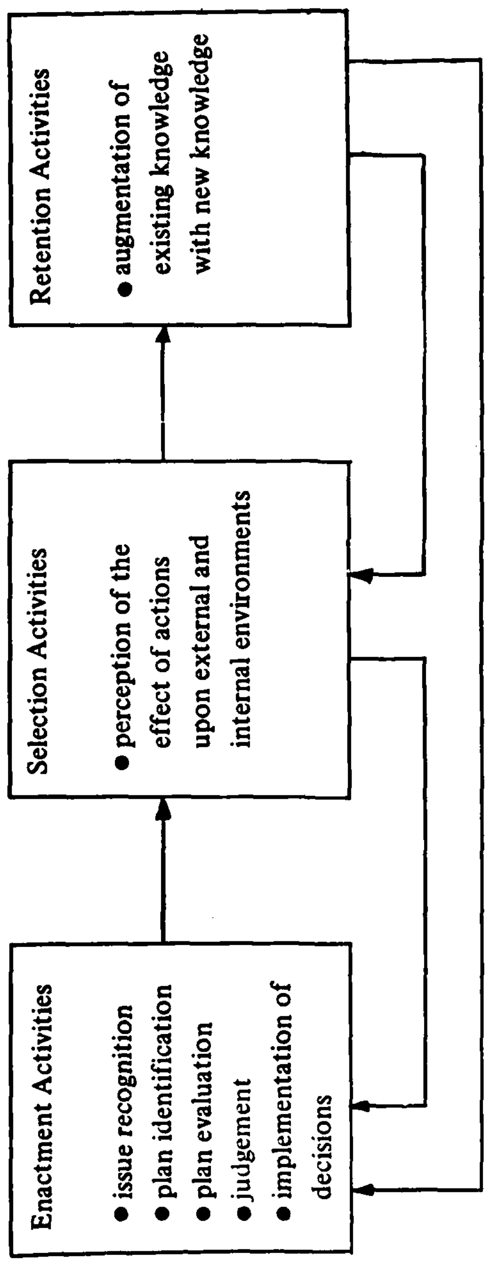

Figure 1. Interpretation of Weick's Social Theory of Organizing

of previously retained organizational knowledge with new knowledge. There are a potentially large number of cycles that may be associated with enactment, selection, and retention. These cycles generally minimize informational equivocality and allow for organizational learning such that the organization is able to cope with very complex and changing environments.

We shall present several other conceptual models of organizations in other sections of this effort. A very important feature of these models is that of *organizational learning.* Much of this organizational learning is not necessarily beneficial or appropriate in a descriptive sense. For example, there is much literature [23,35] which shows that organizations and individuals use improperly simplified and often distorted models of causal and diagnostic inferences, and improperly simplified and distorted models of the contingency structure of environment and task in which these realities are embedded. Individuals often join "groups" to enhance survival possibilities and to enable pursuit of career and other objectives. These coalitions of like minded people pursue interests that result in emotional and intellectual fulfillment and pleasure. The activities that are perceived to result in need fulfillment become objectives for the group. Group cohesion, conformity, and reinforcing beliefs often lead to what has been called "groupthink" [20,21], and an information acquisition and analysis structure that enables processing only in accordance with the belief structure of the group. The resulting selective perceptions and neglect of potentially disconfirming information preclude change of beliefs. A central purpose of a decision support system is to enhance the knowledge base for decisionmaking and the interpretation of this knowledge base to enhance decision quality. To be able to discuss decisionmaking as it might be, we must be aware of decisionmaking and judgement and choice behavior as it is.

3. ORGANIZATIONAL MANAGEMENT

An important aspect of the study of organizations is the role of management and management decisionmaking. In an extraordinarily insightful work, Mintzberg [30] identifies a three-dimensional taxonomy that characterizes managerial paradigms. These paradigms are described in Table 1. The content roles, characteristic roles, and contingencies which influence variations in managerial efforts are obtained from interpretation of the results of the decisionmaking and leadership schools of thought concerning managerial behavior. Mintzberg has identified eight schools of thought concerning management, as indicated in Table 2. The information roles and the decisional roles of the manager are of particular interest here as well as the contingency task structure variables which influence these roles. Especially relevant to our efforts is Mintzberg's discussion of several studies of managerial activities as a programmed system. The need to develop programs, or cognitive maps [4,10], or process tracing models [22], of managerial activity is an essential objective in the development of systems that support management and organizational decisionmaking.

4. MODELS OF ORGANIZATIONAL DECISIONMAKING

The organizational science literature thoroughly discusses the development of conceptual models for decisionmaking by individuals and by organizations based upon various rationality conceptualizations. Among these are: the (economic) rational actor model; the satisficing or bounded rationality model; the bureaucratic politics, incremental, or "muddling through" model; the organizational processes model, and the garbage can model. These are important to our discussions here and we present highlights of each of these models.

a. The Rational Actor Model

The decisionmaker becomes aware of a problem, studies it, carefully weighs alternative means to a solution and makes a choice or decision based on an objective set of values. At first glance, the rational actor model appears to contain much of value. It is especially well matched to the entrepreneurship and decision theory schools of thought as described by Mintzberg. However, we must be aware that it is a normative substantive model. There may be any number of descriptive process realities which may make it infeasible of realization. In rational planning or decisionmaking:

1. The decisionmaker is confronted with an issue that can be meaningfully isolated from other issues.
2. Objectives, which will result in need satisfaction, are identified.
3. Possible alternative activities to resolve needs are identified.
4. The impacts of action alternatives are determined.
5. The utility of each alternative is evaluated in terms of its impacts upon needs.
6. The utilities of all alternatives are compared and the policy or activity with the highest utility is selected for action implementation.

Simon [39-43] was perhaps the first to observe that unaided decisionmakers may not be able to make complete substantive use of the economic rational actor model possible. The concepts of bounded rationality and satisficing represent much more realistic substantive models of actual decision rules and practices. Argyris [1] has presented a definitive recent discussion of the limits to rational man organizational theory. A variety of satisficing heuristic rules have been described in [47]. These are often used as "simple" substitutes for "difficult" rational behavior. Unless very carefully developed and applied however, these heuristic rules may result in very inferior decisions; decisions which are reinforced through feedback and repetition such as to result in experiences (and learnings) that are, by no means, the best teacher. Processes that are only economically rational may be neither desirable nor possible. Social, political, or legal rationality concerns may well prevail. And one of the other decision frameworks we describe here may well be more appropriate if these concerns are dominant over economic rationality concerns.

b. The Satisficing or Bounded Rationality Model

The decisionmaker looks for a course of action that is basically good enough to meet a minimum set of requirements. The goal is to "not shake the system" or "play it safe" by making decisions primarily on the basis of short term acceptability rather than seeking a long term optimum.

Simon introduced the concept of satisficing or bounded rationality as an effort to "... replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and computational capabilities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist." He suggested that decisionmakers compensate for their limited abilities by constructing a simplified representation of the problem and then behaving rationally within the constraints imposed by this simplified model. We may satisfice by finding either optimum solutions in a simplified world or satisfactory solutions in a more realistic world. As Simon says, "neither approach dominates the other" [42].

Satisficing is actually searching for a "good enough" choice. Simon suggested that the threshold for satisfaction, or aspiration level, may change according to the ease or difficulty of search. If many alternatives can be found, the conclusion is reached that the aspiration level is too low and needs to be increased. The converse is true if no satisfactory alternatives can be found. This may lead to a unique solution through iteration.

The principle of bounded rationality and the resulting satisficing model suggest that simple heuristics may well be adequate for complex problem solving situations. While satisficing strategies may well be excellent for repetitive problems by encouraging one to "do what we did last time if it worked last time and the opposite if it didn't," they may also lead to premature choices that result in unforeseen disasterous consequences; consequences which could have been foreseen by more careful analysis. A paper by Thorngate [47] provided useful descriptions of ways in which heuristic decision rules may be used and abused. Development of efficient and effective decision heuristics is a contemporary need for the analysis of decision behavior [8,47] and the modeling of organizational and individual decisions [34,45], as well as for the design of normative systems to aid decisionmaking [37]. We believe that to be effective as well as efficient, heuristics will have to be developed in a very cautious way with due considerations for the many implications of the contingency task structure of a decision situation [32,38].

c. The Bureaucratic Politics, Incrementalism, or "Muddling Through" Model

After problems arise which require a change of policy, policy makers typically consider only a very narrow range of alternatives differing to a small degree from the existing policy. One alternative is selected and tried with unforeseen consequences left to be discovered and treated by subsequent incremental policies. This is the incremental view.

In 1959, Lindblom postulated the approach called incrementalism, or muddling through [26-28], to cope with perceived limitations in the economically rational approach. Marginal values of change only are considered – and these for only a few dimensions of value – whereas the rational approach calls for exhaustive analysis of each identified alternative along all identified dimensions of value. A number of authors have shown incrementalism to be the typical, common, and currently practiced decision process of groups in pluralistic societies. Coalitions of special interest groups make cumulative decisions and arrive at workable compromise through a give and take process that Lindblom calls "partisan mutual adjustment." He indicates that ideological and other value differences do not influence marginal decisions as much as they influence major changes and that, in fact, considering marginal values subject to practical constraints will lead to agreement on marginal programs. Further, incrementalism can result in agreement on decisions and plans even by those who are in fundamental disagreement on values. However, incrementalism appears based on keeping the masses marginally content and thus may not be able to do much to help the greatly underprivileged and unrepresented. There have been a number of studies which indicate incrementalism to be an often used approach in practice. Without doubt, this is a realistic process-oriented descriptive model.

It is important to note [26] that Lindblom rejects (economic) comprehensive rationality even as a normative model, proposes incrementalism as a normative approach, and indicates that systems analysis and economic rationality will often lead to ill-considered, often accidental incompleteness. He indicates the following inevitable limitations to analysis. It is fallible, never rises to infallibility, and can be poorly informed, superficial, biased, or mendacious. It cannot wholly resolve conflicts of value and interest. Sustained analysis may be too slow and too costly compared with realistic needs. Issue formulation questions call for acts of choice or will, and suggest that analysis must allow room for politics.

The main features of the model proposed by Lindblom are as follows. Ends and means are viewed as not distinct. Consequently means-ends analysis is viewed as often inappropriate. Identification of values and goals is not distinct from the analysis of alternative actions. Rather, the two processes are confounded. The test for a good policy is, typically, that various decisionmakers, or analysts, agree on a policy as appropriate without necessarily agreeing that it is the most appropriate means to an end. Analysis is drastically limited, important policy options are neglected, and important outcomes are not considered. By proceeding incrementally and comparing the results of each new policy with the old, decisionmakers reduce or eliminate reliance on theory. There is a greater preoccupation with ills to be remedied rather than positive goals to be sought. Incremental analysis is a good description of political decisionmaking and is sometimes referred to as the "political process" model.

d. The Organizational Processes Model

Plans and decisions are the result of interpretation of standard operating procedures. Improvements are obtained by careful identification of existing standard operating procedures and associated organizational structures and determination of improvements in these procedures and structures.

The organizational process model, originally due to Cyert and March [12], functions by relying on standard operating procedures which constitute the memory or intelligence bank of the organization. Only if the standard operating procedures fail will the organization attempt to develop new standard procedures.

The organizational process model may be viewed as an extension of the concept of bounded rationality to choicemaking in organizations. It is clearly an application of reasoning and rationality, as discovery and application of rules, to cases. There are four main concepts of the behavioral theory of the firm which are suggested as descriptive models of actual choice-making in organizations:

(1) *Quasi-resolution of conflict.* Decisionmakers avoid conflicts arising from noncommensurate and conflicting goals. Major problems are disaggregated and each subproblem is attacked locally by a department. An acceptable conflict resolution between the efforts of different departments is reached through sequential attention to departmental goals and through the formulation of coalitions which seek power and status. When resources are scarce and there must then be unsatisfied objectives, decisions concerning allocations will be met largely on political grounds.

(2) *Uncertainty avoidance* is achieved by reacting to external feedback, by emphasizing short term choices, and by advocating negotiated futures. Generally there will exist uncertainties about the future; uncertainties associated with future impacts of alternatives and uncertainties associated with future preferences. Generally, deficient information processing heuristics and cognitive biases are used to avoid uncertainties. The effects are, of course, suboptimal.

(3) *Problem search* is stimulated by encountering issues and not before issues are surfaced. A form of "satisficing" is used as a decision rule. Search in the neighborhood of the status quo only is attempted and only incremental solutions are considered.

(4) *Organization learning.* Organizations adapt on the basis of experience. They often pay considerable attention to one part of their environment at the expense of another.

The organizational process model may be viewed as suggesting that decisions at time t may be forecasted, with almost complete certainty, from knowledge of decisions at time t-T where T is the planning or forecasting period. Standard operating procedures or "programs," and education motivation and experience or "programming" of management are the critical determinants of behavior for the organizational process model. Cohen and March recommend a strategy of management leadership to cope with organizational process realities. Managers are encouraged to be intimately involved in organizations such that they will be able to strongly influence decisions; to become widely informed such that they will be highly valued in the information-poor organization; to be extraordinarily persistent since unmitigated chutzpah will often have entirely undeserved rewards; to encourage those with opposing views to participate; and to overload organizational systems such as to make themselves more necessary. In this view, the descriptive characteristics of the organization are seen as performance inhibiting factors. They are factors not to be overcome, but to be understood and used to the advantage of the manager.

e. The Garbage Can Model

This relatively new model [11] views organizational decisionmaking as resulting from four variables: problems, solutions, choice opportunities, and people. Decisions result from the interaction of solutions looking for problems, problems looking for solutions, decision opportunities, and participants in the problem-solving process. The model allows for these variables being selected more or less at random from a garbage can. Doubtlessly, this is a realistic descriptive model.

All five of the models, or frameworks, for decisionmaking have both desirable and undesirable characteristics and any of them may be relevant in specific circumstances. If we accept the facts that:

1. Decisionmakers use a variety of methods to select among alternatives for action implementation;
2. These methods are frequently suboptimal; and
3. Most decisionmakers desire to enhance their decisionmaking efficiency and effectiveness;

then we must conclude that there is much more motivation and need for research and ultimate design and development of planning and decision support systems. But these five models make it very clear that improved planning and decision-making efficiency and effectiveness, and aids to this end, can only be accomplished if we understand human decisionmaking as it is as well as how it might be and allow for incorporation of this understanding in the design of systemic process adjuvants. One of the requirements imposed on these adjuvants will be relevance to the individual and group decisionmaking structure. Another requirement is relevance to the information requirements of the decisionmaker. Especially important also is accommodation of organizational learning.

5. INFORMATION PROCESSING IN ORGANIZATIONS

We are particularly interested in describing the decisionmaking process in organizations. This leads naturally to a study of information processing in organizations and a description of how decisionmakers may determine information needs. While there have been a number of studies of group decisionmaking roles, and organizational behavior [44,46], our efforts will be based primarily on those of Vroom and Yetton [49], Huber [17], and Feldman and March [14].

Huber and Vroom and Yetton have indicated a number of potential advantages and disadvantages to group participation in decisionmaking. Since a group has more information and knowledge potentially available to it than any individual in the group, it *should* be capable of making a better decision than an individual. Group decisions are often more easily implemented than individual decisions since participation will generally increase decision acceptance as well as understanding of the decision. Also group participation increases the skills and information that members may need in making future organizational decisions. On the other hand, there are potential disadvantages to groups. They consume more time in decisionmaking than individuals. Their decisions may not fully support higher organizatonal goals. Group participation may lead to unrealistic anticipations of involvement in future decisions and resentment by individuals towards subsequent decisions in which they have not participated. Finally, there is no guarantee that the group will converge on a decision alternative.

Huber asks four primary questions, the answers to which determine guidelines for selection of a particular form of group decisionmaking. These concern involving others, encouraging group activities, delegating authority to the group, and including the leader in the group. The responses to these questions determine an appropriate form of group decisionmaking. There are a number of subsidiary questions concerned with each of the primary questions. For example, we may determine whether or not to involve others by posing questions involving: decision quality, understanding and acceptance, personnel development and relationships, and time required.

Vroom and Yetton have been very concerned with leadership and decisionmaking [49]. Their primary concern is with effective decision behaviors. They develop a number of clearly articulated normative models of leadership style for individual and group decisions. These should be of use to those attempting to structure normative or prescriptive models of the leadership style portion of decision situations which are capable of operational implementation. It is the apparent goal of Vroom and Yetton to move beyond generalities such as the stereotypical leadership style theory X-theory Y [25,49]. They desire to come to grips with, and use explicitly, leadership behavior and situational variables to enhance organizational effectiveness. Other theories and practices of organizational leadership are presented in [7].

Keen [24] acknowledges four causes of inertia relative to organizational information systems. He indicates that: information is only a small component; human information processing is experiential and relies on simplification; organizatonal change is incremental and evolutionary with large changes being avoided; and that data are a political resource affecting particular groups as well as being an intellectual commodity. Each of these factors suggests the importance of a knowledge of the way in which information is processed by organizations.

The purpose of knowledge-based decision support systems is to provide timely, relevant, and accurate information to system users to enhance human judgement and decisionmaking efficiency and effectiveness concerning resource allocations that affect issues under consideration. Among the many concerns that dictate needs and requirements for automated support systems is the fact that decisionmakers must typically make more judgements and associated decisions in a given period of time than they can comfortably make. This creates a stressful situation which can lead to the use of poor information processing and judgemental heuristics, especially since judgements and decisions are typically based on forecasts of the future. Needs and issues associated with the design of information processing and judgement-aiding support systems relate to questions concerning appropriate functions for the decisionmaker and staff to perform. They concern the type of information which should be available and how this information should be acquired, analyzed, stored, aggregated and presented such that it can be used most effectively in a variety of potential operational environments. They concern design of information systems with strong space-time-environmental dependencies. They concern design of information systems that can effectively "train" people to adapt and use appropriate concrete operational heuristics in those environments in which inexperience dictates initial use of formal operational thought. They concern design and use of information systems that support environmentally experienced decisionmakers in the use of a variety of effective concrete operational heuristics. And because of their use by multiple decisionmakers, these tasks must be accomplished in a parallel architectural fashion.

Huber [17-19] and Tushman and Nadler [48] have developed a number of propositions, based on their own research and upon the research of others, reflecting various aspects of information processing in organization. The general conclusion of these studies is that in an effort to enhance efficiency, organizational information processing typically requires selective routing of messages and summarization of messages. In the classical normative theory of decisionmaking, it is easily shown that information about the consequences of alternative courses of action should be "purchased" only if the benefits of the information, in terms of precision, relevance, reliability and other qualities exceed the cost. Feldman and March [14] present a highly symbolic alternate point of view in their descriptive portrait of information use in organizations. Their discussions of information incentives indicate systematic bias in estimating the benefits and costs of information due to the fact that the costs and benefits do not occur at the same place and at the same time such that one group has responsibility for information use whereas another has responsibility for information availability. Also, people are prone to obtain more informaton than is needed since, under uncertainty conditions, the post outcome probabilities of events that do occur will be (descriptively) judged higher than the prior probabilities of these events. This will suggest that less information was obtained than should have been obtained. This will, typically, lead to incentives to obtain too much information.

Feldman and March also indicate that much of the information that is obtained is obtained for surveillance purposes to uncover potential surprises rather than to clarify uncertainties for decisionmaking. Strategic misrepresentation of information, due to interpersonal conflicts and power struggles, is a third factor suggested as decoupling information gathering from decisionmaking. This occurs since information is not innocent and must be suspected of bias. Finally, information is a symbol which indicates a commitment to rationality; and there are incentives to displaying the incentive even if it is not used.

Identification of other variables which influence information processing in and by organizations would represent a desirable activity. To determine how these information processing variables are influenced by the information processing biases of individuals discussed in [23,35] would seem especially desirable in terms of the likely usefulness of the results and the need for an expanded theory of group information processing biases. There appear to have been only limited results obtained in the area of cognitive information processing biases and use of inferior heuristics on the part of groups. The development of appropriate normative strategies for information requirements determination are especially important towards these ends. Davis [9] identifies two major levels of requirements: the organizational information requirements reflected in a planned applications portfolio, and detailed information requirements associated with specific applications. A contingency approach is suggested to yield information requirements at each level and to minimize the deficiencies associated with human information processing.

A major difficulty in cognitive information processing seems to be failure to identify and use an appropriate structure that allows appropriate weighting of observed data. Investigation of the effects of various structured information processing/decision aiding protocols upon the acquisition, analysis, and interpretation of informaton and its integration with judgment and decisionmaking activities would appear to be a contemporary need in information system design. There are a number of studies in expert systems and artificial intelligence [5] relevant to these ends. These and other studies may well provide a structured framework for information processing styles and approaches ranging from the purely qualitative and affective, to reasoning by analogy which may be a blend of qualitative and quantitative, to quantitatively based filtering and detection algorithms.

6. ORGANIZATIONAL LEARNING

Organizational learning results when members of the organization react to changes in the internal or external environment of the organization by detection and correction of errors [1-3]. An error is a feature of knowledge that makes action ineffective and detection and correction of error produces learning. Individuals in an organization are agents of organizational action and organizational learning. The seminal theory of action concept of Argyris [1-3] is based upon these notions which are extraordinarily relevant to decision support system design.

Argyris cites two information related factors that inhibit organizational learning: the degree to which information is distorted such that its value in influencing quality decisions is lessened, and lack of receptivity to corrective feedback. Two types of organizational learning are defined. Single loop learning is learning which does not question the fundamental objectives or actions of an organization. Members of the organization discover sources of error and identify new strategic activities which might correct the error. The activities are analyzed and evaluated and one or more selected for implementation. Environmental control and self protection through control over others, primarily by imposition of power, are typical strategies. The consequences of this approach include defensive group dynamics and low production of valid information.

The lack of information does not result in disturbances to prevailing values. The resulting inefficiencies in decisionmaking encourage frustration and an increase in secrecy and loyalty demands from decisionmakers. All of this is mutually self reinforcing. It results in a stable autocratic state and a self fulfilling prophecy with respect to the need for organizational control.

Double-loop learning involves identification of potential changes in organizational goals and of the particular approach to inquiry that allows confrontation with and resolution of conflict rather than the translation of incompatible objectives into intergroup conflict. Not all conflict resolution is the result of double loop learning however. Good examples of this are conflicts settled through imposition of power rather than inquiry. Thus, double-loop learning is seen to be the result of that organizational inquiry which resolves initially perceived incompatible organizatonal objectives through the restructuring and setting of new priorities and objectives. New understandings are developed which result in updated cognitive maps and scripts of organizational behavior. Organizations are claimed to learn primarily on the basis of single loop learning and, typically, do not engage in double-loop learning.

Individuals act as agents of organizational learning through the processing of initially inaccessible and obscure information and by resolving potential inadequacies associated with individual and organizational theories of action [2]. All human action is said to be based on theories of action. There are two types: espoused theories of action, which are the "official" theories that people claim as a basis for action; and theories in use, which are the descriptive theories of action that may be inferred from actual behavior. While people are often adept at identifying discrepancies between espoused theories of action and theories in use associated with others, they are not equally capable at self diagnosis. However people are programmed with theories in use that suggest that this inconsistent behavior in others not be reported to them by those who detect it. So we see again the presence of social exchanges and customs that inhibit double loop learning.

There are several dilemmas associated with theory of action building [3]. Among these non-mutually exclusive dilemmas, which result in conflicting and intolerable pressures, are:

- *incongruity* between espoused theory and theory in use which are recognized but not corrected
- *inconsistency* between theories in use
- *ineffectiveness* as objectives associated with theories in use become less and less achievable over time
- *disutility* as theories in use become less valued over time
- *unobservability* as theories in use result in suppression of information by others such that evaluation of effectiveness becomes impossible.

Detection and correction of inappropriate espoused theories of action and theories in use is suggested as potentially leading to a reduction in those factors that inhibit double-loop learning. Of course single-loop learning often will be appropriate. The result of double-loop learning is a new set of goals and standard operating policies that become a part of the organization's knowledge base. It is when the environment, or more generally the contingency task structure, changes that double-loop learning is called for. Inability to accommodate double-loop learning is a flaw. Ability to successfully integrate and utilize the appropriate blend of single and double loop learning is called deutero, or dialectic, learning.

Several intervention models, or approaches, are suggested to encourage organizations to adopt a capability for double-loop learning [3]. These include comprehensive intervention; limited intervention through structural mapping of the issue, internalization of the map, testing and validation of the map, simulation and analysis of impacts using the map, and generation of knowledge using the map for use in future designs; and several partial models of intervention. Of particular interest in this seminal work are the several caveats given concerning difficulties in the design of management information and decision support systems such that they support Model II (i.e., double-loop) learning, rather than Model I (single-loop) learning.

7. SUMMARY

We have presented a description and interpretation of some recent results in behavioral and organizational theory that have direct relevance to the design of information systems to aid the human decisionmaker in organizational settings. The primary organizing principles in organizations include: division of labor and task assignment, identifying standard operating principles, top down flow of decisions, formal and informal channels of communication in all directions, the multiple uses of information, and organizational learning. We must be conscious of these descriptive principles in order that we be able to produce normative aids that are realistically grounded in the realities of human desires and capabilities for growth and self actualization.

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An Interactive Procedure for Aiding Multiattribute Alternative Selection[1]

INTRODUCTION

IN THIS PAPER, we examine a procedure for aiding a decisionmaker in the task of selecting a single alternative from a finite set of multiattributed alternatives in an uncertain environment. Key characteristics of this procedure are: it is interactive and it attempts to reduce the number and precision of probability, importance weight and utility values necessary for alternative ranking, thereby reducing the time, stress and effort required by the decisionmaker to use it. The procedure essentially strengthens and extends results in [2, 3] and compliments results in [5].

PROBLEM FORMULATION

The decisionmaker's objective is to select a preferred alternative from A available alternatives. Once an alternative is selected, one of C consequences will occur. If alternative a is selected, consequence c will occur with probability $p_c(a)$ and utility $u_{ic}(a)$ with respect to attribute i, $i = 1, \ldots, I$. We assume that expected utility ranks the alternatives in a manner consistent with the decisionmaker's preferences. If the attributes are additive independent [1, p. 295], then expected utility of alternative a is

$$\sum_{i=1}^{I} w_i \sum_{c=1}^{C} u_{ic}(a) p_c(a) = wu(a)p(a),$$

where $w_i \geq 0$ is the importance weight of attribute i,

$$\sum_{i=1}^{I} w_i = 1.$$

Thus, alternative a' is (weakly) preferred to alternative a if and only if $wu(a')p(a') \geq wu(a)p(a)$.

Let $\Lambda(a', a)$ represent the set of all possible values that the 5-tuple $\{w, u(a'), u(a), p(a'), p(a)\}$ can assume. The set $\Lambda(a', a)$ represents a description of parameter value imprecision and is assumed to be given for every alternative pair (a', a). We remark that if $\Lambda'(a', a) \subseteq \Lambda(a', a)$, then $\Lambda'(a', a)$ represents more information about the parameter values than does $\Lambda(a', a)$ but is presumably more difficult to assess than is $\Lambda(a', a)$.

Define the relation R on the alternative set as follows: $(a', a) \in R$ if and only if $wu'p' \geq wup$ for all $(w, u', u, p', p) \in \Lambda(a', a)$. Thus $(a', a) \in R$ if and only if alternative a' is preferred to alternative a for all possible values of the parameters. Hence, R represents a description of preference on the alternatives in a manner consistent with the expected utility criterion and the set of all possible parameter values and as such represents a potentially key source of alternative selection support. For example, if a^* is such that there is no a' such that $(a', a^*) \in R$ and $(a^*, a') \notin R$, then a^* is nondominated. If a^* is such that $(a^*, a) \in R$ and $(a, a^*) \notin R$ for all $a \neq a^*$, then a^* is the most preferred alternative. Our objective, therefore, is to determine R, given $\Lambda(a', a)$ for all (a', a).

Note that $(a', a) \in R$ if and only if

$$\min_{\Lambda(a', a)} w[u(a')p(a') - u(a)p(a)] \tag{1}$$

is nonnegative. Conditions on $\Lambda(a', a)$ which guarantee that R is transitive for a slightly more general problem can be found in the appendix of [5]; we will assume that R is transitive throughout this paper.

DETERMINATION OF R

The key to determining or approximating R is solving or bounding the mathematical program presented in (1). We now present two cases where (1) can be solved exactly and one case where (1) can be bounded from below. Consideration of the first two cases has been motivated by results in [2].

Case 1

Assume $p(a')$ and $p(a)$ are known precisely, $w \in W$, and $u_i = \{u_{ic}(a)\}_{a,c} \in U_i$, $i = 1, \ldots, I$. Let $u_i(a) = \{u_{ic}(a)\}_c$. Then, (1) becomes

$$\min_{w \in W} \min_{u_1 \in U_1} \ldots \min_{u_I \in U_I} \sum_i w_i[u_i(a')p(a') - u_i(a)p(a)]$$
$$= \min_{w \in W} \sum_i w_i \min_{u_i \in U_i} [u_i(a')p(a') - u_i(a)p(a)].$$

Let

$$v_i(a', a) = \min_{u_i \in U_i} [u_i(a')p(a') - u_i(a)p(a)].$$

Then, $(a', a) \in R$ if and only if

$$\min_{w \in W} \sum_i w_i v_i(a', a) \geq 0. \tag{2}$$

We observe that if U_i is described by linear inequalities (a common way of describing interval estimates and/or ordinal rankings), then the determination of $v_i(a', a)$ can be accomplished by a linear program. If W is described by linear inequalities, then once $v_i(a', a)$ has been determined for all $i = 1, \ldots, I$, (2) can be determined by a linear program. If W and U_i, for all i, are all described by linear inequalities, then R can be constructed by solving $A(A-1)I$ linear programs to determine the $v_i(a', a)$ and then solving $A(A-1)$ linear programs to determine the solutions of (1). We remark that the necessary and sufficient condition for an alternative to be the most preferred for all possible parameter values, given in (2), represents an upper bound on the first two sufficient conditions given in the appendix in [2].

Case 2

Assume w is known precisely, $\{p(a'), p(a)\} \in P(a', a)$ and $u_i \in U_i$, $i = 1, \ldots, I$, and let

$$\hat{u}_{ic}(a) = \max_{U_i} u_{ic}(a)$$

$$\check{u}_{ic}(a) = \min_{U_i} u_{ic}(a).$$

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TABLE 1. ALTERNATIVE RANKING AND WEIGHTS TABLE

Attribute	Weight	Alternative 1	2	3	4
1	0.3	4(0.5) 3(0.5)	2(0.4) 1(0.6)	5	3(0.8) 2(0.2)
2	0.2	2	3(0.5) 5(0.5)	4	2(0.6) 4(0.4)
3	0.05	2	5	1	1
4	0.15	1	3	5	1
5	0.2	3	4(0.5) 5(0.5)	4	2
6	0.1	2	3(0.5) 1(0.5)	4	5

Then, it is easily shown that (1) is bounded from below by

$$\min_{P(a',a)} \left\{ \sum_i w_i \bar{u}_{ic}(a') p(a') - \sum_i w_i \bar{u}_{ic}(a) p(a) \right\}. \quad (3)$$

Clearly, if (3) is nonnegative, then $(a', a) \in R$, and hence we have a sufficient (but not necessary) condition for $(a', a) \in R$. We remark that the sufficient condition given in (3) represents an upper bound on the third sufficient condition given in the appendix in [2].

Case 3

Assume w, $u(a')$, and $u(a)$ are known precisely and $\{p(a'), p(a)\} \in P(a', a)$. Then, (1) becomes

$$\min_{(p',p) \in P(a',a)} \{w u'(a) p' - w u(a) p\}$$

which is a linear program if $P(a', a)$ is described by linear inequalities.

AN INTERACTIVE PROCEDURE

We now present an interactive procedure for solving the problem posed in Case 1. Analogous procedures are easily developed for Cases 2 and 3. This procedure is based on the following three general steps for interactive decision aiding [5]:

(a) Eliminate as many alternatives as possible using currently available parameter value information.
(b) If an alternative can be selected without further alternative elimination, then stop.
(c) If a choice cannot be made, then assess further information about the values of the imprecisely known parameters and return to Step a.

These three steps applied to Case 1 produce the following iterative procedure:

(0) Assess $p(a)$ for all a, W, and U_i for all i.
(1) Determine $v_i(a', a)$ for all i, a', and $a, a' \neq a$.
(2) Construct R from the solutions of (2).
(3) If R provides sufficient information for alternative selection, then stop. If not, go to Step 4.
(4) Assess further parameter value information. If the new information only affects W, then go to Step 2. If the new information affects any U_i, then go to Step 1.

Noting Step 4, we observe that if the new information assessed only affects the importance weights, then need for redetermining the $v_i(a', a)$, and hence for solving $A(A-1)I$ linear programs, is eliminated. This fact may affect the direction of the assessment procedure if interactive time is important and if the solution of the linear programs required to determine the $v_i(a', a)$ is time consuming.

TABLE 2. PROBABILITIES AND UTILITY RANKINGS

$p_c(a)$

c	a = 1	2	3	4
1	0.5	0	0	0
2	0.5	0	0	0
3	0	0.4	0	0
4	0	0.1	0	0
5	0	0.5	0	0
6	0	0	1	0
7	0	0	0	0.6
8	0	0	0	0.2
9	0	0	0	0.2

$u_{ic}(a)$

	Alternative 1		2			3	4		
	Consequence								
i	1	2	3	4	5	6	7	8	9
1	4	3	2	1	1	5	3	3	2
2	2	2	3	3	5	4	2	4	4
3	2	2	5	5	5	1	1	1	1
4	1	1	3	3	3	5	1	1	1
5	3	3	4	4	5	4	2	2	2
6	2	2	3	3	1	4	5	5	5

AN ILLUSTRATIVE EXAMPLE

We now re-examine the juvenile drug treatment and rehabilitation problem presented in [2] using the iterative procedure presented above. For this problem $A = 4$ and $I = 6$. Tables 1 and 2 present the initially relevant information. Note that the importance weights are assumed to be known precisely. All utility scores are ordinal rankings with $k + 1$ being more desirable than k. The figures in parentheses in Table 1 represent the probability of obtaining the ranking to the left of the parentheses. For example, the probability of receiving a ranking of 3 with respect to attribute 1 if alternative 4 is chosen is 0.8. In order to model the consequences, we assume $C = 9$ and define the $p_c(a)$ and $u_{ic}(a)$ as in Table 2. We interpret the matrix entries in Table 2 concerning the $u_{ic}(a)$ to be rankings. For example, for attribute $i = 1$ we interpret the rankings to imply $0 = u_{14}(2) = u_{15}(2) \le u_{13}(2) = u_{19}(4) \le u_{12}(1) = u_{17}(4) = u_{18}(4) \le u_{11}(1) \le u_{16}(3) = 1$, which defines U_1 in terms of a finite set of linear inequalities.

Based on the information contained in Tables 1 and 2, the interactive procedure (approximately simplified due to the fact that W is a singleton) produced $R = \{(3, 1), (3, 4)\}$. Thus, alternative 3 dominates alternatives 1 and 4, leaving alternatives 2 and 3 as contenders for the most preferred. If the decisionmaker prefers 2 to 3 or 3 to 2, then according to the interactive procedure, the alternative selection process can stop (see [4] for an alternative approach). If not, more parameter information would require assessment in order to reduce the nondominated set {2, 3}. In [2], additional parameter value information was iteratively added to the information contained in Table 2 with regard to the $u_{ic}(a)$. The impact of this information on the relation R using our

TABLE 3. DOMINANCE STRUCTURES

Additional information	R
none	$\{(3,1),(3,4)\}$
$u_{11}(1) = 0.8$	$\{(3,1),(3,4)\}$
$u_{12}(1) = 0.7$	$\{(3,1),(3,4)\}$
$u_{13}(2) = 0.2$	$\{(3,1),(3,2),(3,4)\}$
$u_{23}(2) = 0.2$	$\{(3,1),(3,2),(3,4)\}$
$u_{26}(3) = 0.8$	$\{(3,1),(3,2),(3,4)\}$

approach is presented in Table 3. For example, if it is known that $u_{11}(1) = 0.8$, $u_{12}(1) = 0.7$ and $u_{13}(2) = 0.2$, then $R = \{(3,1),(3,2),(3,4)\}$, indicating that alternative 3 is the most preferred. We note that further additional information does not alter the relation.

The approach taken in [2] indicated that (a) no alternative could be ruled out even if $u_{11}(1)$ was assessed and included with the information contained in Table 2 and (b) only after all the additional information listed in Table 3 was provided could alternative 3 be considered the most preferred. Here the results in [2] are less discriminating than our results, and will never be more discriminating since ours are based on necessary and sufficient conditions for an alternative to be the most preferred for all possible parameter values, while in [2] the results are based only on sufficient conditions.

In order to more fully demonstrate the theory, we weakened our assumption on the importance weights to $w_3 \leq w_6 \leq w_4 \leq w_2 = w_5 \leq w_1$ which is consistent with their precise values, and found that $R = \{(3,1),(3,4)\}$ for all levels of additional utility value information (including 'none') listed in Table 3. Then we additionally assumed that $w_1 = 0.3$, $R = \{(3,1),(3,4)\}$ for all levels of information in Table 3 except the last level. The addition of $u_{26}(3) = 0.8$ produced $R = \{(3,1),(3,2),(3,4)\}$. Thus, with only an ordinal ranking of the importance weights consistent with their precise values, we were able to eliminate alternatives 1 and 4 without any utility value information other than that contained in Table 2. If we also assumed that $w_1 = 0.3$, then our approach required the same amount of additional utility value information that the approach in [2] required in order to determine that alternative 3 is the most preferred.

CONCLUSIONS

We have presented an interactive procedure for alternative selection based on utility theory which allows parameter value information to be imprecise. This procedure produces at least as much information regarding alternative ranking as does a recently proposed alternative ranking procedure presented in [2]. An example presented in [2] has been used to illustrate this fact. Determining effective methods for assessing parameter value information is a topic for future research.

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ARIADNE: A Knowledge-Based Interactive System for Planning and Decision Support

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Abstract—The development of an interactive planning and decision support process for multiple criteria alternative selection situations is discussed. Probabilities, utility scores for the lowest level attributes, and attribute trade-off weights, i.e., the parameters, can be imprecisely described by set inclusion. Within a specified structural model of the decision situation, the process allows the decisionmaker to iteratively select the mix of parameter value precision and alternative ranking specificity. By selecting this mix, the decisionmaker is able to direct the alternative selection process in an interactive manner, using alternative selection strategies based on behaviorally meaningful dominance search strategies. Emphasis is placed on the motivation of the research and the behavioral relevance of the support process. References in the bibliography provide further analytical and behavioral discussions related to this process.

I. Introduction and Motivation for the Research

It has been observed that the *process* of choosing among multiattributed alternatives often involves an initial search for a dominance structure and ultimate identification of a set of nondominated alternatives, alternatives which are not worse than any other alternative on any *attribute and which are* better than each other alternative on at least one attribute. In most decision situations, however, no single alternative dominates all other alternatives, at least initially. In such decision situations, the decisionmaker typically "adjusts" the structure of the decision situation and parameter values within this structure so as to identify a dominance structure which contains a single nondominant alternative. This search may involve rational activities, such as aggregation of attributes and compensatory trade-offs through determination of judgmental weights. Alternatively, it may involve various rules which may be quite flawed. Examples of such rules are 1) lexicographic ordering, in which the best alternative on the most important attribute is selected, and 2) sequential pairwise comparison of alternatives using a preference relation that is a function of the two alternatives being compared. In this latter case, nontransitive preferences may easily result due to the fact that the contextual relation used to determine preferences changes from binary comparison to binary comparison.

A variety of holistic, heuristic, and wholistic judgmental activities will typically be involved in the search for a dominance structure among the alternatives. These take on various forms and mixtures of formal knowledge-based, rule-based, or skill-based activities as deemed appropriate for the task at hand [1], [2]. Especially with a large number of alternative courses of action under consideration, the decision process will typically involve mixed scanning, where some noncompensatory rule is first used to eliminate grossly inappropriate alternatives. This is then followed by one or more compensatory information evaluation operations that results in a dominance structure which enables final judgment and alternative selection.

The research discussed here is based upon the hypothesis that people are able to evaluate alternative plans and decisions efficiently and effectively and with low stress when a clear dominance pattern exists among alternatives that allows the establishment of a sufficiently discriminatory priority structure. Our goal is to provide a knowledge-b decision support process that enhances the quality of th dominance structure used for judgment and choice.

Often people process information poorly through various forms of selective perception. A typical flaw involves ignoring potentially disconfirming information in order to perceive a dominance pattern among alternatives when no such pattern exists. Another flaw is to evaluate one nondominant alternative incrementally higher than another one after the introduction of alternatives asymmetrically dominated by the first nondominant alternative but not by the second. Sequential pairwise comparison of alternatives often assumes an implicit contextual relation $\succ$, where $A \succ B$ suggests that choosing A and rejecting B is preferable to choosing B and rejecting A [3].

Thus the preference relation is alternative dependent in general, and nontransitive results can be expected from its use in unaided situations. Agenda-dependent results will typically occur in aided situations if we force transitivity through the use of transitive inference and associated neglect of questions that would have provided results which would have disconfirmed the transitivity assumption. Thus there seems much motivation to provide assistance in this search for a dominance structure that will assist in the process of judgment, choice, and decision [4], [5].

In this paper, we provide an overview of our research to these ends. The next section will present a summary of the features and structural constructs of our decision support system. The following section presents a more detailed discussion of these structural constructs and introduces

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some of the modes in which the support process can be used. Then we discuss some behavioral issues that relate to the conceptual design of ARIADNE. Next we present a brief description of some of the algorithmic constructs that allow and assist the development of dominance structures for the various modes in which use of the support system is possible. Some very simple illustrative examples are followed by a section devoted to conclusions and extensions to our research that are currently in progress.

II. Features of the Decision Support System

We now investigate concepts for the design and evaluation of an interactive knowledge-based planning and decision support system which combines, or allows the combination of, several evaluation rules and contingency structures often used as a basis for evaluation, prioritization, judgment, and choice. We have developed a knowledge-based system to interactively aid planning and decision support processes through encouragement of the search for a dominance structure that is behaviorally realistic and rational, from both a substantive and procedural viewpoint. The support system is called Alternative Ranking Interactive Aid based on DomiNance structural information Elicitation (ARIADNE). The support system allows the use of various integrated forms of wholistic, heuristic, and holistic reasoning in an aided search for dominance information among identified alternatives. We believe it to be flexible enough to match diverse decision situations and environments closely in order to support varying cognitive skills and decision styles, thereby enabling planners and decisionmakers to adapt its use to their own cognitive skills, decision styles, and knowledge.

Our efforts have concerned choice-making situations under certainty and under risk, primarily for the single decision node case. This formulation allows the consideration of a variety of imprecisely known parameters such as attribute trade-off weights, outcome state values on lowest level attributes, event outcome probabilities, and various combinations of these. Parameter needs are determined from the structure of the decision situation, as elicited from the decisionmaker during the formulation and analysis steps of the decision support process. We consider these formulation and analysis steps to be outside the scope of our present software developments but recognize the essential need for them in a complete decision support process.

The decision situation structural model may represent decisions under risk or under certainty. The attribute tree representing the features of decision outcome states may be structured and/or parameterized in a top-down or bottom-up fashion through use of ARIADNE. A single-level structure or a multiple-level hierarchical structure of attributes may be used with the choice of these being at the discretion of the decisionmaker. Multiple decision node situations may be approached through a goal directed decision structuring approach in which the growth of the structure of alternative decisions and event outcomes is guided by sensitivity-like computations obtained through use of the ARIADNE algorithms [6]–[8].

Parameters are elicited from the decisionmaker in the form of equalities and inequality bounds. A variety of mathematical programming approaches and graph theory have been used to generate interactive displays of preference digraphs. These mathematical programming approaches are used to determine dominance structures for alternative prioritization that are based on parameter information elicited from the decisionmaker. At present, only a linear programming approach will yield necessary and sufficient conditions for determination of a priority structure and computational times that are consistent with interactive decision aiding. This requires that we elicit structural parameter information in a slightly restricted form which we denote the "behaviorally consistent information set" (BCIS). Often this BCIS will be in such a form that solution of the generally nonlinear programming problems associated with determination of dominance structures can be replaced by the solution of simple computationally amenable linear programs with bounded variables. The major simplification associated with eliciting parameter imprecision in a prespecified structural format, however, is in the natural language dialogue needed to establish a model of the decision situation.

The purpose of the graph theory algorithm is to allow the construction of a domination digraph or dominance structural model. This digraph is a pictorial representation of the ordinal preferences as determined from a dominance reachability matrix. This matrix is determined by the linear programming algorithms from the decision situation structural model and parameters elicited from the decisionmaker. These domination digraphs encourage either selection of a preferred alternative, or further iteration using the aggregated preference information for feedback learning.

An inverse aiding feature is currently being incorporated into the decision support system. This feature allows the decisionmaker to make wholistic skill-based prioritization among alternatives. These prioritizations may be across some or all identified alternatives, at the top level of the hierarchy of attributes or at some intermediate level. If we elicit numerical bounds on the attribute scores for those attributes which are subordinate to and included within the attribute at which alternatives are prioritized, then bounds on attribute weights, consistent with the wholistic prioritization, may be determined by using a linear programming approach. Alternately, if weights are specified, then it is possible to determine bounds on alternative scores on those attributes subordinate to the attribute at which prioritization was made through the use of linear programming algorithms.

As alternatives are identified and prioritized, updates of these bounds are made available. The results obtained from using the inverse aiding feature are, in may ways, comparable to those obtained from the regression analysis based social judgment theory [9]. This approach provides weight identification only, with a "confidence" measurement concerning the validity of weights; cardinal preferences are assumed. Results in the form of bounds on, or ranges of, weights are available with a very few alternative prioritizations in the inverse aiding approach. The prioritizations needed may involve a mixture of cardinal and ordinal preferences. For a large number of prioritizations, the inverse aiding approach may become cumbersome computationally compared to the regression-based approach, where additional information may be easily processed in a sequential fashion.

The combination of inverse and direct aiding to enhance decisionmaker specification of imprecise values, weights, and probabilities enhances the usefulness of ARIADNE since it allows for judgments and their explanation, using a combination of formal knowledge-based and skill-based modes. This enhanced usefulness will also occur through encouragement to the decisionmaker to become more aware of relevant alternative courses of action and to identify new alternatives on the basis of feedback learning of the impacts of alternatives upon issues and objectives in a behaviorally relevant way that, hopefully, encourages "double-loop learning" [10].

III. Structure of ARIADNE

A complete set of activities envisioned in using the single-stage, or single decision node, version of ARIADNE involves the following set of activities.

A. Formulation of the Decision Situation

1) Define the problem or issue that requires planning and decisionmaking by identification of its elements in terms of a) needs, and b) constraints or bounds on the issue.

2) Identify a value system with which to evaluate alternative courses of action, and identify objectives or attributes of the outcomes of possible decisions or alternative courses of action.

3) Identify possible alternative courses of action or option generation.

B. Analysis of the Decision Situation

1) Determine outcome scenarios.

2) Identify decision structural model elements, that is, those elements or factors from the conceptual formulation framework which appear pertinent for incorporation into a decision situation structural model.

3) Structure decision model elements:

a) structure decision tree,
b) structure information acquisition and processing tree—which may be part of the basic decision tree, and
c) structure attribute tree or objectives hierarchy.

4) Determine independence conditions among elements of the attribute tree and decision alternatives.

5) Identify potential for the use of deficient information processing heuristics, and provide appropriate debiasing procedures.

6) Determine impacts of, or outcomes that may result from, alternative courses of action.

7) Encode uncertainty elements in the form of event outcome probabilities, or bounds on these, to the extent possible.

8) Identify risk aversion coefficients, if needed, to the extent possible.

9) Identify preference or value functions, or bounds on these functions, to the extent possible.

10) Identify attribute weights, or bounds on these functions, to the extent possible.

11) Identify wholistic preferences among alternatives to the extent that this is possible.

12) Identify possible disjunctive and conjuctive aspects, or thresholds for attributes, of identified alternative courses of action.

C. Evaluation and Interpretation of the Outcome of Alternative Courses of Action

1) Identify a decision aiding protocol or plan for evaluation and interpretation of the decision situation.

2) Identify potential for use of deficient judgment heuristics.

3) Use conjunctive and/or disjunctive scanning to eliminate very deficient alternatives and retain alternatives meeting minimum acceptability criteria across attributes.

4) Determine the maximum amount of domination information possible.

a) Display domination digraph.

b) Identify alternative courses of action which could not be among the N most preferred alternatives. Normally, these are deleted from further consideration.

c) If the decisionmaker can select an alternative for implementation by wholistic judgment, or prioritize the remaining alternative set through heuristic elimination, then go to step 6) of evaluation and interpretation (Section III-C).

d) If a choice cannot be made, then assess further information about values of imprecisely known parameters by iterating through steps 6)–11) of analysis (Section III-B), then return to step 1) of the evaluation and interpretation (Section III-C). Many possibilities exist for obtaining greater alternative evaluation specificity such as

i) setting higher aspiration levels or aspects,
ii) moving up the attribute tree by determination of a subset of attribute tradeoff weights,
iii) "tightening" bounds on attribute trade-off weights,
iv) tightening bounds on event outcome probabilities, possibly through information processing updates,
v) tightening bounds on values or preference functions.

5) If the decisionmaker has provided (partial) wholistic preferences as part of the analysis effort, use these with the

inverse aiding feature of the aid to determine bounds on attribute weights implied by these preferences such as to provide learning feedback to decisionmaker.

6) Conduct sensitivity analysis. Provide the decisionmaker with an indication of how sensitive the optimal action alternative, or prioritization of alternatives, is with respect to changes in values and information about impacts.

7) Evaluate validity and veracity of the approach. Encourage judgment concerning whether the formulation, analysis, and interpretation are sound. If not, encourage appropriate modification to structure and parameters associated with the decision situation, including identification of additional attributes and alternative courses of action. Then, iterate back to an appropriate step and continue.

In our work to date, we assume that the details of issue formulation and analysis are accomplished external to the interactive aid itself. A variety of procedures exists for accomplishing these tasks [11]. Our research assumes that, an issue formulation structure exists and that the impacts of alternatives are known. These are provided through various elicitation activities. We *do not* envision that the software we develop for interactive *interpretation*, including evaluation and prioritization, will generally be suitable for use independent of a trained decision analyst. Whether software can be evolved to result in an appropriate "stand alone" aid is very dependent upon the environment and other factors that constitute the contingency task structure for a specific situation. In situations which are repetitive and environments which are stable, such as in health care or equipment fault diagnosis situations, it seems entirely possible to design useful "stand alone" aids. In most strategic, and in many tactical situations there will not be a stable underlying structure that will easily allow this. The activities involved in issue framing and the identification of a dominance structure appropriate for decisionmaking are often very situation dependent.

A number of considerations influence planning and decision support processes. The person using a decision support system should be aware of these considerations if best use of the aiding process is to be obtained. Generally, these considerations involve the operational environment and the familiarity of the decisionmaker with the environment and task at hand. It is the interaction of these factors that influence:

1) behavioral characteristics of the decisionmaker,
2) interaction between decisionmaker and analyst,
3) choice of computer-based support for decisionmaker analyst interaction

Among the behavioral characteristics of the decisionmaker that influence aiding consideration strongly are the facts that the decisionmaker

1) is often impatient with time consuming and stressful assessment procedures;
2) wants to see some preliminary results promptly if these are needed or wanted;
3) may lack interest in interacting directly with complex quantitative procedures for decision aiding that do not seem tailored to the specific contingency task structure of the issue at hand; and, as a consequence,
4) requires a decision aiding approach that adapts to the decisionmaking style appropriate for the decisionmaker in the given contingency task structure.

A number of considerations influence the most desirable interaction between the decisionmaker and the analyst. The interaction must be such that these result:

- a list of objectives and an objectives hierarchy,
- a list of alternatives, and
- a list of outcomes for each alternative.

The extent of the need for the use of these identified lists will vary greatly with the "expertise" of the decisionmaker. A major task of the analyst in the formulation and analysis portion of the aiding effort is to assist the decisionmaker in obtaining these lists in a behaviorally relevant and realistic manner. The analyst must also ensure, to the extent possible, that

1) the foregoing lists are reasonably complete;
2) the lowest level objectives are additively independent;
3) the alternatives are mutually exclusive; and
4) the outcomes that follow from each alternative are mutually exclusive and exhaustive.

The nature of the interactive process is such that iterative changes can be made in terms of addition or deletion of alternatives and attributes. Nevertheless, there are significant advantages in attempting to be reasonably complete at the start of the interpretation portion of the process.

The decisionmaker must provide the analyst, following behaviorally realistic elicitation procedures, with information regarding 1) alternative scores on lowest level attributes, 2) trade-off weights, 3) probabilities, and 4) relative risk aversion coefficients, or else appropriate ratios or bounds on these quantities which represent the precision that the decisionmaker believes appropriate or is capable of providing for the given decision situation.

Many computer-based support perspectives evolve from decisionmaker–analyst interaction considerations. A goal of all decision support system design efforts is to obtain "friendly" software, software that is friendly both to the decisionmaker and the analyst. In particular, the analyst must be able to interpret the decisionmaker's structural and parameter information for input to the computer. To do this may require 1) redefining the outcome space, such as redefinition of attributes to ensure satisfaction of independence considerations, and 2) describing parameter information in terms of inequalities (or more generally set membership).

The analyst must be able to interpret computer output in a fashion that facilitates the decisionmaker's understanding and decisionmaking abilities. The analyst must be able to assist the decisionmaker in responding to the following question that is central in our interactive knowledge-based support system: *Has sufficient preference and structural information been elicited from and provided to the decisionmaker for alternative selection, or is more information required for identification of a dominance structure that is relevant and appropriate for quality decision support*? Change receptivity must therefore be an inherent part of this user friendliness.

If the decisionmaker feels that an alternative can be selected for action implementation at any stage in the interactive aiding effort, the analyst must be able to encourage decisionmaker judgment concerning whether or not the issue formulation, analysis, and interpretation are sound. If the issue formulation, analysis, and/or interpretation are not perceived as sound by the decisionmaker, the analyst must be able to encourage appropriate structural and parameter value modification, typically by means of sensitivity analysis, in order to insure effective, explicable, and valid planning and decision support. If the decisionmaker cannot choose an alternative from among those considered, the analyst must be capable of eliciting further structural and/or parameter information to enhance appropriate selection of alternative courses of action.

One very important feature of a knowledge-based system for planning and decision support is encouragement to the decisionmaker for generating new options, outcomes, and attributes at essentially any point in the aiding effort and ability to evaluate these new options properly. The analyst must be able to cope with this additional information under the assumption that

1) the new information is consistent with previously obtained information, or

2) the new information is *not* consistent with previously obtained information due to a) structural inconsistencies or b) parameter inconsistencies.

Thus the capacity to resolve potential inconsistencies through interaction with the decisionmaker must exist within the planning and decision support process. The indirect or inverse decision aiding feature should be of particular value to this end. In a "policy capture" like fashion, this indirect feature will allow identification of bounds on attribute weights in terms of wholistic preferences among some, or all, alternatives. In the direct aiding feature, values, weights, and probabilities are identified and prioritization of alternatives result from this. Combined use of the direct aiding feature with indirect aiding should result in much learning feedback concerning relations among the various modes of judgment.

ARIADNE, as we have noted, does not contain software to assist in the formulation and analysis portion of the planning and decision support effort. It is in these two steps that alternative choices, attributes, and decision impacts or outcomes are elicited or identified. Our effort is much more concerned with the interpretation part of a decisionmaking effort, that is to say, how information is processed concerning formulation and analysis-based quantities such as probabilities, values, weights, ratios, and bounds upon these. We are concerned also with the way in which this information is aggregated, by any of a variety of formal knowledge-, rule-, or skill-based modes of cognition that result in judgment and choice. We recognize the difficulties in separating the tasks of formulation and analysis from those of interpretation. Difficulties exist at the systems management level since the way in which people cognize a problem, as part of the contingency task structure of a particular situation, determines the way in which they will go about resolving it. Thus the performance objectives, information processing style, and decision style that are most appropriate and likely to be used for a given task are very much dependent upon the task itself. When a particular concrete operational or skill-based strategy has yielded previous satisfactory results, many people will tend to use that strategy unquestioningly and uncritically in new situations perceived to be similar. This can result in very unsatisfactory judgments and choices in decision situations that have changed and that are not recognized as different from familiar past situations. This may result in premature cessation of search and evaluation of alternatives prior to identification of quality strategies, even for familiar situations. The efforts can be devastating in unfamiliar environments that are not so recognized [12].

The strategies which a decisionmaker will desire to use for interactive interpretation will be strongly dependent upon the way in which the task requirements are initially cognized. This will influence the objectives, attributes, and alternatives generated in the formulation step and the value scores or impacts associated with them in the analysis step. The input information to the interpretation step is just this information. Adequacy of the interactive interpretation step will clearly by dependent upon the "quality" of the information input to it.

Many recent studies [13] have indicated that people often construct selectively perceived simple deterministic representations of decision situations that make information processing easy and which do not reflect the complexities and uncertainties that are associated with the actual situation. A goal of a decision support system is to encourage wide scope perceptions and associated information processing. The process used to assess probabilities, utilities, and weights will doubtlessly affect the quantities that are elicited. It is possible, for example, that a poor elicitation procedure may, unknowingly or knowingly, create rather than measure values [14]. An advantage to formal support for planning and decisionmaking processes is that it is possible to conduct a search for inconsistent judgment and perhaps even detect flawed information processing heuristics if process tracing is used. When inconsistencies are discovered, it then becomes possible, at least in principle, to examine the judgment process to determine which judgments imply flawed information processing, and/or incoherent or labile values, and/or deficient decision rules. A major ultimate goal, outside the scope of our present study, is to suggest debiasing and other corrective procedures to enhance the quality of human information processing and decision rule selection.

This mixed scanning based planning and decision support system is based upon rational search for a dominance structure which will enable exposure of some of the processes upon which judgment and choice is based. In particular, it enables determination of the precise point in a dominance structure search process when a decisionmaker is able to select a single nondominated alternative. Thus we should be able, for example, to detect violation of the regularity and similarity hypotheses that often occur when a number of asymmetrically dominated alternatives exists [15]. More importantly, we should be able to correct for this without resorting to a complete elicitation of precise parameter information and prioritization of all alternatives. This activity is often stressful and time consuming, may require perspectives outside the experiential familiarity of the decisionmaker, and allows few results until conclusion of the aiding effort.

The overall process described here appears well suited to accommodating the fact that neither individuals nor groups possess static decision styles capable of being stereotyped and captured by an inflexible support process. It is specifically recognized that an interactive process is needed that is capable of adaptation to a variety of decision styles that are contingency task structure dependent. System design should reflect the realization that it is generally not possible to define a problem or issue fully until one knows potential solutions to the issue. A major cause of this is the fact that information to define the issue fully generally becomes available only as one evaluates potential solutions. Planning and decisionmaking will therefore necessarily be iterative.

IV. Behavioral Relevance

Our decision support system design paradigm is based upon a process model of decisionmaking in which a person perceives an issue which may require a change in the existing course of action. On the basis of a framing of the decision situation, one or more alternative courses of action, in addition to the present option which may be continued, are identified. A preliminary screening of the alternatives, using conjunctive and disjunctive scanning, may eliminate all but one alternative course of action. Unconflicted adherence to the present course of action or unconflicted change to a new option may well be the metastrategy for judgment and choice that is adopted if the decisionmaker perceives that the decision situation is a familiar one and that the stakes are not so high that a more thorough search and deliberation is needed [12].

Alternately, if the decision environment is an unfamiliar one or the stakes associated with judgment and choice are high, more vigilant forms of informative acquisition, analysis, and interpretation are called for. This desire for more vigilant information processing leads to a search for a dominance pattern among alternatives, the search for new alternatives that are not dominated by presently identified alternatives, and the elimination from further consideration of dominated alternatives. If no single nondominated alternative is found, adjustments to the dominance structure of alternatives are made through various forms of cognitive activity such as attribute aggregation, additional information acquisition and analysis, and identification of additional attributes and/or alternatives. This is continued until the structure of needs, objectives, attributes, and alternative action options and their impacts is such that identification of a single nondominated alternative results. This single alternative may well represent a combination of subalternatives. If the time and experience to accomplish these cognitive activities is insufficient, hypervigilance generally results. The decisionmaker is then in a situation where the present course of action is diagnosed as unfortunate, and there is not enough time and experience to allow identification and evaluation of an appropriate alternative course.

Given sufficient time and experience, vigilant information processing often results from the aforementioned tasks. Fig. 1 presents some salient features of this dominance process model for search, discovery, judgment, and choice.

The proper mode of judgment and choice depends upon the decisionmaker's situation diagnosis of the contingency task structure. Here, "proper" decision behavior is based upon the assumption that the environment, the task, the experiential familiarity with the task, and the environment that constitutes the contingency task structure are diagnosed correctly. If this is not the case, then the strategies leading to unconflicted change, adherence, or vigilant information processing may be significantly flawed. The role of the contingency task structure in situation diagnosis and in influencing, at a meta or systems management level the process of judgment and choice is seen to be a very important one. It leads to a four-element representation of situation diagnosis as shown in Table I which also presents a typology of the decisionmaker whose behavior may reflect in judgment activities in either of the four quadrants of this figure. In the upper right quadrant, where true mastery or grand mastery of a decision situation results, it is doubtful that any decision support system will be of much direct and personal use to the decisionmaker. Nevertheless it may have much indirect use in enabling acquisition of a knowledge base and as a useful pedagogical or learning system for others.

Many realistic paradigms have been made of the process of judgment and choice. We believe that the dominance process model described here is not inconsistent with the primary features and intensions of these *descriptive* models. Our purpose, however, is to develop a conceptual design for a *prescriptive* approach to judgment and choice that will aid in the search for better decisions. We recognize that a truly rational approach to prescriptive decisionmaking must be cognizant of the process of decisionmaking as it evolves in a descriptive fashion, that is to say, process rationality

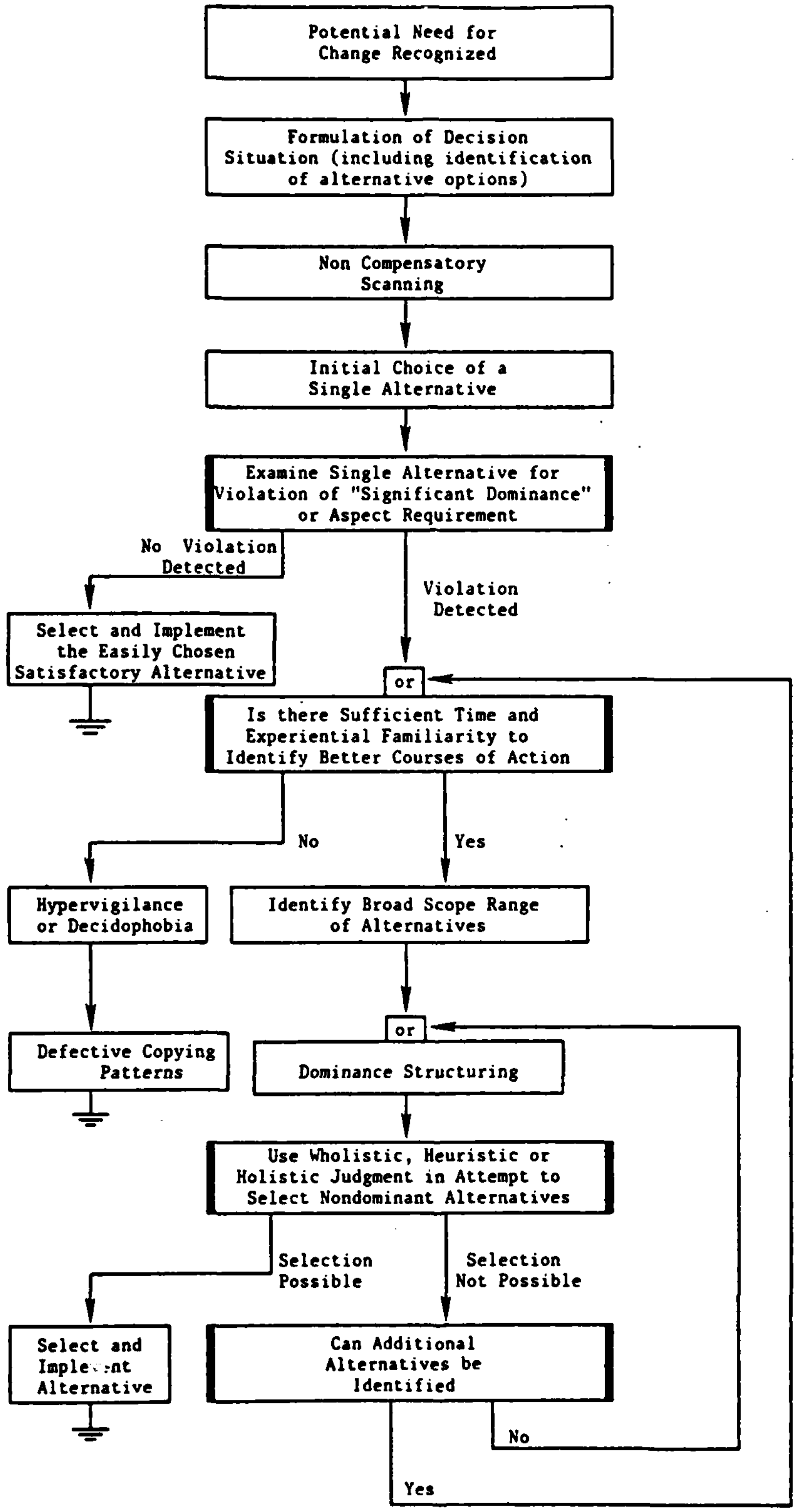


Fig. 1. Descriptive dominance structural model of decision process.

TABLE I
TAXONOMY OF REACTIONS TO DECISION SITUATION IN TERMS OF PERCEIVED VERSUS ACTUAL KNOWLEDGE

Actual Knowledge Level	Perceived Knowledge Level: NOVICE	Perceived Knowledge Level: MASTER
MASTER	Decisionmaker is unaware that considerable judgment ability exists	Wholistic intuitive judgment will likely lead to high quality decision through unconflicted change or unconflicted adherence
NOVICE	Decisionmaker is aware of need for decision support to enable holistic judgment	Decisionmaker may be a master of the art of self deception

or it will not be possible to evolve substantively rational support systems.

It is important that an appropriate decision support system be capable of assisting the decisionmaker through encouragement of full information acquisition, including that which may disconfirm strongly held beliefs, and in the analysis and interpretation of this information such as to avoid a variety of cognitive biases and poor information processing heuristics that may lead to flawed judgment and choice [2], [13].

A realistic decision support process is necessarily iterative. Several desiderata follow from this.

1) We should allow for top-down or bottom-up structuring of the attributes of outcomes, or impacts of decisions. The "true" or "hierarchy" of attributes should be structured to the depth believed appropriate by the decisionmaker.

2) Rather than force a decision situation structural model in the form of a tree, we should encourage the decisionmaker to identify a cognitive map of goals, objectives, needs, attributes, alternatives, and impacts that reflects the way in which he perceives diagnostic and causal inferences to occur. At some later time this cognitive map may be used to structure a multinode decision tree which represents substantive rationality, but not necessarily process rationality.

3) We should encourage identification of alternative courses of action, additional attributes of decision outcomes, and revisions to previously obtained elicitations, at any point in the decision support process as awareness of the decision situation and its structure grows through use of the support system.

4) We should not force a person to quantify parameters to the extent that this becomes overly stressful or behaviorally and physically irrelevant in view of the inherent uncertainty or imprecision associated with the knowledge of parameters characterizing the decision situation structural model or their assessment.

These have two primary implications with respect to our interpretation efforts. We allow for revision in the elicited structure of the decision situation and for the identification of new options as awareness of the decision situation grows. Also, we do not require the decisionmaker to quantify parameters beyond the level felt appropriate for th situation at hand. If the decisionmaker feels comforta e i exercising precision with respect to factual outcomes thi is perfectly acceptable and desirable, but parameter impre cision should be allowed if we are to have a realisti support process.

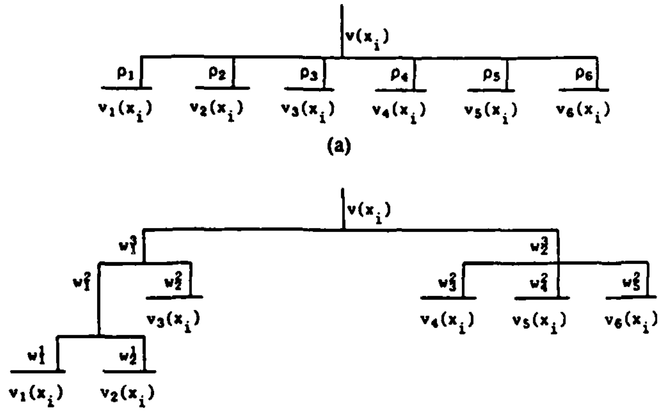
ARIADNE allows parameter imprecision in order t satisfy this quantification relevancy requirement, as d approaches based on fuzzy set theory [16]. We encou ag the decisionmaker to specify precise values or numerica ranges for facts and values. Thus we allow, for exampl expressions for alternative (a) scores on attributes (i the form $0.2 \leqslant v_i(a) \leqslant 0.5$, weights associated with tri bute (i) in the form $0.2 \leqslant w_i \leqslant 0.4$, and probabilities o event (i) resulting from alternative (a) in the form $0. \leqslant P_i(A) \leqslant 0.45$. We allow ordinal representations in th lin ear forms $v_i(a) \leqslant v_i(b) \leqslant v_i(c)$, $2w_i < w_j < w_k$, $P_j(\bar{a}) \leqslant P_i(a) \leqslant 3P_k(a)$, or in similar forms. Quantification of im precision in the form of numerical bounds on param er always leads to behaviorally consistent information set (BCIS). Sometimes totally ordinal information may nee further quantification in order to make the precision in rigidity of the mathematics correspond to the intensio o the decisionmaker in making a purely ordinal specification This is generally not needed to obtain solutions but r he to obtain parametric models that are faithful to the u ler standings of the decisionmaker. For example, that ordina alternative score inequalities $0 \leqslant v_i(a) \leqslant v_i(b) \leqslant v_i(c) \leqslant$ are satisfied by the relations $0 \leqslant v_i(a) \leqslant 1 - 2t$, $w \leqslant (b$ $\leqslant 1 - t$, $2W \leqslant v_i(c) \leqslant 1$ for small positive t and W whicl in the limit become zero. It will generally not be the cas that the decisionmaker would express this much impreci sion and would wish to see it more fully quantifie t reflect (subjective) beliefs. It is, therefore, important that simple and informative display of value scores, weights and probabilities be provided to the decisionmaker. hi will enhance interactive use of the support system and wil enable learning of the impact of these parameters, an associated imprecision, upon decisions.

V. ALGORITHMIC CONTENT

This section will discuss some of the algorithmic co en supporting the decision support system. To facilitate read ing, we will make each subsection of this description more or less independent of other subsections.

A. The Attribute Tree and Decisions Under Certainty

It is possible to use either a hierarchical tree structure o a single-level structure of attributes, each of which ar shown in Fig. 2. Fortunately, the relations between h weights associated with the tree structure and the single level structure are easily determined. They are given in Fig 3(c). Linear inequalities in terms of hierarchical weigh o weight ratios w_i^j become linear inequalities in term o single-level weights ρ_i. It is this fact that allows us to us the single-level representation in the ARIADNE softwar



(a)

(b)

ρ_j = product of weights in path from $v_j(x_i)$ to $v(x_i)$

$$w_k^j = \frac{\text{sum of } \rho_i \text{ weights over those } i \text{ which are subordinate to attribute } a_k^j}{\text{sum of } \rho_i \text{ weights over those } i \text{ which are subordinate to the attribute to which attribute } a_k^j \text{ is subordinate}}$$

Examples $\rho_2 = w_2^1\, w_1^2\, w_1^3$

$$w_2^2 = \frac{\rho_3}{\rho_1+\rho_2+\rho_3}$$

$$w_2^3 = \rho_4 + \rho_5 + \rho_6$$

(c)

Fig. 2. Comparisons between single-level and hierarchical attribute structures. (a) Single-level attribute structure. (b) Hierarchy of attributes. This structure may be equivalent to that of (a). (c) Relationship between weights.

and still to make assessments in terms of the hierarchical weights w_i^j at any level of the hierarchy.

For the case of decisionmaking under outcome certainty, we know that the ith outcome x_i follows from the ith alternative. We therefore have for the value of the ith alternative, $v_k(a_i)$ the expression

$$v(a_i) = \sum_{k=1}^{N} \rho_k v_k(a_i). \tag{1}$$

We say that alternative i has a higher value score than alternative j when

$$\Delta v_{ij\min} = \min_{\Lambda} \Delta v_{ij} = \min_{\Lambda} \sum_{k=1}^{N} \rho_k \left[v_k(a_i) - v_k(a_j) \right]$$
$$= \min_{\Lambda} \rho^T \left[v(a_i) - v(a_j) \right] \geq 0 \tag{2}$$

where Λ denotes the set of imprecise parameters over which the extremization is conducted. This set is restricted through the eliciting process such that $\Lambda \in \Lambda$. ρ and v are vectors with components ρ_k and v_k. For the realistic case where weights and attribute scores for each attribute are functionally independent, (2) becomes

$$\Delta v_{ij\min} = \min_{\rho} \rho^T V(i,j) = \sum_{k=1}^{N} \rho_k V_k(i,j) \geq 0 \tag{3}$$

where

$$V(i,j) = \min_{v(a_i),\, v(a_j)} \left[v(a_i) - v(a_j) \right] \tag{4}$$

is a vector whose components are

$$\min_{v_k(a_i) v_k(a_j)} \left[v_k(a_i) - v_k(a_j) \right], \qquad k = 1, 2, \cdots, N.$$

We emphasize again that the vector minimization to determine $V(i, j)$ is meaningful only when $v_k(a_i)$ is functionally independent of $v_m(a_i)$ and $v_m(a_j)$ for $k \neq m$. In many cases, the $v_k(a_i)$ are elicited so as to be functionally independent of the $v_k(a_j)$, and then we have

$$V(i,j) = \min_{v(a_i)} v(a_i) - \max_{v(a_i)} v(a_j) = \underline{V}(a_i) - \overline{V}(a_j) \tag{5}$$

where $\underline{V}$ and $\overline{V}$ denote the minimum and maximum values of the value score vector on the specified alternative. Determination of the solution to (3) requires solution of a linear program (LP) for each ij pair. If A alternatives exist, we will need to solve no more than $A(A-1)$ LP's to resolve (3). We may need to solve fewer LP's than this since if $\Delta v_{ij\min} > 0$ we can assume that $\Delta v_{ji\min} < 0$ without solving the associated LP and know that $a_i \succ a_j$. Solution of (4) for a specific i and j will involve a single LP. Thus we have $A(A-1)$ LP's to solve to determine $V(i, j)$ for all i and j. Generally, these linear programs are extraordinarily simple to solve and result in necessary and sufficient conditions for $a_i \succ a_j$.

As one simple example, let us consider the alternative score matrix on lowest level attributes as seen in Table II. We assume that the decisionmaker specified a single-level attribute tree and is able to estimate the weights $0.1 \leq \rho_1 \leq 0.2$, $0.2 \leq \rho_2 \leq 0.4$, and $0.3 \leq \rho_3 \leq 0.7$. Of course, these weights must sum to one, and so we have $\rho_1 + \rho_2 + \rho_3 = 1$. We have already specified utilities in the max-min form $[\overline{V}, \underline{V}]$; nothing more is needed here. To see whether $a \succ b$, we need to see whether (3) is satisfied. Thus we

TABLE II
VALUE SCORE ON ATTRIBUTES

Alternative	Attribute 1 max score	Attribute 1 min score	Attribute 2 max score	Attribute 2 min score	Attribute 3 max score	Attribute 3 min score
a	0	0	1	1	0	0
b	1	1	0	0	0.8	0.6
c	0.5	0.3	0.4	0.2	1	1
d	0.4	0.2	0.18	0.1	0.7	0.5

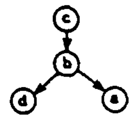


Fig. 3. Preference structure for simple example.

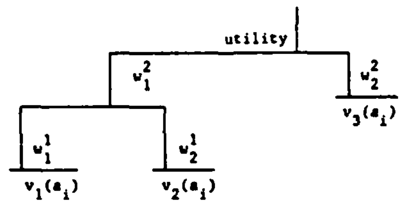


Fig. 4. Hierarchical tree of attributes and associated weights.

examine $\Delta V_{ab\,\min} = \min(\underline{V}_a - \overline{V}_b)^T\rho \geqslant 0$ or

$$\Delta V_{ab\,\min} = \min_{\rho}\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}^T - \begin{bmatrix}1\\0\\0.8\end{bmatrix}^T\right\}\begin{bmatrix}\rho_1\\\rho_2\\\rho_3\end{bmatrix} > 0.$$

This is an LP with bounded variables, a particularly simple form of linear program. We assign maximum weights to the most negative $\underline{V}_a - \overline{V}_b$ components until we are all out of weight. The weights are found by setting all weights at their minimum value and then allocating additional weight where it will do the most good. So we use $\rho_1 = 0.2$, $\rho_3 = 0.6$, $\rho_2 = 0.2$ and get $\Delta V_{ab\,\min} = -1(0.2) + 1(0.2) - 0.8(0.6) = -0.48$, which is not greater than zero. Thus it is not possible to have $a \succ b$. To see if $b \succ a$ we examine

$$\Delta V_{ba\,\min} = \min_{\rho}\left\{\begin{bmatrix}1\\0\\0.6\end{bmatrix}^T - \begin{bmatrix}0\\1\\0\end{bmatrix}^T\right\}\begin{bmatrix}\rho_1\\\rho_2\\\rho_3\end{bmatrix}.$$

We use $\rho_2 = 0.4$, $\rho_3 = 0.5$, $\rho_1 = 0.1$ and get $\Delta V_{ba\,\min} = 0$. So we conclude that $b \succ a$. We determine that $c \succ b$ and $b \succ d$, using just this procedure, such that we have the preference structure of Fig. 3. Thus c is the preferred alternative here.

It may well be, however, that the decisionmaker (DM) visualizes the attributes in a hierarchical form as shown in Fig. 4. The same alternative score matrix used earlier is still appropriate here. If the DM feels comfortable in evaluating weights associated with attributes 1 and 2 (ρ_1 and ρ_2) but not with 3 (ρ_3), then a multilevel hierarchy of attributes may be assumed. We could attempt to assist the DM by aggregating up the attribute tree. Alternately, we could determine whether or not the relationship between attributes 1 and 2 is sufficient to establish a single nondominated alternative by converting to the single-level weight form.

TABLE III

Alternative	Attribute w_1^2 max score	Attribute w_1^2 min score	Attribute 3 (or w_2^2) max score	Attribute 3 (or w_2^2) min score
a	0.8	0.6	0	0
b	0.4	0.2	0.8	0.6
c	0.44	0.24	1	1
d	0.268	0.14	0.7	0.5

We will illustrate calculations using the first approach. Suppose that the DM says $w_1^1 \leqslant w_2^1$. Then the analyst might use this ordinal expression or might convert to a cardinal representation and say that at level 1 the weights are such that $0 \leqslant w_1^1 \leqslant 0.5$, $0.5 < w_2^1 < 1.0$. Following a request to be more explicit, perhaps the decisionmaker indicates that $0.2 \leqslant w_1^1 \leqslant 0.4$ and $0.6 \leqslant w_2^2 \leqslant 0.8$. We now aggregate attributes 1 and 2. Based on this information we can calculate a maximum and minimum score for the utilities of the alternatives on the second level attribute w_1^2. We obtain the aggregated value score matrix shown in Table III. No domination pattern exists at all, so we must go further.

As often occurs in problems of this sort, the level 2 alternative scores are not in proper 0–1 range. If the DM feels more comfortable in seeing these scaled over a 0–1 range, this can easily be done. Otherwise, the DM is asked to consider the difference in scores from max to min on attribute w_1^2 [0.8 on alternative a and 0.14 on alternative d] and express the importance weight of the difference on attribute 3 of the difference between the maximum and minimum scores on alternatives c and a. Suppose that inequalities of the form $0.2 \leqslant w_1^2 \leqslant 0.35$ and $0.65 \leqslant w_2^2 \leqslant 0.8$ finally result. We can then determine a table of maximum and minimum alternative scores:

Alternative	Maximum Score	Minimum Score
a	0.28	0.12
b	0.72	0.46
c	0.888	0.734
d	0.6136	0.374

Thus we have the preference digraph of Fig. 5 which is slightly different from that obtained earlier. The conclusion is the same, however. Alternative c is the best alternative. This particular approach used to aggregate up the attribute tree yields only sufficient conditions for one alternative dominating another alternative. It has the advantage, though, of providing a display of maximum and minimum

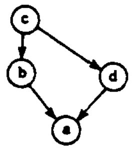


Fig. 5. Domination digraph.

TABLE IV

	Attribute w_1^2		Attribute 3 (or w_2^2)	
Alternative	max score	min score	max score	min score
a	0.8	0.6	0	0
b	0.4	0.2	0.8	0.6
c	0.44	0.24	1	1
d	0.268	0.14	0.7	0.5
ideal best	1	1	1	1
ideal worst	0	0	0	0

scores across alternatives after each aggregation up the tree. This is not obtained through use of the necessary and sufficient conditions of (3) and (4).

One way to avoid the problem with the best and worst alternative scores on each attribute not being uniquely anchored on one and zero after aggregating up the attribute tree may be to define an ideal best and an ideal worst alternative. The ideal best alternative will have a score of one on each lowest level attribute, and the ideal worst attribute will have a score of zero on each lowest level attribute. The DM should still specify the weight bounds $0.2 \leqslant w_1^1 \leqslant 0.4$ and $0.6 \leqslant w_2^1 \leqslant 0.8$ obtained earlier. Now the aggregation up the attribute tree preserves the anchor over zero to one on alternative scores for the ideal alternatives, and we have Table IV. Elicitation of swing weights might now be more comfortably accomplished than in the case where no pair of alternatives is uniquely anchored at zero and one on one or more aggregated attributes.

The question concerning whether the single-level attribute tree or the hierarchical tree is more appropriate in a given situation is difficult to answer. Previous studies on this point have not produced definitive guidelines. Our experience indicates that if the decisionmaker is comfortable with the single-level tree and is willing to express information concerning all attribute weights, then this structure is certainly more convenient to use and very likely is more appropriate as well.

As we have indicated in Fig. 2, we can easily convert from one representation to the other. The only essential difference between the two approaches is that it is "natural" when aggregating up the attribute tree to indicate maximum and minimum scores on each alternative. Use of these to determine preferences results in only sufficient conditions for preference determination as we have

$$\min_{v(a_i), v(a_j), w} \left[v(a_i) - v(a_j) \right]^T w \geqslant \min_{v(a_i), w} v^T(a_i) w - \max_{v(a_j), w} v^T(a_j) w$$

and we use the expressions on the right side of this inequality to determine the minimum and maximum scores on each alternative for each aggregated attribute.

Using necessary and sufficient conditions for preference determination is highly desirable. The way around the potential dilemma noted earlier is to use the best ideal and worst ideal alternative concept as prompts to the decisionmaker. These always propagate through the tree as the best and worst alternatives and will always be anchored at attribute scores of one and zero. We use these as anchors for the weight elicitation effort. All computations are made using (3) and (4) such that we always obtain necessary and sufficient conditions for preference condition determination.

B. Decisions Under Risk

For the decision under risk situation, we calculate the expected utility of alternative a from

$$EU(a) = \sum_{i=1}^{M} P_i(a) U_i(a) \tag{6}$$

where M outcome states can result from alternative a. State x_i occurs with probability $P_i(a)$, and the utility of this state is $U_i(a)$. This utility function will generally be a multiattribute utility function. When additive independence conditions are satisfied we have

$$U_i(a) = \sum_{j=1}^{N} \rho_j U_{ji}(a) \tag{7}$$

where $U_{ji}(a)$ is the utility of the jth attribute of outcome state i associated with alternative a and ρ_j is the trade-off weight associated with the jth attribute. Generally, the decision situation should be structured such that the weights are alternative and outcome state independent. If this is not the case, there is very likely a modeling deficiency in the framing of the decision situation structural model.

Combination of (6) and (7) results in

$$EU(a) = \sum_{i=1}^{M} \sum_{j=1}^{N} P_i(a) U_{ji}(a) \rho_j = P^T(a) U(a) \rho \tag{8}$$

where $\Gamma^T(a)$ and p are vectors of dimension M and N, and $U(a)$ is an $M \times N$ vonNeuman Morgenstern cardinal utility function expressed as a matrix. Alternative a is guaranteed to be preferred to alternative b if

$$\min_{\Lambda} \left[P^T(a) U(a) - P^T(b) U(b) \right] \rho \geqslant 0 \tag{9}$$

where Λ represents the set of all possible values that the parameters $p(a)$, $p(b)$, $U(a)$, $U(b)$, and ρ can assume.

The simplest case occurs when probabilities and utilities are known precisely and only weights are imprecise. We obtain $A(A-1)$ linear programs to solve for all possible alternative preferences is the weight set inclusion is described by linear inequalities. We obtain necessary and sufficient conditions for preference inequalities. In a similar way, if only the probabilities or only the across attributes are imprecise, we may solve a set of $A(A-1)$ linear programs to obtain necessary and sufficient conditions for alternative pair preferences if the precision is expressed as a set of linear inequalities.

If probabilities are known precisely, then we may rewrite (9) as

$$\min_{w} \gamma^T(a,b)\rho > 0 \tag{10}$$

where

$$\gamma^T(a,b) = \min_{U(a),U(b)} \left[P^T(a)U(a) - P^T(b)U(b)\right] \tag{11}$$

and where the utilities of alternative a and b on the ith attribute are functionally independent of those on the jth attribute for $i \neq j$. Thus each column in (11) can be optimized independently of one another. There are $A(A-1)N$ linear programs to solve to specify (11) and $A(A-1)$ linear programs to determine the preference inequalities of (10) if parameter imprecision is expressed by linear inequalities. We then can obtain necessary and sufficient conditions for preferences.

In all other cases,

a) utilities specified precisely, probabilities and weights imprecisely;
b) weights specified precisely, probabilities and utilities imprecisely;
c) weights, probabilities, and utilities specified imprecisely,

obtaining necessary and sufficient conditions for optimality of linear programming solutions will generally not be possible unless the imprecision can be expressed by means of simple numerical inequalities, i.e., $0.2 \leqslant P_i(a) \leqslant 0.35$.

In case b), we can obtain some desirable simplifications. Probabilities and weights are constrained by the sum to one property but utilities are not. When we have simple numerical inequalities on utilities, we can rewrite (9) as

$$\min_{P(a),P(b),w} \left[P^T(a)U(a) - P^T(b)\bar{U}(b)\right]\rho \geqslant 0 \tag{12}$$

where

$$U(a) = \min U(a), \bar{U}(b) = \max U(b). \tag{13}$$

This will enable us to obtain more desirable solution characteristics. In case b), where probabilities are the only other imprecise parameters, we can solve a simple set of linear programming problems to obtain necessary and sufficient conditions for alternative preferences.

In cases a) and c), where both probabilities and weights are imprecise, it seems that no realistic way exists in which to obtain solution for the preference inequalities by solving sets of linear programming problems. This is a considerable complication because of the solution complexity associated with nonlinear (quadratic) programming problems and the fact that we usually will not obtain both necessary and sufficient conditions for preference inequalities to hold. We can place a lower bound on the preference inequalities for (9) in these cases. If this lower bound is greater than zero, then we have sufficient but not necessary conditions for a given alternative preference. Even though we may determine the existence of alternative preferences through solution of linear programs, the fact that we establish only sufficient conditions gives cause for concern as other sufficient conditions may well exist which yield considerably stronger preference relations.

From the foregoing discussion we see that it is a relatively straightforward matter to incorporate imprecision, in the form of linear inequalities, into any combination of utility scores for lowest level attributes, probabilities of event outcomes, and attribute weights as long as probabilities and weights are not simultaneously imprecise. If this occurs, we must solve quadratic programming problems and are no longer able to get necessary and sufficient conditions for preferences.

Appendix

The simplest forms of ARIADNE were designed for use in situations in which alternatives scores on lowest level attributes and probabilities, if appropriate, are precisely known. The intent is to allow the decisionmaker to specify precise attribute weights in a bottom-up fashion so as to be able to aggregate up the attribute tree. Generally, this will result in greater strength to the partial preference ordering among alternatives. Ultimately, a scalar additive multiattribute utility (MAUT) function results. Although the process of obtaining this scalar MAUT function will generally be quite different from that used in conventional MAUT, the substantive results should be the same. A rather complete discussion of this simplest decision-under-certainty version of ARIADNE is contained in [17].

Our initial efforts involving single-stage decision aiding under uncertainty were based on precise specification of not necessarily all alternative scores on lowest level attributes and the use of stochastic dominance concepts. Aggregation up the attribute tree by means of elicitation of partial preference information concerning weights was used to increase the strength of preference specificity. Two forms of stochastic dominance have been considered as described in [18], [19].

The stochastic dominance concepts, especially second-order stochastic dominance, are computationally very time consuming for more than just a few attributes. We have discovered and investigated a strong second-order stochastic dominance bound that greatly reduces computational complexity. However, our research has shown that specification of bounds on parameters, such that linear or quadratic programming techniques may be used to identify a dominance structure, appears behaviorally much more realistic as well as computationally much simpler than stochastic dominance based approaches. Here, expected value stochastic dominance is used in the uncertain case. A rather general description of the analytical constructs supporting this bounded inequality version of ARIADNE is contained in [20].

Development of necessary and sufficient conditions to alternative preferences with parameter information constraints expressed as linear inequalities is contained in [21]. Further extensions of these analytical constructs, including an inverse decision aiding approach to enable learning of judgmental weights in terms of skill-based wholistic preferences, are contained in [22]. The use of structural parameter imprecision concepts in the bottom-up and top-down development of attribute trees is described in [23], [24].

An overview of the concept is presented in [25]. Organizational, behavioral, and methodological concerns which have influenced system design are contained in [26]. Initial analytical and algorithmic developments which serve as the basis for extensions of ARIADNE to the sequential decisionmaking case are contained in [27]–[30]. Finally, an evaluation of the decision support system is discussed in [31].

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PARAMETER IMPRECISION IN FINITE STATE, FINITE ACTION DYNAMIC PROGRAMS*

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ABSTRACT

We examine a finite state, finite action dynamic program having a specially structured one transition value function that is dependent on an imprecisely known parameter. For the finite horizon case, we also assume that the terminal value function is affine in the imprecise parameter. The special structural characteristics of the one transition value function and the terminal value function have been assumed in order to model parameter imprecision associated with the problem's reward or preference structure. We assume that the parameter of interest has no dynamics, no new information about its value is received once the decision process begins, and its imprecision is described by set inclusion. We seek the set of all parameter independent strategies that are optimal for some value of the imprecisely known parameter. We present a successive approximations procedure for solving the finite horizon case and a policy iteration procedure for determining the solution of the discounted infinite horizon case. These algorithms are then applied to a decision analysis problem with imprecise utility function and to a Markov decision process with imprecise reward structure. We also present conditions which guarantee the existence of a parameter independent strategy that maximizes, with respect to all other parameter invariant strategies, the minimum value of its expected reward function, where the minimum is taken over all possible parameter values.

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I INTRODUCTION

Standard models of sequential decisionmaking assume that the parameters of the model are known precisely and do not vary over the problem horizon. These parameters are, for example, terminal node utility values and chance node probabilities for decision analysis problems (Keeney and Raiffa, 1976) and single stage rewards, terminal stage rewards, the discount factor, and transition probabilities for Markov decision processes (Bertsekas, 1976). Such parameters, however,

1. may be only imprecisely known,
2. may change value, perhaps in an uncertain manner, over the problem's planning horizon,
3. may be subject to an estimation process that is based on parameter value information which arrives sequentially.

A standard procedure for considering a parameter subject to dynamic change has been state augmentation (Bertsekas, 1976); probabilistic and nonprobabilistic state dynamics are described in (Bertsekas, 1976) and in(Bertsekas and Rhodes, 1973; Figueras, 1972; White, 1983), respectively. Both probabilistic (Van Hee, 1978; Sagalovsky, 1982; Cyert and DeGroot, 1975) and nonprobabilistic (Sworder, 1966; Bertsekas and Rhodes, 1973; Figueras, 1972) approaches to problem formulation and analysis have been taken that involve (nondynamic) parameter imprecision and sequential estimation. Parametric programming (Viswanathan et al., 1977), successive approximations, and policy iteration (White and Kim, 1980) have been applied to analyze the vector criterion Markov decision process, where the imprecisely known parameter is a vector of importance weights that has no dynamics, is not subject to sequential estimation, and is known only to have nonnegative components and satisfy a sum-to-one property.

In most sequential decisionmaking or control models considered to date, attention has been directed to imprecise, estimated, and/or dynamic parameters that are associated with state dynamics and/or the observation mechanism. Notable exceptions are due to Cyert and DeGroot (1975) and White (1983) for parameters in the utility function and to Smallwood (1966) and Veinott (1969) for the discount factor. We remark that Kreps (1977) and Meyer (1977) have assumed that the utility function is allowed to change in an uncertain manner over time; however, these changes depend on a probabilistically described state. Both Kreps and Meyer assume that the functional relationship between utility and state is precisely known over the entire planning horizon.

In this paper, we examine a finite state, finite action dynamic program having a specially structured one transition value function that is dependent on an imprecisely known parameter. Parameter imprecision is described by set inclusion. For finite horizon problems, we also assume that the terminal value function is affine (linear plus a constant) in the imprecise parameter. The specific functional form of the one transition value function and the terminal value function have been selected in order to model parameter imprecision associated with the reward or preference structure of the problem for the case where no new information regarding the parameter's value becomes available over the planning horizon and where the parameter has no dynamics. Our motivation for examining such a problem formulation is due to the following perceptions:

1. rewards and preferences can be difficult to quantify precisely, particularly when multiple, conflicting, and noncommensurate objectives are being considered and/or preference assessment is involved,
2. set inclusion is a particularly simple description of parameter imprecision that is relatively easy to justify behaviorally in the context of preference assessment.

We remark that set inclusion is an often used model of parameter imprecision in the single stage decisionmaking literature; see for example, Sarin (1977a, 1977b), Fishburn (1965), White et al. (1983), and their references. The results presented here represent to some extent an extension of this work to the sequential case and a generalization of the above referenced results for the vector criterion Markov decision process.

The paper is organized as follows. Results for the finite horizon case are presented in Section 2. Under suitable assumptions on the functional form of the one transition value function, we show that the optimal reward functions are all affine and convex in the unknown parameter. This result is used to develop a simple computational algorithm that is reminiscent of a successive approximation algorithm due to Smallwood and Sondik (1973) for the finite horizon, partially observed Markov decision process. The intent of this algorithm is to identify, on the basis of the given description of parameter imprecision, the set of all strategies that may be optimal, or equivalently, to eliminate all clearly suboptimal strategies. This algorithm is applied to a decision analysis problem and to a Markov decision process problem in Section 3. The infinite horizon, discounted case is examined in Section 4. A policy iteration algorithm, similar to a policy iteration algorithm presented in Sondik (1978), is developed and applied to a Markov decision process problem. Again, the intent of this algorithm is to determine the set of all strategies that are optimal for some feasible parameter value. In Section 5, results are presented for determining a best maximin, parameter invariant strategy for the infinite horizon, discounted case. Conclusions are presented in the final section.

II THE FINITE HORIZON CASE

We now present a finite horizon, finite state and action dynamic program having a specially structured one transition value function and terminal value function. Our approach to problem definition is similar to that of Denardo (1967). Consider the following definitions:

$K < \infty$ represents the number of <u>stages</u> or decision epochs of the problem,

S_k is the finite <u>state space</u> at stage $k = 0,1,\ldots,K$,

$A_k(i)$ is the finite <u>action space</u> at stage $k = 0,1,\ldots,K-1$ when the state at stage k is $i \epsilon S_k$,

$P \subseteq \mathbb{R}^N$ is the set of all possible <u>parameter values</u>, assumed to be convex,

V_k is the set of all bounded, real valued functions on $S_k \times P$, $k=0,1,\ldots,K$,

$h_k: S_k \times P \times A_k \times V_{k+1} \to \mathbb{R}$ is the bounded one <u>transition value function</u> at stage $k=0,1,\ldots,K-1$,

$f_K \epsilon V_K$ is the <u>terminal value function</u>,

Δ_k is the set of all (parameter dependent) <u>policies</u> δ: $S_k \times P \to A_k$, such that $\delta(i,\rho) \epsilon A_k(i)$, $k=0,1,\ldots,K-1$,

$H_{k\delta}: V_{k+1} \to V_k$ is defined as $[H_{k\delta}v](i,\rho) = h_k[i,\rho,\delta(i,\rho),v]$ for each $\delta \epsilon \Delta_k$, $k=0,1,\ldots,K-1$,

$H_k: V_{k+1} \to V_k$ is defined as $H_k v = \sup_\delta H_{k\delta} v$, $k=0,1,\ldots,K-1$,

$f_k: V_k \to R$ is the <u>optimal reward function</u> from stage k to the end of the planning horizon,

a sequence of policies $\pi = \{\delta_0,\ldots,\delta_{K-1}\}$, $\delta_k \epsilon \Delta_k$, $k=0,1,\ldots,K-1$, is called a <u>strategy</u>.

We remark that the dynamic programming equation for this problem is

$$f_k(i,\rho) = \max_{a \epsilon A_k(i)} \{h_k(i,\rho,a,f_{k+1})\} \qquad k=0,1,\ldots,K-1,$$

which we express more succinctly as $f_k = H_k f_{k+1}$, $k=0,1,\ldots,K-1$. If the strategy $\pi^* = \{\delta_0^*,\ldots,\delta_{K-1}^*\}$ is such that $f_k = H_{k\delta_k^*} f_{k+1}$, for all $k=0,1,\ldots,K-1$, then we call π^* an optimal strategy. Reference to the examples in Section 3 may help to explicate our notation.

Our objective is to determine the set of all parameter independent strategies that are optimal for some $\rho\varepsilon P$. This collection of strategies represents the set of all strategies that may be optimal, given that parameter imprecision is described by the set P. We determine this collection of strategies from the optimal strategy $\pi^* = \{\delta_0^*,\ldots,\delta_{K-1}^*\}$ in the following manner: $\{\lambda_k, k=0,1,\ldots,K-1\}$, $\lambda_k : S_k \to A_k$, $\lambda_k(i)\varepsilon A_k(i)$, may not be excluded as a candidate for an optimal parameter independent strategy if there exists a $\rho\varepsilon P$ such that $\lambda_k(i) = \delta_k^*(i,\rho)$ for all $i\varepsilon S_k$, $k=0,1,\ldots,K-1$. We observe that it is sufficient to determine an optimal strategy and the optimal reward functions in order to achieve the above objective. The determination of an *optimal strategy and the optimal reward functions represents* the primary emphasis of this section and of Section 4.

Assume throughout this section that the following assumptions hold:

$h_k^1 : S \times A \to \mathbb{R}$, $h_k^2 : S \times A \to \mathbb{R}^N$, and $h_k^3 : S \times A \to S$

A1. For each $k=0,1,\ldots,K-1$, there are functions $\wedge$ such that

$$h_k(i,\rho,a,v) = h_k^1(i,a) + \sum_{n=1}^{N} h_k^2(i,a)_n \rho_n + \sum_{j\varepsilon S_{k+1}} h_k^3(i,a)_j v(j,\rho).$$

A2. For each $i\varepsilon S_k$, $a\varepsilon A_k(i)$, and $j\varepsilon S_{k+1}$, $h_k^3(i,a)_j \geq 0$, $k=0,1,\ldots,K-1$.

A3. There are functions $\bar{h}^1$ and $\bar{h}^2$ such that

$$f_K(i,\rho) = \bar{h}^1(i) + \sum_{n=1}^{N} \bar{h}^2(i)_n \rho_n.$$

We remark that these assumptions appear to be quite reasonable for a broad class of decision analysis problems and Markov decision processes having imprecisely known parameters associated with their preference or reward structure, a claim supported by the examples in Section 3. We now present two results that will lead to the development of a computational algorithm for this class of dynamic programs.

LEMMA 1: Let $k\varepsilon\{0,1,\ldots,K-1\}$. Assume $v\varepsilon V_{k+1}$ is piecewise affine and convex in ρ on P for each $i\varepsilon S_{k+1}$. Then, $H_k v$ is piecewise affine and convex in ρ on P for each $i\varepsilon S_k$.

Proof: Note that it is sufficient to show that $h_k(i,\rho,a,v)$ is piecewise affine and convex in ρ for each pair $(i,a)\varepsilon S_k\times A_k$. Since $v\varepsilon V_{k+1}$ is piecewise affine and convex in ρ for each $i\varepsilon S_{k+1}$, then for each $i\varepsilon S_{k+1}$, there is a set $A(i)$ such that

$$v(i,\rho) = \max\{\alpha+\gamma\rho:(\alpha,\gamma)\varepsilon A(i)\}.$$

It then follows that

$$h_k(i,\rho,a,v) = h_k^1(i,a)+h_k^2(i,a)\rho+\textstyle\sum_j h_k^3(i,a)_j \max\{\alpha+\gamma\rho:(\alpha,\gamma)\varepsilon A(j)\}. \quad (1)$$

Since the sum of nonnegatively weighted piecewise affine and convex functions is piecewise affine and convex, $h_k(i,\rho,a,v)$ is piecewise affine and convex in ρ for each pair (i,a). $\square$

PROPOSITION 1: For each $k=0,1,\ldots,K$, f_k is piecewise affine and convex in ρ on P for each $i\varepsilon S_k$.

Proof: The result is true by assumption for k=K. Backward induction and the result in Lemma 1 imply that the result is true for k=0,1,...,K-1. $\square$

The above results suggest the following computational procedure for determining the f_k:

0. Define $A_K(i)=\{(\bar{h}^1(i), \bar{h}^2(i))\}$; set k=K-1.
1. Define $A_k(i,a)$ as the set of all pairs (α',γ'), where

$$\alpha'=h_k^1(i,a) + \sum_j h_k^3(i,a)_j\alpha(j) \tag{2a}$$

$$\gamma'=h_k^2(i,a) + \sum_j h_k^3(i,a)_j\gamma(j) \tag{2b}$$

   and where $\underset{\wedge}{(\alpha(j),\gamma(j))}\varepsilon A_{k+1}(j)$. Eliminate all pairs (α',γ') in $A_k(i,a)$ that do not achieve the maximum in $\max\{\alpha'+\gamma'\rho:(\alpha',\gamma')\varepsilon A_k(i,a)\}$ for some value of $\rho\varepsilon P$. (See Smallwood and Sondik (1973) for a related elimination procedure.)
2. Define $A_k(i)=U_a A_k(i,a)$. Eliminate all pairs (α',γ') in $A_k(i)$ that do not achieve the maximum in $\max\{\alpha'+\gamma'\rho:(\alpha',\gamma')\varepsilon A_k(i)\}$ for some value of $\rho\varepsilon P$.
3. If k=0, stop; if not, set k=k-1, and go to Step 1.

We remark that

$$f_k(i,\rho)=\max\{\alpha+\gamma\rho:(\alpha,\gamma)\varepsilon A_k(i)\}$$

and that optimal strategy $\delta_k^*(i,\rho)=a$ if $f_k(i,\rho)=\alpha^*+\gamma^*\rho$ and if $(\alpha^*,\gamma^*)\varepsilon A_k(i,a)$. Thus, the above algorithm can be used to provide both $\{f_k, k=0,1,\ldots,K\}$ and $\{\delta_k^*, k=0,1,\ldots,K-$ Note also that (2) is easily derived from (1) by:

a. replacing $A(i)$ with $A_{k+1}(i)$ in (1),

b. replacing $\max\{\alpha+\gamma\rho:(\alpha,\gamma)\varepsilon A_{k+1}(j)\}$ by $\alpha(j)+\gamma(j)\rho$ for $(\alpha(j),\gamma(j))\varepsilon A_{k+1}(j)$,

c. collecting terms, and

d. considering every combination of pairs in $A_{k+1}(j)$, $j\varepsilon S$.

III EXAMPLES: FINITE HORIZON CASE

We now consider two areas of application of the results presented in Section 2, decision analysis and Markov decision processes, and illuminate these results with two associated numerical examples.

Decision analysis. Assume that the given decision tree has a maximum of K stages. Add the appropriate number of decision nodes with single actions and chance nodes with single outcomes to branches having less than K stages in order to insure all branches have exactly K stages.

Let z_{k+1} be the outcome received after having chosen action a_k, $k=0,1,\dots,K-1$. Define $s_k=\{a_0,z_1,a_1,z_2,\dots,a_{k-1},z_k\}$, and assume all probabilities of the form $p(z_{k+1}|s_k,a_k)$, and hence $p(s_{k+1}|s_k,a_k)$, are known. Let $f_K(s_K)=u(s_K)$, where $u:S_K\to R$ is a utility function. The function u ascribes a utility value to all possible terminal nodes in the decision tree, each branch of which is uniquely associated with an element in S_K.

We consider two cases, the first of which assumes that all there is known about u is that the collection $\{u(s_K)\}$ is a member of a set $U\subseteq R^N$, where $N={}^{\#}S_K$. For this case, $h_k^1=h_k^2=\bar{h}^1=0, h_k^3(i,a)_j=P(s_{k+1}=j|s_k=i,a_k=a)$, and $\bar{h}^2(i)_n=1(=0)$ if $i=n$(if $i\neq n$).

With respect to the second case, we assume that $u(s_K)=\sum_{m=1}^M w_m u_m(s_K)$, where $u_m:S_K\to R$ is the utility function associated with the m^{th} attribute, M is the number of attributes under consideration, the attributes are assumed additive independent, $w_m\geq 0$ is the importance weight of attribute m, and $\sum_m w_m=1$. Assume each u_m is known exactly and all that is known about the vector of importance weights $w=\{w_m\}$ is that it is a member of the set $W\subseteq R^M$. For this case, $h_k^1=h_k^2=\bar{h}^1=0$, $h_k^3(i,a)_j=P(s_{k+1}=j|s_k=i,a_k=a)$, $N=M$, $\bar{h}^2(i)_m=u_m(i)$, and $\rho_m=w_m$.

EXAMPLE 1. Consider the decision tree in Figure 1, which is based on the ore buying example presented in Brown et al. (1974), Chapter 2. Assume all that is known about the single attribute terminal utility values u_i, $i=1,\ldots,9$, is:

$$u_1 \varepsilon [0.7, 1.0]$$
$$u_2=0$$
$$u_3=0.43$$
$$u_4=u_1$$
$$u_5=0$$
$$u_6=0.46$$
$$u_7 \varepsilon [0.7, 1.0],\ u_7 \geq u_1$$
$$u_8=0.01$$
$$u_9=0.43\,.$$

Let $P=\{(\rho_1,\rho_2):\rho_1,\rho_2 \varepsilon [0.7, 1.0]$ and $\rho_2 \geq \rho_1\}$, $S_0=\{01\}$, $S_1=\{11,\ldots,15\}$, and $S_2=\{21,\ldots,29\}$, where these states are defined in Figure 1. Associate ρ_1 with u_1 and ρ_2 with u_7. The functions f_k and the optimal strategy π^*, as a function of state and parameter value, are given in Table 1. We see there are two strategies that are possibly preferred, given the available utility value information:

$$\lambda^1(01)=a_0^1,\ \lambda^1(11)=a_1^1,\ \lambda^1(12)=a_1^4,$$
$$\lambda^2(01)=a_0^2. \qquad \square$$

Markov decision processes. Consider the following stationary, finite horizon Markov decision process. Let $p_{ij}(a)$ be the probability of making transition from state i to state j at the next stage, given action a was just selected. If at stage k the system is in state i and action a was just selected, a reward of $r(i,a)$ is assumed to be accrued, for $k=0,1,\ldots,K-1$. If at the terminal stage K, the system is in state i, then a terminal reward $\bar{r}(i)$ is accrued. The case where the terminal reward is imprecisely known can be treated in a manner analogous to the above decision analysis results. We therefore assume $\bar{r}(i)$ is precisely known, and hence $\bar{h}^1(i)=\bar{r}(i)$ and $\bar{h}^2(i)_n=0$ for all i and n. Assume all that is known about $r=\{r(i,a)\}$ is that it is a member of the given set $R\subseteq\mathbb{R}^N$, where $N=\#S\times\#A$. Then, $h^2(i,a)_n=1(=0)$ if $n=(i,a)$ (if $n\neq(i,a)$).

EXAMPLE 2: Consider the following Markov decision process, which is based on the maintenance-model example presented in Hillier and Lieberman (1980), Chapter 13. Let $K=2$, $S=\{1,\ldots,4\}$, $A=\{1,2,3\}$,

$$\{r(i,a)\}=\begin{bmatrix} 0 & \rho_1 & \rho_2 \\ -1 & \rho_1 & \rho_2 \\ -3 & \rho_1 & \rho_2 \\ -\infty & -\infty & \rho_2 \end{bmatrix}, \quad \bar{r}(i)=0,$$

$$\{p_{ij}(1)\}=\begin{bmatrix} 0 & 7/8 & 1/16 & 1/16 \\ 0 & 3/4 & 1/8 & 1/8 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$p_{i2}(2)=1$ for all i, $p_{i1}(3)=1$ for all i, $\rho_1\in[-6,-2]$, $\rho_2\in[-7,-4]$, and $\rho_2\leq\rho_1$. Table 2 presents the f_k and $\pi^*$for this example. We note that there are three strategies that are possibly preferred, based on the given reward structure information,all three of which select action 1 in states 1 and 2 and action 3 in state 4 for both k=0 and k=1:

$$\lambda_0^1(3)=3, \quad \lambda_1^1(3)=1$$
$$\lambda_0^2(3)=2, \quad \lambda_1^2(3)=1$$
$$\lambda_0^3(3)=2, \quad \lambda_1^3(3)=2$$

where the subscript designates stage. The fourth possibility, $\lambda_0^4(3)=3$ and $\lambda_1^4(3)=2$, produces an optimal expected total reward function identical to the above strategies only when $\rho_1=-4$ and $\rho_2=-3$, is inferior to at least one of the above strategies otherwise, and thus has been deleted. The analysis has therefore eliminated all but three of the 81 possible strategies for this Markov decision process. $\square$

IV THE INFINITE HORIZON, DISCOUNTED CASE.

We now consider the case where $K=\infty$ and where S_k, A_k, h_k, and hence Δ_k, V_k, $H_{k\delta}$, and H_k are all stage invariant. We continue to assume that assumptions A1 and A2 hold and additionally assume the following:

A4. There is a β, $0\leq\beta<1$, such that $\sum_{j\in S}h^3(i,a)_j\leq\beta$, for all $i\in S$ and $a\in A(i)$.

Assumptions A2 and A4 imply that H_δ, for all $\delta\in\Delta$, and H are isotone contractions on V with respect to the supremum norm, and hence there exist unique fixed points f_δ and f of the operators H_δ and H, respectively. Our objective is to determine f and δ^* such that $f=Hf=H_{\delta^*}f$.

A variety of computational procedures have been used to determine f and δ for the $h^2=0$ case: linear programming (d'Epenoux, 1960), successive approximations (White, 1969), policy iteration (Howard, 1960), and various modifications and combinations of the latter two procedures (Puterman and Shin, 1978, 1982; Schweitzer, 1971, Platzman et al., 1982). Viswanathan et al. (1977) have applied parametric linear programming and White and Kim (1981) have applied successive approximations and policy iteration in order to determine f and δ for the case where $h^1=0$, $P=\{\rho:\rho_n\geq 0, \sum_n\rho_n=1\}$, and $\sum_j h^3(i,a)_j=\beta$ for all i and a, i.e., the vector criterion Markov decision process. The latter approaches were based on algorithms due to Smallwood and Sondik (1973) and Sondik (1978). We remark that Henig (1978) has shown that for the vector criterion Markov decision process, the set of all stationary, parameter invariant policies generated by δ produces all extreme points of the convex hull of the nondominated set. We conjecture that the parametric programming approach presented by Viswanathan et al. (1977) is easily extended to consider our more general problem formulation, as is the successive approximations procedure presented earlier. We now motivate and present a generalized policy iteration algorithm for the solution of the infinite horizon case.

(necessarily finite)
Define Λ as the set of all parameter independent policies $\lambda:S\to A, \lambda(i)\in A(i)$ for all $i\in S$, and let f_λ be the fixed point of H_λ. INSERT A

0. Let $\Lambda_0 \subseteq \Lambda$ be given, and set $n=0$.
1. Let $\overline{f}_n(i,\rho) = \max_{\lambda\in\Lambda_n} f_\lambda(i,\rho)$, $i\in S$, $\rho\in P$.
2. Remove any λ from Λ_n that does not achieve the above maximum for some $i\in S$ and $\rho\in P$.
3. Let $\Lambda_{n+1} \subseteq \Lambda$ be composed of the set of all λ that for some $\rho\in P$ achieves the maximum in $H\overline{f}_n$ for all $i\in S$. If Λ_n can be so selected, then set $\Lambda_{n+1}=\Lambda_n$.
4. If $\Lambda_{n+1}=\Lambda_n$, then stop. Otherwise, set $n=n+1$, and go to Step 1.

We remark that by choosing Λ_{n+1} in Step 3 to equal Λ_n whenever possible, we eliminate the possibility of cycling due to any nonuniqueness in achieving the maximum in $H\overline{f}_n$. INSERT B

An alternative approach to describing the above algorithm is to eliminate Step 2 and to ask in Step 3 if there exists a Λ_{n+1} such that $\Lambda_{n+1} \subseteq \Lambda_n$. Step 4 would then be modified to use $\Lambda_{n+1} \subseteq \Lambda_n$ as the stopping rule. We remark that the algorithm viewed in this manner guarantees convergence in one iteration if Λ_0 is chosen to equal Λ since $\Lambda_1 \subseteq \Lambda_0$ for any Λ_1. Of course, the computational impracticality of this choice of Λ_0 limits its usefulness. We now verify that the above algorithm converges to the proper subset of Λ and that this convergence is achieved in a finite number of steps.

PROPOSITION 3: The policy iteration algorithm converges to the fixed point of H in a finite number of iterations.

INSERT A:

The following result provides an important characterization of the fixed point of H that serves to motivate the algorithm presented below for determining this fixed point.

PROPOSITION 2: The fixed point of H, f, is piecewise affine and convex and is given by

$$f(i,\rho) = \max_{\lambda\varepsilon\Lambda} f_\lambda(i,\rho)$$

for all $i\varepsilon S$ and $\rho\varepsilon P$.

Proof: The veracity of the Proposition follows directly from the following well-known (e.g., Bertsekas, 1976) and easily shown result: for fixed $\rho\varepsilon P$, if $\lambda\varepsilon\Lambda$ is such that $f_\lambda(\cdot,\rho) \geq f_\sigma(\cdot,\rho)$ for all $\sigma\varepsilon\Lambda$, then $f_\lambda(\cdot,\rho) = [Hf_\lambda](\cdot,\rho)$. □

We remark that the piecewise affine and convex nature of f allows us to avoid issues related to the concept of finite transcience found in Sondik (1978).

Proposition 2 suggests that an appropriate procedure for calculating f is to determine a set $\Lambda' \subseteq \Lambda$, containing as small a number of parameter independent policies as possible, such that

$$f(i,\rho) = \max_{\lambda\varepsilon\Lambda'} f_\lambda(i,\rho)$$

for all $i\varepsilon S$ and $\rho\varepsilon P$. The intent of the following policy iteration algorithm is to accomplish this objective:

INSERT B:

We also remark that Λ_{n+1} can be determined from $\delta^* \varepsilon \Delta$, where $H_{\delta^*}\overline{f}_n = H\overline{f}_n$, as follows: $\lambda \varepsilon \Lambda_{n+1}$ if and only if $\delta^*(\cdot,\rho) = \lambda(\cdot)$ for some $\rho \varepsilon P$. The parameter dependent policy δ^* can be determined in a manner analogous to the procedure for determining δ_k^* presented in Section 2.

Proof of this result follows the proof of two lemmas. The first lemma indicates that the stopping rule, $\Lambda_{n+1}=\Lambda_n$, is a valid one. The second lemma guarantees that if $\Lambda_{n+1}\neq\Lambda_n$, then Λ_{n+1} produces a strictly better policy than does Λ_n.

LEMMA 2 : If $\Lambda_{n+1}=\Lambda_n$, then $\bar{f}_n$ is the fixed point of H.

INSERT C

Proof: Since $\Lambda_{n+1}=\Lambda_n$, it is sufficient to confine our interest in policies that are members of Λ to those in Λ_n. Assume $\lambda\varepsilon\Lambda_n$, and let $\rho\varepsilon P$ be such that $f_\lambda(\cdot,\rho)\geq f_\sigma(\cdot,\rho)$, for all $\sigma\varepsilon\Lambda_n$. We contend that $[H_\lambda\bar{f}_n](\cdot,\rho)\geq[H_\sigma\bar{f}_n](\cdot,\rho)$, for all $\sigma\varepsilon\Lambda_n$. To show this, assume $\sigma\varepsilon\Lambda_n$ is such that $[H_\sigma\bar{f}_n](\cdot,\rho)\geq[H_\lambda\bar{f}_n](\cdot,\rho)$, where the inequality is strict for at least one $i\varepsilon S$. There are two cases:

(i) $f_\lambda(\cdot,\rho)\neq f_\sigma(\cdot,\rho)$

(ii) $f_\lambda(\cdot,\rho)=f_\sigma(\cdot,\rho)$.

Case (i). Define $H_\delta^{k+1}=H_\delta H_\delta^k$, $f_\sigma^k=H_\sigma^k\bar{f}_n$, and $f_\lambda^k=H_\lambda^k\bar{f}_n$. The isotonicity and contraction properties of H_δ for any $\delta\varepsilon\Delta$ imply that in the limit, f_σ^k and f_λ^k converge to f_σ and f_λ, respectively, and that $f_\sigma(\cdot,\rho)\geq f_\lambda(\cdot,\rho)$, which is a contradiction.

Case (ii). Note that $f_\sigma(\cdot,\rho)=[H_\sigma f_\sigma](\cdot,\rho)=[H_\sigma\bar{f}_n](\cdot,\rho)\geq[H_\lambda\bar{f}_n](\cdot,\rho)=[H_\lambda f_\lambda](\cdot,\rho)=f_\lambda(\cdot,\rho)$, where by assumption the inequality is strict for at least one $i\varepsilon S$. Thus, $f_\sigma(\cdot,\rho)\geq f_\lambda(\cdot,\rho)$ and $f_\sigma(\cdot,\rho)\neq f_\lambda(\cdot,\rho)$, which is a contradiction. $\square$

LEMMA 3. If $\Lambda_{n+1}\neq\Lambda_n$, then $\bar{f}_{n+1}\geq\bar{f}_n$, $\bar{f}_{n+1}\neq\bar{f}_n$.

Proof: It follows from Step 3 of the algorithm that if $\Lambda_{n+1}\neq\Lambda_n$, then $H\bar{f}_n\geq\bar{f}_n$, $H\bar{f}_n\neq\bar{f}_n$. Define $\delta\varepsilon\Delta$ to be such that $H_\delta\bar{f}_n=H\bar{f}_n$. Note that δ is comprised of λ's

INSERT C:

Proof: The proof of Lemma 1 and the assumption $\Lambda_{n+1} = \Lambda_n$ guarantee that for fixed ρ, if $\lambda \varepsilon \Lambda_n$ is such that $f_\lambda(\cdot,\rho) \geq f_\sigma(\cdot,\rho)$ for all $\sigma \varepsilon \Lambda_n$, then $f_\lambda(\cdot,\rho) = [Hf_\lambda](\cdot,\rho) = [H_\lambda f_\lambda](\cdot,\rho)$. Proof of Lemma 2 then follows from the fact that this result holds for all $\rho \varepsilon P$ and from the definition of $\overline{f}_n$.

$\square$

By the isotonicity of H_δ, in Λ_{n+1}. for all $k \geq 1$, $H_\delta^k \overline{f}_n \geq \overline{f}_n$, $H_\delta^k \overline{f}_n \neq \overline{f}_n$; thus, $f_\delta \geq \overline{f}_n$, $f_\delta \neq \overline{f}_n$, where f_δ is the fixed point of H_δ. It is easy to show that $f_\delta = f_\lambda$ whenever $\delta=\lambda$, $\lambda \in \Lambda_{n+1}$. However, $\overline{f}_{n+1}(i,\rho) = \max_\lambda f_\lambda(i,\rho)$, $\lambda \in \Lambda_{n+1}$; thus, $\overline{f}_{n+1} \geq f_\delta$. □

We remark that the algorithm always produces a value function, the $\overline{f}_n$, and an optimal value function, the final $\overline{f}_n$, that are piecewise affine and convex, thus allowing us to avoid having to deal with issues related to the concept of finite transience found in Sondik (1978).

Proof of Proposition 3: Since Λ is finite, it has only a finite number of subsets which can be examined. Each of these subsets can be examined at most once since:

1. if $\Lambda_{n+1} = \Lambda_n$, the algorithm stops (by Lemma 2),
2. if $\Lambda_{n+1} \neq \Lambda_n$, then $\overline{f}_{n+1} \geq \overline{f}_n$, $\overline{f}_{n+1} \neq \overline{f}_n$ (by Lemma 3), insuring that Λ_n will not be considered further.

□

The algorithm determines the set of all parameter independent policies that may be optimal, given P. It may be useful to be able to identify those regions in P where these parameter independent policies are optimal. For example, if further assessment information becomes available which indicates that the parameter of interest is now known to be in the set $P' \subseteq P$, it may be desirable to know if any of the nonexcluded parameter independent policies can be eliminated. It is easily shown that if Λ_n is the nonexcluded subset of parameter independent policies, then the optimal parameter dependent policy δ^* is such that for $\lambda \in \Lambda_n$, $\delta^*(i,\rho) = \lambda(i)$, for all $i \in S$, for all $\rho \in P$ such that $f_\lambda(\cdot,\rho) \geq f_\sigma(\cdot,\rho)$, for any $\sigma \in \Lambda_n$.

EXAMPLE 3. Consider again the maintenance model examined in Example 2, assuming the planning horizon is infinite, the criterion is the expected total discounted reward, and the discount factor is 0.9. Standard results (Bertsekas, 1976) indicate that is is sufficient to examine only strategies that are stage invariant. INSERT D

$$\lambda_1(1)=\lambda_2(1)=\lambda_3(1)=1$$
$$\lambda_1(2)=\lambda_2(2)=\lambda_3(2)=1$$
$$\lambda_1(4)=\lambda_2(4)=\lambda_3(4)=3$$
$$\lambda_1(3)=1,\ \lambda_2(3)=2,\ \lambda_3(3)=3.$$

The functions f_{λ_i}, i=1,2,3, are given in Table 3. It is easily shown that $f_{\lambda_3} \geq f_{\lambda_1}$ on $S \times P$; therefore, Λ_0 can be reduced to the set $\{\lambda_2, \lambda_3\}$. For the set of all $\rho \in P$ such that $f_{\lambda_2}(\cdot,\rho) \geq f_{\lambda_3}(\cdot,\rho)$, $[H_{\lambda_2} f_{\lambda_2}](\cdot,\rho)=[Hf_{\lambda_2}](\cdot,\rho)$. An equivalent result holds for all $\rho \in P$ such that $f_{\lambda_3}(\cdot,\rho) \geq f_{\lambda_2}(\cdot,\rho)$. Thus, $\Lambda_1 = \Lambda_0$, and the algorithm can stop. The regions of P where λ^2 and λ^3 are optimal are presented in Figure 2 and are associated with the statement: $\lambda^2(\lambda^3)$ is optimal if and only if

$$\rho_1 \geq (\leq) 0.92\ \rho_2 + 0.75 \quad . \qquad \square$$

INSERT D

Let $\Lambda_0 = \{\lambda^1\}$, where $\lambda^1(1) = 1$, $\lambda^1(2) = 1$, $\lambda^1(3) = 1$, and $\lambda^1(4) = 3$. The function $f_\lambda 1$ is given in Table 3. Let $\delta^* \varepsilon \Delta$ achieve the maximum in $Hf_\lambda 1$. It is easily shown that $\delta^*(1,\rho) = \delta^*(2,\rho) = 1$ and $\delta^*(4,\rho) = 3$ for all $\rho \varepsilon P$. Straightforward calculations show that

$$h(3,\rho,1,f_\lambda 1) = -13.1968 + 1.96821\rho_2$$

$$h(3,\rho,2,f_\lambda 1) = -10.13630 + \rho_1 + 1.36260\rho_2$$

$$h(3,\rho,3,f_\lambda 1) = -9.12945 + 2.2944\rho_2$$

indicating for $\rho \varepsilon P$,

$$\delta^*(3,\rho) = 2 \text{ if } \rho_1 - 0.9684\rho_2 \geq 1.00685$$

$$= 3 \quad \text{otherwise.}$$

Thus, $\Lambda_1 = \{\lambda^2, \lambda^3\} \neq \Lambda_0$, where

$$\lambda^2(1) = \lambda^3(1) = 1$$

$$\lambda^2(2) = \lambda^3(2) = 1$$

$$\lambda^2(4) = \lambda^3(4) = 3$$

$$^2(3) = 2 \quad , \quad ^3(3) = 3.$$

The resulting functions $f_\lambda 2$ and $f_\lambda 3$ are also given in Table 3. Further calculations indicate that $\Lambda_2 = \Lambda_1$, and hence the algorithm can stop.

V MAXIMIN POLICIES

In earlier sections, we have concentrated on determining the set of all parameter independent strategies that are optimal for some value of the imprecise parameter. If further parameter value assessment can eliminate all but one of these strategies, then strategy selection is trivial. If, however, such elimination is not possible, we are still confronted with the problem of which parameter independent strategy to choose. A likely candidate, and the candidate of interest in this section, is the parameter independent strategy that maximizes, with respect to all other strategies, the minimum value of its expected reward function, where the minimum is taken over all possible parameter values. That is, for the infinite horizon case, we seek a $\tilde{\lambda}\varepsilon\Lambda$ such that

$$\inf_{\rho\varepsilon P} f_{\tilde{\lambda}}(i,\rho) \geq \inf_{\rho\varepsilon P} f_{\lambda}(i,\rho)$$

for all $i\varepsilon S$ and $\lambda\varepsilon\Lambda$. We remark that such a $\tilde{\lambda}$ may not be a member of the set of all parameter independent strategies that are optimal for some value of the imprecise parameter. The infinite horizon problem defined in Section 4 represents the problem of interest; therefore, we will assume assumptions A1, A2, and A4 hold throughout. Extension of the results to be presented to the finite horizon case is straightforward and left to the reader. Let $\|\cdot\| : \mathbb{R}^S \to \mathbb{R}$ be defined as $\|v\| = \sup_i |v_i|$, where $v=\{v_i\}$. Define $h^1_\lambda=\{h^1[i,\lambda(i)],i\varepsilon S\}$, $h^2_\lambda=\{h^2[i,\lambda(i)]_n,i\varepsilon S,n=1\ldots,N\}$, and $h^3_\lambda=\{h^3[i,\lambda(i)]_j,i,j\varepsilon S\}$. We now present our first result of this section.

PROPOSITION 3: Assume that $\tilde{\lambda}\varepsilon\Lambda$ is such that for each $\lambda\varepsilon\Lambda$, $f_{\tilde{\lambda}}(i,\tilde{\rho}) \geq f_{\lambda}(i,\tilde{\rho})$ for all $i\varepsilon S$ for some $\tilde{\rho}\varepsilon \mathbb{R}^N$. Then, for each $\lambda\varepsilon\Lambda$,

$$\inf_{\rho\varepsilon P} f_{\tilde{\lambda}}(i,\rho) + \varepsilon \geq \inf_{\rho\varepsilon P} f_{\lambda}(i,\rho)$$

for all $i\varepsilon S$, where:

- $$\varepsilon = \max_{\lambda\varepsilon\Lambda} \sup_{\rho\varepsilon\tilde{P}} \varepsilon_{\lambda}(\rho,\rho) \quad ,$$

- $\tilde{P}$ is the set of all possible points in the closure of P, $\overline{P}$, that can achieve the infinium in $\inf\{f_{\lambda}(i,\rho), \rho\varepsilon\overline{P}\}$ for any $i\varepsilon S$ and $\lambda\varepsilon\Lambda$,
- $$\varepsilon_{\lambda}(\rho,\rho') = \| (I-h^3_{\tilde{\lambda}})^{-1} h^2_{\tilde{\lambda}} \| \ \| \rho-\tilde{\rho} \| + \| (I-h^3_{\lambda})^{-1} h^2_{\lambda} \| \ \| \tilde{\rho}-\rho' \|.$$

Proof: Note that since

$$f_{\lambda}(\cdot,\rho) = (I-h^3_{\lambda})^{-1}[h^1_{\lambda}+h^2_{\lambda}\rho],$$

the following relationships hold:

$$f_{\tilde{\lambda}}(\cdot,\rho) - f_{\lambda}(\cdot,\rho')$$

$$= [f_{\tilde{\lambda}}(\cdot,\rho)-f_{\tilde{\lambda}}(\cdot,\tilde{\rho})] + [f_{\tilde{\lambda}}(\cdot,\tilde{\rho})-f_{\lambda}(\cdot,\tilde{\rho})] + [f_{\lambda}(\cdot,\tilde{\rho})-f_{\lambda}(\cdot,\rho')]$$

$$\geq [f_{\tilde{\lambda}}(\cdot,\rho)-f_{\tilde{\lambda}}(\cdot,\tilde{\rho})] + [f_{\lambda}(\cdot,\tilde{\rho})-f_{\lambda}(\cdot,\rho')]$$

$$= (I-h^3_{\tilde{\lambda}})^{-1}h^2_{\tilde{\lambda}}(\rho-\tilde{\rho}) + (I-h^3_{\lambda})^{-1}h^2_{\lambda}(\tilde{\rho}-\rho').$$

Thus,

$$f_{\tilde{\lambda}}(i,\rho) + \varepsilon_{\lambda}(\rho,\rho')\geq f_{\lambda}(i,\rho')$$

for all $i\varepsilon S$. The result then follows directly. □

Proposition 3 indicates that we should locate $\tilde{\rho}$, which may or may not be a member of P, as close to all elements in $\tilde{P}$ as possible. We now present a condition that implies that $\tilde{P}$ is a singleton, guaranteeing that $\tilde{\rho}$ can be chosen in P and that $\varepsilon=0$.

A5. There is a $\tilde{\rho}\varepsilon P$ such that

$$h^2(i,a)\tilde{\rho} = \inf_{\rho\varepsilon P} h^2(i,a)\rho$$

for all $i\varepsilon S$ and $a\varepsilon A(i)$.

COROLLARY. Assume $\tilde{\rho}\varepsilon P$ satisfies A5 and that $\tilde{\lambda}\varepsilon\Lambda$ is such that for each $\lambda\varepsilon\Lambda, f_{\tilde{\lambda}}(i,\tilde{\rho})\geq f_{\lambda}(i,\tilde{\rho})$ for all $i\varepsilon S$. The., for each $\lambda\varepsilon\Lambda$,

$$\inf_{\rho\varepsilon P} f_{\tilde{\lambda}}(i,\rho)\geq\inf_{\rho\varepsilon P} f_{\lambda}(i,\rho)$$

for all $i\varepsilon S$.

Proof: It follows from Proposition 3 that it is sufficient to show that A5 implies $f_{\lambda}(i,\tilde{\rho}) = \inf\{f_{\lambda}(i,\rho),\rho\varepsilon P\}$,for all $i\varepsilon S$, for any $\lambda\varepsilon\Lambda$. Clearly, $\inf\{f_{\lambda}(i,\rho),\rho\varepsilon P\}\leq f_{\lambda}(i,\tilde{\rho})$, for all $i\varepsilon S$. In order to show that $f_{\lambda}(i,\tilde{\rho})\leq\inf\{f_{\lambda}(i,\rho),\rho\varepsilon P\}$, for all $i\varepsilon S$, define $f_{\lambda}^{0}(i,\rho)=0$ and $f_{\lambda}^{n}(i,\rho)=h[i,\rho,\lambda(i),f_{\lambda}^{n-1}]$ for all i and ρ. Trivially, $\inf\{f_{\lambda}^{0}(i,\rho),\rho\varepsilon P\}=f_{\lambda}^{0}(i,\tilde{\rho})$ for all i; assume $\inf\{f_{\lambda}^{n-1}(i,\rho),\rho\varepsilon P\}= f_{\lambda}^{n-1}(i,\tilde{\rho})$ for all i. It is then easily shown that $\inf\{f_{\lambda}^{n}(i,\rho),\rho\varepsilon P\}=f_{\lambda}^{n}(i,\tilde{\rho})$ for all i, which by induction holds for all n. Thus, $f_{\lambda}(i,\tilde{\rho})\leq f_{\lambda}(i,\rho)$ for all i and for any $\rho\varepsilon P$, which implies $f_{\lambda}(i,\tilde{\rho})\leq\inf\{f_{\lambda}(i,\rho),\rho\varepsilon P\}$ for all i. □

We remark that although A5 appears to be a strong assumption, there are interesting problem formulations that satisfy it. For example, consider the case where $h^{2}(i,a)_{n}=1$ if $i=n$($=0$ if $i\neq n$), $\rho_{n}\varepsilon[LB_{n},UB_{n}],n=1,\cdots,N\}\varepsilon P$. Then, we would select INSERT E As another example, observe that in Example 3, $\tilde{\rho}=(-6,-7)$, satisfies A5 and hence by the Corollary,

$$\inf_{\rho\varepsilon P} f_{\lambda_3}(i,\rho)\geq\inf_{\rho\varepsilon P} f_{\lambda}(i,\rho)$$

for all $i\varepsilon S$ and $\lambda\varepsilon\Lambda$.

INSERT E

$\tilde{\rho}=\{LB_1,...,LB_N\}^T$, where T = transpose.

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$$f_2(i,\rho) = 0 \text{ for all } i\varepsilon S,\ \rho\varepsilon P$$

$$f_1(1,\rho) = 0,\ \delta_1^*(1,\rho) = 1$$

$$f_1(2,\rho) = -1,\ \delta_1^*(2,\rho) = 1$$

$$f_1(3,\rho) = \max\{-3,\rho_1\},\ \delta_1^*(3,\rho) = \begin{cases} 1 \text{ if } -3 \geq \rho_1 \\ 2 \text{ if } \rho_1 \geq -3 \end{cases}$$

$$f_1(4,\rho) = \rho_2,\ \delta_1^*(4,\rho) = 3$$

$$f_0(1,\rho) = \max\{(-17+\rho_2)/16,\ (-14+\rho_1+\rho_2)/16\},\ \delta_0^*(1,\rho) = 1$$

$$f_0(2,\rho) = \max\{(-17+\rho_2)/8,\ (-14+\rho_1+\rho_2)/8\},\ \delta_0^*(2,\rho) = 1$$

$$f_0(3,\rho) = \max\{-1+\rho_1,\ \rho_2\},\quad \delta_0^*(3,\rho) = \begin{cases} 2 \text{ if } -1+\rho_1 \geq \rho_2 \\ 3 \text{ if } \rho_2 \geq -1+\rho_1 \end{cases}$$

$$f_0(4,\rho) = \rho_2,\quad \delta_0^*(4,\rho) = 3$$

TABLE 2: Optimal Expected Utility Functions and Possibly Optimal Strategies for Example 2.

$f_2(21,\rho) = \rho_1$

$f_2(22,\rho) = 0$

$f_2(23,\rho) = 0.43$

$f_2(24,\rho) = \rho_1$

$f_2(25,\rho) = 0$

$f_2(26,\rho) = 0.46$

$f_2(27,\rho) = \rho_2$

$f_2(28,\rho) = 0.01$

$f_2(29,\rho) = 0.43$

$f_1(11,\rho) = 0.69\rho_1$

$f_1(12,\rho) = 0.46$

$f_1(13,\rho) = \rho_2$

$f_1(14,\rho) = 0.01$

$f_1(15,\rho) = 0.43$

$f_0(01,\rho) = \max\{0.005+0.5\rho_2,\ 0.161+0.4485\rho_1\}$

$$\delta_1^*(1,\rho) = a_1^1,\quad \delta_1^*(2,\rho) = a_1^4$$

$$\delta_0^*(1,\rho) = \begin{cases} a_0^1 \text{ if } 0.161+0.4485\rho_1 \geq 0.005+0.5\rho_2 \\ a_0^2 \text{ if } 0.005+0.5\rho_2 \geq 0.161+0.4485\rho_1 \end{cases}$$

TABLE 1: Optimal Expected Utility Functions and Possibly Optimal Strategies for Example 1.

VI CONCLUSIONS

In this paper, we considered a specially structured class of finite state, finite action dynamic programs having an imprecisely described parameter. It was assumed that the parameter has no dynamics, no new information about its value is received once the problem begins, and its imprecision is described by set inclusion. We developed computational procedures for determining the set of all strategies that may be optimal, given a description of the parameter's imprecision. Applications to decision analysis and Markov decision processes were presented. A condition was also presented which guaranteed that a parameter independent strategy maximizes, with respect to all other parameter independent strategies, the minimum value of its expected reward function, where the minimum is taken over all possible parameter values. Consideration of different functional forms of the one transition value function and different assumptions regarding the dynamics and timing of information availability regarding the unknown parameter are topics for future study. Two other topics for future study are 1) an analysis of the computational feasibility of the algorithms presented in Sections 2 and 4 (related discussions can be found in Smallwood and Sondik, 1973, and Sondik, 1978) and 2) the development of parametric programming and successive approximation procedures for the infinite horizon case and their comparison to the policy iteration algorithm presented in Section 4.

$$f_\lambda 1(\cdot,\rho) = \begin{bmatrix} -9.12945 \\ -10.13630 \\ -12.17710 \\ -8.21650 \end{bmatrix} + \begin{bmatrix} 0 & 1.29447 \\ 0 & 1.36260 \\ 0 & 1.77139 \\ 0 & 2.16503 \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

$$f_\lambda 2(\cdot,\rho) = \begin{bmatrix} -6.57031 \\ -7.44243 \\ -6.69818 \\ -5.91327 \end{bmatrix} + \begin{bmatrix} 0.83782 & 0.83782 \\ 0.88192 & 0.88192 \\ 1.79373 & 0.79373 \\ 0.75404 & 1.75404 \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

$$f_\lambda 3(\cdot,\rho) = \begin{bmatrix} -5.93779 \\ -6.77663 \\ -5.34402 \\ -5.34402 \end{bmatrix} + \begin{bmatrix} 0 & 1.61169 \\ 0 & 1.69651 \\ 0 & 2.45052 \\ 0 & 2.45052 \end{bmatrix} \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}$$

TABLE 3: Expected Total Discounted Reward Functions for Example 3.

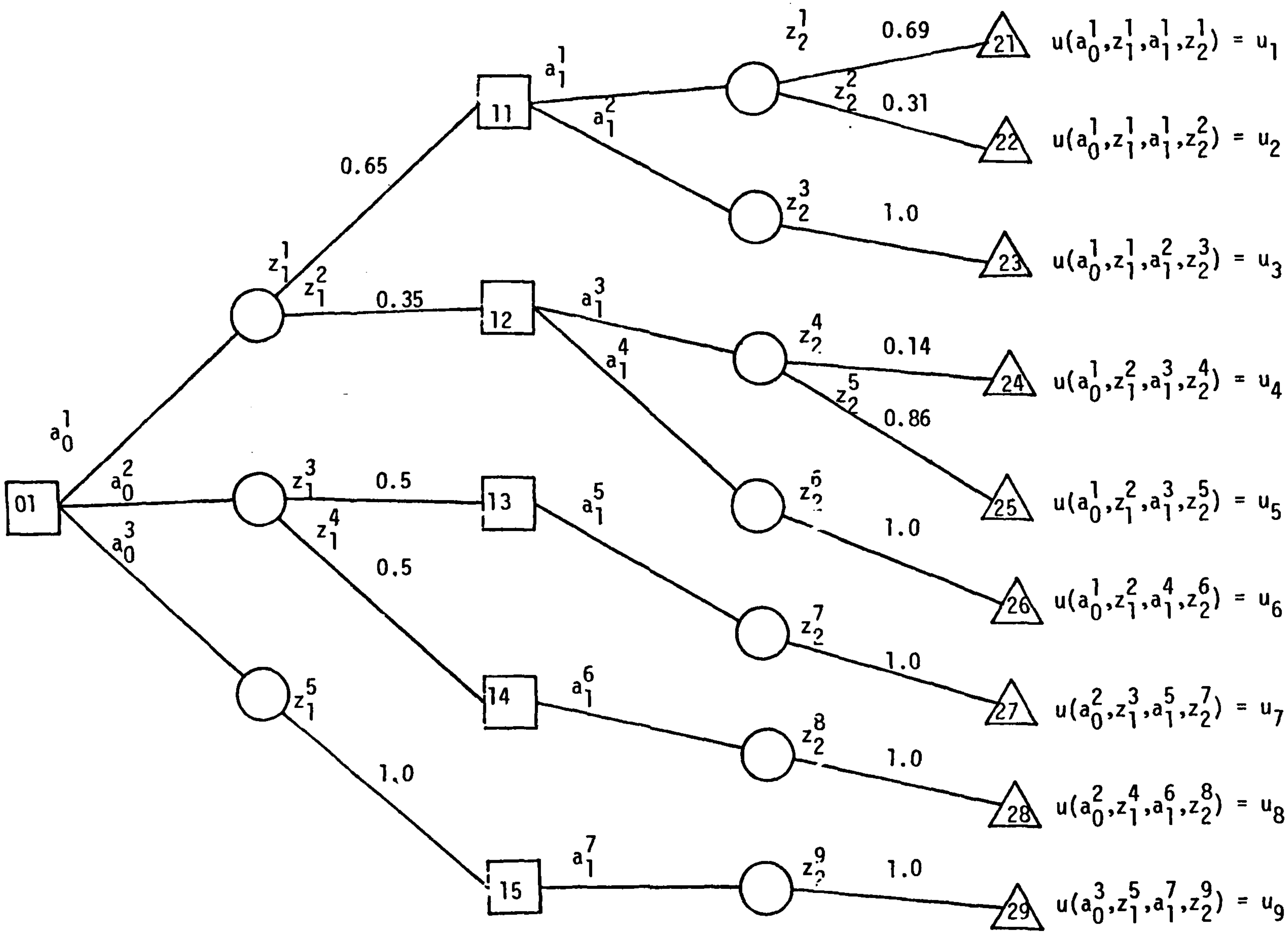

FIGURE [illegible] Decision Tree [illegible] Example 1

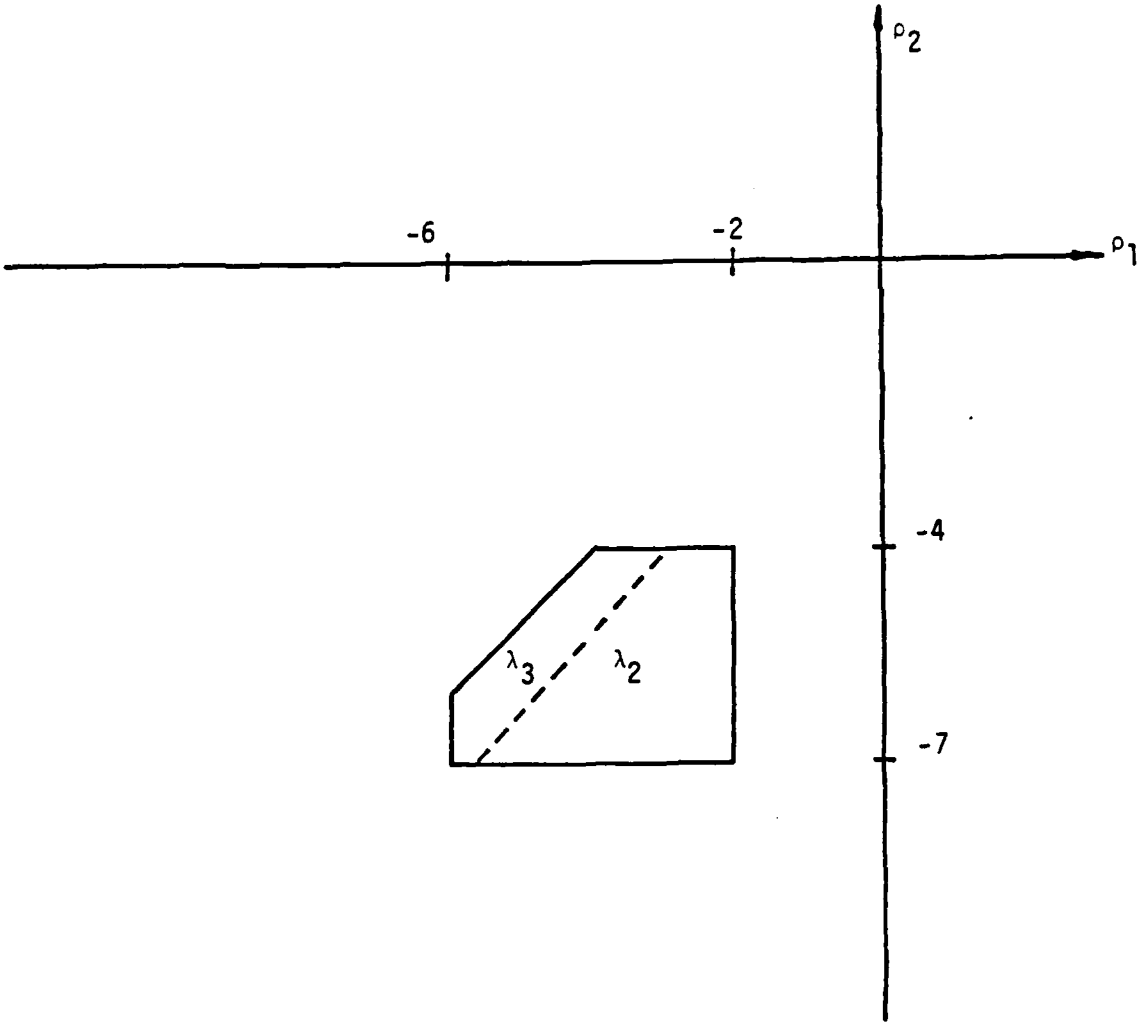

FIGURE 2: The Regions in P where λ_2 and λ_3 are Optimal for Example 3.

ACCEPTED FOR PUBLICATION IN
OPERATIONS RESEARCH

SEQUENTIAL DECISIONMAKING UNDER UNCERTAIN FUTURE PREFERENCES*

by

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ABSTRACT:

We present a model of sequential decisionmaking under uncertain future preferences. The evolution of the tradeoff weight vector is constrained by set inclusion. Three different mechanisms are considered for selecting the next tradeoff weight vector: a mechanism working against the DM (M), a mechanism trying to aid the DM (B), and (briefly) a probabilistic mechanism (P). Piecewise linear, convex upper and lower bounds on the optimal value function are determined for mechanisms M and B. Conditions are given which guarantee that the optimal value function is piecewise linear and convex for all three mechanisms. Procedures for computing these bounds and determining associated strategies are presented. A hypothetical situation involving an individual seeking promotion is used to illustrate the model and the numerical techniques.

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In the usual development of normative decision theories and descriptions of decisionmaking, no provision is made that preferences may change over the planning horizon. Examples of normative, quantitative models of multi-objective, sequential decisionmaking with fixed preference structures can be found in Henig (1978), Mitten (1975), Sobel (1975), Viswanathan, et.al. (1977), and White and Kim (1980). In practice, a decisionmaker may reject an optimal solution determined on the assumption that initial preference is absolute and permanent and accept a solution which offers more flexibility for later adjustment. Such choice behavior would reflect a recognition that preferences may change and that so-called "optimal" solutions can lead to commitments that will be judged harshly in the future. In fact, empirical evidence exists which suggests that ignoring the ambiguities due to changing preferences may lead both to *misinterpreting choice* behavior and to incorrectly modeling the normative decisionmaking problem (see March (1978), p. 590, and related references cited by March).

Several different models of changing preferences have been proposed. Cyert and DeGroot (1975) have proposed that the utility function be viewed as a function of imprecisely known static parameter values, the estimates of which change as a result of learning through experiencing noise corrupted observations of the true parameter values. Learning is assumed to occur according to a Bayesian update, requiring a prior distribution and a probabilistic description of the relationship between outcome and underlying parameter values. Thus, the actual utility function does not change; however, the perceived utility function, the utility function on which the decisionmaker selects alternatives, does change as a function of experience and hence stage or decision epoch.

Witsenhausen (1974) has suggested a suboptimal design procedure which he calls the assumed permanence procedure. This suboptimal design is determined as follows. Determine current preferences, and select the best action for the current stage, assuming current preferences will remain in effect over the remainder of the problem horizon. At the next stage, reassess preferences and select the best action for that stage, assuming that the reassessed preferences will remain in effect over the remainder of the problem horizon, and so forth. Witsenhausen notes that an optimal design could be determined if a stochastic model of preference evolution were available.

Kreps (1977, 1975) enlarged the notion of a state in a Markov decision process with utility criterion to include a so-called summary descriptor. The summary descriptor can be used to model changes in preference, level of aspiration or expectancy, etc. Bodily and White (1982) used this notion to model an investor's evolving attitude toward levels of consumption, as a function of current and past levels of wealth, thus modeling the change of preferences between consuming and investing over time. Related results are presented in Meyer (1977).

In this paper, we consider a finite stage decision analysis problem having a decision tree with no chance nodes (no probabilistic description of uncertainty). We explicitly allow a description of preference, a vector of tradeoff weights, to change from stage to stage. We assume the next tradeoff weight vector, constrained to be a member of a set, is either (1) selected by a mechanism working against the decisionmaker, (2) selected by a mechanism trying to aid the decisionmaker, or (3) selected probabilistically. Emphasis is placed on the former two types of dynamics since (1) we feel that their relative simplicity should enhance assessment of the tradeoff weight dynamical description and (2) the dynamics of variables that can affect preference may not be adequately described probabilistically, e.g. political, behavioral, and organizational issues. The set inclusion description of dynamics has been examined by Cherene (1978) in an economic context and by Bertsekas and Rhodes (1973) in a general control context. A related study can be found in Figueras (1972).

At first glance, the problem to be dealt with in this paper and the iterative procedures for solving multiple criteria problems (see, for example, Zionts and Wallenius (1976), Geoffrion, et.al. (1972), and Musselman and Talavage (1980)) seem closely related. They differ, however, in the following fundamental way. The procedures for solving multiple criteria problems iteratively collect new information to more accurately describe a static description of preference (as is true in the work by Cyert and DeGroot (1975)), whereas the problem to be considered here assumes that the DM knows exactly the current description of preference but that the description of preference is allowed to evolve dynamically.

The paper is outlined as follows. Two models of decisionmaking are presented in Section 2. Both models assume that preference evolution is described by set inclusion and that the decisionmaker bases the current alternative selection on exact knowledge of the current tradeoff weights. The models differ in how the next set of tradeoff weights are chosen. The first model, the maximin problem, assumes the next set of tradeoff weights are selected so as to reduce total value as much as possible. The second model, the maximax problem, assumes selection is intended to increase total value as much as possible.

Dynamic programs and sufficient conditions for optimality for both the maximin and maximax problems are presented in Section 3, and various preliminary results can be found in Section 4.

The maximin problem is studied in Section 5. We determine a piecewise linear, convex lower bound on the optimal value function. This bound is shown to be exact when two objectives are being considered. The desirable feature of this functional form is that it leads to a simple procedure for computing the lower bound and a suboptimal design, which we also present.

In Section 6, a piecewise linear and convex upper bound is determined on the optimal value function for the maximax problem; a procedure for computing this bound and its associated design is presented.

Piecewise linear, convex upper and lower bounds are determined in Section 7 on the optimal value function for the maximin and maximax problems, respectively. Again, a procedure for computing these bounds and their associated designs is presented.

For completeness, a different perspective is taken in Section 8. Emphasis is placed on tractability rather than on the development of models that are intended to have dynamics which are relatively easy to assess. We assume that the number of tradeoff weight vectors that might possibly occur at the next stage is finite and that each of the possible future tradeoff weight vectors has nonnegative elements and satisfies the sum-to-one property. We then show that the previously determined upper bound for the maximax problem is exact. Under the assumption that the next tradeoff weight vector is selected probabilistically, we show the problem becomes equivalent to a partially observed Markov decision process, a solution procedure for which is presented in Smallwood and Sondik (1973). We remark that this solution procedure is based on the fact that the optimal expected value function is piecewise linear and convex and has provided insight into the development of all of the aforementioned computational procedures.

Suboptimal design procedures are discussed in Section 9. A hypothetical example with numerical results is presented in Section 10. Conclusions are presented in Section 11. Proofs of all results are contained in the Appendix.

PROBLEM FORMULATIONS:

Assume that the decisionmaker (DM) must select an alternative a^k from a finite set of alternatives A^k at each stage $k=0,1,\ldots,K-1$. We further assume that the associated decision tree contains no chance nodes; therefore, the collection of alternatives $a^0,\ldots,a^{k-1}$ uniquely identifies a stage k decision node and for the case where $k=K$, identifies a terminal node. Let $\rho_n^k(a^0,\ldots,a^k)$ be the value received at stage k with regard to objective n if a^ℓ was the alternative selected at stage $\ell=0,1,\ldots,k-1$ and if a^k is selected at stage k, $k<K$. Let $\rho_n^K(a^o,\ldots,a^{K-1})$ be the terminal value received at stage K with regard to objective n if a^ℓ was the alternative selected at stage ℓ, $\ell=0,1,\ldots,K-1$. Define $\rho^k(a^o,\ldots,a^k) = \mathrm{col}\{\rho_n^k(a^o,\ldots,a^k)\}$, for $k<K$, and $\rho^K(a^o,\ldots,a^{K-1}) = \mathrm{col}\{\rho_n^K(a^o,\ldots,a^{K-1})\}$.

The DM's impression of the importance of objective n at stage k relative to the available alternatives is expressed by the real number η_n^k which we will refer to as the tradeoff weight of objective n at stage k. We assume that $\eta^k = \mathrm{row}\{\eta_n^k\}\varepsilon N = \{\eta\varepsilon\ R^N:\eta_n\geq 0,n=1,\ldots,N,\sum_{n=1}^N\eta_n=1\}$. The value accrued at stage $k<K$ is assumed to be represented by the inner product $\eta^k\rho^k(a^o,\ldots,a^k) = \sum_{n=1}^N\eta_n^k\ \rho_n^k(a^o,\ldots,a^k)$ if the DM's tradeoff weight vector is η^k and if a^ℓ was the alternative selected at stage $\ell=0,1,\ldots,k$. Similarly $\eta^K\rho^K(a^o,\ldots,a^{K-1})$ is the accrued terminal value. The total

discounted value accrued by the alternatives $a^o,\ldots,a^{K-1}$ and the tradeoff weight vectors $\eta^o,\ldots,\eta^K$ is assumed to be $\sum_{k=0}^{K-1}\beta^k\eta^k\rho^k(a^o,\ldots,a^k)+\beta^K\eta^K\rho^K(a^o,\ldots,a^{K-1})$ where β^k is the discount factor β raised to the k^{th} power. Presumably, $0\leq\beta<1$; however, the analysis will only require $\beta\geq 0$.

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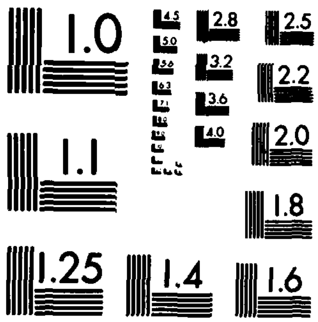
1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

We justify choice of the above criterion as follows. The criterion clearly generalizes the additive value function under the (usual) assumption that the tradeoff weight vector is static and known before alternative selection begins. Note that under this assumption, it is sufficient to set $\beta=1$ and $\rho_k=0$ for all $k=0,\ldots,K-1$. Furthermore, value is allowed to accrue on the basis of descriptions of preference at any stage in the planning horizon. We have assumed that the criterion is temporally additive for technical reasons; we will see that the associated dynamic programs permit the development of relatively tractable computational procedures. Related discussions involving von Neumann Morgenstern utility functions that allow experiential adaptation of preferences can be found in Meyer (1977).

We assume that the DM's preferences, as modeled by the tradeoff vector, may change from stage to stage. Thus, $\{n_k, k=0,1,\ldots,K\}$ is a process, the dynamics of which may depend on, say, its past history, all former alternatives selected, and a variety of other processes that have uncertain dynamics. For example, our future tradeoff weight for gas milage might reasonably depend on our history of automobile purchases, our current and perhaps our past tradeoff weights for gas milage, and processes such as the future cost of gas, future attitudes of the car-buying public, and so forth. Such a perception suggests that the next tradeoff vector may be described by a stochastic difference equation dependent on past and present tradeoff vectors, alternatives, and exogenous variables. The dynamics of the exogenous variables may also be appropriately described by stochastic difference equations. We envision, however, that a description of what value the tradeoff vector may next assume will be assessed from the DM, and hence we view simplicity to be an important characteristic in any description of preference dynamics. A particularly simple description of state dynamics is set inclusion. Such a description requires less information than a probabilistic description. Furthermore, our model will not explicitly consider the impact of various exogenous variables on preference, eliminating a need, for better or for worse, to model such variables. More precisely assume that there is a function $M^k:N \times A^o \times \ldots \times A^k \to 2^N$ (2^N represents the power set of N, the set of all subsets of N), $k=0,1,\ldots,K-1$, such that $\eta^{k+1} \in M^k(\eta^k,a^o,\ldots,a^k)$. Thus, η^{k+1} is constrained to be a member of a set in N which can depend on $\eta^k,a^o,\ldots,a^k$. We will assume throughout that $M^k(\eta^k,a^o,\ldots,a^k) \neq \phi$ for all $\eta^k \in N, a^\ell \in A^\ell, \ell=0,\ldots,k$, and $k=0,\ldots,K-1$.

The actual mechanism that selects η^{k+1} from $M^k(\eta^k,a^o,\ldots,a^k)$ is complex, as we have indicated, and may be related to and to some degree controlled by the DM. The two mechanisms that we will primarily consider here, referred to as "Nature", are:

1. η^{k+1} is chosen to reduce total discounted value as much as possible (mechanism M, associated with a malevolent Nature).

2. η^{k+1} is chosen to increase total discounted value as much as possible (mechanism B, associated with a benevolent Nature).

(A probabilistic mechanism for selecting η^{k+1}, mechanism P, will also be briefly discussed in Section 8.) No attempt will be made to justify the (possibly dubious) claim that these two mechanisms are accurate models of future tradeoff weight selection; their use is intended only to help provide bounds on the total value to be accrued and systematic approaches for alternative selection.

The desired goal of the DM is to select alternatives so as to maximize the total discounted value. In so doing, we assume that the DM knows (or assumes) the following over the entire planning horizon:

(i) the criterion structure, K, β, the sets A^k and $\mathcal{N}^k$, $k=0,\ldots,K-1$, the appropriate probabilities, if required, and the functions $\rho^k, k=0,\ldots,K$,

(ii) the type of mechanism (either M, B, or P) that selects η^{k+1} from $\mathcal{N}^k(\eta^k, a^o,\ldots,a^k), k=0,\ldots,K-1$.

(iii) that η^k and $a^o,\ldots,a^{k-1}$ will be made available to the DM before a^k must be selected, for all $k=0,1,\ldots,K-1$.

Note that (iii) implies that the DM has (a) the current value of the tradeoff vector and the current alternative history on which to base the current alternative selection and (b) knowledge that such information will be made available to him accordingly in the future. Suppressing (i) and (ii) for notational brevity, we, therefore, define a policy at stage k as a function $\delta^k:\mathcal{N}\times A^o\times\ldots\times A^{k-1}\rightarrow A^k$. A strategy is a collection of such policies; i.e., $\pi = \{\delta^o,\ldots,\delta^{K-1}\}$.

The total value accrued by policy π, given initial trade-off weight vector η^o, is:

$$Z_\pi(\eta^o) = \text{ext}\left\{\sum_{k=0}^{K-1} \beta^k \eta^k \rho^k(a^o,\ldots,a^k) + \beta^K \eta^K \rho^K(a^o,\ldots,a^{K-1})\right\},$$

where $a^k = \delta^k(\eta^k, a^o,\ldots,a^{k-1})$ and where extrema are with respect to $\eta^1,\ldots,\eta^K$, subject to the constraints $\eta^{k+1} \epsilon M^k(\eta^k, a^o,\ldots,a^k), k=0,\ldots,K-1$. For the case where selection mechanism M is in effect, the extrema are infima, and we designate $J_\pi = Z_\pi$; when the selection mechanism is B, then the extrema are suprema, and we designate $L_\pi = Z_\pi$. Explicit definition of the criterion for mechanism P will be stated in Section 8. We wish to determine a strategy π^* such that $Z_{\pi^*} \geq Z_\pi$ for all π. Such a strategy will be called a <u>maximin</u> strategy if mechanism M is assumed and a <u>maximax</u> strategy if mechanism B is assumed.

THE DYNAMIC PROGRAMMING EQUATIONS:

We now present dynamic programming equations and sufficient conditions for optimality for both the maximin and maximax problems. Define

$$Z_\pi^k(\eta^k, a^o, \ldots, a^{k-1})$$

$$= \text{ext}\left\{\sum_{t=k}^{K-1} \beta^{t-k}\eta^t\rho^t(a^o, \ldots, a^t) + \beta^{K-k}\eta^K\rho^K(a^o, \ldots, a^{K-1})\right\}$$

where $a^t = \delta^t(\eta^t, a^o, \ldots, a^{t-1})$ for $k \le t \le K-1$ and where the extrema are taken with respect to $\eta^{k+1}, \ldots, \eta^K$, subject to the constraints $\eta^{t+1} \varepsilon M^t(\eta^t, a^o, \ldots, a^t)$, $t=k, \ldots, K-1$. Let

$$\bar{Z}^k(\eta^k, a^o, \ldots, a^{k-1}) = \sup_\pi Z_\pi^k(\eta^k, a^o, \ldots, a^{k-1}) \ .$$

Define for $k=0,1,\ldots,K-1$,

$$h^k(\eta, a^o, \ldots, a^{k-1}, a, v)$$

$$= \eta\rho^k(a^o, \ldots, a^{k-1}, a) + \beta\text{ext}\{v(\eta') : \eta' \varepsilon M^k(\eta, a^o, \ldots, a^{k-1}, a)\}$$

$$\left[H_\delta^k v\right] (\eta, a^o, \ldots, a^{k-1})$$

$$= h^k[\eta, a^o, \ldots, a^{k-1}, \delta(\eta, a^o, \ldots, a^{k-1}), v]$$

$$H_*^k v = \sup_\delta H_\delta^k v.$$

Then, straightforward arguments (see, for example, Bertsekas (1976), p. 65) imply:

(i) Z^k satisfies the dynamic programming equation (DPE)

$$Z^k = H_*^k(Z^{k+1}), k=K-1,\ldots,0,$$

or more explicitly,

$$Z^k(\eta,a^o,\ldots,a^{k-1}) = \sup_{a\varepsilon A^k}\Big\{\eta\rho^k(a^o,\ldots,a^{k-1},a)$$

$$+\beta\text{ext}\{Z^{k+1}(\eta',a^o,\ldots,a^{k-1},a):\eta'\varepsilon M^k(\eta,a^o,\ldots,a^{k-1},a)\}\Big\}$$

(ii) the boundary condition for the DPE is $Z^K(\eta,a^o,\ldots,a^{K-1}) =$

$$\eta^K\rho^K(a^o,\ldots,a^{K-1}),$$

(iii) $Z^o = \sup_\pi Z_\pi$,

(iv) if δ^k is such that $Z^k = H_{\delta^k}^k(Z^{k+1})$, $k=0,\ldots,K-1$, then $\pi=\{\delta^o,\ldots,\delta^{K-1}\}$ is a maximin (maximax) strategy if the extremum in h^k is an infimum (supremum).

PRELIMINARIES

We now present several preliminary results, assumptions, and definitions that will be of benefit in constructing computable upper and lower bounds on J^k and L^k.

Consider the following assumption:

A1. $M: N \to 2^N$ is such that if $\eta' \varepsilon M(\eta^1)$ and $\eta'' \varepsilon M(\eta^2)$, then $\lambda\eta' + (1-\lambda)\eta'' \varepsilon M[\lambda\eta^1 + (1-\lambda)\eta^2]$ for any $\lambda \varepsilon [0,1]$, for all $\eta^1, \eta^2 \varepsilon N$.

Define $g: N \to R$ as $g(\eta) = \inf\{v(\eta') : \eta' \varepsilon M(\eta)\}$, where $v: N \to R$. The following result will be of importance in proving the convexity of J^k on $N, k=0,\ldots,K$.

LEMMA 1. If v is convex and M satisfies A1, then g is convex.

We can now give conditions which guarantee that J^k is convex.

PROPOSITION 1. Assume $M^k(\cdot,a^o,\ldots,a^k)$ satisfies A1 for all $a^\ell \varepsilon A^\ell, \ell=0,\ldots,k$, and $k=0,\ldots,K-1$. Then, $J^k(\cdot,a^o,\ldots,a^{k-1})$ is convex on N for all $k=0,\ldots,K$.

We now present an assumption and a result which will lead to the development of a piecewise linear and convex lower bound on J^k in Section 5.

A2. Let $M(\eta) = N \cap M'(\eta)$ where $M'(\eta) = CH\{\eta Q_e, e=1,\ldots,E\}$ (CH = convex hull) and where E and Q_e, $e=1,\ldots,E$, are given, Q_e is an N×N matrix for each $e=1,\ldots,E$, ar

(i) the rows of Q_e sum to one for all $e \varepsilon \{1,\ldots,E\}$ (note that Q_e may not be stochastic),

(ii) the matrix $Q_e - Q_{e'}$ has identical rows, for each $e,e' \varepsilon \{1,\ldots,E\}$.

Note that in order for $M(\eta) \neq \phi$ for all $\eta \ \varepsilon \ N$ when A2 holds, there must exist at least one e for each $\eta \ \varepsilon \ N$ such that $\eta Q_e \varepsilon N$.

We remark that a $M'(\eta)$ which satisfies A2 does not change shape or rotate as η changes. Two important implications of this characteristic are presented in Lemmas 2 and 3.

LEMMA 2. If A2 holds, then so does A1.

LEMMA 3. Assume $\mathcal{M}'$ satisfies A2. Then, for any $\alpha \varepsilon R^N$, $\mathrm{ext}\{\eta'\alpha : \eta' \varepsilon \mathcal{M}'(\eta)\} = \eta Q_{e^*}\alpha$, where $e^* \varepsilon \{1,\ldots,E\}$ is dependent on α but is independent of $\eta \ \varepsilon\ N$.

We would expect that a typical extreme point of $\mathcal{M}'(\eta)$ that would be assessed from a DM would be of the form $\eta_n' = \eta_n + c_n, n<N$. Such a description of an extreme point allows, for example, $\mathcal{M}'(\eta) = \{\eta' : \ell_n \leq \eta_n' - \eta_n \leq u_n, n<N, \eta_N' = 1 - \sum_{n=1}^{N-1} \eta'_n\}$. It is easy to show that if

$$Q_{nn} = 1 + c_n \qquad , n<N$$

$$Q_{mn} = c_n \qquad , m \neq n,\ n<N$$

$$Q_{mN} = -\sum_{n=1}^{N-1} c_n \qquad , m \neq N$$

$$Q_{NN} = 1-\sum_{n=1}^{N-1} c_n$$

then $\eta Q=\eta'$. Let Q' be defined as is Q except replace the c_n by c_n'. Then, it is easily shown that A2 (ii) holds. We therefore contend that A2 is sufficiently weak to allow relevant descriptions of M.

Consider the following definitions. Let $i:N\to R$ be such that $i(\eta)=\sup\{v(\eta'):\eta'\in M(\eta)\}$ for $v:N\to R$ and $M:N\to 2^N$. Define

$$I(A,E,Q_e,\eta) = \max_{\alpha\in A}\ \max_{1\le e\le E}\ \eta Q_e\alpha.$$

$$G(A,E,Q_e,\eta) =$$

$$\max\left\{\min_{\eta'\in N}\ \max_{\alpha\in A}\eta'\alpha,\ \max_{\alpha\in A}\ \min_{1\le e\le E}\ \eta Q_e\alpha\right\}.$$

Note that if a function $v:N\to R$ is piecewise linear and convex, there is a set of N-vectors A such that $v(\eta)=\max\{\eta\alpha:\alpha\in A\}$. We now present a key result in the development of bounds on both J^k and L^k.

LEMMA 4. Assume v is piecewise linear and convex and that the finite set A is such that $v(\eta)=\max\{\eta\alpha:\alpha\in A\}$. Assume further that M satisfies A2 and that $\eta Q_e, e=1,\dots,E$, are the extreme points of $M'(\eta)$. Then, for all $\eta\in N$:

(a) $G(A,E,Q_e,\eta)$ is piecewise linear and convex and $G(A,E,Q_e,\eta)\le g(\eta)$,

(b) for $N=2, G(A,E,Q_e\eta)=g(\eta)$,

(c) $I(A,E,Q_e,\eta)$ is piecewise linear and convex and $i(\eta)\le I(A,E,Q_e,\eta)$.

LOWER BOUNDS FOR THE MAXIMIN PROBLEM:

In this section, we determine piecewise linear, convex lower bounds on J^k which are exact when N=2. We also present a procedure for computing these lower bounds.

Assume A2 holds for all M^k. We recursively construct the function J_L^k and the set of vectors A_L^k as follows. Let $A_L^K(a^o,\ldots,a^{K-1}) = \{\rho^K(a^o,\ldots,a^{K-1})\}$ and $J_L^K(\eta^K,a^o,\ldots,a^{K-1}) = \eta^K \rho^K(a^o,\ldots,a^{K-1})$. Assume $A_L^{k+1}(a^o,\ldots,a^k)$ is known and that $J_L{}^{k+1}(\eta^{k+1},a^o,\ldots,a^k) = \max\{\eta^{k+1}\alpha:\alpha\varepsilon A_L^{k+1}(a^o,\ldots,a^k)\}$. By Lemma 4(a), $G(A,E,Q_e,\eta)$ is piecewise linear and convex on N. Thus, for each $a^k\varepsilon A^k, k<K$,

$$J_L^k(\eta^k,a^o,\ldots,a^k) = \eta^k\rho^k(a^o,\ldots,a^k) + \beta G(A,E,Q_e,\eta^k)$$

is piecewise linear and convex on N, where $A=A_L^{k+1}(a^o,\ldots,a^k)$, $E=E^k(a^o,\ldots,a^k)$, and $Q_e=Q_e^k(a^o,\ldots,a^k)$. Define

$$J_L^k(\eta^k,a^o,\ldots,a^{k-1}) = \max_{a^k\varepsilon A^k} J_L^k(\eta^k,a^o,\ldots,a^k)$$

which is also piecewise linear and convex on N, and hence there is a set $A_L^k(a^o,\ldots,a^{k-1})$ such that $J_L^k(\eta^k,a^o,\ldots,a^{k-1}) = \max\{\eta^k\alpha:\alpha\varepsilon A_L^k(a^o,\ldots,a^{k-1})\}$.

PROPOSITION 2. Assume $M^k(\cdot,a^o,\ldots,a^k)$ satisfies A2 for all $a^\ell\varepsilon A^\ell, \ell=0,\ldots,k$ and all $k=0,\ldots,K-1$. Then,

(a) $J_L^k(\cdot,a^o,\ldots,a^{k-1})$ is piecewise linear and convex on N for all $k=0,\ldots,K$.

(b) $J_L^k\leq J^k, k=0,\ldots,K$.

(c) for $N=2, J_L^k=J^k, k=0,\ldots,K$.

We now examine in more detail the determination of $A_L^k(a^o,\ldots,a^{k-1})$ from $A_L^{k+1}(a^o,\ldots,a^{k-1},a^k)$, for all $a^k \varepsilon A^k$. Observe that this determination is analogous to determining $\mathcal{A}'$ from $\mathcal{A}(a)$, $E(a)$, and $Q_e(a)$, for all $a \varepsilon A$, where:

$$\max_{\alpha' \varepsilon \mathcal{A}'} \eta\alpha' = \max_{a \varepsilon A}\{\eta\rho(a)+\beta G[\mathcal{A}(a),E(a),Q_e(a),\eta]\}$$

Consider the following procedure:

(1) For each $\alpha \; \varepsilon \; \mathcal{A}(a)$ and for any η, determine $Q_\alpha(a)$ such that $\eta Q_\alpha(a)_\alpha = \min \{\eta Q_e\alpha : 1 \leq e \leq E(a)\}$. Put $\rho(a)+\beta Q_\alpha(a)\alpha$ into $A_T(a)$.

(2) Determine

$$z^*(a) = \min_{\eta' \varepsilon N} \max_{\alpha \varepsilon A(a)} \eta'\alpha$$

and put $\rho(a)+\beta \underline{1} z^*(a)$ into $A_T(a)$. Note that

$$z^*(a) = \text{minimum } z$$

$$\text{subject to: } z \geq \eta\alpha \quad , \quad \alpha \;\varepsilon\; A(a)$$
$$\underline{1}\eta = 1$$
$$\eta \geq 0.$$

(3) Once steps (1) and (2) have been accomplished for all $a \;\varepsilon\; A$, remove all vectors in $\cup_a A_T(a)$ that do not achieve the maximum in max $\{\eta\alpha: \alpha \varepsilon \cup_a A_T(a)\}$ for any $\eta \;\varepsilon\; N$. (See Smallwood and Sondik (1973) for a procedure for removing such vectors.) Then, A' is the collection of all vectors in $\cup_a A_T(a)$ that have not been removed.

UPPER BOUNDS FOR THE MAXIMAX PROBLEM

Piecewise linear and convex upper bounds are now determined for L^k. A procedure for computing these bounds is presented.

Assume A2 holds for all M^k. We will recursively construct the function L_U^k and the set of vectors B_U^k as follows. Let $B_U^K(a^0,\ldots,a^{K-1}) = \{\rho^K(a^0,\ldots,a^{K-1})\}$ and $L_U^K(\eta^K,a^0,\ldots,a^{K-1}) = \eta^K\rho^K(a^0,\ldots,a^{K-1})$. Assume $B_U^{k+1}(a^0,\ldots,a^k)$ is known and that $L_U^{k+1}(\eta^{k+1},a^0,\ldots,a^k) = \max\{\eta^{k+1}\alpha:\alpha \in B_U^{k+1}(a^0,\ldots,a^k)\}$. By Lemma 4(c) $I(B,E,Q_e,\eta)$ is piecewise linear and convex on N. Thus, for each $a^k \varepsilon A^k, k<K$,

$$L_U^k(\eta^k,a^0,\ldots,a^k) = \eta^k\rho^k(a^0,\ldots,a^k) + \beta I(B,E,Q_e,\eta)$$

is piecewise linear and convex on N, where $B=B_U^{k+1}(a^0,\ldots,a^k)$, $E=E^k(a^0,\ldots,a^k)$, and $Q_e=Q_e^k(a^0,\ldots,a^k)$. Define

$$L_U^k(\eta^k,a^0,\ldots,a^{k-1}) = \max_{a^k \varepsilon A^k} L_U^k(\eta^k,a^0,\ldots,a^k),$$

which is also piecewise linear and convex on N, and hence there is a set $B_U^k(a^0,\ldots,a^{k-1})$ such that $L_U^k(\eta^k,a^0,\ldots,a^{k-1}) = \max\{\eta^k\alpha:\alpha \varepsilon B_U^k(a^0,\ldots,a^{k-1})\}$.

PROPOSITION 3. Assume $M^k(\cdot,a^0,\ldots,a^k)$ satisfies A2 for all $a^\ell \varepsilon A^\ell, \ell=0,\ldots,k$ and all $k=0,\ldots,K-1$. Then,

(a) $L_U^k(\cdot,a^0,\ldots,a^{k-1})$ is piecewise linear and convex on N for all $k=0,\ldots,K$,

(b) $L^k \leq L_U^k$, $k=0,\ldots,K$.

We now examine in more detail the determination of $\mathcal{B}_U^k(a^o,\ldots,a^{k-1})$ from $\mathcal{B}_U^{k-1}(a^o,\ldots,a^{k-1},a^k)$, for all $a^k \varepsilon A^k$, which is analogous to determining $\mathcal{B}'$ from $\mathcal{B}(a)$, $E(a)$, and $Q_e(a)$, for all $a \;\varepsilon\; A$, where:

$$\max_{\alpha'\varepsilon\mathcal{B}'} \eta\alpha' = \max_{a\varepsilon A} \{\eta\rho(a) + \beta I[\mathcal{B}(a),E(a),Q_e(a),\eta]\}.$$

Consider the following procedure:

(1) For each $\alpha \;\varepsilon\; \mathcal{B}(a)$ put $\rho(a)+\beta Q_e\alpha$ into $\mathcal{B}_T(a)$, where $e\varepsilon\{1,\ldots,E(a)\}$ is such that for any (and hence all) $\eta \;\varepsilon\; \mathcal{N}$, $\eta(Q_e-Q_{e'})\alpha\geq 0$ for all $e'\varepsilon\{1,\ldots,E(a)\}$.

(2) Once step (1) has been accomplished for all $a \;\varepsilon\; A$, remove all vectors in $\mathcal{U}_a\mathcal{B}_T(a)$ that do not achieve the maximum in $\max\{\eta\alpha : \alpha\varepsilon\mathcal{U}_a\mathcal{B}_T(a)\}$ for any $\eta \;\varepsilon\; \mathcal{N}$. Then, $\mathcal{B}'$ is the collection of all vectors in $\mathcal{U}_a\mathcal{B}_T(a)$ that have not been removed.

UPPER (LOWER) BOUNDS ON $J^k(L^k)$:

Piecewise linear, convex upper (lower) bounds are determined on $J^k(L^k)$; a procedure for computing these bounds is presented. Assume A2 holds for all

Let J_*^k and L_*^k be defined as are J^k and L^k except assume the evolution of the trade-off vectors is constrained by the sets $M_*^k(\eta^k,a^o,\ldots,a^k)$.

PROPOSITION 4. (a) Assume that $M_*^k(\eta^k,a^o,\ldots,a^k) \subseteq M^k(\eta^k,a^o,\ldots,a^k)$ for all $\eta^k,a^o,\ldots,a^k$, and k. Then, $J^k \leq J_*^k \leq L_*^k \leq L^k$, for all $k=0,\ldots,K$.

(b) Assume that there is a matrix $F^k(a^o,\ldots,a^k)$ such that $\{\eta^k F^k(a^o,\ldots,a^k)\} = M_*^k(\eta^k,a^o,\ldots,a^k)$ for all $\eta^k,a^o,\ldots,a^k$, and k. Then, $Z_*^k = J_*^k = L_*^k$ is piecewise linear and convex on N for all k. Furthermore, if $\eta^k F^k(a^o,\ldots,a^k) \varepsilon M^k(\eta^k,a^o,\ldots,a^k)$ for all $\eta^k,a^o,\ldots,a^k$, and k, then there exist a piecewise linear and convex upper bound on J^k and a piecewise linear and convex lower bound on L^k, for all $k=0,\ldots,K$.

We remark that if F is a stochastic matrix, then $\eta \; \varepsilon \; N$ implies $\eta \; F \; \varepsilon \; N$. Therefore, in seeking piecewise linear and convex upper and lower bounds on L^k and J^k, respectively, it is desirable to seek stochastic $F^k(a^o,\ldots,a^k)$. One such matrix is the identity matrix; thus, if the "status quo" is a possibility, i.e. if $\eta \; \varepsilon \; M^k(\eta,a^o,\ldots,a^k)$ for all $\eta,a^o,\ldots,a^k$, and k, then such bounds exist.

Observe that when there is a matrix $F^k(a^o,\ldots,a^k)$ such that $\{\eta^k F^k(a^o,\ldots,a^k)\} = M_*^k(\eta^k,a^o,\ldots,a^k)$ for all η^k, $a^o,\ldots,a^k$, and k, then L_*^k can be determined recursively as follows:

$$L_*^k(\eta^k) = \max_{a^k \varepsilon A^k} \left\{ \eta^k \rho^k(a^o,\ldots,a^k) \right.$$

$$\left. + \beta \max\{\eta^k F^k(a^o,\ldots,a^k)\alpha : \alpha \varepsilon A_*^{k+1}(a^o,\ldots,a^k)\} \right\},$$

where $L_*^K(\eta^K,a^o,\ldots,a^{K-1}) = \eta^K \rho^K(a^o,\ldots,a^{K-1})$ and $A_*^{k+1}(a^o,\ldots,a^k)$ is such that $L_*^{k+1}(\eta^{k+1},a^o,\ldots,a^k) = \max\{\eta^{k+1}\alpha : \alpha \varepsilon A_*^{k+1}(a^o,\ldots,a^k)\}$. Determination of $A_*^k(a^o,\ldots,a^{k-1})$ from $A_*^{k+1}(a^o,\ldots,a^{k-1},a^k)$, for all $a^k \varepsilon A^k$, is analogous to determining A' from $A(a)$ and $F(a)$, for all $a \ \varepsilon \ A$, where:

$$\max_{\alpha' \varepsilon A'} \eta\alpha' = \max_{a \varepsilon A} \left\{ \eta\rho(a) + \beta \max\{\eta F(a)\alpha : \alpha \varepsilon A(a)\} \right\}.$$

Consider the following procedure:

(1) For each $\alpha \ \varepsilon \ A(a)$, put $\rho(a)+\beta F(a)\alpha$ into $A_T(a)$.

(2) Once step (1) has been accomplished for $a \ \varepsilon \ A$, remove all vectors in $U_a A_T(a)$ that do not achieve the maximum in $\max\{\eta\alpha : \alpha \varepsilon U_a A_T(a)\}$ for some $\eta \ \varepsilon \ N$. Then, A' is the collection of all vectors in $U_a A_T(a)$ that have not been removed.

THE FINITE M^k CASE:

Thus far, our assumptions describing the form of the M^k have been primarily motivated by what information a DM might be able to provide with regard to their form. We then developed various analytical results that provided insight into how alternatives should be selected and information regarding what should be expected if the alternatives were so selected. For completeness, we now reexamine the basic model with the primary concern being tractability. This reexamination is accomplished in two ways, both involving finite M^k sets. First, we consider the maximax problem under the assumption that all elements in $M^k(\eta^k,\ldots)$ are linear in η^k and are members of N. We then show that L^k is piecewise linear and convex and hence easily computable. Second, we assume that there exists a probability distribution on M^k and develop conditions which guarantee that the solution of the resulting DPE is piecewise linear and convex. Interest in such an approach is due to the possibility that mechanisms M and B may be highly unrealistic ways of modeling the selection of η^{k+1} from M^k. Interesting and direct analogies are drawn between the probabilistic results and Bayes' Rule and an algorithm due to Smallwood and Sondik (1973). We remark that the maximin problem is not considered under the assumption that the M^k are finite. Unfortunately, J^k is no longer convex under this assumption and hence is relatively intractable.

<u>The Maximax Problem</u>. Assume $M^k(\eta^k,a^o,\ldots,a^k) = \{\eta^k Q_e{}^k(a^o,\ldots,a^k):e=1,\ldots,E^k(a^o,\ldots,a^k)\}$ and that $Q_e{}^k(a^o,\ldots,a^k)$ is stochastic for each e, for all $\eta^k,a^o,\ldots,a^k$, and k. A straightforward reexamination of the proofs of Lemma 4(c) and Proposition 3 in light of these assumptions implies the following result.

PROPOSITION 5. $L^k = L_U^k, k=0,\ldots,K$.

Thus, L^k is now easily computable and is as computable as its upper bound was under the hypotheses of Proposition 3.

<u>A Probabilistic Approach</u>. Assume probabilities of the form $P^k[\eta^{k+1}(e)|\eta^k,a^o,\ldots,a^k], e=1,\ldots,E^k(a^o,\ldots,a^k)$, are given, where $M^k(\eta^k,a^o,\ldots,a^k) = \{\eta^{k+1}(e), e=1,\ldots,E^k(a^o,\ldots,a^k)\}$. We shall call this mechanism for selecting η^{k+1} from M^k mechanism P. The associated DPE is $Z^k = H_*^k Z^{k+1}$, where:

- $Z^K(\eta^K,a^o,\ldots,a^{K-1}) = \eta^K\rho^K(a^o,\ldots,a^{K-1})$,

- $h^k(\eta^k,a^o,\ldots,a^{k-1},a^k,v)$

  $= \eta^k\rho^k(a^o,\ldots,a^k)$

  $+\beta\sum_e P[\eta^{k+1}(e)|\eta^k,a^o,\ldots,a^k]v[\eta^{k+1}(e)]$,

- $[H_\delta^k v](\eta^k,a^o,\ldots,a^{k-1})$

  $=h^k[\eta^k,a^o,\ldots,a^{k-1},\delta(\eta^k,a^o,\ldots,a^{k-1}),v]$,

- $H_*^k v = \sup_\delta H_\delta^k v$.

Call the strategy composed of policies which achieve the supremum in $Z^k = H_*^k Z^{k+1}$ an <u>optimal expected</u> strategy.

In order to insure piecewise linearity and convexity of the Z^k, assume that if $\eta^{k+1}(e) \in M^k(\eta^k,a^o,\ldots,a^k)$, then there is a $Q_e^k(a^o,\ldots,a^k)$ such that

$$\eta^{k+1}(e) = \eta^k Q_e(a^o,\ldots,a^k) \Big/ P[\eta^{k+1}(e)|\eta^k,a^o,\ldots,a^k] \ .$$

In order to insure that $\eta^{k+1}(e) \varepsilon N$ for all η^k, assume further that $Q_e(a^o,\ldots,a^k)$ has all nonnegative elements and that $P[\eta^{k+1}(e)|\eta^k,a^o,\ldots,a^k]$ acts as a normalizing term; i.e.,

$$\eta^k Q_e(a^o,\ldots,a^k)\underline{1} = P[\eta^{k+1}(e)|\eta^k,a^o,\ldots,a^k]$$

Since $\sum_e P[\eta^{k+1}(e)|\eta^k,a^o,\ldots,a^k] = 1$,

$$\sum_j \sum_e Q_e^k(a^o,\ldots,a^k)_{ij} = 1$$

for all $i=1,\ldots,N$. Under these assumptions, $Q_e^k(a^o,\ldots,a^k)_{ij}$ can be interpreted as the probability that a controlled Markov process will go from state i at stage k to state j at stage k+1 and e is a presumably noise corrupted observation of the state at stage k and/or k+1. Note also that under these assumptions η^k can be interpreted as an a priori probability, where $\eta^{k+1}(e)$ is the a posteriori associated with observation e. Thus, the problem is a slight generalization, at least in form, of the partially observed Markov decision process, a solution procedure for which is presented in Smallwood and Sondik (1973).

We remark that it is unlikely that the Q_e^k could be assessed directly and that probably what could be assessed for a given $\eta^k,a^o,\ldots,a^k$ is $E^k(a^o,\ldots,a^k)$ and $\eta^{k+1}(e),e=1,\ldots,E^k(a^o,\ldots,a^k)$. We would therefore have the following linear feasibility problem: given $\eta \ \varepsilon \ N$, E, and $\eta'(e)\varepsilon N$, $e=1,\ldots,E$, determine $Q_e(i,j)$ such that

$$Q_e(i,j) \geq 0 \text{ for all } e,i,j$$

$$\sum_e \sum_j Q_e(i,j) = 1 \text{ for all } i$$

$$\eta'(e) = \eta Q_e \Big/ \eta Q_e \underline{1} \text{ for all } e$$

Note that there are N^2E unknowns and $N+(N-1)E$ constraints. If we wished to assume values for $P[\eta'(e)|\eta] = \eta Q_e \underline{1}, e=1,\ldots,E$, e.g., $P[\eta'(e)|\eta] = 1 \big/ E$, then there would be an additional $E-1$ constraints. Procedures for determining a solution(s) to the above problem are discussed and presented in Matneiss and Rubin (1980).

SUBOPTIMAL DESIGNS

Throughout this paper, we have presented approaches for determining Z^k or bounds on Z^k for three different mechanisms for selecting η^{k+1} from M^k:

1. mechanism M, which resulted in strategies that provide upper and lower bounds on $J^k, k=0,\ldots,K$.
2. mechanism B, which resulted in strategies that provide upper and lower bounds on $L^k, k=0,\ldots,K$, and which for the finite M^k case considered resulted in strategies that provide $L^k, k=0,\ldots K$.
3. mechanism P, which for the finite M^k case considered resulted in strategies that provide $Z^k, k=0,\ldots,K$.

A procedure for selecting alternatives based on a given preference evolution mechanism is:

1. At stage k, select alternative $a^k=\delta^k(\eta^k,a^0,\ldots,a^{k-1})$, where $\pi=\{\delta^k, k=0,\ldots,K\}$ is a strategy associated with the given preference dynamics.
2. Let the actual (which is, of course, likely to differ from the assumed) preference dynamics provide the DM with η^{k+1}. Let $k=k+1$, and return to step 1.

This procedure is identical to Witsenhausen's assumed permanance procedure where now what is assumed to be permanent is the mechanism which determines the path of the trade off weight process. Note also that this procedure generates a "closed-loop" feedback strategy. We remark that bounds determination in Section 7 which assumed that M^k was a single point produced an open-loop strategy; use of the above procedure with such a strategy produces an open-loop feedback strategy (see Bertsekas (1976), Sections 5.3, 5.4, and 5.5 for further discussion).

An alternate, and perhaps more behaviorally compatible, procedure would be to select the preference evolution mechanism to be used for alternative selection at each stage, depending, perhaps, on the mood of the DM and/ or his impression of under what mechanism preference is evolving. Obviously, many other alternate approaches for alternative selection are possible. Of course, none of the descriptions of preference dynamics considered here are likely to be entirely accurate; therefore, the resulting strategies are perhaps best viewed as suggestions or "rules of thumb" rather than procedures for selecting alternatives without further thought.

A HYPOTHETICAL EXAMPLE

We now present an example and associated numerical results for mechanisms M and B. Consider a newly certified individual who is faced with promotion or dismissal in three years. (The number of years and hence stages considered has been chosen for computational ease; in the context of this example, a more realistic number would be six.) Assume the two objectives under consideration are (1) research productivity and (2) teaching proficiency. The individual wishes to take a course of action that will maximize his (or her) perceived value at the time of the "up or out" decision. The individual is faced with the likelihood that the current relative importance of the objectives will change over the planning horizon. In the context of this example, our model requires us to assume that the individual knows precisely the current relative importance of the objectives and that there is only one outcome for each alternative selected.

Assume that $A^k=\{0,1\}, k=0,1,2$, where 0 = emphasize teaching and 1= emphasize research. Assume also that $\beta=1$, $\rho^k=0$ for all $k=0,1,2$, and that the terminal value ρ^3 is such that

$$\rho^3(a^0,a^1,a^2) = \rho(a^0)+\rho'(a^1)+\rho''(a^2)$$

where:

$$\rho(0) = \begin{bmatrix} 0 \\ .81 \end{bmatrix} \qquad \rho(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\rho'(0) = \begin{bmatrix} 0 \\ .9 \end{bmatrix} \qquad \rho'(1) = \begin{bmatrix} .9 \\ 0 \end{bmatrix}$$

$$\rho''(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \rho''(1) = \begin{bmatrix} .81 \\ 0 \end{bmatrix}$$

These values reflect two apparently realistic perceptions. First, research accomplished three years ago (presumably resulting in a very recent publication) is more valuable now than research accomplished two years ago (presumably awaiting publication) which is in turn more valuable than research accomplished last year (presumably awaiting reviewer comments). Second, recent, high quality teaching is better remembered and hence more valuable than less recent, high quality teaching.

Let $M^k(\eta^k, a^o, \ldots, a^k) = M^k(\eta^k) = \{\eta' \varepsilon N : -\varepsilon \le \eta_1^k - \eta_1' \le \varepsilon\}$. We consider three cases:

(a) the maximin case for ε=0.1

(b) the maximax case for ε=0.1

(c) the ε=0.0 case (i.e, the trade-off weights are assumed to be static).

Use of the algorithms developed in Sections 5, 6, and 7 provide the following results:

$$A_L^0 = \left\{ \begin{bmatrix} -0.813 \\ 1.897 \end{bmatrix}, \begin{bmatrix} 1.312 \\ 1.312 \end{bmatrix}, \begin{bmatrix} 0.73 \\ 1.63 \end{bmatrix}, \begin{bmatrix} 1.63 \\ 0.73 \end{bmatrix}, \begin{bmatrix} 1.897 \\ -0.813 \end{bmatrix} \right\}$$

$$B_U^0 = \left\{ \begin{bmatrix} 0.813 \\ 3.523 \end{bmatrix}, \begin{bmatrix} 3.523 \\ 0.813 \end{bmatrix} \right\}$$

$$A_*^0 = \left\{ \begin{bmatrix} 0 \\ 2.71 \end{bmatrix}, \begin{bmatrix} 1 \\ 1.9 \end{bmatrix}, \begin{bmatrix} 1.9 \\ 0 \end{bmatrix}, \begin{bmatrix} 2.71 \\ 0 \end{bmatrix} \right\}$$

Graphs of the associated functions are presented in Figure 1. Note that

$J^0 \leq Z_*^0 \leq L_U^0$, as expected. The strategies associated with these cases are presented in Table I.

We remark that these strategies exhibit significantly different behavioral characteristics. Note that both the maximin strategy and the permanent preference strategy encourage an early interest in research and a later interest in teaching, where the encouragement on the part of the maximin strategy to have "all bases covered" is comparatively quite strong. The strategy determined from the upper bound on L^k, however, suggests initially determining a preference (presumably, on the part of a promotions committee) toward one of the two objectives and then emphasize the attainment of value associated with that objective over the entire problem horizon. Such a strategy makes sense under the assumption that mechanism B is in effect since mechanism B, i.e., the promotions committee, is watching the alternatives selected by the individual and changing its view of the relative importance of the objectives so as to increase the individual's perceived value as much as possible at the end of the three year period. We suspect that many would feel that mechanism B is an unrealistic model of how a typical promotions committee establishes criteria for promotion.

CONCLUSIONS

We have presented a model of sequential decisionmaking under uncertain future preferences. We have claimed that the dynamic preference model which has received primary attention, set inclusion and mechanisms B and M, provides few constraints on, and hence enhances, the assessment of the dynamic structure. The validity of this claim requires future examination; appropriate assessment procedures have yet to be developed.

Although we have attempted to justify the decisionmaking model's validity, primary emphasis has been placed on examining the model's tractability. Algorithms have been developed which provide upper and lower bounds on the optimal value functions of interest, and strategies associated with these bounds have been obtained. Circumstances have been investigated as to when these bounds are exact. We remark that the maximax results can be used to give the DM an insight as to how to exert control over the tradeoff weight vector evolution, if he is able to exert such control.

Two directions for future research appear to be particularly worthwhile from the perspective of model validity: (1) generalizing the model to include a probabilistically evolving, perhaps partially observed state, and (2) relaxing the requirement that the current tradeoff weight vector is precisely known. The former direction would allow the insertion of chance nodes in the decision tree, which would permit the combination of the model of future preference uncertainty presented in this paper and the model considered by Kreps (1975, 1977). The latter direction would allow the decisionmaker to be unsure and/or purposely vague about current tradeoff weights.

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	(a)	(b)	(c)
$\delta^0(\eta^0)=$	$\begin{cases} 0 \text{ if } \eta_1^0 \le 0.15 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^0 \le 0.50 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^0 \le 0.45 \\ 1 \text{ otherwise} \end{cases}$
$\delta^1(\eta^1,0)=$	$\begin{cases} 0 \text{ if } \eta_1^1 \le 0.30 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^1 \le 0.63 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^1 \le 0.50 \\ 1 \text{ otherwise} \end{cases}$
$\delta^1(\eta^1,1)=$	$\begin{cases} 0 \text{ if } \eta_1^1 \le 0.55 \\ 1 \text{ if otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^1 \le 0.43 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^1 \le 0.50 \\ 1 \text{ otherwise} \end{cases}$
$\delta^2(\eta^2,0,0)=$	$\begin{cases} 0 \text{ if } \eta_1^2 \le 0.45 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^2 \le 0.65 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^2 \le 0.55 \\ 1 \text{ otherwise} \end{cases}$
$\delta^2(\eta^2,0,1)=$	$\begin{cases} 0 \text{ if } \eta_1^2 \le 0.55 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^2 \le 0.55 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^2 \le 0.55 \\ 1 \text{ otherwise} \end{cases}$
$\delta^2(\eta^2,1,0)=$	$\begin{cases} 0 \text{ if } \eta_1^2 \le 0.55 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^2 \le 0.55 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^2 \le 0.55 \\ 1 \text{ otherwise} \end{cases}$
$\delta^2(\eta^2,1,1)=$	$\begin{cases} 0 \text{ if } \eta_1^2 \le 0.65 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^2 \le 0.45 \\ 1 \text{ otherwise} \end{cases}$	$=\begin{cases} 0 \text{ if } \eta_1^2 \le 0.55 \\ 1 \text{ otherwise} \end{cases}$

TABLE 1: The (a) Maximin Strategy for ε=0.1, (b) Strategy Associated with the Maximax Problem for ε=0.1, and (c) ε=0.0 Case, for the Example.

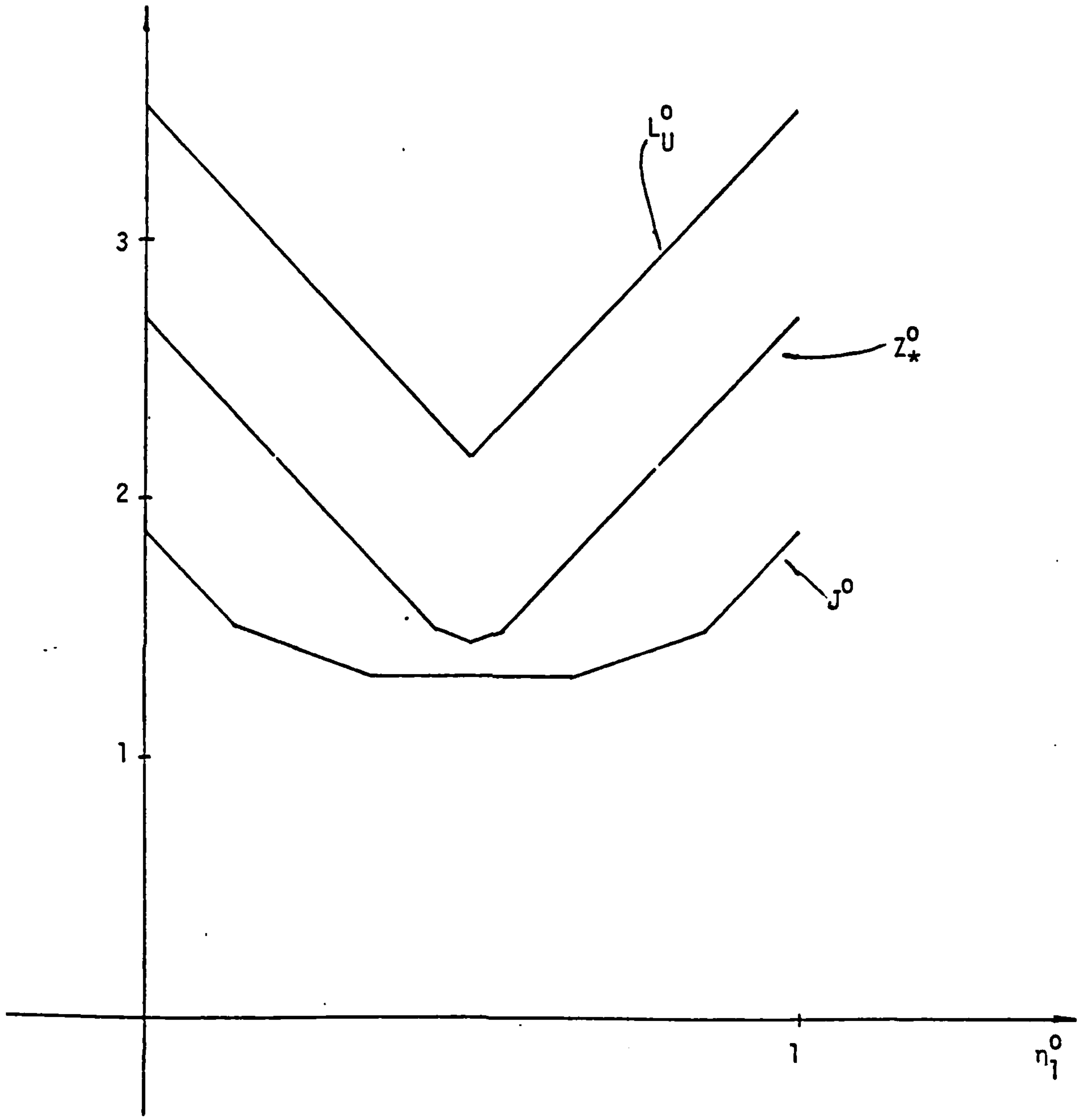

FIGURE 1: J^o, Z^o, and L_U^o for the Example.

APPENDIX

Proof of Lemma 1:

Assume $\bar{\eta}^{mi} \varepsilon M(\eta^i)$ for all m and that $\lim_m v(\bar{\eta}^{mi}) = g(\eta^i), i=1,2$. We note that

$$g[\lambda\eta^1+(1-\lambda)\eta^2]$$

$$\leq \lim_m \inf v\,[\lambda\bar{\eta}^{m1}+(1-\lambda)\bar{\eta}^{m2}]$$

$$\leq \lambda \lim_m v(\bar{\eta}^{m1})+(1-\lambda)\lim_m v(\bar{\eta}^{m2})$$

$$= \lambda g(\eta^1)+(1-\lambda)g(\eta^2),$$

where the second inequality is due to the convexity of v. □

Proof of Proposition 1:

Clearly, J^K is convex on N; assume J^{k+1} is also. By Lemma 1,

$$\inf\{J^{k+1}(\eta',a^o,\ldots,a^{k-1},a):\eta'\varepsilon M^k(\eta,a^o,\ldots,a^{k-1},a)\}$$

is convex in η on N for each $(a^o,\ldots,a^{k-1},a)\varepsilon A^o x\ldots xA^k$, and thus $h^k(\eta,a^o,\ldots a^{k-1},a,J^{k+1})$ is convex in η on N for each $(a^o,\ldots,a^{k-1},a)\varepsilon A^o x\ldots xA^k$. The result follows from the fact that the supremum of convex functions is convex and from induction. □

Proof of Lemma 2:

Since N is convex, it is sufficient to show that M' satisfies A1. Let $\eta' \varepsilon M(\eta^1)$ and $\eta'' \varepsilon M(\eta^2)$. Then, there exist $\{\sigma_e'\}$ and $\{\sigma_e''\}$ such that

$\eta' = \sum_e \sigma_e' \eta^1 Q_e$ and $\eta'' = \sum_e \sigma_e'' \eta^2 Q_e$, where $\sigma_e' \geq 0, \sigma_e'' \geq 0, \sum_e \sigma_e' = 1$, and $\sum_e \sigma_e'' = 1$. Define $\bar{\eta}' = \sum_e \sigma_e'' \eta^1 Q_e$ and $\bar{\eta}'' = \sum_e \sigma_e' \eta^2 Q_e$. Assumption 2A(ii) implies that $\eta^1(Q_e - Q_{e'}) = \eta^2(Q_e - Q_{e'})$ for all e, e', or equivalently, that $(\eta^2 - \eta^1)Q_e$ is independent of e. Thus, $\sum_e \sigma_e'(\eta^2 - \eta^1)Q_e = \sum_e \sigma_e''(\eta^2 - \eta^1)Q_e$, or equivalently, $\bar{\eta}' - \eta' = \eta'' - \bar{\eta}''$. A standard algebraic argument then implies that $\lambda\eta' + (1-\lambda)\eta'' = \lambda[\lambda\eta' + (1-\lambda)\,\bar{\eta}''] + (1-\lambda)[\lambda\,\bar{\eta}' + (1-\lambda)\eta'']$ But $\lambda\eta' + (1-\lambda)\,\bar{\eta}''$ and $\lambda\,\bar{\eta}^1 + (1-\lambda)\eta''$ are easily shown to be members of $M'[\lambda\eta^1 + (1-\lambda)\eta^2]$ and hence so is $\lambda\eta' + (1-\lambda)\eta''$. $\square$

Proof of Lemma 3:

Note $\eta Q_e \alpha - \eta Q_{e'} \alpha = \eta(Q_e - Q_{e'})\alpha$, which is independent of η due to the fact that $\eta \,\varepsilon\, N$. $\square$

Proof of LEMMA 4: (a) Piecewise linearity and convexity. It is sufficient to show that

$$\max_\alpha \min_e \eta Q_e \alpha$$

is piecewise linear and convex. From Lemma 3, $\min_e Q_e \alpha$ is linear for each $\alpha \varepsilon A$. The result then follows from the fact that the maximum of a finite collection of linear functions is piecewise linear and convex.

$G(A, E, Q_e, \eta) \leq g(\eta)$. Let

$\mathcal{D}^*$ be the set of all points in N that achieve the minimum in

$$\min_{\eta \varepsilon N} \max_{\alpha \varepsilon A} \eta\alpha,$$

and define $N^* = \{\eta : \mathcal{D}^* \cap M(\eta) \neq \phi\}$.

Assume $\eta \varepsilon N^*$. Then there exists an $\eta^* \varepsilon \mathcal{D}^* \cap M(\eta)$ such that

$$\max_{\alpha \varepsilon A} \eta^* \alpha = \min_{\eta \varepsilon N} \max_{\alpha \varepsilon A} \eta\alpha.$$

For each $\alpha \varepsilon A$,

$$\min_{\eta' \varepsilon M(\eta)} \eta' \alpha = \min_e \eta Q_e \alpha \leq \eta^* \alpha;$$

thus,

$$\max_\alpha \min_e \eta Q_e \alpha \leq \max_\alpha \eta^* \alpha = g(\eta).$$

Let $\eta \notin N^*$. Then,

$$\max_\alpha \min_e \eta Q_e \alpha$$
$$\leq \min_e \max_\alpha \eta Q_e \alpha$$
$$= \min_{\eta' \varepsilon M(\eta)} \max_{\alpha \varepsilon A} \eta' \alpha = g(\eta),$$

where the inequality is due to Rockafellar (1970), Lemma 36.1, and the first equality follows from the assumption $\eta \notin N^*$.

(b) Since the lower bound is exact for all N on N^*, we wish to show that the lower bound is exact on N^* complement when N=2. The sum-to-one property of elements in N allows us to assume that $\eta \varepsilon$ R and hence $M(\eta)$, for any $\eta \varepsilon N$, will have (at most) two extreme points, η^1 and η^2. In general,

$$\max_\alpha \min_e \eta Q_e \alpha$$
$$= \max_\alpha \min_{\eta' \varepsilon M(\eta)} \eta' \alpha \leq g(\eta)$$
$$\leq \min_e \max_\alpha \eta Q_e \alpha$$

where the equality and the second inequality are obvious and the first inequality is due to Rockafellar (1970), Lemma 36.1. Thus, it is sufficient to show that

$$\max_\alpha \min_e \eta Q_e \alpha = \min_e \max_\alpha \eta Q_e \alpha.$$

Assume $\eta^* \leq \eta^1 \leq \eta^2$ for any $\eta^* \varepsilon D^*$. Then, $\min_e \max_\alpha \eta^e \alpha = \max_\alpha \eta^1 \alpha = \eta^1 \alpha^*$.

We now wish to examine $\max_\alpha \min_e \eta^e \alpha$. Note that $\eta^e \alpha^* \geq \eta^e \alpha$, e=1,2, for any α such that $\eta\alpha$ has negative slope. ($\eta\alpha^*$ has nonnegative slope since $\eta^* \leq \eta^1$ and $\min_e \max_\alpha \eta^e \alpha = \eta^1 \alpha^*$.) Thus, we can eliminate consideration of any $\alpha \varepsilon A$ such that $\eta\alpha$ has negative slope. Now, for α's such that $\eta\alpha$ has nonnegative slope,

$\min_e \eta^e\alpha = \eta^1\alpha$, and hence $\max_\alpha \min_e \eta^e\alpha = \max_\alpha \eta^1\alpha = \eta^1\alpha^*$. A similar argument holds if $\eta^1 \leq \eta^2 \leq \eta^*$, for any $\eta^* \varepsilon D^*$.

(c) Clearly, $I(A,E,Q_e,\eta)$ is piecewise linear and convex. A simple argument proves that $\max_{\eta'} \max_\alpha \eta'\alpha = \max_\alpha \max_{\eta'} \eta'\alpha$. Note that when η is such that $\eta Q_e \varepsilon N$ for all e, $\max_{\eta'} \eta'\alpha = \max_e \eta Q_e\alpha$. When there is an e such that $\eta Q_e \notin N$, $M(\eta) \subseteq CH\{\eta Q_e, 1 \leq e \leq E\}$ and $M(\eta) \neq CH\{\eta Q_e, 1 \leq e \leq E\}$; therefore, $\max_{\eta'} \eta'\alpha \leq \max_e \eta Q_e \alpha$. □

Proof of Proposition 2:

(a) The construction of J_L^k and induction guarantee that J_L^k is piecewise linear and convex on N for all $k=0,\ldots,K$.

(b) Clearly, $J_L^K \leq J^K$; assume $J_L^{k+1} \leq J^{k+1}$. Application of Lemma 4(a) implies that

$$J_L^k(\eta^k, a^o, \ldots, a^k)$$

$$\leq h^k(\eta^k, a^o, \ldots, a^{k-1}, a^k, J_L^{k+1})$$

$$\leq h^k(\eta^k, a^o, \ldots, a^{k-1}, a^k, J^{k+1})$$

for all $a^k \varepsilon A^k$. Thus, $J_L^k(\eta^k, a^o, \ldots, a^{k-1}) \leq J^k(\eta^k, a^o, \ldots, a^{k-1})$, and the result follows by induction.

(c) An argument similar to part (b) and use of Lemma 4(b) provides the result. □

Proof of Proposition 3:

The proof of part (a) is identical to the proof of Proposition 2(a). Proof of part (b) is analogous to the proof of Proposition 2(b), using Lemma 4(c) rather than Lemma 4(a).

Proof of Proposition 4:

(a) Follows directly from the definitions.

(b) The fact that $J_*^k = L_*^k$ for all k is a result of the fact that $G(A, E, Q_e, n) = I(A, E, Q_e, n)$ when $E=1$ and $nQ_1 \in N$ for all n. $\square$

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A GENERALIZED MODEL OF SEQUENTIAL DECISIONMAKING UNDER RISK*

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ABSTRACT

We present and analyze a generalization of the standard decision analysis model of sequential decisionmaking under risk. The decision tree is assumed given and all probabilities are assumed to be known precisely. Utility values are assumed to be affine in an imprecisely known parameter. The affine form is sufficiently general to allow importance weights or the utility values themselves to be represented by the imprecise parameter. Parameter imprecision is described by set inclusion. A relation on all available alternatives is assumed given for each decision node. The intent of each (not necessarily complete) relation is to model the decisionmaker's directly expressed preferences among the available alternatives at the associated decision node. A numerical procedure is developed to determine the set of all strategies that may be optimal and the corresponding set of all possible parameter values. An example illustrates the procedure.

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I. INTRODUCTION

The standard decision analysis model of sequential decisionmaking under risk (Keeney and Raiffa, 1976) assumes that the decision tree, fact information in the form of probabilities, and preference information in the form of utilities are known precisely. Given such structural and parameter value information, the decision analysis paradigm provides an optimal strategy, i.e., a best alternative to select at each decision node in order to maximize expected utility.

In reality, utilities and probabilities may be difficult to precisely assess (due to, for example, the time and stress associated with their assessment) and may not need to be precisely assessed in order to determine information sufficient for strategy selection; see Fishburn (1965), Sarin (1977a,b), and White, et al., (1980,1982a) for further discussion. Furthermore, the decisionmaker may be willing and able to directly provide information about the relative desirability of the available alternatives for at least some of the decision nodes in the tree; e.g., a physician may be willing and able to directly express a preference for selecting diagnostic test A over diagnostic test B in diagnosing patients having a specific collection of signs, symptoms, and laboratory test results. Thus, the input assumptions associated with the standard decision analysis model may be sufficiently demanding to act as a barrier to the acceptability of the decision analysis paradigm and yet may ignore readily available information that could be of significant value in strategy selection.

In this paper, we assume that the decision tree is given and that all probabilities are known precisely. We then diverge from the aforementioned input assumptions for the standard decision analysis model by assuming that utility values may be imprecisely described as being members of given sets and that a (possibly null, not necessarily complete) preference relation on the set of all available alternatives is given for each decision node. Set inclusion has been selected as a model of parameter imprecision due to its simplicity and behavioral justifiability. The intent of the relations is to model the decisionmaker's directly expressed preferences among the available alternatives at each decision node. The problem formulation to be examined thus includes two interesting cases:

(i) the standard decision analysis model of sequential decisionmaking; i.e., the case where the utilities are described precisely and where all relations are null, and

(ii) the inverse decision analysis problem; i.e., the case where the utilities, or some aspect of the utilities such as the importance weights, are unknown and where the relations are sufficiently discriminating to provide an optimal policy.

We remark that the output of the inverse decision analysis problem is the set of all utilities that permit the given strategy to be optimal and that the general problem of determining the set of all parameter values for which a given strategy is optimal has been examined in a variety of contexts, e.g., control theory (Casti, 1980), mathematical programming (Bitran et al., 1981), and decision analysis (White, et al., 1982b). We then develop an analytic procedure for determining the set of all strategies that may be optimal and the corresponding set of utility values. This analytic procedure serves as the basis of a numerical procedure that is a generalization of results presented in (White and El Deib, 1982; White, et al., 1982b).

This paper is outlined as follows. The generalized sequential decision analysis model is formulated in Section II and analysed in Section III. A numerical example is presented in Section IV. Conclusions are discussed in the final section.

II. PROBLEM FORMULATION

Assume that a K stage decision tree is given. For notational simplicity, we will assume that the appropriate number of decision nodes with single actions and chance nodes with single outcomes have been added to branches having less than K stages in order to insure all branches have exactly K stages. Let S_k be the finite set of all k^{th} stage decision nodes, $k=0,1,\ldots,K-1$, and let S_K be the finite set of all terminal nodes. The finite set $A(s_k)$ represents the set of all alternatives available for selection at decision node $s_k \varepsilon S_k$, $k=0,1,\ldots,K-1$. If alternative $a_k \varepsilon A(s_k)$ is selected at decision node $s_k \varepsilon S_k$, then the resulting realization of the random variable associated with the appropriate chance node is restricted to be a member of the finite set $Z(s_k,a_k)$, $k=0,1,\ldots,K-1$. The probability that realization $z_{k+1} \varepsilon Z(s_k,a_k)$ will result after selecting alternative $a_k \varepsilon A(s_k)$ at decision node $s_k \varepsilon S_k$ is $p(z_{k+1}|s_k,a_k)$, which we assume is given precisely. We assume throughout for each $k=0,1,\ldots,K-1, s_k \varepsilon S_k$, and $a_k \varepsilon A(s_k)$, that $p(z_{k+1}|s_k,a_k) \geq 0$ for all $z_{k+1} \varepsilon Z(s_k,a_k)$ and

$$\sum_{z_{k+1} \varepsilon Z(s_k,a_k)} p(z_{k+1}|s_k,a_k) = 1.$$

Note that $s_{k+1} = (s_k,a_k,z_{k+1})$ with probability $p(z_{k+1}|s_k,a_k)$. Thus $\{s_k, k=0,1,\ldots\}$ is a controlled Markov chain.

Let $u(s_K)$ be the utility accrued at terminal node $s_K \varepsilon S_K$. Assume for each $s_K \varepsilon S_K$ there is a given scalar $u^0(s_K)$ and a given N-vector $u^1(s_K) = \text{row}\{u_1^1(s_K),\ldots,u_N^1(s_K)\}$ such that

$$u(s_K) = u^0(s_K) + u^1(s_K)\rho,$$

where the N-vector $\rho = \text{col}\{\rho_1,\ldots,\rho_N\}$, the imprecisely known parameter, is assumed to be a member of the given set $P \subseteq \mathbb{R}^N$.

The above description of utility imprecision is sufficiently general to allow for several interesting cases of parameter imprecision. For example, assume ρ is the vector of importance weights for a multiattribute, sequential decision analysis problem, N is the number of attributes, ρ_n is the importance weight of the n^{th} attribute, $u^0(s_K) = 0$, and $u_n^1(s_K)$ is the utility accrued at terminal node s_K with respect to attribute n. Then $P \subseteq \{\rho \varepsilon \mathbb{R}^N : \rho_n \geq 0, \sum_n \rho_n = 1\}$ represents the set of all possible importance weight vector values. As another example, assume for each $s_K \varepsilon S_K$, $u(s_K)$ is imprecisely known. Let N equal the number of elements in the set S_K, $u^0(s_K) = 0$, $u_n^1(s_K) = 1$ if $n = s_K$ $(=0$ if $n \neq s_K)$, for all $s_K \varepsilon S_K$. Then P $\{\rho \varepsilon R^N : 0 \leq \rho_n \leq 1\}$ represents the set of all possible utility values.

Assume that the decisionmaker has provided a relation on the alternatives at each decision node. That is, assume that there exists a set $R(s_k) \subseteq A(s_k) \times A(s_k)$ for each $s_k \varepsilon S_k$, $k=0,1,\ldots,K-1$, where $(a_k', a_k) \varepsilon R(s_k)$ indicates that the decisionmaker perceives alternative a_k' to be at least as preferred as alternative a_k at decision node s_k. If $R(s_k) = \phi$, then the decisionmaker has expressed no preference with respect to the alternatives at decision node s_k.

A (parameter independent) strategy π is a rule that selects an alternative in $A(s_k)$ for each $s_k \varepsilon S_k$, $k=0,1,\ldots,K-1$; i.e., $\pi(s_k) \varepsilon A(s_k)$ for each $s_k \varepsilon S_k$, $k=0,1,\ldots,K-1$.

The problem objective is as follows. Given:

1. the decision tree structure, i.e., K, S_k for all k, $A(\cdot)$, and $Z(\cdot,\cdot)$,
2. the probability structure, i.e., $p(\cdot|\cdot,\cdot)$,

III. A SOLUTION PROCEDURE

We now examine a procedure for achieving the problem objective. Let ξ be a <u>parameter</u> <u>dependent</u> strategy; i.e., let $\xi(s_k,\rho)\varepsilon A(s_k)$ for every $s_k \varepsilon S_k$, $k=0,1,\ldots,K-1$, $\rho\varepsilon P$. Define $f_k^{\xi}(s_k,\rho)$ to be the expected utility accrued if strategy ξ is used, if ρ is the value of the parameter, and if the problem were to commence at decision node $s_k \varepsilon S_k$. Define $f_k(s_k,\rho) = \sup_{\xi} f_k^{\xi}(s_k,\rho)$; i.e., $f_k(s_k,\rho)$ represents the optimal expected utility accrued if ρ is the parameter value and if the problem were to begin at decision node s_k. It is straightforward to show that f_k satisfies the following dynamic programming equation and boundary condition:

$$f_k(s_k,\rho) = \max_{a_k \varepsilon A(s_k)} \{h(s_k, \rho, a_k, f_{k+1})\}$$

where

$$h(s_k,\rho,a_k,v) = \sum_{z_{k+1}} p(z_{k+1}|s_k,a_k)v(s_k,a_k,z_{k+1},\rho)$$

and

$$f_K(s_K,\rho) = u^0(s_K) + u^1(s_K)\rho.$$

It has been shown in White and El Deib (1982) that for each $s_k \varepsilon S_k$ and each k $f_k(s_k,\rho)$ is piecewise affine and convex in ρ, and hence there exists a set $A(s_k)$ of pairs (α,γ) such that

$$f_k(s_k,\rho) = \max\{\alpha + \gamma\rho : (\alpha,\gamma)\varepsilon A(s_k)\}.$$

It is assumed that $A(s_k)$ represents the smallest set of such pairs.

Clearly, determining $f_k(s_k,\rho)$ from f_{k+1} is equivalent to determining $A(s_k)$ from $A(s_k,a_k,z_{k+1})$ for all $z_{k+1}\varepsilon Z(s_k,a_k)$ and $a_k\varepsilon A(s_k)$. The latter determination can be accomplished by noting that

$$f_k(s_k,\rho) = \max_{a_k}\{\max\{\alpha' + \gamma'\rho\}\}$$

where the first maximization is over all $a_k\varepsilon A(s_k)$ and the second maximization is over all pairs (α',γ') of the form

$$\alpha'=\alpha'(s_k,a_k) = \sum_{z_{k+1}} p(z_{k+1}|s_k,a_k)\alpha(s_k,a_k,z_{k+1})$$

$$\gamma'=\gamma'(s_k,a_k) = \sum_{z_{k+1}} p(z_{k+1}|s_k,a_k)\gamma(s_k,a_k,z_{k+1}),$$

for $\{\alpha(s_k,a_k,z_{k+1}),\gamma(s_k,a_k,z_{k+1})\}\varepsilon A(s_k,a_k,z_{k+1}),z_{k+1}\varepsilon Z(s_k,a_k)$. We remark that if

$$\alpha'(s_k,a_k')+\gamma'(s_k,a_k')\rho \geq \alpha'(s_k,a_k)+\gamma'(s_k,a_k)\rho$$

for all $a_k\varepsilon A(s_k)$, then a_k' is an optimal alternative at stage k for parameter value ρ. Define ξ^* to be such that $\xi^*(s_k,\rho) = a_k'$; that is, ξ^* represents an optimal parameter dependent strategy. Note that ξ^* satisfies

$$f_k(s_k,\rho) = \sum_{z_{k+1}} p[z_{k+1}|s_k,\xi^*(s_k,\rho)]\ f_{k+1}[s_k,\xi^*(s_k,\rho),z_{k+1},\rho]$$

for all $s_k\varepsilon S_k,k=0,1,\ldots,K-1$.

The above algorithm for computing $A(s_k)$ from $A(s_{k+1})$, $s_{k+1}\varepsilon S_{k+1}$, becomes clearly more tractable as the number of pairs in each of the $A(s_{k+1})$

3. the a priori parameter set $P \subseteq \mathbb{R}^N$,

4. the utility structure $u^0: S_K \to \mathbb{R}$ and $u^1: S_K \to \mathbb{R}^N$, and

5. the preference structure $R(\cdot)$,

determine:

1. the set of all strategies that may be optimal and

2. the set of all parameter values consistent with these strategies.

becomes smaller. The number of pairs in any of the $A(s_k)$ sets increases as the imprecision of the parameter ρ increases. That is, let P' and P'' represent two set descriptions of parametric imprecision, and assume $P' \subseteq P'' \subseteq \mathbb{R}^N$. Let $A'(s_k)$ and $A''(s_k)$ be the smallest sets of pairs defined above that generate $f_k(s_k,\rho)$ for all ρ in P' and P'', respectively. Then, $A'(s_k) \subseteq A''(s_k)$. We now indicate how the information contained in the $R(s_k)$ can be used to efficiently decrease parameter imprecision as the $A(s_k)$ are iteratively determined.

Define

$$P(s_k,a_k',a_k) = \{\rho : h(s_k,\rho,a_k',f_{k+1}) \geq h(s_k,\rho,a_k,f_{k+1})\}.$$

Thus, $P(s_k,a_k',a_k)$ represents the set of all parameter values which imply that $(a_k',a_k)\varepsilon R(s_k)$, assuming that the decisionmaker will select alternatives in the future in a manner consistent with expected utility maximization. Let

$$P(s_k) = \bigcap P(s_k,a_k',a_k),$$

where the intersection is with respect to all $(a_k',a_k)\varepsilon R(s_k)$. If $R(s_k) = \phi$, let $P(s_k) = \mathbb{R}^N$. Define $\Delta P_k = \bigcap P(s_k)$, where the intersection is with respect to all $s_k \varepsilon S_k$. Assume $P_K = P$, and let

$$P_k = \Delta P_k \cap P_{k+1}.$$

Thus, P_k represents the set of all values that the parameter vector can assume, given the decision tree, preference, and probability structures for stages t=k,...,K, and the utility structure. Once P_k is known, it is sufficient to determine $A(s_k)$ for all $s_k \varepsilon S_k$ in terms of P_k.

The above discussion suggests the following algorithm for determining P_0 and the set of all strategies that are optimal for parameter vectors in P_0:

0. Define $P_K = P$ and $A(s_K) = \{u^0(s_K), u^1(s_K)\}$ and $f_K(s_K, \rho) = u^0(s_K)+u^1(s_K)\rho$ for all $s_K \varepsilon S_K$ and $\rho \varepsilon P_K$. Set $k=K-1$.

1. Determine P_k.

2. Determine $A(s_k)$ for all $s_k \varepsilon S_k$ with respect to P_k. Set $k=k-1$.

3. If the strategy π is such that $\pi(s_k) = \xi(s_k, \rho)$ for some $\rho \varepsilon P_0$ for all $s_k \varepsilon S_k$, $k=0,1,\ldots,K-1$, then π may be an optimal strategy. The collection of such strategies represents the sought after set of possibly optimal strategies.

IV. A NUMERICAL EXAMPLE

We now present a numerical example to illustrate the algorithm developed in the previous section. Consider the decision tree, and probability and utility structures presented in Figure 1, where we assume $N=3$, $u^0(s_3) = 0$ for all $s_3 \epsilon S_3 = \{301,\ldots,315\}$, $u^1(s_3)$ is the 3-tuple next to the appropriate terminal node, and $P = \{\rho \epsilon \mathbb{R}^3 : \rho_n \geq 0,\ \rho_1 + \rho_2 + \rho_3 = 1\}$. We interpret ρ to be the vector of importance weights for a 3-attribute, additive utility function. Assume also that

$$R(01) = \{(a_{01}^1, a_{01}^3),\ (a_{01}^2, a_{01}^3)\}$$

$$R(11) = \phi$$

$$R(21) = \{(a_{21}^1, a_{21}^4),\ (a_{21}^2, a_{21}^3),\ (a_{21}^2, a_{21}^4)\}$$

$$R(25) = \{(a_{25}^1, a_{25}^3),\ (a_{25}^2, a_{25}^3),\ (a_{25}^3, a_{25}^4)\}.$$

Reference to Figure 1 indicates there are ten (10) available strategies:

$$\pi_1 = \{a_{01}^1, a_{11}^1, a_{21}^1\}$$

$$\pi_2 = \{a_{01}^1, a_{11}^1, a_{21}^2\}$$

$$\pi_3 = \{a_{01}^1, a_{11}^1, a_{21}^3\}$$

$$\pi_4 = \{a_{01}^1, a_{11}^1, a_{21}^4\}$$

$$\pi_5 = \{a_{01}^1, a_{11}^2\}$$

$$\pi_6 = \{a_{01}^2, a_{25}^1\}$$

$$\pi_7 = \{a_{01}^2, a_{25}^2\}$$

$$\pi_8 = \{a_{01}^2, a_{25}^3\}$$

$$\pi_9 = \{a_{01}^2, a_{25}^4\}$$

$$\pi_{10} = \{a_{01}^3\}.$$

In the context of the notation introduced in Section 2, π_1, for example, is equivalent to $\pi_1(s_{01}) = a^1_{01}$, $\pi_1(s_{11}) = a^1_{11}$, and $\pi_1(s_{21}) = a^1_{21}$. Note that the preference information contained in the R sets indicates that:

1. at least one of the 5 strategies π_1, π_2, π_3, π_4, π_5 is at least as preferred as π_{10} $((a^1_{01}, a^3_{01}) \varepsilon R(01))$,

2. at least one of the 4 strategies π_6, π_7, π_8, and π_9 is at least as preferred as π_{10} $((a^2_{01}, a^3_{01}) \varepsilon R(01))$,

3. π_1 is at least as preferred as $\pi_4((a^1_{21}, a^4_{21}) \varepsilon R(21))$,

4. π_2 is at least as preferred as $\pi_3((a^2_{21}, a^3_{21}) \varepsilon R(21))$,

5. π_2 is at least as preferred as $\pi_4((a^2_{21}, a^4_{21}) \varepsilon R(21))$,

6. π_6 is at least as preferred as $\pi_8((a^1_{25}, a^3_{25}) \varepsilon R(25))$,

7. π_7 is at least as preferred as $\pi_8((a^2_{25}, a^3_{25}) \varepsilon R(25))$,

8. π_8 is at least as preferred as $\pi_9((a^3_{25}, a^4_{25}) \varepsilon R(25))$.

Strategies π_3, π_4, π_8, π_9, and π_{10} are therefore likely to be eliminated as contenders for most preferred.

Boundary conditions and the values of the h functions at stage 2 are presented in Table 1. Recall that $(a^1_{21}, a^4_{21}) \varepsilon R(21)$ implies that

$$h(21, \rho a^1_{21}, f_3) \geq h(21, \rho, a^4_{21}, f_3).$$

Thus, from Table 1

$$(7,7,7)\rho \geq (8,1,6)\rho,$$

or equivalently

$$\rho_2 \geq 0.4\rho_1 - 0.2.$$

A summary of this and other constraints on ρ implied by conditions associated with the second stage, is presented in Table 2. These constraints are illustrated graphically in Figure 2; the darkened region is P_2. We note that only constraints 2,3, and 6 are necessary to describe P_2.

The above analysis indicates that

$$f_2(21,\rho) = \max\{(7,7,7)\rho,(5,5,9)\rho\}$$
$$A(21) = \{(7,7,7),(5,5,9)\},$$

where (7,7,7) is associated with a_{21}^1 and (5,5,9) is associated with a_{21}^2. It is straightforward to show that a_{21}^1 is at least as preferred as a_{21}^2 if and only if $\rho_2 \geq -\rho_1+0.5$. Note also that

$$f_2(25,\rho) = \max\{(7.5,3,1.5)\rho,\ (5,5,6)\rho\} = (5,5,6)\rho,$$

and hence $A(25) = \{(5,5,6)\}$, where (5,5,6) is associated with a_{25}^2. We note that P_2 is a subset of the set of all parameter values such that a_{25}^2 is at least as preferred as a_{25}^1, i.e. the set of all ρ such that $\rho_2 \geq -2.8\rho_1+1.8$. Thus, due to the inequality on P generated by the statement $(a_{25}^3,a_{25}^4)\varepsilon R(25)$, it also follows that $(a_{25}^2, a_{25}^1) \varepsilon R(25)$ and hence π_7 is at least as preferred as π_6.

With respect to stage 1, note that

$$h(11,\rho,a_{11}^1,f_2) = 0.6f_2(21,\rho)+0.4f_2(22,\rho) = \max\{(7.0,4.2,5.8)\rho,(5.8,3.0,7.0)\rho\}.$$

Also, $h(11,\rho,a_{11}^2,f_2) = (6,3,2)\rho$. Since $(7.0,4.2,5.8) \geq (6,3,2)$, a_{11}^1 is always at least as preferred as a_{11}^2; therefore,

$$f_1(11,\rho) = \max\{(7.0,4.2,5.8)\rho,(5.8,3.0,7.0)\rho\}$$

and at least one of the strategies π_1, π_2, π_3, and π_4 is at least as preferred as π_5. No new information about the value of the imprecisely known parameter has been obtained; hence, $P_1 = P_2$.

With respect to stage 0, note that

$$h(01,\rho,a_{01}^1,f_1) = 0.5f_1(11,\rho)+0.5f_1(12,\rho) = \max\{(4.5,4.1,5.9)\rho,(3.9,3.5,6.5)\rho\},$$
$$h(01,\rho,a_{01}^2,f_1) = 0.6f_1(13,\rho)+0.4f_1(14,\rho) = (5.0,7.0,3.6)\rho$$
$$h(01,\rho,a_{01}^3,f_1) = (5,5,1)\rho.$$

Observe that since $(5,7,3.6) \geq (5,5,1)$, the statement $(a_{01}^2,a_{01}^3)\varepsilon R(01)$ is uninformative with respect to further reducing the set of possible parameter values. We see that $(a_{01}^1,a_{01}^3)\varepsilon R(01)$ is equivalent to

$$\max\{(4.5,4.1,5.9)\rho,(3.9,3.5,6.5)\rho\} \geq (5,5,1)\rho.$$

The set of all ρ satisfying the above inequality is the union of the set of all ρ satisfying at least one of the following two inequalities, both easily derived from the above inequality:

$$\rho_2 \leq -0.93\rho_1 + 0.84$$
$$\rho_2 \leq -0.94\rho_1 + 0.79.$$

The union is described by the first of these two inequalities and has no effect on the set of possible parameter values. Thus, $P_0 = P_2$.

The set of all values of ρ such that a_{01}^1 is at least as preferred as a_{01}^2 is the set of all ρ such that

$$\max\{(4.5,4.1,5.9)\rho,\ (3.9,3.5,6.5)\rho\} \geq (5.0,7.0,3.6)\rho.$$

This (nonconvex) set is the union of all ρ that satisfy at least one of the following two inequalities:

$$\rho_2 \leq -0.539\rho_1 + 0.44$$
$$\rho_2 \leq -0.625\rho_1 + 0.45.$$

Observe that this set contains P_0. Therefore, a_{01}^1 is least as preferred as a_{01}^2 for all $\rho \varepsilon P_0$, and hence at least one of the strategies π_1 through π_5 is at least as preferred as each of the strategies π_6 through π_9. We have already determined that of the first five (5) strategies, only strategies π_1 and π_2 may be optimal. We therefore conclude that the set of all possibly optimal strategies is $\{\pi_1,\pi_2\}$, $P_0 = P_2$, and

$$f_0(01,\rho) = \max\{(4.5,4.1,5.9)\rho,(3.9,3.5,6.5)\rho\}.$$

We remark that the set of all possibly optimal strategies may not contain the strategy π^*, where π^* is such that

$$\min_{\rho \varepsilon P_0} f_0^{\pi^*}(01,\rho) \geq \min_{\rho \varepsilon P_0} f_0^{\pi}(01,\rho)$$

for any other strategy π and where $f_0^{\pi}(01,\rho)$ is the expected utility accrued by strategy π given parameter value ρ. We first note that $f_0^{\pi}(01,\rho)$ can be determined by the dynamic programming equation and boundary condition

$$f_k^{\pi}(s_k,\rho) = h[s_k,\rho,\pi\ (s_k),f_{k+1}^{\pi}]$$

$$f_K^{\pi}(s_K,\rho) = u^0(s_K) + u^1(s_K)\rho\ .$$

The values of the $f_0^{\pi}(01,\rho)$ are presented in Table 3. The values of the criterion for each π of the linear program

$$\min_{\rho \varepsilon P_0} f_0^{\pi}(01,\rho)$$

are also presented in Table 3, indicating that $\pi^* = \pi_1$. Further discussion of this type of objective in a more general context can be found in (White and El Deib, 1982).

V. CONCLUSIONS

We have presented and analyzed a generalization of the standard decision analysis model of sequential decisionmaking under risk. The form of the information assumed to be available a priori and the problem objective have been generalized. We have only assumed that the vector of utilities is a member of a given set rather than requiring it to be known precisely. We have also assumed that a relation on the set of available alternatives is given for each decision node. The problem objective has been generalized to include the determination of all possibly optimal strategies and the determination of the set of all possible utility values. We have noted that the standard decision analysis problem and the inverse decision analysis problem represent special cases of the problem presented here. A numerical procedure, illuminated by a numerical example, has been developed to satisfy the problem objective. One other problem objective was considered within the context of the numerical example, the determination of a strategy that maximizes the minimum expected utility to be accrued, where the maximum is with respect to the set of all strategies and the minimum is with respect to the set of all possible parameter values.

A currently outstanding question is how to cope with the situation where $P_0 = \phi$. If $P_0 = \phi$, then the relations are ranking a dominated strategy higher than a strategy that is dominating the first strategy. Such a situation indicates that at least one inconsistency exists between the relations and P and/or the problem structure is fundamentally inaccurate.

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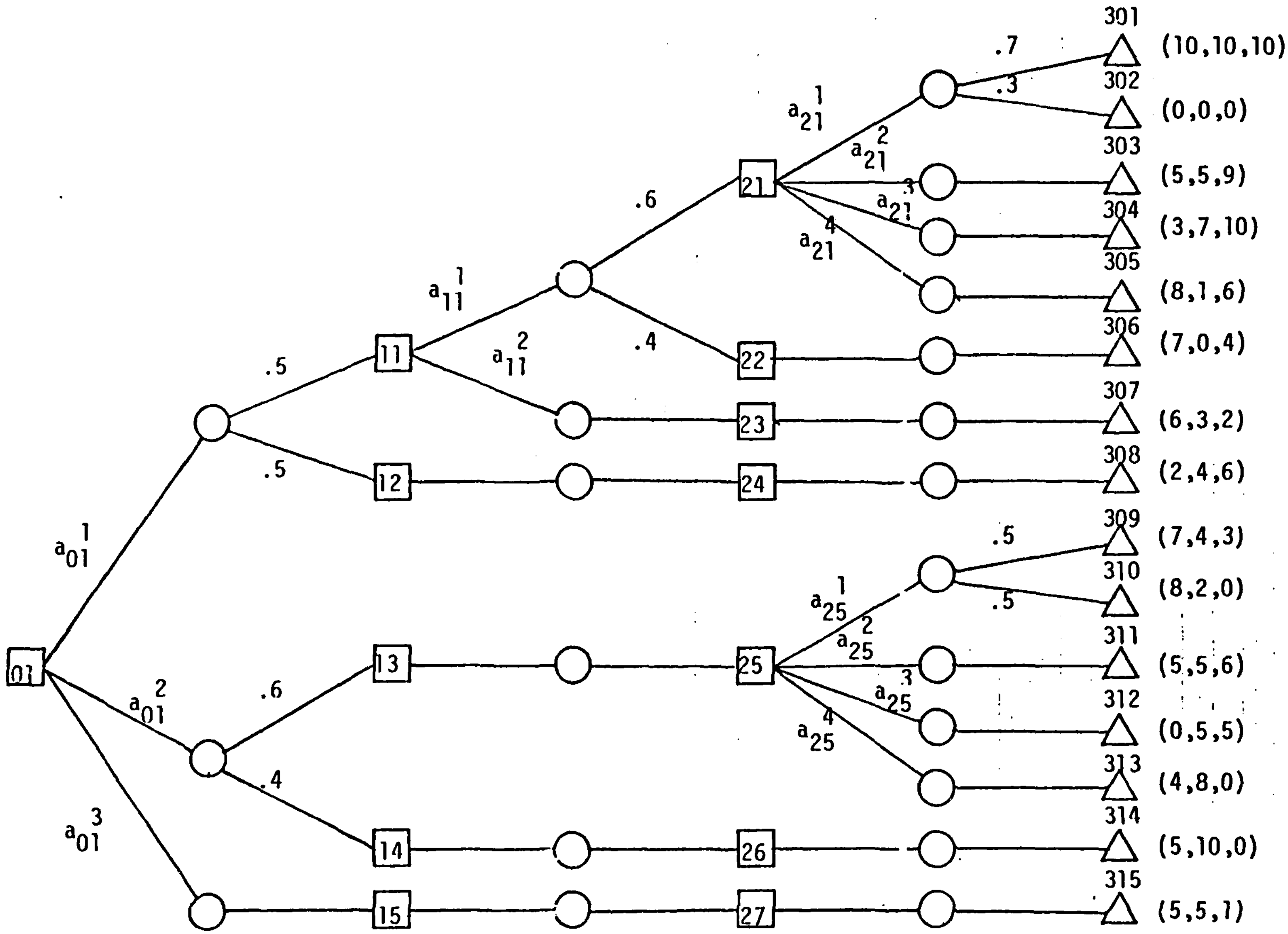

FIGURE 1: Decision Tree and Probability and Utility Structures for Example Problem

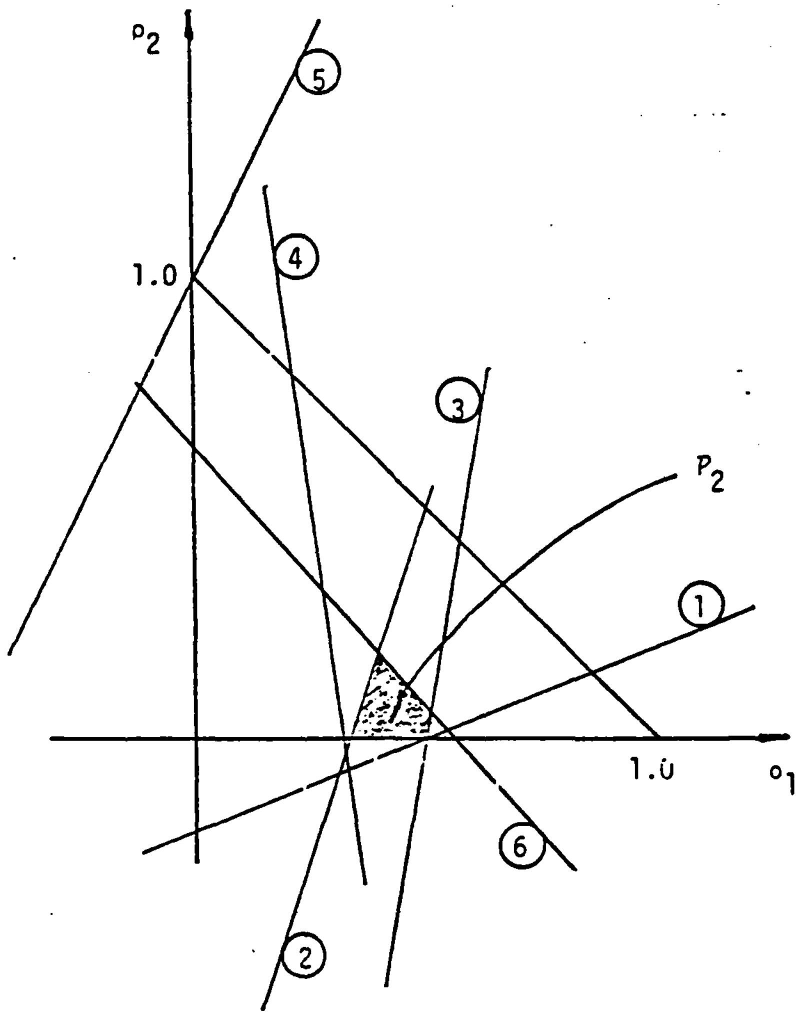

FIGURE 2: Region of Parameter Imprecision
After Stage 2 for Example Problem.

$f_3(301,\rho) = (10,10,10)\rho$	$h(21,\rho,a_{21}^1,f_3) = (7,7,7)\rho$
$f_3(301,\rho) = (0, 0, 0)\rho$	
$f_3(303,\rho) = (5, 5, 9)\rho$	$h(21,\rho,a_{21}^2,f_3) = (5,5,9)\rho$
$f_3(304,\rho) = (3, 7,10)\rho$	$h(21,\rho,a_{21}^3,f_3) = (3,7,10)\rho$
$f_3(315,\rho) = (8, 1, 6)\rho$	$h(21,\rho,a_{21}^4,f_3) = (8,1,6)\rho$
$f_3(306,\rho) = (7, 0, 4)\rho$	$h(22,\rho,a,f_3) = (7,0,4)\rho$
$f_3(307,\rho) = (6, 3, 2)\rho$	$h(23,\rho,a,f_3) = (6,3,2)\rho$
$f_3(308,\rho) = (2, 4, 6)\rho$	$h(24,\rho,a,f_3) = (2,4,6)\rho$
$f_3(309,\rho) = (7, 4, 3)\rho$	$h(25,\rho,a_{25}^1,f_3) = (7.5,3,1.5)\rho$
$f_3(310,\rho) = (8, 2, 0)\rho$	
$f_3(311,\rho) = (5, 5, 6)\rho$	$h(25,\rho,a_{25}^2,f_3) = (5,5,6)\rho$
$f_3(312,\rho) = (0, 5, 5)\rho$	$h(25,\rho,a_{25}^3,f_3) = (0,5,5)\rho$
$f_3(313,\rho) = (4, 8, 0)\rho$	$h(25,\rho,a_{25}^4,f_3) = (4,8,0)\rho$
$f_3(314,\rho) = (5,10, 0)\rho$	$h(26,\rho,a,f_3) = (5,10,0)\rho$
$f_3(315,\rho) = (5, 5, 1)\rho$	$h(27,\rho,a,f_3) = (5,5,1)\rho$

TABLE 1: Boundary Conditions and h Function Values at Stage 2 for Example Problem.

1. $(a_{21}^1, a_{21}^4) \varepsilon R(21) \iff \rho_2 \geq 0.4\rho_1 - 0.2$
2. $(a_{21}^2, a_{21}^3) \varepsilon R(21) \iff \rho_2 \leq 3\rho_1 - 1$
3. $(a_{21}^2, a_{21}^4) \varepsilon R(21) \iff \rho_2 \geq 6.0\rho_1 - 3.0$
4. $(a_{25}^1, a_{25}^3) \varepsilon R(25) \iff \rho_2 \geq -7.\overline{3}\rho_1 + 2.\overline{3}$
5. $(a_{25}^2, a_{25}^3) \varepsilon R(25) \iff \rho_2 \leq 4.0\rho_1 + 1.0$
6. $(a_{25}^3, a_{25}^4) \varepsilon R(25) \iff \rho_2 \leq -1.125\rho_1 + 0.625$

TABLE 2: Constraints on the Parameter from the Second Stage for Example Problem.

π	$f_0^\pi(01,\rho)$	$\min\{f_0^\pi(01,\rho):\rho\varepsilon P_0\}$
π_1	$(4.5,4.1,5.9)\rho$	5.021
π_2	$(3.9,3.5,6.5)\rho$	4.930
π_3	$(3.3,4.1,6.8)\rho$	4.877
π_4	$(4.8,2.3,5.6)\rho$	4.685
π_5	$(4.0,3.5,4.0)\rho$	3.909
π_6	$(6.5,5.8,0.9)\rho$	2.767
π_7	$(5.0,7.0,3.6)\rho$	4.067
π_8	$(2.0,7.0,3.0)\rho$	2.500
π_9	$(4.4,8.8,0.0)\rho$	1.467
π_{10}	$(5.0,5.0,1.0)\rho$	2.333

TABLE 3: Values for $f_0^\pi(01,\rho)$ and $\min\{f_0^\pi(01,\rho):\rho\varepsilon P_0\}$ for Example Problem.

Note that the preference information contained in the R sets indicates that:

1. at least one of the 5 strategies π_1, π_2, π_3, π_4, π_5 is more preferred than π_{10} $((a_{01}^1, a_{01}^3) \varepsilon R(01))$,

2. at least one of the 4 strategies π_6, π_7, π_8, and π_9 is more preferred than π_{10} $((a_{01}^2, a_{01}^3) \varepsilon R(01))$,

3. π_1 is more preferred than $\pi_4((a_{21}^1, a_{21}^4) \varepsilon R(21))$,

4. π_2 is more preferred than $\pi_3((a_{21}^2, a_{21}^3) \varepsilon R(21))$,

5. π_2 is more preferred than $\pi_4((a_{21}^2, a_{21}^4) \varepsilon R(21))$,

6. π_6 is more preferred than $\pi_8((a_{25}^1, a_{25}^3) \varepsilon R(25))$,

7. π_7 is more preferred than $\pi_8((a_{25}^2, a_{25}^3) \varepsilon R(25))$,

8. π_8 is more preferred than $\pi_9((a_{25}^3, a_{25}^4) \varepsilon R(25))$.

Strategies π_3, π_4, π_8, π_9, and π_{10} are therefore eliminated as contenders for most preferred.

Boundary conditions and the values of the h functions at stage 2 are presented in Table 1. Recall that $(a_{21}^1, a_{21}^4) \varepsilon R(21)$ if and only if

$$h(21, \rho, a_{21}^1, f_3) \geq h(21, \rho, a_{21}^4, f_3).$$

I. INTRODUCTION

The standard decision analysis model of sequential decisionmaking under uncertainty (Keeney and Raiffa, 1976) assumes that the decision tree, fact information in the form of probabilities, and preference information in the form of utilities are known precisely. Given such structural and parameter value information, the decision analysis paradigm provides an optimal strategy, i.e., a best alternative to select at each decision node in order to maximize expected utility.

In reality, utilities and probabilities may be difficult to precisely assess due to, for example, the time and stress associated with their assessment. Furthermore, the decisionmaker may be willing and able to directly provide information about the relative desirability of the available alternatives for at least some of the decision nodes in the tree; e.g., a physician may be willing and able to directly and intuitively express a preference for selecting diagnostic test A over diagnostic test B in diagnosing patients having a specific collection of signs, symptoms, and laboratory test results. Thus, the input assumptions associated with the standard decision analysis model may be sufficiently demanding to act as a barrier to the acceptability of the decision analysis paradigm and yet may ignore readily available information that could be of significant value in strategy selection.

In this paper, we assume that the decision tree is given and that all probabilities are known precisely. We then diverge from the aforementioned input assumptions for the standard decision analysis model by assuming that the utilities may be imprecisely described by set inclusion and that a (possibly null) relation on the set of all available alternatives is given for each decision node. Set inclusion has been selected as a model of parameter imprecision due to its simplicity and behavioral justifiability; see Fishburn (1965), Sarin (1977a,b), and White et al. (1982a) for further discussion. The intent of the relations is to model the decisionmaker's directly expressed preferences among the available alternatives at each decision node. We then develop an analytic procedure for determining:

(i) the set of all strategies that may be optimal, and

(ii) the set of all values that the utilities can take.

This analytic procedure serves as the basis of a numerical procedure that is a generalization of results presented in (White and El Deib, 1982). We note that a symbiotic relationship exists between the given set of all utility values and the given relations associated with the decision nodes in that:

(i) information contained in the relations tend to shrink the set of all possible utility values, and

(ii) the set of all possible utility values tends to increase the number of pairwise elements in the relations.

The problem formulation to be examined includes two interesting cases:

(i) the standard decision analysis model of sequential decision-making; i.e., the case where the utilities are described precisely and where all relations are null,

(ii) the inverse decision analysis problem; i.e., the case where the utilities, or some aspect of the utilities such as importance weights, are unknown and where the relations are sufficiently discriminating to provide an optimal strategy.

We remark that the output of the inverse decision analysis problem is the set of all utilities that permit the given strategy to be optimal.

A variety of researchers, e.g. Fishburn (1965), Sarin (1977a,b), White et al. (1982a,b), have recognized that for the single decision node case, precise parameter value information:

(i) may represent a barrier to the use of decision analysis, and

(ii) may not be necessary to determine what strategy ranking information is required for strategy selection.

This research has been extended to the sequential case by White and El Deib (1982). The problem determining the set of all parameter values for which a given strategy is optimal, the inverse control problem, has been examined in a variety of contexts; e.g., control theory, (Casti, 1980), mathematical programming (Bitran et al., 1981) and decision analysis (White et al., 1982c). The research to be reported here therefore represents an extension of the results presented in (White and El Deib, 1982) and (White et al., 1982c).

This paper is outlined as follows. The generalized sequential decision analysis model is formulated in Section II and analyzed in Section III. A numerical example is presented in Section IV. Conclusions are discussed in the final section.

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47.1

HIERARCHICAL INFERENCE IN LARGE SCALE SYSTEMS

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Abstract. Often there exists one or more information items believed relevant but not fully definitive to allow error free diagnosis of the state of the environment. The process of extracting the (maximum amount of) information concerning the state of the environment and the effect of various decisions or actions is called inference. Bayes' theorem yields an optimum procedure for the sequential aggregation of information across the independent samples of information for cases in which it is possible to identify a mutually exclusive and exhaustive set of hypotheses about the state of the environment. An investigation of the use of state variable concepts for sequential inference analysis with dependent multicue information in possible nonstationary, that is changing over time, environments is described. The central object of the research is to develop extensions to the theory of hierarchical inference in large scale decentralized systems that will allow better use of inference analysis in operational planning and decision support settings. To these ends, the concluding section of this paper describes a possible use of hierarchical inference structures as structured protocols for communication, command, and control system design. These protocols allow identification of various forms of cognitive bias, and appropriate debiasing procedures, in the formulation, analysis, and interpretation of various information and judgment inputs to decisionmaking.

I. INTRODUCTION

For maximum efficiency and effectiveness, available planning and decision support resources must be allocated and coordinated in space, over a hierarchy of decisionmakers, and in time, as new information arrives and the environmental situation extant changes. Associated information acquisition, processing, and evaluation typically must, as a consequence, be distributed both in space and in time. Information acquisition, processing, and evaluation must be accomplished selectively in space and time since different decisionmakers have different information needs. In addition, it will be physically impossible and behaviorally undesirable to supply all relevant information in a planning and decision support system to each decisionmaker in the hope that it will be effectively cognized and utilized. Further, differences in education, motivation, familiarity with the environmental situation extant, and stress will influence cognitive information processing style. Consequently, a central task in the design of effective systems is that of selection and choice of appropriate information system architecture to enhance selective information acquisition, processing, and evaluation at the most appropriate time. Thus, questions of information selection, information aggregation in space and in time, and the contingency task structure which is a function of the environment and the decisionmakers, become of major importance.

As is true with systems engineering [1-4] efforts generally, the structural components of a decision support system (DDS) consist of adjuvants for issue formulation, analysis, and interpretation. As a consequence, an issue formulation adjuvant will enable decisionmakers to acquire, process, and evaluate information in order to perceive the current state of the environment; to compare that perception with a desired state; and to identify possible action alternatives which might cause the environmental state extant to change such as to be more in conformity with a desired environmental state. The issue analysis adjuvant will enable acquisition, processing, and evaluation of information such as to allow determination or analysis of the impacts of the proposed action alternatives in terms of environmental state changes. Finally the interpretation adjuvant will allow valuation, in accordance with a value system, of the identified action alternatives in terms of their impacts upon the environment such as to enable selection of one or more of the proposed action alternatives for deployment or implementation.

A variety of questions make design and

application of the aforementioned adjuvants a non routine task. Perhaps central among these questions and concerns is the fact that decisionmakers may differ considerably in their education, motivation, and prior experience with particular operational environment conditions extant. Whether a formal operational style or concrete operational style of cognition is appropriate for a particular contingency task, and whether or not this will be used, is strongly dependent upon these factors and the type of stress that they produce. Automated aids must, consequently be flexible in the sense of being capable of adaptation to a variety of information processing and judgmental styles that are difficult to specify a priori.

Expert decisionmaking is typically done in a concrete operational mode of cognition, and involves use of one or more of a variety of judgmental heuristics including:

- reasoning by analogy
- standard operating procedures
- pure intuitive affect
- incremental adjustment

These decision styles are doubtlessly appropriate for, and potentially capable of excellent results in, environments where the decisionmakers diagnosis of situationally caused stress, and when other components of the contingency task structure, are appropriate for these forms of concrete operational behavior. In unstructured situations in unfamiliar environments, a formal operational mode of cognition is generally appropriate. One task of an automated decision support system is to assist in acquiring the experience and situational familiarity appropriate for cognizing the formal operational thought process into a situation where concrete operational thought is efficient, effective, and otherwise appropriate. Concrete operational thought typically involves use of forward processed judgmental heuristics based upon a perceptive mode of information processing. It is the preferred cognitive style when and if it is "fully appropriate" for information processing and judgment. Inferior cognition and/or poorly perceived concrete environmental situations may, however, result in a combination of information processing biases, poor judgmental heuristics, and value incoherencies which may result in extremely poor use of information and poor aggregation of facts and values, perhaps accomplished intuitively, to form judgments. Extraordinarily poor concrete operational thought and associated judgment may well be the result of these maladies which may be very effectively reinforced through feedback.

Thus it is desirable that an appropriately designed automated aid, or adjuvant, for planning and decision support systems be capable of:

- assisting in the evaluation of alternative plans and courses of action that involve formal operational thought processes,
- assisting in the transfer of formal operational situations to concrete operational situations,
- assisting in evaluation of alternative plans and courses of action that involve concrete operational thought processes,
- assisting in the avoidance of information processing biases and poor judgmental heuristics,
- assisting in the use of variety of judgmental heuristics appropriate for given operational environments as natural extensions of a decisionmaker's normal cognitive style,
- assisting, to the extent possible, in the determination of whether a formal or concrete style of cognition is most appropriate in a given situation,
- assisting decisionmakers who need to use formal operational thought, and those whose expertise allows appropriate and effective use of concrete operational thought, to function together in a symbiotic and mutually supportive way.

Clearly there is a space-time dependency associated with these desired capabilities.

Also among the many concerns that dictate needs and requirements for automated decision support systems is the fact that more judgments and associated decisions must be made in a given period of time than can be comfortably made. This creates a stressful situation which can lead to the use of poor information processing and judgmental heuristics, especially since judgments and decisions are typically based on forecasts of the future. A certain level of stress is also necessary for optimal information processing and decisionmaking.

An "obvious" aid to judgment and decision making consists of using optimization based approaches. It is easily shown that a multistage stochastic decision and control framework is "proper" in these situatiosn. Even more proper would be the use of decentralized hierarchical procedures to cope with the space-time distributed nature of the problems.

Unfortunately there exists potentially serious problems associated with the use of substantively "optimum" procedures in many typical decision situations. A time in the future, or planning horizon, must be specified before problem solution, using precise optimization techniques, is begun. In effect a "two point boundary value problem" must be solved even though the presentation format used for solution display may not make this fully evident. The "folding back" procedure associated with many multistage decision analysis procedures, in which the decision situation is analyzed backwards in time from the terminal time to the starting time, represents the equivalent of the backward sweep procedure necessary to solve a two point boundary value problem of dynamic optimization theory. Another major problem with this procedure is that there is no useful product until the entire effort is completed.

As a consequence, one of several approaches which sacrifice substantive optimality, that is in practice often unobtainable, are often used.

Not only is there a need to solve problems over a particular planning horizon, but it is often necessary to update the resulting solutions "periodically" as better information is obtained. Thus it is necessary to cope not only with planning horizons and the need to, in effect, solve two point boundary value problems backwards in time; but it is also necessary to update solutions, or recommended alternative courses of actions, at various planning periods. To do this in a "substantively optimum" fashion would require solution of a new two point boundary value problem at each planning period with processing of the new state change and value change information that has been obtained since the last update.

These difficulties are further confounded with space and multiple decisionmaker, and related organizational issues; with the cognitive style, experience, and contingency task structure related determinants of human information processing and judgmental mechanisms; and with potential information processing bias and judgment heuristics.

Thus we see that there are indeed formidable needs and issues to be resolved that are associated with the design of information processing and judgment aiding support systems. These relate to questions concerning appropriate functions for the decisionmaker and staff to perform. They concern the type of information which should be available and how this information should be acquired, processed, stored, aggregated and presented such that it can be used most effectively in a variety of potential operational environments. They concern design of information systems with strong space-time environmental dependencies. They concern design of information systems that can effectively "train" decisionmakers to adapt and use appropriate concrete operational heuristics in those environments in which inexperience dictates initial use of formal operational thought. They concern design and use of information systems that support environmentally experienced decisionmakers in the use of a variety of effective concrete operational heuristics. And because of use of decision support systems by multiple decisionmakers, these tasks must be accomplished in a parallel architectural fashion.

Structure is vitally important in each of the steps of a systems engineering effort [3-7]. There are a number of structural models, generally in the form of a tree, that are very useful for the analysis and interpretation steps, including models:

(1) of the objectives or of the attributes which proposed policies or decisions should satisfy. This structure is generally called an objectives tree or an attribute tree [3-7].

(2) of the impacts on aleatory states of nature of proposed action alternatives, policies or decisions. This structure is generally called a decision tree or decision diagram [3,8].

(3) of the way in which inferences about the state variables, or states of nature, and influenced by information and observable events [5,6,8]. This structure is generally called an inference or influence tree, or inference diagram [9].

This paper reports on research that extends previous efforts concerned with hierarchical policy and decision analysis, studies in large scale systems. To what is believed are some interesting, unsolved and application relevant issues involving hierarchical inference. Our efforts involve the mathematical concepts of systems science and operational research; integration of these with organizational human and behavioral factors concerns; and application to information processing requirements for planning and decision support in systems engineering and large scale systems.

II. HIERARCHICAL INFERENCE STRUCTURES

Hierarchical inference structures are useful in decisionmaking when the decisionmaker has observed available data and wishes to interpret the implications of the data when it is combined with prior experience. Let us consider that the decisionmaker has only limited information available. More is accessible but only at the expense of an incurred cost. Two options available to the decisionmaker are maximum likelihood estimation and hypothesis testing. In the former the decisionmaker attempts to obtain and utilize information to determine which decision strategy has the maximum likelihood of being the most authentic in the sense of the one that most accurately and fully expresses beliefs. To accomplish hypothesis testing, the decisionmaker assigns worth scores to each of the information items and uses the likelihood estimates to obtain utilities for each strategy. If the decisionmaker is rational, the hypothesis that maximizes espected utility is selected and assumed correct.

In the initial phase of inference analysis the alternate decision strategies d_i available to the decisionmaker are proposed, as are the state or activity variables s_i and information items v_i. Through the structuring process, these elements are related to form a model of the hierarchical inference structure. It is assumed that the decision strategies are mutually exclusive and exhaustive and that the prior probabilities of being the authentic strategy, $p(d_i)$, $i=1,\dots,n$, are selected such that they sum to 1, and that $D = \{d_1, d_2, \dots, d_n\}$ $S = \{s_1, s_2, \dots\}$ and $V = \{V_1, V_2, \dots\}$ are discrete and finite. We also require independence among the state variables at any level of the structure. Upon completing the assignment of prior probabilities to the decision strategies, we proceed downward into the structure, making nominal conditional probability assessments on the remaining variables. The encoding of the nominal probability assignments is complete when the probability of each available information item v_i conditioned on the state and decision variables has been determined. The structure is now called a deductive hierarchical inference structure.

The inductive hierarchical inference structure is formed from the deductive structure by making inferences of two types: (k) relating a conditional probability distribution of an element to an element two levels higher, and (ii) coalescing several conditional probability distributing into an element on which they are conditioned. The algorithms for doing this are:

$$\text{(i) } p(v|d,e) = \sum_s p(v|d,s,e)p(s\ d,e) \quad (1)$$

$$\text{(ii) } p(S|d,e) = \prod_s p(s|d,e) \quad (2)$$

where v is an information item, d is a decision strategy, s is a state variable, e is prior experience, and where p( ) is the conditional probability mass function. The maximum likelihood estimate of the authentic decision strategy is obtained using these algorithms and an application of Bayes' rule.

With this preliminary inductive inference structure, a conventional sensitivity analysis may be performed to determine the impact of the prior probabilities of the decision strategies, conditional probabilities of the state variables, and conditional probabilities of the information items on the maximum likelihood estimate of the authentic decision strategy. When a maximum likelihood decision strategy is relatively unaffected by perturbations in prior probabilities, state variable probabilities, and informational probabilities, then the decisionmaker gains additional confidence due to the decision strategy's robustness. If the number of decision strategies considered is large, then the posterior probabilities may offer the decisionmaker flexibility in the sense that a strategy may have to be only nominally altered with the acquisition of additional information. This approach to flexibility in decision analysis has previously been proposed by Merkhofer [10]. If this flexibility condition is not available, then aleatory variables and potent decision variables causing this may be readily identified.

When performing an inference analysis, we attempt to describe all state and decision variables relevant to the problem. Often achievement of an ideal model, which expresses accurately our uncertainty about the dependence relationship between information items and state and decision variables, is restricted by economics as well as cognitive bias and limitations of prior experience. For similar reasons uncertainty of the conditional behavior of state variables exists. Uncertainty due to finite considerations of prior experience has been called primary uncertainty due to incomplete consideration of prior experience [11]. Primary uncertainty is resolved only by obtaining additional information on the variables and is expressed by authentic probabilities. Secondary uncertainty is resolved by further introspection and perhaps computation and is expressed by probabilities based on partial consideration of prior experience.

The use of experts in supplying information and conditional probabilities necessary to make an inference has previously been presented by Morris [12]. If several experts are used and a joint assessment is done, procedures such as the Delphi approach have proved useful. Also considered, though with less practical success, is the case where multiple experts independently assess probabilities but where the decisionmaker values the credibility of some more than that of others. Here redundancy occurs if some experts use the same indicators as others. If, however, each expert uses a different indicator and if opinions of experts are similar, the decisionmaker's confidence is enhanced. But a problem remains with encoding, and that is how to elicit nearly authentic probabilities from experts.

In a situation where there do not exist any experts who are capable and willing to assess informational probabilities or where no direct empirical evidence has been collected, it is possible to use stochastic simulation modeling to generate useful and needed conditional probabilities. This may be practical when, for example, there is knowledge available or understanding of the dependence relationship between the state variables and information items. Through this model appropriate conditional probabilities based on relative frequencies may be generated and encoded for use as informational items in maximum likelihood estimation.

The incorporation of dynamics into the decision situation structure appears potentially very useful, if it can be done in a reasonably simple way. For example, in problems with a long time horizon, it is not possible to expect either encoded probabilities or utility functions to remain constant. The concept of using a stochastic simulation model poses other interesting questions. Whereas the expert elicited conditional probabilities are direct subjective estimates, those from the stochastic model are also subjective, but in this case indirect. They are related to the conditional probabilities through the model's structure, parameters, and, possible boundary conditions.

III. INFORMATION PROCESSING STRUCTURE AND BIASES

We describe contemporary research investigating the effects of various structured information processing/decision aiding protocols upon the formulation, analysis, and interpretation of information and its integration with judgment and decision making activities. Our basic information formulation, analysis, and interpretation structure will be based upon the 6 elements found in explicit argument [13]. These elements are:

1. claims or hypotheses

2. grounds or foundations to support the claims
3. warrants or justification for the grounds or foundations
4. backing or the general body of information that is presupposed by the warrant
5. modal qualifiers or circumstances contingencies or restrictions which will have to exist in order that the warrant truly supports the grounds
6. possible rebuttals or circumstances, contingencies, or restrictions which, if they exist will refute or diminish the force of the warrant.

A simplified block diagram of the interaction among these elements is shown.

The information processing "structure" for the information processor, consisting in part of the decisionmakers view of possible and probable action courses and the "decision situation model", is specified by elements 3-6. Element 2, the "grounds", comprises the situational data pertaining to the operational conditions extant.

Toulmin shows, through examples, that the six elements for logical argument and reasoning can be used as a model for rational reasoning in a number of areas including: law, science, the arts, management, and ethics. Clearly it will apply to military intelligence and a number of other relevant areas as well. This structured information processing model is also sufficiently general to accommodate analytical hierarchical inference. Thus it will provide a structured framework for information processing that can accommodate a variety of information processing styles and approaches ranging from the purely qualitative and affective and quantitative, to quantitatively based filtering and detection algorithms.

Information summarization is needed in DSS for a variety of reasons. Huber [14] and Geiselman and Samet [15] discuss the needs for information summarization and procedures to condense and organize information into a form that can be managed and used in an efficient manner. We postulate that the structured information processing model suggested here provides organizational support for message aggregation and integration that will accommodate and encourage effective information summarization. Information summarization guidelines, such as these, can be modeled as a special case of the structured framework presented here which will accommodate both receptive and preceptive styles of processing and summarization of information, and which will also accommodate non numerical and numerical information, thus hopefully enabling rapid conversion from one to the other as needed or desired in different contingency task structures. Whether information summarization using this framework as a guideline can produce more useful and accurate summaries than
this framework is a subject currently under investigation.

The recent literature discusses many of the processing biases that occur during formulation, analysis and interpretation of information [16-21]. These processing biases include:

Formulation bias: frequency, base rate, availability, selective perception, concrete information, spurious cues, spurious correlations, data presentation, conservatism,
Analysis bias: anchoring and adjustment, representativeness, law of small numbers, selective perception, hindsight, and
Interpretation bias: lexicographic semiorders, wishful thinking, illusions of control, alterations of multi-stage decision structure.

These biases are influenced by a number of factors; with stress and the contingency task-structure being among the dominant influences. Also, there are a number of feedback mechanisms involved which influence these biases.

Use of structured information processing models, such as the model for reasoning presented here, will allow recognition of many of the biases of analysis and interpretation, and debiasing of those that do occur. Information formulation biases are generally due to the use of agenda dependent "editing" or "framing" rules. It is on the basis of scenario descriptions and summaries that issues and prior statistics for information processing are determined. One of the fundamental claims of prospect theory and related theories [16-21] is that these editing or framing rules lead to agenda dependent information formulation and related information acquisition. Knowledge of editing rules used in a given situation will generally allow determination of a set of debiasing procedures. Various structured procedures and protocols, based upon a set of required specifications to determine information formulation needs for the 6 element reasoning model displayed earlier, should serve as useful adjuvants to the effective determination of "proper" editing rules which avoid various error formulation biases.

IV. HIERARCHICAL INFERENCE ALGORITHMS AND ISSUES

In a very large number of contemporary areas, there are issues involving the possible occurrence of uncertain-to-occur events. Often it is much easier to determine, by a combination of subjective elicitation of appropriate parties at interest and objective physical measurement, probabilities and likelihoods for the events at issue if they are conditioned upon the occurrence of other events and/or decisions. This approach forms the basis for cross impact analysis, hierarchical inference analysis and other related approaches in which probability structures or probability diagrams have proven to be of considerable value.

In these approaches, it is assumed that observed information can be directly related to postulated hypotheses and that the impact of observed information on the probability of a given hypothesis being true, or occuring, evolves sequentially according to Bayes rule. The complexity of many contemporary large scale issues is such that the amount and type of cognitive skills and technical knowledge, required to express all appropriate probabilities or likelihood ratios, which infer or link all information elements to postulated hypotheses, is beyond the unaided capability of any single individual, and often even the aided capability. A possible approach to ameliorate this situation is to disaggregate the complex issue into a hierarchical structure and to determine the structure and parameters within the structure [22]. In the hierarchical approach to information structuring and associated inference, a number of intermediate elements are identified. These elements typically represent activity states or activity indicators which are relevant to the postulated hypotheses. These activity states or activity indicators are presumed to be meaningful representations of a portion of the issue. Probabilities which relate observed information to these intermediate elements are assessed or elicited, as well as the probabilities which related these intermediate elements to the postulated hypotheses.

The rationale behind this approach is behaviorally and organizationally compelling. Parties at interest to a given issue may be expert in diverse portions of the complete issue and a single individual, or group, will probably not have sufficient experience and knowledge to relate lowest level information to highest level hypotheses. If a complex issue is hierarchically decomposed, it may be possible to utilize the abilities and knowledge of various groups in an efficient and logically consistent fashion. Presumably, organizations are structured in a hierarchical manner to take advantage of opportunities such as these.

There is no unique structure for a hierarchical model of a given issue. These models are necessarily subjective contingency structures in that they can only represent a conceptual model of a particular issue, and the way in which a particular issue is disaggregated. The influence of various cognitive styles of the individual or group constructing the model and the influence of various constraints, such as environmental constraints, are generally the strongest determinants affecting the choice of a particular hierarchical model.

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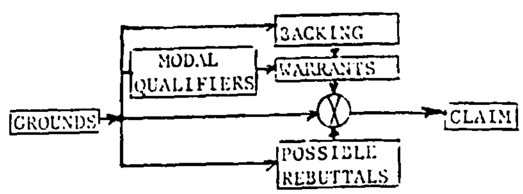

Figure 1. Structured Protocol for Information Processing

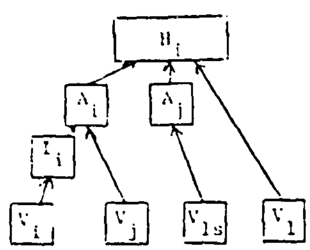

Figure 2. Prototypical Hierarchical Inference Structure

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