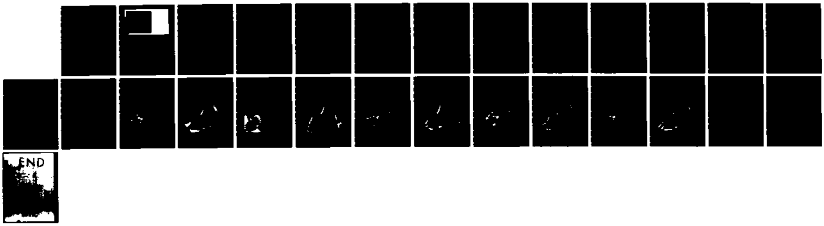
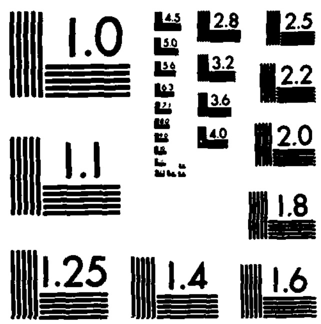
AD-A144 660 TRIANGULAR EXTRAPOLATION(U) WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER P ALFELD JUN 84 MRC-TSR-2707 DAAG29-80-C-0041 1/1

UNCLASSIFIED F/G 12/1 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

2

AD-A144 660

MRC Technical Summary Report # 2707

TRIANGULAR EXTRAPOLATION

Peter Alfeld

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

DTIC FILE COPY

June 1984

(Received May 22, 1984)

DTIC
ELECTE
AUG 27 1984
S
A

Approved for public release
Distribution unlimited

Sponsored by

U.S. Army Research Office
P.O. Box 12211
Research Triangle Park
North Carolina 27709

U.S. Department of Energy
Washington, DC 20545

84 08 24 040

UNIVERSITY OF WISCONSIN—MADISON

MATHEMATICS RESEARCH CENTER

Triangular Extrapolation

Peter Alfeld*

Technical Summary Report #2707
June 1984

Abstract

This paper describes several methods of extending a bivariate interpolant defined on triangles smoothly beyond the triangulation. Existing methods are reviewed, new methods are introduced, and the methods are compared.

AMS (MOS) Subject Classification: 65D05

Key Words: Scattered Data, Trivariate Interpolation

Work Unit Number 3 - Numerical Analysis and Scientific Computing

A1

* University of Utah, Salt Lake City, Utah 84112

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041 and the Department of Energy under Contract No. DE-AL02-82ER12046A000

SIGNIFICANCE AND EXPLANATION

The general problem area addressed in this report is that of the interpolation of scattered bivariate data (x_i, y_i, z_i) as ~~they~~ these arise for example in the Computer Aided Geometric Design of geometric objects like the shape of a ship, or the fuselage and wings of an aircraft. Many existing techniques proceed by triangulating the domain and then defining the interpolant piecewise on each triangle. This approach has the drawback that the interpolant cannot be readily evaluated outside of the triangulated domain. This report deals with the problem of extending such a triangular interpolant. Existing methods are reviewed, new methods are introduced, and the methods are compared. It appears that at present no completely satisfactory techniques exist and that the subject requires more research.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

Triangular Extrapolation

Peter Alfeld*

List of Contents

1. Introduction

The interpolation of multivariate scattered data is a rich and difficult problem area of increasing importance, e.g. in the Computer Aided Geometric Design of geometric objects like the shape of a vehicle, or in the modeling and representation of measured data. For an introduction to this area see e.g. the survey by Barnhill, 1984, the proceedings edited by Barnhill and Nielson, 1984, and the references quoted therein.

In this paper, we restrict ourselves to interpolation of scattered data in two independent variables. Thus we assume that we are given *points* (*vertices*) $V_i = (x_i, y_i) \in \Re^2$, $i = 1, \cdots, N$ and *data* at those points. The data are either part of the problem under consideration, or else they have to be made up suitably. Most frequently, the user supplies only *function* (*positional*) values. Most interpolation schemes, however, also require values of certain *derivatives* for their construction, which have to be generated from the given information.

A widely used approach to the interpolation problem (*Triangular Interpolation*) consists of first triangulating the convex hull of the given points, D, and then defining the interpolant piecewise on each triangle, taking care to maintain desired global properties such as differentiability of a required order. This technique has several advantages. The interpolants are usually of a simple structure (e.g. polynomial), and they can be defined and evaluated locally (using data only on the triangle containing the point of evaluation). Much of the analysis and construction of the scheme can therefore be confined to a single triangle. Many triangular interpolation schemes have been constructed, see the above

---

* University of Utah, Salt Lake City, Utah 84112

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041 and the Department of Energy under Contract No. DE-AL02-82ER12046A000

references for details. For information on the nontrivial problem of constructing a triangulation see the papers by Lawson, 1977, Barnhill and Little, 1984, and Cline and Renka, 1984.

A drawback of triangular interpolation, however, is that the interpolant cannot be readily evaluated in the exterior, E, of D. The notable exception to this general statement is provided by Akima's technique (Akima, 1978), which is very efficent and widely available, but receives only a "C" grade in visual quality (Franke, 1978). Akima proceeds by dividing the exterior of D into semi-infinite rectangular and triangular elements and then defining a special structure of the interpolant piecewise on those elements. The approach is described in section 2.2 as **Discrete Extension**.

In the same survey, Franke extended Nielson's minimum norm network interpolant (Nielson, 1983) beyond D, but in the process had to reduce the smoothness of the interpolant (from first order differentiability to mere continuity). Franke uses the same semi-infinite elements as Akima, and extrapolates by Taylor polynomials, either by a univariate one perpendicularly outward from edges of D, or by a bivariate one about a boundary vertex of D. This technique is described below as **Transfinite Extension**. Franke did not intend his extension to be an answer to the triangular extrapolation problem, rather he had to modify an existing technique to facilitate comparison with others. This required only very little extrapolation.

In this paper, we describe two more techniques of triangular extrapolation. One of them, **Blending by Visible Edges**, can be implemented without having to utilize the particular structure of the interpolant. All that is required is the ability to extend (smoothly) the representation of the interpolant on any particular triangle to all of $\Re^2$. *This is possible for all bivariate triangular interpolants known to this author.*

The second new technique, **Triangular Extension**, consists of adding vertices to the domain structure, redefining the triangulation, generating all needed data at the new vertices, and then redefining the interpolant on the new triangulation.

Before discussing the techniques in detail we mention some possible issues in extending a triangular interpolant. The following questions may be relevant. (Φ is the interpolant.)

-1- Is the degree of differentiability of Φ on E at least the same as that on D?

-2- Is the extended domain of Φ finite or does it cover all of $\Re^2$?

-3- Is Φ of the same structure (e.g. piecewise polynomial of a certain degree) on E as on D?

-4- Is the class of functions that are reproduced exactly by the scheme maintained in extending from D to E?

-5- Is the extended version of Φ identical to the original one on D?

-6- Is the extension technique general or does it depend on the special structure of Φ on D? For example, a technique is not general if it requires derivatives of Φ since these may be hard to supply if Φ is complicated or if it is defined by a black box routine.

-7- What is the quality of the technique (e.g. with respect to accuracy or visual pleasantness)?

The following table lists the answers to the first six of the above questions for what in this author's opinion constitutes an "ideal technique" and the three techniques described in detail below. (Not even remotely precise criteria exist for answering the seventh question.)

Question	Ideal Technique	Transf. Ext. (Franke)	Discr. Ext. (Akima)	Blending by vis. Edges	Triang. Ext.
-1-	yes	no	yes	yes	yes
-2-	all of $\Re^2$	all of $\Re^2$	all of $\Re^2$	all of $\Re^2$	finite
-3-	yes	no	no	no	yes
-4-	yes	no	no	yes	yes
-5-	yes	yes	yes	yes	depends
-6-	general	special	special	general	special

Table 1: Properties of Methods

It is apparent that none of our techniques is "ideal". The choice of an extrapolation technique depends on the problem under investigation and the underlying interpolant on D. Indeed, if evaluation outside of D is necessary, then an alternative method that does not require a triangulation (e.g. one of those described in Franke, 1982) may be preferable in spite of the advantages and attractive features of triangular interpolation. Note, however, that the mere ability to evaluate an interpolant outside of D does not by itself make that method a worthy extrapolation technique.

2. Extrapolation Techniques

2.0 Geometry and Notation

Before entering the discussion of the individual extrapolation techniques we need to introduce some additional notation. We assume that D is the convex hull of the given vertices V_i, $i = 1, \cdots, N$, and that it has been triangulated. Of particular interest are boundary vertices of D and triangles having at least one edge on the boundary of D. Without restriction of generality we assume that the vertices have been labeled such that V_i, $i = 1, \cdots, M$ are the boundary vertices in counterclockwise order. The boundary segment with endpoints V_i and V_{i+1} (where $V_{M+1} := V_1$) is denoted by e_i. The unique triangle of which e_i is an edge is $\triangle_i$. (If a triangle has two boundary edges, e_i and e_{i+1}, then $\triangle_i = \triangle_{i+1}$.)

We denote the extended interpolant by Φ. On a triangle $\triangle_i$, it is represented by Φ_i. We assume that Φ_i can be evaluated anywhere in $\Re^2$. This stipulation is necessarily somewhat vague because of the abundance and variety of available interpolation schemes. For any given method, it will usually be obvious how to proceed. For example, if a scheme is polynomial on each triangle, then Φ_i will be the same polynomial in all of $\Re^2$.

The geometric concepts introduced in this paragraph are illustrated in figure 1. The exterior of D is E. We divide E into semi-infinite regions by drawing two lines from each boundary vertex that are perpendicular to the adjoining two boundary edges. The semi-infinite rectangular element corresponding to e_i is R_i, and the wedge corresponding to V_i is W_i. A wedge W_i is empty if the two boundary segments joining at V_i are parallel. The triangles in D and the set of R_i and W_i tesselate the plane.

For a given point $P \in R_i$ we denote by B the perpendicular projection of P onto e_i. We describe the location of P in terms of new coordinates s and t as

$$P = B + \frac{t(P - B)}{\|P - B\|}$$

where B is the convex combination

$$B = sV_{i+1} + (1 - s)V_i = V_i + s(V_{i+1} - V_i)$$

and s and t are given explicitly by

$$s = \frac{(P - V_i) \circ e_i}{\|e_i\|^2} \quad \text{and} \quad t = \|P - B\| \tag{1}$$

In any implementation of a scheme based on the above tesselation one must be able to compute in which element a given point resides. This can be accomplished as follows:

Location Algorithm

Note: All index arithmetic is *modulo*(M).

1. Given a point $P \in \Re^2$. Determine a specific triangle in D, call it the ***current triangle***.
2. Compute the barycentric coordinates b_1, b_2, and b_3 of P with respect to the current triangle (barycentric coordinates are discussed in several of the articles in Barnhill and Nielson, 1984).
3. *IF* all three barycentric coordinates are non-negative, *THEN* P resides in the current triangle, so *STOP*, *ELSE* consider that edge e of the current triangle that corresponds to the most negative barycentric coordinate. *IF* $e = e_i$ is a boundary edge, *THEN* P lies outside of D, *GO TO* step 4. *IF* e is an interior edge, *THEN* replace the current triangle by the triangle across e and *GO TO* step 2.
4. Let $s_{\text{old}} = 1/2$. *GO TO* step 5.
5. Using (1) compute s. *IF* $s \in [0,1]$ *THEN* $P \in R_i$ hence *STOP*. *IF* $s < 0$ and $s_{\text{old}} > 1$ *THEN* $P \in W_{i-1}$ hence *STOP*. *IF* $s > 1$ and $s_{\text{old}} < 0$ *THEN* $P \in W_{i+1}$ hence *STOP*. *IF* none of the above, *THEN* *GO TO* step 6.
6. Let $s_{\text{old}} = s$. *IF* $s < 0$ *THEN* $i := i - 1$ *ELSE* $i := i + 1$. *GO TO* step 5.

2.1 Transfinite Extension

The general idea of Franke's extension is as follows. In each wedge W_i define Φ by bivariate Taylor interpolation of order m, say, to the derivatives of Φ (as they are defined on D) at the vertex V_i. In each rectangle R_i define Φ by Taylor extrapolation to derivatives of Φ along lines perpendicular to the boundary edge. Note that it must be possible to evaluate derivatives of Φ at any point on the boundary (hence the name ***transfinite extension***). The degree of the transfinite extrapolation must equal m, for otherwise the extended interpolant would be discontinuous along rays joining a wedge and a rectangle. Also, the derivative data must be well defined, i.e. Φ must be at least m times differentiable at the boundary vertices. (We assume that it is arbitrarily often differentiable across a boundary edge, so that all needed derivatives can be supplied.)

To be explicit, we list the appropriate formulas. Let $P = (x,y) \in \Re^2$ be a general point. If $P \in W_i$ we define

$$\Phi(P) := \sum_{\mu+\nu\leq m} \frac{\partial^{\mu+\nu}\Phi}{\mu!\nu!\partial^{\mu}x\partial^{\nu}y}(V_i)(x-x_i)^{\mu}(y-y_i)^{\nu} \tag{2}$$

If $P \in R_i$ we express the location of P in terms of the new coordinates t and s as described in (1) and define

$$\Phi(P) = \sum_{i=0}^{m} \frac{t^i\partial\Phi}{i!\partial(P-b)}(B) \tag{3}$$

where partial differentiation denotes the standard normalized directional derivative.

The interpolant extended in this fashion will be m times differentiable everywhere on the boundary of D. However, the smoothness of Φ may be reduced across the rays separating semi-infinite elements. Assume that P lies on a ray emanating from the boundary vertex V_i and that a k-th order partial derivative $\mathcal{D}\Phi$ is to be evaluated at P. Differentiating in (2) and (3) yields derivatives of Φ through $(m+k)-th$ order at $B = V_i$. Thus, in order for $\mathcal{D}\Phi$ to be continuous, Φ must be at least $(m+k)$ times differentiable at V_i.

Consider for example Akima's interpolant, which is twice differentiable at the vertices. There one could pick $m = 1$, i.e. interpolate linearly in triangular wedges, and linearly along rays emanating perpendicularly from the boundary segements. The resulting extended interpolant would then be once differentiable everywhere in $\Re^2$. This property, however, is crucially dependent upon the fact that Akima's interpolant, while only once differentiable on edges separating triangles, is actually *twice* differentiable at the vertices of the triangulation. Akima does extend his interpolant differentiably to all of $\Re^2$, but he proceeds in a different manner which is described in the next subsection.

2.2 Discrete Extension

The basic idea of this technique is to interpolate to discrete data at the boundary vertices and to construct a special extension on each of the two types of semi-infinite elements. The vertex data are read off the interpolant as it is defined on D. We illustrate the technique by considering Akima's interpolant.

In a triangular wedge W_i Akima simply uses bivariate quadratic Taylor interpolation as defined in (2). In a rectangular strip R_i, however, he interpolates by the unique polynomial in $span\{1, s, s^2, s^3, s^4, s^5, t, st, s^2t, s^3t, t^2, \text{and } 3s^2t^2 - 2s^3t^2\}$ that interpolates to the twelve data given at V_i and V_{i+1}.

Akima's extended interpolant is continous across e_i because it can be represented there as a univariate quintic that is determined uniquely by the six tangential data at V_i and V_{i+1}. Similarly, it is continuous across the rays separating semi-infinite regions because there it can be represented uniquely as a univariate quadratic function that is determined by the vertex data. Differentiability follows because a perpendicular cross-boundary derivative is a unique univariate cubic across boundary edges, and a univariate linear function across rays.

Note that the construction of Akima's interpolant depends upon the special structure of Φ on D. There is no obvious way of generalizing this approach.

2.3 Blending by Visible Edges

Here we proceed as follows: For a given point $P \in E$ we consider all visible edges e_i, i.e. all edges for which the determinant of the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ V_i & P & V_{i+1} \end{pmatrix}$$

is positive. The weight attached to a triangle $\triangle_i$ is larger the larger the angle α_i which is formed by V_i, P and V_{i+1} (see figure 1). The weights are chosen to force a certain degree of global smoothness and to maintain the degree of precision of the original interpolant.

We define

$$\Phi(P) = \sum_i \Psi(\alpha_i(P))\Phi_i(P) \tag{4}$$

where the summation is taken over all visible edges and the blending functions $\Psi(\alpha_i)$ satisfy

$$\sum_i \Psi(\alpha_i(P)) = 1. \tag{5}$$

Obviously, if $\Phi_i(P) = F(P)$ for some given function F and all i, then $\Phi(P) = F(P)$ so that the precision of the interpolant on D is preserved.

We next address the question of the smoothness of Φ as defined in (4). Let us assume that the individual terms in (4) are m times differentiable. Then, applying any m-th order differentiation operator to Φ yields a sum of products of derivatives through order m of Ψ, Φ_i. and α_i, all of which are continuous by assumption. However, if some of the angles $\alpha_i(P)$ approach zero, then the corresponding term will be added or omited from equation (4). In order to maintain continuity of the derivative, all corresponding terms must be zero when this happens. Each term contains some derivative of Ψ as a factor. Thus we require that

$$\Psi^{(i)}(0) = 0 \qquad \forall i = 0, 1, ..., m. \tag{6}$$

Note that by picking m larger than the degree of smoothness of Φ on D it is possible to achieve a degree of smoothness in the exterior E that exceeds that in D!

The task now remains of constructing blending functions that satisfy the requirements (5) and (6). The simplest approach is a s follows. We force (6) by defining functions

$$\chi(\alpha) = \alpha^{m+1}$$

and then define the blending functions Ψ_i by the standard trick of dividing by the sum of the appropriate terms, i.e.

$$\Psi_i(P) := \frac{\chi(\alpha_i(P))}{\sum_j \chi(\alpha_j(P))} \tag{7}$$

where the sum in the denominator is taken over all visible edges.

2.4 Triangular Extension

The general philosophy is as follows: Virtually all triangular interpolants require for their construction certain auxiliary data that are not supplied by the user. Hence there must be a method of generating such information from user supplied data. So extend the triangulated domain by adding points in the domain and generate the data needed at the additional points (including the kind usually supplied by the user) by using the data generation method that is already in existence. The triangulation of the extended domain can be accomplished by adding triangles to the existing triangulation of D, or by triangulating the convex hull of the new extended point set.

Of course, not every method for the generation of auxiliary data can actually be generalized to work on the expanded point set. Most frequently, the user supplies C^0 data and the interpolation scheme must then make up some kind of derivative data. This process amounts to numerical differentiation, and is clearly not suitable or generalizable to the generation of additional positional data. However, here we describe a specific approach that generalizes easily and naturally.

In Alfeld, 1984, as well as in earlier references (Schmidt, 1982; Nielson, 1983) the interpolation problem was solved by picking that function in a certain space of functions that minimized a particular functional subject to interpolation. The fundamental idea is to consider needed data (such as derivative values at given vertices or positional values at additional vertices) to be parameters that enter the minimization.

The details of the algorithm used for the numerical examples in section 3. will be reported elsewhere. However, we sketch its main ingredients. The algorithm is applicable for any triangulated domain where, at any vertex V, some, none, or all of the following data may be given: $\{F(V),\ F_x(V),\ F_y(V)\}$. Since it is possible to have no data at a given vertex, it is simple to extend the given domain and define the interpolant on the retriangulated extended domain. The new domain is of the same structure as the old one, and the distinction between D and E no longer applies.

The interpolant is *cubic* on each triangle (as in Schmidt, 1982), and is globally once differentiable. The number of data supplied must not exceed the total number of available parameters. Among all interpolants q, the one is chosen that minimizes the *thin plate functional*

$$I(q) = \iint_D q_{xx}^2 + 2q_{xy}^2 + q_{yy}^2 dxdy$$

The minimizing solution is found by applying the finite element technique of setting up element stiffness matrices and assembling them into a global stiffness system. The interpolant has the advantage of greatest possible simplicity (i.e. piecewise cubic) but has the drawback that the differentiability conditions cannot be imposed locally (as they could in Alfeld, 1984) but rather have to be incorporated as global constraints. It is well known that the C^1 conditions may be redundant (Schumaker, 1984, Morgan and Scott, 1977). The only known source of redundancy are singular vertices, i.e. points at which exactly four triangles meet at the intersection of the two diagonals of the quadrilateral defined by the four triangles. If a singular vertex (or nearly singular vertex) occurs then the singularity can be destroyed by introducing an additional vertex (without any data attached to it) nearby and considering the data needed there as three more parameters that enter the minimization problem.

The resulting linear system is solved directly, rather than iteratively. Powerful methods for this task exist (George and Liu, 1981).

3. Numerical Examples

In this section, we illustrate the extrapolation techniques described in the preceding section by interpolating to Franke's well-known test function

$$\begin{aligned} f(x,y) = {} & \frac{3}{4} e^{-((9x-2)^2+(9y-2)^2)/4} \\ & + \frac{3}{4} e^{-(9x+1)^2/49-(9y+1)/10} \\ & + \frac{1}{2} e^{-((9x-7)^2+(9y-3)^2)/4} \\ & - \frac{1}{5} e^{-(9x-4)^2-(9y-7)^2} \end{aligned}$$

The domain data and the triangulation of D are given in table 1. At each vertex, the function value, and no derivative values, are given. The domain D is the unit square covered by 36 (essentially random) points and 54 triangles. The extended domain was taken to be a square with twice the area of D, with D in its center.

There are five groups of plots (which were generated by ⟨PLOT79⟩, see Beebe, 1979).

1. **Primitive Function**
2. **IMSL Interpolant** This interpolant was generated by the IMSL routine IQHSCV, which is an implementation of Akima's technique.
3. **Blending by Visible Edges** This is the result of the technique described in section 2.3.
4. **36 + 4 point extension** Here the domain D was extended by adding the four vertices of the extended domain E, and retriangulating. Then the technique described in section 2.4 was applied. In this example it so happened that the triangulation of the original domain D was preserved, although this cannot be expected to be true in general.
5. **36 + 12 point extension** Similar to 4., except that 12 points equally spaced on the boundary of E were added. As in 4., the triangulation of D was preserved.

For each of the above groups there are two plots, namely a contour plot with the triangulation superimposed, and a hidden line view of the surface from a Northeast direction.

The plotting parameters (viewpoint, vertical scaling, interval between contour lines, and range covered by the contour lines) are identical in all five groups.

It is apparent that none of the extrapolated interpolants matches the simplicity and beauty of the primitive function. The individual surfaces differ substantially, both quantitatively and qualitatively. Blending by Visible Edges seems to give rise to the worst undulations, introducing several spurious saddle points. This is unfortunate because it offsets the other advantages of this method, namely generality, versatility, and the ability to generate any degree of smoothness in E (by choosing the exponent in (7) suitably large). It is an open question whether better results can be obtained by a different choice of the blending functions. Akima's method performs better, but it suffers from poor quality even in D due to deficiencies in the derivative generation. Triangular Extension gives rise to surfaces with few undulations and seems to be the best of all methods. The quality of the surface seems to improve with the number of points added to the original set.

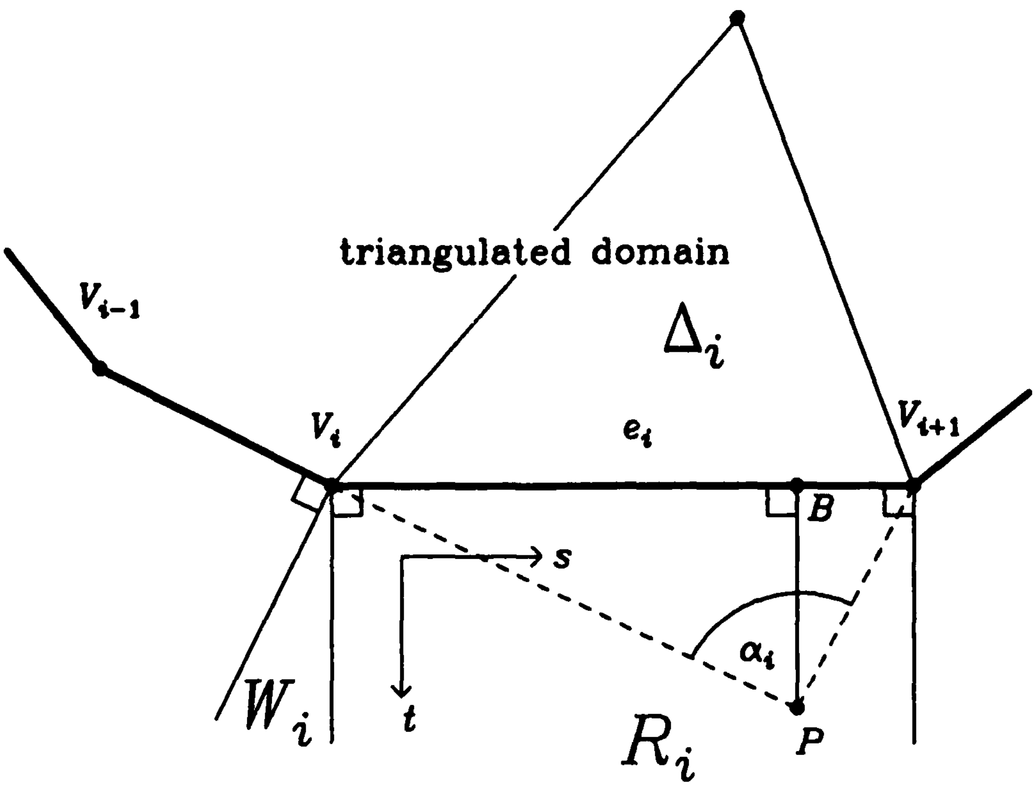


Figure 1: Geometry of Triangular Extrapolation

Points

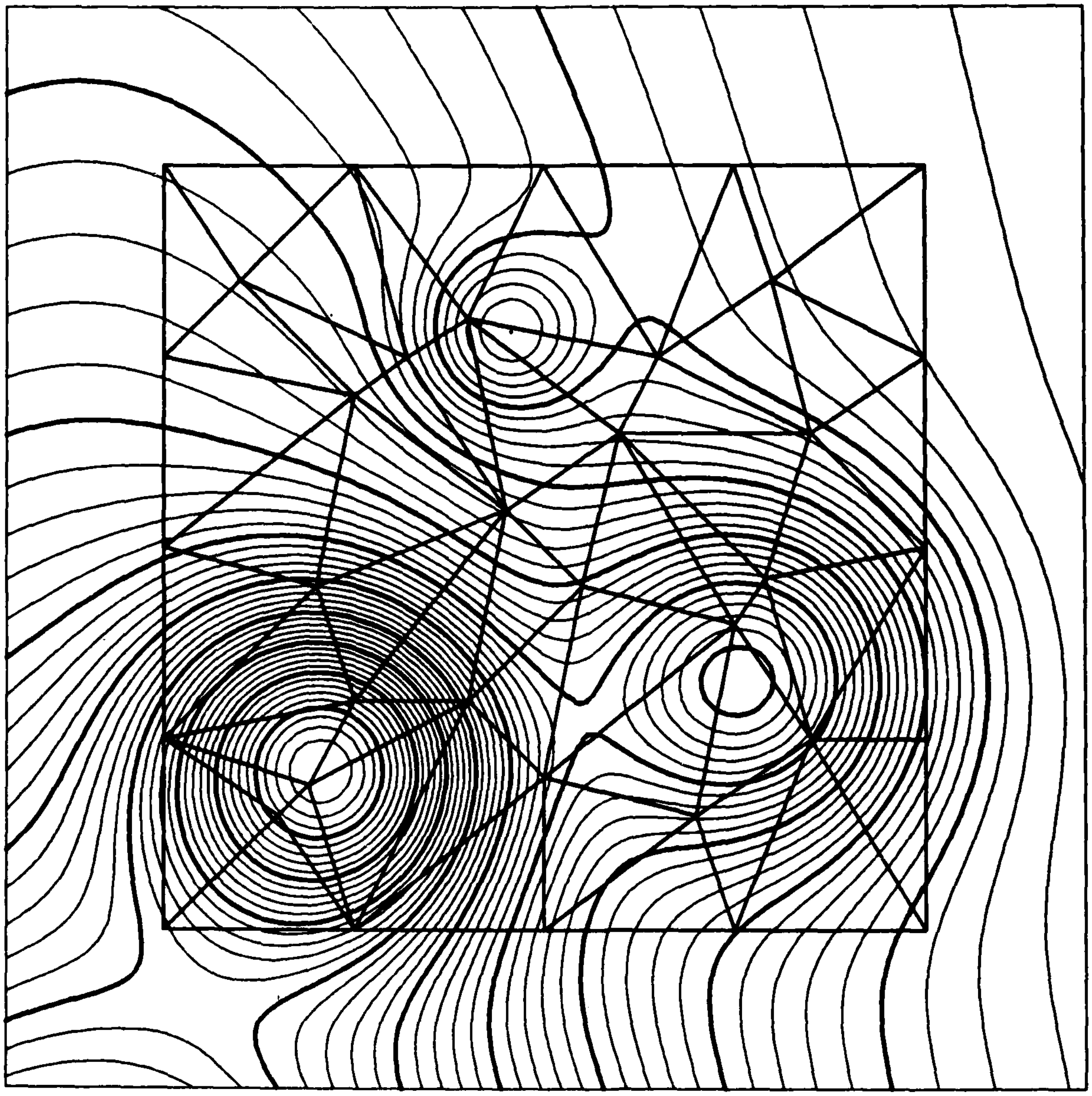
1	0.00	0.00
2	0.50	0.00
3	1.00	0.00
4	0.15	0.15
5	0.70	0.15
6	0.50	0.20
7	0.25	0.30
8	0.40	0.30
9	0.75	0.40
10	0.85	0.25
11	0.55	0.45
12	0.00	0.50
13	0.20	0.45
14	0.45	0.55
15	0.60	0.65
16	0.25	0.70
17	0.40	0.80
18	0.65	0.75
19	0.80	0.85
20	0.85	0.65
21	1.00	0.50
22	1.00	1.00
23	0.50	1.00
24	0.10	0.85
25	0.00	1.00
26	0.25	0.00
27	0.75	0.00
28	0.25	1.00
29	0.00	0.25
30	0.75	1.00
31	0.00	0.75
32	1.00	0.25
33	1.00	0.75
34	0.19	0.19
35	0.32	0.75
36	0.79	0.46

Triangulation

1	11	15	14
2	14	8	11
3	15	9	36
4	11	6	9
5	6	11	8
6	14	16	13
7	14	7	8
8	8	26	6
9	15	20	18
10	14	17	35
11	9	15	11
12	17	14	15
13	7	14	13
14	16	14	35
15	17	15	18
16	20	15	36
17	6	5	9
18	9	10	36
19	36	21	20
20	7	29	34
21	34	8	7
22	6	2	5
23	10	9	5
24	18	23	17
25	20	19	18
26	34	29	4
27	6	26	2
28	16	12	13
29	29	7	13
30	26	8	34
31	5	27	10
32	18	30	23
33	35	28	24
34	35	24	16
35	16	31	12
36	21	36	10
37	20	33	19
38	28	35	17
39	29	13	12
40	4	1	26
41	26	34	4
42	31	16	24
43	27	5	2
44	10	32	21
45	28	17	23
46	10	3	32
47	19	22	30
48	30	18	19
49	33	20	21
50	1	4	29
51	28	25	24
52	24	25	31
53	3	10	27
54	22	19	33

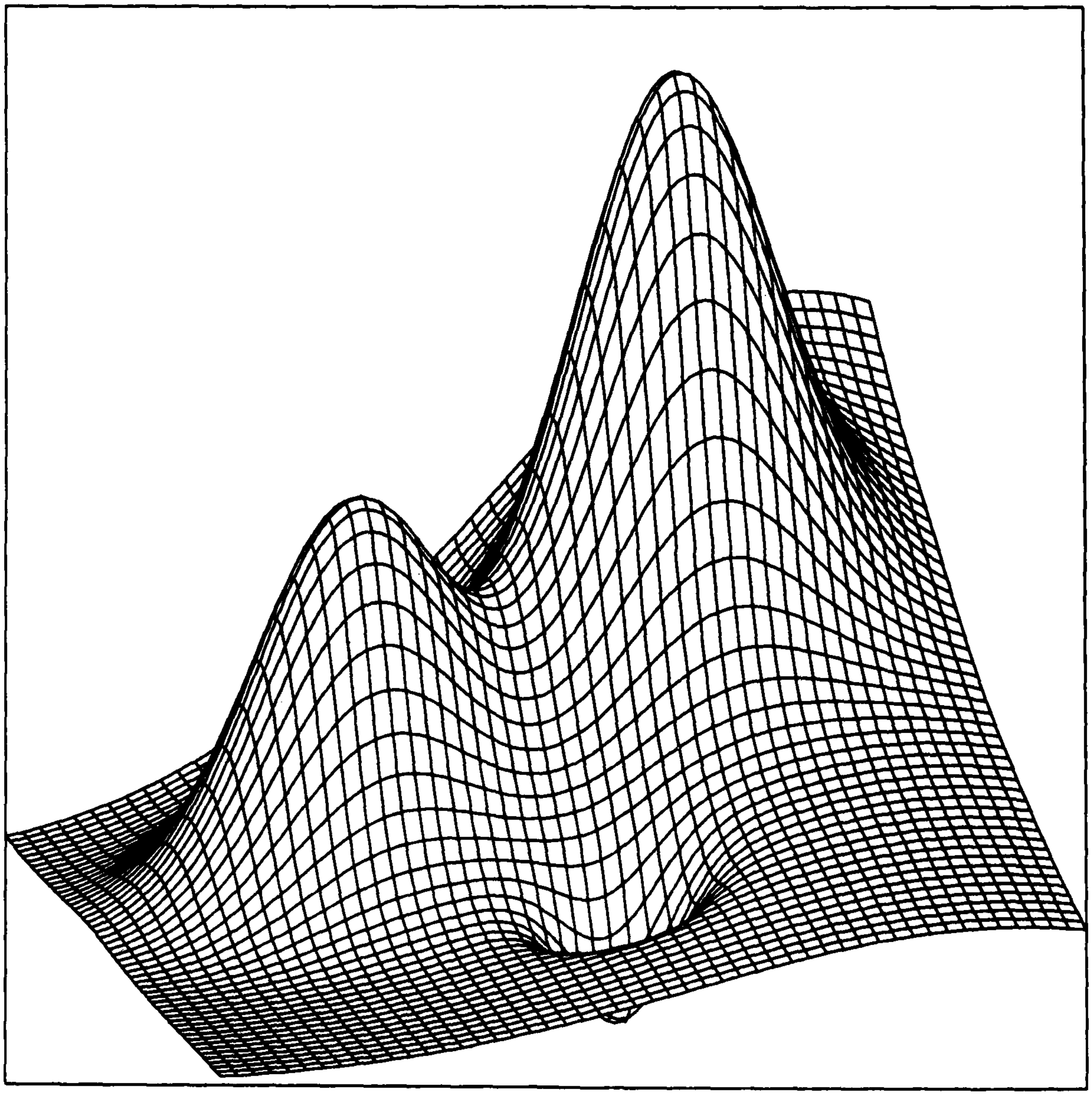
Table 1: triangulated Domain

Primitive Function



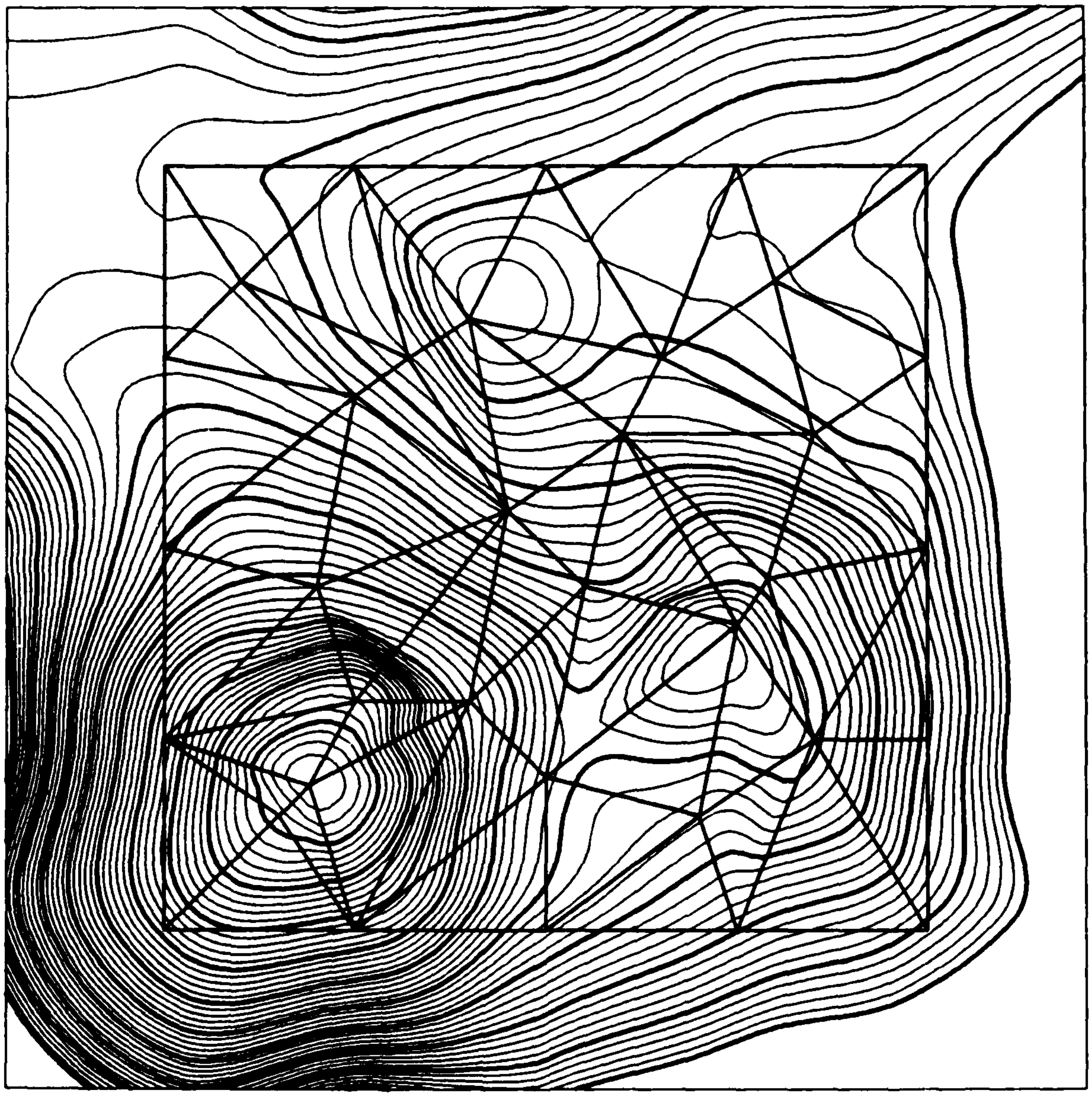
Contour Plot

Primitive Function



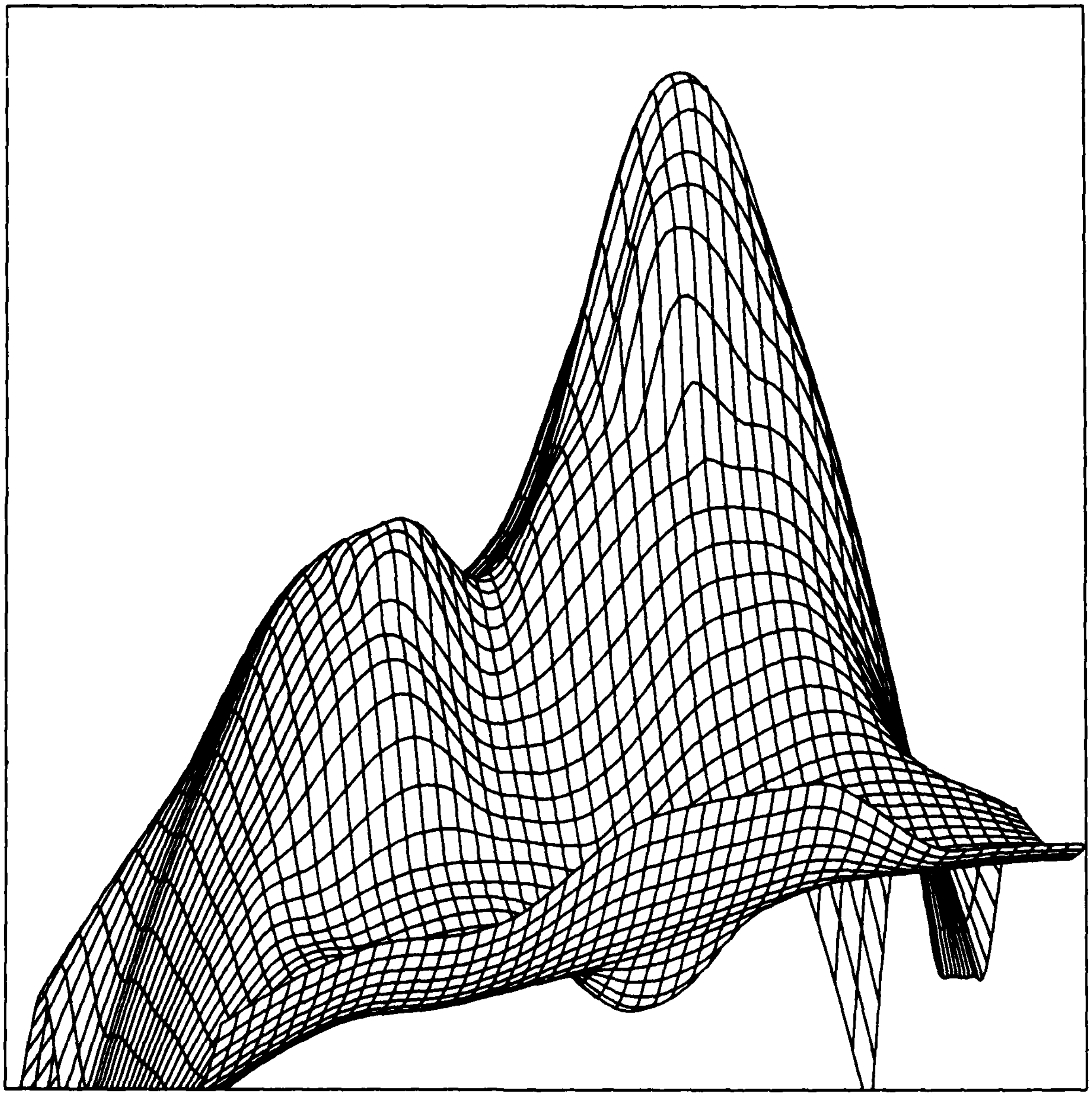
Northeast View

IMSL interpolant



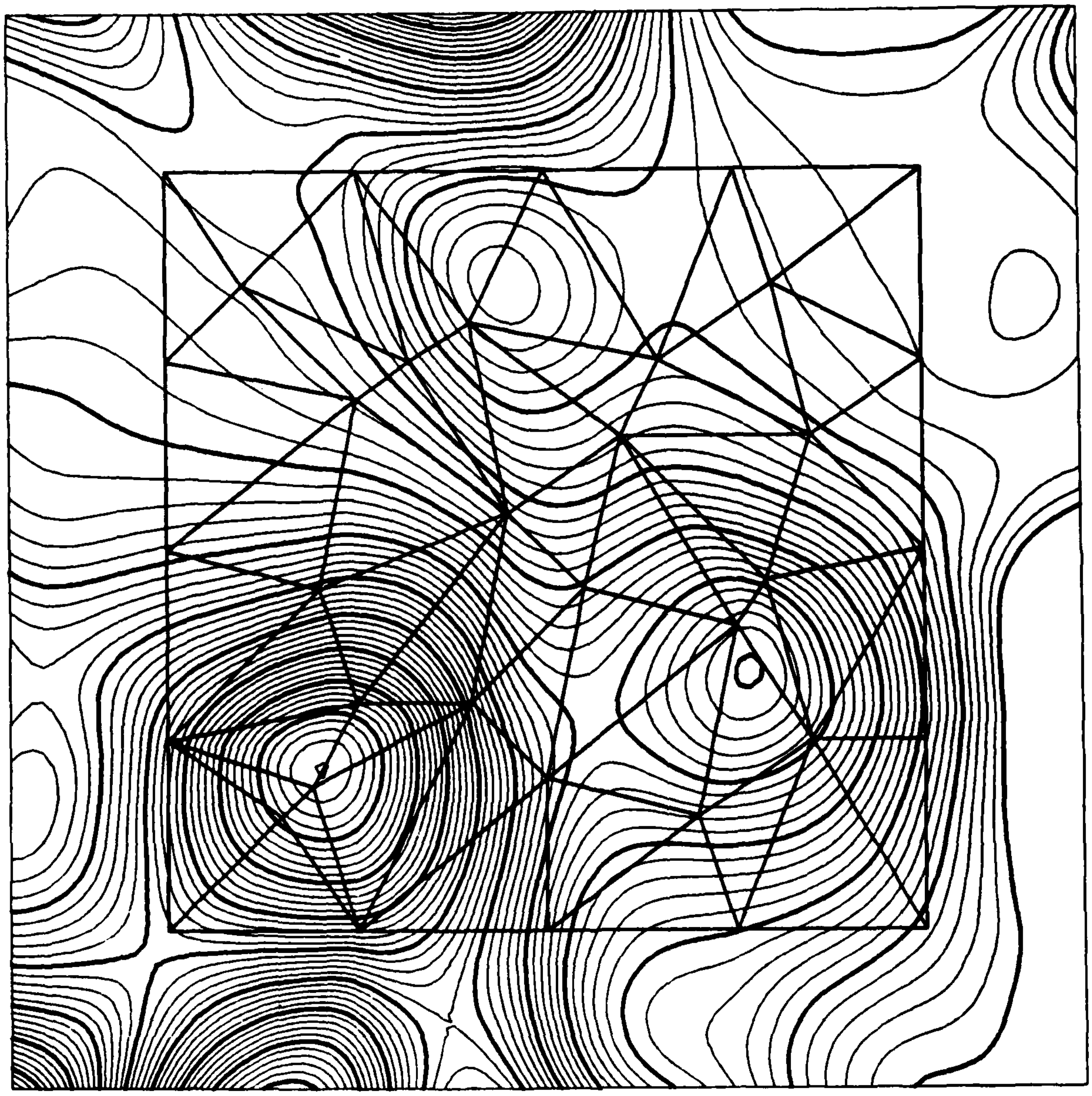
Contour Plot

IMSL interpolant



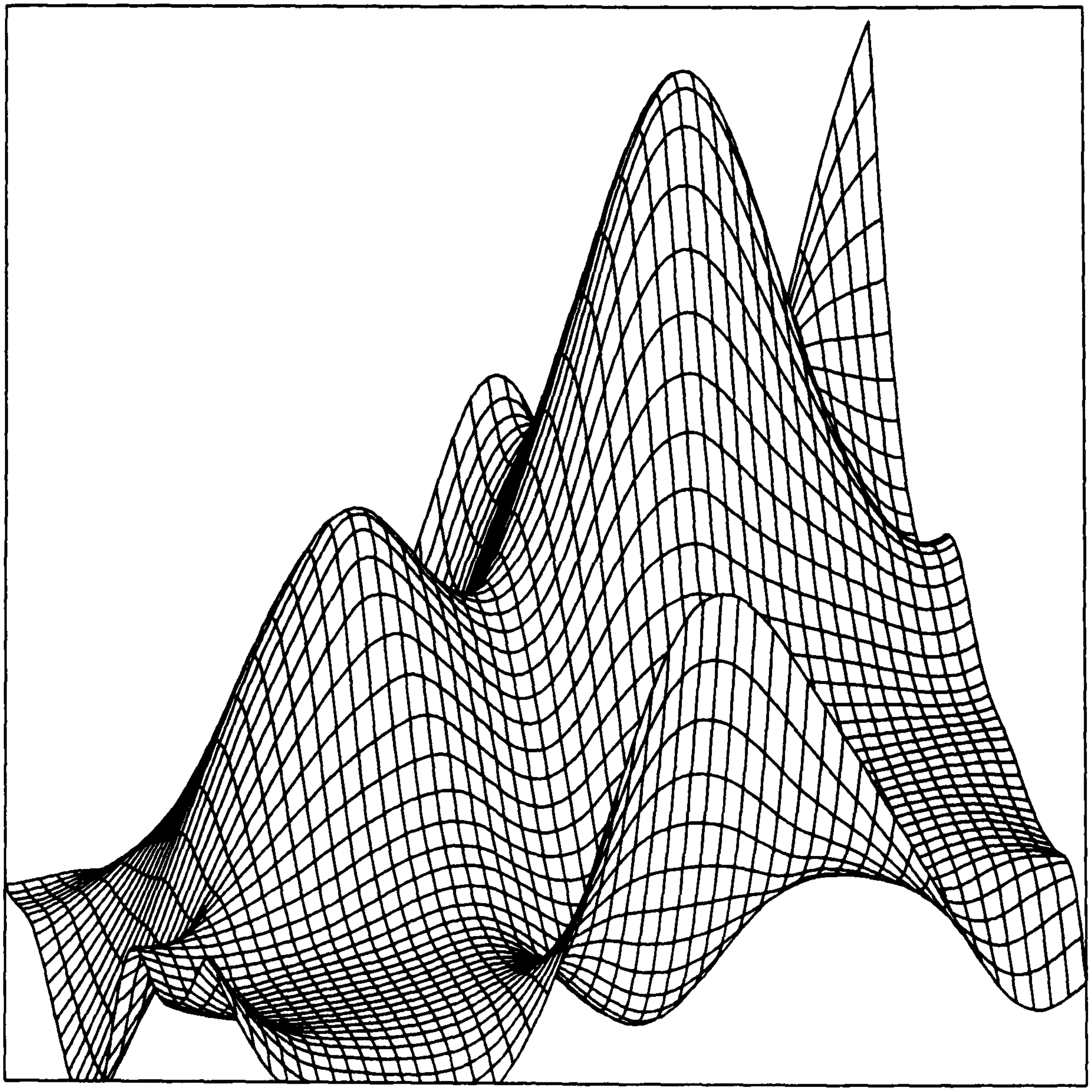
Northeast View

Blending by Visible Edges



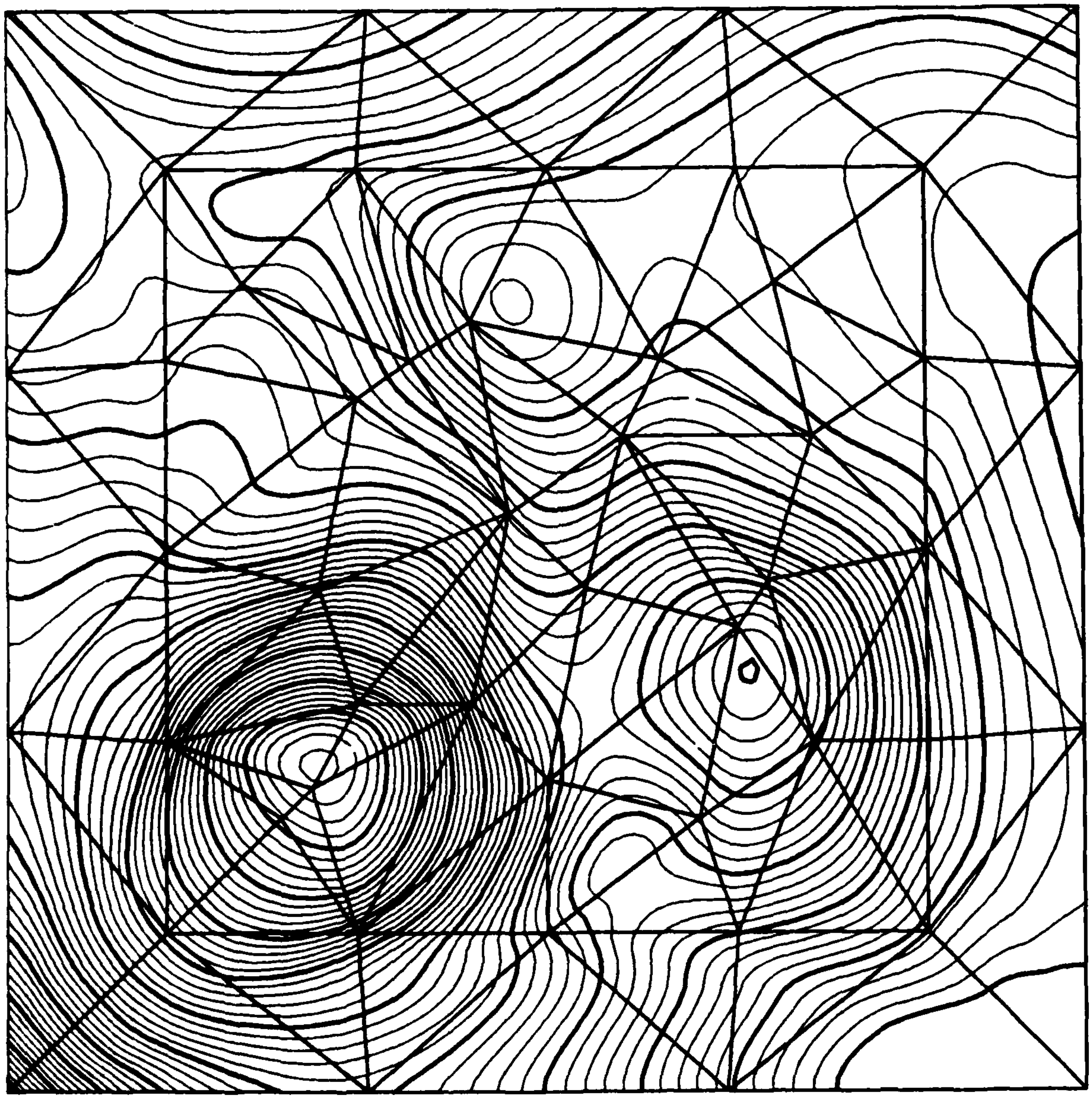
Contour Plot

Blending by Visible Edges



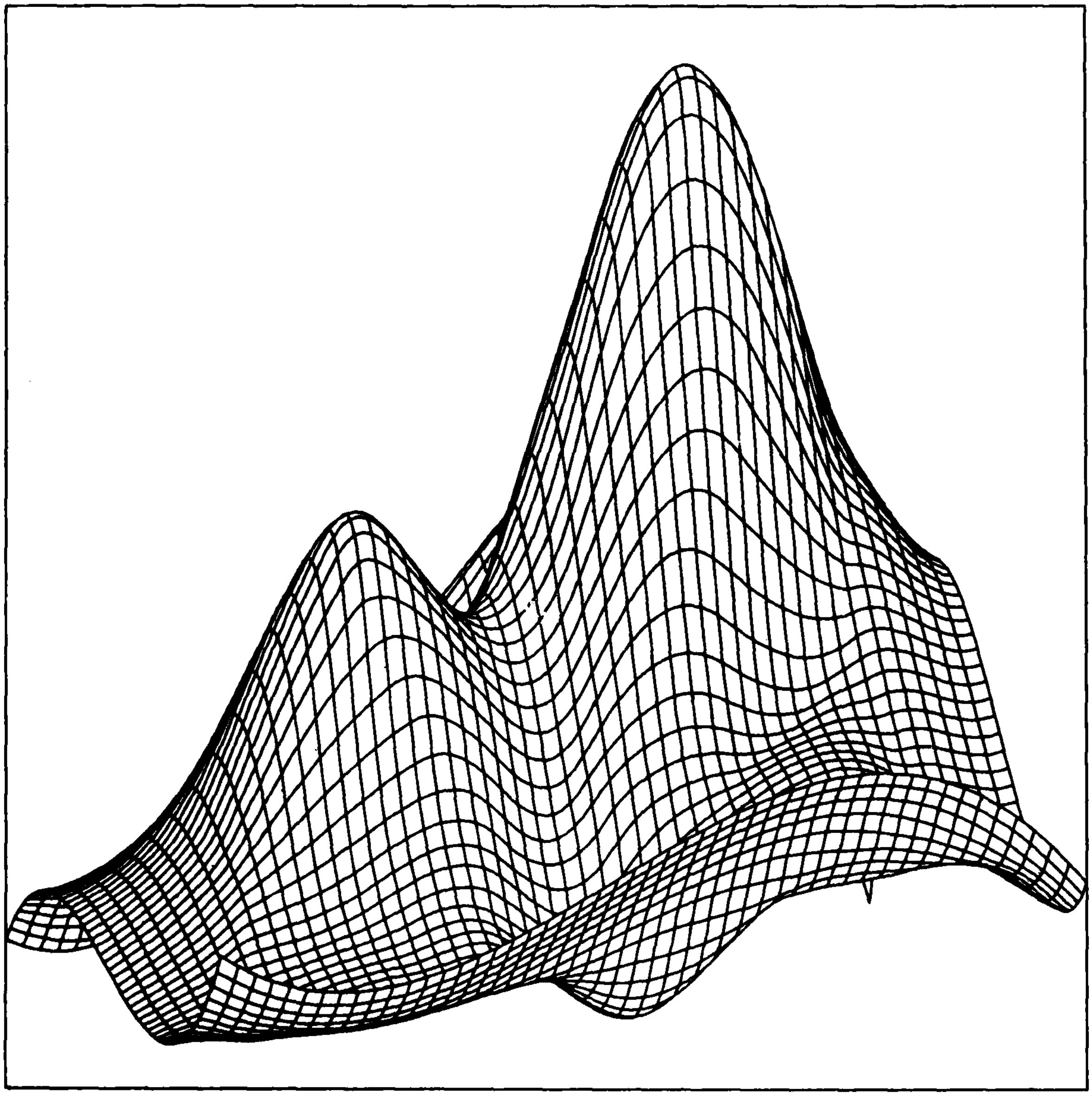
Northeast View

36 + 4 point extension



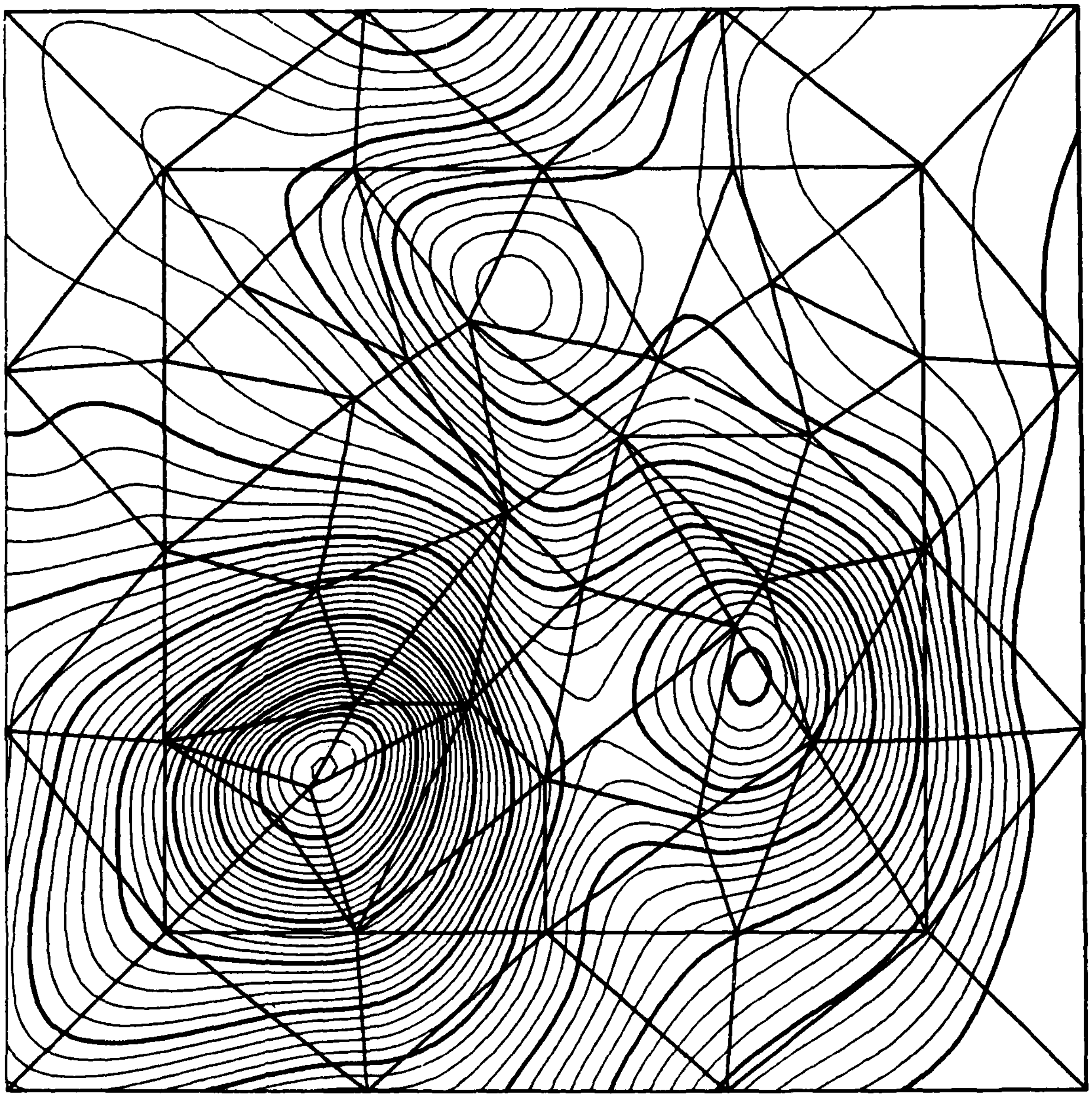
Contour Plot

36 + 4 point extension



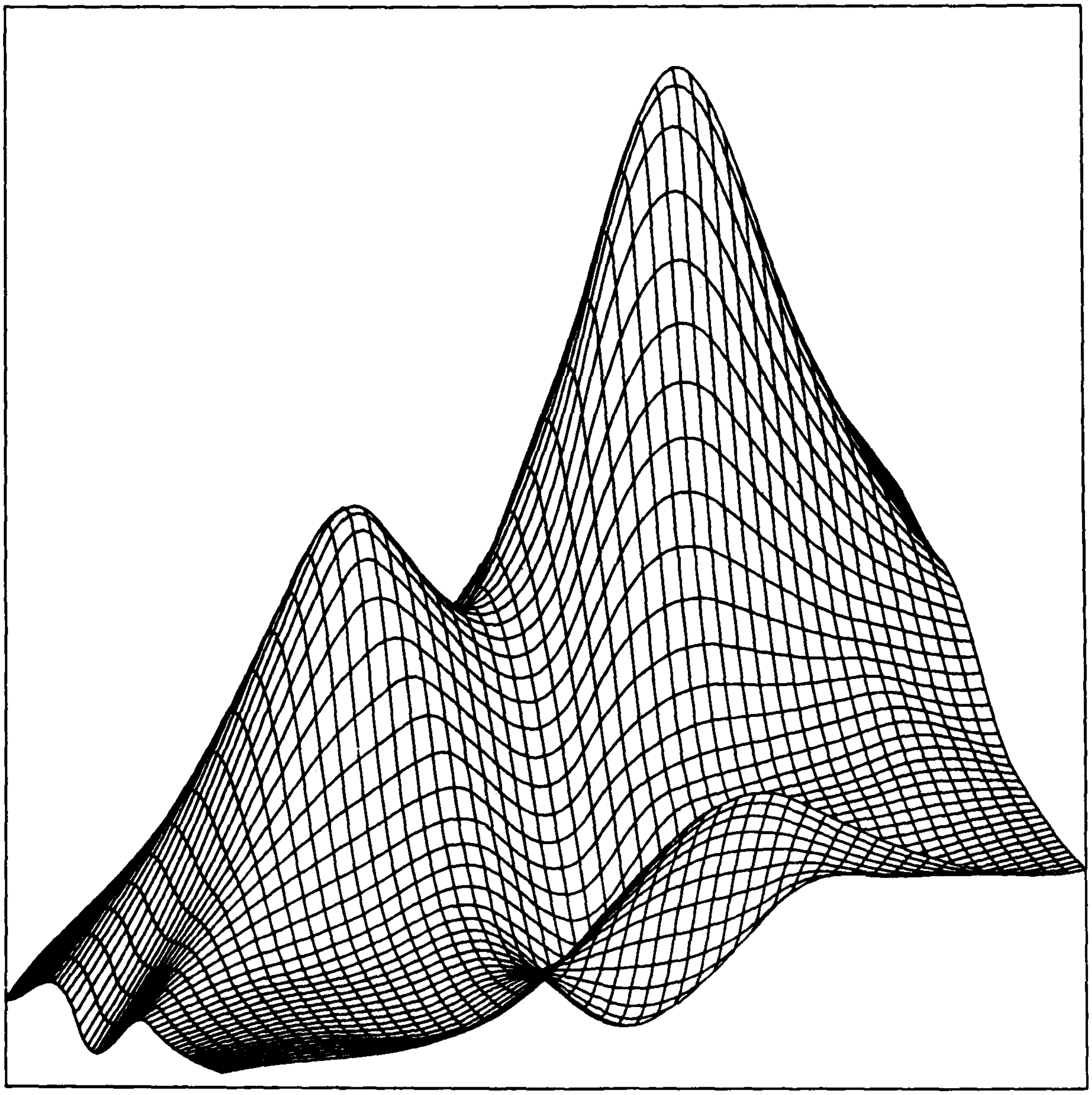
Northeast View

36 + 12 point extension



Contour Plot

36 + 12 point extension



Northeast View

Conclusions

Triangular Extrapolation, as well as multivariate extrapolation based on interpolants other than triangular ones, is a difficult area that requires more research. At present, several techniques exist, but none of them is entirely satisfactory. Of the existing ones, Triangular Extension seems to generate the visually most pleasing surfaces.

References

H. Akima, 1978; *A method of bivariate interpolation and smooth surface fitting for irregularly distributed data points*; ACM Trans. Math. Software 4, pp. 148-159.

P. ALfeld, 1984; *Multivariate Scattered Data Derivative Generation by Functional Minimization*; submitted for publication.

N.F.F. Beebe, 1979; *A User's Guide to ⟨PLOT79⟩*; Department of Physics, University of Utah, Salt Lake City, Utah 84112.

R.E. Barnhill, 1984; *A Survey of the Representation and Design of Surfaces*; IEEE Computer Graphics and Applications, v. 3, No. 7, pp. 9-17.

R.E. Barnhill and G.M. Nielson, 1984 (ed.); *Surfaces*; Special issue of the Rocky Mountain Journal of Mathematics, v. 14, No. 1.

A.K. Cline and R.L. Renka, 1984; *A storage-efficient method for construction of a Thiessen triangulation*; in Barnhill and Nielson, 1984, pp. 119-140.

R. Franke, 1982; *Scattered Data Interpolation: Tests of some methods*; Math. Comp. v. 38, pp. 1377-1393.

A. George and J.W. Liu, 1981; *Computer Solution of Large Sparse Positive Definite Systems*; Prentice Hall.

C.L. Lawson, 1977; *Software for C^1 Surface Interpolation*; in John R. Rice (ed.), *Mathematical Software III*, Academic Press.

J. Morgan and R. Scott; *The dimension of piecewise polynomials*; Manuscript.

G.M. Nielson, 1983; *A Method for Interpolating Scattered Data Based Upon a Minimum Norm Network*; Math.Comp. v. 40, pp. 253-271.

R. Schmidt, 1982; *Eine Methode zur Konstruktion von C^1 Flächen zur Interpolation unregelmässig verteilter Daten*; in W. Schempp and K. Zeller (ed.), Multivariate Approximation Theory II, Birkhäuser Verlag.

L.L. Schumaker, 1984; *Bounds on the dimension of spaces of multivariate piecewise polynomials*; in BArnhill and Nielson, 1984, pp. 251-264.

Acknowledgements

The author has benefited from the stimulating environments provided by the Mathematics Research Center and the Computer Aided Geometric Design Group at the University of Utah.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 2707	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) TRIANGULAR EXTRAPOLATION		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Peter Alfeld		8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041 DE-AL02-82ER12046A000
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 3 - Numerical Analysis and Scientific Computing
11. CONTROLLING OFFICE NAME AND ADDRESS (See item 18 below)		12. REPORT DATE June 1984
		13. NUMBER OF PAGES 22
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

U.S. Army Research Office
P.O. Box 12211
Research Triangle Park
North Carolina 27709

U.S. Department of Energy
Washington, DC 20545

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Scattered Data
Bivariate Interpolation
Bivariate Extrapolation
Triangular Interpolation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper describes several methods of extending a bivariate interpolant defined on triangles smoothly beyond the triangulation. Existing methods are reviewed, new methods are introduced, and the methods are compared.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

END

FILMED

DTIC