


# LATERALLY LOADED PILES AND COMPUTER PROGRAM COM624G 

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## 20. ABSTRACT (Continued).

A computer program called COM624, along with documentation, was developed at the University of Texas (UT) at Austin, to analyze laterally loaded pile problems. Analysis performed by Program COM624 is dependent upon soil parameters input to the program. These soil parameters take the form of curves which simulate the nonlinear interaction of the pile and the surrounding soil. The UT Report also presented criteria for developing these soil response curves in various types of soils.

This report consolidates the information available on laterally loaded pile analysis and provides supplementary data on Program COM624 (redesignated as COM624G). It describes modifications made in the input procedures and the addition of graphics options. Several examples of laterally loaded pile problems encountered in the Corps are added. Also included is a procedure for nondimensional analysis of laterally loaded piles which can be used to perform companion hand calculations to verify the results of the computer solutions.

## PREFACE

This report reviews soil-structure interaction analyses of laterally loaded piles and provides supplementary documentation on a computer program COM624 developed by Prof. Lymon C. Reese, Nasser Al Rashid Professor, Civil Engineering Department, University of Texas (UT) at Austin, and Mr. W. R. Sullivan who was a graduate student at UT. Liberal use is made herein of material previously published by Prof. Reese and his graduate students.

Mr. Reed L. Mosher and Mr. Michael E. Pace of the Computer-Aided Design Group, Automatic Data Processing (ADP) Center, U. S. Army Engineer Waterways Experiment Station (WES), modified the original program to run in the timesharing mode, added graphics options, and also restructured the input to the program. The modified program has been designated as COM624G. Messrs. Mosher and Pace prepared Appendix $C$ which contains the input to the modified program. Mr. A. E. Templeton, Vicksburg District (VXD), ran all of the computer and hand-derived examples contained in this report. Contributions of all of the above are gratefully acknowledged.

Funds for this work were authorized by the U. S. Army Engineer Division, Lower Mississippi Valley (LMVD), as part of the analysis support provided by the WES ADP Center. Mr. James A. Young, Geology, Soils, and Materials Branch, LMVD, was the technical point of contact.

The work was accomplished during the period July 1981 through April 1983. This report was written by Prof. Reese, Mr. Larry A. Cooley, Chief, Foundation and Materials Branch, VXD, and Dr. N. Radhakrishnan, Special Technical Assistant, ADP Center, WES.

COL Nelson P. Conover, CE, and COL Tilford C. Creel, CE, were Commanders and Directors of WES during the course of the work and the preparation of this report. Mr. F. R. Brown was Technical Director.


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## CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply
cubic inches
feet
feet per second
feet per second squared
foot-kips (force)
foot-pounds (force)
inches
inches per pound inches to the fourth power
kips
kips per square inch
pounds per inch
pounds per cubic inch
pounds per square inch
pounds per cubic foot
pounds per square foot
tons (force)
tons (mass) per square foot

By
16.3871
0.3048
0.3048
0.3048
4.448222
1.355818
2.54
0.1129848
0.4162
4.4482
6.8497
175.1268

27,679.9000
6.8948
16.0185
4.8824
8.8964

9,764.856

To Obtain
cubic micrometers
meters
meters per second
meters per second squared
kilonewtons
joules
centimeters
newton meters
micrometers to the fourth power
kilonewtons
megapascals
newtons per meter
kilograms per cubic meter millipascals
kilograms per cubic meter
kilograms per square meter kilonewtons
kilograms per square meter

# LATERALLY LOADED PILES AND COMPUTER PROGRAM COM624G 

PART I: INTRODUCTION

## Need for Soil-Structure Interaction Analyses in Design of Pile Foundations

1. Pile foundations are frequently used to support structures when the soil immediately below the base will not provide adequate bearing capacity. Piles transfer load from the structure to soil strata which can support the applied load. The behavior of such a system depends on the interaction of the piles with both the structure and the soil. Rational analysis of a problem involving pile design must take into consideration the effects of these interactions. Equilibrium of forces and compatibility of displacements throughout the total system must be achieved in the analysis. This report deals with analysis of the lateral interaction of the pile shaft and the soil. The problem of satisfying equilibrium between the pile shaft and superstructure is outside the scope of this report. A number of references are available on this topic for the interested reader (CASE Task Group on Pile Foundations 1980; Martin, Jones, and Radhakrishnan 1980; Awoshika and Reese 1971; Radhakrishnan and Parker 1975; Haliburton 1971; and Dawkins 1982).

## Acknowledgments

2. A major portion of the material presented herein is excerpted or summarized from reports published by Prof. Lymon C. Reese and his students/ associates at The University of Texas at Austin (UT). The computer program presented herein (COM624G) was developed under the direction of Prof. Reese and modified by the Automatic Data Processing (ADP) Center at the U. S. Army Engineer Waterways Experiment Station (WES) to provide interactive capability and graphics.
3. Excellent summaries of the methods used in analysis of laterally loaded piles are available (Reese and Sullivan 1980, Reese and Allen 1977). It is suggested that the user study these references before becoming deeply involved in pile design using the method of analysis presented herein. Excerpts from these two references appear throughout this report and are acknowledged where included.

## Example Applications

4. If a structure is supported on vertical piles and if all loads from the structure are also vertical, then the loads transmitted to the piles will all be axial. If some horizontal component of load is present, a lateral force will also be transmitted to the piles. If some of the piles are battered, an axial and lateral force will be transmitted to the piles regardless of the direction of the applied load. For most structures, particularly hydraulic structures, both horizontal and vertical components of load are present. The theory and the computer program presented in this report consider the response of individual piles to lateral loads. The program is not directly applicable to problems where group effects must be considered, such as pile-supported retaining structures where the piles are closely spaced. Several methods to analyze such problems are available ( $O^{\prime}$ Neill, Hawkins, and Mahar 1980; Reese 1980; and Davisson 1970) but will not be addressed herein. Axially loaded pile behavior and a computer program for analyzing such behavior will be the subject of another report.
5. The method of analysis presented in this report is directly applicable to problems in which the lateral response of single-pile foundation elements is analyzed. Examples of such problems encountered by the Corps are single-pile dolphins (Figure 1) and baffles for grade control structures (Figure 2). The method can also be extended and used in multiple-pile foundation elements such as in the continuous frame pile-supported pumping station shown in Figure 3. To solve problems of this type, the user must ensure in the analysis that the predicted behavior of the structural frame is compatible with the predicted behavior of each of the foundation elements. Thus, the problem is analyzed in two parts: (a) a frame analysis using methods which may vary from a finite element analysis to a moment distribution analysis depending on the level of sophistication desired by the user, and (b) a laterally loaded pile analysis. The analysis is performed on an idealized frame resting on piles which are subjected to horizontal and vertical loads. The frame is separated from the piles at the groundline as shown by the insert in Figure 3 . Final results of the analysis must show the lateral deflection, rotation, shear, axial load, and moment to have the same values at the points where the piles connect to the frame.
6. Because analysis of this problem must be performed in two parts, the


Figure 1. Single pile mooring dolphin



Figure 3. Idealized continuous frame pile-supported pumping station
analysis is iterative. One approach is to assume the reactions of each pile on the frame, apply these reactions to the frame, and analyze. Results of this analysis are then applied to the piles. Then the results of the pile analysis are compared to the assumptions made for the frame analysis, the inputs for the frame analysis are revised, and the process is repeated until compatible forces, moments, and deflections result from both analyses. This approach is discussed in more detail by Reese and Allen (1977).

Methods of Analysis
7. Many different methods have been used in analysis of laterally loaded piles, where the analysis in general consists of computing pile deflection,
bending moment, and shear as a function of depth below the top of the pile. Figure 4 presents the results of a laterally loaded pile analysis. Several of the methods of analysis are based on the theory of subgrade reaction in which the soil around the pile shaft is replaced by a series of discrete springs. Solution of the problem involves solution of a fourth-order differential equation. Most researchers utilizing this approach solve the equation using either a closed-form or a power series solution which requires numerous simplifying assumptions. The more critical of these assumptions are: (a) a constant or linear variation of subgrade modulus with depth, (b) linearly elastic soil behavior, and ( $c$ ) constant flexural stiffness of the pile. Examples of these methods of analysis are given in Davisson (1970), Terzaghi (1955), Winkler (1967), Broms (1964a), and Broms (1964b).


Figure 4. Form of the results obtained from a laterally loaded pile (Reese and Cox 1968)
8. An entirely different approach (Poulos 1971) assumes the soil to be an elastic, homogeneous, isotropic half-space with a constant Young's modulus and Poisson's ratio. The pile is modeled as a thin, rectangular, vertical strip with soil pressures constant across the pile width. This method suffers from the critical limitation of the other methods previously discussed; i.e., the soil response is assumed to be linear.
9. The method utilized in the laterally loaded pile program, COM624G. is
based on the theory of subgrade reaction discussed above. However, the method used for solution of the fourth-order differential equation is the finite difference technique. This solution method, which is presented in Part II, offers several advantages over the conventional methods: (a) the soil modulus can be varied both with depth and pile deflection, (b) stratified soil deposits can be analyzed, (c) the pile stiffness with depth can be considered, (d) the flexural stiffness of the pile can be varied, and (e) several types of boundary conditions can be employed.

## Nonlinear Interaction Curves

10. Program COM624G presents mathematical solutions of physical models which are capable of describing the actions and reactions of the pile shaftsoil systems. However, as with most geotechnical engineering applications, the analysis is only as reliable as the soil parameters input to the problem. In this case, the soil parameters take the form of curves which simulate the nonlinear interaction of the pile and the surrounding soil.
11. A family of curves describes the behavior of the soil around a laterally loaded pile in terms of lateral soil reaction versus lateral pile movement for a number of locations along the pile. Each curve represents lateral force (per unit length) transferred to the soil by a given lateral movement at a given location.
12. Criteria used in developing these nonlinear pile shaft-soil interaction curves are presented in Part III. These criteria are thought to yield conservative estimates of soil response; however, the user must always bear in mind that the criteria are based on limited data and there are many inevitable uncertainties in estimating soil response. Nevertheless, the criteria presented here represent the current state of the art. In Part IV of an earlier report by Radhakrishnan and Parker (1975), soil criteria are provided for laterally and axially loaded piles. The material presented herein updates these criteria for laterally loaded piles. Soil criteria for axially loaded piles presented in Radhakrishnan and Parker (1975) will be updated in a separate report.

## Purpose and Scope

13. The primary purpose of this report is to present background
information on laterally loaded pile analaysis and to provide supplementary documentation of computer program COM624G. The subject area covered is rich in technical literature, and no attempt is made herein to discuss the methods of analysis in detail. However, enough theory and background are presented to explain the basis of the method used in the computer program. Examples of problems encountered by the Corps of Engineers are used where appropriate for illustrative purposes.
14. Background and theory for laterally loaded pile analysis (the basis for program COM624G) are presented in Part II. Part lll presents criteria toi developing soil response curves. Appendix A presents a procedure for nondimensional analysis of laterally loaded piles which can be used to perform companion hand calculations to verify the results of the computer solutions. Appendix $B$ presents a design example which illustrates the importance of engineering judgment in analysis of laterally loaded piles. A user's guide for COM624G is presented in Appendix C. A complete and well-documented user's guide for COM624 is presented by Reese and Sullivan (1980). Appendix D presents examples of problems particularly applicable to Corps of Engineers projects. The notations used in the report are summarized in Appendix $E$.

## PART IT: BACKGROUND AND 'PHEORY FOR T.ATERAIJ,Y IOADED PIIE ANALYSIS

15. Two steps are involved in obtaining the response of a given pile to a lateral load: (a) the soil response must be determined as a function of depth, pile deflection, pile geometry, and nature of loading; and (b) the equations must be solved that yield pile deflection, slope, bending moment, and shear. In this part of the report, the theory involved in developing and solving the equations will be reviewed. The procedures for developing the nonlinear curves which predict the soil response will be presented in Part III.

## Review of Basic Beam-Column Relations

16. The method of analysis used in COM624G is based on the theory of a beam on an elastic foundation. In this case, however, the beam is inserted vertically into the ground instead of being placed horizontally on the surface and is treated as a beam-column. The basic concepts of bean-column relations are covered in detail in numerous engineering mechanics texts (see Higdon et al. 1967); therefore, a review of them will not be presented here.
17. The basic relationships between deflection, slope, moment, shear, and load for a beam (Figure 5, without the axial load, $\mathrm{P}_{\mathrm{x}}$ ) \% of constant flexural rigidity are

$$
\begin{align*}
& S=\frac{d y}{d x}  \tag{1}\\
& M=E I \frac{d^{2} y}{d x^{2}}  \tag{2}\\
& V=\frac{d M}{d x}=E I \frac{d^{3} y}{d x^{3}} \tag{3}
\end{align*}
$$

and

$$
\begin{equation*}
q=\frac{d V}{d x}=E I \frac{d^{2} M}{d x^{2}}=E I \frac{d^{4} y}{d x^{4}} \tag{4}
\end{equation*}
$$

[^0]
$$
p=-E_{s} v
$$


Figure 5. Relationships between deflection, shear, and load for a typical beam-column
where
S = slope
$M=$ moment
$E I=$ flexural rigidity
$V=$ shear
$q$ = uniformly distributed vertical load on beam
$y=$ deflection at point $x$ along the length of the column
Writing these equations in terms of load and deflection gives

$$
\begin{equation*}
q=\frac{d^{2} M}{d x^{2}} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
y=\frac{1}{E I} \iint M d x \tag{6}
\end{equation*}
$$

The differential equation for a beam-column subjerted to loads only at its ends can be obtained by taking the equation for bending due to flexure and adding to it the bending due to a constant axial load $\mathrm{P}_{\mathrm{x}}$

$$
\begin{equation*}
E I \frac{d^{4} y}{d x^{4}}+P \frac{d^{2} y}{d x^{2}}=0 \tag{7a}
\end{equation*}
$$

If the beam-column is resting on or embedded in soil, a soil reaction $p$ will be resisting the movement of the system and Equation 7 a will be transformed to

$$
\begin{equation*}
E I \frac{d^{4} y}{d x^{4}}+P \frac{d^{2} y}{d x^{2}}=q+p \tag{7b}
\end{equation*}
$$

where $p$ is the soil resisting pressure applied to the beam.

## $\mathrm{p}-\mathrm{y}$ Concepts of Lateral Load Transfer

18. When the basic beam-column is inserted vertically as a pile shaft, the method of analysis in COM624G considers the soil surrounding the shaft as a set of nonlinear elastic springs as depicted in Figure 6. This assumption is attributed to Winkler (1967), and it states that each spring acts independently; i.e., the behavior of one spring has no effect on any of the adjacent springs. Intuitively, this assumption does not seem correct for describing the nonlinear response of soils. Consequently, this approach has been criticized by some. However, available experimental data (Matlock 1970; Reese, Cox, and Koop 1975) suggest that, for the range of boundary conditions a pile is normally subjected to, the soil response at a point is affected only marginally by the changes in deflected shape.
19. In the analysis, the response of the springs can be taken as either linear or nonlinear. The approach in program COM624G is to treat the springs as nonlinear with their response represented by curves which relate soil resistance $p$ to pile deflection $y$. In general, these curves are nonlinear and depend on several parameters including depth, pile geometry, shear strength of the soil, and type of loading (static or cyclic). The response of a pile to sustained or dynamic loading is not treated in this report.
20. The concept of a p-y curve can be defined graphically by considering a thin slice of a pile and surrounding soil, as shown in Figure 7a. The earth pressures which act on the surface of the pile prior to lateral loading


Figure 6. Model of pile-soil system with soil represented as a set of nonlinear elastic springs (Reese 1978)


> b. Section $A-A$. Earth pressure distribution prior to lateral loading
c. Section A-A. Earth pressure distribution after lateral loading

Figure 7. Graphical definition of $p$ and $y$ (Reese and Sullivan 1980)

are assumed to be uniform (Figure 7b). For this condition, the resultant force, obtained by integrating the pressures, is zero. If the pile is given a lateral deflection $y_{i}$, as shown in Figure 7 c , a net soil reaction $p_{i}$ will be obtained upon integrating the pressures. This process can be repeated in concept for a series of deflections $y$, resulting in a series of forces per unit length of pile $p$, which can be combined to define a $p-y$ curve. In a similar manner, $p-y$ curves may be generated for a number of depths. A family of p-y curves for different depths is shown in Figure 8. The curves are plotted in the second and fourth quadrants to indicate that the soil resistance $p$ is opposite in sign to the deflection $y$. The user should note that $p$ stands for a force per unit length of pile and is expressed in units

$x=$ DEPTH BELOW GROUNDLINE
Figure 8. Possible family of p-y curves
(Reese and Sullivan 1980)
of pounds per linear inch or pounds per linear foot. It is not a soil pressure which is stated in un:ts of pounds per square inch or pounds per square foot.
21. A typical p-y curve is shown in Figure 9. The curve is plotted in the first quadrant for convenience. The soil modulus $\mathrm{E}_{\mathrm{s}}$ is defined as $-\mathrm{p} / \mathrm{y}$ and is taken as the secant modulus to a point on the $\mathrm{p}-\mathrm{y}$ curve as shown in Figure 8. Because the curve is strongly nonlinear, the soil modulus changes from an initial stiffness $E_{s}$ to an ultimate stiffness $p_{u} / y_{u}$. As can be seen, the soil modulus $E_{s}$ is not a constant except for a small range of deflections. The soil modulus has units of force per length squared, which is the force per unit length of the pile per unit of movement of the pile into the soil. The soil modulus should not be confused with Young's modulus which has the same units but a different meaning.


Figure 9. Characteristic shape of p-y curve (Reese and Sullivan 1980)
22. The soil modulus is introduced into the analysis with the relationship:

$$
\begin{equation*}
p=-E_{s} y \tag{8}
\end{equation*}
$$

By substituting this relationship in Equation $7 b$, the basic equation for laterally loaded piles becomes

$$
\begin{equation*}
E I \frac{d^{4} y}{d x^{4}}+P \frac{d^{2} y}{d x^{2}}+E s_{s} y=q \tag{9}
\end{equation*}
$$

Also,

$$
\begin{equation*}
V=\frac{d M}{d x}+p_{x} \frac{d y}{d x} \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
M=\operatorname{EI} \frac{d^{2} y}{d x^{2}} \tag{11}
\end{equation*}
$$

Equation 9 is developed in the following paragraphs of this part of the report and its solution is presented.

## Solution of Governing Differential Equation

23. Computer program COM624G utilizes central difference approximations to describe the load-deformation response of laterally loaded piles. In the following paragraphs, central difference approximations describing the elastic curve of a laterally loaded pile will be derived and used in formulating a set of simultaneous equations for describing the load-deformation response of a laterally loaded pile.

Formulation of finite difference approximations
24. The finite difference approach to the solution of laterally loaded piles was first suggested by Gleser (1953). The idea was extended by a number of investigators including Reese and Matlock (1956, 1960).
25. The first step in the formulation is the derivation of the central difference approximations for the elastic curve (Figure 10). It can be seen from this figure that the slope of the curve at station $i$ may be approximated as a secant drawn through the points on the curve of the two adjacent stations Mathematically, this step is expressed as

$$
\begin{equation*}
\left(\frac{d y}{d x}\right)_{i} \approx \frac{y_{i+1}-y_{i-1}}{2 h} \tag{12}
\end{equation*}
$$

where $h$ denotes the increment length. For higher derivatives, the process could be repeated by taking simple differences and dividing by $2 h$ each time. However, to keep the system more compact, temporary stations $j$ and $k$ are considered and the slopes at these points computed on the basis of the deflection


Figure 10. Geometric basis for central difference approximations (Reese and Sullivan 1980)
of the station on each side. The second derivative for each permanent station is then written as the difference between these slopes divided by one increment length in the following equation:

$$
\begin{align*}
\left(\frac{d^{2} y}{d x^{2}}\right)_{i} & =\frac{\left(\frac{d v}{d x}\right)_{k}-\left(\frac{d y}{d x}\right)_{j}}{h} \\
& =\frac{y_{i+1}-2 y_{i}+y_{i-1}}{h^{2}} \tag{13}
\end{align*}
$$

Similarly, the third derivative is expressed as

$$
\begin{align*}
\left(\frac{d^{3} y}{d x^{3}}\right) & =\frac{\left(\frac{d^{2} y}{d x^{2}}\right)_{i+1}-\left(\frac{d^{2} y}{d x^{2}}\right)_{i-1}}{2 h} \\
& =\frac{y_{i+2}-2 y_{i+1}+2 y_{i-1}-y_{i-2}}{2 h^{3}} \tag{14}
\end{align*}
$$

and the fourth derivative as

$$
\begin{align*}
\left(\frac{d^{4} y}{d x^{4}}\right)_{i} & =\frac{\left(\frac{d^{3} y}{d x^{3}}\right)_{k}-\left(\frac{d^{3} y}{d x^{3}}\right)_{j}}{h} \\
& =\frac{y_{i+2}-4 y_{i+1}+6 y_{i}-4 y_{i-1}-y_{i-2}}{h^{4}} \tag{15}
\end{align*}
$$

Formulation of finite difference approximations for equations of bending of laterally loaded piles
26. In the development of the equations, consideration must be given to the assumptions regarding the variation in pile bending stiffness ( $E I=R$ ). For the case of pure bending and constant bending stiffness, the second derivative of moment is usually written as

$$
\begin{equation*}
\frac{d^{2} M}{d x^{2}}=\text { EI } \frac{d^{4} y}{d x^{4}} \tag{16}
\end{equation*}
$$

For the case of pure bending and a variable bending stiffness, the second derivative of moment is expressed as

$$
\begin{equation*}
\frac{d^{2} M}{d x^{2}}=E I \frac{d^{4} y}{d x^{4}}+2 \frac{d}{d x} \text { (EI) } \frac{d^{3} y}{d x}+\frac{d^{2}}{d x^{2}} \text { (EI) } \frac{d^{2} y}{d x^{2}} \tag{17}
\end{equation*}
$$

However, in formulating the finite difference equations, the assumption was made that the moment was a smooth continuous function of $x$ and that the second derivative of moment could be approximated by the expression

$$
\begin{equation*}
\frac{d^{2} M}{d x^{2}} \approx \frac{M_{i+1}-2 M_{i}+M_{i-1}}{h^{2}} \tag{18}
\end{equation*}
$$

where $M_{i+1}, M_{i}$, and $M_{i-1}$ are the moments at joints $i+1, i$, and $i-1$, respectively. For a variable stiffness, Equation 18 is a somewhat cruder approximation than Equation 20. However, it permits the bending stiffness to vary from station to station.
27. Equations 9,10 , and 11 may now be written in finite difference form by using the central difference approximations for the first and second of the elastic curves. The equations will be written for a general point referred to as station i. Station numbering increases from the bottom to the top of piles. The equations obtained for station $i$, formulated from Equation 11 , are as follows:

$$
\begin{equation*}
M_{i}=R_{i}\left(\frac{y_{i+1}-2 y_{i}+y_{i-1}}{h^{2}}\right) \tag{19}
\end{equation*}
$$

where $R=$ flexural rigidity (EI). Equations 8, 13, 16, 18, and 19 can be employed. and Equation 20 can be formulated from Equation 9 .

$$
\begin{align*}
& y_{i+2}\left(R_{i+1}\right)+y_{i+1}\left(-2 R_{i+1}-2 R_{i}+P_{x} h^{2}\right) \\
& \quad+y_{i}\left(R_{i+1}+4 R_{i}+R_{i-1}-2 P_{x} h^{2}+E_{s i} h^{4}\right)  \tag{20}\\
& \quad+y_{i-1}\left(-2 R_{i}-2 R_{i-1}+P_{x} h^{2}\right)+y_{i-2}\left(R_{i-1}\right)-q=0
\end{align*}
$$

Equation 21 can be formulated from Equation 10 in a similar manner.

$$
\begin{align*}
& v_{i}=\frac{1}{2 h^{3}}\left[y_{i+2}\left(R_{i+1}\right)+y_{i+1}\left(-2 R_{i+1}+P_{x} h^{2}\right)\right] \\
& +y_{i}\left(R_{i+1}-R_{i-1}\right)+y_{i-1}\left(-P_{x} h^{2}\right)+y_{i-2}\left(-R_{i-1}\right) \tag{21}
\end{align*}
$$

Solution of the finite difference equations (extracted from Reese and Sullivan 1980)
28. The final step is the formulation of a set of simultaneous equations which when solved yield the deflected shape of the pile. The solution
requires the application of four boundary conditions, since Equation 9 is actually a fourth-order differential equation in terms of the dependent variable $y$. If values of deflection are found, moment, shear, and soil reaction can be obtained for any location along the pile by backsubstitution of appropriate values of deflection into appropriate equations.
29. The pile is divided into equal increments of length $h$ (Figure 11). In addition, two fictitious increments are added to both the top and bottom of the pile. The four fictitious stations are used in formulating the set of equations, but they will not appear in the solution or influence the results. The coordinate system and numbering system used are also illustrated in Figure 11.
30. Using the notation shown in Figure 11, the two boundary conditions at the bottom of the pile (point 0 ) are zero bending moment,

$$
\begin{equation*}
R_{0}\left(\frac{d^{2} y}{d x^{2}}\right)_{0}=0 \tag{22a}
\end{equation*}
$$

and zero shear,


Figure 11. Finite difference representation of a pile (Reese and Sullivan 1980)

$$
\begin{equation*}
R_{0}\left(\frac{d^{3} y}{d x^{3}}\right)_{0}+P_{x}\left(\frac{d y}{d x}\right)_{0}=0 \tag{22b}
\end{equation*}
$$

For simplicity it is assumed that

$$
\begin{equation*}
R_{-1}=R_{0}=R_{1} \tag{22c}
\end{equation*}
$$

These boundary conditions are, in finite difference form,

$$
\begin{gather*}
y_{-} \quad 2 y_{0}+y_{1}=0  \tag{23a}\\
y_{-2}=y_{-1}\left(2-\frac{P_{x} h^{2}}{R_{0}}\right)-y_{1}\left(2-\frac{P_{x} h^{2}}{R_{0}}\right)+y_{2} \tag{23b}
\end{gather*}
$$

respectively. Substituting these boundary conditions in finite difference form in Equation 20 where $i$ is equal to zero, and rearranging terms, results in the following equations:

$$
\begin{equation*}
y_{0}=a_{0} y_{1}-b_{0} y_{2} \tag{24a}
\end{equation*}
$$

where

$$
\begin{align*}
& a_{0}=\frac{2 R_{0}+2 R_{1}-2 P P_{x} h^{2}}{R_{0}+R_{1}+E_{s o} h^{4}-2 P_{x} h^{2}}  \tag{24b}\\
& b_{0}=\frac{R_{0}+R_{1}}{R_{0}+R_{1}+E_{s o} h^{4}-2 P_{x} h^{2}}  \tag{24c}\\
& d_{0}=\frac{g h^{4}}{R_{0}+R_{1}+E_{s o} h^{4}-2 P_{x} h^{4}} \tag{24d}
\end{align*}
$$

31. Equation 20 can be expressed for all values of $i$ other than 0 and the top of the pile by the following relationships:

$$
\begin{gather*}
y_{i}=a_{i} y_{i+1}-b_{i} y_{i+2}+d_{i}  \tag{25a}\\
a_{i}=\frac{-2 b_{i-1} R_{i-1}+a_{i-2} b_{i-1} R_{i-1}+2 R_{i}-2 b_{i-1} R_{i}+2 R_{i+1}-P_{x} h^{2}\left(1-b_{i-1}\right)}{c_{i}} \tag{25b}
\end{gather*}
$$

$$
b_{i}=\frac{R_{i+1}}{c_{i}}
$$

and
$c_{i}=R_{i-1}-2 a_{i-1} R_{i-1}-b_{i-2} R_{i-1}+a_{i-2} a_{i-1} R_{i-1}+4 R_{i}$

$$
\begin{equation*}
-2 a_{i-1} R_{i}+R_{i+1}+k_{i} h^{4}-P_{x} h^{2}\left(2-a_{i-1}\right) \tag{25d}
\end{equation*}
$$

$d_{i}=\frac{q_{i} h^{4}-d_{i-1}\left(a_{i-2} R_{i-1}-2 R_{i-1}-2 R_{i}+P_{x} h^{2}\right)-d_{i-2} R_{i-1}}{c_{i}}$
32. The top of the pile ( $i=t$ ) is shown in Figure 11. Three sets of boundary conditions are considered.
a. The lateral load $\left(P_{t}\right)$ and the moment $\left(M_{t}\right)$ at the top of the piles are known.
b. The lateral load ( $P_{t}$ ) and the slope of the elastic curve ( $S_{t}$ ) at the top of the pile are known.
c. The lateral load ( $P_{t}$ ) and the rotational-restraint constant $\left(M_{t} / S_{t}\right)$ at the top of the pile are known.
33. For convenience in establishing expressions for these boundary conditions, the following constants are defined.

$$
\begin{align*}
& J_{1}=2 h S_{t}  \tag{26a}\\
& J_{2}=\frac{M_{t} h^{2}}{R_{t}}  \tag{26b}\\
& J_{3}=\frac{2 P_{t} h^{3}}{R_{t}}  \tag{26c}\\
& J_{4}=\frac{h}{2 R_{t}} \frac{M_{t}}{S_{t}} \tag{26d}
\end{align*}
$$

and

$$
\begin{equation*}
\mathrm{U}=\frac{-\mathrm{P}_{\mathrm{x}} \mathrm{~h}^{2}}{\mathrm{R}_{\mathrm{t}}} \tag{26e}
\end{equation*}
$$

34. The difference equations expressing the first of the boundary conditions for the top of the pile are:

$$
\begin{gather*}
\frac{R_{t}}{2 h^{3}}\left(y_{t-2}-2 y_{t-1}+2 y_{t+1}-y_{t+2}\right)+\frac{P_{t}}{2 h}\left(y_{t-1}-y_{t+1}\right)=P_{t}  \tag{27a}\\
\frac{R_{t}}{h^{2}}\left(y_{t-1}-2 y_{t}+y_{t+1}\right)=M_{t} \tag{27b}
\end{gather*}
$$

After some substitutions the difference equations for the deflection at the top of the pile and at the two imaginary points above the top of the pile are:

$$
\begin{align*}
y_{t} & =\frac{Q_{2}}{Q_{1}}  \tag{28a}\\
y_{t+1} & =\frac{J_{2}+G_{1} y_{t}-d_{t-1}}{G_{2}}  \tag{28b}\\
y_{t+2} & =\frac{a_{t} y_{t+1}-y_{t}+d_{t}}{b_{t}} \tag{28c}
\end{align*}
$$

where

$$
\begin{align*}
& Q_{1}=H_{1}+\frac{G_{1} H_{2}}{G_{2}}+\left(1-a_{t} \frac{G_{1}}{G_{2}}\right) \frac{1}{b_{t}}  \tag{28d}\\
& Q_{2}=J_{3}+\frac{a_{t}\left(J_{2}-d_{t-1}\right)}{b_{t} G_{2}}+\frac{H_{2}\left(d_{t-1}-J_{2}\right)}{G_{2}}+\frac{d_{t}}{b_{t}}+d_{t-1}\left(2+U-a_{t-2}\right)-d_{t-2} \\
& G_{1}=2-a_{t-1}  \tag{28e}\\
& G_{2}=1-b_{t-1}  \tag{28f}\\
& H_{1}=-2 a_{t-1}-U_{t-1}-b_{t-2}+a_{t-1} a_{t-2} \tag{28g}
\end{align*}
$$

and

$$
\begin{equation*}
H_{2}=-a_{t-2} b_{t-1}+2 b_{t-1}+2+u\left(1+b_{t-1}\right) \tag{28i}
\end{equation*}
$$

35. The difference equations for the second set of boundary conditions are Equations 27a and 29:

$$
\begin{equation*}
y_{t-1}-y_{t+1}=J_{1} \tag{29}
\end{equation*}
$$

36. The resulting difference equations for the deflections at the three points at the top of the pile are:

$$
\begin{align*}
y_{t} & =\frac{Q_{4}}{Q_{3}}  \tag{30a}\\
y_{t+1} & =\frac{a_{t-1} y_{t}-J_{1}+d_{t-1}}{G_{4}}  \tag{30b}\\
y_{t+2} & =\frac{a_{t} y_{t+1}-y_{t}+d_{t}}{b_{t}} \tag{30c}
\end{align*}
$$

where

$$
\begin{align*}
& Q_{3}=H_{1}+\frac{H_{2}{ }_{2} t-1}{G_{4}}-\frac{a^{\prime} t^{a} t-1}{b_{t} G_{4}}+\frac{1}{b_{t}}  \tag{30d}\\
& Q_{4}=J_{3}+\frac{J_{1} H_{2}}{G_{4}}-\frac{J_{1} a_{t}}{b_{t} G_{4}} \tag{30e}
\end{align*}
$$

and

$$
\begin{equation*}
G_{4}=1+b_{t-1} \tag{30f}
\end{equation*}
$$

and the other constants are as previously defined.
37. The difference equations for the third set of boundary conditions are Equations 27a and 31:

$$
\begin{equation*}
\frac{y_{t-1}-2 y_{t}+y_{t+1}}{y_{t-1}-y_{t+1}}=J_{4} \tag{31}
\end{equation*}
$$

38. The resulting difference equations for the deflections at the three points at the top of the pile are:

$$
\begin{gather*}
y_{t}=\frac{J_{3}-\frac{a_{t} d_{t-1}\left(1-J_{4}\right)}{\left.b_{t} G_{2}+J_{4} G_{4}\right)}+\frac{d_{t}}{b_{t}}+d_{t-1}\left(2+E-a_{t-2}\right)-d_{t-2}+\frac{d_{t-1} H_{2}\left(1-J_{4}\right)}{G_{2}+J_{4} G_{4}}}{H_{1}+H_{2} H_{3}-\frac{a_{t}}{b_{t}} H_{3}+\frac{1}{b_{t}}}  \tag{32a}\\
y_{t+1}=\frac{y_{t}\left(G 1+J_{4} a_{t-1}\right)-d_{t-1}\left(1-J_{4}\right)}{G_{2}+J_{4} G_{4}}=H_{3} y_{t}-\frac{d_{t-1}\left(1-J_{4}\right)}{G_{2}+J_{4} G_{4}}  \tag{32b}\\
y_{t+2}=\frac{1}{b_{t}}\left(a_{t} y_{t+1}-y_{t}+d_{t}\right) \tag{32c}
\end{gather*}
$$

where

$$
\begin{equation*}
H_{3}=\frac{G_{1}+J_{4}{ }^{a} t-1}{G_{2}+J_{4} G_{4}} \tag{32d}
\end{equation*}
$$

The other constants have been previously defined.
39. Using the above equations, the behavior of a pile under lateral load may be obtained by using COM624G.

## PART III: CRITERIA FOR DEVELOPING SOIL RESPONSE CURVES FOR LATERALLY LOADED PILES

40. The methods of constructing $p-y$ curves as presented in this report were developed at UT. The methods were derived largely from results obtained in field tests of piles under lateral loading. The approach was to take the experimental field curves and correlate them empirically with simple, basic soil mechanics theory and experience. By combining soil mechanics theory with experimental results, correlations could be made between soil properties, pile diameter, and depth. This gives generality to the methods used in construction of the $p-y$ curves.
41. McClelland and Focht (1958) were the first to report p-y itieria which considered the nonlinearity of the soil. Since their work, numerous re. searchers have contributed to $p-y$ curve development; however, most of the developmental work has been performed at UT. A history of the development will not be presented here; however, the interested reader can refer $t$, Mever and Reese (1979) for more detailed information.
42. The methods presented herein represent the current stat. wt $\quad \mathrm{F}$ curve development; hovever, it is expected that this development mil, nt in. as more field tests are performed and as more experience is gained the ....: must remain abreast of these changes in order to ensure that the alialuw. flect the state of the art at the particular time they are perturmed
43. Recommended methods for computing p-y curves are haseit in 1.1 , tests presented in five different references for four difterent types it conditions. These are:
a. Soft clay below the water table (Matlock 1970)
b. Stiff clay below the water table (Reese, Cox, and Koop 19751
c. Stiff clay above the water table (Keese and Welch 1975)
d. Unified clay criteria developed for combined soft and stiff clays below the water table, (Sullivan, Reese, and Fenske 1979)
e. Sands (Reese, Cox, and Koop 1974).
44. These references describe field experiments, the soil conditions in which they were performed, the rationale and considerations involved in evaluating the data, and conclusions from the experiments presented in the form of recommended $p-y$ curve criteria. As can be seen from the descriptive names, the criteria were developed separately for clays above and below the
water table and for sands. Other soil types would be expected to exhibit characteristics falling between the extremes of the soils and conditions in these tests.
45. The criteria for the conditions listed in subparagraphs 43a, b, c, and $e$ have been combined into summary form and are presented in Reese and Sullivan (1980) and Reese and Allen (1977). The material presented herein is extracted primarily from these two references. However, the user of COM624G is strongly encouraged to study the references cited in paragraph 42 before becoming deeply involved in the analysis of laterally loaded piles. Also, the user should bear in mind that any one set of $p-y$ curves is strongly related to only one or two lateral load tests, and this fact should be considered when using the curves for design.

> Factors Influencing p-y Curves
46. Factors that most influence $p-y$ curves are soil properties, pile geometry, nature of loading, and pile spacing. The correlations that have been developed for predicting soil response have been based on best estimates of soil properties determined from borings, laboratory tests, and field in situ tests. Thus far, no investigations have been performed to determine the effect which the method of pile installation has on these soil properties. The logic supporting this approach is that the effects of pile installation on soil properties are principally confined to a zone of soil close to the pile wall, while a mass of soil several diameters from the pile is stressed as lateral deflection occurs. There are instances where the method of pile installation must be considered; e.g., if a pile is jetted into place, a considerable volume of soil could be removed with a considerable effect on the soil response. In such instances, the user must rely on experience in adjusting the $p-y$ curves to account for the effect of pile installation.
47. The principal dimension of the pile which affects the soil response is its diameter. All recommendations for developing $p-y$ curves include the term for the diameter of the pile: if the cross section of the pile is not circular, the width of the pile perpendicular to the direction of loading is usually taken as the diameter. Field tests have been performed on piles with a limited range of diameters. Experience indicates that, for the normal range of pile diameters encountered in practice, the criteria adequately represent
the effect of pile diameter. However, additional research is needed on largediameter piles ( $30 \mathrm{in} . \dot{*}$ and larger) to determine the effect of pile diameter on large pile behavior (Meyer and Reese 1979). Stevens and Audibert (1979) have presented evidence that, for piles 50 in . and larger, the observed groundline deflections are approximately half the predicted deflections.
48. p-y curves can be greatly affected by the type of loading. This report summarizes recommendations for short-term static loads and for cyclic (or repeated) loading. The curves do not consider any consolidation effects that would occur under sustained loading. Nor do they consider cases where the loadings are dynamic, as would occur during an earthquake.
49. Because the field tests were run on single piles, the p-y criteria do not consider group effects. Unfortunately, the designer is often faced with the problem of analyzing the lateral response of pile groups. Although several methods are available in the literature, there is no one established, widely used method which considers the group effect on soil response. Four available methods which address group effect are presented in O'Neill, Hawkins, and Mahar (1980), Davisson (1970), Focht and Koch (1973), and Poulos (1971a and b).
50. Another factor which can influence $p-y$ criteria is the effect of pile batter. The criteria were derived from experiments on vertical piles. As the batter of a pile is increased, some point will eventually be reached where the criteria for vertical piles are no longer applicable. Information for specific recommendations on this problem is not available; however, some comparison studies performed by Meyer and Reese (1979) indicate that by applying adjustment factors recommended by Kubo (1967), reasonable estimates of pile deflection for laterally loaded batter piles can be obtained.

Analytical Basis for $p-y$ Curves
51. As discussed previously, the methods of constructing p-y curves were derived from results obtained in field tests of piles under lateral loading. Results were then correlated with soil properties, pile diameter, and depth to give generality to the methods. Soil resistance-pile deflection

[^1]curves are generally considered to be composed of an initial elastic portion and an ultimate failure value. Principles of the theory of elasticity are generally applied for the definition of the initial portion. Several failure mechanisms are postulated and used to define the ultimate values. The following paragraphs briefly describe the analytical concepts which were correlated with the experimental curves.
52. The theory of elasticity is only applicable to linearly elastic materials; however, use has been made of the theory of elasticity and related approaches in describing certain concepts which have been incorporated into the nonlinear $p-y$ curves.

Initial Portion of p-y Curve

## Terzaghi

53. In his classic paper "Evaluation of Coefficients of Subgrade Reaction," Terzaghi (1955) proposed coefficients of lateral subgrade reaction which used a straight-line relationship between deflection of the pile $y$ and resistance offered by the soil $p$. Terzaghi recognized the limitations of this approach and stated that the linear relationship between $p$ and $y$ was valid for values of $p$ that were smaller than about half the ultimate bearing capacity of the clay.
54. For stiff clays, Terzaghi gave the relationship

$$
\begin{equation*}
k_{h}=\frac{\bar{k}_{s l}}{1.5 b}(1 \mathrm{ft}) \tag{33}
\end{equation*}
$$

where

| $\mathbf{k}_{h}$ | $=$ coefficient of horizontal subgrade reaction |
| ---: | :--- |
| $\bar{k}_{s l}$ | $=$ coefficient of vertical subgrade reaction for a $1-f t$-wide beam |
| $b$ | $=$ width of the pile, $f t$ | Adapting the coefficient of lateral subgrade reaction to fit the soil modulus Es yields

$$
\begin{equation*}
E_{s}=k_{h} b \tag{34}
\end{equation*}
$$

55. Terzaghi proposed that the coefficient of horizontal subgrade reaction for piles in stiff clay was constant with depth and recommended the values of $\bar{k}_{s l}$ given in Table 1.

Table 1
Terzaghi's Recommendations for Soil Modulus $\overline{\mathbf{k}}_{\mathbf{s l}}$
for Laterally Loaded Piles in Stiff Clay

|  | Consistency of Clay |  |  |
| :---: | :---: | :---: | :---: |
|  | Stiff | Very Stiff | Hard |
| Value of $\mathrm{q}_{\mathbf{u}}$, tsf | 1-2 | 2-4 | 4-7 |
| Range for $\overline{\mathbf{k}}_{\mathbf{s l}}$, pci | 58-116 | 116-232 | 232-464 |
| Proposed values for $\bar{k}_{\text {sl }}$, pci | 87 | 174 | 348* |

* Higher values should be used only if estimated on the basis of adequate test results.

56. For sands, Terzaghi recognized that the stiffness increases with depth (or confining pressure). Thus, the family of $p-y$ curves recommended for sand consisted of a series of straight lines with slopes horizontal at the ground surface and increasing linearly with depth. The linear relationship between $p$ and $y$ can be expressed in terms of $E_{s}$ as:

$$
\begin{equation*}
E_{s}=k x \tag{35}
\end{equation*}
$$

where

$$
\begin{aligned}
& k=\text { constant giving variation of soil modulus with depth } \\
& x=\text { depth below ground surface }
\end{aligned}
$$

Table 2 gives Terzaghi's recommendations for $k$. Terzaghi also recognized that, as for clay, the assumed linear relationship between $p$ and $y$ was valid only for values of $p$ smaller than about one-half the ultimate bearing capacity of the sand.

Table 2
Terzaghi's Recommendations for Values of $k$ for
Laterally Loaded Piles in Sand

|  | Relative Density of Sand |  |  |
| :--- | :--- | :--- | :--- |
|  |  | Loose | Medium |

57. Even though Terzaghi's work assumed a linear relationship between pile deflection and soil resistance, it provided a useful concept for defining the initial soil reactions for the portions of certain $p-y$ curves where the soil reaction is less than half the ultimate soil reaction. This concept was utilized in defining the $p-y$ curves for stiff clay below the water table (Reese, Cox, and Koop 1975), for the unified soil criteria (Sullivan, Reese, and Fenske 1979), and for sands (Reese, Cox, and Koop 1974), except that the values were adjusted slightly to reflect the results from the individual field tests.

## Skempton

58. Skempton (1951) suggested a relationship between load and settlement for various footing shapes bearing on clay. By combining the theory of elasticity with field observations from full-scale foundations, Skempton related settlements of footings to strains obtained from unconsolidated, undrained (Q) triaxial tests with the equation

$$
\begin{equation*}
\rho_{1}=2 \varepsilon b \tag{36}
\end{equation*}
$$

where

```
\(\rho_{1}=\) mean settlement of the foundation for the particular case
    \(\varepsilon=\) strain in laboratory triaxial test for the deviator stress corre-
        sponding to the mean foundation pressure under the footing
    b \(=\) footing width
```

Equation 36 involves numerous approximations; nevertheless, because of the experimental evidence presented by Skempton, the method is frequently used in predicting foundation settlements. However, further assumptions are necessary before the equation can be used in predicting $p-y$ curves. The concept is extended to the $p-y$ curve for a laterally loaded pile by assuming that the depth is such that the behavior is not affected by the free surface of the soil.
59. As an example of the use of Skempton's concept, Equation 36 was extended to define the deflection of the pile, $y_{50}$, at one-half the ultimate soil resistance (Matlock 1970; Reese, Cox, and Koop 1975; Reese and Welch 1975; and Sullivan, Reese, and Fenske 1979). The equation is

$$
\begin{equation*}
y_{50}=A \varepsilon_{50} b \tag{37}
\end{equation*}
$$

where

$$
\begin{aligned}
& A= \text { factor varying from } 0.35 \text { to } 2.5 \text { based on experimental results from } \\
& \text { the pile tests for the different soil conditions } \\
& \varepsilon_{50}= \text { strain from an undrained soil test corresponding to half the maxi- } \\
& \text { mum principal stress difference }
\end{aligned}
$$

## McClelland and Focht

60. McClelland and Focht (1958) presented work which paralleled the work of Skempton (1951), although their work was not as strongly based on the theory of elasticity as his. Their paper represented the first report of experimental p-y curves from a full-scale load test. They attempted to relate soil resistance and pile deflection directly to stress-strain curves from consolidated undrained ( $R$ ) triaxial tests with confining pressure equal to overburden pressure. To obtain values of soil resistance $p$ from the laboratory tests, they recommended the following equation

$$
\begin{equation*}
p=5.5 b \sigma_{\Delta} \tag{38}
\end{equation*}
$$

where

$$
\begin{aligned}
b & =\text { pile diameter } \\
\sigma_{\Delta} & =\text { deviator stress }\left(\sigma_{1}-\sigma_{3}\right)
\end{aligned}
$$

To obtain values of pile deflection $y$ from stress-strain curves, McClelland and Focht proposed

$$
\begin{equation*}
y=0.5 \varepsilon b \tag{39}
\end{equation*}
$$

where the 0.5 corresponds to a value of 2 suggested by Skempton.
61. McClelland and Focht's work has been superseded by additional research on $p-y$ curves because it has since been proven that the appropriate soil modulus cannot be determined directly from a shear test. Nevertheless, theirs was a very important step because it was the first effort to relate the nonlinearity of $p-y$ curves to an analytical approach utilizing soil shear strength and stress-strain properties.

## Soil Models for Predicting Ultimate Soil Resistance

62. This section reviews the concepts involved in determining the ultimate resistance $p_{u}$ that can be developed against a pile near the ground
surface and at some depth below the surface. This review was extracted from Reese and Sullivan (1980) and Reese and Allen (1977).
Saturated clay
63. Theoretical values for ultimate resistance against piles in saturated clay employ the use of two models which assume that the clay around the pile shaft fails as either a group of sliding blocks or a wedge, depending on the depth below the surface. The soil is assumed to be saturated and to fail under undrained conditions so the shear strength is represented by cohesion $c$ with the angle of internal friction $\phi$ equal to zero.
64. The failure of the clay as the pile shaft moves laterally into the soil is considered in two parts. At some depth in the ground, failure will occur by flow of the soil around the pile without vertical displacement; i.e., plane strain conditions. This type of failure is depicted in Figure 12. Near the surface, a wedge-shaped block of sail is assumed to form which is moved upward and outward by the force of the pile. Figure 13 illustrates this theoretical wedge of soil.
65. The blocks in Figure 12 can be considered to be samples of unit height which fail under plane strain conditions. If it is assumed that blocks 1, 2, 4, and 5 fail by shear and that block 3 develops resistance by sliding, the stress conditions are represented hy Figure 12 b . If $\sigma_{1}$ is taken to be some small stress equal to the active pressure, then block 1 must move in the direction of pile movement. $\sigma_{2}$ must be approximately $2 c$ in order to cause failure of block 1. If $\sigma_{2}$ is considered to be the confining stress on block 2 , then $\sigma_{3}$ must be approximately 4 c . If block 3 slides due to the stress $\sigma_{3}$, then block 3 must have a resistance to sliding of 2 c . By assuming thal blocks 4 and 5 fail by the same line of reasoning as blocks 1 and 2 (i.e., $\sigma_{4}=6 c$, it can be found that $\sigma_{6}=10 c$. By examining a free body of a section of the pile (Figure 12c), it can be concluded that the total force exerted by the pile segment on the soil during failure is

$$
\begin{equation*}
p_{u}=11 c b \tag{40}
\end{equation*}
$$

66. The wedge in Figure 13 offers resistance to lateral movement of the pile by means of cohesion along the sides and bottom and its weight. Summing components of the forces in the horizontal direction, the resultant force $F_{p}$ is

a. Section through pile

b. Mohr-Coulomb diagram

c. Forces acting on pile

Figure 12. Model of lateral flow-around type of failure for clay (Reese and Sullivan 1980)


Figure 13. Assumed passive wedge type of failure for clay (Reese and Sullivan 1980)

$$
\begin{equation*}
F_{p}=c_{a} b H \quad \tan \alpha+(1+m) \cot \alpha+\frac{1}{2} \gamma b H^{2}+c_{a} H^{2} \sec \alpha \tag{41}
\end{equation*}
$$

where

$$
\begin{aligned}
c_{a}= & \text { average undrained shear strength } \\
H= & \text { depth to the point under consideration } \\
m= & \text { reduction factor to be multiplied by } c \text { to yield the average } \\
& \text { sliding stress between the pile and the }{ }^{\text {stiff clay }} \\
\gamma= & \text { average unit weight of the soil (submerged unit weight if the soil } \\
& \text { is below the water table) }
\end{aligned}
$$

The remaining terms are defined in Figure 13. It is possible to take the partial derivatives of Equation 41 with respect to the angle $\alpha$ and set the equation equal to zero to find the angle at which the equation is minimized. However, as an approximation, the angle $\alpha$ can be taken as $45^{\circ}$ and $m$ can be assumed equal to zero. Differentiation of the resulting expression with respect to $H$ yields an expression for the ultimate resistance per unit length of pile as follows:

$$
\begin{equation*}
p_{u}=2 c_{a} b+\gamma b H+2.83 c_{a} H \tag{42}
\end{equation*}
$$

67. Equations 40 and 42 are approximate in that the two models give a greatly simplified picture of how saturated clay behaves in resistance to lateral loading. However, the theoretical expressions give a point of departure for using the results of experiments to arrive at more realistic expressions. The two equations can be solved simultaneously to find the depth at which the failure would change from the wedge type to the flow-around type.

## Sands

68. The expressions for determining the ultimate resistance of sand to the lateral movement of a pile can again be divided on the basis of two different failure mechanisms (group of sliding blocks or wedge).
69. The model for computing the ultimate soil resistance at a depth where the overburden is sufficient to enforce a plane strain condition is given in Figure 14. The stress $\sigma_{1}$ is obtained by assuming a ankine active failure condition. This assumption is based on two-dimensional behavior and is subject to some uncertainty. However, the assumption should be adequate for present purposes because the developed equations will subsequently be adjusted to reflect observed conditions from field tests. If $\sigma_{1}$ is imposed as


Figure 14. Assumed mode of soil failure by lateral flow around the pile (Reese and Sullivan 1980)
the confining stress on block 1 , the stress required to cause the failure of block 1 along the dashed lines would be approximately

$$
\begin{equation*}
\sigma_{2}=\sigma_{1} \tan ^{2}\left(45+\frac{\phi}{2}\right) \tag{43}
\end{equation*}
$$

where $\phi$ is the angle of internal friction of the sand. Assuming the states of stress shown in Figure 14 b , block 2 would be required to fail along the dashed line because of the imposed stress of $\sigma_{3}$. Block 3 could be assumed
to move as a rigid unit. Continuing this line of reasoning leads to the es tablishment of the net force on the segment of pile as

$$
\begin{align*}
& p_{u}=b\left(\sigma_{6}-\sigma_{1}\right) \\
& p_{u}=K_{a} b \gamma H\left(\tan ^{8} \beta-1\right)+K_{o} b \gamma H \tan \phi \tan ^{4} \beta \tag{44}
\end{align*}
$$

where

$$
\begin{aligned}
K_{a} & =\text { Rankine active earth pressure coefficient }=\tan ^{2} 45-(\phi / 2) \\
H & =\text { depth to the point under consideration } \\
\beta & =45+(\phi / 2) \\
K_{0} & =\text { at-rest earth pressure coefficient }
\end{aligned}
$$

70. The ultimate soil resistance near the ground surface is computed using the free body shown in Figure 15. As can be seen in Figure 15c, the total ultimate lateral resistance $F_{p t}$ on the pile is equal to the passive force $F_{p}$ minus the active force $F_{a}$. The force $F_{a}$ is computed from Rankine's theory using the minumum coefficient of active earth pressure. The passive force $F_{p}$ is computed from the geometry of the wedge, assuming the Mohr-Coulomb failure theory to be valid for sand. The directions of the forces are shown in Figure 15b. By summing forces in the horizontal and vertical directions, the magnitudes of the forces $F_{a}$ and $F_{p}$ can be determined. No frictional force is assumed to be acting on the face of the pile. The equation for $F_{p t}$ is

$$
\begin{array}{r}
\mathrm{F}_{\mathrm{pt}}=\gamma \mathrm{H}^{2}\left[\frac{\mathrm{~K}_{0} \mathrm{H} \tan \phi \sin \beta}{3 \tan (\beta-\phi) \cos \alpha}+\frac{\tan \beta}{\tan (\beta-\phi)}\left(\frac{\mathrm{b}}{2}+\frac{\mathrm{H}}{3} \tan \beta \tan \alpha\right)\right. \\
\left.+\mathrm{K}_{\mathrm{o}} \mathrm{H} \frac{\tan \beta}{3}(\tan \phi \sin \beta-\tan \alpha)-\frac{\mathrm{K}_{\mathrm{a}} \mathrm{~b}}{2}\right] \tag{45}
\end{array}
$$

where
$K_{o}=$ coefficient of earth pressure at rest
$K_{a}=$ minimum coefficient of active earth pressure
71. The ultimate soil resistance per unit length of the pile at any depth can be obtained by differentiating the force $F_{p t}$ with respect to the depth $H$. The result of that differentiation is given by


Figure 15. Assumed passive wedge type of failure (Reese and Sullivan 1980)

$$
\begin{align*}
p_{u}=\gamma H & {\left[\frac{K_{0} H \tan \phi \sin \beta}{\tan (\beta-\phi) \cos \alpha}+\frac{\tan \beta}{\tan (\beta-\phi)}\right.} \\
& \left.\times(b+H \tan \beta \tan \alpha)+K_{0} H \tan \beta(\tan \phi \sin \beta-\tan \alpha)-K_{a} b\right] \tag{46}
\end{align*}
$$

72. The values of the parameters in Equation 46 must be estimated using soil mechanics theory. Selection of the parameters will be discussed in the subsequent section on $p-y$ curves.
73. Equations 44 and 46 can be solved simultaneously to find the approximate depth at which the soil changes from the wedge type to the flow-around type. Again, it should be emphasized that the equations are not expected to give perfect predictions of the ultimate soil resistance. However, correlating the equations with experimental results allows practical use of them and lends generality to the experimental results.

## Experimental Techniques for Developing p-y Curves

74. The preceding paragraphs have described the basic theory utilized in correlating observed experimental p-y curves with theory. The following section describes several methods for obtaining experimental p-y curves. Direct measurement
75. Direct measurement of p-y curves in the field would involve measuring the pile deflection at some predetermined points and then measuring the soil response corresponding with the measured deflection. Deflection can be measured by installing slope inclinometer casings either on the inside or on the surface of a pile and taking readings with a slope inclinometer. Alternatively, sighting down a hollow pile from a fixed position at scales that have been placed at intervals along the length of the pile has been used. This method is cumbersome in practice, however, and has not been very successful.
76. Measuring the soil response $p$ is considerably more involved and difficult than measuring the deflection. The distribution of pressure acting on the pile must first be determined and then the pressure diagram integrated to determine soil response. Pressure meters of many different types are available and have been utilized in measuring pressures (Bierschwale, Coyle, and Bartoskewitz 1981). This approach requires measurement of the soil pressure at a few points around the exterior of a pile and estimation of soil
pressures between the pressure meters to obtain the pressure distribution. Whether or not this procedure yields accurate pressure distribution is a subject of debate (Reese and Sullivan 1980; Bierschwale, Coyle, and Bartoskewitz 1981).

## Experimental moment curves

77. The method used most successfully at UT for determining $p-y$ curves involves the placement of electrical resistance straia gages at points along the pile shaft. Before the field test is performed, strain readings are correlated with moment by placing the pile horizontally on simple supports and applying known moments. During the lateral load test, strain readings are taken at each point at each increment of load and converted to moment values by use of the moment calibration curves. Deflection values are obtained by use of Equation 47:

$$
\begin{equation*}
y=\iint \frac{M}{E I} \tag{47}
\end{equation*}
$$

where

$$
\begin{aligned}
M & =\text { measured moment } \\
E I & =\text { flexural stiffness of the pile }
\end{aligned}
$$

The deflection can be obtained with considerable accuracy using numerical procedures to doubly integrate the moment curves.
78. The computation of soil resistance is somewhat more difficult than determining deflections. It is obtained by double differentiation of the mr ent curves using Equation 48:

$$
\begin{equation*}
p=\frac{d^{2} M}{d x^{2}} \tag{48}
\end{equation*}
$$

The difficulty in differentiating the moment curves lies in the fact that a curve fitted through data points is not necessarily accurate except at the data points and differentiation results can be erratic, particularly for double differentiation.
79. Taking the family of curves showing the distribution of deflection and soil resistance, $p-y$ curves can be plotted as shown in Figure 16. The curves can be checked by performing an analysis using the field loads and comparing the results with the experimental moment curves as illustrated in Figure 17.


Figure 16. Examples of experimental $p-y$ curves from field test (Reese, Cox, and Koop 1975)


Figure 17. Computed and measured values of moment versus depth from a laterally loaded pile test (Welch and Reese 1972)

## Nondimensional methods

80. Nondimensional methods have been used fairly successfully to obtain p-y curves from a lateral load test (Reese, Cox, and Koop 1974). The basis for this method is described in Appendix A. The procedure does not result in p-y curves which are as accurate as the curves obtained using strain gage data. The main advantage is that costly instrumentation is not required.
81. Deflection and slope are measured at the top of the pile after each increment of load is applied. The $p-y$ curve is computed by first assuming a variation of soil modulus with depth for a particular load and then performing a nondimensional solution. This procedure is repeated until the assumed variation of soil modulus yields computed results which agree with the measured deflection and slope at the top of the pile. When the calculated slope and deflection agree with those measured, the assumed variation is taken to be correct. This "correct" modulus is used for the computer solution from which the deflection is obtained with depth. Given the soil modulus and the deflection, the value of resistance at desired depths can then be computed. One complete solution gives one point on the p-y curve at each depth being considered. The entire procedure is then repeated for each load to obtain additional points on the $p-y$ curve.

Recommendations on Use of $p-y$ Curves
82. Ideally, fully instrumented testing should be performed for each design involving laterally loaded piles. Unfortunately, the cost of load tests can often only be justified for large projects. On projects where fully instrumented lateral load tests can be justified, the tests should be performed at the specific site using the pile types and installation procedures to be utilized in construction. On intermediate-sized projects for which sitespecific data are needed, but a fully instrumented lateral load test cannot be justified, the nondimensional methods for obtaining p-y curves presented by Reese and Cox (1968) are recommended. These methods are approximate; however, they require only pile head measurements which are relatively easy and economical to obtain and they provide project-specific data not available otherwise. In certain situations, the designer may also consider using a combination of instrumented pile testing and nondimensional methods. This can be accomplished by utilizing the slope inclinometer to obtain pile deflections while using
nondimensional methods to obtain soil resistance.
83. The $p-y$ criteria presented in the remaining sections of this part of the report are provided for the purpose of assisting the designer in situations where laterally loaded pile tests cannot be justified. The designer must use the $p-y$ criteria with extreme caution and a clear understanding of their limitations. Under no circumstances should a design be undertaken without a sufficient number of borings to define the subsurface profile and a sufficient number of soil tests to define the shear strength and the unit weight versus depth profile. Also, the designer should be ever mindful of the fact that any one set of $p-y$ construction methods presented herein is strongly related to only one or two lateral load tests.
84. In performing analyses, the designer should, at a minimum, perform parametric studies to investigate the sensitivity of the results to the input parameters. For example, the load, boundary conditions, and parameters specific to developing the individual $p-y$ curves should be varied to determine the parameters most critical to the design. The results of the parametric studies should then be considered in making design decisions. An example design problem is presented in Appendix B.
Curves for clays
85. The recommended p-y curves for clays were developed from three major test programs on three different types of clay soils: (a) soft clays below the water table, (b) stiff clays below the water table, and (c) stiff clays above the water table. In each test program, the piles were subjected to short-term static loads and to repeated (cyclic) loads. The test program is described briefly for each set of $p-y$ criteria in the following paragraphs. In addition, step-by-step procedures are given for computing the p-y curves, recommendations are given for obtaining the necessary data on soil properties, and example curves are presented.
86. The final portion of this section on clays presents a method that has been developed for predicting $p-y$ curves for clays below the water table of any shear strength. This "unified" method (Sullivan, Reese, and Fenske 1979) is based on all of the major experiments in clay below the water table.

Response of soft
clay below the water table
87. Field experiments. The research program leading to the development of p-y criteria for soft clay was carried out and reported by Matlock (1970).

The research involved extensive field testing with an instrumented pile, experiments with laboratory models, and parallel development of analytical meth~ ods and correlations.
88. There were two test sites: one at Lake Austin in Austin, Tex., and the other at the mouth of the Sabine River, which forms much of the TexasLouisiana border. The soils at the Lake Austin site consisted of clays and silts, somewhat jointed and fissured due to desiccation during periods of low water with vane shear strengths averaging about 800 pcf. The Sabine clay appeared to be a more typical, slightly overconsolidated marine deposit with vane shear strengths averaging about 300 pcf in the significant upper zone.
89. A steel test pile 12.75 in. in diameter with an embedded length of 42 ft was used at both test sites. The pile contained 35 pairs of electrical resistance strain gages which were calibrated to provide extremely accurate determinations of bending moment. Gage spacings varied from 6 in. near the top to 4 ft in the lowest section. Tests were performed (a) with the pile head free to rotate and (b) with the pile head restrained against rotation to determine what difference there might be in the soil response due to different boundary conditions. The free-head tests were performed with only a lateral load applied at the mudline. The restrained head tests utilized a framework to simulate the effect of a jacket-type structure, as shown in Figure 18. Short-time static loading and cyclic loading were used in testing the pile. The moment curves obtained in the tests were differentiated to determine soil resistance and integrated to obtain pile deflection.
90. In addition to field experiments, some laboratory experiments were performed which were of value in explaining the nature of deterioration of soil resistance. These experiments were not utilized directly in constructing the $p-y$ criteria, but were of use in explaining and interpreting the field data. Principal conclusions from the tests are listed below:
a. The resistance-deflection characteristics of the soil were highly nonlinear and inelastic.
b. Within practical ranges, the degree of pile head restraint appeared to have no effect on the $p-y$ relationship.
c. Cyclic loading produced a permanent physical displacement of the soil away from the pile in the direction of loading.
d. The permanent displacement of the soil away from the pile produced a slack zone in the $p-y$ relationship. Upon reloading

the pile, this slack zone was reflected in bending moments which were much higher than those produced by equal loads during the initial cyclic series.
e. During cyclic loading with a constant load, the deflections and moments would gradually increase with each repetition, but the rate of increase diminished to the point where the soil-pile system practically stabilized and no further increases in deflections or moments occurred with continued repetitions of load. It can be intuitively seen that some upper limit of load must exist for any pile above which the system would not stabilize under cyclic loading, and this conclusion was borne out by the tests. Below this upper limit, stabilization generally occurred in less than 100 cycles.
f. The measured ultimate resistance near the surface was similar to the theoretical ultimate resistance as expressed in Equation 42 .
g. If the $p-y$ data resulting from the tests are plotted in nondimensional form on $\log -\log$ paper, a relatively smooth straight line can be fitted to the data up to the value of ultimate resistance. This result will be illustrated in the directions for constructing the $p-y$ curves.
91. The details of the experiments for the soft-clay criteria are discussed more thoroughly here than will be the case for the remaining criteria. The discussion is primarily intended to provide the user with a clearer understanding of the experiments which provide the basis for the p-y criteria.
92. Recommendations for computing $p-y$ curves. The following procedure is for short-term static loading and is illustrated by Figure 19a.
a. Obtain the best possible estimate of the variation of undrained shear strength $c$ and submerged unit weight with depth $x$. Also, obtain the values of $\varepsilon_{50}$, the strain corresponding to half the maximum principal stress difference. If no stressstrain curves are available, typical values of $\varepsilon_{50}$ given in Table 3 can be used.

Table 3

| Representative Values of | $\varepsilon_{50}$ |
| :---: | :---: |
| Shear Strength <br> c psf | $\varepsilon_{50}$ |
| $250-500$ | percent |
| $500-1000$ | 2 |
| $1000-2000$ | 1 |
| $2000-4000$ | 0.7 |
| $4000-8000$ | 0.5 |

b. Compute the ultimate soil resistance per unit length of pile, using the smaller of the values given by the equations below:

$$
\begin{align*}
& p_{u}=\left(3+\frac{\gamma^{\prime}}{c} x+\frac{J}{b} x\right)(c b)  \tag{49}\\
& p_{u}=9 c b \tag{50}
\end{align*}
$$



Figure 19. Characteristic shapes of the p-y curves for soft clay below the water surface (Matlock 1970)
where

$$
\gamma^{\prime}=\text { average effective unit weight from the ground surface }
$$ to the p-y curve

$\mathrm{c}=$ shear strength at depth x
$x=$ depth from the ground surface to the $p-y$ curve
$b=$ width of the pile

Matlock (1970) states that the values of $J$ were determined experimentally to be 0.0 for a soft clay and about 0.25 for a medium clay. A value of 0.5 is frequently used. The value of $p_{u}$ is computed at each depth where a p-y curve is desired, based on shear strength at that depth.
c. Compute the deflection $y_{50}$ at half the ultimate soil resistance from the following equation:

$$
\begin{equation*}
y_{50}=2.5 \varepsilon_{50} b \tag{51}
\end{equation*}
$$

d. Points describing the $p-y$ curve are now computed from the following relationship:

$$
\begin{equation*}
\frac{\mathrm{p}}{\mathrm{p}_{\mathrm{u}}}=0.5\left(\frac{\mathrm{y}}{\mathrm{y}_{50}}\right)^{1 / 3} \tag{52}
\end{equation*}
$$

The value of $p$ remains constant beyond $y=8 y_{50}$
93. The following procedure is for cyclic loading and is illustrated in Figure 19b.
a. Construct the $p-y$ curve in the same manner as for short-term static loading for values of $p$ less than $0.72 p_{u}$.
b. Solve Equations 49 and 50 simultaneously to find the depth $\mathbf{x}_{\mathbf{r}}$ where the transition occurs. If the unit weight and shear strength are constant in the upper zone, then

$$
\begin{equation*}
x_{r}=\frac{6 c b}{(y b+J c)} \tag{53}
\end{equation*}
$$

If the unit weight and shear strength vary with depth, the value of $x_{r}$ should be computed with the soil properties at the depth where the $p-y$ curve is desired.
c. If the depth to the $p-y$ curve is greater than or equal to $x_{r}$, then $p$ is equal to $0.72 p_{u}$ for all values of $y$ greater than $3 y_{50}$.
d. If the depth to the $p-y$ curve is less than $x_{r}$, then the value of $p$ decreases from $0.72 p_{u}$ at $y=3 y_{50}$ to the value given by the following expression at $y=15 y_{50}$ :

$$
\begin{equation*}
p=0.72 p_{u}\left(\frac{x}{x_{r}}\right) \tag{54}
\end{equation*}
$$

The value of $p$ remains constant beyond $y=15 y_{50}$.
94. Recommended soil tests. For determining the various shear strengths of the soil required in the p-y construction, Matlock (1970) recommended the following tests in order of preference.
a. In situ vane-shear tests with parallel sampling for soil identification.
b. Unconsolidated, undrained triaxial compression tests having a confining stress equal to the overburden pressure, with c being defined as half the total maximum principal stress difference.
c. Miniature vane tests of samples in tubes.
d. Unconfined compression tests.

Tests must also be performed to determine the unit weight of the soil.
95. Example curves. An example set of $p-y$ curves was computed for soft clay for a pile with a diameter of 48 in . The soil profile that was used is shown in Figure 20. In the absence of a stress-strain curve for the soil, $\varepsilon_{50}$ was taken as 0.01 for the full depth of the soil profile. The loading was assumed to be both static and cyclic.
96. $p-y$ curves were computed for the following depths below the mudline: $0,1,2,4,8,12,20,40$, and 60 ft . The plotted curves are shown in Figure 21 for static loading and in Figure 22 for cyclic loading.

Response of stiff clay below the water table
97. Field experiments. Reese, Cox, and Koop (1975) performed lateral load tests employing steel pipe piles that were 24 in . in diameter and 50 ft long. The piles were driven into stiff clay at a site near Manor, Tex. The clay had an undrained shear strength ranging from about 1 tsf at the ground surface to about 3 tsf at a depth of 12 ft .
98. Recommendations for computing p-y curves. The following procedure is for short-term static loading and is illustrated by Figure 23.
a. Obtain values for undrained soil shear strength $c$, soil submerged unit weight $\gamma^{\prime}$, and pile diameter $b$.
b. Compute the average undrained soil shear strength $c$ over the depth $x$.
c. Compute the ultimate soil resistance per unit length of pile using the smaller of the values given by the equations


Figure 21. Example p-y curves for soft clay below the water table; Matlock criteria, static loading


Figure 22. Example p-y curves for soft clay below the water table; Matlock criteria, cyclic loading


e. Establish the initial straight-line portion of the p-y curve

$$
\begin{equation*}
p=(k x) y \tag{57}
\end{equation*}
$$

Use the appropriate value of $k_{s}$ or $k_{c}$ from Table 4 for $k$.

Table 4
Representative Values of $\mathbf{k}$ for Stiff Clays

|  | Ave | $\begin{aligned} & \text { ed She } \\ & \text { tsf } \end{aligned}$ | , ${ }_{\text {\% }}$ |
| :---: | :---: | :---: | :---: |
|  | 0.5-1 | 1-2 | 2-4 |
| $\mathrm{k}_{\mathrm{s}}$ (static), pci | 500 | 1000 | 2000 |
| $\mathrm{k}_{\mathrm{c}}$ (cyclic), pci | 200 | 400 | 800 |

* The average shear strength should be computed from the shear strength of the soil to a depth of five pile diameters. It should be defined as half the total maximum principal stress difference in an unconsolidated undrained triaxial test. (Also see Table 6.)
f. Compute the following:

$$
\begin{equation*}
y_{50}=\varepsilon_{50} b \tag{58}
\end{equation*}
$$

Use an appropriate value of $\varepsilon_{50}$ from results of laboratory tests or, in the absence of laboratory tests, from Table 3.
g. Establish the first parabolic portion of the $p-y$ curve using the following equation and obtaining $p_{c}$ from Equation 55 or 56:

$$
\begin{equation*}
p=0.5 p_{c}\left(\frac{y}{y_{50}}\right)^{0.5} \tag{59}
\end{equation*}
$$

Equation 59 could define the portion of the $p-y$ curve from the point of the intersection with Equation 59 to a point where $y$ is equal to $A_{s} y_{50}$ (see note after step $j$ ).
h. Establish the second parabolic portion of the $p-y$ curve,

$$
\begin{equation*}
p=0.5 p_{c}\left(\frac{y}{y_{50}}\right)^{0.5}-0.055 p_{c}\left(\frac{y-A_{s} y_{50}}{A_{s} y_{50}}\right)^{1.25} \tag{60}
\end{equation*}
$$

Equation 60 should define the portion of the $p-y$ curve from the point where $y$ is equal to $A_{s} y_{50}$ to a point where $y$ is equal to $6 A_{s} y_{50}$ (see note after step $j$ ).
i. Establish the next straight-line portion of the $p-y$ curve,
$p=0.5 p_{c}\left(6 A_{s}\right)^{0.5}-0.411 p_{c}-\frac{0.0625}{y_{50}} p_{c}\left(y-6 A_{s} y_{50}\right)$

Equation 61 should define the portion of the $p-y$ curve from the point where $y$ is equal to $6 A_{s} y_{50}$ to a point where $y$ is equal to $18 \mathrm{~A}_{\mathrm{s}} \mathrm{y}_{50}$ (see note after step j ).
j. Establish the final straight-line portion of the $p-y$ curve,

$$
\begin{align*}
& p=0.5 p_{c}\left(6 A_{s}\right)^{0.5}-0.411 p_{c}-0.75 p_{c} A_{s}  \tag{62}\\
& p=p_{c}\left(1.225 \sqrt{A_{s}}-0.75 A_{s}-0.411\right) \tag{63}
\end{align*}
$$

Equation 62 should define the portion of the $p-y$ curve from the point where $y$ is equal to $18 A_{s} y_{50}$ and for all larger values of $y$ (see following note).
(Note: The step-by-step procedure is outlined, and Figure 23 is drawn, as if there is an intersection between Equations 57 and 59. However, there may be no intersection of Equation 57 with any of the other equations defining the $p-y$ curve. Equation 57 defines the $p-y$ curve until it intersects with one of the other equations or, if no intersection occurs, Equation 57 defines the complete $p-y$ curve.)
99. The following procedure is used for computing p-y curves in which loading is cyclic (see Figure 25).
a. Steps $a, b, c, e$, and $f$ are the same as for the static case.
d. Choose the appropriate value of $A_{c}$ from Figure 24 for the particular nondimensional depth.

$$
\begin{equation*}
y_{p}=4.1 A_{c} y_{50} \tag{64}
\end{equation*}
$$

Compute the following.

g. Establish the parabolic portion of the p-y curve,

$$
\begin{equation*}
p=A_{c} p_{c}\left(1-\left|\frac{y-0.45 y_{p}}{0.45 y_{p}}\right|^{2.5}\right) \tag{65}
\end{equation*}
$$

Equation 65 should define the portion of the $p-y$ curve from the point of the intersection with Equation 57 to the point where $y$ is equal to $0.6 y_{p}$ (see note after step i).
h. Establish the next straight-line portion of the $p-y$ curve,

$$
\begin{equation*}
p=0.936 A_{c} p_{c}-\frac{0.085}{y_{50}} p_{c}\left(y-0.6 y_{p}\right) \tag{66}
\end{equation*}
$$

Equation 66 should define the portion of the $p-y$ curve from the point where $y$ is equal to $0.6 y_{p}$ to the point where $y$ is equal to $1.8 y_{p}$ (see note after step i).
i. Establish the final straight-line portion of the $p-y$ curve,

$$
\begin{equation*}
p=0.936 A_{c} p_{c}-\frac{0.102}{y_{50}} p_{c} y_{p} \tag{67}
\end{equation*}
$$

Equation 67 should define the portion of the $p-y$ curve from the point where $y$ is equal to $1.8 y_{p}$ and for all larger values of $y$ (see following note).
(Note: The step-by-step procedure is outlined, and Figure 25 is drawn, as if there is an intersection between Equations 57 and 65. However, there may be no intersection of those two equations, and there may be no intersection of Equation 57 with any of the other equations defining the $p-y$ curve. If there is no intersection, the equation should be employed that gives the smallest value of $p$ for any value of $y$.
100. Recommended soil tests. Triaxial compression tests of the unconsolidated, undrained ( $Q$ ) type with confining pressures conforming to in situ pressures are recommended for determining the shear strength of the soil. The value of $\varepsilon_{50}$ should be taken as the strain during testing which corresponds to a stress equalling one-half the maximum total principal stress difference. The shear strength $c$ should be interpreted as half of the maximum total stress difference. Values obtained from the triaxial tests might be somewhat conservative but would represent more realistic strength values than any from other tests. The unit weight of the soil must also be determined.
101. Example curves. Example sets of $p-y$ curves were computed for stiff clay using a pile with a diameter of 48 in . The soil profile that was used is shown in Figure 26. The submerged unit weight of the soil was assumed to be 50 pcf for the entire depth. In the absence of a stress-strain curve, $\varepsilon_{50}$ was taken as 0.005 for the full depth of the soil profile. The slope of the initial portion of the $p-y$ curves was established by assuming a value of $k_{s}$ of 1000 pci and a value of $k_{c}$ of 400 pci. The loading was assumed to be both static and cyclic.
102. The p-y curves were computed for the following depths below the mudline: $0,1,2,4,8,12,20,40$, and 60 ft . The plotted curves are shown in Figure 27 for static loading and in Figure 28 for cyclic loading.


Figure 26. Soil profile used for example p-y curves for stiff clay


Figure 27. Example p-y curves for stiff clay below the water table; Reese criteria, static loading



Figure 29. Characteristic shape of p-y curve for static loading in stiff clay above the water table (Reese and Sullivan 1980)
b. Compute the ultimate soil resistance per unit length of shaft $p_{u}$ using the smaller of the values given by Equations 49 and 50. (In the use of Equation 49, the shear strength is taken as the average from the ground surface to the depth being considered, and $J$ is taken as 0.5 . The unit weight of the soil should reflect the position of the water table.)
c. Compute the deflection $y_{50}$ at half the ultimate soil resistance from Equation 51.
d. Points describing the $p-y$ curve may be computed from the relationship below.

$$
\begin{equation*}
\frac{p}{p_{u}}=0.5\left(\frac{y}{y_{50}}\right)^{1 / 4} \tag{68}
\end{equation*}
$$

e. Beyond $y=16 y_{50}$, $p$ is equal to $p_{u}$ for all values of $y$
105. The following procedure is for cyclic loading and is illustrated in Figure 30:
a. Determine the $p-y$ curve for short-term static loading by the procedure previously given.
b. Determine the number of times the design lateral load will be applied to the pile.
c. For several values of $p / p_{u}$, obtain the value of $C$, the parameter describing the effect of repeated loading on deformation, from a relationship developed through laboratory tests (Welch and Reese 1972) or, in the absence of tests, from the following equation:

$$
\begin{equation*}
C=9.6\left(\frac{p}{p_{u}}\right)^{4} \tag{69}
\end{equation*}
$$



Figure 30. Characteristic shape of p-y curve for cyclic loading in stiff clay above the water table (Reese and Sullivan 1980)
d. At the value of $p$ corresponding to the values of $p / p_{u}$ selected in step $c$, compute new values of $y$ for cyclic loading from

$$
\begin{equation*}
y_{c}=y_{s}+\left(y_{50}\right) c \log N \tag{70}
\end{equation*}
$$

where

$$
\begin{aligned}
y_{c} & =\text { deflection under } N \text { cycles of load } \\
y_{s}= & \text { deflection under a short-term static load } \\
y_{50}= & \text { deflection under a short-term static load at half } \\
& \text { the ultimate resistance } \\
N= & \text { number of cycles of load application }
\end{aligned}
$$

e. The $p-y$ curve defines the soil response after $N$ cycles of load.
106. Recommended soil tests. Triaxial compression tests of the unconsolidated, undrained ( $Q$ ) type with confining stresses equal to the overburden pressures at the elevations from which the samples were taken are recommended to determine the shear strength. The values of $\varepsilon_{50}$ should be taken as the strain during the test corresponding to the stress equal to half the maximum total principal stress difference. The undrained shear strength $c$ should be defined as half the maximum total principal stress difference. The unit weight of the soil must also be determined.
107. Example curves. An example set of $p-y$ curves was computed for stiff clay above the water table for a pile with a diameter of 43 in . The soil profile that was used is shown in Figure 26 . The unit weight of the soil was assumed to be 112 pcf for the entire depth. In the absence of a stressstrain curve, $\varepsilon_{50}$ was taken as 0.005 . The $p-y$ curves were computed for both static and cyclic loadings. Equation 69 was used to compute values for the parameter $C$ for cyclic loadings, and it was assumed that there are to be 100 cycles of load application.
108. p-y curves were computed for the following depths below the ground surface: $0,1,2,4,8,12,20,40$, and 60 ft . The plotted curves are shown in Figure 31 for static loading and in Figure 32 for cyclic loading.

Unified criteria for
clays below the water table
109. Introduction. As was noted in the previous section, no recommendations were made for ascertaining the range of undrained shear strength in


Figure 31. Example p-y curves for stiff clay above the water table; Reese and Welch criteria, static loading
which the criteria for soft clay versus those for stiff clay should be used. Sullivan (1977) and Sullivan, Reese, and Fenske (1979) examined the original experiments and developed a set of recommendations that yield computed behaviors in reasonably good agreement with the experimental results from the Sabine River tests reported by Matlock (1970) and with those from the Manor, Tex., tests reported by Reese, Cox, and Koop (1975). However, as will be seen from the following presentation, there is a need for the user to employ some judgment in selecting appropriate parameters for use in the prediction equations.
110. Recommendations for computing p-y curves. The following procedure is for short term static loading and is illustrated in Figure 33:
a. Obtain values for the undrained shear strength $c$, the submerged unit of weight $\gamma^{\prime}$, and the pile diameter $b$. Also, obtain values of $\varepsilon_{50}$ from stress-strain curves. If no stress-strain curves are available, the values in Table 3 can be used as guidelines for selection of $\varepsilon_{50}$.


Figure 32. Example p-y curves for stiff clay above the water table; Reese and Welch criteria, cyclic
loading
b. Compute $c_{a}$ and $\bar{\sigma}_{v}$, for $x<12 b$,
where

$$
\begin{aligned}
c_{\mathbf{a}} & =\text { average undrained shear strength } \\
\overline{\mathbf{\sigma}}_{\mathbf{v}} & =\text { average effective stress } \\
\mathbf{x} & =\text { depth }
\end{aligned}
$$

c. Compute the variation of $p_{u}$ with depth using the equation below:
(1) For $x<12 b, p_{u}$ is the smaller of the values computed from

$$
\begin{align*}
& p_{u}=\left(2+\frac{\bar{\sigma}_{v}}{c_{a}}+0.833 \frac{x}{b}\right) c_{a}^{b}  \tag{71}\\
& p_{u}=\left(3+0.5 \frac{x}{b}\right) c b \tag{72}
\end{align*}
$$

(2) For $x>12 b$,

$$
\begin{equation*}
p_{u}=9 \mathrm{cb} \tag{73}
\end{equation*}
$$



The steps below are for a particular depth $x$.
d. Select the coefficients $A$ and $F$ as indicated below. The coefficients $A$ and $F$, determined empirically for the load tests at the Sabine River and Manor sites, are given in Table 5. The terms used in Table 5, not defined previously, are defined below:

$$
\begin{aligned}
& W_{L}=\text { liquid limit } \\
& P I=\text { plasticity index } \\
& L I=\text { liquidity index } \\
& O_{R}=\text { overconsolidation ratio } \\
& S_{t}=\text { sensitivity }
\end{aligned}
$$

The recommended procedure for estimating $A$ and $F$ for other clays is:
(1) Determine as many of the following properties of the clay as possible: $C, \varepsilon_{50}, O_{R}, S_{t}$, degree of fissuring, ratio of residual to peak undrained shear strength $W_{L}$, PI , and LI .
(2) Compare the properties of the soil in question to the properties of the Sabine and Manor clays listed in Table 5.
(3) If the properties are similar to those of either the Sabine or the Manor clay, use $A$ and $F$ for the similar clay.
(4) If the properties are not similar to either, the user should estimate $A$ and $F$ using his judgment and Table 5 as guides.
e. Compute

$$
\begin{equation*}
y_{50}=A \varepsilon_{50} b \tag{74}
\end{equation*}
$$

f. Obtain $\left(E_{s}\right)_{\max }$. When no other method is available, Equation 75 and Table 6 may be used as guidelines:

$$
\begin{equation*}
\left(E_{s}\right)_{\max }=k x \tag{75}
\end{equation*}
$$

Table 5

## Curve Parameters for the Unified Criteria (Reese and Sullivan 1980)

|  | Clay Description | A | F |
| :---: | :---: | :---: | :---: |
| Sabine | River site | 2.5 | 1.0 |
|  | Inorganic, intact |  |  |
|  | $c=300 \mathrm{lb} / \mathrm{ft}^{2}$ |  |  |
|  | $\varepsilon_{50}=0.7 \%$ |  |  |
|  | $0_{R}=1$ |  |  |
|  | $S_{t} \simeq 2$ |  |  |
|  | $\mathrm{w}_{\mathrm{L}}=92$ |  |  |
|  | $\mathrm{PI}=68$ |  |  |
|  | $\mathrm{LI}=1$ |  |  |
| Manor, | Tex., site | 0.35 | 0.5 |
|  | Inorganic, very fissured |  |  |
|  | $c \simeq 2400 \mathrm{lb} / \mathrm{ft}^{2}$ |  |  |
|  | $\varepsilon_{50}=0.5 \%$ |  |  |
|  | $0_{R}>10$ |  |  |
|  | $S_{t} \simeq 1$ |  |  |
|  | $\mathrm{w}_{\mathrm{L}}=77$ |  |  |
|  | PI $=60$ |  |  |
|  | $L I=0.2$ |  |  |

Table 6
Representative Values for $k$

| Shear Strength <br> c, psf | k <br> pci |
| :--- | ---: |
| $250-500$ | 30 |
| $500-1000$ | 100 |
| $1000-2000$ | 300 |
| $2000-4000$ | 1000 |
| $4000-8000$ | 3000 |

(Also see Table 4.)
g. Compute the deflection at the intersection between the initial linear portion and curved portion from the equation

$$
\begin{equation*}
y_{k}=\left[\frac{0.5 p_{u}}{\left(E_{s}\right)_{\max }}\right]^{3 / 2}\left(y_{50}\right)^{-1 / 2} \tag{76}
\end{equation*}
$$

( $y_{k}$ can be no larger than $8 y_{50}$.)
h. (1) For $0<y<y_{k}$

$$
\begin{equation*}
p=\left(E_{s}\right)_{\max } y \tag{77}
\end{equation*}
$$

(2) For $y_{k}<y<8 y_{50}$

$$
\begin{equation*}
p=0.5 p_{u}\left(\frac{y}{y_{50}}\right)^{1 / 3} \tag{78}
\end{equation*}
$$

(3) For $8 y_{50}<y<30 y_{50}$

$$
\begin{equation*}
p=p_{u}+\frac{p_{R}-p_{u}}{22 y_{50}}\left(y-8 y_{50}\right) \tag{79}
\end{equation*}
$$

where

$$
\begin{equation*}
p_{R}=p_{u}\left[F+(1-F) \frac{x}{12 b}\right] \tag{80}
\end{equation*}
$$

( $p_{R}$ will be equal to or less than $p_{u}$ )
(4) For $y>30 y_{50}$

$$
\begin{equation*}
p=p_{R} \tag{81}
\end{equation*}
$$

111. The following procedure is for cyclic loading and is illustrated in Figure 34:

$$
\begin{align*}
& \text { a. Repeat steps a through } h(1) \text { for static loading. } \\
& \text { b. Compute } \\
& \qquad p_{C R}=0.5 p_{u} \frac{x}{12 b} \leq 0.5 p_{u}  \tag{82}\\
& \text { c. (1) For } y_{g}<y<y_{50} \\
& \qquad p=0.5 p_{u}\left(\frac{y}{y_{50}}\right)^{1 / 3}  \tag{83}\\
& \text { (2) For } y_{50^{\circ}}<y<20 y_{50} \\
& \text { (3) For } y>20 y_{50},  \tag{84}\\
& p=0.5 p_{u}+\frac{p_{C R}-0.5 p_{u}}{19 y_{50}}\left(y-y_{50}\right) \\
& \text { (3) } \tag{85}
\end{align*}
$$

112. Comments. The procedures outlined above for both static and cyclic loading assume that an intersection of the curve defined by Equations 77 and 78 occurs. If that intersection does not occur, the $p-y$ curve is defined by Equation 77 until it intersects a portion of the curve defined by Equations 79 and 81 for static loading and Equations 83 or 84 for cyclic loading.
113. Example curves. Example sets of $p-y$ curves were computed using the unified criteria and the soil profiles in Figures 20 and 26. The soil profile in Figure 20 represents a soft clay, and the profile in Figure 26 represents a stiff clay, both below the water table. The p-y curves for both soil profiles were computed for static and cyclic loadings using a pile 48 in . in diameter and the following depths: $0,1,2,4,8,12,20,40$, and 60 ft .
114. For the soft clay profile in Figure 20 , the value of $\varepsilon_{50}$ was assumed to be 0.02 from the mudline to a depth of 20 ft and to decrease to 0.01

at a depth of 90 ft . The value of $A$ was assumed to be 2.5 , and the value of $F$ was assumed to be 1.0. The value of $k$ for computing the maximum value of the soil modulus was assumed to be 200,000 pcf. Figure 35 shows the set of


Figure 35. Example p-y curves for soft clay below the water table; unified criteria, static loading
p-y curves for static loading, and Figure 36 shows curves for cyclic loading.
115. For the stiff clay profile in Figure 26, the value of $\varepsilon_{50}$ was assumed to be 0.005 and $y$ was taken as 50 pcf for the entire depth. The value of $A$ was assumed to be 0.35 , the value of $F$ to be 800,000 pcf. Figure 37 shows the set of p-y curves for static loading, and Figure 38 shows curves for cyclic loading.


Figure 36. Example p-y curves for soft clay below the water table; unified criteria, cyclic loading


Figure 37. Example p-y curves for stiff clay below the water table; unified criteria, static loading


Figure 38. Example p-y curves for stiff clay below the water table; unified criteria, cyclic loading

Recommendations for $\mathrm{p}-\mathrm{y}$ Curves for Sand
116. As shown below, a major experimental program was conducted on the behavior of laterally loaded piles in sand below the water table. The results can be extended to sand above the water table.
Response of sand below the water table
117. Field experiments. An extensive series of tests was performed at a site on Mustang Island, near Corpus Christi, Tex. (Cox, Reese, and Grubbs 1974). Two steel pipe piles, 24 in . in diameter, were driven into sand in a manner simulating the driving of an open-ended pipe. The piles were then subjected to lateral loading. The embedded length of the piles was 69 ft . One of the piles was subjected to short-term loading and the other to repeated loading.
118. The soil at the site was a uniformly graded fine sand with an angle of internal friction of 39 deg. The submerged unit weight was 66 pcf. Tlie water surface was maintained a few inches above the mud line throughout the test program.
119. Recommendations for computing $\mathrm{p}-\mathrm{y}$ curves. The following
procedure is for both short-term static loading and cyclic loading and is illustrated in Figure 39 (Reese, Cox, and Koop 1974).
a. Obtain values for the angle of internal friction $\phi$, the soil unit weight $\gamma$, and pile diameter $b$.
b. Make the following preliminary computations.

$$
\begin{equation*}
\alpha=\frac{\phi}{2} ; \quad \beta=45+\frac{\phi}{2} ; \quad K_{0}=0.4 ; \quad K_{a}=\tan ^{2}\left(45-\frac{\phi}{2}\right) \tag{86}
\end{equation*}
$$

c. Compute the ultimate soil resistance per unit length of pile using the smaller of the values given by the equations below.

$$
\begin{align*}
p_{s t}= & \gamma x\left[\frac{K_{0} x \tan \phi \sin \beta}{\tan (\beta-\phi) \cos \alpha}+\frac{\tan \beta}{\tan (\beta-\phi)}\right. \\
& \times(b+x \tan \beta \tan \alpha)+K_{0} x \tan \beta  \tag{87}\\
& \left.\times(\tan \phi \sin \beta-\tan \alpha)-K_{a} b\right]
\end{align*}
$$



Figure 39. Characteristic shape of a family of p-y curves for static and cyclic loading in sand (Reese, Cox, and Koop 1974)

$$
\begin{equation*}
P_{s d}=K_{a} b \gamma x\left(\tan ^{8} \beta-1\right)+K_{0} b \gamma x \tan \phi \tan ^{4} \beta \tag{88}
\end{equation*}
$$

d. In making the computations in step $c$, find the depth $x_{t}$ at which there is an intersection between Equations 87 and 88. Above this depth, use Equation 87. Below this depth, use Equation 88.
e. Select a depth at which a $p-y$ curve is desired.
f. Establish $y_{u}$ as $3 b / 80$. Compute $p_{u}$ from

$$
\begin{equation*}
p_{u}=\bar{A}_{s} p_{s} \quad \text { or } \quad p_{u}=\bar{A}_{c} p_{s} \tag{89}
\end{equation*}
$$

Use the appropriate value of $\bar{A}_{s}$ or $\bar{A}_{c}$ from Figure 40 for the particular nondimensional depth, and for either the static or cyclic case. Use the appropriate equation for $p_{s}$ from Equation 87 or Equation 88 by referring to the computation in step d.


Figure 40. Values of the coefficients $\bar{A}_{c}$ and $\bar{A}_{s}$ (Reese and Sullivan 1980)
g. Establish $y_{m}$ as $b / 60$. Compute $p_{m}$ from

$$
\begin{equation*}
p_{m}=B_{s} p_{s} \text { or } p_{m}=B_{c} p_{s} \tag{90}
\end{equation*}
$$

Use the appropriate value of $B_{s}$ or $B_{c}$ from Figure 41 for the particular nondimensional depth, and for either the static or the cyclic case. Use the appropriate equation for $p_{s}$. The two straight-line portions of the $p-y$ curve, beyond the point where $y$ is equal to $b / 60$, can now be established.


Figure 41. Nondimensional coefficient $b$ for soil resistance versus depth (Reese and Sullivan 1980)
h. Establish the initial straight-line portion of the $p-y$ curve,

$$
\begin{equation*}
p=(k x) y \tag{91}
\end{equation*}
$$

Use the appropriate value of $k$ from Table 7 or 8.
i. Establish the parabolic section of the $p-y$ curve,

$$
\begin{equation*}
p=\bar{C} y^{1 / n} \tag{92}
\end{equation*}
$$

Table 7
Representative Values of $k$ for Submerged Sand

|  |  | Relative Density |  |
| :--- | :--- | :---: | :---: | :---: |
| Recommended $k$, pci | $\underline{\text { Loose }}$ | Medium | Dense |

Table 8
Representative Values of $k$ for Sand Above the Water Table

|  |  | Relative Density |  |
| :--- | :---: | :---: | :---: |
| Recommended $k$, pci | $\underline{\text { Loose }}$ | $\underline{\text { Medium }}$ | $\frac{\text { Dense }}{2}$ |
| 25 | 90 | 225 |  |

Fit the parabola between points $k$ and $m$ as follows:
(1) Determine the slope of the line between points $m$ and $u$ from

$$
\begin{equation*}
\mathrm{m}=\frac{\mathrm{p}_{\mathrm{u}}-p_{\mathrm{m}}}{\mathrm{y}_{\mathrm{u}}-\mathrm{y}_{\mathrm{m}}} \tag{93}
\end{equation*}
$$

(2) Obtain the power of the parabolic section from

$$
\begin{equation*}
n=\frac{p_{m}}{m y_{m}} \tag{94}
\end{equation*}
$$

(3) Obtain the coefficient $\overline{\mathrm{C}}$ from

$$
\begin{equation*}
\overline{\mathrm{C}}=\frac{\mathrm{p}_{\mathrm{m}}}{\mathrm{y}_{\mathrm{m}}^{1 / \mathrm{n}}} \tag{95}
\end{equation*}
$$

(4) Determine point $k$ from

$$
\begin{equation*}
y_{k}=\left(\frac{\bar{C}}{k x}\right)^{n / n-1} \tag{96}
\end{equation*}
$$

(5) Compute the appropriate number of points on the parabola by using Equation 92.
Note: The step-by-step procedure is outlined, and Figure 39 is drawn, as if there is an intersection between the initial straight-line portion of the $p-y$ curve and the parabolic portion of the curve at point $k$. However, in some instances, there may be no intersection with the parabola. Equation 91 defines the $p-y$ curve until there is an intersection with another branch of the $p-y$ curve, or, if no intersection occurs, Equation 91 defines the complete $p-y$ curve. This completes the development of the $p-y$ curve for the desired depth. Any number of curves can be developed by repeating the above steps for each desired depth.
120. Recommended soil tests. Triaxial compression tests are recommended for obtaining the angle of internal friction of the sand. Confining pressures should be used which are close or equal to those at the depths being considered in the analysis. If samples cannot be obtained, correlations between $d$ and results from penetration tests can be used. Tests must be performed to determine the unit weight of the sand.
121. Example curves. An example set of $p-y$ curves was computed for sand below the water table for a pile with a diameter of 48 in . The soil profile used is presented in Figure 42. The submerged unit weight was assumed to be 57.5 pcf , and $k$ was taken to be 80 pci . The loading was assumed to be both static and cyclic.
122. p-y curves were computed for the following depths below the mud line: $0,1,2,4,8,12,20,40$, and 60 ft . The plotted curves are shown in Figure 43 for static loading and in Figure 44 for cyclic loading. Response of sand above the water table
123. The procedure described in the previous section can be used for sand above the water table if appropriate adjustments are made to the unit weight and angle of internal friction of the sand. Some small-scale experiments were performed by Parker and Reese (1971), and recommendations for p-y curves for dry sand were developed from those experiments. The results of the Parker and Reese experiments should be useful in checking solutions which were obtained using results from the test program for full-scale piles.

## Summary

124. This part of the report has described procedures which can be used in developing soil response curves for laterally loaded piles in soft clay,


Figure 42. Soil profile used for example p-y curves for sand below the water table; Reese criteria
stiff clay, or sands. Most of the material covered in this part of the report was extracted from reports of work done and documented at UT by Prof. Reese and his associates. The examples are selected from Corps of Engineers' files.
125. It must be emphasized that development of proper soil-response curves requires experience and a feel for the problem. At best, the procedures described in this part should only be used as guidelines. In every case, a user is responsible for developing these curves, and it is assumed that he will apply judgment in using the guidance provided here.


Figure 43. Example p-y curves for sand below the water table, static loading

Figure 44. Example p-y curves for sand below the water table, cyclic loading

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# APPENDIX A: NONDIMENSIONAL SOLUTIONS FOR ANALYSIS OF Laterally loaded piles 

## Introduction

1. The principle of dimensional analysis is usually applied to physical models; however, Reese and Matlock (1956)* applied the principle to mathematical models as well. They used the principle of dimensional analysis to produce a set of nondimensional coefficients which can be used to solve the governing differential equation for laterally loaded piles.
2. The development of the nondimensional solution method was a result of extensive experience gained at The University of Texas at Austin through manual use of the difference equation method. Parts of the method were done a few times for each boundary condition, using a range of values for the variables. It was found that these solutions could then be applied to many similar problems. The theoretical legitimacy of this method of approach was confirmed by applying the principles of engineering similitude to derive the method.
3. At the time of the development of nondimensional methods of analysis, computers were available to few engineers outside of research. The nondimensional methods were developed because they included many of the advantages of the finite difference solutions, yet could be performed relatively easily by using a hand calculator. Their primary advantage was that the nonlinear soil response could be taken into account through successive iterations of the solution. The main disadvantage was that a predetermined variation of soil modulus with depth must be assumed. Today, the nondimensional methods are important because they: (a) provide a hand solution method to verify computer results by the finite difference technique, (b) prc de a better understanding of the mechanics of the response of a pile under lateral loading, and (c) can be used on occasion to obtain results for use in design if a computer is not avalable.
4. Readers are referred to Reese and Sullivan (1980), Reese and Allen (1977), Reese and Matlock (1956) and Matlock and Reese (1960) for the concept and theory of nondimensional solutions and the details of the solution procedure for analyses of laterally loaded piles. This appendix presents a

[^3]step-by-step procedure and an example solution, including the manual generation of a $p-y$ curve using soft clay criteria.

Solution Procedure (Extracted from Reese and Sullivan 1980)
5. The solution procedure is described below for three sets of boundary conditions at the top of the pile: (a) pile head free to rotate, (b) pile head fixed against rotation, and (c) pile head restrained against rotation. These boundary conditions are shown in Figure Al along with the sign convention used in the solutions.
6. Limitations imposed by the nondimensional solutions are as follows:
a. The effect on bending moment of the axial load cannot be investigated.
b. A constant value of flexural rigidity of the pile must be used.
c. The nondimensional curves included herein are valid only for the case of a linearly varying soil modulus with zero at the groundline.

## Case I: Pile head free to rotate

7. The solution procedure for Case $I$ is as follows:
a. Construct $p-y$ curves at various depths by procedures recommended in the main text, with the spacing between $p-y$ curves being closer near the ground surface than near the bottom of the pile.
b. Assume a value of $T$, the relative stiffness factor, from

$$
\begin{equation*}
T=\sqrt[5]{\frac{E I}{k}} \tag{Al}
\end{equation*}
$$

where

$$
\begin{align*}
E I= & \text { flexural rigidity of pile } \\
k= & \text { constant relating the secant modulus of soil reaction } \\
& \text { to depth }\left(E_{s}=k x\right) \tag{A2}
\end{align*}
$$

c. Compute the depth coefficient $z_{\max }=L / T$.
d. Compute the deflection $y$ at each depth $x$ along the pile where a $p-y$ curve is available from

$$
\begin{equation*}
y=A_{y} \frac{P_{T} T^{3}}{E I}+B_{y} \frac{M_{T} T^{2}}{E I} \tag{A3}
\end{equation*}
$$



Figure A1. Sign convention and boundary conditions considered in the solution procedure (Reese and Sullivan 1980)
where

$$
\begin{aligned}
A_{y} & =\text { deflection coefficient (from Figure } A 2) \\
P_{T} & =\text { shear at top of pile } \\
T & =\text { relative stiffness factor } \\
B_{y} & =\text { deflection coefficient (from Figure } A 3) \\
M_{T} & =\text { moment at top of pile }
\end{aligned}
$$

The particular curves to be employed in determining the $A_{y}$ and $B_{y}$ coefficients depend on the value of $z_{\text {max }}$ computed in step c .
e. From a p-y curve, select the value of soil resistance p that corresponds to the pile deflection value $y$ at the depth of the p-y curve. Repeat this procedure for every p-y curve that is available.
f. Compute a secant modulus of soil reaction $E_{s}$ using the equation

$$
E_{s}=\frac{p}{y}
$$

Plot the $E_{s}$ values versus depth.
g. From the $E_{s}$-versus-depth plot in step $f$, compute the constant $k$ which relates $E_{s}$ to depth $\left(k=E_{s} / x\right)$. Give more weight to the $E_{s}$ values near the ground surface.
h. Compute a value of the relative stiffness factor $T$ from the value of $p$ found in step g. Repeat steps $b$ through $g$ using the new value of $T$ each time, until the assumed value of $T$ equals the calculated value of $T$.
i. When the iterative procedure has been completed, the values of deflection along the pile are known from step d of the final iteration. Values of soil reactions may be computed from the basic expression

$$
p=E_{s} y
$$

Values of slope, moment, and shear along the pile can be determined from

$$
\begin{equation*}
S=A_{s} \frac{P_{t} T^{2}}{E I}+B_{s} \frac{M_{t} T}{E I} \tag{A4}
\end{equation*}
$$



Figure A2. Pile deflection produced by lateral load at mud line (Reese and Sullivan 1980)


$$
\begin{equation*}
M=A_{m} P_{t} T+R_{M} M_{t} \tag{A5}
\end{equation*}
$$

and

$$
\begin{equation*}
v=A_{v} P_{t}+B_{v} \frac{M_{t}}{T} \tag{A6}
\end{equation*}
$$

The appropriate coefficients to be used in the above equations may be obtained from Figures A4 through A9.

Case II: Pile head fixed against rotation
8. Case II may be used to obtain a solution for the case where the superstructure translates under load but does not rotate and where the superstructure is very stiff in relation to the pile.
a. Perform steps $a, b$, and $c$ of the solution procedure for freehead piles (Case $I$ ).
b. Compute the deflection $y$ at each depth along the pile where a p-y curve is available from

$$
\begin{equation*}
y_{F}=F_{y} \frac{P_{t} T^{3}}{E I} \tag{A7}
\end{equation*}
$$

The deflection coefficients $F_{y}$ may be found by entering Figure Al0 with the appropriate value of $z_{\text {max }}$
c. The solution proceeds in manner similar to steps e through $h$ for the free-head case (Case I).
d. Compute the moment at the top of the pile $M_{T}$ from

$$
\begin{equation*}
M_{t}=F_{M T} P_{t} T \tag{A8}
\end{equation*}
$$

The value of $F_{M T}$ may be found by entering Table Al with the appropriate value of $z_{\text {max }}$.
e. Compute values of slope, moment, shear, and soil reaction along the pile by following the procedure in step $i$ for the free-head pile.


$s_{A}+x$

$$
S_{A}=A_{s}\left(\frac{P_{t} T^{2}}{E I}\right) \quad x=z(T)
$$

WHERE T-(EI/k)'5

$$
\left\{p_{t}\right.
$$

Figure A4. Slope of pile caused by lateral load at mud line (Reese and Sullivan 1980)


Figure A5. Slope of pile caused by moment applied at mud line (Reese and Sullivan 1980)



Figure A7. Bending moment produced by moment applied at mud line (Reese and Sullivan 1980)




$$
V_{A}=A_{V}\left(P_{t}\right) \quad x=\pi(T)
$$

$$
\text { WHERE } T=(E L / k)^{1 / 5}
$$

Figure A8. Shear produced by lateral load at mud line (Reese and Sullivan 1980)





$$
v_{F}=F_{V}\left(\frac{P_{1} T^{3}}{E I}\right) \quad x=v(T)
$$

WHERE $T=(E L / k)^{1.5}$

Figure A10. Deflection of pile fixed against rotation at mud line (Reese and Sullivan 1980)

Table Al

| Moment Coefficients at Top of |  |
| :--- | ---: |
| Pile for Fixed-Head Case |  |
| $\mathrm{Z}_{\max }$ | $\frac{\mathrm{F}_{\mathrm{Mt}}}{}$ |
| 2 | -1.06 |
| 3 | -0.97 |
| 4 | -0.93 |
| 5 and above | -0.93 |

## Case III: Pile head

 restrained against rotation9. Case III may be used to obtain a solution for the case where the superstructure translates under load but does not rotate.
a. Perform steps $a, b$, $c$ of the solution procedure free-head piles (Case I).
b. Obtain the value of the spring stiffness $k_{\theta}$ of the pile superstructure system. The spring stiffness is defined as

$$
\begin{equation*}
k_{\theta}=\frac{M_{t}}{S_{t}} \tag{A9}
\end{equation*}
$$

where

$$
\begin{aligned}
& M_{t}=\text { moment at top of pile } \\
& S_{t}=\text { slope at top of pile }
\end{aligned}
$$

c. Compute the slope at the top of the pile $S_{t}$ from

$$
\begin{equation*}
S_{t}=A_{s t} \frac{P_{T} T^{2}}{E I}+B_{s t} \frac{M_{T} T}{E I} \tag{A10}
\end{equation*}
$$

where

$$
\begin{aligned}
& A_{s t}=\text { slope coefficient (From Figure A4) } \\
& B_{s t}=\text { slope coefficient (from Figure A5) }
\end{aligned}
$$

d. Solve Equations $A 9$ and $A l 0$ for the moment at the top of the pile $M_{t}$
e. Perform steps a through i of the solution procedure for freehead piles (Case I).
10. This process completes the solution of the laterally loaded pile problem for three sets of boundary conditions. The solution gives values of deflection, slope, moment, shear, and soil reaction as a function of depth. To illustrate the nondimensional method, an example solution is presented next.

## Example Solution

11. The following paragraphs present an example analysis using the nondimensional method and a comparison of the results with the computer solution of the same problem.

Problem statement
12. Figure All illustrates the problem to be solved by the nondimensional method as well as pertinent soils data. This same problem, as solved by COM624G, is presented in Appendix D as example problem l. A comparison of the two solutions is presented following the nondimensional solution. Nondimensional solution
13. The solution will proceed in the step-by-step manner described for Case I.
14. Step 1. Compute and construct $p-y$ curves. The p-y curves for the example problem as generated by COM624G (using the soft clay criteria) art presented in Appendix D, example problem 1. These same curves are generated manually in the following steps to illustrate the hand procedure. The computations follow the step-by-step procedure given for soft clay criteria in Part III of the main report. Computations for both static and cyclic curves are presented; however, only cyclic curves are utilized in the pile analysis. The depths for which curves are to be computed are: $0,16,32,48,80,128$, 154 , and 240 in. Only the static and cyclic curves for $x=48$ in. are computed in the following example:
a. Static curves:
(1) Obtain the variation of shear strength and submerged unit weight with depth and determine $\varepsilon_{50}$. (See Table 3, Part III of the main text.)

The following properties are used:

$$
\begin{aligned}
c & =500 \mathrm{ps} f=3.47 \mathrm{psi} \\
\boldsymbol{\gamma}^{\prime} & =30 \mathrm{pcf}=0.0168 \mathrm{pci}
\end{aligned}
$$



Figure Al1. Example problem for solution by nondimensional methods

$$
\begin{aligned}
\varepsilon_{50} & =0.010 \\
b & =16 \mathrm{in} . \\
x & =48 \mathrm{in} .
\end{aligned}
$$

(2) Compute $p_{u}$ using the smaller of the values from

$$
p_{u}=\left(3+\frac{y^{\prime}}{c} x+\frac{0.5}{b} x\right) c b
$$

and

$$
p_{u}=9 \mathrm{cb}
$$

$$
p_{u}=\left[3+\frac{0.0168}{3.47}(48)+\frac{0.5}{16}(48)\right] 3.47(16)
$$

$$
=262.7 \mathrm{lb} / \mathrm{in} .
$$

$$
p_{u}=9(3.47)(16)=499.7 \mathrm{lb} / \mathrm{in}
$$

Therefore, use

$$
p_{u}=262.7 \mathrm{lb} / \mathrm{in} .
$$

(3) Compute $y_{50}$ at half $p_{u}$ :

$$
y_{50}=2.5 \varepsilon_{50} b
$$

$$
y_{50}=2.5(0.010)(16)=0.40 \mathrm{in} .
$$

(4) Compute points describing the $p-y$ curve:

$$
\begin{aligned}
& \frac{p}{p_{u}}=0.5\left(\frac{y}{y_{50}}\right)^{1 / 3} \\
& p \text { is constant beyond } y=8 y_{50}
\end{aligned}
$$

| $y$, in. | $\mathrm{p}, \mathrm{lb} / \mathrm{in}$. |
| :---: | :---: |
| 0.2 | 104.3 |
| 0.4 | 131.4 |
| 0.8 | 165.5 |
| 1.2 | 189.4 |
| 2.0 | 224.6 |
| 3.2 | 262.7 |
| $8 y_{50}=8$ | $=3.2 \mathrm{in}$. |



Figure Al2. Computed static and cyclic p-y curves for $x=48 \mathrm{in}$.
(5) The computed static $p$-y curve is plotted in Figure A12.
b. Cyclic curves:
(1) The cyclic curve is the same as the static curve for $p$ less than $0.72 \mathrm{p}_{\mathrm{u}}$
(2) Solve for $X_{r}$ :

$$
x_{r}=\frac{6 c b}{y^{\prime} b+0.5 c}
$$

$$
\begin{aligned}
& x_{r}=\frac{6(3.47)(16)}{0.0168(16)+0.5(3.47)} \\
& x_{r}=166.2 \mathrm{in} .
\end{aligned}
$$

(3) If $x \geq x_{r}, p=0.72 p_{u}$ for $y>3 y_{50}$
(4) If $x<x_{r}, p$ decreases from $0.72 p_{u}$ at $y=3 y_{50}$ to $p$ in the following equation at $y=15 y_{50}$ :

$$
p=0.72 p_{u} \frac{x}{x_{r}}
$$

$$
\begin{aligned}
& p=0.72(262.7) \frac{48}{166.2}=54.6 \mathrm{lb} / \mathrm{in} \\
& y=15 y_{50}=15(0.40)=6.0 \mathrm{in} . \\
& p=0.72 p_{u}=0.72(262.7)=189.1 \mathrm{lb} / \mathrm{in} . \\
& y=3 y_{50}=3(0.40)=1.2 \mathrm{in} .
\end{aligned}
$$

(5) The computed cyclic p-y curve is plotted in Figure A12. c. The remainder of the $p-y$ curves for the other values of $x$ are computed using the same procedure. These computed curves are presented in Figure A13.


Figure A13. Plot of p-y curves for example problem solved by nondimensional method; soft clay criteria, cyclic loading
15. Step 2. Assume $T: T=95 \mathrm{in}$.
16. Step 3. Compute $z_{\max }$ :

$$
z_{\max }=\frac{L}{T}=\frac{720}{95}=7.58
$$

17. Step 4. Compute the deflection $y$ at depths of $0,16,32,48,80$, 128, 154, and 240 in. using Equation $A 3$ and Figures A2 and A3. The computations are presented in tabular form in Table A2.
18. Step 5. From the set of $p-y$ curves (Figure Al3) the values of $p$ are determined corresponding to the $y$ values computed in step 4 (see the tabulation in Table A2).
19. Step 6. Compute the $E_{s}$ value at each depth (see the tabulation in Table A2).
20. Step 7. Prepare a plot of $E_{s}$ versus depth as shown in Figure Al4. In fitting the straight line to the plotted points, more weight should be given to the points near the ground surface. The $k$ value is determined as the slope of this line:

$$
k=\frac{E_{s}}{x}=\frac{500}{142}=3.52 \mathrm{lb} / \mathrm{in}^{3}
$$

21. Step 8. Compute $T$ :

$$
T=5 \frac{E I}{k}=\sqrt[5]{\frac{(3.14) 10^{10}}{3.52}}=97.9 \mathrm{in} .
$$

Step 8 completes the first iteration of the solution procedure. Before proceeding to the next iteration, the results thus far should be examined to provide guidance in further computations. It is evident from Figure Al4 that $E_{s}=k x$ is not a good representation of the variation of the soil modulus with depth. A straight line through the origin does not fit the plotted points. However, the constraints of the method required that the line pass through the origin to satisfy the assumption that $\mathrm{E}_{\mathrm{s}}=\mathrm{kx}$. Figure Al4 also reveals that the solution has not been found because the $k$ value of 4.0 pci that was assumed is not equal to the $k$ of 3.52 pci that was obtained. Correspondingly, the assumed value of $T$ was not equal to the $T$ value obtained. From comparisons, it appears that the value of $k$ will decrease and $T$ will increase with successive iterations. The iterations are continued until the desired degree of convergence is achieved. In the example problem, the computations were continued for three additional iterations. The additional computations are shown in Tables A3-A5; the corresponding plots of $E_{s}$ versus $x$ are shown in Figures A15-A17. For this example, the computations were continued until the deflections at the groundline agreed within 5 percent for the
Table A2
Nondimensional Analysis of Laterally Loaded Piles with Pile Head Free to Rotate Computations for Iteration No. 1

| Depth in. | Depth Coefficient | Deflection Coefficient | Deflection Coefficient | $\begin{gathered} \text { Deflection } \\ \text { in. } \end{gathered}$ | Soil <br> Resistance lb/in. | $\begin{aligned} & \text { Soil } \\ & \text { Modulus } \\ & \text { lb/in. }{ }^{2} \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x | $z=\frac{x}{T}$ | $A_{y}, \quad \text { from }$ Figure A2 | $B_{y}$, from <br> Figure A3 | $y=A_{y} \frac{P_{t} T^{3}}{E I}+B_{y} \frac{M_{t} T^{2}}{E I}$ | $\begin{aligned} & p, \text { from } \\ & p-y \text { Curve } \end{aligned}$ | $E_{s}=-\frac{p}{y}$ |
| 0 | 0.0 | 2.40 | 1.60 | 1.72 | 110 | 64 |
| 16 | 0.17 | 2.15 | 1.33 | 1.56 | 138 | 88 |
| 32 | 0.34 | 1.85 | 1.10 | 1.35 | 163 | 121 |
| 48 | 0.51 | 1.60 | 0.85 | 1.20 | 195 | 163 |
| 80 | 0.84 | 1.15 | 0.50 | 0.89 | 220 | 247 |
| 128 | 1.35 | 0.58 | 0.13 | 0.48 | 233 | 485 |
| 154 | 1.62 | 0.32 | 0.02 | 0.27 | 220 | 815 |
| 240 | 2.53 | -0.03 | -0.10 | 0.00 |  |  |

[^4]Figure A14. Plot of $\begin{aligned} & \text { E versus } x \text { for example problem; } \\ & \text { first iteration }\end{aligned}$

Table A3
Nondimensional Analysis of Laterally Loaded Piles with Pile Head Free to
Rotate Computations for Iteration No. 2

| Depth in. | Depth Coefficient | Deflection Coefficient | Deflection Coefficient | $\begin{gathered} \text { Deflection } \\ \text { in. } \\ \hline \end{gathered}$ | $\begin{gathered} \text { Soil } \\ \text { Resistance } \\ \text { lb/in. } \\ \hline \end{gathered}$ | $\begin{aligned} & \text { Soil } \\ & \text { Modulus } \\ & \text { lb/in. } 2 \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X | $z=\frac{x}{T}$ | $A_{y}$, from <br> Figure A2 | $\begin{array}{r} \mathrm{B}_{\mathrm{y}}, \text { from } \\ \text { Figure A3 } \\ \hline \end{array}$ | $y=A_{y} \frac{P_{t} T^{3}}{E I}+B_{y} \frac{M_{t} T^{2}}{E I}$ | $\begin{array}{ll} \mathbf{p}, \quad \text { from } \\ \mathrm{p}-\mathrm{y} & \text { Curve } \end{array}$ | $E_{s}=-\frac{p}{y}$ |
| 0 | 0.00 | 2.40 | 1.60 | 1.89 | 103 | 54 |
| 16 | 0.16 | 2.17 | 1.36 | 1.73 | 132 | 76 |
| 32 | 0.33 | 1.86 | 1.07 | 1.51 | 160 | 106 |
| 48 | 0.49 | 1.61 | 0.83 | 1.33 | 190 | 126 |
| 80 | 0.82 | 1.17 | 0.52 | 0.99 | 225 | 227 |
| 128 | 1.31 | 0.62 | 0.15 | 0.56 | 250 | 446 |
| 154 | 1.58 | 0.35 | 0.03 | 0.33 | 240 | 727 |
| 240 | 2.46 | -0.03 | -0.10 | 0.00 |  |  |
| $k=\frac{E_{s}}{x}$ | $3.14$ | $T_{\text {obtained }}=$ | $1 / 5=100.0$ |  |  |  |

Table A4
Nondimensional Analysis of Laterally Loaded Piles with Pile Head Free to
Nondimensional Analysis of Laterally Loaded Piles with Pile Head Free to

$$
\text { Rotate Computations for Iteration No. } 3
$$ Rotate Computations for Iteration No. 3

| Depth in. | Depth Coefficient | Deflection Coefficient | Deflection Coefficient | Deflection in. | $\begin{gathered} \text { Soil } \\ \text { Resistance } \\ \text { lb/in. } \\ \hline \end{gathered}$ | $\begin{aligned} & \text { Soil } \\ & \text { Modulus } \\ & \text { lb/in. }{ }^{2} \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X | $z=\frac{x}{T}$ | $\begin{aligned} & A_{y}, \text { from } \\ & \text { Figure A2 } \end{aligned}$ | $\begin{array}{r} B_{y}, \text { from } \\ \text { Figure } \mathrm{A} 3 \\ \hline \end{array}$ | $y=A_{y} \frac{P_{t} T^{3}}{E I}+B_{y} \frac{M_{t} T^{2}}{E I}$ | $\begin{aligned} & \mathrm{p}, \text { from } \\ & \mathrm{p}-\mathrm{y} \text { Curve } \\ & \hline \end{aligned}$ | $E_{s}=-\frac{p}{y}$ |
| 0 | 0.00 | 2.40 | 1.60 | 2.02 | 100 | 50 |
| 16 | 0.16 | 2.20 | 1.35 | 1.89 | 128 | 68 |
| 32 | 0.32 | 1.87 | 1.10 | 1.62 | 160 | 99 |
| 48 | 0.48 | 1.63 | C. 85 | 1.44 | 190 | 132 |
| 80 | 0.80 | 1.20 | 0.55 | 1.08 | 237 | 219 |
| 128 | 1.28 | 0.65 | 0.15 | 0.62 | 250 | 403 |
| 154 | 1.54 | 0.37 | 0.05 | 0.36 | 24. | 667 |
| 240 | 2.40 | 0.00 | 0.10 | 0.03 | 75 | 2500 |

[^5]\[

$$
\begin{aligned}
& E I=3.14 \times 10^{10} \mathrm{lb} \text {-in. }{ }^{2} \\
& \text { (or } \mathrm{T}_{\text {assumed }}=100.0 \mathrm{in.} \text { ) } \\
& \underline{100.0} \text { in. } \quad z_{\text {max }}=\frac{L}{T}=\underline{7.20} \\
& \begin{array}{c}
M_{t}=-827,130 \text { in. }-1 \mathrm{~b} \\
k_{\text {assumed }}=3.14 \text { lb-in. }{ }^{3}
\end{array} \\
& P_{t}=32,000 \mathrm{lb}
\end{aligned}
$$
\]

Table A5
Nondimensional Analysis of Laterally Loaded Piles with Pile Head Free to
Rotate Computations for Iteration No. 4


| $\begin{gathered} \text { Depth } \\ \text { in. } \end{gathered}$ | Depth Coefficient | Deflection Coefficient | Deflection Coefficient | Deflection in. | Soil Resistance lb/in. | Soil Modulus lb/in. ${ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X | $z=\frac{x}{T}$ | $A_{y}$, from Figure A2 | $\begin{array}{r} B_{y}, \text { from } \\ \text { Figure A3 } \\ \hline \end{array}$ | $y=A_{y} \frac{P_{t} T^{3}}{E I}+B_{y} \frac{M_{t} T^{2}}{E I}$ | $\begin{array}{ll} p, & \text { from } \\ p-y & \text { Curve } \end{array}$ | $E_{s}=-\frac{p}{y}$ |
| 0 | 0.00 | 2.40 | 1.60 | 2.12 | 95 | 45 |
| 16 | 0.16 | 2.20 | 1.35 | 1.98 | 125 | 63 |
| 32 | 0.32 | 1.87 | 1.10 | 1.69 | 158 | 93 |
| 48 | 0.47 | 1.63 | 0.85 | 1.51 | 187 | 124 |
| 80 | 0.79 | 1.20 | 0.55 | 1.13 | 240 | 212 |
| 128 | 1.26 | 0.65 | 0.15 | 0.65 | 257 | 395 |
| 154 | 1.52 | 0.37 | 0.05 | 0.38 | 243 | 639 |
| 240 | 2.36 | 0.00 | -0.10 | 0.03 | 75 | 2500 |
| 480 | 4.73 | 0.00 | 0.00 | 0.00 |  |  |
| 720 | 7.09 | 0.00 | 0.00 | 0.00 |  |  |

[^6]

Figure A15. Plot of $E$ versus $x$ for example problem; second iteration


Figure Al6. Plot of $\underset{\text { third iteration }}{\text { thersus }}$ for example problem;


Figure Al7. Plot of $E$ versus $x$ for example problem; fourth iteration
last two iterations. However, the number of iterations for a particular problem should be determined by the user after giving due consideration to the degree of accuracy required and to the limitations inherent in the method. After the final iteration is complete, continue with step 9.
22. Step 9. The final step in the computation procedure is to determine the results of the analysis as follows:
a. The value of deflection $y$ and soil reaction $p$ along the pile are known from step 4 of the final iteration (Table A5). These results are presented in Figures A18 and A19 and are compared with the computer solution of example problem 1 from Appendix $D$.
b. Compute slope $S$ versus depth from Equation A4:

$$
\begin{equation*}
S=A_{s} \frac{P_{t} T^{2}}{E I}+B_{s} \frac{M_{t} T}{E I} \tag{A4bis}
\end{equation*}
$$

where $A_{s}$ and $B_{s}$ are slope coefficients taken from Figures A4 and A5, respectively. Results of the computations are presented in tabular form in Table A6 and in graphic form in Figure A20.
c. Compute moment $M$ versus depth from Equation A5:

$$
\begin{equation*}
M=A_{m} P_{t} T+B_{m} M_{t} \tag{A5bis}
\end{equation*}
$$

where $A_{m}$ and $B_{m}$ are moment coefficients taken from Figures A6 and A7, respectively. Results of these computations are presented in tabular form in Table A7 and in graphic form in Figure A21. Also plotted in Figure A21 are results from the computer solution.
d. Compute shear $V$ versus depth from Equation A6:

$$
\begin{equation*}
v=A_{v} P_{t}+\frac{B_{v} M_{t}}{T} \tag{A6bis}
\end{equation*}
$$

where $A_{v}$ and $B_{v}$ are shear coefficients taken from Figures $A 8$ and $A 9$, respectively. Results of these computations are presented in tabular form in Table A8 and in graphic form in Figure A 22.



Figure A19. Plot of soil resistance $p$ versus depth $x$ for example problem

Table A6
Computed Slopes

| Depth in. | Depth Coefficient | $\begin{gathered} \text { Slope } \\ \text { Coefficient } \end{gathered}$ | $\begin{gathered} \text { Slope } \\ \text { Coefficient } \end{gathered}$ | Slope |
| :---: | :---: | :---: | :---: | :---: |
| x | $z=\frac{x}{T}$ | $\begin{aligned} & A_{s}, \text { from } \\ & \text { Figure } A 4 \\ & \hline \end{aligned}$ | $\mathrm{B}_{\mathrm{s}}, \mathrm{from}$ Figure A5 | $\mathrm{S}=\mathrm{A}_{\mathrm{s}} \frac{\mathrm{P}_{\mathrm{T}} \mathrm{~T}^{2}}{\mathrm{EI}}+\mathrm{B}_{\mathrm{s}} \frac{\mathrm{M}_{\mathrm{T}} \mathrm{~T}}{\mathrm{EI}}$ |
| 0 | 0.0 | -1.625 | -1.750 | -0.0124 |
| 16 | 0.16 | -1.600 | -1.625 | -0.0125 |
| 32 | 0.32 | -1.560 | -1.425 | -0.0126 |
| 48 | 0.47 | -1.510 | -1.285 | -0.0124 |
| 80 | 0.79 | -1.350 | -0.975 | -0.0116 |
| 128 | 1.26 | -1.000 | -0.575 | -0.0090 |
| 154 | 1.52 | -0.800 | -0.400 | -0.0073 |
| 240 | 2.36 | -0.260 | -0.048 | -0.0026 |
| 480 | 4.73 | 0.035 | 0.025 | 0.0003 |
| 720 | 7.09 | 0.000 | 0.000 | 0.0000 |



Figure A20. Plot of slope versus depth for example problem

Table A7
Computed Moments

| $\begin{aligned} & \text { Depth } \\ & \text { in. } \end{aligned}$ | Depth Coefficient | $\begin{gathered} \text { Moment } \\ \text { Coefficient } \end{gathered}$ | Moment Coefficient | Moment <br> in. -lb |
| :---: | :---: | :---: | :---: | :---: |
| $x$ | $z=\frac{x}{T}$ | $\begin{aligned} & \mathrm{A}_{M} \text {, from } \\ & \text { Figure } \mathrm{A} 6 \\ & \hline \end{aligned}$ | $\begin{aligned} & \mathrm{B}_{\mathrm{M}} \text {, from } \\ & \text { Figure } \mathrm{A} 7 \\ & \hline \end{aligned}$ | $M=A_{M} P_{t} T+B_{M} M_{t}$ |
| 0 | 0.0 | 0.00 | 1.00 | $-8.27 \times 10^{5}$ |
| 16 | 0.16 | 0.16 | 1.00 | $-3.07 \times 10^{5}$ |
| 32 | 0.32 | 0.32 | 0.99 | $2.21 \times 10^{5}$ |
| 48 | 0.47 | 0.44 | 0.98 | $6.19 \times 10^{5}$ |
| 80 | 0.79 | 0.65 | 0.92 | $1.35 \times 10^{6}$ |
| 128 | 1.26 | 0.77 | 0.75 | $1.88 \times 10^{6}$ |
| 154 | 1.52 | 0.76 | 0.63 | $1.95 \times 10^{6}$ |
| 240 | 2.36 | 0.49 | 0.25 | $1.38 \times 10^{6}$ |
| 480 | 4.73 | -0.01 | -0.02 | $-1.59 \times 10^{4}$ |
| 720 | 7.09 | 0.00 | 0.00 | 0.0 |



Table A8
Computed Shears

| $\begin{aligned} & \text { Depth } \\ & \text { in. } \\ & \hline \end{aligned}$ | Depth Coefficient | Shear Coefficient | Shear Coefficient | Shear $1 \mathrm{~b}$ |
| :---: | :---: | :---: | :---: | :---: |
| x | $z=\frac{x}{T}$ | $\begin{aligned} & \mathrm{A}_{\mathrm{v}} \text {, from } \\ & \text { Figure } \mathrm{A} 8 \end{aligned}$ | $\mathrm{B}_{\mathrm{v}}$, from <br> Figure A9 | $v=A_{v} P_{t}+B_{v} \bar{T}^{\frac{M}{t}}$ |
| 0 | 0.00 | 1.00 | 0.00 | 32,000 |
| 16 | 0.16 | 0.97 | -0.02 | 30,400 |
| 32 | 0.32 | 0.89 | -0.07 | 29,050 |
| 48 | 0.47 | 0.78 | -0.13 | 26,019 |
| 80 | 0.79 | 0.50 | -0.26 | 18,119 |
| 128 | 1.26 | 0.05 | -0.43 | 5,104 |
| 154 | 1.52 | -0.15 | -0.47 | -970 |
| 240 | 2.36 | -0.43 | -0.39 | -10,582 |
| 480 | 4.73 | 0.0 | 0.02 | -163 |
| 720 | 7.09 | 0.0 | 0.00 | 0 |



Figure A22. Plot of shear versus depth for example problem
23. Tables A9 through All present forms which are included for convenience of the user when making nondimensional analyses.

Comparison between nondimensional and computer solutions
24. Comparisons between the nondimensional solution and the computer solution (Appendix D, example problem 1) are presented in Figures A18, A19, and A21. Figure A18 presents a comparison of deflection versus depth. As is shown, the maximum variation occurs at the ground surface and is approximately 12 percent. Figure Al9 presents a comparison of soil resistance versus depth. The maximum percentage difference occurs at the ground surface and is approximately 10 percent. The maximum numerical difference occurs at the depth of maximum soil resistance ( 120 in. ) and is approximately $12 \mathrm{lb} / \mathrm{in}$. Figure A2l presents a comparison of moment versus depth. The maximum variation is approximately 6 percent and occurs at a depth of approximately 100 in. The maximum moment occurs at a depth of approximately 150 in . and the two methods yield essentially equal results.
25. The comparisons presented above indicate good to excellent agreement between the nondimensional and computer solutions. However, the user should be aware that the variations presented above apply only to this particular problem and variations for other problems may be larger or smaller. When considering whether or not the nondimensional solution yields a satisfactory degree of accuracy, the user should consider the variables inherent in computing the response of a laterally loaded pile.


Table A10
Nondimensional Analysis of Laterally Loaded Piles with Pile Head Restrained Against Rotation



Table All
Nondimensional Analysis of Laterally Loaded Piles with Pile Head Fixed Against Rotation

$k=\frac{E}{x}=$ $\qquad$ lb/in. ${ }^{3}$
$T_{\text {obtained }}=\left(\frac{E I}{k}\right)^{1 / 5}=$ $\qquad$ in.

## appendix b: example design problem

## Introduction

1. The behavior of a laterally loaded pile is a complex function of soil and pile parameters and loading conditions. In many cases, complexity of behavior combined with the uncertainty of loading conditions requires the designer to investigate a range of parameters and loading conditions before arriving at a final design. This appendix presents a design problem in which soil and loading conditions are not known with certainty and illustrates some of the decisions that must be made by the designer. Meyer and Reese (1979): present an excellent study on the effects of variations in soil parameters on computed pile behavior which should provide the user with further insight. From the example in this appendix and the study by Meyer and Reese (1979), the user should be aware of the sensitivity of the analysis to variations in parameters and loading conditions and the necessity for sound engineering judgment based on a thorough understanding of the design variables and analysis procedures.

## Example Design Problem

2. The example problem, which is illustrated in Figure B1, is taken from design studies of mooring dolphin facilities for Columbia Lock and Dam on the Ouachita River in central Louisiana. The example considers one particular load case for a single-pile dolphin.

## Loading case

3. The loading case presented in the example is one of several cases that might be analyzed. The specific case is for collision impact between the end of a barge and the dolphin. Other cases that might be analyzed are mooring forces from current and wind, berthing impact from the end and side of a barge, and collision impact between the end and side of a barge and the dolphin.

[^7]

Figure B1. Example design problem, single-pile mooring dolphin
4. Loads for the case presented were computed as follows:
a. Energy. Barge impact energy was computed from

$$
\begin{equation*}
E=f \frac{W v^{2}}{2 g} \tag{B1}
\end{equation*}
$$

where
$E=$ impact energy, ft-lb
$f=$ dissipation factor
$W=$ weight of barge (tow and cargo), lb
$V=$ velocity, normal to the dolphin, at impact, $f t / s e c$
$\mathrm{g}=$ acceleration of gravity, $\mathrm{ft} / \mathrm{sec}^{2}$
The factor $f$ reflects the energy dissipation created by the swing of the vessel about the dolphin after impact and is calculated from

$$
\begin{equation*}
\mathrm{f}=\frac{1}{1+16 \frac{\mathrm{~d}^{2}}{\mathrm{~L}^{2}}} \tag{B2}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{d}= & \text { distance from point of contact, measured tangent to the } \\
& \text { point of contact, to the center of gravity of the } \\
& \text { barge, ft } \\
\mathrm{L}= & \text { length of the barge, } \mathrm{ft}
\end{aligned}
$$

Equation $B 2$ for the dissipation factor reveals that, for end impact, an 80 percent reduction in energy is effected.
b. Normal force. Barge impact force was computed from

$$
P_{\max }=\frac{2 E}{\delta}
$$

where

$$
\begin{aligned}
P_{\max } & =\text { maximum normal force required to resist impact, lb } \\
E & =\text { impact energy, ft-lb } \\
\delta & =\text { deflection of dolphin, } \mathrm{ft}
\end{aligned}
$$

5. Computing the force $P_{\max }$ involves an iterative procedure in which a deflection is assumed, a trial $P_{\text {max }}$ is computed, the analysis is performed using the trial $P_{\max }$ to obtain a new deflection, and the procedure is
continued until the trial deflection and the computed deflection agree. The forces, moments, shears, etc., are then taken from the final iteration. $P_{\text {max }}$ can also be determined by computing a curve of $P_{\max }$ versus $\delta$, plotting the curve, and integrating the area under the curve by trial until an energy balance is obtained.
6. Because of the dependence of $P_{\max }$ on deflection and the fact that deflection is a function of the bending moment and stiffness of the pile, a pile with a larger section modulus will not necessarily have smaller bending stresses than a pile with a smaller section modulus.

## Design conditions

7. Surveys indicated the mud line to be at el $40, *$ as indicated in Figure B1. The top of the dolphin was set by the design criteria which required 8 ft of stickup above the 10 -year frequency high-water stage (el 70). The lowwater stage is el 52 which is controlled by the minimum upper pool of the lock. The design considered the force $P_{\max }$ to be applied 3 ft above the water surface. Because of the dependence of $P_{\text {max }}$ on deflection, which in $t u r n$ was dependent on bending moment and pile stiffness, it was necessary to perform analyses with $P_{\text {max }}$ applied as a low-level force ( 3 ft above low water) and as a high-level force ( 3 ft above high water). The example presented herein considers only the high-level force. Another important variable in the design was the velocity of the barge upon impact. Based on the hydraulic analysis for the design, a velocity of $1.0 \mathrm{ft} / \mathrm{sec}$ was selected as the best estimate. Design soil parameters
8. Borings at the site indicated the soil to be silts from the river bottom down to a depth of 15 ft . Below this, sands are indicated to extend beyond the penetration of the piling. Because $p-y$ criteria are not available for silts, it was necessary to make a design decision as to the appropriate p-y criteria to use. The decision was to use soft clay criteria for the silts, then vary the criteria to determine the influence of the variation on the pile behavior. Sand criteria were used for the sands. The soil profile used and the design parameters are shown in Figure B2. Figure B3 presents the generated $p-y$ curves. Cyclic $p-y$ curves were used for both soils.

[^8]

Figure B2. Pile and soil properties; single-pile mooring dolphin



Figure B3. p-y curves; single-pile mooring dolphin
9. The various conditions investigated under the load case are tabulated in Table B1. Results of the analysis are presented in tabular form in Table B2 and in graphical form in Figures B4 and B5.
Conclusions
10. As can be seen in Figures $B 4$ and $B 5$ and Table $B 2$, the results from an analysis can vary considerably depending on the input assumptions. For this particular example, the variation in shear strength of $\pm 40$ percent did not have a significant effect. The conditions which exhibit the most influence are the assumed 10 ft of scour and the increase in the barge velocity, with the combined effect of scour and increased barge velocity yielding the most critical condition. As shown in Table B2, the factor of safety for the combined condition drops drastically. This response is caused by the fact that the location of the maximum moment dropped into a segment of the pile which had a reduced section modulus. Obviously, this pile would not have an adequate section modulus if the conditions of scour and/or increased barge velocity were considered realistic. The final decisions in an example of this type must be made by the designer after considering the degree of certainty with which the design conditions are known.
11. A detailed input and output for computer analysis of one load case is presented in Appendix D, example 2.

Table B1
Description of Conditions Analyzed for Load Case IIIA

| Condition No. | Description of Condition |
| :---: | :---: |
| 1 | Analyzed with a barge velocity of $1.0 \mathrm{ft} / \mathrm{sec}$, groundline at mud line, and conventionally generated $p-y$ curves |
| 2 | Loaded as in Condition 1 except 10 ft of scour assumed below mud line |
| 3 | Loaded as in Condition 1 except 40 percent reduction in estimated strength of the silts |
| 4 | Loaded as in Condition 1 except 40 percent increase in estimated strength of the silts |
| 5 | Velocity of barge assumed to be $1.5 \mathrm{ft} / \mathrm{sec}$. All other factors same as in Condition 1 |
| 6 | Same as Condition 5 except 10 ft of scour assumed below mud line |

Table B2
Summary of Analysis

| Condition $\qquad$ | Pile Head Deflection in. | ```Deflection``` | Maximum <br> Bending <br> Moment <br> ft-kips | ```Factor of Safety*``` |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 20.3 | 7.5 | 7,442 | 1.62 |
| 2 | 28.4 | 12.3 | 4,417 | 0.98 |
| 3 | 20.9 | 7.9 | 7,642 | 1.62 |
| 4 | 19.6 | 7.2 | 7,258 | 1.62 |
| 5 | 28.1 | 10.7 | 10,083 | 1.21 |
| 6 | 41.0 | 18.2 | 11,250 | 0.67 |

* Yield strength of steel $=60 \mathrm{ksi}$.



## Introduction

1. COM624G is a computer progran that facilitates analysis of laterally loaded piles for various boundary conditions. The program was originally written by Prof. L. C. Reese and W. R. Sullivan at The University of Texas at Austin and was labelled COM624 (Reese and Sullivan 1980).\% In the COM624G version of the program, the input format was changed, a conversational mode for inputting data loads added, and graphical options were provided for plotting both input and output data. The program was also double-precisioned for use on the Honeywell DPS-1 computer. These modifications were programmed by Messrs. Michael Pace and Reed L. Mosher of the Automatic Data Processing Center, U. S. Army Engineer Waterways Experiment Station (WES).
2. Complete documentation of COM624 is provided in Reese and Sullivan (1980), and the reader should refer to this source for detailed information on the program. This appendix provides an input guide only to COM624G. The order of the input data by major groups (identified by a keyword) is immaterial, although input within each major group should be together in sequential order. All major groups are not required for problem solution, and within each group some data are optional. The optional data are indicated by inclosing them in parentheses.
3. Example problems are included at the end of the input guide. These problems are the same as those used in Reese and Sullivan (1980) for COM624 and are included so that verification is possible.

## Accessing the Program

4. To run COM624G on the WES or Office of Personnel Management, Macon, Ga., computer systems, sign on to the particular system. Then
$\div$ FORT

* OLD WESLIB/CORPS/IO012,R
* GCS2D
* device - TK4 (4014)

ALP (Alphanumeric Terminal)

[^9]
## Cybernet System

5. /OLD,CORPS/UN = CECELB
/CALL, CORPS,I0012

Input Guide for COM624G

Keyword [Line Number] (Optional Information)
I. Title

TITLE One line for identifying the individual problem in a computer run. It may be any alphanumeric information up to 72 characters including the line number and embedded blanks.
[LN] TITLE
[LN] Any alphanumeric information up to 72 characters.
II. System Units

UNITS One line identifying the units to be used in the program. This information is only used to insure proper unit identification on output (i.e., no conversions are made in the program).
[LN] UNITS
[LN] ISYSTM (IDUM1 IDUM2 IDUM3)
ISYSTM $=$ ENGL - for English units (L=inches, F=lbs.)
$=$ METR - for metric units or any other system
(IDUM1 IDUM2 IDUM3) = Alphanumeric information describing the system of units selected. (i.e., feet and kips, cm and grams, etc.)
III. Pile Descriptions

PILE Two to eleven lines that describe the pile geometry and properties.
[LN] PILE NI NDIAM LENGTH EPILE XGS
[LN] XDIAM(I) DIAM(I) MINER(I) (AREA(I)) (I = 1, NDIAM)
1st Group

| NI $=$ | Number of increments into which pile is divided |
| :--- | :--- |
| NDIAM $=$ | Number of segments of pile with different |
|  | diameters |
| LENGTH $=$ | Length of pile |
| EPILE $=$ | Modulus of elasticity |
| XGS | $=$ Depth below top of pile to ground surface |

2nd Group

| XDIAM | $=$ Depth below top of pile |
| :--- | :--- |
| DIAM | $=$ Diameter of pile at XDIAM |
| MINERT | $=$ Moment of inertia at XDIAM |
| (AREA) | $=$Cross-sectional area of pile ( $\mathrm{L}^{2}$ ) (If left blank,$\quad$computed assuming a pipe section) |

IV. Soil Description

SOIL Two to ten lines that describe soil system and its properties.
[LN] SOIL NL
[LN] LAYER(I) KSOIL(I) XTOP(I) XBOT(I) K(I) (AE(I) FR(I)) ( $\mathrm{I}=1$, NL)
1st Group
NL $\quad=$ Number of layers of soil.
2nd Group
LAYER(I) = Layer number
KSOIL(I) $\quad=$ Code to control the type of $p-y$ curves
$=1$ to have $p-y$ curves computed internally using Matlock's (1970) criteria for soft clay
$=2$ to have $\mathrm{p}-\mathrm{y}$ curves computed internally using Reese's and Welch's (1975) criteria for stiff clay below the water table
$=3$ to have p -y curves computed internally using Reese's and Welch's (1975) criteria for stiff clay above the water table
$=4$ to have p -y curves computed internally using Reese et al. (1974) criteria for sand
$=5$ to use linear interpolation between input $p-y$ curves
$=6$ to have $\mathrm{p}-\mathrm{y}$ curves computed internally using Sullivan et al. (1979) unified clay criteria
XTOP(I) $\quad=X$-coordinate of top of layer
XBOT(I) $\quad=X$-coordinate of bottom of layer
$K(I) \quad=$ Constant $\left(F / L^{3}\right)$ in equation $E_{S}=K x$. This is used to define initial soil moduli for the first iteration and to determine initial slope of $p-y$ curve where $K S O I L=2,4$, or 6
(AE(I)) = Factor "A" in uniform clay criteria
(FR(I))
= Factor " F " in uniform clay criteria. (Leave blank unless $\operatorname{KSOIL}(\mathrm{I})=6$ )


1st Group
NPY $\quad=$ Number of $\mathrm{p}-\mathrm{y}$ curves (maximum 30)
NPPY $\quad=$ Number of points on $p-y$ curves (maximum 30)
2nd Group
XPY(I) $\quad=X$-distance from top of pile to input p-y curve
3rd Group (Defines the $p-y$ curve at distance $=X P Y(I)$. )
$Y P(I, J) \quad=$ Deflection of a point on a $p-y$ curve
$\mathrm{PP}(\mathrm{I}, \mathrm{J}) \quad=$ Soil resistance corresponding to YP
VIII. Boundary Conditions at the Pile Head

BOUNDARY Specifies the boundary condition at the pile head
[LN] BOUNDARY KBC NRUN
[LN] KOPSUB(I) PTSUB(I) BC2SUB(I) PXSUB(I) ( $\mathrm{I}=1$, NRUN)

1st Group
KBC $\quad=$ Code to control boundary condition at top of pile
$=1$ for free head (user specified lateral load and moment)
$=2$ for specified lateral load and slope at pile head. (Slope is 0 for fixed-head pile)
$=3$ for a specified lateral load and rotational restraint at the pile head

NRUN
$=$ Number of sets of boundary conditions (load cases)

2nd Group
KOPSUB(I) $\quad=$ Pile head printout code
$=0$ if only the pile head deflection and slope, maximum bending moment, and maximum combined stress are to be printed for the associated loads
$=1$ if complete output is desired for the associated loads

PTSUB(I) $\quad=$ Lateral load at top of pile
BC2SUB(I) $\quad=$ Value of second boundary condition
$=$ Moment (if KBC = 1)
$=$ Slope (if KBC = 2)
$=$ Rotational stiffness (if KBC = 3)
PXSUB(I)
$=$ Axial load on pile (assumed to be uniform over whole length of pile)
IX. Distributed Lateral Load on Pile (Optional)

LOAD Describes a distributed lateral load applied to the pile.
[LN] LOAD NLD NW(J)
[LN] XW(J,I) WW(J,I)
( $\mathrm{I}=1, \mathrm{NW}$ ); ( $\mathrm{J}=1$, NRUN)
NLD $\quad=$ Load case number
NW $\quad=$ Number of points on plot of distributed lateral load on pile versus depth for specified NLD

XW(I) $\quad=\mathrm{X}$-coordinate where distributed loads are specified
WW(I) $\quad=$ Distributed lateral load
X. For Cyclic Load (Optional)

CYCLIC Specifies if the loading is cyclic or static.
[LN] CYCLIC KCYCL RCYCL
KCYCL
$=0$ for cyclic loading
$=1$ for static loading
RCYCL $\quad=$ Number of cycles of loading (need only for $p-y$ curves generated criteria for stiff clay above the water table)
XI. Control of output

OUTPUT Describes the amount of output to be printed.
[LN] OUTPUT KOUTPT INC KPYOP NNSUB
[LN] XNSUB (I) ... XNSUB(NNSUB)

| KOUTPT $=$ | 0 if data are to be printed only to depth where |
| ---: | :--- |
| moment first changes sign |  |
| $=$ | 1 if data are to be printed for full length of |
|  | pile |
| $=$ | 2 for extra output to help with debugging |
| $=$ | Increment used in printing output |
| $=$ | 1 to print values at every node |
| $=$ | 2 to print values at every second node |
| $=$ | 3 to print values at every third node, etc. |
|  | (up to NI +1 ) |
| $=$ | 0 if no p-y curves are to be generated and |
|  | printed for verification purposes |
| $=$ | 1 if p-y curves are to be generated and printed |
|  | for verification |
| $=$ | Number of depths for which internally generated |
| NNSUB $\quad$ |  |

> 2nd Group
> XNSUB(I) $\quad=\quad X$-coordinate at which internally generated $p-y$ curves are to be generated for printing solution convergence maximum deflections.
> [LN] CONTROL MAXIT YTOL EXDEFL case
> YTOL $\quad=$ Tolerance on solution convergence
> EXDEFL $\quad=$ Value of deflection of pile head that is considered grossly excessive and which stops the run. Default to pile diameter
XIII. Termination of Input Sequence

END Terminates the input sequence and initiates the analysis.
[LN] END

## Example Problems

6. Pile properties and the soil profile to be used in all four problems are shown in Figure C1.

Example problem 1
7. A free-head pile will be analyzed for lateral loads of $5,000,10,000$, 15,000 , and $20,000 \mathrm{lb}$. An axial load of $100,000 \mathrm{lb}$ will be used, and no moment will be applied at the pile head. The $\mathrm{p}-\mathrm{y}$ curves shown in Figure Cl will be used in this analysis.




[^10]

```
(p-y Data)
```

    NO. POINTS FOIR
    STRENGTH FARAMETERS
VE. LIEFTH
0
***** P-Y LIATA. *****
NCI. OF
P-Y CURVES
7
NO. POINTS IN
P-Y CURVES
B
$x$-GOGRD. TG
INFUT P-Y CIJRVE
0.600 E 02

IIEFLECTION
0.
0.200 E 00
0.400 E 00
0.300 E 00
$0.120 E 01$
0.600 E 01

X-COGRLI TO
INPUT P-Y CURVE
0.760 E 02

DEFLECTION
0.
$0.200 E 00$
0.400 E 00
$0.300 E 00$
0.120 E 01
0.800 E 01
$X$-CGORD. TG
INPIJT F-Y CURVE 0.920 O O2

DEFLECTION
0.
0.200 E 00
0.400 E 00
0.800 E 00
0.120 E 01
$0.800 E 01$
X-COORD. TO

SOIL FEEISTANCE
0.
$0.661 E$ O2
$0 . E 32 E 02$
$0.105 E 03$
0.120 E 03
0.

GOIL RESISTANCE
0.
0.79 EE 02
0.100 E 03
$0.127 E 03$
$0.145 E$ OS
0.150 E O2

SOIL RESISTANCE
0.
0.933 E 02
$0.117 E 03$
0.14 EE O
0.169 E 03
$0.340 E 02$

| INFIIT F-Y LURVE 0.10 E OS |  |  |
| :---: | :---: | :---: |
| DEFLECTION | EOIL RESISTANIE |  |
| 0. | 0. |  |
| $0.200 E 00$ | 0.107E 0\% |  |
| 0.400 EO | O.13EE 03 |  |
| 0.300800 | O.170E 0S |  |
| $0.120 E 01$ | $0.1 \%$ E OS |  |
| O. SOOE O1 | 0.610 E O2 |  |
| $x$ ECORFL TO INFIIT F-Y CURVE $0.140 E 03$ |  |  |
|  |  |  |
|  |  |  |
| LEFLEGTION | GOIL FESISTANIE |  |
| O. | O. |  |
| 0.200 E 00 | 0.134 E O3 |  |
| O. 400E 00 | 0.16\%E 03 |  |
| 0.800 E 00 | 0.213E 03 |  |
| O. 120 O O1 | 0.243E 05 |  |
| 0.600 E 01 | 0.123 E 03 |  |
|  INFIT F-Y GURVE 0.18 EE 03 |  |  |
|  |  |  |
|  |  |  |
| DEFLECTION | GOIL FESISTANEE |  |
| 0. | O. |  |
| 0.200 E 00 | 0.175E 03 |  |
| 0.400 E 00 | 0.221 E OS |  |
| 0.800 EO | 0.278509 |  |
| $0.120 E 01$ | 0.31 EE 05 |  |
| 0.800 E 01 | $0.264 E 03$ |  |
| $X-G O O R D . T O$ INFIIT F-Y EIIRVE 0.214 E 03 |  |  |
|  |  |  |
| DEFLEETION | GOIL RESISTANIE |  |
| 0. | O. |  |
| 0.200 E 00 | 0.19 EE 03 |  |
| 0.400 E 00 | $0.250 E 03$ |  |
| O. 8OOE 00 | 0.315E 03 |  |
| 0.120 E O1 | 0. SEOE 03 |  |
| 0.600 E 01 | 0.360103 |  |
| ***** GUJTPUT | İATA. ***** |  |
| DATA GUT | PUT F-Y | NO. LIEFTHS TO |
| OUTPUT INCRE | EMENT PRINTEIT | FRINT FOR |



| 2 | LIAL VE. DEFTH |
| :---: | :---: |
| EIOININARY | NO. FOINTS FOR |
| SET NCI. | IISTRIE. LATERAL LIALI VE. LIEFTH |
| 3 | 0 |
| bolindiary | NO. PGINTE FOR |
| SET NO. | [IISTRIB. LATERAL |
| 4 | 0 |




IINITS--ENITL

OUTFUTINFGRMATIGN

(Load Case 1)

| NO. OF ITERATIGNE | $=$ |
| :--- | :--- |
| MAXIMUM DEFLECTION ERROR | $=0.40 \% \mathrm{E}-03 \mathrm{IN}$ |

File loalinnti conditigin LATERAL LGAD AT FILE HEAI $=0.500 E$ O4 LES
$=0$.
$=0.100 E$ LES-IN
$=0 E S E$ AFFLIEL MIIMENT AT FILE HEAL axial laal at file heali IN

## GUITFITT VEFIFIGATION

THE MAXIMUM MOMENT IMEALANEE FGR ANY ELEMENT $=-0.29 E-02$ IN-LEG THE MAX. LATERAL FORIE IMEALANEE FOR ANY ELEMENT $=0.3 E-0 Z$ LEG


OUITFIUT SLIMMARY
PILE HEAL IEFLEETION $=0.4 .6 E$ OO IN
MAXIMUM BENDING MOIMENT $=0.475 E O 6$ IN-LES
MAXIMUM TOTAL STREGS



PILE LOADING EONLITION LATERAL LGAI AT PILE HEAD $=0.100 E$ OS LES APFLIEI MOMENT AT FILE HEAI $=0 . \quad$ LESG-IN AXIAL LGAII AT FILE HEAI $=0.100 E 06$ LES


## GUTPIIT VERIFICATIIIN

```
THE MAXIMLIM MOMENT IMEALANCE FGR ANY ELEMENT = -0. 5%4E-0Z IN-LES
THE MAX. LATERAL FGRCE IMEALANCE FOR ANY ELEMENT = 0.10GE-02 LEG
COMPUTEI LATERAL FORCE AT FILE HEAII =0.100OOE 0S LES
    CGMFIITEL MUMENT AT FILE HEALI
    COMFUTEI BLGFE AT FILE HEAII
THE GVERALL MOMMENT IMEALANCE
=0.102E-0Z IN-LES
THE OVERALL LATERAL FOFIEE IMEALANG:E
= -0.135E-0E LES
```

OUITFUIT EUMMARY

```
FILE HEAI [IEFLECTIGN = 0.11EE O1 IN
MAXIMLMM EENDING MIMENT =0.10EE 07 IN-LEE
MAXIMUM TOTAL STRESG = 0.146E OS LEE/IN**2
MAXIMUM SHEAF FOREE = 0.10SE OS LES
```



```
NO. OF ITERATIONS = 11
MAXIMUM IEFLECTION ERRGIR = 0.96SE-03 IN
```

PILE LDALING CGNLIITION LATERAL LGAI AT FILE HEAD APFLIEI MOMENT AT FILE HEALI AXIAL LGAL AT FILE HEAII
$=0.150 E O E$ LES
$=0$.
$=0.100 E O E$ LES IN


## GUTPPUT VERIFIEATION



OUTFUT SUMMARY
PILE HEAD DEFLEGTIGN $=0.226 E$ O1 IN
MAXIMUM EENDING MDMENT $=0.177 E$ OT IN-LES
MAXIMUM TOTAL STRESS $=0.227 E$ OS LES/IN* 2
MAXIMUM SHEAR FGRCE $=0.164 E$ OS LES


C26
（Load Case 4）
NO．OF ITERATIGNE $=0.25$
MAXIMUM DEFLECTION ERFIOR $=0.318 E-O S$ IN

FILE LOADING CONDITIUN
LATERAL LGAL AT PILE HEALI $=0.200 \mathrm{O}$ OS LES APPLIEL MOMENT AT FILE HEAI AXIAL LUAI AT PILE HEAII

```
=0.2OOE OS LES
O. LES-IN
=0.100E OG LES
```

| X IN | DEFLEC IN | MOMENT | TOTAL STRESE： ES／IN＊＊2 | LISTR． <br> LOAII | SOIL | FLEXIIRAL FIGIIITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IN | IN | ES－IN | LES／IN＊＊2 | LES／IN | LEG／IN＊＊2 | LEG－IN＊＊2 |
| ＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊れせ＊＊ | せれがせ＊＊＊＊ |
| 0 ． | $0.456 E 01$ | 0. | 0．273E 04 | O． | 0. | O．304E 11 |
| 12.00 | $0.427 E 01$ | O．270E O6 | $0.484 E 04$ | O． | 0. | 0.304 E 11 |
| 24.00 | $0.397 E 01$ | 0.539 E 06 | 0．690E 04 | O． | 0. | $0.304 E 11$ |
| 36.00 | $0.368 E 01$ | 0．80\％E 06 | 0.896 E 04 | 0. | 0. | 0.304 E 11 |
| 48.00 | $0.339 E$ 01 | 0．108E 07 | 0.110 E OS | 0. | 0. | 0.304 E 11 |
| 60.00 | $0.3100^{01}$ | $0.135 E 07$ | $0.131 E 05$ | 0. | 0.234 E 02 | O． 304 E 11 |
| 72.00 | 0.232 E 01 | 0．161E 07 | $0.151 E 05$ | 0. | 0.339 E O2 | 0．304E 11 |
| 84.00 | $0.255 E 01$ | 0.185 E 07 | 0.169 E O5 | 0. | 0．469E O2 | $0.304 E 11$ |
|  |  |  |  |  |  |  |
| 636.00 | 0．662E－02－0 | 0．254E 05 | 0．43EE． 04 | 0. | 0.990 OS | 0.212 E 11 |
| 648.00 | 0．695E－02－0 | 0．187E O5 | 0.431 E 04 | 0. | 0.990 E 0． | 0．212E 11 |
| 680.00 | $0.714 \mathrm{E}-02-0$ | 0.130 E 05 | $0.425 E 04$ | O． | $0.9 \% 0 E 03$ | $0.212 E 11$ |
| 872.00 | $0.725 E-02-0$ | $0.834 E 04$ | $0.420 E 04$ | 0. | 0．9\％OE OS | $0.212 E 11$ |
| 684.00 | 0．730E－02－0． | 0．470E 04 | 0.416 E 04 | 0. | 0．990E 03 | 0.212 EE 11 |
| 696.00 | 0．732E－02－0 | $0.209 E 04$ | O．413E 04 | 0. | O． 990 E 03 | $0.212 E 11$ |
| 708.00 | 0．73：3E－02－0 | 0.522 E 03 | $0.411 E 04$ | 0. | $0.990 E 03$ | $0.212 E 11$ |
| 720.00 | $0.733 E-020$ |  | $0.411 E 04$ | 0. | 0.99 OE OS | 0.212 E 11 |

OIITFUT VERIFICATIGN
THE MAXIMUM MOMENT IMEALANCE FOR ANY ELEMENT $=-0.23 \Xi E-01$ IN-LEG THE MAX. LATERAL FORIE IMEALANIE FGR ANY ELEMENT $=0.266 E-02$ LES


THE GVEFALL MOMENT IMEALANEE $=0.546 E-02$ IN-LES THE GVERALL LATERAL FOFLE IMEALANEE $=-0.480 E-0 \xi$ LEG

IUTFIIT ELIMMAFY


 so.

E UMMAFY TAELE


## Example problem 2

8. A free-head pile with no applied moment and a lateral load of $10,000 \mathrm{lb}$ will be analyzed. An axial load of $100,000 \mathrm{lb}$ will be applied at the pile head. p-y curves will be generated internally using the soft clay criteria for the soft clay, sand criteria for the sand, and unified clay criteria for the medium clay $(A=2.0$ and $F=0.7$ for the unified criteria). Loading will be assumed to be cyclic. Output will include points on the p-y curves at $x$ coordinates of $60,80,100,150,200,250,300$, and 500 in .
```
10 TITLE
20 EX. FRO. 2 FFOMM LUOUMENTATION GF GOM. FFOG. EOMEZ4 EY L.G. FEEEE, 19EO.
30 UNITS
40 ENGL
SO FILE 12O 2 720 29.E6 60
SO FILE 120 2
70 180 16 732
BO SIIL 3
#0 1 1 60 240 30
100 2 4 200 560 25
11036560 600 100 1.00.7
120 ETFENGTH 6
13060 3.5 0 .02
130 60 3.5 0.02 
150 240 0 30 .02
160 360 0 30 .02
170 360 70.01
180 30070.01
190 WEIGHT 6 (Unit Weight Profile - NGI)
200 60.02
210 240.02
220 240 .032
230 360.032 Where I=1,NGI
240 360 .026
250 800 .026
260 OHITFUTT 1 2 1 s (Output Control - KOUTPT,INC, KPYOP,NNSUB)
270 60 #0 100 150 200 250 300 500 (XNSUB(I) .... XNSUB (NNSUB)
2%O EOUNLIARY 1 1 (Boundary Condition at Pile Head - KBC,NRUN)
2%0 1 10000 0 1.ES (KOPSUB(I),PTSUB(I), BC2SUB(I),PXSUB(I), Where I = 1,NRUN)
300 EYELIC O O (Cyclic Load Indicator - KCYCL,RCYCL)
310 CONTROL 100.001 24 (Program Control - MAXIT,YTOL,EXDEFL)
320 END
(Pile Properties - NI,NDIAM,LENGTH,EPILE,XGS)
(XDIAM(I),DLAM(I),MINERT(I)
Where I = 1,NDIAM
    Where I = 1,NDIAM
    LAYER(I),KSOIL)I),XTOP)I), XBOT(I),K(I), (AE(I),FR(I))
    Where I = 1,NL
    (Soil Strength Profile - NSTR)
XSTR(I),Cl(I),PHI1(I),EE50(I)
                                Where I = 1,NSTR
    XGI(I),GAM1(I)
```

(Input Echo)
**** UNIT IIATA. \#\#\#\#*

EYSTEM OF UNITS
(LIF TG 16 CHAF.)
ENGL
***** PILE IIATA. ** * *

NO. INC:REMENTS
FILE IS LIVITEED
NG. SEGMENTS
LENGTH
DEPTH

120
WITH [IIFFERENT
OF
Monlulis of
EHARACTERISTIES
FILE
0.720 E 05 0.2\%OE OS
0.600 E 02

TOF IF EEGMENT
0.
O. 180E OS O. 160 OE

MOMENT GF INERTIA
0.105 E 04
0.732 E 03

CRUSE-SECT.
AFEA
$0.35 \%$ O2
$0.243 \mathrm{E} \quad 02$
***** SIIIL IIATA. *****

NUMEEE GF LAYERG 3

***** UNIT WEISHT IIATA. \#\#***

NO. FOINTE FOR FLOT
IIF EFF. LINIT WEIGHT
VE. DEFTH
6

EIEPTH BELOW TGF
EFFECTIVE
TO FOINT
$0.600 E 02$
UNIT WEIGHT
0.240 E 03
$0.200 \mathrm{E}-01$
0.240 E 03
$0.200 \mathrm{E}-01$
0. $320 \mathrm{E}-01$

| $0.360 E ~ 03$ | $0.320 E-01$ |
| :--- | :--- |
| $0.360 E ~ 03$ | $0.260 E-01$ |
| $0.300 E 03$ | $0.260 E-01$ |

***** FROFILE IIATA. *****

NO. PGINTS FOR
STRENGTH PARAMETEFE
VS. HEFTH
6

| LEPTH EELOW | UNLIRAINED EHEAR | ANGLE GF INTEFNAL | STRAIN AT 50\% |
| :---: | :---: | :---: | :---: |
| TGF GF FILE | ETRENGTH OF SOIL | FRICTION IN FALIIANE: | STRESE LEVEL |
| 0.600 E O2 | O. SEOE O1 | O. | 0.200E-01 |
| $0.240 E 03$ | $0.350 E 01$ | 0. | $0.200 E-01$ |
| 0.240 E 03 | 0. | O. ETAE OO | 0. 200E-01 |
| 0.360503 | 0. | $0.524 E 00$ | 0. $200 \mathrm{E}-01$ |
| $0.360 E 03$ | 0.700 E 01 | 0. | $0.100 \mathrm{E}-01$ |
| O.EOOE 03 | 0.700 E 01 | 0. | 0. 100E-01 |


| NO. OF F-Y CLIRVES |  |  |  |
| :---: | :---: | :---: | :---: |
| ***** OLITPUT [IATA. ***** |  |  |  |
| IIATA | OUITFUT | $F-Y$ | NO. IEFTHE TG |
| OUITFLIT | INCFEMENT | FRINTOUT | FRINT FGR |
| COLIE | CODE | COLE | F-Y CURVES |
| 1 | 20 | 1 | 3 |

DEFTH FOR
PRINTING
P-Y CUFVES
$0.600 E ~ O 2$
$0.800 E ~ 02$
$0.100 E ~ 03$
$0.150 E ~ 03$
$0.200 E ~ 03$
$0.250 E ~ 03$
$0.300 E ~ 03$
$0.500 E ~$
0.3
***** FILE HEAL (ECILINLIARY) IIATA.





|  |  | $\begin{array}{r} 0.244 \\ 0.267 \\ 0.600 \\ 5.733 \\ 10.867 \\ 16.000 \end{array}$ |  | $\begin{aligned} & 1339.206 \\ & 1396.323 \\ & 2234.117 \\ & 2234.117 \\ & 2234.117 \\ & 2234.117 \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { DEFTH } \\ & \text { IN } \\ & 440.00 \end{aligned}$ | $\begin{aligned} & \text { IIIAM } \\ & \text { IN } \\ & 16.000 \end{aligned}$ | $\begin{gathered} \mathrm{C} \\ \mathrm{LE}=/ \operatorname{IN*} 2 \\ 0.7 \mathrm{E} 01 \end{gathered}$ | EAVG <br> LES/IN**S <br> $0.4 E 01$ | GAMMA $\begin{array}{r} \text { LES/IN*K } \\ 0.3 E-01 \end{array}$ | $\begin{gathered} E E 0 \\ 0.100 E-01 \end{gathered}$ |
|  |  | $Y$ |  | $F$ |  |
|  |  | IN |  | LES/IN |  |
|  |  | 0. |  | O. |  |
|  |  | 0.013 |  | 220.142 |  |
|  |  | 0.027 |  | 277.362 |  |
|  |  | 0.040 |  | 317.500 |  |
|  |  | 0.053 |  | 349.454 |  |
|  |  | 0.067 |  | 376.43: |  |
|  |  | 0.080 |  | 400.025 |  |
|  |  | 0.093 |  | 421.117 |  |
|  |  | 0.107 |  | 440.285 |  |
|  |  | 0.120 |  | 457.914 |  |
|  |  | 0.13 c |  | 474.232 |  |
|  |  | 0.147 |  | 439.592 |  |
|  |  | 0.160 |  | 504.000 |  |
|  |  | 1.173 |  | 504.000 |  |
|  |  | 2.137 |  | 504.000 |  |
|  |  | 3.200 |  | 504.000 |  |
|  |  | 4.800 |  | 504.000 |  |




EX. PRG. 2 FFOM IMOGMENTATION OF COM. FFOU. COMEZ4 EY L.E. FEESE, 19 80.

LINITS--ENGL

> GUTPUT INFGRMATION


NO. OF ITERATIGNS $=14$
MAXIMUM DEFLECTIGN ERROR $=0.562 E-03 \mathrm{IN}$

PILE LOAIING CONDITION
LATERAL LGAI AT PILE HEAII $=0.100 E 05$ LEE
AFPLIEI MOMENT AT PILE HEAI $=0$. LES-IN
AXIAL LGAD AT PILE HEAD $=0.100 \mathrm{O} 06 \mathrm{LES}$


## OUITPUT VERIFIIGATION

```
THE MAXIMUM MOMENT IMEALANGE FGR ANY ELEMENT = 0.10GE-OI IN-LES
THE MAX. LATEFAL FOFCE IMBALANLE FGR ANY ELEMENT = 0.14:EEOZ LES
EOMFUTEI LATERAL FORCE AT FILE HEALI
    G:GMFUTEL MGMENT AT FILE HEAD
    COMPUTEI SLOPE AT FILE HEAII
THE GVERALL MOMMENT IMEALANCE = O. ZGEE-OQ IN-LES
THE OVEFALL LATEFAL FORCE IMEALANEE
=0.10000E OS LES
    =0.
        IN-LBE
    = -0.34314E-02
=-0.131E-0G LES
```


## GUTPEUT SIIMMARY

```
FILE HEAD DEFLECTION = 0.13SE OI IN
MAXIMIUM BENIIINOG MIMENT = 0.116E OT IN-LEE
MAXIMLMM TOTAL STRESS = 0.141E OS LES/IN**2
MAXIMIIM SHEAR FOREE =0.1OEE O5 LES
```

 80.

> GUMMARY TAELE


| LATERAL | ECIINLIAFY | AXIAL |  |  | MAX. | MAX. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LGALI | GONLITION | LGALI | YT | ET | MOMENT | STRESS |
| (LES) | EIC: | (LES) | (IN) | (IN/IN) | ( IN-LES) | (LEE/IN**2) |
| 0.100 E | 0. | O. 100 O O6 | 13 EE | . 54 SE-O | O. 116E O | 0.141 E O5 |





MICROCOPY RESOLUTION TEST CHART
national bureau of standards-1963-A

## Example problem 3

9. A fixed-head pile will be analyzed under a lateral load of $10,000 \mathrm{lb}$ and an axial load of $100,000 \mathrm{lb}$. p-y curves will be generated internally using the soft clay criteria for both clay layers and sand criteria for the sand layer. A p-y curve will be output at $x=500 \mathrm{in}$.
```
10 TITLE
20 EX. PRO. G FFOM IIOCIMENTATION GF IUM. FRG. GOMEZ4 EY L.E. FEESE, 1%GO.
SO UNITS
40 ENGL
SO FILE 120 2 720 2%.EG GO (Pile Properties - NI,NDIAM,LENGTH,EPILE,XGS)
60 O 16 1047 (XDIAM(I),DIAM(I),MINERT(I)
70 130 16 732
BO ETRENGTH &
90 60 3.5 0.0.02
100 240 3.5 0.0 .02
110 240 0.0 30. .02
120 360 0.0 %0. .02
130 360 7.0 0.0 .01
140 300 7.0 0.0 .01
15O WEIGHT 6
160 60.02
170 240.02
180 240 .032
190 360.032
200 360 .026
210 800 .026
220 SOIL S
230
240 2 4 240 360 25
250 3 1 360 800 100
260 ELINNLIARY 2 }
270 1 10000 0.0 1.E5
250 OUITFUT 1 2 1 1
290 E00
300 CYCLIC O O
310 END
```

```
(Input Echo)
```

***** UNIT LIATA.

EYSTEM OF LINITS (LIF TO 16 CHAR.)
ENGL
***** PILE IIATA. *****

| NO. | NC:REMENTS | NG. SEGMENTE | LENGTH | MOLHMUS GF | LIEPTH |
| :---: | :---: | :---: | :---: | :---: | :---: |
| FILE | IS LIVIIEEI | WITH DIFFERENT CHARAETEFISTIES | $\begin{array}{r} \text { OF } \\ \text { Fin } \end{array}$ | ELASTIEITY |  |
|  | 120 | 2 | O.72OE 03 | 0.290 E 08 | 600E 02 |


| OF | LII AMETEF | MOMENT Of | CROSG-EECT. |
| :---: | :---: | :---: | :---: |
| SEGMENT | OF FILE | INEFTIA | AREA |
| 0. | $0.160 E 02$ | O. 1 OEE 04 | 0.359E 02 |
| 0.180 O 0 | 0.160 E 02 | 0.732 E 03 | $0.243 E 02$ |

***** SUIL LIATA.

NUMEER GIF LAYERS
3

| LAYER NUMEEF | P-Y GURVE |  | TGF OF |  | EOT | TIUM | INITIAL SOIL |  |  | FACTGR"A" | FAC:TOR "F" |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | OF L | LAYER | MOLuL |  |  |  |  |
| 1 | 1 | 0.6 | OOE | 02 | 0.240E | OS | 0.300E | 02 | Q |  | 0. |
| 2 | 4 | 0.2 | 40E | 03 | 0. 360 E | az | 0.250 E |  | 0 |  | 0. |
| 3 | 2 | 0.3 | GOE | 03 | O. BOOE | 03 | 0.100E | 03 |  | OOE O1 | 0.700 E |

***** LINIT WEIGHT LIATA.

```
NO. FOINTS FOR FLOT
OF EFF. UNIT WEIGIHT
        VS. DEPTH
            6
```

DEFTH BELOW TOF TO PGINT $0.600 E 02$ $0.240 E 03$ $0.240 E 03$

EFFECTIVE UNIT WEIGHT $0.200 \mathrm{E}-01$ $0.200 \mathrm{E}-01$
0. 32OE-01

| Pile heaid | LATERAL LGAI AT | VALUE GF EECONA | AXIAL LGAL |
| :---: | :---: | :---: | :---: |
| FRINTGUT CODE 1 | TOF OF FILE 0.100 E OS | EOUNLIARY GONLITIGN 0. | $\begin{gathered} \text { ON FPILE } \\ 0.100 E 0 \Leftrightarrow \end{gathered}$ |
| ***** CYCLIC [ATA. \#**** |  |  |  |
| ```EYCLIE(O) OR STATIC(1) LOADING O``` | NO. CYCLES |  |  |
|  | OF LGALIING |  |  |
|  | $0.100 E 03$ |  |  |
| ***** PROGRRAM CONTRIL DATA. \#\#*** |  |  |  |
| MAX. NO. GF ITERATICINE | TOLERENCE ON P | FILE HEAI IEFLECTIUN |  |
|  | EGLUTION F | FLAG(STGFE FILIN) |  |
|  | CCINVERGENC:$0.100 \mathrm{E}-02$ |  |  |
| 100 |  | $0.240 E 02$ |  |
| \#**** LOAD IATA. ***** |  |  |  |
| EGUINDARY | NO. FOINTS FOR |  |  |
| EET NO. | DISTRIE. LATERAL |  |  |
| 1 | LOAL VS. [IEFTH <br> 0 |  |  |

GENERATED P-Y CURVES
THE NUMBER OF CLIRVES
$=1$
THE NUMBEF OF FOINTS ON EAGH CLIRUE
IEPTH
IN
440.00

| IIIAM IN | $\begin{gathered} \text { C: } \\ \text { LES/IN*2 } \end{gathered}$ |
| :---: | :---: |
| 16.000 | O.7E O1 |
|  | Y, IN |
|  | 0. |
|  | 0.003 |
|  | 0.100 |
|  | 0.200 |
|  | 0.300 |
|  | 0.400 |
|  | 0.500 |
|  | 0.600 |
|  | 0.700 |
|  | 0.800 |
|  | 0.900 |
|  | 1.000 |
|  | 1.100 |
|  | 1.200 |
|  | 3.200 |
|  | 6.000 |
|  | S.000 |


| GAMMA | ESO |
| :--- | :---: |
| LEE/IN\#\#3 |  |
| $0.3 E-01$ | $0.100 E-01$ |

F,LES/IN 0.
100.800
317.500
400.025
457.914
504.000
542.916
576.936
607.356
6.35 .000
660.427
684.033
706.114
726.894
725.760
725.760
725.760


EX．FRG．S FFGM LMGUMENTATIGN GF EGM．FFGU．GOMEZ4 EY L．I．REEEE， $1 \%$ 80.

UNITE－－ENIGL

pile loaning coniltion LATERAL LGAL AT FILE HEAI ELOFE AT FILE HEAL AXIAL LGAI AT FILE HEAI

```
= 0.100E OS LES
=0. IN/IN
=0.10OE OG LEE
```

| x | LIEFLEE： | ：MOMENT | TOTAL STRESG | LIETF． LGAII | GOIL MOLILUE | FLEXIIFAL FIGIGITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IN | IN | LES－IN | LES／IN＊＊2 | LES／IN | LES／IN＊＊？ | LES－IN＊＊2 |
| \＃\＃\＃＊＊＊＊ | ＊＊＊＊＊\＃\＃ | ＊＊＊＊＊＊＊＊＊＊ |  | ＊れそれそれそれ |  | ＊＊＊＊\＃\＃\＃れ |
| 0. | 0．269E | 00－0． 9 BCE OS | 0.10 EE OS | 0. | 0. | O．304E 11 |
| 12.00 | 0．2B7E | OO－0． 66.6 E O6 | 0.940 O 4 | 0. | 0. | O．3O4E 11 |
| 24.00 | $0.261 E$ | OO－0．74EE OE | O． 348 EE 04 | 0. | 0. | $0.304 E 11$ |
| 36.00 | 0．251E | 00－0．624E 06 | $0.755 E 04$ | 0. | 0. | O． 304 E 11 |
| 43.00 | 0.238 E | OO－0．EOSE Ot | 0.66 EE 04 | 0. | 0. | O．304E 11 |
| 60.00 | 0.223 E | 00－0．3E1E 06 | 0．570E 04 | 0. | 0．247E 0： | O．304E 11 |
| 72.00 | $0.206 E$ | 00－0．266E 06 | 0．451E 04 | 0. | 0．2\％\％0\％ | 0．304E 11 |
| 84.00 | $0.187 E$ | 00－0．159E OE | 0.400 E 04 | 0. | $0.35 \% \mathrm{E}$ 0． | O．3OAE 11 |

$636.000 .100 E-360$. $648.000 .100 \mathrm{E}-360$. $660.000 .100 \mathrm{E}-360$. $672.000 .100 E-3 B 0$. $684.000 .100 E-360$. $696.000 .100 E-360$. $703.000 .100 \mathrm{E}-360$. $720.000 .100 \mathrm{E}-360$.

| $0.411 E$ | 04 | 0. |
| :--- | :--- | :--- |
| $0.411 E$ | 04 | 0. |
| $0.411 E$ | 04 | 0. |
| $0.411 E$ | 04 | 0. |
| $0.411 E$ | 04 | 0. |
| $0.411 E$ | 04 | 0. |
| $0.411 E$ | 04 | 0. |
| $0.411 E ~ 04$ | 0. |  |

$0.156 \mathrm{E} \quad 120.212 \mathrm{E} 11$
$0.196 E 120.212 E 11$
0.156 E 120.212 E 11
0.196 E 120.212 E 11
$0.19 E \mathrm{E}$ 12 0．212E 11
$0.198 E 120.212 E 11$
$0.196 E 120.212 E 11$
0.196 E 12 $0.212 E 11$

## GUITPUT VERIFICATION

The maximum moment imealance for any element $=0.481 \mathrm{E}-02$ In-LES the max. lateral force imealande fur any element $=-0.743 \mathrm{E}-03 \mathrm{LE}$ l

```
COMFLITED LATERAL FORCE AT PILE HEADI = 0.10000E 0S LES
    COMPUTEI SLOPE AT FILE HEAD = 0. IN/IN
```

THE OVERALL MOMENT IMEALANCE $\quad=-0.179 E-0 Z$ IN-LES
THE OVERALL LATERAL FORCE IMEALANEE $\quad=-0.406 E-0 \%$ LES

## OUTPUT SUMMARY

```
FILE HEAD DEFLECTION = 0.269E 00 IN
MAXIMIMM EENDING MOMENT = -0.986E OS IN-LEG
MAXIMLMM TOTAL STRESS =0.10SE OS LES/IN**Z
MAXIMUM SHEAR FORCE =0.101E OS LES
```

EX. PRO. 3 FRGM DOCUMENTATION OF COM. PRO. COMBZ4 EY L.C. REESE, 19 30.

> SuMMARYTAELE
> *************************

| LATERAL | ECUUNDARY | AXIAL |  |  | MAX. | MAX. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LGAD | CONLIT I ION | LGAII | YT | ST | MOMENT | STRESS |
| (LES) | BC2 | (LES) | (IN) | (IN/IN) | (IN-LES) | (LES/IN**2) |
| O. 1 OOE | 0. | $0.100 E$ | 269E | . | -0.986E 0 | 0.10 EEO |



## Example problem 4

10. A pile with a rotational restraint of $M_{S} / S_{t}=1 \times 10^{6}$ in. - lb will be analyzed under a lateral load of $10,000 \mathrm{lb}$ and an axial load of $100,000 \mathrm{lb}$ p-y curves will be generated internally using soft clay criteria for the soft clay, sand criteria for sand, and the criteria for stiff clay below the water table for the medium clay. Coordinates of a $p$-y curve at $x=500$ in. will be output.
```
10 TITLE
ZO EX. FFOO. }4\mathrm{ FFGM [HIOMENTATIGN GF EGM. FFIG, GOMEZ4 FY L.E. FEESE, 1GGO.
SO UNITS
nO ENGL
SO FILE 120 = 720 2%.E& 6O (Pile Properties - NI,NDIAM, LENGTH,EPILE,XGS)
60 0 16 1047 (XDIAM(I),DIAM(I),MINERT(I)
70 1:00 16 732
80 EOIL 3
90 1 1 60 240 30
100 2 4 240 560 2%
110 3 2 360:000 100
120 OUITFUIT 1 2 1 1
130 500
140 EnINN:1
150 1 10000 1.EG 1.EE
160 LONTFOL 100 .001 24
170 STFENGTH &
150 60 3.5 0 .02
100 240 3. 50,02
200 240 0 30 .02
210 360 0 30 .02
220 300 7 0 .01
250 800 70.01
240 WEIGHT 6
250 60 .02
260 240 .02
270 240 .052
230 360 .032
2%0 360 .026
300 300 .026
310 CYOLIG O O
320 ENJ
```

(Input Echo)
***** UNIT LIATA.

SYSTEM GF UNITS
(UP TO 16 CHAR.)
ENGL
***** PILE LIATA. *****

| NO. | INC:REMENTS | NO. SEGMENTE | LENGTH | MOCHILUE OF | LIEF.TH |
| :---: | :---: | :---: | :---: | :---: | :---: |
| File | IS LIVIUEI | WITH IIIFFERENT | OIF | ELASTIEITY |  |
|  |  | CHARALTERISTIES | Fille |  |  |
|  | 120 | 2 | 2OE OS | $0.290 E O S$ | gOOE O |


| TOF OF | IIIAMETER | MOMENT OF | GRUGG-EEGT. |
| ---: | :---: | :---: | :---: |
| EEGMENT | OF FILE | INERTIA | AREA |
| 0. | $0.160 E 02$ | $0.105 E 04$ | $0.35 .9 E 02$ |
| $0.180 E 03$ | $0.160 E 02$ | $0.732 E 03$ | $0.243 E 02$ |


***** UNIT WEIGHT IIATA. *****

NO. FIINTS FOR FLOT
OF EFF. LINIT WEIGHT
VS. DEFTH
6

| IEPTH BELOW TOF | EFFECTIVE |
| :---: | :---: |
| TO FOINT | UNIT WEIGHT |
| $0.600 E 02$ | $0.200 E-01$ |
| $0.240 E ~ 03$ | $0.200 E-01$ |
| $0.240 E ~ 03$ | $0.320 E-01$ |


| $0.360 E 03$ | $0.320 E-01$ |
| :--- | :--- |
| $0.360 E ~ 03$ | $0.260 E-01$ |
| $0.300 E 03$ | $0.260 E-01$ |

**** PROFILE IIATA. ****

NO. FOINTS FOR
ETRENGTH FAFAMETEFE
VS. LEFTH
$\theta$

| IEFFTH EELOW | LINLIFAINELI EHEAF | ANGLE OF INTEFINAL | STFAIN AT 50\% |
| :---: | :---: | :---: | :---: |
| TOF OF FILE | ETRENGTH EF EOIL | FFIITION IN FALIIAN: | ETRES' LEVEL |
| O. AOOE O2 | 0.350 O O1 | 0. | O. OOE-01 |
| 0.240 E OS | O.350E O1 | 0. | O. $200 \mathrm{E}-01$ |
| 0.240E 03 | 0. | O. E4E OO | O. $200 \mathrm{E}-01$ |
| 0.360 E 03 | 0. | O. 5-2E OO | 6. $200 \mathrm{E}-0.1$ |
| $0.360 E 03$ | 0.700 E 01 | 0. | $0.100 \mathrm{E}-01$ |
| O. BOOE 0: | $0.700 E 01$ | 0. | O. 100E-01 |

**** F-Y [IATA. ****

NiO. DF
F-Y CLIFVES
0
***** IUITFUIT [IATA. **

| [IATA | OUITFUT | F-Y | NO. LIEFTHE TE |
| :---: | :---: | :---: | :---: |
| QuITFIIT | INEFEMENT | FFi I NTOIT | FFiINT FOR |
| EOLIE | COLIE | OOLE | F-Y Llifles |
| 1 | 20 | 1 | 1 |

LEFTH FIR
PRINT ING:
F-Y GIRVE:
0. SOOE OS
***** Pile heal (EDINNLIGFY) liata.

| EGIINILAFTY | Ni. OF EET'S |
| :---: | :---: |
| GONLITIGN | GF - NLAFIY |
| coue | GONLITIDNE |
| 3 | 1 |



GENERATED F-Y CLIRVES
THE NUMEER OF CIIRVES
THE NUMBER OF PGINTE ON EACH CILRVE

| $\begin{aligned} & \text { IEFTH } \\ & \text { IN } \\ & 440.00 \end{aligned}$ | $\begin{aligned} & \text { IIIAM } \\ & \text { IN } \\ & 16.000 \end{aligned}$ | $\begin{gathered} C \\ \text { LES/IN**2 } \\ 0 \text { TF } 01 \end{gathered}$ | CAVE LES/IN**2 $0.4 E 01$ | GAMMA <br> LEE/IN\#\#3 <br> 0. 3E-O1 | $\begin{aligned} & E S O \\ & 0.100 E-01 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A S=0.60$ | $A C=0.30 \quad Y, I N$ |  |  | F, LEE/ |  |
|  | 0. |  |  | 0. |  |
|  | 0.020 |  |  | 94.272 |  |
|  | 0.035 |  |  | 172.416 |  |
|  | 0.059 |  |  | 235.477 |  |
|  | 0.079 |  |  | 284.574 |  |
|  | $0.0 \%$ |  |  | 320.726 |  |
|  | 0.118 |  |  | 345.890 |  |
|  | 0.139 |  |  | 360.996 |  |
|  | 0.157 |  |  | 366.079 |  |
|  | 0.177 |  |  | 36\%.600 |  |
|  | 0.197 |  |  | 368.079 |  |
|  | 0.216 |  |  | 360.996 |  |
|  | 0.236 |  |  | 345.890 |  |
|  | 0.394 |  |  | 242.801 |  |
|  | 0.551 |  |  | 135. 357 |  |
|  | 0.708 |  |  | 36.812 |  |
|  | 7.872 |  |  | 36.812 |  |



$s 0$.

LINITS--ENGL

> GUTFUT INFGFMATIGN


NO. OF ITERATIUNS $=\quad 14$
MAXIMUM DEFLECTION ERFOR $=0.56 E E-03 \mathrm{IN}$

FILE LOADING EONLITIDN LATEFAL LGAL AT FILE HEAI $=0.100 E 05$ LES
$=0.100 E 07$ LES-IN
$=0.100 E 06$ LES
FGITATIGNAL RESTRAINT

| $x$ | IIEFLEC: | MAMENT |  | TITAL STRESS |  | LIISTR. LGALI | SOIL Modulus | FLEXII FIGII |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IN | IN | LES-IN |  | LES/IN\#\#2 |  | LEG/IN | LES/IN**2 | LES-IN**2 |  |
| *******.****************** |  |  |  |  |  | ********* | ********* | ******** |  |
| 0. | 0.135 E 01 | -0. ES7E | 04 | $0.121 E$ | $04$ | O. | 0. | $0.304 E$ | 11 |
| 12.00 | $0.125 E 01$ | 0.122E | 06 | 0. 208 E | 04 | 0. | 0. | 0.304 E | 11 |
| 24.00 | $0.115 E 01$ | 0.252E | 06 | $0.307 E$ | 04 | 0. | 0. | O. 304E | 1 |
| 36.00 | $0.105 E 01$ | 0. 382E | 06 | 0.406 E | 04 | 0. | 0. | $0.304 E$ | 11 |
| 45.00 | O.950E 00 | 0.511 E | 06 | O. 505E | 04 | 0. | 0. | 0. 304E | 1 |
| 80.00 | $0.356 E 00$ | $0.641 E$ | 06 | $0.604 E$ | 04 | O. | 0.100 E 03 | $0.304 E$ | 1 |
| 72.00 | $0.765 E 00$ | 0.760 E | 06 | $0.696 E$ | 04 | 0. | 0.124 E OS | 0. 304E | 11 |
| 34.00 | 0.677E OO | 0. $866 E$ | 06 | 0.776 E | 04 | 0. | 0.152 E 03 | O. 304E |  |
| $1$ |  |  |  |  |  |  |  | $1$ |  |
| -36.00 0.744E-05 0.105E 04 0.200E 04 0. O.522E 04 0.212E 11 |  |  |  |  |  |  |  |  |  |
| 64E.00-0.147E-04 |  | 0.817 E | 03 | 0.199E | 04 | 0. | 0.522 E 04 | 0. 212 E |  |
| 660.00-0.312E-04 |  | 0.596E | 03 | 0.199 E | 04 | 0. | 0.522 E 04 | $0.212 E$ |  |
| 672.00-0.438E-04 |  | 0.398E | 03 | $0.19 \% \mathrm{E}$ | 04 | 0. | O. 522 E 24 | $0.212 E$ |  |
| $684.00-0.536 E-04$ |  | 0.232E | 03 | 0.199E | 04 | 0. | $0.522 E 04$ | 0. 212 E |  |
| 696.00-0.618E-04 |  | 0.107 E | 03 | 0.199E | 04 | 0. | 0. 522 E O4 | $0.212 E$ |  |
| $708.00-0.673 \mathrm{E}-04$ |  | 0.273 E | 02 | 0.198 E | 04 | 0. | 0.522 E 04 | $0.212 E$ |  |
|  |  |  |  | 0.198 E |  | ). | O.522E 04 | $0.212 E$ |  |

GUTFITT VEFIFII:ATIGN
IH: MAXIMUM MOMENT IMEALANEE FOK ANY ELEMENT $=0.104 E-01$ IN-LEE
THE MAX. LATEFAL FORLE IMEALANEE FOF ANY ELEMENT $=-0.145 E-02$ LEE
 $\because$

GOMFIITEI SLOFE AT FILE HEALI
$=-0.5710 \mathrm{E}-02$
rhe inefall mioment imgalanie
$=-0.324 E-02$ IN-LES
THE IVEFALL LATEFAL FGFIE IMEALANEE $=-0.1$ EE-OE LES

IIITFIIT EIIMMAFY

PILE HEAI IIEFLEETIGN = 0.135E OI IN
MAXIMUM EENIING MOMENT $=0.115 E$ O7 IN-LES
MAXIMUM TOTAL STRESS $=0.140 E$ OS LES/IN*W 2 MAXIMLM SHEAF FORLE $=0.10 E E$ OS LEE

EX. FRO. 4 FROM [IOLUMENTATIGN OF GOM. FFIO. GOME 24 BY L. G. REESE, 19 80.
EUMMAFY TAELE

| LATEFAL | EgIINDIAFY | AXIAL |  |  | MAX. |  | MAX. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LOAD | GONDITION | LOALI | $Y T$ | ET | MIMENT |  | ETFES\% |
| (LES) | EC: | (LES) | (IN) | (IN/IN) | (IN-LES) |  | E $=$ / IN**2) |
| 0.100 E | 0.100 E 07 | 100 E | 1 SE | . 637 E | 0.115 E |  |  |



## APPENDIX D: ADDITIONAL EXAMPLE PROBLEMS

## Example 1

1. This example is provided to illustrate program sequence and also for comparison to the problem analyzed earlier by nondimensional methods in Appendix A. Pile properties and soil description are shown in Figure D1. Prompts, data and output echoes, and graphics are presented as they would appear at the user's terminal. Input is from a data file, and p-y curves will be generated for verification at $x$ coordinates of $0,16,32,48,80,128,154$, 240,480 , and 720 in.

```
10 TITLE
2O GOMPARIGON GOLIITION FOF EXAMFLE SOLVELI BY NGN-[I]MENEIGNAL METHIIL
3O INNITS
4 0 ~ E N G L
SO FILE 72 1 7%O 2%.EE O (Pile Properties - NI,NDIAM,LENGTH,EPILE,XGS)
60 0 16 10E2.7% (XDIAM(I),DIAM(I),MINERT(I), Where I=1,NDIAM)
70 EGIL 1 (Soil Description - NL)
80 1 1 0 720 25
FO WEIGHT 2
100 0 .0174
110 720.0174
120 ETRENGTH ב
130 0 3.472 0 .01
140 720 3.4720.01
150 OLITFUIT 1 2 1 10 (Output Control - KOUTPT,INC,KPYOP,NNSUB)
160 0 16 32 4E EO 12G 154 240 400 720 (XNSUB(I) ... XNSUB(NNSUB)
170 EGllN 1 1 (Boundary Conditions at Pile Head - KBC,NRUN)
180 1 32000 -E27130 O (KOPSUB(I),PTSUB(I),BC2SUB(I),PXSUB(I), Where I = 1,NRUN)
1%O CYELIG O O (Cyclic Load Indicator - KCYCL,RCYCL)
200 CONTROLL 100 .001 40 (Program Control - MAXIT,YTOL,EXDEFL)
210 END
```

```
IS INFIIT FFOM TERMINAL OF A FILE
ENTEF: T OF F
=F
ENTER LIATA FILE NAME
=ELIGOMNLI
```

GOMFAKISON GOLUTION FGR EXAMFLE GILVELI EYY NGN-LIMENEIGNAL METHOL INFUT GOMFLETE. [UI YOUl WANT INFITT LIATA ECHIFFFINTEL TG YEIIF TEFIMINAL, A FILE, EITH, IF NEITHEF? (ENTER $T, F, E$, DR N) = F
ENTEF NAME FOR INFIT EGHOFRINT FILE $=$ I NFUIT

THIE FILE ALFEEALIY EXIETE: INFUTT ENTEF ANOTHEF NAME= INEX
***** UNIT LIATA.

EYSTEM GF INITE
(IIF TG 16 EHAR.)
ENGL
***** Pile LIATA.


NUMEEF OF LAYERE 1

| LAYEF | F－Y Clifive | TOF OF | EOTTOM | INITIAL EIIL | FACTOR | FALTOL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMEEF | CONTFOGL GIE | LAYEF | Gif LAyEf | MOLULI EONFT． | ＂A＂ | ＂F＂ |
| 1 | 10. |  | 720E OS | O．ご心E Oこ O． |  | a． |


| NG．FOINTS FGF FLGT |  |
| :---: | :---: |
| GF EFF．IINIT WEIGHT |  |
| VE．LIEFTH |  |
| 2 |  |
| LEEFTH EELOW TGF． | EFFEETIVE |
| TO FGINT | INIT WEIEHT |
| 0. | $0.174 \mathrm{E}-01$ |
| O．720E 0： | $0.174 \mathrm{E}-01$ |

＊＊＊＊FROIFILE LIATA．＊れれま

NO．FOLNTE FOR：
ETFENGTH FAFAMETEFE
VE．LEFTH
2

| DEFTH EELOW | IINLIFAINEL SHEAF | ANGLE GF INTEFNAL | ETRAIN AT EO\％ |
| :---: | :---: | :---: | :---: |
| TOF DF FILE | STEENGTH IFF EOIL | FFIETIUN IN FAALIANS | STFESS LEVEL |
| 0. | 0.347 E 01 | O． | $0.100 E-01$ |
| 0.720 E 03 | O． 347 E O1 | 0. | O．100E－01 |


0

| LIATA | EUITFIT | $F-Y$ | NO．LIEFTHE TO |
| :---: | :---: | :---: | :---: |
| GUTPIT | INIGEMENT | FFiINTIUIT | FRINT FOR |
| COLE | COLE | COLDE | F－Y EliRVES |
| 1 | 2 | 1 | 10 |

```
LIEPTH FGR
FRINTING：
F－Y CINFVES
0.
```



WILL OUITFUT GO TO THE TEFMINAL, FILE OF EUITH? ENTEF $T, F$, GR $E$ $=\mathrm{B}$

ENTER NAME FGF GUITFIIT FILE =GUTEX
( $P-Y$ curves generated for verification)

GENERATED F-Y CURVES

THE NLIMEEFI OF CURVES
THE NUMBER GF PUINTS IN EAIG CURVE

| IIEPTH | LIIAM | C |
| :---: | :---: | :---: |
| IN | IN | LES/IN**Z |
| 0. | 16.000 | O.3E O1 |
|  |  | $Y, I N$ |
|  |  | 0. |
|  |  | 0.003 |
|  |  | 0.100 |
|  |  | 0.200 |
|  |  | 0.300 |
|  |  | 0.400 |
|  |  | 0.500 |
|  |  | 0.600 |
|  |  | 0.700 |
|  |  | 0.800 |
|  |  | 0.900 |
|  |  | 1.000 |
|  |  | 1.100 |
|  |  | 1.200 |
|  |  | 3.200 |
|  |  | 6.000 |
|  |  | E.000 |

LEFTH
IN
$1 \in .00$

| IIAM |  |
| :---: | ---: |
| IN | LEG/INHZ |
| IG.000 | $0.3 E 01$ |

$=10$
$=17$

| EAMMA | EEO |
| :--- | :---: |
| LEG/IN**S |  |
| $0.2 E-01$ | $0.100 E-01$ |

F,LEG/IN
0.
16.666
$52.4 \% 3$
66.137
75.709
6.3 .38
89.762
95.367
100.416

104 . 987
109.191
113.096
116.744
120.130
6.9 .96
0.000
0.
$\begin{array}{cc}\text { GAMMA } & E 50 \\ 0 . Z / 1 N * * 3 & 0.100 E-01\end{array}$
F'LES:IN
0.
$19.83 \%$
62.645
76.929
90.850
9.443
107.122
113.334
119.336
125.291
130.307
134.965
0.
0.003
0.100
0.200
0.000
0.400
0.000
0.600
0.700
0.800
0.000
1.000
1.100
1.200
3.200
6.000
8.000

| F，LESGIN |
| :---: |
| G。 |
| －2． 781 |
| 103 － 2 |
| 130.01 |
| 14E． 217 |
| 16三． 604 |
| 176．5心0 |
| 1：7， |
| 197．516 |
| 20太，「心大 |
| 214．775 |
| 22こ．45： |
| $22 \%$－\％ |
| 23600 |
| 185．－7 |
| 114.113 |
| 114．113 |

LIEFTH
IN
12.00
LIAM
IN
16.000
$C$
LES／IN＊＊2
$O . S E O 1$

0.700
0.800
0.000
1.000
1.100
1.200
3.200
6.000
3.000
LIEPTH
IN
240.00

| LIAM | $E$ |
| :---: | :---: |
| IN | LES/IN*Z |
| 16.000 | $0.3 E O 1$ |


| EAMMA | ESO |
| :---: | :---: |
| LES/IN**3 |  |
| O. 2E-01 | 0.100E-01 |
|  | $\begin{aligned} & \text { F', LES/IN } \\ & \text { O. } \end{aligned}$ |
|  | 4\%.997 |
|  | 157.430 |
|  | 195.412 |
|  | 227.126 |
|  | 249.984 |
|  | $26 \% .287$ |
|  | 286.160 |
|  | 301.249 |
|  | 314.960 |
|  | 327.572 |
|  | 359.230 |
|  | 350.232 |
|  | 360.509 |
|  | 359.977 |
|  | 359.977 |
|  | 359.977 |

DEFTH
IN
480.00
IIIAM
IN
16.000
C
LES/IN**Z
$0.3 E 01$
Y, IN
0.003
0.100
0.200
0.300
0.400
0.500
0.600
0.700
0.300
0.900
1.000
1.100
1.200
3.200
6.000
8.000

| GAMMA <br> LESTIN**: | ESO |
| :---: | :---: |
| O. $2 \mathrm{E}-01$ | 0. 100E-01 |
|  | $\begin{gathered} \text { F, LES/IN } \\ 0 . \\ 4 \% .997 \end{gathered}$ |
|  | 157.480 |
|  | 195.412 |
|  | 227.126 |
|  | 249.984 |
|  | 269.267 |
|  | 266.160 |
|  | 301. $24 \%$ |
|  | 314.760 |
|  | 327.572 |
|  | 339.280 |
|  | 350.232 |
|  | 360.539 |
|  | 359.977 |
|  | 359.977 |
|  | 359.977 |

287.33
300.412
312.441
323.609
334.05
343. 885
333.437

31\%.55\%
319.5 .5

F', LES/IN
0.
$45 \cdot 97$
198.412
227.126
269.287
286.160
314.960
327.572
350.232
360.539
359.977
35.977

| $\begin{aligned} & \text { LIEFTH } \\ & \text { IN } \\ & 720.00 \end{aligned}$ | IIIAM <br> IN <br> 16.000 | $\begin{gathered} \mathrm{C} \\ \text { LES/INKた } \end{gathered}$ | БAMMA LEG/INk* O. ZE - 11 | $E G 0$ $0.100 E-01$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Y, IN |  | $F$, LES/IN |
|  |  | 0. |  | 0. |
|  |  | 0.003 |  | $4 \% 7$ |
|  |  | 0.100 |  | 157.480 |
|  |  | 0.200 |  | 1\%\%.412 |
|  |  | 0.300 |  | 227.126 |
|  |  | 0.400 |  | 24\%.9\%4 |
|  |  | 0.500 |  | 269.267 |
|  |  | 0.600 |  | 286.160 |
|  |  | 0.700 |  | $301.24 \%$ |
|  |  | 0.800 |  | 314.960 |
|  |  | 0.900 |  | 327.572 |
|  |  | 1.000 |  | $3 \% .280$ |
|  |  | 1.100 |  | 350.232 |
|  |  | 1.200 |  | 560.50 |
|  |  | 3.200 |  | 359.977 |
|  |  | 6.000 |  | 35\%.977 |
|  |  | 8.000 |  | \%5.977 |

EOMFARISON EGLUTIGN FGR EXAMFLE GOLVEI EY NON-IIMENSIGNAL METHOL LIO YOU WANT TO FLOT INFUT DATA: (Y IR N) $=\mathrm{V}$



D14

- 11 TFF T I NFGRMATI日N


File lóaling goniition LATERAL LGALI AT FILE HEAII AFFLIEI MOMENT AT FILE HEAE AXIAL LGAI AT FILE HEAL

$$
\begin{aligned}
& =0.320 E \text { OE LES } \\
& =-0.327 E \text { OG LEE-IN } \\
& =0 .
\end{aligned}
$$




## OUTFIIT VERIFICATION

THE MAXIMUM MOMENT IMEALANEE FOR ANY ELEMENT = 0.ES2E-02 IN-LEE THE MAX. LATEFAL FOROE IMEALANGE FOR ANY ELEMENT $=-0.652-0 S$ LES

```
COMFITEI LATEFAL FORLE AT PILE HEAI = 0.32OOOE OS LES
    COMFIITED MQMENT AT FILE HEAII = -0.32713E OE IN-LES
    COMPUTEI ELOFE AT FILE HEAI = -0.116EOE-01
THE GVEFALL MGMENT IMEALANEE THE GVERALL LATERAL FORCE IMBALANEE
\[
\begin{aligned}
& =0.9 \mathrm{BE}-0 \mathrm{IN} \text { INEG } \\
& =-0.26 E-09 \text { LES }
\end{aligned}
\]
```


## IIJTFUT SIIMMAFY

FILE HEAI DEFLEGTION = $0.19 E E 01 \mathrm{IN}$ MAXIMUM EENLING MOMENT $=0.200 E 07$ IN-LES MAXIMUM TOTAL ETREES $=0.14 E E$ OE LEE/IN*Z MAXIMLM SHEAR FOREE $=0.320 E$ OS LES

S -1 MAFY TAELE



GOMFARISON GOLUTION FGR EXAMFLE GOLVEI BY NON-IIMENEIGNAL METHOII LIG YOUI WANT TO FLGT OHITFUIT: (Y OR N) $=Y$


D18

## Example 2

2. This example is taken from the example design of a single-pile dolphin at Columbia Lock and Dam on the Ouachita River presented earlier in Appendix B. The analysis presented here is for one particular load case for a single-pile dolphin as shown in Figure D2. Pile properties and soil stratification are shown in Figure D3.


Figure D2. Example design problem; single-pile mooring dolphin
370 END

```
\(\qquad\)
```

010 TITLE

```
010 TITLE
020 COLUMBIA LOCK & DAM - SINGLE PILE DOLPHIN
020 COLUMBIA LOCK & DAM - SINGLE PILE DOLPHIN
0 3 0 ~ U N I T S
0 3 0 ~ U N I T S
040 ENGL
040 ENGL
050 PILE 100 3 1236 29.E6 516 (PILE PROPERTIES-NI,NDIAM,LENGTH,EPILE,XGS)
050 PILE 100 3 1236 29.E6 516 (PILE PROPERTIES-NI,NDIAM,LENGTH,EPILE,XGS)
070 0 48 31077
070 0 48 31077
080 360 48 59287
080 360 48 59287
090 9244831077
090 9244831077
100 SOIL 2 (SOIL DESCRIPTION-NL)
100 SOIL 2 (SOIL DESCRIPTION-NL)
120115516 696 25 (LAYER(I),KSOIL(I),XTOP(I),XBOT (I) ,K(I)
120115516 696 25 (LAYER(I),KSOIL(I),XTOP(I),XBOT (I) ,K(I)
13024696124040} where I=1,NL)
13024696124040} where I=1,NL)
140 WEIGHT }
140 WEIGHT }
160516.0304
160516.0304
170696.0304
170696.0304
180 696 . 0333
180 696 . 0333
190 1240 . 0333)
190 1240 . 0333)
200 STRENGTH 4 (SOIL STRENGTH PROFILE-NSTR
200 STRENGTH 4 (SOIL STRENGTH PROFILE-NSTR
220516 2.778 0 . 02
220516 2.778 0 . 02
230696 2.778 0 . 02
230696 2.778 0 . 02
240 696 0 30 . 01
240 696 0 30 . 01
250 1240 0 30 . 01
250 1240 0 30 . 01
260 OUTPUT 12 1 10 (OUTPUT CONTROL-KOUTPT,INC,KPYOP,NNSUB)
260 OUTPUT 12 1 10 (OUTPUT CONTROL-KOUTPT,INC,KPYOP,NNSUB)
280 516 540 564 588 612 636 695 708 1116 1236 (XNSUB(I)....XNSUB(NNSUB))
280 516 540 564 588 612 636 695 708 1116 1236 (XNSUB(I)....XNSUB(NNSUB))
290 BOUN 1 1 (BOUNDARY CONDITION AT PILEHEAD-KBC,NRUN)
290 BOUN 1 1 (BOUNDARY CONDITION AT PILEHEAD-KBC,NRUN)
3101 1340000 0 (KOPSUB(I),PTSUB(I),BC2SUB(I),PXSUB(I),
3101 1340000 0 (KOPSUB(I),PTSUB(I),BC2SUB(I),PXSUB(I),
                where I=1,NRUN)
                where I=1,NRUN)
330 CYCLIC O O
330 CYCLIC O O
(CYCLIC LOAD INDICATOR-KCYCL,RCYCL)
(CYCLIC LOAD INDICATOR-KCYCL,RCYCL)
350 CONTROL 100 .001 100
350 CONTROL 100 .001 100
                                XDIAN(I),DIAM(I),MINERT(I)
                                XDIAN(I),DIAM(I),MINERT(I)
                                where I=1,NDIAM
                                where I=1,NDIAM
(UNIT WEIGHT PROFILE-NGI)
(UNIT WEIGHT PROFILE-NGI)
(PROGRAM CONTROL-MAXIT,YTOL,EXDEFL)
```

(PROGRAM CONTROL-MAXIT,YTOL,EXDEFL)

```

\section*{(Input Echo for Mooring Dolphin Analysis)}
```

        ##*** INNIT LIATA
    SYETEM GF INITE
    (IIF TG 1今 DHAF.)
    ENBL
**** FILE [IATA. *****

```
\begin{tabular}{|c|c|c|c|c|c|}
\hline NO. & INCFEMENTS & NG. SEGMENTS & LENGTH & MOLMILUS OF & [EEFTH \\
\hline FILE & IE LIVILEE & WITH LIIFFEFENT & OF & ELASTIEITY & \\
\hline & 100 & CHAFAGTEFIBTIGS & FILE & 0.20050 & 516 E \\
\hline
\end{tabular}
\begin{tabular}{|c|c|}
\hline TGiF IF & II \\
\hline SEGMENT & OF FiILE \\
\hline , & 0.480 E O \\
\hline O. 360E 03 & O. 4EOE \\
\hline .92E 0z & O. 4EOE \\
\hline
\end{tabular}
MOMENT OF
INEFTIA
\(0.311 E\) OS
\(0.5 S E\) OS
\(0.311 E\) OS
\begin{tabular}{|c|}
\hline CROGGEEGT \\
\hline 0.111 E 03 \\
\hline 0.21\%E 03 \\
\hline O.111E 03 \\
\hline
\end{tabular}


NLIMEER OF LAYERE
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline LAYER & F-Y İRive & TOF OF & EOTTIM & INITIAL EOIL & FACTGR & FAC:TEF \\
\hline NuMEEF & CONTFOLL COLE & LAYER & If LAYEF: & MOGLLI CONET. & "A" & "F" \\
\hline
\end{tabular}
\begin{tabular}{llllllll}
1 & 1 & \(0.516 E\) & 03 & \(0.69 E E\) & 03 & \(0.250 E\) & 02 \\
2 & 4 & \(0.65 E\) & 03 & \(0.124 E\) & 04 & \(0.400 E\) & 02 \\
0. & 0. & 0.
\end{tabular}
***** UNIT UEIGHT LIATA. *****

NO. PGINTE FOR FLOT
OF EFF. UNIT WEIGHT
VE. LEFTH
4

DEPTH EELOW TOF TG FIINT

EFFECTIVE UNIT WEIGHT
\begin{tabular}{|c|c|}
\hline \(0.516 E 08\) & 0. 304E-01 \\
\hline \(0.696 \mathrm{E} ~ 09\) & O. 304E-01 \\
\hline 0.69EE OE & 0.35E-01 \\
\hline O. 124E 04 & O. 35E-01 \\
\hline
\end{tabular}


LIEFTH FGIR
FRINTING;
F-Y CURYES
\(0.516 E 03\)
0.540 E 0.3
\(0.564 E 03\)
0. sese 03
\(0.412 E 05\)
\(0.636 E 03\)
\(0.6 \% \mathrm{EE} 03\)
0.708 E 03
0.112 E 04
\(0.124 E 04\)
```

    PILE HEALI (ENIIINLIGRIY) IIATA.
    ```

EGIINLIAF:Y CONLITION GODE
1

FILE HEAI
FRINTGIT GOLE 1
```

NO. OF EETS GF EIUINLIAFY CONLITIGNS
1

```
LATEFAL LGAII AT TOF OF FILE 0.134 E 06

\section*{VALIIE GF EEGGNL EGUNLIAFY EONNLITIGN}
``` 0.
\begin{tabular}{|c|c|}
\hline EYCLIC(O) & NO. EYELES \\
\hline OF STATIC(1) & GF LOALINGO \\
\hline LOALING: & \\
\hline 0 & O. 100 E OS \\
\hline
\end{tabular}
```

MAX. NOI. IF ITERATIONE

100
TILERENE: ON GOLUTION C:INVERGENCE $0.100 \mathrm{E}-02$
***** LOIAI LIATA. *
EGOINDIAFiY EET NO.

1

NG. FGINTS FGR
UISTFIE. LATEFIAL LCIAL VE. IIEFTH 0

## FILE HEAII IIEFLECTIGN FLAG (ETOFS RUN)

$0.100 E$ OS

## (P-Y curves for Mooring Dolphin Analysis)

## GENERATED F-Y GURVES

THE NUMEER GF EURVES
THE NUMEER OF FGINTS GN EACH GURVE

| LIEF•TH IN 0. | $\begin{aligned} & \text { IIIAM } \\ & \text { IN } \\ & 48.000 \end{aligned}$ | $\begin{gathered} \mathrm{C} E / \text { IN**2 } \\ 0.3 E 01 \end{gathered}$ | GAMMA <br> LES/INk*S <br> $0.3 E-01$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Y, IN |  | F, LE: / IN |
|  |  | 0. |  | O. |
|  |  | 0.019 |  | 40.005 |
|  |  | 0.600 |  | 126.002 |
|  |  | 1.200 |  | 15\%.753 |
|  |  | 1.600 |  | 181.727 |
|  |  | 2.400 |  | 200.016 |
|  |  | 3.000 |  | 215.4\%1 |
|  |  | 3.600 |  | 226.961 |
|  |  | 4.200 |  | 241.054 |
|  |  | 4.800 |  | 252.004 |
|  |  | 5.400 |  | $262.0 \% 5$ |
|  |  | 6.000 |  | 271.463 |
|  |  | 6.600 |  | 260.226 |
|  |  | 7.200 |  | 206. 473 |
|  |  | 19.200 |  | 168.013 |
|  |  | 36.000 |  | 0.000 |
|  |  | 4:3.000 |  | 0. |
| $\begin{aligned} & \text { IIEPTH } \\ & \text { IN } \\ & 24.00 \end{aligned}$ | III AM <br> IN | $\frac{C}{\text { LES/IN**2 }}$ | GAMMA <br> LES/IN**S | ESO |
|  | 48.000 | O. BE 01 | O. SE-01 | 0. 200E-01 |
|  |  | Y, IN |  | F',LES/IN |
|  |  | 0. |  | O. |
|  |  | $0.01 \%$ |  | 4E. 85 |
|  |  | 0.600 |  | 147. 5\% |
|  |  | 1.200 |  | 185.80 |
|  |  | 1.800 |  | 212.780 |
|  |  | 2.400 |  | 294.194 |
|  |  | E.000 |  | 252. 278 |
|  |  | 3.600 |  | 26.3 .086 |
|  |  | 4.200 |  | $2 \mathrm{E2.221}$ |
|  |  | 4.800 |  | 295.086 |
|  |  | 5.400 |  | 306.881 |
|  |  | 6.000 |  | 317.851 |
|  |  | S. 6.80 |  | 323.111 |
|  |  | 7.200 |  | 367.767 |


|  |  | $\begin{aligned} & 19.200 \\ & 31.000 \\ & 48.000 \end{aligned}$ |  | $\begin{array}{r} 20 E \cdot 720 \\ 2 \Theta .313 \\ 20.310 \end{array}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { LEFTH } \\ & \text { IN } \\ & 4 E .00 \end{aligned}$ | $\begin{aligned} & \text { III AM } \\ & \text { IN } \\ & 4 E .000 \end{aligned}$ | $\begin{gathered} C \\ \text { LEE/IN* } 2 \\ O . Z E O 1 \end{gathered}$ | GAMMA LEG/ INが O. SE-O1 | $E E 0$ $0.200 E-01$ |
|  |  | Y, IN |  | F, LEG/IN |
|  |  | 0. |  | O. |
|  |  | 0.019 |  | 56.67 |
|  |  | 0.600 |  | $16 \% .064$ |
|  |  | 1. 200 |  | 213.005 |
|  |  | 1. 600 |  | 243.335 |
|  |  | 2.400 |  | 263.373 |
|  |  | 3.000 |  | $26 \% .0 \% 6$ |
|  |  | 3.600 |  | 307.210 |
|  |  | 4.200 |  | $2 \% .405$ |
|  |  | 4.800 |  | $35.12 \%$ |
|  |  | 5.400 |  | 351.663 |
|  |  | 6.000 |  | 364.235 |
|  |  | 6.600 |  | 375.996 |
|  |  | 7.200 |  | 387.061 |
|  |  | 19.200 |  | 25.2 .949 |
|  |  | 36.000 |  | $6 \leqslant .037$ |
|  |  | 4E.000 |  | 66.037 |
| $\begin{aligned} & \text { LEFFTH } \\ & \text { IN } \\ & 72.00 \end{aligned}$ | $\begin{aligned} & \text { LIIAM } \\ & \text { IN } \end{aligned}$ | $\frac{\mathrm{E}}{\mathrm{LESI} / \mathrm{IN} * 2}$ | GAMMA LEE/IN** | ESO |
|  | 4S.000 | 0.3 E 01 | O.SE-01 | 0.200E-01 |
|  |  | Y,IN |  | F,LEE/IN |
|  |  | 0. |  | 0. |
|  |  | 0.019 |  | 60.510 |
|  |  | 0.600 |  | 190.55 |
|  |  | 1.200 |  | 240.135 |
|  |  | 1.800 |  | 274.856 |
|  |  | 2.400 |  | 302.551 |
|  |  | 3.000 |  | \%25.913 |
|  |  | 3.600 |  | 346.355 |
|  |  | 4.200 |  | 364. 5.96 |
|  |  | 4.800 |  | $3 ¢ 1.1 \% 1$ |
|  |  | 5.400 |  | 396.454 |
|  |  | 6.000 |  | 410.625 |
|  |  | 6.600 |  | 423. 680 |
|  |  | 7.200 |  | 451. 354 |
|  |  | 19.200 |  | 300.673 |
|  |  | 56.000 |  | 111.671 |
|  |  | 43.000 |  | 111.671 |
| $\begin{aligned} & \text { LIEFTH } \\ & \text { IN } \end{aligned}$ | $\begin{aligned} & \text { IIAM } \\ & \text { IN } \end{aligned}$ | $\begin{gathered} C \\ \text { LES/IN**2 } \end{gathered}$ | GAMMA <br> LEE/IN**3 | E50 |
|  |  |  | D26 |  |


| 96.00 | 4E.000 | $0.3 E 01$ | $0.3 E-01$ | $0.200 E-01$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $Y, I N$ |  | $F \cdot$, LES $/$ IN |
|  |  | O. |  | 0. |
|  |  | $0.01 \%$ |  | 67.346 |
|  |  | 0.600 |  | 212.126 |
|  |  | 1. 200 |  | 267.262 |
|  |  | 1.800 |  | 505.930 |
|  |  | 2.400 |  | 356.730 |
|  |  | 3.000 |  | 3K2.731 |
|  |  | 3.1000 |  | 85c. 45 |
|  |  | 4.200 |  | 405.785 |
|  |  | 4. 800 |  | 424.25 |
|  |  | 5.400 |  | 441.241 |
|  |  | 6.000 |  | 457.012 |
|  |  | 6.600 |  | 471.765 |
|  |  | 7.200 |  | 4:5. 48 |
|  |  | $1 \% .200$ |  | 351.601 |
|  |  | 36.000 |  | 165.715 |
|  |  | 45.000 |  | 165.715 |
| $\begin{aligned} & \text { IIEFTH } \\ & \text { IN } \\ & 120.00 \end{aligned}$ | III AM | 6 | GAMMA | ESO |
|  | IN |  | LES/IN**S |  |
|  | 45.000 | O. SE O1 |  | 0. 200E-01 |
|  |  | $Y, I N$ |  | F,LES/IN |
|  |  | 0. |  | O. |
|  |  | 0.019 |  | 74.182 |
|  |  | 0.600 |  | 235.657 |
|  |  | 1.200 |  | 2\%4. 390 |
|  |  | 1.800 |  | $356.9 \%$ |
|  |  | 2.400 |  | 370.908 |
|  |  | 3.000 |  | 399.54\% |
|  |  | 3.600 |  | 424. 564 |
|  |  | 4.200 |  | 446.971 |
|  |  | 4.800 |  | 467. 315 |
|  |  | 5.400 |  | 4S6.027 |
|  |  | 6.000 |  | 503.400 |
|  |  | 6.600 |  | $51 \% .1549$ |
|  |  | 7.200 |  | 594.942 |
|  |  | 1\%.200 |  | 406.635 |
|  |  | 36.000 |  | 225.16\% |
|  |  | 4S.000 |  | $228.16 \%$ |
| $\begin{aligned} & \text { LEFFTH } \\ & \text { IN } \\ & 17 F .00 \end{aligned}$ |  |  |  | ESO |
|  | IN | LES/IN**Z | LES/IN** |  |
|  | 48.000 | O. 3E 01 | O. BE -01 | $0.200 E-01$ |
|  |  | $\begin{aligned} & Y, I N \\ & 0 . \end{aligned}$ |  | $\begin{gathered} \text { P,LES:IN } \\ 0 . \end{gathered}$ |
|  |  | $0.01 \%$ |  | 90.736 |
|  |  | 0.600 |  | 286.563 |
|  |  | 1.200 |  | 361.075 |


|  |  |  |  | 300 |  | 413.331 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 400 |  | 454.580 |  |
|  |  |  |  | 00 |  | 490.05s |  |
|  |  |  |  | 00 |  | 520.76 |  |
|  |  |  |  | 00 |  | 543.223 |  |
|  |  |  |  | 00 |  | 575.171 |  |
|  |  |  |  | 00 |  | 5\%6.127 |  |
|  |  |  |  | 00 |  | 617.435 |  |
|  |  |  |  | 00 |  | 637.366 |  |
|  |  |  |  | 00 |  | 656.122 |  |
|  |  |  | 15. | 00 |  | 5 E6.079 |  |
|  |  |  |  |  |  | 417.451 |  |
|  |  |  |  |  |  | $417.451$ |  |
| $\begin{aligned} & \text { LIEFTH } \\ & \text { IN } \end{aligned}$ | $\begin{aligned} & \text { IIAM } \\ & \text { IN } \end{aligned}$ | $\begin{aligned} & \mathrm{FHI} \\ & \mathrm{LIEG} \end{aligned}$ | GAMMA <br> LES/IN**3 | $A$ | E | FCT | FCLI |
| 192.00 | 43.00 | 50.0 | $0.3 E-01$ | 0.90 | 0.55 | 0. 29 E 04 | O.E1E O4 |
|  |  |  | $Y$ |  |  | F |  |
|  |  |  |  |  |  | LES/IN |  |
|  |  |  | 0. |  |  | 0. |  |
|  |  |  |  |  |  | $451.1 \%$ |  |
|  |  |  |  |  |  | 642.120 |  |
|  |  |  |  |  |  | 76\%. 357 |  |
|  |  |  |  |  |  | 913.E5 |  |
|  |  |  |  |  |  | 1023.77\% |  |
|  |  |  |  |  |  | 1123. 34 |  |
|  |  |  |  |  |  | 1215.056 |  |
|  |  |  |  |  |  | 1300. 507 |  |
|  |  |  |  |  |  | 1350.85 |  |
|  |  |  |  |  |  | 1456.956 |  |
|  |  |  |  |  |  | 1529.424 |  |
|  |  |  |  |  |  | 1598.696 |  |
|  |  |  |  |  |  | 2616.04E |  |
|  |  |  | 17. |  |  | 2616.04: |  |
|  |  |  | 32. |  |  | 2616.04: |  |
|  |  |  | 4E. |  |  | 2616.043 |  |
| $\begin{gathered} \text { LIEFTH } \\ \text { IN } \end{gathered}$ | IIIAM IN | FHI LIEG | GAMMA LEG/IN**S | $A$ | E | FG:T | Ficil |
| 800.00 | 48.00 | 30.0 | O. BE-O1 | 0.3 | O. 5.5 | $0.25 E 0$ | 0.27E OE |
|  |  |  | $Y$ |  |  | $F$ |  |
|  |  |  | IN |  |  | LES/IN |  |
|  |  |  | 0. |  |  | 0. |  |
|  |  |  | 0. |  |  | 1600.000 |  |
|  |  |  |  |  |  | 3200.000 |  |
|  |  |  | 0. |  |  | 4800.000 |  |
|  |  |  | 0. |  |  | 6400.000 |  |
|  |  |  | 0. |  |  | 8000.000 |  |
|  |  |  |  |  |  | \$1800.000 |  |
|  |  |  | 0. |  |  | 10534.620 |  |
|  |  |  | 0.5 |  |  | 11231.746 |  |





## COLUMEIA LOEK \& IAAM - EINGLE FILE [MILFHIN

UNITS--ENGL

G 1 TFGUTINFGFMATIGN


| NO. OF ITEFATIONS | $=$ |
| :--- | :--- |
| MAXIMLIM IIEFLEGTION EFFORR | $=0.410 E-\sigma E ~ I N$ |

Pile loaling cinitition

| LATERAL LGAI AT FILE HEAI | $=0.134 E O E$ LES |  |
| :--- | :--- | :--- |
| AFFLIEI MGMENT AT FILE HEAI | $=0$. | LES-IN |
| AXIAL LOAI AT FILE HEAI | $=0$. | LES |


| X | IIEFLEC: | MOMENT | TIITAL STRESS | LIETR. LOAL | EMIL <br> MODHLLIE | FLEXIIFAAL RIGIEITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IN | IN | LES-IN | LES/IN** | LEG/IN | LFG/ INれた2 | LES-IN*2 |
| ****** | *** | ******** |  |  |  |  |
| 0. | 0.13\%E 02 | O. | 0. | 0. | 0. | 0.901E 12 |
| 24.72 | $0.150 \mathrm{O} ~ 02$ | 0.3S1E 07 | 0.25EE 04 | 0. | 0. | O.OOIE 12 |
| 49.44 | 0.132 E 02 | 0.662E 07 | O.512E 04 | 0. | 0. | $0.901 E 12$ |
| 74.16 | 0.173E 02 | 0.7\%4E OZ | $0.7 \leqslant 7 E 04$ | 0. | 0. | O.OOE 12 |
| 93.83 | $0.164 \mathrm{E} ~ 02$ | 0.132E OS | 0.102 E OS | 0. | 0. | $0.901 E 12$ |
| 123.60 | 0.156 E 02 | 0.16\%E OS | 0.12SE 05 | 0. | 0. | $0.901 E 12$ |
| 143.32 | 0.147 E 02 | 0.19\%E OS | 0.15 BE 05 | 0. | 0. | $0.901 E 12$ |
| 173.04 | 0.13\%E 02 | $0.2 E 2 E$ OS | $0.17 \% \mathrm{E}$ 05 | 0. | 0. | $0.901 E 12$ |
| 197.76 | 0.131 E 02 | 0.26 SE OS | $0.205 E 05$ | 0. | 0. | 0.901 E 12 |
| 222.43 | 0.12 EE 02 | 0.29\%E OS | 0.230 E O5 | 0. | 0. | $0.901 E 12$ |
| 247.20 | $0.115 E 02$ | 0.331 E 03 | 0. $25 \leqslant 6$ O5 | 0. | 0. | 0.901 E 12 |
| 271.92 | 0.107E 02 | 0.364E 0s | $0.281 E$ OS | 0. | 0. | $0.901 E 12$ |
| 296.64 | $0.100 \mathrm{E} ~ 02$ | $0.377 E$ 0S | 0.307 E 05 | 0. | 0. | $0.901 E 12$ |
| 321.36 | 0. F2\%E 01 | $0.431 E \quad 03$ | 0.33 EE O5 | 0. | 0. | $0.901 E 12$ |
| 346.0E | $0.862 E 01$ | $0.464 E$ 0. | $0.35 E E$ OS | 0. | 0. | 0.901 E 12 |
| 370.80. | 0.797E 01 | $0.497 E$ OE | $0.201 E$ O5 | 0. | 0. | 0.172E 13 |
| 395.52 | 0.734 E O1 | O.5.30E 03 | $0.215 E 05$ | 0. | 0. | 0.172 E 13 |
| 420.24 | 0.675 E 01 | 0.563E 0 | 0.220 O | 0. | 0. | 0.172 E 13 |
| 444.96 | 0.614 E 01 | O. 596E OS | 0.241E 05 | 0. | 0. | $0.172 E 13$ |
| 469.63 | $0.557 E$ O1 | 0.629E 03 | $0.255 E 05$ | 0. | 0. | $0.172 E 13$ |
| 474.40 | $0.503 E 01$ | 0.662E 08 | 0.26EE O5 | 0. | 0. | 0.172 E 13 |
| 519.12 | 0.4EOE 01 | 0.6F6E OE | $0.282 E 05$ | 0. | $0.560 E 02$ | 0.172E 13 |
| 543.34 | 0.400 E 01 | 0.728 EE OE | $0.295 E$ 05 | 0. | 0.710 E O2 | $0.172 E 13$ |
| 563.56 | 0.353 E 01 | 0.753 E 03 | $0.307 E 05$ | 0. | $0.865 E 02$ | 0.172E 13 |




## OITTFIT VEFIFIGATIEN

THE MAXIMIMM MOMENT IMEALANEE FOF ANY ELEMENT $=0.750 \mathrm{O}$ OO IN-LEE THE MAX. LATEFAL FGFILE IMEALANGE FGF ANY ELEMENT $=-0$. EGGE-01 LES

```
GOMFIITEI LATEFAL FGREE AT FILE HEALI
    GIMFIITEI MOMENT AT FILE HEALI
    GOMFIITEI ELOFE AT FILE HEAL
```

```
=0.1%400E OS LES
```

=0.1%400E OS LES
=0. IN-LEE
=0. IN-LEE
= -0, 556%E-01

```
= -0, 556%E-01
```

THE GVEFALL MOIMENT IMEALANLE $\quad=-0.92 \mathrm{OE}$ OO IN-LEE
THE OVEFGLL LATEFAL FGROE IMEALANEE $=-0.111 E-0$ LES

IIITFIIT EILMMAFY
FILE HEAL DEFLEETIGN = 9. $9 \%$ O2 IN
MAXIMIM EENLIING MOMENT $=0.3 G E$ OS IN-LES
MAXIMIM TEITAL STFESE = O. 371E OS LES/IN**Z
MAXIMIM EHEAR FQROE $=0.1 \Xi 4 E$ OS LES

3. The pile shown in Figure D 4 will be analyzed under various loads and pile head boundary conditions. The soil profile used is shown in Figure D5. Four variations will be analyzed in a single run.

Free-head pile: p-y curves by soft clay criteria, Example 3a
4. The pile is treated as a free-head pile with an applied moment of $300,000 \mathrm{in} .-\mathrm{lb}$. Lateral loads of $25,000,30,000$, and $35,000 \mathrm{lb}$, along with an axial load of $15,000 \mathrm{lb}$, will be analyzed. $\mathrm{p}-\mathrm{y}$ curves will be generated internally using the soft clay criteria and cyclic loading. The strain at 50 percent of the maximum deviator stress is assumed to be a constant 0.02 to a depth of 336 in . and to decrease linearly to 0.01 at a depth of 1176 in .

Free-head pile: p-y curves
by unified criteria, Example $3 b$
5. This problem is identical with Example 3a except that the p-y curves will be generated by the unified criteria with cyclic loading, and a lateral load of $25,000 \mathrm{lb}$ will be analyzed. Values of $A=2.5, F=1.0$, and $k=116$ pci are assumed. Output will include points on the $p-y$ curves at $x$ coordinates of $96,120,144,192,240,336,576$, and 960 in .

Fixed-head pile: p-y curves by unified criteria, Example 3c
6. This problem is identical with Example 3 b for unified criteria except that the pile head is fixed against rotation. A p-y curve will be output at a depth of $x=576$ in. for verification.
Rotational restraint at pile
head of $1.5 \times 10^{6} \mathrm{in} .-1 \mathrm{~b}$, Example 3d
7. This problem is identical with Example 3 b for unified criteria except that the boundary condition at the pile head will be one of rotational restraint with $M_{t} / S_{t}=1.5 \times 10^{6}$ in. $-1 b$. A p-y curve will be output at depth of $x=576 \mathrm{in}$. for verification.

Comparison of Examples 3a, 3b, 3c, and 3d
8. Comparisons between soil resistance, moment, and deflection for examples 3a, 3b, 3c, and 3d for a lateral load of $25,000 \mathrm{lb}$ are shown in Figure D6.






COMPARISON OF RESULTS FOR EXAMPLES 30-3a


```
    O TITLE
20 FREE HEAI FILE - F-Y ENFVES EY EOFT ELAY LFITERIA
30 UNITS
40 ENGL
EO FILE F6 2%60 2%.E< %6 (Pile properties - NI,NDIAM,LENGTH,EPILE,XGS)
60 0 24 5675.7 (XDIAM(I),DIAM(I),MINERT(I)
70 5%0 24 3425.8 Where I = 1,NDIAM
BO SDIL 1 (Soil Description - NL)
901 1 96 1176 116 (LAYER(I),KSOIL(I),XTOP(I),XBOT(I),K(I) where I = l,NL)
100 WEIGHT 6 (Unit Weight Profile - NGI)
11096 .015%
120 336. .015%
130 336.0246 XG1(I), GAM1(I)
140 %00.024t Where I = 1,NGI
150 900.0504
160 1176 .0504
170 STRENGTH 3 (Soil Strength Profile - NSTR)
180 96 1.35% 0.0 .02 XSTR(I),Cl(I),PHIl(I),EE50(I)
190 336 1.389 0.0 .02 Where I = 1,NSTR
200 1176 6.250 0.0.01
2 1 0 ~ E O U I N D A F Y ~ 1 ~ 3 ~ ( B o u n d a r y ~ C o n d i t i o n ~ a t ~ P i l e ~ H e a d ~ - ~ K B C , N R U N ) ~
2201 25.ES 3.E5 1.5E4 KOPSUB(I),PTSUB(I),BC2SUB(I),PXSUB(I)
2:011 30.E3 3.E5 1.5E4 Where I = 1,NRUN
240 1 S5.E3 3.ES 1.SE4
25O EYCLIC O O (Cyclic Load Indicator - KCYCL, RCYCL)
260 IUITPUIT 1 2 1 % (Output Control - KOUTPT,INC,KPYOP,NNSUB)
270 7E 120 144 192 240 3% 5% 5% 960 (XNSUB(I) ... XNSUB(NNSUB))
250 CONTFIGL 100 .001 40 (Program Control - MAXIT,YTOL, EXDEFL)
290 ENII
300 TITLE
310 FFEE HEAII FILE - P-Y CURVES EY UNIFIEII CFITERIA
320 SOIL 1 (Soil Description - NL)
3001 b %6 117E 11\leqslant2.5 1.0 (LAYER(I),KSOIL(I),XTOP(I),XBOT(I),K(I) Where I=1,NL)
340 BOUNLIAFiY 1 1 (Boundary Condition at Pile Head - KBC,NRUN)
S50 1 25.E3 3.ES 1.EE4 (KOPSUB(I),PTSUB(I),BC2SUB(I),PXSUB(I) where I=1,NRUN)
360 EUITFUTT 1 2 1 8 (Output Control - KOUTPT, INC, KPYOP, NNSUB)
```



```
380 END
390 TITLE
40G FIXED HEALI FILE - F-Y EURVES EY INIFIEII EFITEFIA
410 EOLINLIARY 2 1 (Boundary Condition at Pile Head - KBC,NRUN)
420 1 25.ES 0.0 1. SE4 (KOPSUB(I),PTSUB(I),BC2SUB(I),PXSUB(I) Where I=1,NRUN)
430 GUTTPUT 1 2 1 1 (output Control - KOUTPT, INC, KPYOP, NNSUB)
440 576
450 ENII
460 TITLE
470 RGITATIONAL RESTRAINT AT FILE HEALI OF 1.5 EG IN-LEE
430 EOUNLIARY 3 1 (Boundary Condition at Pile Head - KBC,NRUN)
490 1 25.E3 1.SEG 1.SE4(KOPSUB(I),PTSUB(I),BC2SUB(I),PXSUB(I) Where I-1,NRUN)
500 END
```

(Input Echo for Problem 1 - Free head pile - P-Y curves by Soft Clay Criteria)
\#\#\#* UNIT LIATA
SYSTEM OF UNITS
(UIF TO 16 EHAR.)
*\#*** PILE LIATA. *****


| TOF OF | filameter | MGMENT OF | CRUES-SECT. |
| :---: | :---: | :---: | :---: |
| SEGMENT | Of File | INERTIA | AREA |
| 0. | $0.240 E 02$ | 0. 56\%E 04 | 0.872 E 02 |
| $0.530 E 03$ | 0.240 E 02 | 0.34EE 04 | $0.504 E 02$ |


NUMEER DF LAYERS
1

| LAYER | F-Y Curve | TGP OF | E0 | INITIAL SOI | FACTIF | T |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMEER | CONTFOL COLE | LAYEF | OF LAYER | MOLMLI GOMET. | "A"' | "F:" |
| 1 | 10. | E O2 | 116E O4 | . 11 fe OS |  |  |

***** LINIT WEIGHT LIATA. *****
NO. PGINTS FGR PLOT
OF EFF. UNIT WEIGHT VS. DEFTH
$\leqslant$

| IEFTH EELOW TOF | EFFECTIVE IUNIT WEIIGHT |
| :---: | :---: |
| 0.760802 | $0.15 \% E-01$ |
| 0.336 E 03 | 0.15\%E-01 |
| 0.336 E 03 | $0.246 \mathrm{E}-01$ |
| 0.900 O | $0.24 \mathrm{EE}-01$ |
| 0.900 e 0 | 0.304E-01 |

NO. POINTS FOR
STRENGTH PARAMETERS
VS. DEPTH

| DEFTH BELOW | UNDRAINEL GHEAR | ANGLE OF INTERNAL | STRAIN AT SO\% |
| :---: | :---: | :---: | :---: |
| TOP OF PILE | STRENGTH OF SOIL | FRICTION IN RADIANS | STRESS LEVEL |
| $0.960 E ~ O 2 ~$ | $0.13 \% E ~ O 1$ | 0. | $0.200 E-01$ |
| $0.336 E ~ 03$ | $0.139 E ~ 01$ | 0. | $0.200 E-01$ |
| $0.118 E 04$ | $0.625 E ~ 01$ | 0. | $0.100 E-01$ |

NO. OF
P-Y CIJRVES
0
***** OUTPUT DATA.
IATA
OUTPUT
CODE
1

## QuTFIUT <br> INC:FEMENT CODE

2

## $P-Y$

PRINTOUT CODE

1
NO. DEPTHS T

PRINT FOR FPY GURVES

8

DEPTH FOR
PRINTING
P-Y CURVES
0.860 E 02
$0.120 E 03$
$0.144 E 03$
0.192 E 03
0.240 E 03
$0.336 E 03$
0.57 EE 03
0.960 E 03

(P-Y Curves generated for verification - Problem 1)

GENERATED F-Y EURVES

THE NUMEER GF DURVES
THE NUMEER IF FGINTS ON EAL:H CURVE

| $\begin{gathered} \text { LEFFTH } \\ \text { IN } \\ 0 . \end{gathered}$ | $\begin{aligned} & \text { LIIAM } \\ & \text { IN } \\ & 24.000 \end{aligned}$ | $\begin{gathered} \mathrm{C} \\ \text { LES/IN*2 } \end{gathered}$ |
| :---: | :---: | :---: |
|  |  | Y, IN |
|  |  | 0. |
|  |  | 0.010 |
|  |  | 0.300 |
|  |  | 0.600 |
|  |  | 0.900 |
|  |  | 1.200 |
|  |  | 1.500 |
|  |  | 1.300 |
|  |  | 2.100 |
|  |  | 2.400 |
|  |  | 2.700 |
|  |  | 3.000 |
|  |  | 3.300 |
|  |  | 3.600 |
|  |  | 9.600 |
|  |  | 13.000 |
|  |  | 24.000 |

LIEF•TH
IN
24.00
IIIAM
IN

C
LES/IN*2
$0.1 E 01$

| GAMMA | $E 50$ |
| :---: | :---: |
| LES/INK |  |
| $0.2 E-01$ | $0.200 E-01$ |

$Y, I N$
0.
0.010
0.300
0.600
0.900
1.200
1.500
1.800
2.100
2.400
2.700
3.000
3.300
3.600
7.600
3.000

F,LES/IN
0.
12.58
39.65
49.957
57.164
62.917
67.775
72.022
75.820
79.271
82. 445
$35.3 \%$
ES. 148
90.742
57.725
11.6 .9
$11.6 \%$

| $\begin{aligned} & \text { IIEF•TH } \\ & \text { IN } \\ & 4: .00 \end{aligned}$ | $\begin{aligned} & \text { IIAM } \\ & \text { IN } \\ & 24.000 \end{aligned}$ | $\begin{gathered} E \\ \text { LES/IN**2 } \\ 0.1 E \text { OI } \end{gathered}$ | $\begin{aligned} & \text { GAMMA } \\ & \text { LEG/IN**S } \\ & 0.2 E-01 \end{aligned}$ | ESO $0.200 E-01$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $Y$ Y, IN |  | F,LES/IN |
|  |  | O. |  | O. |
|  |  | 0.010 |  | 15.166 |
|  |  | 0.300 |  | 47.770 |
|  |  | 0.600 |  | 60.137 |
|  |  | 0.900 |  | 68.896 |
|  |  | 1.200 |  | 75.630 |
|  |  | 1.500 |  | E1.6ES |
|  |  | 1.800 |  | E6: 804 |
|  |  | 2.100 |  | 91.381 |
|  |  | 2.400 |  | 95.540 |
|  |  | 2.700 |  | 9\%, 36\% |
|  |  | 3.000 |  | 102.91E |
|  |  | 3.300 |  | 106.240 |
|  |  | 3.600 |  | 109.366 |
|  |  | 9.600 |  | 75. 447 |
|  |  | 13.000 |  | 23.19\% |
|  |  | 24.000 |  | 28.199 |
| $\begin{aligned} & \text { LIEFTH } \\ & \text { IN } \\ & \xi 6.00 \end{aligned}$ | LIAM | ¢ | GAMMA | ESO |
|  | IN | LES/IN**2 | LES/IN**3 |  |
|  | 24.000 | O.1E O1 | 0.2E-01 | 0. $200 \mathrm{E}-01$ |
|  |  | Y, IN |  | P,LES/IN |
|  |  | 0. |  | 0. |
|  |  | 0.010 |  | 20.351 |
|  |  | 0.300 |  | 64.040 |
|  |  | 0.600 |  | 80.68 |
|  |  | 0.900 |  | 92.861 |
|  |  | 1.200 |  | 101.657 |
|  |  | 1.500 |  | 109.506 |
|  |  | 1.800 |  | 116.365 |
|  |  | 2.100 |  | 122.504 |
|  |  | 2.400 |  | 128.080 |
|  |  | 2.700 |  | 13: 20 |
|  |  | 3.000 |  | 137.970 |
|  |  | 3.300 |  | 142.425 |
|  |  | 3.800 |  | 146.6.14 |
|  |  | 9.600 |  | 116.394 |
|  |  | 18.000 |  | 75.606 |
|  |  | 24.000 |  | 75.606 |
| $\begin{aligned} & \text { LIEFTH } \\ & \text { IN } \\ & 144.00 \end{aligned}$ | IIIAM | 0 | GAMMA | E50 |
|  | IN | LES/IN**2 | LES/IN**S |  |
|  | 24.000 | O.1E O1 | O. 2E-01 | 0. $200 \mathrm{E}-01$ |
|  |  | $\begin{aligned} & \mathrm{Y}, \mathrm{IN} \\ & 0 . \end{aligned}$ |  | $\begin{aligned} & \text { Fi, LES/IN } \\ & 0 . \end{aligned}$ |
|  |  | 0.010 |  | 25.497 |
|  |  | 0.300 |  | 80.309 |


|  |  | 0.600 |  | 101.135 |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 0.900 |  | $115.82 \%$ |
|  |  | 1.200 |  | 127.483 |
|  |  | 1.500 |  | 137.327 |
|  |  | 1.800 |  | 145.932 |
|  |  | 2.100 |  | 156.62 |
|  |  | 2.400 |  | $160.61 \%$ |
|  |  | 2.700 |  | 167.050 |
|  |  | 3.000 |  | 173.021 |
|  |  | 8.300 |  | 178.606 |
|  |  | 8.600 |  | $1: 5.86$ |
|  |  | 9.600 |  | 166.345 |
|  |  | 18.000 |  | 142.222 |
|  |  | 24.000 |  | 142.222 |
| LEFPTH | IIIAM | $\underline{\square}$ | GAMMA | ESO |
| IN | IN | LEG/IN**2 | LEE/IN** |  |
| 240.00 | 24.000 | O.1E O1 | O. $2 \mathrm{E}-\mathrm{O} 1$ | 0.200E-01 |
|  |  | $Y, I N$ |  | F,LES/IN |
|  |  | 0. |  | 0. |
|  |  | 0.010 |  | 30.002 |
|  |  | 0.300 |  | 94.502 |
|  |  | 0.600 |  | 11\%.065 |
|  |  | 0.900 |  | 136.295 |
|  |  | 1.200 |  | 150.012 |
|  |  | 1.500 |  | 161.56 |
|  |  | 1.600 |  | 171.721 |
|  |  | 2.100 |  | 180.77E |
|  |  | 2.400 |  | 18\%.00\% |
|  |  | 2.700 |  | $1 \% 6.571$ |
|  |  | 3.000 |  | 205.5 |
|  |  | 3.300 |  | 210.170 |
|  |  | 3.600 |  | 21E. 56 |
|  |  | 9.600 |  | 216.017 |
|  |  | 18.000 |  | 216.017 |
|  |  | 24.000 |  | 216.017 |
| LIEFTH | LIIAM | C | GAMMA | EGO |
| IN | IN | LEG/IN**2 | LES/JN*れ3 |  |
| 480.00 | 24.000 | O. SE O1 | O.2E-01 | $0.171 \mathrm{E}-01$ |
|  |  | Y,IN |  | $F, L E S / I N$ |
|  |  | $0 .$ |  | 0. |
|  |  | $0.008$ |  | 60.002 |
|  |  | 0.257 |  | 1E6. 94 |
|  |  | 0.514 |  | 253.117 |
|  |  | 0.771 |  | 272.576 |
|  |  | 1.029 |  | $300.00 \%$ |
|  |  | 1.253 |  | 323.174 |
|  |  | 1.543 |  | 343.424 |
|  |  | 1.800 |  | 361.52 |
|  |  | 2.057 |  | 377.957 |
|  |  | 2.314 |  | $3 \%$. 122 |
|  |  | 2.5 .71 |  | 407.174 |
|  |  | 2.829 |  | 420. 313 |




free hean pile－p－y clifves ey goft ilay gfitefia

UNITS－－ENGL
（Load Case 1 －Problem 1）


File LGALING CONDITIGN
LATERAL LCIAI AT FILE HEAII＝0．2SOE OS LES AFFLIEL MOMENT AT FILE HEALI $=0.300 E 0 \Leftrightarrow$ LES－IN AXIAL LGAD AT PILE HEAL

```
=0.SOOE OE LEG-IN
= O.1SOE OS LEG
```

| $x$ | DEFLEC： | MOMENT | TITAL STRESS | LISTR． LOAII | SOIL MOLuLLES | FLEXIIFAL FIGILITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IN | IN | LES－IN | LES／IN＊＊2 | LES／IN | LES／IN韦2 | LES－IN＊ |
| ＊＊＊れ＊＊＊ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊＊ | \＃＊＊＊＊＊＊＊＊ | ＊＊＊＊＊せれ゙＊ | せれせれぜれそれ | ＊＊＊＊＊＊＊＊＊ |
| 0. | 0.454 E 01 | 0．300E 06 | $0.306 E 03$ | 0. | 0. | O．165E 12 |
| 20.00 | $0.425 E 01$ | $0.804 E 06$ | $0.167 E 04$ | 0. | 0. | 0.16512 |
| 40.00 | $0.397 E 01$ | 0.131 E 07 | $0.2 \% 4 E 04$ | 0. | 0. | $0.165 E 12$ |
| 60.00 | $0.36 \% E 01$ | 0．181E 07 | 0.400 E O4 | 0. | 0. | 0.165 E 12 |
| 80.00 | 0.341 E 01 | 0．232E 07 | O．EO7E O4 | 0. | 0. | 0．16EE 12 |
| 100.00 | 0.314 E 01 | 0.282 E 07 | $0.614 E 04$ | 0. | $0.229 E 02$ | 0．16EE 12 |
| 120.00 | $0.287 E 01$ | 0．330E 07 | 0.716 E O4 | 0. | 0.293 E 02 | O．165E 12 |
|  |  | $0.375 E 07$ | O． 810 E O4 | 0. | $0.365 E 02$ | $0.165 E \quad 12$ |
| $820.00-$ | －0．315E－03 | －0．131E OG | 0．932E 03 | 0. | 0.994 E OS | $0.9 \% \mathrm{SE} 11$ |
| E40．00 | 0．266E－03－ | O．111E 06 | 0.6 E 7 E 03 | 0. | 0.129 E 06 | 0.96 EE 11 |
| 860.00 | 0． $376 \mathrm{E}-03-$ | －0．546E 05 | 0．48\％E 03 | 0. | 0.102 E O6 | $0.993 E 11$ |
| 880.00 | $0.302 \mathrm{E}-03$ | －0．141E 05 | 0．347E 03 | 0. | O．131E 06 | $0.9 \% \mathrm{SE} 11$ |
| 900.00 | 0．147E－0．3 | 0.107 E 05 | $0.335 E 03$ | 0. | O．241E 06 | $0.993 E 11$ |
| 920.00 | 0． $317 \mathrm{E}-04$ | $0.213 E$ O5 | $0.372 E 03$ | 0. | 0．10EE 07 | 0． 9 GEE 11 |
| $940.00-$ | －0．123E－05 | 0.672 E 04 | $0.321 E 03$ | 0. | O．E76E OS | $0.9 \% \mathrm{EE} 11$ |
| 960.00 | $0.633 \mathrm{E}-09$ | O． | $0.298 E 03$ | 0. | 0． $57 \% \mathrm{E} 11$ | $0.9 \%$ E 11 |

## QUTPUT VERIFIEATION

```
THE MAXIMUM MOMENT IMEALANIE FOR ANY ELEMENT = 0.365E-01 IN-LES
THE MAX. LATEFAL FORCE IMBALANEE FOR ANY ELEMENT = -0. S7:E-0Z LEG
GOMFIITEL LATERAL FGRIE AT FILE HEALI = 0.25000E 0S LES
    EOMFITEI MOMENT AT FILE HEAI =0.30000E OG IN-LES
    COMFUITEI SLGFE AT PILE HEAII
    = -0.14385E-01
THE OVERALL MOMENT IMEALANCE = =0.134E-01 IN-LES
THE OVERALL LATEFAL FGRIEE IMEALANCE
= -0.75OE-0E LES
```

IUITFUTT SUMMARY

| FILE HEALI LIEFLELTION $=0.454 E$ OI IN |
| :--- |
| MAXIMIM EENDING MOMENT |$=0.566 E$ O7 IN-LES


(Load Case 2 - Problem 1)
NO. OF ITERATIUNS $=14$

MAXIMLIM IIEFLEGTION ERROR $=0.35 E E-0 S I N$

File Loalingio ginnition
LATEFAL LGAI AT FILE HEAII

```
=0.3OOE OE LEE
```

```
=0.3OOE OE LEG-IN
```

=0.3OOE OE LEG-IN
=0.15OE OS LES

```
=0.15OE OS LES
```

AFFLIEI MOMENT AT FILE HEALI
AXIAL LCIAI AT FILE HEAII

QUTPUT VERIFICATION

OUTFUT SUMMARY
FILE HEAL DEFLECTION $=0.616 E$ OI IN
MAXIMUM GENIING MOMENT $=0.699 E$ O7 IN-LES
MAXIMUM TOTAL STRESS
MAXIMUM SHEAF FORLE

（Load Case 3 －Problem 1）

| NO．DF ITERATIUNS | $=$ |
| :--- | :--- |
| MAXIMUM LIEFLEETION ERFRGR | $=0.754 E-0.3 \mathrm{IN}$ |

file loaling coindition LATERAL LGAI AT FILE HEAI APFLIEI MOMENT AT FILE HEAII $=0.3 E O E$ OE LES
$=0.300 E$ OE LES－IN
$=0.150 E$ OS LES AXIAL LIAL AT FILE HEAI

| X | DEFLEC： | MOMENT | TUTAL STRESS | DISTR． LGALI | SGIL MOLULUS | FLEXURAL RIGILITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IN | IN | LES－IN | LBS／IN＊＊2 | LBS／IN | LES／INを＊こ | LES－1N＊＊2 |
| ＊＊＊＊＊＊＊ | ＊＊＊＊＊＊\＃\＃\＃ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊ |  |
| O． | 0． 836 E O1 | O．300E 06 | 0． $306 \pm 03$ | 0. | 0. | O．16．5E 12 |
| 20.00 | $0.788 E 01$ | O．101E 07 | O．2SOE 04 | 0. | 0. | 0.16 或 12 |
| 40.00 | 0．740E O1 | Q．171E 07 | O．3EOE 04 | 0. | 0. | 0.16 SE 12 |
| 60.00 | $0.6 \% 3 E 01$ | O．242E 07 | 0． 529 E O4 | 0. | 0. | 0.165 EE 12 |
| 30.00 | O． 046 E O1 | O．313E 07 | 0．679E 04 | 0. | 0. | 0.16 GE 12 |
| 100.00 | $0.601 E 01$ | O．3E4E 07 | O．BCBE 04 | 0. | O． 10 FE 02 | O． 165 E E 12 |
| 120.00 | $0.556 E 01$ | 0．45\％E 07 | 0.973 E 04 | 0. | 0．144E 92 | 0．165E 12 |
| 140.00 | $0.512 E 01$ | O． $513 E 07$ | O． 111 ES | 0. | $0.1 \% 1 \mathrm{E}$ O2 | O．165E 12 |
| 880.00 | 0．291E－03－ | －0．252E 06 | 0.11 EE 04 | 0. | $0.121 E 06$ | 0.998 EL |
| 800.00 | 0．129E－02 | －0．149E 06 | 0.819 E 03 | 0. | O．4EEE OS | 0.99 EE 11 |
| 920.00 | 0．2こ7E－02 | －0．656E OS | 0．53EE 03 | 0. | O．SGE OS | O． 9 FE 11 |
| 940．00 | 0．297E－02 | －0．178E O5 | O．360E 03 | 0. | 0．27\％E OS | $0.9 \%$ OE 11 |
| 960.00 | 0．359E－02 | 0. | 0．298E 0．3 | O． | $0.25 E 6$ OS | O．\％\％ 11 |

## OUTPUT VERIFICATION

THE MAXIMUM MIMENT IMBALANCE FGR ANY ELEMENT $=-0.5 S 1 E-01$ IN－LES THE MAX．LATERAL FORCE IMEALANEE FOR ANY ELEMENT $=0.692 E-02$ LES

CGMPITEL LATERAL FGRCE AT PILE HEAD
COMPUTED MOMENT AT FILE HEALI
COMPUTED SLGPE AT PILE HEAL
$=0.35000 E$ OS LES
$=0.30000 \mathrm{E} 06$ IN－LBS $=-0.23999 E-01$

THE OVERALL MOMENT IMEALANCE $=0.426 \mathrm{E}-01$ IN－LES THE OVERALL LATERAL FGRCE IMEALANCE $=-0.187 E-07$ LES

## OUTPUT SUMMARY

PILE HEAD DEFLECTION $=0.836 E$ O1 IN
MAXIMUM BENDING MOMENT $=0.357 E$ O7 IN－LBS
MAXIMUM TOTAL STRESS $=0.190 E$ O5 LES／IN＊\＃Z
MAXIMUM SHEAR FORCE $=0.354 E$ O5 LBS
(
free head file - f-y curves gy soft clay dfiteria
SUMMAKY TAELE

$* * * * *$ IIVIT [IATA.

SYSTEM GF UNITS (UP TO is CHAR.)
ENGL

*\#*** SIIL LIATA. *****

NLIMEEF OF LAYERE
1

|  | E |  |  | AL EOIL | - |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMEER | ONTFIOL COL | LAYER | IF LAYER | MOLLIL C CONET. | "A" |  |



```
NO. FOINTS FOR FLOT
DF EFF. IINIT WEIGHT
    VS. IEIPTH
            6
```

    UNIT WEIGHT IIATA.
    DIEFTH EELOW TGIF TG FGINT 0.960 F .92 0.336403 $0.336 E 03$ 0.700 E 03 0.900 E OS

EFFECTIVE IUNIT WEIEHT
0. 159E-01
$0.159 E-01$
$0.246 E-01$
$0.246 E-01$
$0.304 E-01$

$$
\begin{aligned}
& \text { FILE HEAD } \\
& \text { FRINTOUIT COLIE } \\
& 1
\end{aligned}
$$

## LATERAL LOALI A TGF OF FILE 0.250 E O5

VALUE GIF SEC:ONL EUUNLIARY EGNDITION 0.800 E OE

AXIAL LOADI ON FILE 0.150 E 05

```
            ***** CYILLIC DATA.
                NO. EYCLES
                            GF LOADING
                            0.100E 03
\begin{tabular}{cc} 
GVELIC(O) & NG. EYELES \\
ORTATIG(1) & OF LOADING \\
LOADMiS & \(0.100 E 03\) \\
0 & 0.
\end{tabular}
PROGRAM CONTRGI DATA.
```

MAX. NO. OF ITERATIONS 100
***** LOAI IIATA. *****
EGIINLIARY SET NO.

1
NO. FOINTS FOR DISTRIE. LATERAL LDAD VS. DEFTH 0

$$
\text { ( } \mathrm{P}-\mathrm{Y} \text { curves generated by verification - Problem 2) }
$$

GENEFATEL P-Y CURVES

| THE NUMEER GF CUFVES | $=8$ |
| :--- | :--- |
| THE NUMEEF GF FUINTS GN EAL:H CLIFVE | $=17$ |



| $\begin{aligned} & \text { IEFFTH } \\ & \text { IN } \end{aligned}$ | $\begin{aligned} & \text { IIIAM } \\ & \text { IN } \end{aligned}$ | $\frac{G}{\text { LES } / \mathrm{IN**2}}$ | $\begin{gathered} \text { CAVG } \\ L E G / I N * E \end{gathered}$ | GAMMA $\text { LE } E / 1 N \# \# S$ | ESO |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 48.00 | 24.000 | O. 1E 01 | 0.1 EO | O. 2E-O1 | $0.200 \mathrm{E}-01$ |
|  |  | Y |  | F. |  |
|  |  | IN |  | LES/IN |  |
|  |  | 0. |  | 0. |  |
|  |  | 0.100 |  | 29.122 |  |
|  |  | 0.200 |  | 36.691 |  |
|  |  | 0.800 |  | 42.001 |  |
|  |  | 0.400 |  | 46.229 |  |
|  |  | 0.500 |  | $4 \% .797$ |  |
|  |  | 0.600 |  | 52.918 |  |
|  |  | 0.700 |  | 55.709 |  |
|  |  | 0.800 |  | 5.243 |  |
|  |  | 0.900 |  | 60.576 |  |
|  |  | 1.000 |  | 62. 741 |  |
|  |  | 1.100 |  | 64.765 |  |
|  |  | 1.200 |  | 66.6 .72 |  |
|  |  | 8.800 |  | 48.152 |  |
|  |  | 16.400 |  | 29.632 |  |
|  |  | 24.000 |  | 11.112 |  |
|  |  | 36.000 |  | 11.112 |  |
| $\begin{aligned} & \text { LEFTH } \\ & \text { IN } \\ & 96.00 \end{aligned}$ | IIIAM | C | Eavis | GAMMA | E50 |
|  | IN | LEE/IN**2 | LES/IN**S | LES/IN**S |  |
|  | 24.000 | 0.1 EO | O.1E O1 | $0.2 \mathrm{E}-01$ | $0.200 \mathrm{E}-01$ |
|  |  | $\begin{aligned} & Y \\ & \text { IN } \end{aligned}$ |  | $\begin{gathered} \text { F' } \\ \text { LES/IN } \end{gathered}$ |  |
|  |  | 0. |  | 0. |  |
|  |  | 0.100 |  | 36.402 |  |
|  |  | 0.200 |  | 45.864 |  |
|  |  | 0.300 |  | 52.501 |  |
|  |  | 0.400 |  | 57.785 |  |
|  |  | 0.500 |  | 52. 247 |  |
|  |  | 0.600 |  | 66.147 |  |
|  |  | 0.700 |  | 69.635 |  |
|  |  | 0.800 |  | 72.804 |  |
|  |  | 0.900 |  | 75.719 |  |
|  |  | 1.000 |  | 78.426 |  |
|  |  | 1.100 |  | 80.95\% |  |
|  |  | 1.200 |  | 83.340 |  |
|  |  | E. 800 |  | 64.820 |  |
|  |  | 16.400 |  | 46.300 |  |
|  |  | 24.000 |  | 27.780 |  |
|  |  | 36.000 |  | 27.780 |  |






NO．OF ITERATIONE $=\quad 27$ MAXIMUM LIEFLEGTIGN EFIFOR $=0.765 E-0 S \mathrm{IN}$

Fille loaliing cinditigin

LATEFAL LIAII AT FILE HEAII AFFLIEL MIMENT AT FILE HEAL AXIAL LÖAL AT FILE HEAL

```
```

= O.2SOE OS LES

```
```

= O.2SOE OS LES
=0.300E 0G LES-IN
=0.300E 0G LES-IN
= O.15OE OS LES

```
```

= O.15OE OS LES

```
```

| $x$ | LIEFLET： | miment | TITAL GTRESS | LIETF． LGAL | SOIL MOLULIE | FLEXifial FiIGIIITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IN | IN | LES－IN | LEG／IN＊＊2 | LBS／IN | LES／IN＊＊2 | LES－IN＊＊2 |
| ＊＊＊＊＊ | 长如＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊＊ | ＊＊＊＊＊＊＊＊ | ＊\＃＊れ＊＊＊れ＊ |  | ＊＊れそれだだ |
| O． | 0.685 E 01 | 0．300E 06 | O．EOGE 03 | O． | 0. | O．16EE 12 |
| 20.00 | 0.6 SOE O1 | O．BOEE OG | O．1EsE O4 | 0. | 0. | O．165E 12 |
| 40.00 | O．611E 01 | 0.131 E 07 | 0．294E 04 | 0. | 0. | O．16EE 12 |
| 60.00 | $0.574 E 01$ | $0.132 E 07$ | O．401E 04 | 0. | 0. | 0.165 ES 12 |
| 80.00 | 0．536E 01 | 0．z3こE 07 | O．SOEE 04 | 0. | 0. | $0.165 E 12$ |
| 00.00 | 0．4\％\％ 01 | $0.23 S E 07$ | 0.61 EE 04 | G． | 0.610 E 01 | 0．16EE 12 |
| 20.00 | 0.463 E 01 | 0.332 E 07 | 0.720 E 04 | 0. | $0.964 E 01$ | 0.165 E 12 |
| 40.00 |  | 0．380E 07 |  |  |  |  |



320．00－0．755E－02－0．363E 06 0．157E 04 0．
0.124 E O． 0.5 OE 11 E40．00－0．371E－02－0．331E OE 0．14EE 04 O． E60．00－0．12ЗE－02－0．26\％E 06 0．124E 04 0．


O．2OEE OE O．$\because E 11$
$0.445 E$ O「 O．OSGE 11
0．$\sigma \sigma \mathrm{E}$ O与 O．$\sigma \mathrm{GE} 11$
O． 60 OE OE O．OOE 11
$0 . \operatorname{SEE}$ OS O．OGE 11
$0.543 E$ OE O． $9.9 E 11$
$0.570 E$ OS O．OGE 11

## OUTPUT VERIFIEATION



OUTFUT GUMMARY
FILE HEAL 'EFLECTION = O. GESE OI IN MAXIMUM EENLING MOMENT $=0.634 E 07$ IN-LES MAXIMLIM TOTAL STRESS $=0.164 E$ OS LES/IN**2 MAXIMUM SHEAR FOFLE $=0.2 S E E$ OS LES

```
FREE HEAL FILE - F-Y CURVES BY UNIFIEL CRITERIA
```

GUMMARYTAELE



(Input Echo tor froblem 3-Fixed head pile - P-Y curves by Unified Criteria)

```
    ***#* UNIT IIATA. #####
            #**** PILE IIATA. *****
```

```
    GYETEM OF LNNTS
```

    GYETEM OF LNNTS
    (UP TO 16 CHAR.)
    (UP TO 16 CHAR.)
    ENGL

```
ENGL
```



```
\begin{tabular}{|c|c|c|c|}
\hline TGF GIF & LIAMETEF & MOMENT GF & CROSS-SE \\
\hline EEGMENT & OF PILE & INERTIA & AREA \\
\hline 0. & 0.240 E O2 & O. 563E 04 & 0. 372 E O2 \\
\hline 0. 530E OS & \(0.240 E 02\) & 0.343 E 04 & \(0.504 E 02\) \\
\hline
\end{tabular}
```

***** SOIIL [IATA. *

NIMMEER OF LAYERE:
1

| LAYER | P-Y GURVE <br> ONTRGU CO |  | EIOTTOM <br> IF LAYER | IAL BOIL | $\begin{aligned} & \text { FAETGF } \\ & \text { "A" } \end{aligned}$ | "F" |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 O. | OE O\% | 11 EE 04 | OS |  |  |

\#\#*** IJNIT WEIGHT LIATA. *****

NO. FOINTS FOR PLOT
GF EFF. UNIT WEIGHT
VS. DEPTH
6

LIEPTH BELOW TGF TO POINT $0.960 \mathrm{E} \mathrm{O2}$ $0.336 E$ OS $0.336 E 03$ 0.900 E 03 $0.900 E 03$

EFFECTIVE
UNIT WEIGHT
0. 159E-01
$0.159 \mathrm{E}-01$
$0.246 E-01$
$0.246 E-01$
$0.304 E-01$

$$
0.118 \mathrm{E} 04 \quad 0.304 \mathrm{E}-01
$$

***** FROIFILE IIATA.

NO. FGINTS FGR
STRENGTH FARAMETERS
VS. LIEFTH
3

| DEPTH EELOW | IINIFAAINEI SHEAF | ANGLE GF INTERNAL | ETRAIN AT 50\% |
| :---: | :---: | :---: | :---: |
| TGF OF FILE | ETFENITH GF EIIL | FFISTION IN FALIIANE: | ETFESE LEVEL |
| 0.960 E O2 | 0.139 E 01 | 0. | $0.200 \mathrm{E}-01$ |
| $0.356 E 03$ | $0.13 \% \mathrm{Ol}$ | 0. | $0.200 \mathrm{E}-01$ |
| 0.11 EE 04 | $0.625 E 01$ | 0. | 0. $100 \mathrm{E}-01$ |

NO. IF
F-Y CURVES
0

| IIATA | OUITFUIT | F-Y | NO. DEFPTHS TO |
| :---: | :---: | :---: | :---: |
| GuITFIIT | INCFEMENT | FFINTEIIT | FRINT FGR |
| CODE | GOIE | COIE | P-Y ELIRVES |
| 1 | 2 | 1 | 1 |

LIEFTH FOR
FFINTING:
F-Y Cllives
0. 576E 0:
***** PILE HEAL (EDUNLIAFY) LIATA. *****

| EOUNDARY | NG. OF EETE: |
| :---: | :---: |
| CONLITION | OF EOUNDARIY |
| COLIE | CONDITIONS |
| 2 | 1 |

File head
PRINTOUIT COLE
1

LATERAL LGAII AT
TOF GF FILE $0.250 E O S$

VALUE GF SECOINLI EOUNDARY GONLITION 0.

AXIAL LOALI ON FILE O. 150 OE



MICROCOPY RESOLUTION TEST CHART national bureau of standards-1963-a

## ( $\mathrm{P}-\mathrm{Y}$ curves generated for verification - Problem 3)

GENEFATEII F-Y GUFVES
THE NIMEER OF CUIFVES
$=1$
THE NIMEEF OF FGINTS IN EAGH GIIFVE
$=17$

| [IEF'TH | IIAM IN | $\begin{gathered} \mathrm{C} \\ \text { LES/IN* } \end{gathered}$ | CAVE $L E S / \text { I } N *: S$ | GAMMA LEG/1N** | $E E O$ $0.171 E-01$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 480.00 | 24.000 | $0 . \mathrm{SE}_{Y} \mathrm{Ol}$ | O. 2 E O1 | $\begin{gathered} 0.2 E-01 \\ F \end{gathered}$ | $0.171 \mathrm{E}-01$ |
|  |  | IN |  | LEG/ IN |  |
|  |  | 0. |  | O. |  |
|  |  | O.03E |  | 131.041 |  |
|  |  | O. 171 |  | 165.101 |  |
|  |  | 0. 2¢7 |  | 188.9\%4 |  |
|  |  | 0.343 |  | 208.014 |  |
|  |  | 0.429 |  | 224.077 |  |
|  |  | O. 514 |  | 236.117 |  |
|  |  | 0.600 |  | 250.672 |  |
|  |  | 0.60 |  | 262.0E2 |  |
|  |  | 0.771 |  | 272.576 |  |
|  |  | U.857 |  | 232.319 |  |
|  |  | 0.943 |  | 291.432 |  |
|  |  | 1.029 |  | 300.009 |  |
|  |  | 7.543 |  | $300.00 \%$ |  |
|  |  | 14.057 |  | $300.00 \%$ |  |
|  |  | 20.571 |  | $300.00 \%$ |  |
|  |  | 30.857 |  | $800.00 \%$ |  |




FIXEI HEAL FILE - F-Y LURVES EY LINIFIEL EFITEFIA

UNITS--ENBL



E20.00 0.212E-0S-0.137E OS O. 3ESE OS O. O. O4OE OE O. OE 11




 $\because 40.00-0.13 E E-040.626 E 030.300 E 030 . \quad 0.97 \% E$ OE O. 0.0 OE 11


OUTFUT VERIFIGATIUN

```
THE MAXIMUM MOMENT IMEALANEE FOIR ANY ELEMENT \(=0.403 E-01\) IN-LEE THE MAX. LATERAL FOREE IMEALANGE FGF ANY ELEMENT \(=-6.24 E E-62\) LES
CQMF'LITEI LATERAL FORIGE AT FILE HEAI \(=0.2 E 000\) OL LEE GOMFITEL SLOIFE AT FILE HEAI \(\quad=0.21634 E-1 \%\) IN/IN
THE GVEFALL MOMENT IMEALANCE \(=0.147 E-01\) IN-LES
```



GUTFUT SLIMMAFY



FIXEI HEAI FILE - F-Y LURVES BY LINIFIEI CRITEFIA

(Input Echo for Problem 4 - Rotational Restraint at Pile Head)

```
            **#** IINIT LIATA.*####
    GYETEM QF UNITS
    (UF TO 16 CHAR.)
ENGL
    ***** FILE IIATA. ###**
\begin{tabular}{|c|c|c|c|c|c|}
\hline Nil. & INCREMENTS & NO. EEGMENTS & LENGTH & MOLulus & LIEFTH \\
\hline FILE & is mivined & WITH [IIFFERENT & OF & ELAETIEITY & \\
\hline & & CHARAOTERISTILS & FILE & & \\
\hline & 96 & & \%OE & 0.200608 & \%OE O2 \\
\hline
\end{tabular}
```

| TOF OF | IIAMETEF | MUMENT OF | EROES-EE |
| :---: | :---: | :---: | :---: |
| SEGMENT | OF FILE | INEFTIA | AFEA |
| 0. | 0.240 E O2 | O. E6SE 04 | $0.372 E 02$ |
| Q. sion os | O.240E O2 | 0.343E 04 | 0. 504E O2 |


***** UNIT WEIGHT LIATA. *****
NII. PGINTS FOR FLOT
OF EFF. IINIT WEIGHT
VE. LEFTH
$b$

IIEFTH BELOW TGF TO FOINT 0.960 E 02 $0.336 E 03$ $0.336 E 03$ 0.700 E 03 $0.900 E$ os

EFFEGTIVE IUNIT WEIGHT $0.159 \mathrm{E}-01$ $0.15 \mathrm{~F}-01$
0.24 人E-O1
$0.246 \mathrm{E}-01$
0. 304E-01

```
0.11EE 04 0. 304E-01
#**** FFOOFILE IIATA. *****
NO. FOINTS FOR
STRENGTH PARAMETERE
    VS. IIEFTH
                3
    DEFTH EELIOW IINLRAINEII SHEAR ANGLE OF INTEFNAL ETFAINS AT EO%
    TGF OF FILE ETRENGTH OF GOIL FFIGTION IN FAIIANE
        0.9BOE O2
        0.33BE 03
        0.11GE 04
            INLIRAINEI SHEAR ANGLE OF INTEFNAL
            O.
        ETFAIN AT EO%
        ETFEGE LEVEL
        0. 200E-01
        0.200E-01
        O.100E-01
            ####* F-Y IIATA. *****
        NO. IF
    F-Y CURVES
                O
\begin{tabular}{|c|c|c|c|}
\hline IIATA & OUTFPIT & F'-Y & NO. LIEFTHS TO \\
\hline OUTFUT & I NCREMENT & FRINTOUIT & FRINT FGR: \\
\hline COLE & COIE & COILE & F-Y CufVes \\
\hline 1 & 2 & 1 & 1 \\
\hline
\end{tabular}
```


## DEFTH FGR

```
FRINTING F-Y CURVES \(0.575 E 03\)
***** PILE HEAII (ECUUNLIAFY) LIATA. *****
\begin{tabular}{cc} 
EGUNLARY & NO. GIF SETE \\
CONDITIGN & OF BOINLIARY \\
CGIE & CONLITIGNS \\
3 & 1
\end{tabular}
```

FILE HEAD PRINTGUT COLE

1

LATERAL LGALI AT
TGP OIF PILE 0.250 E 05

VALUE OF SECONT EIOUNLIARY GONLIITIEN 0.150 EO

AXIAL LOAL ON FILE O. 1 EOE OS




D86


# FITATIONAL RESTRAINT AT FILE HEAL GF 1．E ES IN－LES 

IINITE——ENGL

GUTFUTINFGFMATIGN



File loading EONDITIGN LATEFAL LGAD AT FILE HEAL FigTaTIGNAL FEETFAINT AXIAL LOAII AT FILE HEAM

$$
\begin{aligned}
& =0.2 G E \text { OS LES } \\
& =0.1 G O E O 7 L E S-I N \\
& =0.1 S O E O S L E S
\end{aligned}
$$

| $x$ | LIEFLEE： | MOMENT | TgTAL STFESS | GIGTE． <br> LGAB | GOIL MGLにルば | FLEYMFAL FIGIIITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IN | IN | LES－1N | LES；IN＊＊こ | LES／IM | LES，INk\％ 2 | LGS－IN土大き |
| ＊＊れれがれ |  |  | 先れそれそれ゙せれ |  |  | れだれだれだが |
| 0. | O．641E | O1－0．267E OS | O．22EE 0S | O． | 0. | O．16EE 12 |
| 20.00 | $0.606 E 0$ | $010.47 \% \mathrm{E} 06$ | O． 11 EE 04 | 0. | 0. | O． 165 E |
| 40.00 | Q．570E O | 01 O．984E Ot | O． $2 \boldsymbol{C E}$ G 04 | 0. | 0. | O．1 SEE 1 |
| 60.00 | O．SGEE 0 | 01 O．14\％E 07 | O． 32 E O4 | 0. | 0. | $0.165 E 12$ |
| 80.00 | O．SOOE O | 01 O． 10 OE O7 | 0．43E 04 | 0. | 0. | 0．1达 12 |
| 100.00 | 0.46 OE O | $010.2 E O E 07$ | 0．54tE O4 | 0. | 0.665601 | 0.165 E 12 |
| 120.00 | 0.432 E 0 | 01 O．－9E 07 | O．650E O4 | 0. | 0.10 OE O2 | O．165E 1 |
| 140.00 | O． $0^{\circ} \mathrm{GE}$ | 010.347 O | $0.751 E 04$ | 0. | $0.147 E$ O2 | O．16SE 1 |

 340．00－0．2た2 $02-301 E 060.135 \mathrm{O} 040$.


 $\because 20.00$ O． $76 E-0 \xi-0 . E 4 E$ OE O． $4 \Xi E$ OG $\because$



| O． 14 SE | OS O．大马E |
| :---: | :---: |
| O． | O¢ O． |
|  | い $\because$ 为気 |
| O．$\%$ \％ | ¢5 O． |
| O．S6OE | OE O．-SE |
| O．太大比 | OL O． O OE |
| O． 7 OEE |  |
| 0.100 O | い6 リ． |

## DHITPUT VERIFIGATIEN


THE MAX. LATEFIG FGFIE IMBALANGE FGF ANY ELEMENT = - $0.42 \mathrm{CE}-\mathrm{GE}$ LEE

```
GOMFITTEI LATEFAL FGRIEE AT PILE HEAN = 0.2SOOOE OE LEE
    GIMFUTEI FGTATIONAL ETIFFNESS AT FILE HEAL = 0.1SOOOE O7 1N-LE
5
    GOMFUTEL SLOFE AT FILE HEAI =-0.17E1%E-O1
THE OVEFALL MOMENT IMEALANEE = 0.152E-0: IN-LEE
THE GVEFALL LATEFAL FGRIE IMEALANEE = -0.06GE-0E LES
```

DUITFUT GIIMMAFY
FILE HEAII IEFLEOTION $=0 . E 41 E 01 \mathrm{IN}$ MAXIMIM EENLING MOMENT $=0.64 E E 07$ IN-LE: MAXINUM TOTAL ETFESG = O.1EGE OG LEG/IN\&だ MAXIMIM SHEAR FDFIEE $=0.2 G E E$ OE LES


| LATEFAL | EIOIINLIAFiY | AXIAL |  |  | MAX. | MAX |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LIAL | CINIITIGN | LiAl | $Y 7$ | ST | MIMENT | ETFE: |
| (LES) | ECO | (LES) | (IN) | (IN/IN) | (IN-LES) | (LEC/1N**こ |
| -.250E | O. 1 SOE 07 | 1 SOE | 41E | 17 EE | $0.645 E$ | , |

## APPENDIX E: NOTATION



|  |
| :--- | :--- | :--- |


| Symbol | Definition | Definition on Page |
| :---: | :---: | :---: |
| $\bar{\sigma}_{v}$ | Average effective stress | 71 |
| $\sigma_{\Delta}$ | Deviator stress | 35 |
| Y | Average unit weight of the soil (submerged unit weight if the soil is below the water table) | 39 |
| $y^{\prime}$ | Average effective unit weight from the ground surface to the $p-y$ curve | 52 |
| $\phi$ | Angle of internal friction | 36 |




[^0]:    * For convenience, symbols and unusual abbreviations are listed and defined in the Notation (Appendix E).

[^1]:    A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

[^2]:    - 1980. "Lecture on Analysis of Pile Groups," Corps of Engineers Training Soil-Structure Interaction, Huntsville, Ala.
    Reese, L. C., and Allen, J. D. 1977. Drilled Shaft Design and Construction Manual, Vol 2, U. S. Department of Transportation, Federal Highway Administration, Washington, D. C.

    Reese, L. C., and Cox, W. R. 1968. "Soil Behavior from Analysis of Tests of Uninstrumented Piles Under Lateral Loading," Proceedings, American Society for Testing and Materials, San Francisco, Calif., pp 161-176.
    Reese, L. C., and Matlock, H. 1956. "Non-Dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth," Proceedings, Eighth Texas Conference on Soil Mechanics and Foundation Engineering, Austin, Tex.

[^3]:    * References cited in this appendix are included in the References at the end of the main text.

[^4]:    97.9 in.
    $T_{\text {obtained }}=\left(\frac{E I}{k}\right)^{1 / 5}$
    $\mathrm{k}=\frac{\mathrm{s}}{\mathrm{x}}=3.5$

[^5]:    $$
    k=\frac{E_{s}}{x}=
    $$

    $T_{\text {obtained }}=\left(\frac{\mathrm{EI}}{\mathrm{k}}\right)^{1 / 5}=101.5 \mathrm{in}$.

[^6]:    $T_{\text {obtained }}=\left(\frac{E I}{k}\right)^{1 / 5}=102$ in.
    2.9
    $k=\frac{E}{x}=$

[^7]:    * References cited in this appendix are included in the References at the end of the main text.

[^8]:    * All elevations (el) cited herein are in feet referenced to the National Geodetic Vertical Datum (NGVD).

[^9]:    * References cited in this appendix are included in the References at the end of the main text.

[^10]:    (Output Control - KOUTPT, INC, KPYOP, NNSUB) (Boundary Conditions at Pile Head -KBC,NRUN)
    (KOPSUB (I) , PTSUB (I) , BC2SUB (I) , PXSUB (I)
    where $I=1$, NRUN
    (Program Control - MAXIT,YTOL, EXDEFL)

