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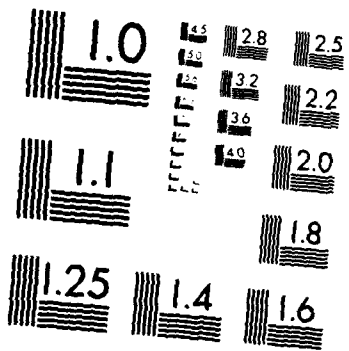
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Electrostatic Equations for Large Scale Plasma Simulation Studies

K. HAIN AND J. FEDDER

*Geophysical and Plasma Dynamics Branch
Plasma Physics Division*

July 19, 1984

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ELECTROSTATIC EQUATIONS FOR LARGE SCALE PLASMA SIMULATION STUDIES

I. INTRODUCTION

In recent years, global numerical simulation of plasma dynamics has become an area of active research interest. A large portion of this effort has been based on the equations of magnetohydrodynamics, MHD. The equations of MHD include a plasma continuity equation, a momentum equation, an energy equation, Maxwell's equations, and Ohm's law. These equations allow a simulation of the temporal evolution of a fluid plasma and the electromagnetic field. Perpendicular to the magnetic field the simulations maintain a balance between plasma forces and field forces, while parallel to the field the balance depends only on fluid quantities. The balance between plasma and field forces is responsible, in part, for the reliability and accuracy of the simulation results. In certain problems of interest the plasma has a relatively low temperature and the magnetic forces dominate the motion perpendicular to the field. This situation leads to difficulties in accurately computing the plasma forces and therefore the dynamics of the plasma. In such situations the difficulty can be alleviated by restricting the MHD equations to the so-called electrostatic approximation. In the electrostatic approximation one retains the hydrodynamic equations parallel to the magnetic field but perpendicular to the magnetic field balances plasma forces against $\mathbf{J} \times \mathbf{B}$, where \mathbf{B} is a known invariant magnetic field and \mathbf{J} is calculated from the difference between electron and ion velocities.

The purpose of this report is to derive a set of electrostatic equations which can be used to develop a simulation code for large scale low pressure plasma systems. This is done in a general coordinate

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system. The organization of the paper is as follows. In the Section II the meaning of the electrostatic approximation is discussed. In Section III, the physical equations are derived. In Section IV, the electrostatic equations are written for a general coordinate system. In Section V, the application of these equations to a plasma system in a dipole coordinate system appropriate for the earth's ionosphere-magnetosphere is presented. Finally, in Section VI we briefly summarize our results.

II. THE ELECTROSTATIC APPROXIMATION

The electrostatic approximation consists of four assumptions:

- 1) All forces are small compared to the magnetic force so that $\partial \underline{B} / \partial t = 0$ and $\nabla \times \underline{B} = 0$. In order for \underline{B} not to change in time it is sufficient and necessary that the electric field, is the gradient of a scalar, i.e., $\underline{E} = -\nabla \Psi$.
- 2) The current \underline{J} is given explicitly by the difference in the ion and electron velocities, i.e., $\underline{J} = en(\underline{V}_i - \underline{V}_e)$. The requirement that $\nabla \cdot \underline{J} = 0$ provides an equation for the electric potential Ψ .
- 3) The electrostatic potential Ψ is constant along magnetic field lines, i.e., the field lines are equipotentials.
- 4) In order to compute the velocities perpendicular to the magnetic field lines one uses the approximation $\underline{E} + \underline{V}_i \times \underline{B} = 0$. This assumption is not necessary but greatly simplifies the analysis.

Within the context of these assumptions, i.e., the electrostatic approximation, we derive an appropriate set of equations for performing plasma simulations in a general coordinate system.

III. DERIVATION OF EQUATIONS

The momentum equations for the electrons and ions are, respectively,

$$-\frac{e}{m_e} (\underline{E} + \underline{v}_e \times \underline{B}) - \nu_{en}(\underline{v}_e - \underline{v}_n) - \nu_{ei}(\underline{v}_e - \underline{v}_i) = \underline{g}_e \quad (1)$$

$$\frac{e}{m_i} (\underline{E} + \underline{v}_i \times \underline{B}) - \nu_{in}(\underline{v}_i - \underline{v}_n) - \nu_{ie}(\underline{v}_i - \underline{v}_e) = \underline{g}_i \quad (2)$$

where \underline{B} is the magnetic induction, \underline{E} is the electric field, \underline{v}_e , \underline{v}_i , and \underline{v}_n are the electron, ion, and neutral velocities, respectively, ν_{en} and ν_{ei} are the electron-neutral and electron-ion collision frequencies, ν_{in} and ν_{ie} are the ion-neutral and ion-electron collision frequencies, respectively, and \underline{g}_α is the total force (including inertia) acting on species α .

Equations (1) and (2) can be written in the form

$$-\underline{v}_{en} \times \underline{\Omega} - (\lambda \nu_{en} + \nu) \underline{v}_{en} + \nu \underline{v}_{in} = \underline{g}_e \lambda + \underline{\varepsilon} \quad (3)$$

$$\underline{v}_{in} \times \underline{\Omega} - (\nu_{in}/\lambda + \nu) \underline{v}_{in} + \nu \underline{v}_{en} = \underline{g}_i/\lambda - \underline{\varepsilon} \quad (4)$$

where

$$\underline{v}_{en} = \underline{v}_e - \underline{v}_n$$

$$\underline{v}_{in} = \underline{v}_i - \underline{v}_n$$

$$\underline{\varepsilon} = e(m_e m_i)^{-1/2} (\underline{E} + \underline{v}_n \times \underline{B})$$

$$\underline{\Omega} = e\mathcal{E}/(m_e m_i)^{1/2} \quad \lambda = (m_e/m_i)^{1/2}$$

$$v = \lambda v_{ei} = v_{ie}/\lambda.$$

Eliminating the cross terms from Eqs. (3) and (4) one finds that

$$\begin{aligned} d_e \underline{v}_{en} + v b \underline{v}_{in} &= -\lambda v_{en} \underline{\varepsilon} + \underline{\varepsilon} \times \underline{\Omega} \\ &\quad - \lambda^2 g_e v_{en} - v(\lambda g_e + g_i/\lambda) + \lambda g_e \times \underline{\Omega} \end{aligned} \quad (5)$$

$$\begin{aligned} d_i \underline{v}_{in} - v b \underline{v}_{en} &= (v_{in}/\lambda) \underline{\varepsilon} + \underline{\varepsilon} \times \underline{\Omega} \\ &\quad - g_i/\lambda^2 v_{in} - v(\lambda g_e + g_i/\lambda) - g_i/\lambda \times \underline{\Omega} \end{aligned} \quad (6)$$

where

$$d_e = \Omega^2 + (\lambda v_{en} + v)^2 - v^2$$

$$d_i = \Omega^2 + (v_{in} + v)^2 - v^2$$

$$b = v_{in}/\lambda - \lambda v_{en} \quad \text{and} \quad \Omega^2 = |\underline{\Omega}|^2.$$

Defining the quantity D

$$D = d_i d_e + v^2 b^2$$

one can solve for \underline{v}_{en} and \underline{v}_{in}

$$\begin{aligned}
\mathbf{v}_{en} = \frac{1}{D} \{ & -(d_i \lambda v_{en} + vb v_{in}/\lambda) \underline{\varepsilon} + (d_i - vb)(\underline{\varepsilon} \times \underline{\Omega}) \\
& -(\lambda^2 g_e v_{en} d_i - vb g_i v_{in}/\lambda^2) - v(d_i - vb)(\lambda g_e + g_i/\lambda) \\
& + (\lambda d_i g_e + vb g_i/\lambda) \times \underline{\Omega} \} \quad (7)
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{v}_{in} = \frac{1}{D} \{ & (d_e v_{in}/\lambda - vb \lambda v_{en}) \underline{\varepsilon} + (d_e + vb)(\underline{\varepsilon} \times \underline{\Omega}) \\
& -(g_i v_{in} d_e/\lambda^2 + g_e vb \lambda^2 v_{en}) - v(d_e + vb)(\lambda g_e + g_i/\lambda) \\
& -(g_i d_e/\lambda - g_e vb \lambda) \times \underline{\Omega} \} \quad (8)
\end{aligned}$$

We now derive an expression for the current \mathbf{J} using Eqs. (7) and (8). We note that

$$\mathbf{J} = en_e(\mathbf{v}_i - \mathbf{v}_e) = en_e(\mathbf{v}_{in} - \mathbf{v}_{en}) \quad (9)$$

so that

$$\begin{aligned}
\mathbf{J} = \sigma_p (\underline{\mathbf{E}} + \mathbf{v}_n \times \mathbf{B}) + \sigma_h (\underline{\mathbf{E}} \times \mathbf{B}/B - B \mathbf{v}_n) \\
+ \frac{\rho \lambda}{\Omega B} \{ S_e \lambda^2 g_e v_{en} - S_i g_i v_{in}/\lambda^2 + v(S_e - S_i)(\lambda g_e + g_i/\lambda) \\
- (S_e \lambda g_e + S_i g_i/\lambda) \times \underline{\Omega} \} \quad (10)
\end{aligned}$$

where

$$S_e = \frac{\Omega^2}{D} (d_i - vb)$$

$$S_i = \frac{\Omega^2}{D} (d_e + vb)$$

$$\sigma_p = (\rho/B^2) (S_e v_{en} \lambda^2 + S_i v_{in})$$

$$\sigma_h = (\rho/B^2) \Omega \lambda (S_i - S_e)$$

and $\rho = n_e m_i$ and $B = |B|$

In order to get an equation for the potential one has to first isolate the inertial terms. Neglecting electron inertia and defining

$$g_i = \frac{d\tilde{v}_i}{dt} - \tilde{f}_i ; g_e = -\tilde{f}_e / \lambda^2$$

one finds that

$$\begin{aligned} \tilde{j} - \sigma \frac{E}{r^2} - \sigma \frac{E}{r^2} \times \frac{B}{B} &= \sigma_p (\tilde{v}_n \times B) - \sigma \frac{v}{r^2} |B| \\ &- \frac{\rho}{B\Omega} \{ S_i \frac{d\tilde{v}_i}{dt} - \tilde{f}_i - S_e \tilde{f}_e \} \times \tilde{\Omega} \\ &- \frac{\rho}{B\Omega\lambda} \{ [S_i v_{in} + (S_i - S_e) v \lambda] \frac{d\tilde{v}_i}{dt} \\ &- [S_i v_{in} + (S_i - S_e) v \lambda] \tilde{f}_i + [S_e v_{en} \lambda^2 + (S_e - S_i) v \lambda] \tilde{f}_e \} \end{aligned} \quad (11)$$

We rewrite Eq. (11) using the following notation

$$T_{pi} = S_i \quad T_{pe} = S_e$$

$$T_{hi} = \frac{S_i v_{in} + (S_i - S_e) v \lambda}{\Omega \lambda} \quad T_{he} = \frac{S_e v_{en} \lambda^2 + (S_e - S_i) v \lambda}{\Omega \lambda}$$

so that one finally obtains

$$\begin{aligned} \underline{J} - \sigma_p \underline{E} - \sigma_h (\underline{E} \times \underline{B})/B &= \sigma_p (\underline{v}_n \times \underline{B}) - \sigma_h \underline{v}_n B \\ &- (\rho/B^2) [T_{pi} (d\underline{v}_i/dt - \underline{f}_i) - T_{pe} \underline{f}_e] \times \underline{B} \\ &- (\rho/B) [T_{hi} (d\underline{v}_i/dt - \underline{f}_i) + T_{he} \underline{f}_e] \end{aligned} \quad (12)$$

The total time derivative d/dt also contains all coriolis and centrifugal terms.

Using $\nabla \cdot \underline{J} = 0$ and integrating along the fields with

$$\underline{E} = -\nabla \Psi ; \quad \underline{v}_i = (\underline{E} \times \underline{B})/|B|^2$$

one can derive the following two dimensional potential equation for Ψ

$$\begin{aligned} &\int dl \nabla \cdot [\sigma_p \nabla \Psi + (\rho/B) T_{pi} d(\nabla \Psi/B)/dt] \\ &+ \int dl \nabla \cdot [(-\sigma_h \nabla \Psi + (\rho/B) T_{hi} d(\nabla \Psi/B)/dt) \times \underline{B}]/B] \\ &= \int dl \nabla \cdot [\sigma_p (\underline{v}_n \times \underline{B})] - \int dl \nabla \cdot [\sigma_h \underline{v}_n B] \\ &+ \int dl \nabla \cdot [(\rho/B^2) \underline{F}_p \times \underline{B}] + \int dl \nabla \cdot [(\rho/B) \underline{F}_h] \end{aligned} \quad (13)$$

where

$$\underline{F}_p = T_{pi} \underline{f}_i + T_{pe} \underline{f}_e \quad \underline{F}_h = T_{hi} \underline{f}_i - T_{he} \underline{f}_e$$

This equation will be used in the subsequent analysis. The first integral represents the Pedersen conductance owing to collisions and the capacitance associated with plasma inertia. The next integral represents the analogous Hall terms. The first two terms on the right hand side are associated with collisional coupling to the neutral gas. They are, in many instances, the main driving terms.

One should remark also that the first integral in Eq. (13) must to be larger than the second integral for the potential equation to be diagonally dominant. By looking at the formulae for σ_p and σ_h one finds that the field line integral of the product of the plasma density and the Larmor frequency must be larger than the comparable integral of the collision frequency.

IV. GEOMETRY

We now derive an expression for the potential equation in a generalized geometry. An orthogonal coordinate system (x_1, x_2, x_3) is defined by the geometrical factors (h_1, h_2, h_3) such that the length element ds is given by

$$ds^2 = h_1^2 dx_1^2 + h_2^2 dx_2^2 + h_3^2 dx_3^2 \quad (14)$$

We define

$$\partial_{\alpha} = \frac{1}{h_{\alpha}} \frac{\partial}{\partial x_{\alpha}} \quad (15)$$

so that

$$\nabla \cdot \underline{A} = \frac{1}{g} \sum_{\alpha} \frac{\partial}{\partial x_{\alpha}} (h_{\alpha+1} h_{\alpha+2} A_{\alpha}); \quad g = h_1 h_2 h_3 \quad (16)$$

The covariant derivative of the velocity is given by

$$\sum_{\alpha} (v_{\alpha} \nabla_{\alpha}) v_{\beta} = \sum_{\alpha} \{ (v_{\alpha} \partial_{\alpha}) v_{\beta} + c_{\beta\alpha} v_{\alpha} v_{\beta} - c_{\alpha\beta} v_{\alpha}^2 \} \quad (17)$$

with

$$c_{\beta\alpha} = \frac{1}{h_{\beta}} \partial_{\alpha} h_{\beta} \quad (18)$$

Since the electrostatic approximation assumes a vacuum magnetic field, one of the coordinates can be the field lines. Here x_3 is chosen. It follows then that

$$\nabla \cdot \underline{B} = \frac{1}{g} \frac{\partial}{\partial x_3} (h_1 h_2 B_3) = 0 \quad (19)$$

which means

$$h_1 h_2 B_3 = f(x_1, x_2) \quad (20)$$

is a function of x_1 and x_2 only.

With the definition of

$$ds_3 = h_3 dx_3 \quad (21)$$

the potential equation (Eq. (13)) can be written explicitly as

$$\begin{aligned}
& \frac{\partial}{\partial x_1} \int \left(\frac{h_2}{h_1} \sigma_p ds_3 \right) \frac{\partial}{\partial x_1} \Psi + \frac{\partial}{\partial x_2} \int \left(\frac{h_1}{h_2} \sigma_p ds_3 \right) \frac{\partial}{\partial x_2} \Psi \\
& + \frac{\partial}{\partial x_1} \int (\sigma_h ds_3) \frac{\partial}{\partial x_2} \Psi - \frac{\partial}{\partial x_2} \int (\sigma_h ds_3) \frac{\partial}{\partial x_1} \Psi \\
& + \frac{\partial}{\partial x_1} \int \left\{ h_2 \frac{\rho}{B} T_p \frac{d}{dt} \left(\frac{1}{B h_1} \frac{\partial}{\partial x_1} \Psi \right) ds_3 \right\} + \frac{\partial}{\partial x_2} \int \left\{ h_1 \frac{\rho}{B} T_p \frac{d}{dt} \left(\frac{1}{B h_2} \frac{\partial}{\partial x_2} \Psi \right) ds_3 \right\} \\
& - \frac{\partial}{\partial x_1} \int \left\{ \frac{\rho}{B} T_h h_2 \frac{d}{dt} \left(\frac{1}{B h_2} \frac{\partial}{\partial x_2} \Psi \right) ds_3 \right\} + \frac{\partial}{\partial x_2} \int \left\{ \frac{\rho}{B} T_h h_1 \frac{d}{dt} \left(\frac{1}{B h_1} \frac{\partial}{\partial x_1} \Psi \right) ds_3 \right\} \\
& = \frac{\partial}{\partial x_1} \int B \sigma_p h_2 v_2^n ds_3 - \frac{\partial}{\partial x_2} \int B \sigma_p h_1 v_1^n ds_3 \\
& - \frac{\partial}{\partial x_1} \int B \sigma_h h_2 v_1^n ds_3 - \frac{\partial}{\partial x_2} \int B \sigma_h h_1 v_2^n ds_3 \\
& + \frac{\partial}{\partial x_1} \int \frac{\rho}{B} F_p h_2 G_2 ds_3 - \frac{\partial}{\partial x_2} \int \frac{\rho}{B} F_p h_1 G_1 ds_3 \\
& + \frac{\partial}{\partial x_1} \int \frac{\rho}{B} h_2 F_h G_1 ds_3 + \frac{\partial}{\partial x_2} \int \frac{\rho}{B} h_1 F_h G_2 ds_3 \quad (22)
\end{aligned}$$

It is the goal to separate geometrical quantities from fluid quantities. To this end we define normalized quantities at a point x_3^0 . We define λ_u , B^0 , and g^0 as follows:

$$\lambda_\alpha = \frac{h_\alpha}{h_\alpha^0} = \frac{h_\alpha(x_1, x_2, x_3)}{h_\alpha(x_1, x_2, x_3^0)} \quad (23)$$

$$B^0 = B(x_1, x_2, x_3^0)$$

$$g^0 = h_1^0 h_2^0 h_3^0 \quad (24)$$

so that

$$B = \frac{B^0}{\lambda_1 \lambda_2} \quad (25)$$

We also define the length elements

$$ds^{\alpha\beta} = h_1^\alpha h_2^\beta ds_3, \quad (26)$$

and split the total time derivative d/dt

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\underline{v} \cdot \nabla), \quad (27)$$

and define

$$S_p = \sigma_p B^2 \quad S_h = \sigma_h B^2 \quad (28)$$

With these definitions, the potential equation becomes

$$\begin{aligned} \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{h_2^0}{h_1^0 B_0^2} \int S_p ds^{13} \frac{\partial \Psi}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\frac{h_1^0}{h_2^0 B_0^2} \int S_p ds^{31} \frac{\partial \Psi}{\partial x_2} \right) \right\} \\ + \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{1}{B_0^2} \int S_h ds^{22} \frac{\partial \Psi}{\partial x_2} \right) - \frac{\partial}{\partial x_2} \left(\frac{1}{B_0^2} \int S_h ds^{22} \frac{\partial \Psi}{\partial x_1} \right) \right\} \end{aligned} \quad (29)$$

$$\begin{aligned}
& + \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{h_2^0}{h_1^0 B_0} \int \rho T_p ds^{13} \frac{\partial^2 \psi}{\partial x_1 \partial t} \right) + \frac{\partial}{\partial x_2} \left(\frac{h_1^0}{h_2^0 B_0} \int \rho T_p ds^{31} \frac{\partial^2 \psi}{\partial x_2 \partial t} \right) \right. \\
& - \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{1}{B_0} \int \rho T_h ds^{22} \frac{\partial^2 \psi}{\partial x_2 \partial t} \right) + \frac{\partial}{\partial x_2} \left(\frac{1}{B_0} \int \rho T_h ds^{22} \frac{\partial^2 \psi}{\partial x_1 \partial t} \right) \right\} \\
& = - \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left[\frac{h_2^0}{B_0} \int \rho T_p (\underline{v} \cdot \nabla) \frac{ds^{12}}{h_1 B} \frac{\partial \psi}{\partial x_1} \right] - \frac{\partial}{\partial x_2} \left[\frac{h_1^0}{B_0} \int \rho T_p (\underline{v} \cdot \nabla) \frac{ds^{21}}{h_2 B} \frac{\partial \psi}{\partial x_2} \right] \right\} \\
& + \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left[\frac{h_2^0}{B_0} \int \rho T_h (\underline{v} \cdot \nabla) \frac{ds^{12}}{h_2 B} \frac{\partial \psi}{\partial x_2} \right] - \frac{\partial}{\partial x_2} \left[\frac{h_1^0}{B_0} \int \rho T_h (\underline{v} \cdot \nabla) \frac{ds^{21}}{h_1 B} \frac{\partial \psi}{\partial x_1} \right] \right\} \\
& + \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{h_2^0}{B_0} \int S_p v_2^n ds^{12} \right) - \frac{\partial}{\partial x_2} \left(\frac{h_1^0}{B_0} \int S_p v_1^n ds^{21} \right) \right\} \\
& - \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{h_2^0}{B_0} \int S_h v_1^n ds^{12} \right) - \frac{\partial}{\partial x_2} \left(\frac{h_1^0}{B_0} \int S_h v_2^n ds^{21} \right) \right\} \\
& + \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{h_2^0}{B_0} \int \rho F p_2 ds^{12} \right) - \frac{\partial}{\partial x_2} \left(\frac{h_1^0}{B_0} \int \rho F p_1 ds^{21} \right) \right\} \\
& + \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{h_2^0}{B_0} \int \rho F h_1 ds^{12} \right) + \frac{\partial}{\partial x_2} \left(\frac{h_1^0}{B_0} \int \rho F h_2 ds^{21} \right) \right\}
\end{aligned}$$

The only integrals which are not cast in the standard form are the transport terms. We define

$$T_\alpha = - (\underline{v} \cdot \nabla) v_\alpha = - (\underline{v} \cdot \partial) v_\alpha - \sum_\beta (c_{\alpha\beta} v_\alpha v_\beta - c_{\beta\alpha} v_\beta^2) \quad (30)$$

using equation (4).

Furthermore, we note that

$$v_1 = - \frac{1}{B h_2} \frac{\partial \psi}{\partial x_2} = - f h_1 \frac{\partial \psi}{\partial x_2} \quad (31)$$

$$v_2 = \frac{1}{B h_1} \frac{\partial \Psi}{\partial x_1} = f h_2 \frac{\partial \Psi}{\partial x_1} \quad (32)$$

and define

$$u_1 = - f \frac{\partial \Psi}{\partial x_2} \quad (33)$$

$$u_2 = f \frac{\partial \Psi}{\partial x_1} \quad (34)$$

The u_1, u_2 are independent of x_3 . It follows that

$$v_1 \frac{\partial}{\partial x_1} + v_2 \frac{\partial}{\partial x_2} = u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2} \quad (35)$$

and

$$(v_1 \frac{\partial}{\partial x_1} + v_2 \frac{\partial}{\partial x_2}) \Psi = 0 \quad (36)$$

since Ψ is constant along flow lines. With this Eq. (30) becomes

$$T_\alpha = - h_\alpha (u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2}) u_\alpha - \sum_\beta (2 c_{\alpha\beta} v_\alpha v_\beta - c_{\beta\alpha} v_\beta^2). \quad (37)$$

Defining the following integration elements

$$ds_{\gamma\delta}^{\alpha\beta} = c_{\gamma\delta} ds^{\alpha\beta}, \quad (38)$$

the transport terms can then be written in the form

$$\begin{aligned}
T_1^p = & -\frac{h_1^0}{B_0} \left\{ \int \rho T_p ds^{31} \left(u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2} \right) u_1 \right. \\
& + h_1^0 \int \rho T_p ds^{41} u_1^2 + 2 h_2^0 \int \rho T_p ds^{32} u_1 u_2 + 2 \int \rho T_p v_3 ds^{31} u_1 \\
& \left. - \frac{h_2^0}{h_1^0} \int \rho T_p ds^{23} u_2^2 - \frac{1}{h_1^0} \int \rho T_p v_3^2 ds^{21} \right\} \quad (39)
\end{aligned}$$

$$T_2^p = -\frac{h_2^0}{B_0} \left\{ \int \rho T_p ds^{13} \left(u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2} \right) u_2 \right. \quad (40)$$

$$\begin{aligned}
& + h_2^0 \int \rho T_p ds^{14} u_2^2 + 2 h_1^0 \int \rho T_p ds^{23} u_1 u_2 + 2 u_2 \int \rho T_p v_3 ds^{13} \\
& \left. - \frac{h_1^0}{B_0 h_2^0} \int \rho T_p ds^{32} u_1^2 - \frac{1}{h_2^0} \int \rho T_p v_3^2 ds^{12} \right\} \\
T_1^h = & -\frac{h_2^0}{B_0} \left\{ \int \rho T_h ds^{22} \left(u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2} \right) u_1 \right. \quad (41)
\end{aligned}$$

$$\begin{aligned}
& + h_1^0 \int \rho T_h ds^{32} u_1^2 + 2 h_2^0 \int \rho T_h ds^{23} u_1 u_2 + 2 \int \rho T_h v_3 ds^{22} u_1 \\
& \left. - \frac{h_2^0}{h_1^0} \int \rho T_h ds^{14} u_2^2 - \frac{1}{h_1^0} \int \rho T_h v_3^2 ds^{12} \right\} \\
T_2^h = & -\frac{h_1^0}{B_0} \left\{ \int \rho T_h ds^{22} \left(u_1 \frac{\partial}{\partial x_1} + u_2 \frac{\partial}{\partial x_2} \right) u_2 \right. \quad (42)
\end{aligned}$$

$$\begin{aligned}
& + h_2^0 \int \rho T_h ds^{23} u_2^2 + 2 h_1^0 \int \rho T_h ds^{32} u_1 u_2 + 2 \int \rho T_h v_3 ds^{22} u_2 \\
& \left. - \frac{h_1^0}{h_2^0} \int \rho T_h ds^{41} u_1^2 - \frac{1}{h_2^0} \int \rho T_h v_3^2 ds^{21} \right\} .
\end{aligned}$$

We define the quantities

$$E_{11} = \frac{1}{B_0} \int S_p ds^{13} \quad E_{22} = \frac{1}{B_0} \int S_p ds^{31} \quad E_{12} = \frac{1}{B_0} \int S_h ds^{22} \quad (43)$$

$$C_{11} = \frac{1}{B_0} \int \rho T_p ds^{13} \quad C_{22} = \frac{1}{B_0} \int \rho T_p ds^{31} \quad C_{12} = \frac{1}{B_0} \int \rho T_h ds^{22} \quad (44)$$

$$D_1^p = \frac{1}{B_0} \int S_p v_1^n ds^{21} \quad D_2^p = \frac{1}{B_0} \int S_p v_2^n ds^{12} \quad (45)$$

$$D_1^h = \frac{1}{B_0} \int S_h v_1^n ds^{12} \quad D_2^h = \frac{1}{B_0} \int S_h v_2^n ds^{21} \quad (46)$$

$$F_1^p = \frac{1}{B_0} \int \rho F_{p1} ds^{21} \quad F_2^p = \frac{1}{B_0} \int \rho F_{p2} ds^{12} \quad (47)$$

$$F_1^h = \frac{1}{B_0} \int \rho F_{h1} ds^{12} \quad F_2^h = \frac{1}{B_0} \int \rho F_{h2} ds^{21} \quad (48)$$

The potential equation for Ψ can then be written in the final compact form

$$\begin{aligned} & \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{h_2^0}{h_1} E_{11} \frac{\partial \Psi}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\frac{h_1^0}{h_2} E_{22} \frac{\partial \Psi}{\partial x_2} \right) \right\} \\ & + \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(E_{12} \frac{\partial \Psi}{\partial x_2} \right) - \frac{\partial}{\partial x_2} \left(E_{12} \frac{\partial \Psi}{\partial x_1} \right) \right\} \\ & + \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} \left(\frac{h_2^0}{h_1} C_{11} \frac{\partial^2 \Psi}{\partial x_1 \partial t} \right) + \frac{\partial}{\partial x_2} \left(\frac{h_1^0}{h_2} C_{22} \frac{\partial^2 \Psi}{\partial x_2 \partial t} \right) \right\} \\ & - \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_1} C_{12} \frac{\partial^2 \Psi}{\partial x_2 \partial t} \right\} - \frac{\partial}{\partial x_2} \left\{ C_{12} \frac{\partial^2 \Psi}{\partial x_1 \partial t} \right\} \end{aligned} \quad (49)$$

$$= \frac{1}{g_0} \frac{\partial}{\partial x_1} \{h_2^0 [D_2^p + T_2^p + F_2^p - D_1^h + T_1^h + F_1^h]\}$$

$$- \frac{1}{g_0} \left\{ \frac{\partial}{\partial x_2} [h_1^0 [D_1^p + T_1^p + F_1^p + D_2^h - T_2^h - F_2^h]] \right\}$$

V. APPLICATION TO DIPOLE COORDINATES

For the electrostatic approximation we assume an undisturbed geomagnetic field which is taken to be a pure dipole field. Therefore, we use the appropriate dipole coordinate system (η, ϕ, σ) where

$$\eta = \frac{r}{\sin^2 \theta} ; \phi ; \sigma = \frac{\cos \theta}{r^2} \quad (50)$$

where r is the radius, ϕ the magnetic longitude, θ the magnetic colatitude. An orthogonal coordinate system is defined by the geometrical h factors, which we derive next.

We note that

$$\frac{\partial \eta}{\partial r} = \frac{1}{\sin^2 \theta} \quad \frac{\partial \sigma}{\partial r} = -\frac{2 \cos \theta}{r^3}$$

$$\frac{\partial \eta}{\partial \theta} = -\frac{2 r \cos \theta}{\sin^3 \theta} \quad \frac{\partial \sigma}{\partial \theta} = -\frac{\sin \theta}{r^2}, \quad (51)$$

which gives the Jacobian D

$$D = -\frac{1}{r^2 \sin^3 \theta} (1 + 3 \cos^2 \theta). \quad (52)$$

The inverse matrix is

$$\frac{\partial r}{\partial \eta} = \frac{\sin^4 \theta}{1 + 3 \cos^2 \theta}; \quad \frac{\partial r}{\partial \sigma} = -\frac{2 r^3 \cos \theta}{1 + 3 \cos^2 \theta}$$

$$\frac{\partial \theta}{\partial \eta} = -\frac{2 \sin^3 \theta \cos \theta}{r (1 + 3 \cos^2 \theta)}; \quad \frac{\partial \theta}{\partial \sigma} = -\frac{r^2 \sin \theta}{1 + 3 \cos^2 \theta} \quad (53)$$

Using this matrix one finds that

$$h_\eta = \frac{\sin^3 \theta}{(1 + 3 \cos^2 \theta)^{1/2}}; \quad h_\phi = r \sin \theta; \quad h_\sigma = \frac{r^3}{(1 + 3 \cos^2 \theta)^{1/2}} \quad (54)$$

Furthermore it can be shown that

$$\partial_\eta r = \frac{\sin \theta}{(1 + 3 \cos^2 \theta)^{1/2}}; \quad \partial_\sigma r = -\frac{2 \cos \theta}{(1 + 3 \cos^2 \theta)^{1/2}}$$

$$r \partial_\eta \theta = -\frac{2 \cos \theta}{(1 + 3 \cos^2 \theta)^{1/2}}; \quad r \partial_\sigma \theta = -\frac{\sin \theta}{(1 + 3 \cos^2 \theta)^{1/2}} \quad (55)$$

Since the geometry is independent of ϕ the $c_{\alpha\phi}$ are zero. After some algebra one finds the remaining c 's,

$$c_{\eta\eta} = -\frac{12 \cos^2 \theta (1 + \cos^2 \theta)}{r \sin \theta (1 + 3 \cos^2 \theta)^{3/2}} \quad (56)$$

$$c_{\eta\sigma} = -\frac{6 \cos \theta (1 + \cos^2 \theta)}{r (1 + 3 \cos^2 \theta)^{3/2}} \quad (57)$$

$$c_{\phi\eta} = \frac{1 - 3 \cos^2 \theta}{r \sin \theta (1 + 3 \cos^2 \theta)^{1/2}} \quad (58)$$

$$c_{\phi\sigma} = -\frac{3 \cos \theta}{r (1 + 3 \cos^2 \theta)^{1/2}} \quad (59)$$

$$c_{\sigma\eta} = \frac{3 \sin \theta (1 + \cos^2 \theta)}{r (1 + 3 \cos^2 \theta)^{3/2}} \quad (60)$$

The coefficient for the gravity is

$$g_n = -g_{00} \left(\frac{r_0}{r}\right)^2 \frac{\sin \theta}{(1 + 3 \cos^2 \theta)^{1/2}}; \quad g_0 = g_{00} \left(\frac{r_0}{r}\right)^2 \frac{\cos \theta}{(1 + 3 \cos^2 \theta)^{1/2}} \quad (61)$$

For high altitude magnetospheric problems one can set $v_{en} = 0$. Then with $\Omega_i = eB/m_i = \Omega \lambda$ and $b = (v_{in}/\lambda)\eta_{ie} = v_{ie}/\Omega_i = v/(\Omega_i \lambda^2)$ and $\omega_{ie} = \Omega_i + \eta_{ie} v_{in}$ one finds

$$T_p = \frac{\Omega_i \omega_{ie}}{\omega_{ie}^2 + v_{in}^2}; \quad T_{pe} = \frac{\Omega_i \omega_{ie} + v_{in}^2}{\omega_{ie}^2 + v_{in}^2} \quad (62)$$

$$T_h = \frac{\Omega_i v_{in}}{\omega_{ie}^2 + v_{in}^2}; \quad T_{he} = \frac{\eta_{ie} v_{in}^2}{\omega_{ie}^2 + v_{in}^2} \quad (63)$$

$$S_p = \rho v_{in} T_p; \quad S_h = -\rho v_{in} T_h \quad (64)$$

Furthermore we note that

$$\underline{f}_i = \underline{\xi} - \frac{1}{\rho} \nabla p_i; \quad \underline{f}_e = -\frac{1}{\rho} \nabla p_e \quad (65)$$

which leads to

$$\underline{F}_p = T_p \underline{\xi} - \frac{1}{\rho} (T_p \nabla p_i + T_{pe} \nabla p_e); \quad \underline{F}_h = T_h \underline{\xi} - \frac{1}{\rho} (T_h \nabla p_i - T_{he} \nabla p_e). \quad (66)$$

We choose the geomagnetic equator ($\sigma = 0$) as a reference plane. At the equator one has

$$h_1^0 = 1; \quad h_2^0 = n; \quad h_3^0 = 1; \quad g^0 = n; \quad B^0 = A/n^3 \quad (67)$$

and

$$u_1 = -\frac{\eta^2}{A_0} \frac{\partial \Psi}{\partial \Phi}; \quad u_2 = \frac{\eta^2}{A_0} \frac{\partial \Psi}{\partial \eta}. \quad (68)$$

Introducing the earth's rotation ω , which means replacing v_ϕ by $v_\phi + \omega r \sin\phi$ or u_ϕ by $u_\phi + \omega$, one obtains for the transport terms

$$\begin{aligned} T_\eta^p = & -\frac{1}{B_0} \left\{ \int \rho T_p ds^{31} (u_\eta \frac{\partial}{\partial \eta} + u_\phi \frac{\partial}{\partial \phi}) u_\eta + \int \rho T_p ds_{\eta\eta}^{41} u_\eta^2 \right. \\ & \left. + 2 \int \rho T_p v_\sigma ds_{\eta\sigma}^{31} u_\eta - \eta^2 \int \rho T_p ds_{\phi\eta}^{23} (u_\phi + \omega)^2 - \int \rho T_p v_\sigma^2 ds_{\sigma\eta}^{21} \right\} \end{aligned} \quad (69)$$

$$\begin{aligned} T_\phi^p = & -\frac{\eta}{B_0} \left\{ \eta \int \rho T_p ds^{13} (u_\eta \frac{\partial}{\partial \eta} + u_\phi \frac{\partial}{\partial \phi}) u_\phi \right. \\ & \left. + 2 \int \rho T_p ds_{\phi\eta}^{23} (u_\phi + \omega) u_\eta + 2 \int \rho T_p v_\sigma ds_{\phi\sigma}^{13} (u_\phi + \omega) \right\} \end{aligned} \quad (70)$$

$$\begin{aligned} T_\eta^h = & -\frac{1}{B_0} \left\{ \int \rho T_h ds^{22} (u_\eta \frac{\partial}{\partial \eta} + u_\phi \frac{\partial}{\partial \phi}) u_\eta + \int \rho T_h ds_{\eta\eta}^{32} u_\eta^2 \right. \\ & \left. + 2 \int \rho T_h v_\sigma ds_{\eta\sigma}^{22} u_\eta - \eta^2 \int \rho T_h ds_{\phi\eta}^{14} (u_\phi + \omega)^2 - \int \rho T_h (v_\sigma^2 + T_i + T_e) ds_{\sigma\eta}^{12} \right\} \end{aligned} \quad (71)$$

$$\begin{aligned} T_\phi^h = & -\frac{\eta}{B_0} \left\{ \int \rho T_h ds^{22} (u_\eta \frac{\partial}{\partial \eta} + u_\phi \frac{\partial}{\partial \phi}) u_\phi \right. \\ & \left. + 2 \int \rho T_h ds_{\phi\eta}^{32} (u_\phi + \omega) u_\eta + 2 \int \rho T_h v_\sigma ds_{\phi\sigma}^{22} (u_\phi + \omega) \right\}. \end{aligned} \quad (72)$$

The final equation can now be written as

$$\begin{aligned}
 & \frac{1}{n} \frac{\partial}{\partial n} \left(n E_{nn} \frac{\partial \Psi}{\partial n} \right) + \frac{1}{2} \frac{\partial}{\partial \phi} \left(E_{\phi\phi} \frac{\partial \Psi}{\partial \phi} \right) \\
 & - \frac{1}{n} \left(\frac{\partial E_{n\phi}}{\partial n} \frac{\partial \Psi}{\partial \phi} - \frac{\partial E_{n\phi}}{\partial \phi} \frac{\partial \Psi}{\partial n} \right) \\
 & + \left[\frac{1}{n} \frac{\partial}{\partial n} \left(n C_{nn} \frac{\partial^2 \Psi}{\partial n \partial t} \right) + \frac{1}{2} \frac{\partial}{\partial \phi} \left(C_{\phi\phi} \frac{\partial^2 \Psi}{\partial \phi \partial t} \right) \right] \\
 & + \frac{1}{n} \left[\frac{\partial C_{n\phi}}{\partial n} \frac{\partial^2 \Psi}{\partial \phi \partial t} - \frac{\partial C_{n\phi}}{\partial \phi} \frac{\partial^2 \Psi}{\partial n \partial t} \right] \\
 & = \frac{1}{n} \frac{\partial}{\partial n} \left[n \left(D_{\phi}^p + T_{\phi}^p + F_{\phi}^p - D_n^h + T_n^h + F_{\phi}^h \right) \right] \\
 & - \frac{1}{n} \frac{\partial}{\partial \phi} \left[D_n^p + T_n^p + F_n^p + D_{\phi}^h - T_{\phi}^h - F_{\phi}^h \right],
 \end{aligned} \tag{73}$$

where all symbols have been previously defined.

VI. SUMMARY

We have presented a set of equations for large scale simulation studies of plasma systems in the electrostatic approximation. The equations are written for a generalized geometry as well as for a dipole geometry which is suitable for the earth's magnetosphere.

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