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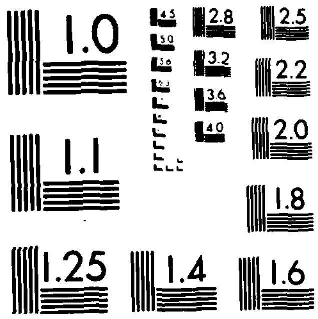
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The purpose of this proposal was to define and determine an R-reliable interval and its associated confidence on the sum of two continuous, independent, random variables with different scale parameters. The relationship between the confidence of the R-reliable interval on the sum and the confidences of the R-reliable intervals on the summand variables was also investigated.

The confidence of the R-reliable interval on the sum was defined, and the largest order statistics were chosen which simplified the expression for the confidence of the R-reliable interval on the sum. The choice of the largest order statistics led to nonparametric results for the confidences of the R-reliable intervals on the summand variables.

Exponential and folded normal continuous distributions were considered, and numerical values of the confidences associated with the R-reliable interval on the sum were (CONTINUED)

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ABSTRACT

ONE-SIDED R-RELIABLE INTERVALS AND THEIR ASSOCIATED CONFIDENCES ON THE SUM OF TWO CONTINUOUS, INDEPENDENT, RANDOM VARIABLES

Dr. S. A. Patil

The purpose of this proposal is to define and determine an R-reliable interval and its associated confidence on the sum of two continuous, independent, random variables with different scale parameters. The relationship between the confidence of the R-reliable interval on the sum and the confidences of the R-reliable intervals on the summand variables was also investigated.

The confidence of the R-reliable interval on the sum was defined, and the largest order statistics were chosen which simplified the expression for the confidence of the R-reliable interval on the sum. The choice of the largest order statistics led to nonparametric results for the confidences of the R-reliable intervals on the summand variables.

Exponential and folded normal continuous distributions were considered, and numerical values of the confidences associated with the R-reliable interval on the sum were obtained for selected sample sizes and selected ratios of the scale parameters. A distribution-free bound for the confidence of the R-reliable interval on the sum in terms of the confidences associated with the R-reliable intervals on the summand variables was obtained. The monotonicity of the confidence of the R-reliable interval on the sum was established for specific values of sample sizes.

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1. INTRODUCTION

Wilks [1] investigated the R-reliable interval [L,U] with Neyman-Peason confidence coefficient C such that

$$P\left[\int_{L}^{U} f(x)dx \geq R\right] = C(R) \tag{1.1}$$

where L,U are functions of a random sample from the distribution of x and f(s) is the probability density function (pdf) of X. He obtained a nonparametric R-reliable interval for X. Wilks [2] has given an extensive survey of order statistics and their applications to R-reliable intervals and has given an extensive list of references on the subject. Guenther [3] discussed the problem of R-reliable intervals for Normal, Gamma, and Poisson distributions. More recently, this subject has also been investigated by Locasso [4] and Bevensee [5]. If L and U are statistics (functions of a random sample), then a definition of a Neyman-Pearson R-reliable interval is given by

$$P[P(L \leq x \leq U) \geq R] = C(R). \tag{1.2}$$

This definition is equivalent to the definition given by Wilks [1]. He referred to this interval as a "100 R% tolerance interval." Guenther [3] called it "R-content interval." In the present development, we prefer to follow the more recent notation of Locasso [4] and Bevensee [5], and so we refer to it as an R-reliable interval. Suppose Y denotes a random variable which is the (negative of the logarithm of the) stress on a component of a complex system (such as an airplane), and Z denotes the random variable which is the (logarithm of the) threshold of failure of components of that type to that kind of stress. Also, the random variables Y and Z are independent. Then $X = Y + Z$ is the safety margin of components of that kind to stress of that type. Also, if Y,Z denote the lifetime of two identical components of a system in which the second component takes over when the first fails, then the reliability of the system can be related to the reliability of the distribution of X. We are interested in finding an R-reliable interval on the sum and its associated confidence, knowing the R-reliable intervals for Y and Z and their associated confidences.

Suppose $Y_1, Y_2, \ldots, Y_m$, $Z_1, Z_2, \ldots, Z_m$ denote random samples, from the distributions of Y and Z, respectively. Let T_y and T_z be some statistics from these samples. Then for reliabilities R_y, $R_z > 0$, the R-reliable interval for Y is $(0, T_y)$ and its associated confidence is given by

$$P[P(0 < y < T_y) \geq R_y] = C_y(R_y). \tag{1.3}$$

Similarly, the R-reliable interval for Z is $(0, T_z)$ and its confidence is determined by

$$P[P(0 < Z < T_Z) \geq R_Z] = C_Z(R_Z). \qquad (1.4)$$

If the statistic T is a function of order statistics, then the confidences in the equations (1.3) and (1.4) can be determined irrespective of the underlying distributions of Y and Z; and therefore, they are distribution free. Our objective is to find an R-reliable interval on the sum X and its associated confidence for different distributions and determine a non-parametric bound on the confidence in terms of the confidences of the R-reliable intervals of the summand variables. Ashley [6] stated a non-parametric bound on the confidence for the R-reliable interval on the sum X as

$$C_X(R_YR_Z) = P[P(0 < X < T_Y + T_Z) \geq R_YR_Z] \geq C_Y(R_Y)C_Z(R_Z). \quad (1.5)$$

This inequality relates the confidences on R-reliable intervals of Y and Z with the confidence on the R-reliable interval of X. Also it is non-parametric because it does not assume anything about the distribution of Y and Z. However, the product $C_Y(R_Y)$ (C_ZR_Z) is not a satisfactory bound because $C_Y(R_Y)$ $C_Z(R_Z)$ becomes too small. Also, the product R_YR_Z decreases rapidly and the reliability on X becomes too small. The above R-reliable interval could be strengthened if in the inequality (1.5), the product R_YR_Z is replaced by the significantly larger value $\min\{R_Y, R_Z\}$. Thus, inequality (1.5) becomes

$$C_X(\min\{R_Y, R_Z\}) = P[P(0 < X < T_m + T_n)] \geq \min\{R_Y, R_Z\}$$
$$\geq C_Y(R_Y)C_Z(R_Z) \quad . \qquad (1.6)$$

The inequality (1.6) increases the reliability but the associated confidence $C_Y(R_Y)C_Z(R_Z)$ is unchanged. Hence, a larger confidence such as $\min(C_Y(R_Y), C_Z(R_Z))$ would be more useful.

In this development, we focus our attention on the problems of obtaining non-parametric bound on the confidence of the sum C_X and determining the exact confidence on the R-reliable interval of X for various distributions of Y and Z. Sepcifically, section 2 deals with the mathematical development of determining C_X when Y and Z have different scale parameters.

In the following sections, a procedure for comparing the confidence of the sum with those of the summands is discussed. The exact confidences are obtained and tabulated for various distributions and the confidences are compared numerically. The R-reliable interval for distribution on $(-\infty,\infty)$ are briefly discussed. Finally, a conclusion is given and some unsolved problems are mentioned.

2. MATHEMATICAL DEVELOPMENT

In this section, we consider the mathematical theory that leads to the development of the R-reliable interval and its associated confidence.

Suppose the random variable Y is distributed on $(0,\infty)$

with the probability density function (pdf) $1/\theta_y f(y/\theta_y)$, where θ_Y is the scale parameter. Similarly, the random variable Z is distributed on $(0,\infty)$ with pdf $1/\theta_z f(z/\theta_z)$. Further, Y and Z are independent. We define X to be the sum of Y and Z, then an R-reliable interval on X is to be determined. Let Y_1, Y_2, . . ., Y_m, and Z_1, Z_2, . . ., Z_n be random samples of sizes m,n from the distribution with pdf's $1/\theta_y f(y/\theta_y)$ and $1/\theta_z f(z/\theta_z)$ respectfully. Let the statistics T_Y and T_Z be defined as

$$T_Y = T_Y(Y_1, Y_2, \ldots, Y_m)$$

and

$$T_Z = T_Z(Z_1, Z_2, \ldots, Z_n).$$

Then for a given reliability $R > 0$, the R-reliable interval for Z is $(0, T_Y + T_Z)$ and its associated confidence is determined by

$$C_X(A,m,n) = P[P[0 < X < T_Y + T_Z] \geq R] \qquad (2.1)$$

where $A = \theta_Z/\theta_Y \geq 1$. Note that if $\theta_Z/\theta_Y \leq 1$, Y and Z are interchanged and this interchange results in $A \geq 1$. Since $X = Y + Z$, we have

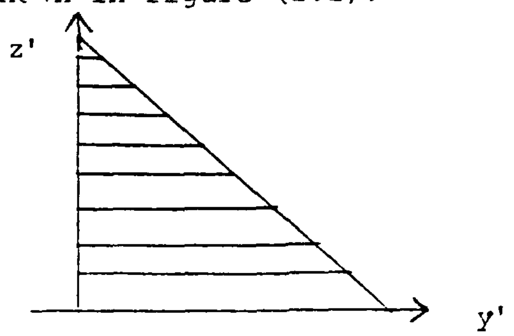
$$\begin{aligned} C_X(A,m,n) &= P[P[0 < Y + Z < T_Y + T_Z] \geq R] \\ &= P[P[0 < \frac{Y}{Y} + \frac{Z}{Y} < \frac{T_Y}{Y} + \frac{T_Z}{Y}] \geq R] \\ &= P[P[0 < \frac{Y}{Y} + \frac{AZ}{Z} < \frac{T_Y}{Y} + \frac{AT_Z}{Z}] \geq R] \end{aligned}$$

$$C_X(A, m, n) = P[P[0 < Y' + AZ' < T_Y' + AT_Z'] \geq R] \tag{2.2}$$

where $Y' = \frac{Y}{\theta_Y}$, $Z' = \frac{Z}{\theta_Z}$, $T_Y' = \frac{T_Y}{\theta_Y}$, and $T_Z' = \frac{T_Z}{\theta_Z}$. The random variables Y' and Z' are independent and identically distributed. The distributions of T_Y' and T_Z' do not depend on θ_Y, θ_Z and are the statistics from the random samples from the distributions of Y' and Z'. For fixed value of $T_Y' + AT_Z'$, equation (2.2) can be expressed in terms of quantile of the distribution of $Y' + AZ' = X'$. The distribution of X' can be written as

$$G(x') = P[X' \leq x'] = P[Y' + AZ' \leq x'] \,, \tag{2.3}$$

which is the probability of y' and z' lying in the triangular region shown in figure (2.1).



Since Y' and Z' are independent and identically distributed, the equation (2.3) is expressed in an integral form, as

$$G(x') = \int\int_{0 \leq y' + Az' \leq x'} f(y) f(z) \, dy dz \,. \tag{2.4}$$

Integrating on y, we obtain

$$G(x') = \int_0^{x'/A} f(z)F(x' - Az)dz \;. \tag{2.5}$$

Similarly, integrating equation (2.4) with respect to z, first we find

$$G(x') = \int_0^{x'} f(y)F\left(\frac{x'-y}{A}\right)dy \tag{2.6}$$

where F is the distribution function of Y'. Equations (2.5) and (2.6) can be used to find the quantile of the distribution of X. Let $\xi_A(R) = \xi$ be the R^{th} quantile of the distribution of X'. Then ξ can be determined by

$$G(\xi) = P[0 < X' \leq \xi] = R \;. \tag{2.7}$$

For a given T_Y' and T_Z', the statement

$$P[0 < Y' + AZ' < T_Y' + AT_Z'] \geq R \tag{2.8}$$

implies $T_Y' + AT_Z' > \xi$. Hence, the confidence $C_X(A,m,n)$ can be written as

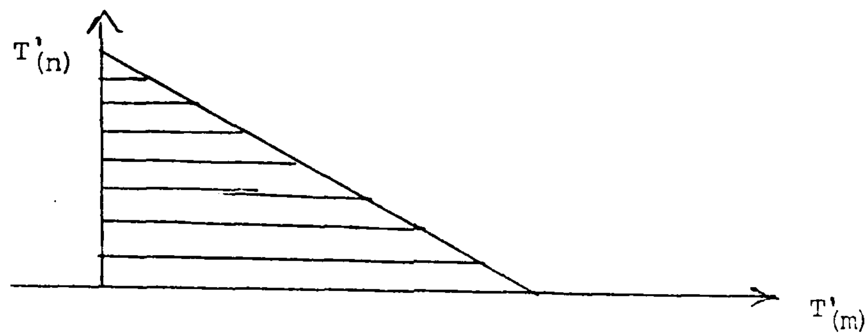
$$\begin{aligned} C_X(A,m,n) &= P[T_Y' + AT_Z' > \xi] \\ &= 1 - P[T_Y' + AT_Z' \leq \xi] \;. \end{aligned} \tag{2.9}$$

There are a number of choices for the statistics T_Y' and T_Z' such as range, midrange, order statistics, mean, etc. However, one is usually interested in knowing what percentage of the probability lies below the mean or the largest order statistics.

In this development, the largest order statistics are chosen because the corresponding R-reliable interval for Y and Z are distribution free. Also, the associated confidence of the sum X could be compared with those of Y and Z. Then, the R-reliable interval for X becomes $(0, T_{(m)} + T_{(n)})$, where $T_{(m)}$ and $T_{(n)}$ are the largest order statistics from the random sample from the distributions of Y and Z. The confidence associated with this interval is

$$C_X(A,m,n) = 1 - P(T'_{(m)} + AT'_{(n)} \leq \xi) \quad . \qquad (2.?)$$

where $T'_{(m)} = \frac{T_{(m)}}{\theta_Y}$ and $T'_{(n)} = \frac{T_{(n)}}{\theta_Z}$. The probability $P[T'_{(m)} + AT'_{(n)} \leq \xi]$ implies that $T'_{(m)}$ and $T'_{(n)}$ lie inside the triangular region of figure (2.2).



Then equation (2.10) reduces to

$$C_X(A,m,n) = 1 - n \int_0^{\xi/A} [F(z)]^{n-1} [F(\xi - Az)]^m f(z)\,dz \qquad (2.11)$$

or equivalently

$$C_X(A,m,n) = 1 - m \int_0^{\xi} [F(y)]^{m-1}[F(\frac{\xi-y}{A})]^n f(y)dy \qquad (2.12)$$

Remark: For given sample sizes m and n, the confidence $C_X(A,m,n)$ depends only on the parameter A, and therefore, $C_X(A,m,n)$ could be computed for different values of m,n,A and various distributions.

3. COMPARISON OF CONFIDENCES

In this section, we compare the R-reliable intervals for Y and Z with X. The R-reliable interval depends on reliability R and the sample sizes. For this reason, we compare the R-reliable intervals for a given value of R. For a given value of R, this is accomplished by comparing the corresponding confidences associated with these intervals.

The distribution free R-reliable interval for Y for a given sample size m is $(0, Y_{(m)})$ with the associated confidence C_Y determined by

$$C_Y = P[P(0 < Y < Y_{(m)}) \geq R] = 1 - R^m . \qquad (3.1)$$

Similarly the R-reliable interval for Z is $(0, \xi_{(m)})$ with confidence C_Z as

$$C_Z = 1 - R^n . \qquad (3.2)$$

For comparison purposes, we define the following:

Definition 1. Two R-reliable intervals are equivalent, for a given value of R, if the corresponding confidences are equal.

Definition 2. An R-reliable interval I_1 with confidence $C_1(A)$ is said to be better than another R-reliable I_2 with confidence $C_2(A)$ if for a given value of R, $C_1(A) \geq C_2(A)$ for all A, and $C_1(A) > C_2(A)$ for some A.

Definition 3. Two-R-reliable intervals I_1 and I_2 with associated confidences $C_1(A)$ and $C_2(A)$ are asymptotically equivalent if

$$\lim_{A\to\infty} C_1(A) = \lim_{A\to\infty} C_2(A) \quad . \tag{3.3}$$

<u>Lemma 1</u>. The limit as A approaches ∞ of ξ_A/A is $F_Z^{-1}(R)$.

Proof: We have

$$\int_0^{\xi_A} f(z)F(\frac{\xi_A}{A} - \frac{z}{A})dz = R$$

$$\lim_{A\to\infty} \int_0^{\xi_A} f(z)F(\frac{\xi_A}{A} - \frac{z}{A})dz = R \; .$$

Let $\lim_{A\to\infty} \frac{\xi_A}{A} = \xi_Z$, then

$$\lim_{A\to\infty} \int_0^{\xi_A} f(z)F(\frac{\xi_A}{A} - \frac{z}{A})dz = \int_0^{\infty} f(z)F(\xi_Z)dz \tag{3.4}$$

$$= F(\xi_Z) = R,$$

hence

$$F^{-1}(R) = \xi_Z \; . \tag{3.5}$$

We use lemma 1 to prove the following theorem.

<u>Theorem 1</u>. The R-reliable interval on the sum X is asymptotically equivalent to the R-reliable interval of Z.

Proof: From equation (2.11), we have

$$C_X(A,r,n) = 1 - n \int_0^{\xi_A} f(z)[F(z)]^{n-1}[F(\xi_A - Az)]^r dz \tag{3.6}$$

and from lemma 1, $\lim_{A\to\infty} (\xi_A/A) = \xi_Z$.

Taking the limit as $A\to\infty$ of equation (3.6),

$$\lim_{A\to\infty}(C_X(A,m,n)) = 1 - n \lim_{A\to\infty} \int_0^{\xi_A/A} f(z)[F(z)]^{n-1}[F(A(\frac{\xi_A}{A} - z))]^m dz$$

$$= 1 - n \int_0^{\xi_Z} f(z)[F(z)]^{n-1}F(\infty)dz$$

$$= 1 - n \int_0^{\xi_Z} f(z)[F(z)]^{n-1}dz$$

$$= 1 - [F(\xi_Z)]^n = 1 - R^n = C_Z(n) \quad . \qquad (3.7)$$

Lemma 2: If $h(X)$ and $g(X)$ are increasing functions of a random variable X, then the covariance between $h(X)$ and $g(X)$ is positive.

Proof: Let x_0 be a value of x such that

$$g(X) - E[g(X)] \leq 0 \text{ for } x \leq x_0 \ .$$

Then

$$\text{cov}(h(X),g(X)) = \int_{-\infty}^{\infty} h(X)(g(X) - E[g(X)])dF(X)$$

$$= \int_{-\infty}^{x_0} h(x)(g(X) - E[g(X)])dF(X)$$

$$+ \int_{x_0}^{\infty} h(x)(g(X) - E[g(X)])dF(X) \ .$$

Since $h(x)$ is increasing then

$$\text{cov}(h(X),g(X)) \geq \int_{-\infty}^{x_0} h(x_0)(g(X) - E[g(X)])dF(x) -$$

$$-h(x_0)\int_{x_0}^{\infty}(g(x)-E[g(X)])dF(X)=h(x_0)\int_{-\infty}^{\infty}(g(X)-E[g(X)])dF(X)=0$$

where $F(X)$ denotes the distribution function of X.

Corallary 1. If $h(X)$ is an increasing function of X and $g(X)$ is a decreasing function of X then the covariance between $h(X)$ and $g(X)$ is negative.

Proof: We note that $-g(X)$ is an increasing function of X and $\mathrm{cov}(h(X), -g(X)) \geq 0$ from lemma 1. Hence, $\mathrm{cov}\ (h(X),g(X)) \geq 0$.

Theorem 2. For any sample size m and for n = 1, the R-reliable interval for X is better than the R-reliable interval for Z.

Proof: From equation (2.11) with n = 1, we obtain

$$C_X(A,m,1) = 1 - \int_0^{\xi_A/A} f(z)[F(\xi_A - A\ )]^m dz\ . \qquad (3.8)$$

In order to show that R-reliable interval for X is better than that of Z, we show that $C_X(A,m,n)$ is a decreasing function of A.

$$\frac{d}{dA}C_X(A,m,1) = -m\int_0^{\xi_A/A} f(z)[F(\xi_A-Az)]^{m-1}\ f(\xi_A-Az)(\frac{d\xi}{dA}-z)dz\ . \qquad (3.9)$$

To find $d\xi_A/dA$, we let m = 1 in equation (3.8). Then

$$C_X(A,1,1) = 1 - \int_0^{\xi_A/A} f(z)F(\xi_A - Az)dz$$

$$= 1 - R \qquad \text{for all } A. \qquad (3.10)$$

$$\frac{d}{dA} C_X(A,1,1) = -\int_0^{\xi_A/A} f(z)f(\xi_A - Az)(\frac{d\xi_A}{dA} - z)dz = 0.$$

Therefore

$$\frac{d\xi_A}{dA} = \frac{\int_0^{\xi_A/A} zf(z)f(\xi_A - Az)dz}{\int_0^{\xi_A/A} f(z)f(\xi_A - Az)dz} . \qquad (3.11)$$

Now $\frac{d}{dA}(C_X(A,m,1)) \le 0$ if

$$\frac{d\xi_A}{dA} \ge \frac{\int_0^{\xi_A/A} zf(z)[F(\xi_A - Az)]^{m-1} f(\xi_A - Az)dz}{\int_0^{\xi_A/A} f(z)[F(\xi_A - Az)]^{m-1} f(\xi_A - Az)dz} \qquad (3.12)$$

or

$$\frac{\int_0^{\xi_A/A} zf(z)f(\xi_A - Az)dz}{\int_0^{\xi_A/A} f(z)f(\xi_A - Az)dz} \ge \frac{\int_0^{\xi_A/A} z[F(\xi_A - Az)]^{m-1}f(z)f(\xi_A - Az)dz}{\int_0^{\xi_A/A} [F(\xi_A - Az)]^{m-1}f(z)f(\xi_A - Az)dz} \qquad (3.13)$$

equivalently

$$\frac{\int_0^{\xi_A/A} [F(\xi_A - Az)]^{m-1}f(z)f(\xi_A - Az)dz}{\int_0^{\xi_A/A} f(z)f(\xi_A - Az)dz}$$

$$- \frac{\int_0^{\xi_A/A} [F(\xi_A - Az)]^{m-1}f(z)f(\xi_A - Az)dz}{\int_0^{\xi_A/A} f(z)f(\xi_A - Az)dz}$$

$$+ \frac{\int_0^{\xi_A/A} zf(z)f(\xi_A - Az)dz}{\int_0^{\xi_A/A} f(z)f(\xi_A - Az)dz} - 0$$

or $\operatorname{cov}(z,[F(\xi_A - Az)]^{m-1}) \leq 0$. Since $[F(\xi_A - Az)]^{m-1}$ is a decreasing function of z, from Corollary 1, $\operatorname{cov}(z,[F(\xi_A - Az)]^{m-1}) \leq 0$. Therefore $\frac{d}{dA}(C_X(A,m,1)) \leq 0$ and $C_X(A,m,1)$ is a decreasing function of A. Also from Theorem 1, $C_X(A,m,1)$ is asymptatically equivalent to $C_Z(1)$. Hence R-reliable interval for X is better than that of Z.

Remark: For n = 1 and any m, $C_X(A,m,1)$ is a decreasing function of A for $A \geq 1$. Hence $C_X(1,m,1)$ is the maximum value that $C_X(A,m,1)$ can attain. Also, A = 1 implies that Y and Z are identically distributed, therefore, $C_X(1,m,1)$ is the upper bound for $C_X(A,m,1)$.

<u>Theorem 3</u>. For any sample size n and for m = 1, the R-reliable interval for X is better than the R-reliable interval for Y.

Proof: We find

$$1 - C_X(A,1,n) = n \int_0^{\xi_A/A} [F(z)]^{n-1} [F(\xi_A - Az)] f(z) dz$$

$$= \int_0^{\xi_A} [F(\frac{\xi_A - y}{A})]^n f(y) dy \; . \tag{3.14}$$

From the right hand side of equation (3.14), it is clear that the integral is a decreasing function of n, hence

$$n \int_0^{\xi_A/A} [F(z)]^{n-1} [F(\xi_A - Az)] f(z) dz < \int_0^{\xi_A/A} F(\xi_A - Az) f(z) dz$$

or

$$1 - n \int_0^{\xi_A/A} [F(z)]^{n-1} F(\xi_A - Az) f(z) dz > 1 - \int_0^{\xi_A/A} F(\xi_A - Az) f(z) dz$$

$$= 1 - R.$$

Thus,

$$C_X(A,1,n) > 1 - R = C_Y(1).$$

Remark: Theorem 2 could be proved the same way as Theorem 3. It can be shown the $C_X(A,1,n)$ is an increasing function of A, and $C_X(\Lambda,1,n)$ is the minimum value that $C_X(A,1,n)$ can attain.

Now the general case where the sample sizes m and n are both greater than one is considered. For this purpose, we prove the following lemma.

<u>Lemma 3.3</u>. For any m and n, $C_X(A,m,n)$ is an increasing function of m and n.

Proof:

$$1 - C_X(A,m,n) = n \int_0^{\xi_A/A} [F(z)]^{n-1} [F(\xi_A - Az)]^m f(z)dz. \qquad (3.15)$$

Since $F(\xi_A - Az) < 1$ for all $z < \frac{\xi_A}{A}$, then $[F(\xi_A - Az)]^m$ is a decreasing function of m. Hence, $C_X(A,m,n)$ is an increasing function of m. Equivalently from equation (2.12), we can write

$$1 - C_X(A,m,n) = m \int_0^{\xi_A} [F(y)]^{m-1} [F(\frac{\xi_A - y}{A})]^n f(y)dy. \qquad (3.16)$$

Again, since $F(\frac{\xi_A - y}{A}) < 1$ for all $y < \xi_A$, then $[F(\frac{\xi_A - y}{A})]^n$ is a decreasing function of n. Therefore $C_X(A,m,n)$ is an increasing function of n. Thus, $C_X(A,m,n)$ is an increasing function of m and n.

<u>Theorem 4</u>. For any sample sizes m and n greater than 1, the R-reliable interval for the sum is better than the R-reliable interval for the summand with the smaller sample size.

Proof: $s = \min(m,n)$, then from Lemma 3.3 we get

$$C_X(A,m,n) \geq C_X(A,s,s) \text{ for all } A. \tag{3.17}$$

Since
$$\begin{aligned} C_X(A,m,n) &= P[P[0 < X < Y_{(m)} + Z_{(n)}] \geq R] \\ &= P[P[0 < X' < Y'_{(m)} + AZ'_{(n)}] \geq R] \\ &= P[Y'_{(m)} + AZ'_{(n)} > \xi_0] \\ &= 1 - P[Y'_{(m)} + AZ'_{(n)} \leq \xi] \end{aligned}$$

and
$$\begin{aligned} C_X(A,s,s) &= P[P[0 < X' < Y'_{(s)} + AZ'_{(s)}] \geq R] \\ &= P[Y'_{(s)} + AZ'_{(s)} > \xi] \\ &= 1 - P[Y'_{(s)} + AZ'_{(s)} \leq \xi]. \end{aligned}$$

Then the inequality (3.17) becomes

$$C_X(A,m,n) = 1 - P[Y'_{(m)} + AZ'_{(n)} \leq \xi] \geq 1 - P[Y'_{(s)} + AZ'_{(s)} \leq \xi]$$

$$1 - C_X(A,m,n) = P[Y'_{(m)} + AZ'_{(n)} \leq \xi] \leq P[Y'_{(s)} + AZ'_{(s)} \leq \xi]. \tag{3.18}$$

Since $Y'_{(s)} + AZ'_{(s)} \geq X'_{(s)}$ where $X'_{(s)}$ is the largest of the set $Y'_1 + AZ'_1$, $Y'_2 + AZ'_2$, ..., $Y'_m + AZ'_n$, then

$$[Y'_{(s)} + AZ'_{(s)} \leq \xi] \subseteq [X'_{(s)} \leq \xi].$$

Hence,

$$1-C_X(A,m,n) \leq P[Y'_{(s)} + AZ'_{(s)} \leq \xi] \leq P[X'_{(s)} \leq \xi] = 1-C_X(A,s,s).$$

4. CONFIDENCES ON THE R-RELIABLE INTERVALS FOR EXPONENTIAL AND NORMAL DISTRIBUTION

The bounds considered in section three are nonparametric and can be used for all continuous distributions on $(0,\infty)$. However the bounds do not give the confidences for specific values of A and other parameters. In this section, confidences $C_X(A, m, n)$ of the R-reliable interval for the sum $X = Y + Z$ is calculated when Y, Z have exponential or normal distributions. These distributions are widely used in applications and hence are chosen in finding confidences. Since we are considering the distribution on $(0,\infty)$ for the normal case, we consider the folded normal distribution. The p. d. f. of the folded normal equation with scale $1/\sigma$ is given by

$$f(y) = \frac{2}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(y/\sigma)^2} \quad , \ 0 < y > \infty.$$

In order to find $C_X(A, m, n)$ at $R = .9$, the 90th percentile of the distribution of $X' = Y' + AZ'$ is found using distributions of Y' and Z'. Here Y', Z' are distributions with scale parameters one. The integral expression for (2.11) is used to calculate the confidences $C_X(A, m, n)$. The sample sizes m and n chosen to be $m = 1, 2, 4, 6, 8, 10$, and $n = 1, 2, 4, 6, 8$, and 10, and A is taken from 1 to 100 for exponential distribution and A is taken from 1 to 20 for normal distributions. For larger values of A, in the case of normal distribution, the confidences $C_X(A, m, n)$ do not vary much. We first consider the exponential distribution.

4.1. Exponential Distribution

The pdf of the random variable Y' distributed as exponential is given by

$$f(y') = \exp(-y') , \qquad 0 < y' < \infty ,$$

$$= 0 \text{ elsewhere} , \qquad (4.1)$$

and the distribution function of Y' is

$$F_{Y'}(y') = 0, \qquad y' < 0 ,$$

$$= 1 - \exp(-y') , \qquad 0 \leq y' < \infty . \qquad (4.2)$$

The distribution of Z' is identical to that of Y'. Using the independence of Y' and Z' and their pdf's, the distribution function of $X' = Y' + AZ'$ is found to be

$$G_{X'}(x') = \int_0^{x'/A} \int_0^{x'-Az'} \exp(-y') \exp(-z') \, dy' \, dz'$$

$$= 1 + \frac{\exp(-x') - A \exp(-x'/A)}{A - 1} , \qquad 0 \leq x' < \infty , \qquad (4.3)$$

and $G_{X'}(x')$ is zero for $x' < 0$. With $x' = \xi_{.9}$, the 90^{th} percentile of X' is found by

$$G_{X'}(\xi_{.9}) = 1 + \frac{\exp(-\xi_{.9}) - A \exp(-\xi_{.9}/A)}{A - 1} = .9 , \qquad (4.4)$$

and the confidence $C_X(m,n)$ using Equation (2.11) and the distribution functions and pdf's of Y' and Z' becomes

$$C_X(m,n) = 1 - n\int_0^{\xi_{.9}/A} [1 - \exp(-z')]^{n-1} \left\{1 - \exp[-(\xi_{.9} - Az')]\right\}^m$$

$$\exp(-z')\,dz' . \tag{4.5}$$

Values of $C_X(m,n)$ and $\xi_{.9}$ for selected A, m, and n are tabulated in Table 1.

4.2 Folded Normal Distribution

Let Y' and Z' be independent and identically distributed as folded normals on $(0,\infty)$. The pdf of Y' is given by

$$f(y') = \frac{2}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}y'^2) , \qquad 0 < y' < \infty ,$$

$$= 0 \text{ elsewhere} , \tag{4.6}$$

and the distribution function of Y' is

$$F_{Y'}(y') = 0 , \qquad y' < 0 ,$$

$$= \int_0^{y'} \frac{2}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}t^2)\,dt , \qquad 0 \le y' < \infty . \tag{4.7}$$

Table 1. The Confidence $C_X(A,m,n)$ of the R-Reliable Interval on the Sum of Two Continuous Random Variables with Exponential Distributions with 90 Percent Reliability

n = 1 A	ξ	m = 1	2	4	6	8	10
1.	3.89	0.10	0.16	0.25	0.33	0.39	0.44
2.	5.94	0.10	0.13	0.18	0.21	0.24	0.26
3.	8.12	0.10	0.12	0.15	0.17	0.18	0.20
4.	10.36	0.10	0.11	0.13	0.15	0.16	0.16
5.	12.63	0.10	0.11	0.13	0.13	0.14	0.15
6.	14.91	0.10	0.11	0.12	0.13	0.13	0.14
7.	17.20	0.10	0.11	0.12	0.12	0.13	0.13
8.	19.49	0.10	0.11	0.11	0.12	0.12	0.13
9.	21.78	0.10	0.11	0.11	0.12	0.12	0.12
10.	24.08	0.10	0.11	0.11	0.12	0.12	0.12
20.	47.08	0.10	0.10	0.11	0.11	0.11	0.11
30.	70.09	0.10	0.10	0.10	0.10	0.11	0.11
40.	93.12	0.10	0.10	0.10	0.10	0.10	0.10
50.	116.14	0.10	0.10	0.10	0.10	0.10	0.10
100.	231.26	0.10	0.10	0.10	0.10	0.10	0.10
∞	∞	0.10	0.10	0.10	0.10	0.10	0.10

n = 2 A	ξ	m = 1	2	4	6	8	10
1.	3.89	0.16	0.24	0.36	0.45	0.52	0.57
2.	5.94	0.18	0.23	0.30	0.35	0.39	0.42
3.	8.12	0.19	0.22	0.27	0.30	0.32	0.34
4.	10.36	0.19	0.21	0.25	0.27	0.28	0.30
5.	12.63	0.19	0.21	0.23	0.25	0.26	0.27
6.	14.91	0.19	0.21	0.23	0.24	0.25	0.26
7.	17.20	0.19	0.20	0.22	0.23	0.24	0.25
8.	19.49	0.19	0.20	0.22	0.23	0.23	0.24
9.	21.78	0.19	0.20	0.21	0.22	0.23	0.23
10.	24.08	0.19	0.20	0.21	0.22	0.22	0.23
20.	47.08	0.19	0.19	0.20	0.20	0.21	0.21
30.	70.09	0.19	0.19	0.20	0.20	0.20	0.20
40.	93.12	0.19	0.19	0.20	0.20	0.20	0.20
50.	116.14	0.19	0.19	0.19	0.20	0.20	0.20
100.	231.26	0.19	0.19	0.19	0.19	0.19	0.19
∞	∞	0.19	0.19	0.19	0.19	0.19	0.19

Table 1. (continued)

n = 4 A	ξ	m = 1	2	4	6	8	10
1.	3.89	0.25	0.36	0.50	0.60	0.67	0.72
2.	5.94	0.32	0.39	0.48	0.54	0.58	0.62
3.	8.12	0.33	0.38	0.45	0.49	0.52	0.55
4.	10.36	0.34	0.38	0.42	0.46	0.48	0.50
5.	12.63	0.34	0.37	0.41	0.43	0.45	0.47
6.	14.91	0.34	0.37	0.40	0.42	0.43	0.45
7.	17.20	0.34	0.36	0.39	0.41	0.42	0.43
8.	19.49	0.34	0.36	0.38	0.40	0.41	0.42
9.	21.78	0.34	0.36	0.38	0.39	0.40	0.41
10.	24.08	0.34	0.36	0.38	0.39	0.40	0.40
20.	47.08	0.34	0.35	0.36	0.37	0.37	0.37
30.	70.09	0.34	0.35	0.35	0.36	0.36	0.36
40.	93.12	0.34	0.35	0.35	0.35	0.36	0.36
50.	116.14	0.34	0.35	0.35	0.35	0.35	0.36
100.	231.26	0.35	0.35	0.35	0.35	0.35	0.35
∞	∞	0.34	0.34	0.34	0.34	0.34	0.34

n = 6 A	ξ	m = 1	2	4	6	8	10
1.	3.89	0.33	0.45	0.60	0.69	0.76	0.80
2.	5.94	0.42	0.50	0.60	0.66	0.71	0.74
3.	8.12	0.45	0.51	0.58	0.62	0.65	0.68
4.	10.36	0.46	0.50	0.56	0.59	0.62	0.64
5.	12.63	0.46	0.50	0.54	0.57	0.59	0.61
6.	14.91	0.46	0.49	0.53	0.55	0.57	0.58
7.	17.20	0.47	0.49	0.52	0.54	0.56	0.57
8.	19.49	0.47	0.49	0.52	0.53	0.55	0.56
9.	21.78	0.47	0.49	0.51	0.53	0.54	0.55
10.	24.08	0.47	0.49	0.51	0.52	0.53	0.54
20.	47.08	0.47	0.48	0.49	0.49	0.50	0.50
30.	70.09	0.47	0.47	0.48	0.49	0.49	0.49
40.	93.12	0.47	0.47	0.48	0.48	0.48	0.49
50.	116.14	0.47	0.47	0.48	0.48	0.48	0.48
100.	231.26	0.47	0.47	0.47	0.47	0.47	0.48
∞	∞	0.47	0.47	0.47	0.47	0.47	0.47

Table 1 . (continued)

n = 8	A	ξ	m = 1	2	4	6	8	10
	1.	3.89	0.39	0.52	0.67	0.76	0.82	0.86
	2.	5.94	0.51	0.59	0.69	0.75	0.79	0.82
	3.	8.12	0.54	0.60	0.67	0.72	0.75	0.77
	4.	10.36	0.56	0.60	0.66	0.69	0.72	0.73
	5.	12.63	0.56	0.60	0.64	0.67	0.69	0.71
	6.	14.91	0.56	0.60	0.63	0.66	0.67	0.69
	7.	17.20	0.57	0.59	0.63	0.65	0.66	0.67
	8.	19.49	0.57	0.59	0.62	0.64	0.65	0.66
	9.	21.78	0.57	0.59	0.61	0.63	0.64	0.65
	10.	24.08	0.57	0.59	0.61	0.62	0.63	0.64
	20.	47.08	0.57	0.58	0.59	0.60	0.60	0.61
	30.	70.09	0.57	0.58	0.58	0.59	0.59	0.59
	40.	93.12	0.57	0.57	0.58	0.58	0.59	0.59
	50.	116.14	0.57	0.57	0.58	0.58	0.58	0.58
	100.	231.26	0.57	0.57	0.57	0.57	0.58	0.58
	∞	∞	0.57	0.57	0.57	0.57	0.57	0.57

n = 10	A	ξ	m = 1	2	4	6	8	10
	1.	3.89	0.44	0.57	0.72	0.80	0.86	0.89
	2.	5.94	0.58	0.66	0.76	0.81	0.84	0.87
	3.	8.12	0.62	0.68	0.75	0.79	0.81	0.83
	4.	10.36	0.63	0.68	0.73	0.76	0.79	0.80
	5.	12.63	0.64	0.68	0.72	0.75	0.77	0.78
	6.	14.91	0.64	0.68	0.71	0.73	0.75	0.76
	7.	17.20	0.65	0.67	0.71	0.72	0.74	0.75
	8.	19.49	0.65	0.67	0.70	0.72	0.73	0.74
	9.	21.78	0.65	0.67	0.69	0.71	0.72	0.73
	10.	24.08	0.65	0.67	0.69	0.70	0.71	0.72
	20.	47.08	0.65	0.66	0.67	0.68	0.68	0.69
	30.	70.09	0.65	0.66	0.67	0.67	0.67	0.68
	40.	93.12	0.65	0.66	0.66	0.67	0.67	0.67
	50.	116.14	0.65	0.66	0.66	0.66	0.66	0.67
	100.	231.26	0.65	0.65	0.66	0.66	0.66	0.66
	∞	∞	0.65	0.65	0.65	0.65	0.65	0.65

Let $G_{X'}(x')$ be the distribution function of $X' = Y' + AZ'$; then $G_{X'}(x')$ is found to be

$$G_{X'}(x') = 0 \;, \qquad x' < 0 \;,$$

$$= \int_0^{x'/A} \int_0^{x'-Az'} \frac{2}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}y'^2) \cdot \frac{2}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}z'^2) \quad dy'\, dz' \;,$$

$$0 \leq x' < \infty \;. \tag{4.8}$$

Equation (4.8) is used to find the 90^{th} percentile of the distribution of X'. The expression for $C_X(m,n)$ using Equation (2.11) and the pdf and the distribution function of Z' becomes

$$C_X(m,n) = 1 - m \int_0^{\xi_{.9}/A} \left[\int_0^{z'} \frac{2}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}t^2) \; dt \right]^{n-1}$$

$$\cdot \left[\int_0^{\xi_{.9}-Az'} \frac{2}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}t^2) \, dt \right]^m \quad \frac{2}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}z'^2) \quad dz' \;. \tag{4.9}$$

The values for $\xi_{.9}$ and $C_X(m,n)$ are calculated for selected A, m, and n through Equations (4.8) and (4.9). These values are seen in Table 1. The normal distribution is the last of the symmetric distributions which are considered here, because it is by far the most widely used symmetric distribution. This distribution is also folded so that it could be used in this study.

Table 2. The Confidence $C_X(A,m,n)$ of the R-Reliable Interval on the Sum of Two Continuous Random Variables with Folded Normal Distributions with 90 Percent Reliability

n = 1 A	ξ	m = 1	2	4	6	8	10
1.	2.77	0.10	0.17	0.26	0.32	0.37	0.42
2.	4.27	0.10	0.14	0.18	0.21	0.23	0.25
3.	5.87	0.10	0.13	0.15	0.17	0.18	0.19
4.	7.49	0.10	0.12	0.14	0.15	0.16	0.17
5.	9.13	0.10	0.12	0.13	0.14	0.15	0.15
6.	10.77	0.10	0.11	0.13	0.13	0.14	0.14
7.	12.42	0.10	0.11	0.12	0.13	0.13	0.14
8.	14.06	0.10	0.11	0.12	0.13	0.13	0.13
9.	15.71	0.10	0.11	0.12	0.12	0.13	0.13
10.	17.36	0.10	0.11	0.12	0.12	0.12	0.13
20.	33.88	0.10	0.11	0.11	0.11	0.11	0.11
∞	∞	0.10	0.10	0.10	0.10	0.10	0.10

n = 2 A	ξ	m = 1	2	4	6	8	10
1.	2.77	0.16	0.26	0.39	0.47	0.54	0.59
2.	4.27	0.18	0.24	0.31	0.36	0.39	0.42
3.	5.87	0.18	0.23	0.27	0.30	0.32	0.34
4.	7.49	0.19	0.22	0.25	0.27	0.29	0.30
5.	9.13	0.19	0.21	0.24	0.25	0.27	0.28
6.	10.77	0.19	0.21	0.23	0.24	0.25	0.26
7.	12.42	0.19	0.20	0.22	0.23	0.24	0.25
8.	14.06	0.19	0.20	0.22	0.23	0.24	0.24
9.	15.71	0.19	0.20	0.22	0.22	0.23	0.23
10.	17.36	0.19	0.20	0.21	0.22	0.23	0.23
20.	33.88	0.19	0.19	0.20	0.20	0.21	0.21
∞	∞	0.19	0.19	0.19	0.19	0.19	0.19

Table 2 (continued)

n = 4 A	ξ	m = 1	2	4	6	8	10
1.	2.77	0.25	0.39	0.55	0.65	0.72	0.76
2.	4.27	0.31	0.40	0.51	0.57	0.61	0.64
3.	5.87	0.33	0.39	0.46	0.51	0.53	0.56
4.	7.49	0.33	0.38	0.44	0.47	0.49	0.51
5.	9.13	0.34	0.37	0.42	0.44	0.46	0.47
6.	10.77	0.34	0.37	0.40	0.43	0.44	0.45
7.	12.42	0.34	0.37	0.40	0.41	0.42	0.43
8.	14.06	0.34	0.36	0.39	0.40	0.41	0.42
9.	15.71	0.34	0.36	0.38	0.40	0.41	0.41
10.	17.36	0.34	0.36	0.38	0.39	0.40	0.41
20.	33.88	0.34	0.35	0.36	0.36	0.37	0.37
∞	∞	0.34	0.34	0.34	0.34	0.34	0.34

n = 6 A	ξ	m = 1	2	4	6	8	10
1.	2.77	0.32	0.47	0.65	0.75	0.82	0.85
2.	4.27	0.42	0.52	0.64	0.70	0.74	0.77
3.	5.87	0.44	0.52	0.60	0.64	0.67	0.70
4.	7.49	0.45	0.51	0.57	0.61	0.63	0.65
5.	9.13	0.46	0.50	0.55	0.58	0.60	0.61
6.	10.77	0.46	0.50	0.54	0.56	0.58	0.59
7.	12.42	0.46	0.49	0.53	0.55	0.56	0.57
8.	14.06	0.46	0.49	0.52	0.54	0.55	0.56
9.	15.71	0.46	0.49	0.51	0.53	0.54	0.55
10.	17.36	0.46	0.48	0.51	0.52	0.53	0.54
20.	33.88	0.46	0.47	0.49	0.49	0.50	0.50
∞	∞	0.47	0.47	0.47	0.47	0.47	0.47

Table 2 (continued)

n = 8 A	ξ	m = 1	2	4	6	8	10
1.	2.77	0.37	0.54	0.72	0.81	0.86	0.90
2.	4.27	0.50	0.61	0.72	0.78	0.82	0.85
3.	5.87	0.53	0.61	0.70	0.74	0.77	0.79
4.	7.49	0.55	0.61	0.67	0.71	0.73	0.75
5.	9.13	0.55	0.60	0.65	0.68	0.70	0.72
6.	10.77	0.56	0.60	0.64	0.67	0.68	0.69
7.	12.42	0.56	0.59	0.63	0.65	0.67	0.68
8.	14.06	0.56	0.59	0.62	0.64	0.65	0.66
9.	15.71	0.56	0.59	0.62	0.63	0.65	0.65
10.	17.36	0.56	0.59	0.61	0.63	0.64	0.65
20.	33.88	0.56	0.58	0.59	0.60	0.60	0.61
∞	∞	0.57	0.57	0.57	0.57	0.57	0.57

n = 10 A	ξ	m = 1	2	4	6	8	10
1.	2.77	0.41	0.59	0.76	0.85	0.90	0.93
2.	4.27	0.56	0.68	0.79	0.84	0.87	0.90
3.	5.87	0.61	0.69	0.77	0.81	0.83	0.85
4.	7.49	0.62	0.69	0.75	0.78	0.80	0.82
5.	9.13	0.63	0.68	0.73	0.76	0.78	0.79
6.	10.77	0.64	0.68	0.72	0.74	0.76	0.77
7.	12.42	0.64	0.67	0.71	0.73	0.75	0.76
8.	14.06	0.64	0.67	0.70	0.72	0.73	0.74
9.	15.71	0.64	0.67	0.70	0.71	0.73	0.73
10.	17.36	0.64	0.67	0.69	0.71	0.72	0.73
20.	33.88	0.64	0.66	0.67	0.68	0.68	0.69
∞	∞	0.65	0.65	0.65	0.65	0.65	0.65

5. CONCLUSION AND RECOMMENDATIONS

The numerical calculations enable us to find the values for the confidences on the sum of two continuous, independent, random variables for various distributions. These values of $C_X(A,m,n)$ are useful when the exact confidences are required. But, to obtain these values not only the ratio of the scale parameters of the summand variables have to be estimated but also their distributions have to be known.

A nonparametric bound for the confidence of the R-reliable interval on the sum was found based on the largest order statistics. This was the result of Theorem 4. stating that

$$C_X(A,m,n) \geq \min\ [C_Y(m),\ C_Z(n)] = \min\ (1 - R^m,\ 1 - R^n)$$

which depends only on the sample sizes and the reliability but not on the estimate of the variances or the distributions of Y and Z. Also, the monotonicity of $C_X(A,m,n)$ with A was proved for $n = 1$, $m > 1$ and for $m = 1$, $n > 1$. This research could be extended by considering a two-sided R-reliable interval for Y and Z and, therefore, a two-sided R-reliable interval for their sum X. Other statistics such as the range or the max $[|Y_{(1)}|, |Y_{(m)}|]$ can be used which might lead to a better bound for the confidence of the R-reliable interval on the sum. The tables of the values of $C_X(A,m,n)$ suggest that the confidence on the sum is decreasing with A for equal sample sizes greater than one. This result would be useful because it suggests that for equal sample sizes greater than one, the best confidence on the sum X is obtained when the distributions of summand variables Y and Z are identical. Study of particular classes of distributions such as the Exponential class defined on $(0, \infty)$ or the Symmetric class defined on $(-\infty, \infty)$ might lead to more specific results for the confidence on the sum which may be useful for those distributions.

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