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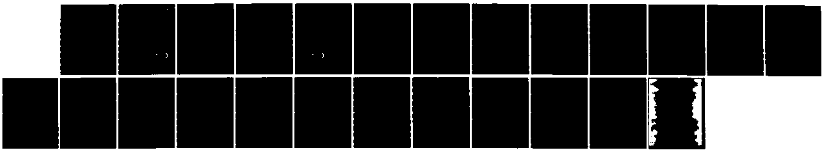

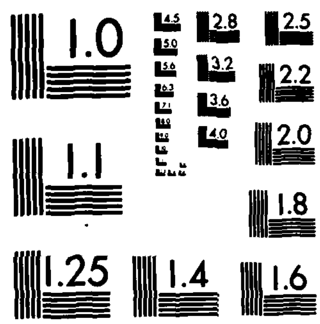

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Constructing and Evaluating Models for Predicting Visibility for Data-Void Locations in Norway Using Weighted Least Squares.

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AIR FORCE GEOPHYSICS LABORATORY
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UNITED STATES AIR FORCE
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The Weibull distribution was used to model visibility at 51 Norway stations. Weighted least squares was used to investigate which variables, when incorporated into the coefficients of α and β, would give a better fit. The model demonstrated that, as in Germany, incorporation of elevation and relative elevation into the variables model gives dramatically lower RMS over the constants model. Inclusion of latitude and longitude did not seem to improve the model significantly. The prediction ability of the model at data-void locations was estimated using sample re-use and indicated the model will be reasonably accurate.

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Constructing and Evaluating Models for Predicting Visibility for Data-Void Locations in Norway Using Weighted Least Squares

by

L. A. Franklin, P. N. Somerville, and S. J. Bean
University of Central Florida

1. INTRODUCTION

To be able to state the probability that a weather element (e.g., visibility, ceiling, etc.) will have a value above a specified threshold for any location is a goal of the Air Weather Service. Many models have been developed where records exist. However, it is more difficult to construct models to estimate weather elements where no data exists. Some models for estimating visibility for Germany have been developed by Bean and Somerville that require only knowledge of the elevation and the average elevation at 20 kilometers. These models were developed using the method of non-linear regression. Also, the accuracy of predicting visibility at data-void locations in Germany was measured by sample re-use again utilizing non-linear regression. This paper models visibility at 51 Norway locations utilizing the method of weighted least squares to explore many more possible models including the possible incorporation of latitude and longitude as variables. Furthermore, the models are examined for their ability to estimate visibility at data-void locations in Norway by sample re-use utilizing weighted least squares.

2. BACKGROUND OF PREVIOUS WORK ON VISIBILITY

Bean and Somerville in AFGL-TR-81-0144 "Some Models for Visibility for German Stations" demonstrated that the visibility data was fit well by the Weibull distribution given by

$$F(x) = 1 - e^{-\alpha x^{\beta}}$$

where F(x) is the probability that visibility is less than x miles. Different values of α and β were derived for each month and each of eight 3-hour periods. The values α and β were chosen to give F(x) the closest fit to the empirical cumulative distribution in the least squares sense. That is, if $E_j (x_i)$ is the empirical probability (step function) or empirical cumulative distribution that the visibility is less than x_i at the j^{th} station as recorded in the RUSSWO's (Revised Uniform Summary of Surface Weather Observations), we then choose α_j and β_j so that the following expression is minimized:

$$\sum_j \sum_i [E_j (x_i) - F_j (x_i; \alpha_j, \beta_j)]^2 \tag{2.1}$$

That is, the values of α_j and β_j for each station are those that minimize the sum of squares of the distances between the empirical and model probabilities over all distances for which data is available (i.e., 14 different distances) and over all stations involved (i.e., 30 for Germany). This is done for each 3-hour period and each of the 12 months for which data is available in the RUSSWO's.

The parameters of the Weibull distribution, α_j and β_j, may themselves depend upon other variables. These variables may include other weather elements or information which (if known) would give better models for visibility. Such work was done by Somerville and Bean in AFGL-TR-81-0313, "Modeling Visibility for Locations in Germany When No Records Exist." However, in that paper many of the variables incorporated into α_j and β_j are information which would not normally be available at a "data-void" location. Hence, Bean and Somerville in AFGL-TR-82-0335, "Some New Practical Models for Visibility for Germany Locations," found that by incorporating the elevation of the location and relative elevation measured at 20 kilometers from the location good models for visibility were possible. The parameters for the Weibull distribution at the j^{th} station were given in that paper as

$$\alpha_j = \gamma_o + \gamma_1 * EL_j + \gamma_2 * AE_j$$
$$\beta_j = \delta_o + \delta_1 * EL_j + \delta_2 * AE_j$$

where EL_j is the cube of the elevation in feet, divided by 10^9, and AE_j is the cube of the average elevation in feet, divided by 10^9, as measured at 20 equispaced loca ons on a circle with radius 20 kilometers. For each 3-hour period and month a set of γ_o, γ_1, γ_2 and δ_o, δ_1, δ_2 were determined by minimizing expression (2.1). If all six constants are present, the model has been called the "variables model" and the constants have been found by non-linear regression and are recorded in AFGL-TR-82-0335. If, however, we have only $\alpha_j = \gamma_o$ and $\beta_j = \delta_o$, the model has been called the "constants model" and these have been found by non-linear regression and are recorded in AFGL-TR-81-0313 (when calculated from 30 German locations) and in AFGL-TR-82-0187 (when calculated from 60 German locations), which is another paper by Bean and Somerville entitled "Evaluation of An Observation-Based Climatology Model for Predicting Visibility for Data-Void Locations in Germany." The constants model effectively fits only one model to all stations, hence ignoring any geographical features.

The method of minimizing expression (2.1) has been the method of non-linear regression and has been discussed in detail in AFGL-TR-80-0362 "Least Squares Fitting of Distributions Using Non-Linear Regression." While the method has been extremely successful in fitting models and seems to display very robust features, it is based on an iterative solution incorporating an initial estimate of the parameters and hence is rather time consuming in the calculations.

Sample re-use has been used to evaluate the ability of a model to predict visibility at data-void locations and has been discussed in AFGL-TR-82-0335. Briefly, sample re-use takes a single station and uses all the other stations to obtain the fitted model. Then the fitted model is used to predict visibility at the omitted station and the root mean square error is calculated between that single station's empirical and predicted visibility. This is repeated for each station in turn and hence for the 30 Germany stations results in 30 times as many non-linear regressions as would be needed to fit all 30 stations at once.

In AFGL-TR-83-0248, "A Comparison of Several Alternatives to Maximum Likelihood for the Weibull Distribution," several other methods of estimation were compared to non-linear regression. In that simulation study the method of non-linear regression appeared to be the best method since it usually provided the lowest RMS. Also it seemed more robust than all other methods considered in that it provided a better model when that data was contaminated or when the true underlying distribution was not the form of the distribution chosen to model it. However, the method of weighted least squares, first suggested by Major Al Boehm, USAF, showed promise as being most cost effective since it provided reasonably good models and used only a fraction of the computer time that non-linear regression required.

In AFGL-TR-84-0132, "A Comparison of Non-Linear Regression and Weighted Least Squares for Predicting Visibility in Germany," Franklin, Somerville and Bean demonstrated that non-linear regression provided better models than weighted least squares whether measured by fitting all stations at once or by estimating through sample re-use. However, the authors felt that the time-saving features of weighted least squares could be utilized to advantage in the preliminary stages of model building and testing. The purpose of this report is to examine the data from 51 stations in Norway and utilize weighted squares to examine models and the possibility of incorporating other variables into the model to improve the ability to predict visibility at data-void regions.

3. METHODOLOGY OF MODELING VISIBILITY IN NORWAY UTILIZING WEIGHTED LEAST SQUARES

The method of weighted least squares is based on the log-linearization method. If E(x) is the empirical cumulative distribution function and x_1, x_2, ..., x_n are the ordered observations of the distances for visibility, then let

$$q_i = 1 - E(x_i) \tag{3.1}$$

and

$$\hat{q}_i = \exp\,(-\hat{\alpha}\ x_i^{\hat{\beta}}) \tag{3.2}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are estimates of α and β.

Then

$$\ln(-\ln \hat{q}_i) = \ln \hat{\gamma} + \hat{\beta} \ln x_i \tag{3.3}$$

We may regard this as a simple linear regression model with ln(-ln q) as the dependent variable and ln x as the independent variable and ordinary least squares can be used to obtain coefficients from which α and β may be estimated. Using this notation non-linear regression sought to minimize the expression (2.1) but written as

$$\sum_{i=1}^{n} (q_i - \hat{q}_i)^2 \tag{3.4}$$

The log-linearization method coupled with ordinary least squares seeks to minimize the expression

$$\sum_{i=1}^{n} (\ln(-\ln q_i) - \ln(-\ln \hat{q}_i))^2 \tag{3.5}$$

Since the sums of squares being minimized are different in equation (3.4) and (3.5) the estimates of α and β from log-linearization can be very different from the estimates derived from non-linear regression.

The method of weighted least squares seeks to weight equation (3.5) so it has the same value as (3.4). That is we seek w_i so that

$$w_i^2\,(\ln(-\ln q_i) - \ln(-\ln \hat{q}_i\,))^2 = (q_i - \hat{q}_i)^2 \qquad (3.6)$$

for each i.

Solving for w_i we find

$$\frac{1}{w_i} = \frac{\ln(-\ln q_i) - \ln(-\ln \hat{q}_i)}{q_i - \hat{q}_i} \qquad (3.7)$$

for each i.

Now as $q_i \to q_i$ we have

$$\frac{1}{w_i} \to \frac{d}{dq_i}\,(\ln(-\ln q_i)) \qquad (3.8)$$

thus, taking the derivative, we obtain

$$\frac{1}{w_i} = \frac{1}{-\ln q_i}\,\frac{1}{q_i} \qquad (3.9)$$

Hence using

$$w_i = -q_i \ln q_i \text{ for each } i \qquad (3.10)$$

we have approximate weights, w_i, that make the weighted least squares approximately equivalent to non-linear regression.

Now when we use weighted least squares and assume the α_j and β_j are functions of other variables, because of the form of the equation (3.3) we really have $\ln \alpha_j = \mu_0 + \mu_1 z_1 + \text{---} + \mu_\kappa z_\kappa$ (3.11)

and $\qquad \beta_j = \gamma_0 + \gamma_1 z_1 + \text{---} + \gamma_\kappa z_\kappa$

where $z_1, \ldots, z_\kappa$ are the variables in the model (e.g., relative elevation, elevation, latitude, longitude, etc.). Hence the form of these coefficients and their use is different from the coefficients derived by non-linear regression. The coefficients are <u>NOT</u> interchangeable.

4. RESULTS OF MODELING VISIBILITY IN NORWAY UTILIZING WEIGHTED LEAST SQUARES

The use of weighted least squares allowed many variables to be considered as possible variables in equations (3.11) for α_j and β_j in Norway. The variables tested for both α_j and β_j were:

Latitude, longitude, relative elevation at 10, 15, 20 and 25 kilometers respectively, (elevation), $(\text{elevation})^2$, $(\text{elevation})^3$, $(\text{elevation})^4$, $(\text{elevation})^5$, $(\text{elevation})^{\frac{1}{2}}$, $(\text{elevation} + 100)^{\frac{1}{2}}$, $(\text{elevation} + 500)^{\frac{1}{2}}$, ln(elevation + 1), ln(elevation + 500), ln(elevation + 1000). The variables were tested in groups by a stepwise regression program based on weighted least squares for each of the 12 months and each of the 8 three-hour periods. Those variables that frequently appeared significant in the models were carried over and included with the next group of variables.

Among the noteworthy variables were the following:

Latitude and longitude appeared as significant relatively often in all regressions. Of all the possible relative elevation variables, the measurements taken at 20 and 25 kilometers were significant quite often, with the measurements at 25 kilometers occurring slightly more frequently. However, since the model for Germany had already been worked out using relative elevation at 20 kilometers, it was decided to maintain that as the variable also for Norway. Of all the possible powers of elevation, the variable $(\text{elevation})^3$ appeared far more frequently than any other. It also was the variable that was chosen previously in the Germany study and hence was kept here too for Norway. Of all the other variables only two others seem to warrant further attention: $(\text{elevation} + 500)^{\frac{1}{2}}$ and ln(elevation + 1000).

Because of time constraints and a desire to keep some degree of comparability with the previous study on visibility in Germany, it was decided to only investigate models that would include relative elevation at 20 kilometers, elevation, latitude and longitude. There were five such models considered and their results are contained in this report:

Model 1: constants model.

Model 2: Variables model with relative elevation and elevation.

Model 3: Variables model with relative elevation, elevation, and latitude.

Model 4: Variables model with relative elevation, elevation, and longitude.

Model 5: Variables model with relative elevation, elevation, latitude and longitude.

Using weighted least squares the above five models were fitted to the data for all 51 Norway stations for each month and three-hour period of data. Exhibit 4.1 has the five RMS's calculated for each month and 6 of the hour combinations. The first two three-hour periods (00-02 LST and 03-05 LST) are excluded due to frequently bad data in those periods. The reader should note that while in general the more complex the model the lower the RMS, this is not always true in exhibit 4.1. This is in part due to the approximating nature of weighted least squares and the presence of the weights w_i as already discussed in the computation of the "approximately" best halves of α_j and β_j. The exhibit shows clearly that there is a dramatic improvement in Model 2 (variables model) over Model 1 (constants model) and that, surprisingly, the inclusion of latitude, longitude or both brings little if any further improvement in the RMS. Exhibit 4.2 displays the RMS for the five models but for each of the 51 stations averaged over all months and the 6 three-hour periods. The overall average RMS for the constants model was .272 while for the variables model it dropped to .108 which is nearly one-third as large. The inclusion of latitude and/or longitude decreases the RMS only negligibly.

When weighted least squares was used to calculate sample re-use the results were consistent with the results from the model fitted to all the stations. Exhibit 4.3 displays the average RMS for the five models utilizing sample re-use for all months and 6 three-hour periods. Exhibit 4.4 displays the average RMS for the five models utilizing sample re-use but for each of the 51 stations in Norway. Again, RMS from the constants model is more than twice the RMS of the variables model with little improvement upon addition of latitude and/or longitude to the model. The reader should also note that the RMS's obtained by sample re-use are generally larger than the corresponding RMS obtained by fitting all 51 stations. As noted in other reports, this implies that the RMS obtained by fitting all stations is actually an optimistic estimate when estimating visibility at a data-void location and that the RMS obtained by sample re-use is much more realistic in that capacity.

5. SUMMARY

The model developed for estimating probabilities of visibilities less than a specified distance at data-void locations in Germany incorporates elevation and relative elevation of the location of interest. The same model seems most effective in estimating probabilities of visibilities for Norway as well. Inclusion of elevation and relative elevation into the variables model brings a substantial decrease in RMS error when compared to the constants model. Further inclusion of latitude and/or longitude does not seem to help significantly in Norway. Two other variables appear that bear possible investigation in a Norway visibility model: (elevation + 500 feet)$^{\frac{1}{2}}$ and ln(elevation + 1000). These were not included in this study due to time considerations and a desire to test the same model as already had been fitted in Germany.

Values of the α and β coefficients are not included since it has been established that non-linear regression gives better estimates than the weighted least squares technique that was used here to explore potential models.

6. SUGGESTED DIRECTIONS FOR FURTHER RESEARCH

It is the unanimous position of the authors that much has been accomplished in modeling weather elements both for where data exists and for data-void locations. It is also their unanimous position that much remains that can be done.

First, weighted least squares can be utilized to reexamine models of visibility in Germany for possible improvement by inclusion of promising variables that were discovered in the Norway study.

Second, the modeling of visibility needs to be extended to other countries in Europe, first for individual countries and then to develop a single unified model of Europe, if possible, and, if not, to cluster similar countries by modeling similarities.

Third, the modeling of other important weather elements (e.g., ceiling, windspeed, precipitation) should be developed for data-void regions just as visibility has.

Fourth, while latitude and longitude did not seem to improve modeling of visibility in Norway, the model which includes one or both should be examined by non-linear regression to determine their true usefulness in Norway.

Fifth, weighted least squares should be utilized to examine the possibility of yet untried variables for inclusion in visibility modeling. For example, prevailing winds and their relationship to the nearest body of water and nearest mountain chain. It is recommended by the authors that this be undertaken using sample re-use to show which stations are most poorly predicted by the present model and then to have those stations examined for common properties that may be omitted from the present model.

It is hoped that these recommendations will stimulate continued research in the modeling of weather elements.

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Month & Method		Hour Period (LST) 06-08	09-11	12-14	15-17	18-20	21-23
	1	.253064	.248888	.246818	.269374	.254401	.129791
	2	.110570	.120881	.109833	.112210	.098715	.166648
Jan	3	.106017	.111576	.104082	.107388	.095049	.163621
	4	.110289	.121481	.109227	.112457	.098926	.174353
	5	.108364	.113610	.105489	.110194	.095196	.167991
	1	.227635	.209958	.208727	.211561	.217743	.129741
	2	.108583	.128388	.093011	.103345	.095260	.126104
Feb	3	.102473	.119324	.091673	.103095	.093233	.125138
	4	.108544	.128787	.093110	.103060	.093678	.128075
	5	.103910	.121761	.093001	.103885	.093755	.126930
	1	.230627	.226929	.252096	.302739	.273663	.169528
	2	.123319	.119347	.119311	.203400	.132920	.162719
Mar	3	.114549	.112419	.114337	.181690	.128716	.166282
	4	.122261	.121019	.118153	.214697	.131015	.174618
	5	.117686	.119279	.116125	.191570	.129965	.172902
	1	.224057	.259461	.319427	.347760	.344578	.069799
	2	.051774	.058173	.053374	.054573	.045326	.066576
Apr	3	.054384	.061833	.051018	.049770	.044142	.064570
	4	.050441	.055516	.052306	.054830	.044711	.066073
	5	.047331	.057262	.036942	.040162	.035542	.056011
	1	.270299	.301961	.319421	.301010	.280717	.101357
	2	.074346	.086795	.098549	.096463	.085430	.124105
May	3	.076075	.073423	.078128	.073994	.072472	.082039
	4	.070104	.084307	.094673	.095241	.082488	.135489
	5	.050370	.042772	.037886	.039855	.036334	.053079
	1	.262605	.309545	.283131	.277450	.250687	.124775
	2	.085548	.097746	.106089	.104020	.100754	.116186
Jun	3	.072500	.078729	.082551	.080196	.078758	.082895
	4	.085917	.099188	.105256	.105152	.100711	.119008
	5	.048704	.067091	.048578	.045815	.047675	.053875
	1	.262132	.305901	.285443	.280230	.260363	.170111
	2	.122031	.143580	.118336	.119965	.117444	.162186
Jul	3	.096135	.153028	.081563	.075854	.072778	.102448
	4	.123818	.145527	.120199	.121039	.116733	.163696
	5	.067718	.149198	.056561	.055148	.052593	.082227
	1	.208943	.289733	.258858	.245072	.249131	.140458
	2	.107875	.111710	.105078	.105916	.104886	.133494
Aug	3	.099531	.120341	.068684	.057438	.060778	.083809
	4	.109169	.113002	.105312	.103867	.108828	.140423
	5	.089873	.124323	.051617	.044211	.041834	.066882

(continued)

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Month & Method		Hour Period (LST) 06-08	09-11	12-14	15-17	18-20	21-23
	1	.287152	.368153	.423931	.415632	.415049	.108593
	2	.097935	.088830	.075652	.082019	.083092	.105744
Sep	3	.105475	.075891	.056830	.054580	.062542	.078128
	4	.098111	.093473	.076013	.083095	.084487	.107252
	5	.103702	.063294	.039163	.036725	.050440	.062808
	1	.268062	.298610	.367281	.357101	.362542	.136636
	2	.098111	.078021	.059119	.063121	.064186	.191779
Oct	3	.091034	.077961	.060244	.063251	.064092	.186323
	4	.094296	.074718	.056220	.060176	.058618	.187708
	5	.091088	.075525	.055773	.060288	.058688	.187493
	1	.317370	.322251	.326891	.348600	.339612	.113046
	2	.093381	.092529	.079215	.093359	.093516	.169961
Nov	3	.093512	.092641	.079183	.092702	.093272	.170600
	4	.096926	.094991	.081423	.097676	.095292	.173659
	5	.096794	.094802	.080711	.094390	.094599	.172823
	1	.286167	.291562	.284062	.314253	.300978	.116590
	2	.083520	.089119	.076121	.087866	.082674	.155249
Dec	3	.083215	.088835	.074979	.086700	.080722	.144582
	4	.086995	.094589	.080856	.093617	.089582	.163134
	5	.084860	.092264	.076356	.088914	.081241	.147036

Exhibit 4.1

RMS from Weighted Least Squares Fitting of Visibility Data for All 51 Norway Stations for All 5 Models.

WMO	Station	RMS1	RMS2	RMS3	RMS4	RMS5
10010	Jan Mayen	.181069	.101530	.095017	.097540	.063527
10100	Andoya/Andenes	.275615	.106986	.098623	.095022	.081824
10230	Bardufoss	.281356	.106516	.102554	.092405	.074875
10250	Tromso/Langnes	.283541	.108491	.109121	.094092	.080057
10280	Bjornoya	.211852	.077104	.052214	.070432	.038374
10330	Torsvag	.275940	.109117	.106853	.093142	.075793
10470	Kautokeino	.283831	.110234	.107846	.092556	.065395
10490	Alta Lufthavn	.288561	.116105	.119169	.097441	.073900
10530	Hammerfest Radio	.275427	.112660	.116621	.093699	.067212
10550	Fruholmen	.281672	.117466	.118485	.092030	.072556
10610	Brennelv	.296295	.129629	.131065	.106826	.083231
10780	Bletnes Fyr	.279586	.115545	.117435	.087082	.059509
10890	Kirkenes Lufthavn	.269202	.097592	.092496	.078235	.029442
10980	Vardo	.254712	.077512	.075510	.055744	.028685
11020	Sklinna Fyr	.270224	.100635	.081492	.100851	.082815
11050	Skomvaer Fyr	.282403	.119649	.104266	.117528	.106650
11150	Myken	.276388	.111601	.092827	.107728	.089774
11210	Nord-Solvaer	.276158	.108711	.089173	.104143	.084457
11520	Bodo	.280028	.112167	.094714	.103727	.082710
11600	Skrova	.281863	.119557	.105949	.111247	.095760
11650	Grotoy	.286376	.123862	.108705	.114592	.096682
12050	Svindy Fyr	.285635	.126755	.109538	.149056	.130361
12100	Vigra	.287072	.121662	.099337	.139542	.116840
12120	Ona/Husoy	.284151	.120720	.099944	.137247	.116129
12150	Hustad	.287230	.123618	.103116	.137705	.116130
12280	Sula Fyr	.286374	.122976	.101590	.131689	.110912
12380	Fokstua	.274149	.071137	.145193	.070756	.173484
12410	Orland	.284332	.119625	.096067	.124172	.099673
12650	Tynset	.268284	.063966	.065984	.062943	.064312
12710	Vaernes	.286484	.122404	.098129	.122763	.095651
12880	Roros	.273171	.061920	.065460	.060080	.062474
13060	Hellisoy Fyr	.259017	.094722	.083002	.121250	.101449
13090	Kinn	.284976	.121996	.100397	.146479	.122625
13110	Bergen/Flesland	.270188	.096277	.074993	.119559	.092241
13170	Bergen/Florida	.271802	.098669	.079509	.121603	.095973
13610	Fanaraken	.285482	.211998	.203993	.228921	.207896
13720	Nesbyen	.284768	.081317	.092534	.081198	.094479
13820	Kise Pa Hedmark	.272806	.100261	.074565	.099946	.069656
13810	Oslo/Gadermoen	.247491	.097867	.071973	.095975	.071872
14030	Utsira	.264757	.096472	.085565	.121891	.101897
14060	Slatteroy	.277981	.116502	.100922	.139268	.116635
14150	Stavanger/Sola	.273996	.101754	.084394	.122959	.097442
14270	Lista	.263732	.093338	.077132	.108794	.085439
14420	Byglandsf Jord-Sol	.259438	.086260	.066099	.092773	.067916
14450	Skafsa	.273291	.058077	.056809	.060647	.058384
14480	Oksoy	.264175	.093693	.070947	.102252	.073406
14650	Torungen Fyr	.261883	.098780	.075526	.104718	.075616
14700	Gvarv	.266496	.097905	.074032	.101835	.072488
14820	Ferder	.279616	.133329	.099871	.132786	.095346
14880	Oslo/Fornebu	.263294	.093053	.057568	.092211	.053131
14940	Rygge	.245764	.087976	.058066	.086626	.059689
	Overall	.272808	.108470	.097185	.110086	.092187

Exhibit 4.2

Weighted Least Squares Fitting of Visibility Data for All 51 Norway Stations for All 5 Models.

Month & Method		Hour Period (LST) 06-08	09-11	12-14	15-17	18-20	21-23
	1	.260671	.255765	.253878	.276852	.261949	.131165
	2	.147357	.15111	.136302	.124818	.120282	.174654
Jan	3	.140621	.115400	.134860	.122725	.117092	.173997
	4	.142522	.144529	.129432	.115180	.115430	.169929
	5	.138825	.124657	.120627	.121158	.117663	.177174
	1	.233813	.214203	.214521	.216759	.223680	.130921
	2	.128526	.144892	.121804	.125912	.118254	.127725
Feb	3	.124337	.138728	.122178	.127591	.117328	.128180
	4	.124594	.141432	.118263	.119781	.110353	.122012
	5	.126860	.142124	.122997	.125410	.115648	.128286
	1	.235985	.231640	.254910	.306403	.278853	.171616
	2	.138681	.147250	.140507	.227515	.145563	.167722
Mar	3	.132299	.143848	.135588	.211475	.145306	.173500
	4	.134258	.142936	.133428	.243717	.137599	.166735
	5	.132133	.141482	.120781	.213193	.138275	.169324
	1	.231687	.266547	.326917	.355273	.352635	.070603
	2	.077996	.106744	.078252	.077998	.093288	.069292
Apr	3	.092695	.117411	.075720	.075220	.099283	.067983
	4	.082652	.105587	.073589	.074271	.090711	.064175
	5	.088242	.114062	.065475	.065529	.094937	.056931
	1	.276839	.307356	.324662	.305790	.285889	.103154
	2	.098468	.114069	.121577	.120951	.110741	.126744
May	3	.097876	.108545	.113011	.105491	.105684	.086183
	4	.072435	.090904	.094209	.098726	.089960	.087831
	5	.083561	.084488	.092073	.086413	.088640	.056737
	1	.268737	.315317	.287626	.281549	.254717	.126685
	2	.109043	.126960	.128423	.128976	.121947	.118938
Jun	3	.109631	.116978	.110772	.111268	.104583	.088255
	4	.100169	.115356	.109081	.111019	.111085	.104045
	5	.095818	.111234	.094976	.117248	.114718	.064300
	1	.268133	.311238	.290207	.284506	.264623	.172128
	2	.139261	.170322	.133701	.139484	.138393	.166247
Jul	3	.129624	.172899	.112078	.104931	.101994	.106897
	4	.137038	.167864	.130587	.140027	.140608	.165784
	5	.125860	.180137	.106100	.105453	.102952	.096467
	1	.213648	.294844	.262868	.248410	.252786	.141971
	2	.133407	.137185	.123200	.135913	.133394	.136572
Aug	3	.132424	.147182	.094975	.101451	.086709	.089541
	4	.132930	.136247	.123517	.137798	.132677	.132789
	5	.139046	.153351	.087324	.099943	.080324	.071352

(continued)

(continued from previous page)

Month & Method		Hour Period (LST) 06-08	09-11	12-14	15-17	18-20	21-23
	1	.295144	.375860	.430701	.422407	.412871	.109374
	2	.119705	.125679	.091718	.099800	.124102	.106897
Sep	3	.136300	.131041	.105772	.084097	.117457	.080352
	4	.115609	.125322	.085593	.093043	.121451	.103659
	5	.132976	.139451	.108556	.124318	.149577	.101763
	1	.276278	.307197	.376207	.365677	.372821	.138189
	2	.142293	.123213	.083979	.079258	.118913	.201826
Oct	3	.138047	.123921	.088462	.080126	.119630	.197397
	4	.140102	.118819	.080572	.073941	.113523	.214818
	5	.137955	.120266	.085390	.075060	.115071	.218988
	1	.325599	.330649	.335674	.356801	.347834	.114104
	2	.125555	.103046	.094636	.105214	.123244	.216022
Nov	3	.124792	.104598	.098631	.106063	.123983	.212154
	4	.123908	.100244	.090717	.100077	.121043	.216566
	5	.125473	.104172	.096289	.101319	.121011	.218899
	1	.295343	.300354	.292805	.323412	.309544	.117784
	2	.128270	.112463	.090399	.098437	.129756	.167626
Dec	3	.133973	.113033	.091071	.098950	.240421	.160315
	4	.130777	.107268	.088372	.094015	.127020	.160212
	5	.133644	.111180	.090920	.097155	.129332	.164798

Exhibit 4.3

RMS from Weighted Least Squares Fitting of Visibility Data Using Sample Re-use in Norway for All 5 Models.

WMO	Station	RMS1	RMS2	RMS3	RMS4	RMS5
10010	Jan Mayen	.201657	.124205	.137245	.185611	.270795
10100	Andoya/Andenes	.276769	.108078	.100268	.087509	.072492
10230	Bardufoss	.282116	.107175	.103667	.083418	.071476
10250	Tromso/Langnes	.284103	.109077	.110155	.084666	.076446
10280	Bjornoya	.223829	.092315	.091549	.125443	.104157
10330	Torsvag	.277284	.110161	.108631	.083954	.072219
10470	Kautokeino	.284651	.110652	.108527	.080231	.067916
10490	Alta Lufthavn	.288974	.116814	.119896	.084716	.075913
10530	Hammerfest Radio	.276936	.113448	.115184	.082050	.069810
10550	Fruholmen	.282689	.118429	.120436	.082526	.071050
10610	Brennelv	.296459	.129816	.131411	.092644	.083512
10780	Bletnes Fyr	.280837	.116590	.119390	.073696	.060526
10890	Kirkenes Lufthavn	.270830	.098723	.094222	.063914	.040563
10980	Vardo	.257486	.079356	.078484	.051955	.034439
11020	Sklinna Fyr	.271989	.102099	.082500	.093651	.073721
11050	Skomvaer Fyr	.283421	.120558	.105176	.110247	.092866
11150	Myken	.277576	.112504	.093773	.100251	.079237
11210	Nord-Solvaer	.277092	.109413	.089805	.096486	.074576
11520	Bodo	.280849	.112996	.095764	.095632	.074236
11600	Skrova	.282928	.120341	.106989	.103329	.086062
11650	Grotoy	.286942	.124322	.109356	.106364	.087150
12050	Svindy Fyr	.286737	.127460	.109818	.135874	.120028
12100	Vigra	.287499	.122252	.099881	.127073	.106565
12120	Ona/Husoy	.284911	.121450	.100512	.125290	.105957
12150	Hustad	.287786	.124177	.103497	.125851	.106716
12280	Sula Fyr	.287008	.123624	.102096	.121808	.100983
12380	Fokstua	.276145	.169574	.219654	.179830	.256838
12410	Orland	.284941	.120277	.096730	.114923	.091788
12650	Tynset	.270592	.074656	.077932	.073022	.075946
12710	Vaernes	.287094	.122927	.098714	.114213	.089976
12880	Roros	.274875	.064424	.067918	.060588	.064010
13060	Hellisoy Fyr	.261920	.095833	.084788	.103435	.096801
13090	Kinn	.285623	.122495	.100840	.131117	.112440
13110	Bergen/Flesland	.271420	.097385	.076445	.103280	.088033
13170	Bergen/Florida	.272961	.099555	.080643	.104828	.091252
13610	Fanaraken	.442660	.509366	.511978	.504702	.514746
13720	Nesbyen	.285586	.083415	.094443	.084054	.097414
13820	Kise Pa Hedmark	.274190	.100873	.075232	.093776	.069846
13810	Oslo/Gadermoen	.253534	.102839	.082259	.099649	.083438
14030	Utsira	.267242	.098100	.087530	.106708	.101847
14060	Slatteroy	.279619	.117930	.102492	.125671	.114293
14150	Stavanger/Sola	.274987	.102754	.086142	.107741	.097999
14270	Lista	.266064	.094899	.080359	.097892	.090549
14420	Byglandsf Jord-Sol	.262805	.087035	.067405	.086983	.070500
14450	Skafsa	.274404	.077342	.076994	.078274	.078178
14480	Oksoy	.266383	.095003	.074275	.093279	.079147
14650	Torungen Fyr	.264676	.099772	.077710	.096311	.079588
14700	Gvarv	.269191	.098470	.075074	.095158	.074225
14820	Ferder	.281361	.143462	.120330	.140114	.119227
14880	Oslo/Fornebu	.265791	.094568	.060334	.087083	.056913
14940	Rygge	.250530	.091452	.065377	.086750	.067229
	Overall	.278763	.129961	.122581	.125313	.121550

Exhibit 4.4
Weighted Least Squares Fitting of Visibility Data Using
Sample Re-use in Norway for All 5 Models.

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