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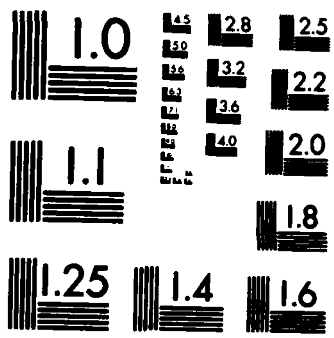

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FINAL REPORT

AUTHOR: F. W. J. OLVER

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The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Asymptotic analysis; computer arithmetic; difference equations; elementary functions; elliptic functions; error analysis; extended-range arithmetic; floating-point system; Gaussian elimination; Legendre functions; level-index arithmetic, Liouville-Green approximation; mathematical functions; multiple precision; Newton's rule; ordinary differential equations; overflow; (continued)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Problems were studied and solved in five main areas: asymptotic solutions of second-order differential equations; numerical solution of difference equations; algorithms for the generation of mathematical functions; error analysis of numerical algorithms; computer arithmetic.

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19. Key Words (continued)

polynomial evaluation; turning points; underflow; unrestricted algorithms; Whittaker functions; WKBJ approximation

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RESEARCH FINDINGS

I. Asymptotic solutions of second-order differential equations.

Publications†: [1], [4], [5], [6], [16].

Three main problems in this category were studied and solved successfully.

First, general connection formulas were obtained for the Liouville-Green (or WKBJ) approximate solutions

$$\{f(u,z)\}^{-1/4}\exp\left\{\pm\int\{f(u,z)\}^{1/2}dz\right\}$$

of differential equations of the form

$$d^2w/dz^2 = \{u^2f(u,z)+g(u,z)\}w \ , \qquad (1)$$

in which u is a large real or complex variable and the independent variable z ranges over the complex plane. The differential equation may have any number of turning points (zeros of $f(u,z)$) of arbitrary (including fractional) multiplicity, and any number of singularities. There are many potential physical applications of this work, including scattering problems, transmission of radio waves, trapping of water waves and hydrodynamic instability.

Second, asymptotic approximations were obtained for standard solutions of Whittaker's form of the confluent hypergeometric equation:

$$\frac{d^2W}{dz^2} = \left(\pm\frac{1}{4} \mp \frac{\kappa}{z} - \frac{\frac{1}{4} \mp \mu^2}{z^2}\right)W$$

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†References are listed on pages 6-8.

for large positive values of the parameter μ that are uniform with respect to unrestricted values of the argument z in the open interval $(0,\infty)$ and bounded real values of the ratio κ/μ. The approximations are in terms of parabolic cylinder functions, complete with error bounds. This work closes one of the two important gaps in the asymptotic theory of these frequently occurring functions in problems of mathematical physics [25].

The third problem is the construction of uniform asymptotic solutions of equation (1) in cases in which $f(u,z)$ has a simple pole and a simple turning point, the locations of which depend on a second parameter and may coalesce for a critical value of that parameter. This is another major unsolved problem in asymptotics that was described in [25]. J. J. Nestor, a graduate student in the Applied Mathematics Program at the University of Maryland has solved the problem fully for real variables, under the direction of the principal investigator, and the results are given in Nestor's Ph.D. thesis [16].

II. Numerical solution of difference equations.

In 1967 the principal investigator published a stable algorithm for the computation of solutions of inhomogeneous linear difference equations of the second order [24]. This has since been incorporated in various packages for computing special functions; see, for example, [19], [22], [27]-[31]. The extension of this algorithm to linear difference equations of any order, or systems of such equations, was far from a trivial problem. Working under the direction of the principal investigator, D. W. Lozier solved this problem and was awarded a Ph.D. degree in the Applied Mathematics Program at the University of Maryland for this work.

So far Lozier's results have been issued as a U.S. Department of Commerce Report [20]; some of them have also been reproduced in a recent text [32]. [Note. Lozier's work was not supported by the AROD since he was (and still is) a full-time employee of the National Bureau of Standards. The principal investigator's work on this problem was supported by AROD.]

III. Algorithms for the generation of mathematical functions.

Publications: [2], [3], [7], [11], [12], [13].

The construction of high-quality portable software for the generation of mathematical functions is an ongoing project at several universities and government laboratories around the world. The main contribution by the principal investigator and his co-workers, D. W. Lozier and J. M. Smith, has been the construction of comprehensive packages for the Legendre functions [7], [13], [21]. It was possible to cover extremely large ranges of the parameters for these functions by the introduction of an "extended-range" arithmetic, in which a whole word is allotted to the exponents of floating-point numbers, without undue sacrifice of speed. The new packages have been incorporated in the GAMS library at the National Bureau of Standards [22], and one is also available from the National Technical Information Service [23].

A second type of problem that was studied is the construction of unrestricted algorithms for the generation of mathematical functions, that is, algorithms that will generate function values to any guaranteed precision for any values of the variables. The object here is twofold. First, we seek, in effect, constructive numerical algorithms for the functions which can then be used to produce day-to-day (restricted) algorithms, based, for example, on expansions in series of Chebyshev polynomials or rational functions. Second, it was believed (correctly) that new mathematical and computational tools would need to be developed in the course of constructing the unrestricted algorithms, which might be applicable to other computational problems.

So far, most of the elementary functions have been treated. An unrestricted algorithm for the exponential function was described in [2], [3]. Unrestricted algorithms for reciprocals, square roots, logarithms and the trigonometric functions await testing at the National Bureau of Standards, pending the development of a considerable extension to R. P. Brent's multiple-precision package [17], [18].

A third area in the general study of algorithms for the generation of mathematical functions consisted of work on some miscellaneous topics, especially elliptic functions, that was initiated by B. C. Carlson during his visit to Maryland in 1980-81; see [11], [12].

IV. Error analysis of numerical algorithms.

Publications: [8], [9], [10], [14].

In the course of the work on unrestricted algorithms described in III, a new version of error analysis was developed, based on a logarithmic definition of relative error, and applicable to real and complex arithmetic. It was soon realized that this analysis might also be applicable to other problems in numerical analysis. An application to Gaussian elimination was made in [9] in collaboration with J. H. Wilkinson, and new explicit error bounds of a posteriori type were obtained. Other successful applications so far include polynomial evaluation and Newton's rule, and this work is being readied for publication under the new AROD Contract (DAAG 29-84-K-0022).

V. Computer arithmetic.

Publication: [15].

Overflow and underflow are continual problems in the construction of comprehensive computing programs [26]. This is especially true for robust algorithms for the generation of mathematical functions. It has already been noted in III that a special form of computer arithmetic was developed in connection with packages for Legendre functions, and this eased considerably the problems of overflow and underflow in these cases.

The floating-point system (including extended-range arithmetic) can be regarded as a mapping from the real line onto an $\mathbb{R}^2$-space. The principal investigator and his co-workers, C. W. Clenshaw and D. W. Lozier, have been addressing the following problem. Are there other forms of mapping onto an $\mathbb{R}^2$-space in which the phenomena of overflow and underflow are eliminated, in the sense that arithmetic operations or numbers that lie outside the mapping essentially become trivial? An affirmative answer, describing a number system based on iterated (or generalized) exponential and logarithmic functions, was supplied recently in [15]. Since this work was completed algorithms have been devised and tested for the new arithmetic, called level-index arithmetic, and some test runs have also been made at the National Bureau of Standards for generating Bessel and Legendre functions. In consequence, level-index arithmetic is already a feasible proposition for multiple-precision computations for which speed is not an important consideration. Its feasibility for ordinary single- and double-precision work may hinge upon the possibility of constructing silicon chips to implement the arithmetic operations.

Publications Supported by AROD Grant DAAG 29-77-G-0003 and Contract DAAG 29-80-C-0032

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Note. It is expected that J. J. Nestor will be awarded a Ph.D. degree in the Applied Mathematics Program at the University of Maryland during the summer of 1984. Part of his thesis work was supported by the AROD contract.

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