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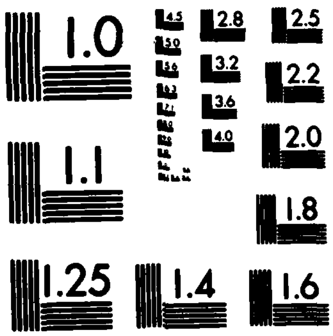

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ASYMMETRICAL COPLANAR TRANSMISSION LINES

TECHNICAL REPORT

T. KITAZAWA
R. MITTRA

JUNE 1984

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Variational expressions of the line capacitances of the asymmetrical coplanar waveguide and the asymmetrical coplanar strip line are derived for the first time. An efficient numerical method is employed to compute the line capacitance and is shown to yield accurate results for both isotropic and anisotropic substrates.

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ASYMMETRICAL COPLANAR TRANSMISSION LINES

by

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I. INTRODUCTION

Coplanar transmission-line structures have received considerable attention due to their easy adaptation to shunt elements. The characteristics of symmetrical coplanar transmission lines have been investigated by many authors for both isotropic and anisotropic substrates [1], [2], [3], [4], [5]. Recently, asymmetrical versions of these coplanar lines have been introduced [6], [7], because of additional flexibilities offered by the asymmetric configuration in the design of microwave integrated circuits (MIC).

This paper derives two variational expressions for the line capacitances of the asymmetrical coplanar waveguide (ACPW) and the asymmetrical coplanar strip line (ACSL) that are general enough to be applicable to either the isotropic or uniaxially anisotropic substrates. These expressions are then employed in conjunction with an accurate and efficient numerical method which computes the line characteristics, viz., the effective dielectric constant and the characteristic impedance of ACPW and ACSL with anisotropic substrates.

Numerical results are compared with the exact analytical solutions for the special case in which air is a substrate material; excellent agreement is found.

II. THEORY

Figure 1 shows the cross sections of the asymmetrical coplanar waveguide (ACPW) and the asymmetrical coplanar strip line (ACSL) to be analyzed. The substrate is a uniaxially anisotropic medium, whose optical axis is assumed to be inclined at an angle of γ measured from the x axis. The permittivity tensor of this anisotropic substrate is given by

$$\hat{\epsilon} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{xy} & \epsilon_{yy} \end{bmatrix} \epsilon_o \quad (1)$$

where

$$\begin{aligned} \epsilon_{xx} &= \epsilon_{\parallel}\cos^2\gamma + \epsilon_{\perp}\sin^2\gamma \\ \epsilon_{yy} &= \epsilon_{\parallel}\sin^2\gamma + \epsilon_{\perp}\cos^2\gamma \\ \epsilon_{xy} &= (\epsilon_{\parallel} - \epsilon_{\perp})\sin\gamma\cos\gamma \end{aligned} \quad (2)$$

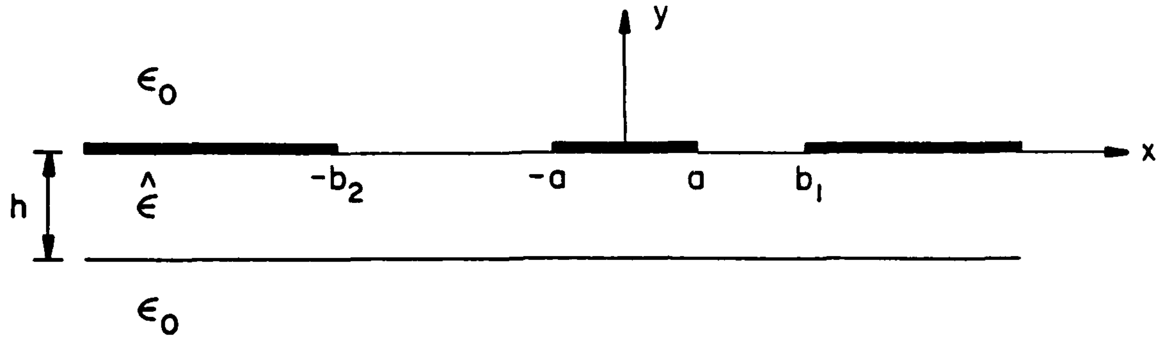
where $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are the relative permittivities, longitudinal and transverse to the optical axis, respectively. The conductors are assumed to be of zero thickness.

A. Variational Expression for the Line Capacitance of ACPW

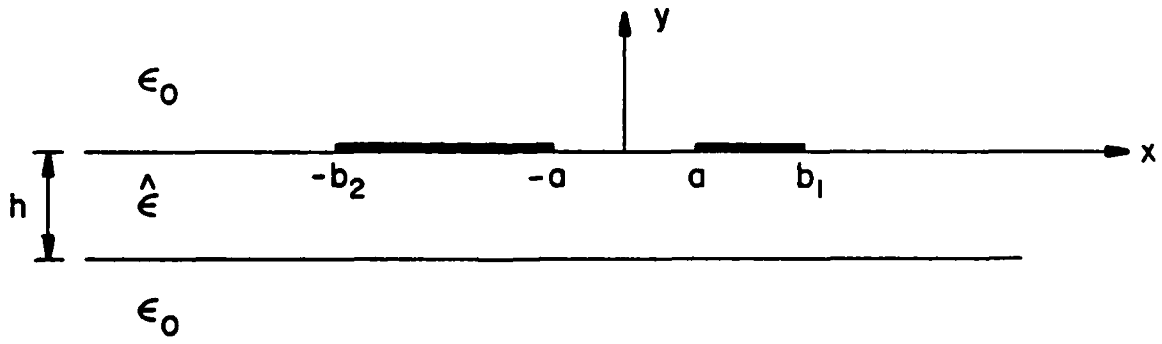
The derivation of the variational expression for the line capacitance of ACPW (shown in Figure 1(a)) is based on an extension of the procedure followed for the symmetrical case [4], [5], [8], and is outlined below.

From a solution to Laplace's equation, the charge distribution on the conductor at y = 0 can be expressed in terms of the aperture field e(x) at y = 0 as

$$\sigma(x) = \frac{-j\epsilon_o}{2\pi} \int \int_{-\infty}^{\infty} \alpha f(\alpha)\, e(x')\, e^{j\alpha(x'-x)}\, dx'\, d\alpha \quad (3)$$



(a) Asymmetrical Coplanar Waveguide (ACPW)



(b) Asymmetrical Coplanar Strip Line (ACSL)

Figure 1. Asymmetrical coplanar transmission lines.

$$W_i = b_i - a \quad (i = 1, 2)$$

$$S_{\substack{1\\2}} = \pm (a + b_{\substack{1\\2}})$$

$$f(\alpha) = \left\{ 1 + \frac{1 + \epsilon_e \tanh(Kh|\alpha|)}{1 + \frac{1}{\epsilon_e}\tanh(Kh|\alpha|)} \right\} \frac{1}{|\alpha|}$$

$$K = \sqrt{\frac{\epsilon_{xx}}{\epsilon_{yy}} - \left(\frac{\epsilon_{xy}}{\epsilon_{yy}}\right)^2} \qquad (4)$$

$$\epsilon_e = \sqrt{\epsilon_{xx}\,\epsilon_{yy} - \epsilon_{xy}2}$$

The total charge Q_o on the center strip $|x| < a$ can be expressed as follows:

$$Q_o = \int_{x_2}^{x_1} \sigma(x)\,dx \qquad (5)$$

where x_1 and x_2 can be arbitrary values in the right slot $a < x_1 < b_1$ and in the left slot $-b_2 < x_2 < -a$, respectively.

Multiplying (5) by $e(x_1)$ and integrating over the right slot located at $a < x_1 < b_1$, we get

$$Q_o V_o = \int_a^{b_1} \left\{ \int_{x_2}^{x_1} \sigma(x)\,dx \right\} dx_1 \qquad (6\text{-a})$$

$$(-b_2 < x_2 < -a)$$

Similarly,

$$-Q_o V_o = \int_{-b_2}^{-a} \left\{ \int_{x_2}^{x_1} \sigma(x)\,dx \right\} dx_2 \qquad (6\text{-b})$$

$$(a < x_1 < b_1)$$

where V_o is the potential difference between the center strip and the ground conductors, i.e.,

$$V_o = \int_a^{b_1} e(x)\,dx = -\int_{-b_2}^{-a} e(x)\,dx \tag{7}$$

Substituting (3) into (6), subtracting (6-b) from (6-a), and rearranging the resulting expression, we obtain the line capacitance as follows

$$C = \frac{Q_o}{V_o} \tag{8}$$

$$= \frac{\int \int_{-\infty}^{\infty} \int_{o}^{\infty} e(x)\,F(\alpha;x\,|\,x')\,e(x')\,d\alpha\,dx'\,dx}{\left[\int_a^{b_1} e(x)\,dx\right]^2}$$

where the Green's function F is given by

$$F(\alpha;x\,|\,x') = \frac{\epsilon_o}{\pi}\, f(\alpha) \cos\{\alpha(x'-x)\} \tag{9}$$

It can be verified that equation (8) has the stationary property and provides an upper bound to the line capacitance C. As expected, equation (8) reduces to (2) in [5] for the symmetrical case $b_1 = b_2$. It is also worthwhile to mention that equation (8), together with (9) and (4), suggests the transformation from the anisotropic case $(\epsilon_{\|}, \epsilon_{\perp}, h)$ to the isotropic case (ϵ_e, Kh).

B. Variational Expression for the Line Capacitance of ACSL

The derivation of the variational expression for the line capacitance of ACSL shown in Figure 1(b) is quite similar to that of ACPW. In this case, the potential distribution $\phi(x)$ on the strip surface y = 0 is the basic quantity as opposed to the charge distribution in the ACPW case. The potential distribution can be expressed in terms of the charge distribution $\sigma(x)$ on the strip conductor by solving Laplace's equation. The expression is

$$\phi(x) = \frac{1}{2\pi\epsilon_o} \int \int_{-\infty}^{\infty} g(\alpha)\, \sigma(x')\, e^{j\alpha(x'-x)}\, dx'\, d\alpha \tag{10}$$

$$g(\alpha) = \frac{1 + \epsilon_r \coth(Kh\,|\alpha|)}{1 + \epsilon_r^2 + 2\epsilon_r \coth(Kh\,|\alpha|)} \bullet \frac{1}{|\alpha|} \tag{11}$$

$\phi(x)$ should be constant on the strip conductors, that is,

$$\phi(x) = \frac{V_o}{2} \qquad (a < x < b_1) \tag{12-a}$$

$$\phi(x) = -\frac{V_o}{2} \qquad (-b_2 < x < -a) \tag{12-b}$$

where V_o is the potential difference between the strip conductors. By multiplying (12) with $\sigma(x)$ and integrating over the right strip $(a < x < b_1)$ and the left strip $(-b_2 < x < -a)$, respectively, we obtain

$$\frac{V_o}{2} \int_a^{b_1} \sigma(x)\, dx = \frac{V_o Q_o}{2} \tag{13-a}$$

$$= \frac{1}{2\pi\epsilon_o} \int_a^{b_1} \int \int_{-\infty}^{\infty} \sigma(x)\, g(\alpha)\, \sigma(x')\, e^{j\alpha(x'-x)}\, dx'\, d\alpha\, dx$$

and

$$\frac{-V_o}{2} \int_{-b_2}^{-a} \sigma(x)\, dx = \frac{V_o Q_o}{2} \tag{13-b}$$

$$= \frac{1}{2\pi\epsilon_o} \int_{-b_2}^{-a} \int \int_{-\infty}^{\infty} \sigma(x)\, g(\alpha)\, \sigma(x')\, e^{j\alpha(x'-x)}\, dx'\, d\alpha\, dx$$

where Q_o is the total charge on the strip conductor

$$Q_o = \int_a^{b_1} \sigma(x)\, dx = - \int_{-b_2}^{-a} \sigma(x)\, dx \tag{14}$$

The line capacitance of ACSL can be obtained from (13) and can be expressed as follows

$$\frac{1}{C} = \frac{V_o}{Q_o} \tag{15}$$

$$= \frac{\int \int_{-\infty}^{\infty} \int_{o}^{\infty} \sigma(x)\, G(\alpha; x \mid x')\, \sigma(x')\, d\alpha\, dx'\, dx}{\left[\int_a^b \sigma(x)\, dx \right]^2}$$

where Green's function G is given by

$$G(\alpha; x \mid x') = \frac{1}{\pi \epsilon_o}\, g(\alpha) \cos \{\alpha(x' - x)\} \tag{16}$$

Equation (15) gives an upper bound to 1/C, hence, a lower bound to the line capacitance C.

C. Method of Solution

The line capacitance can be evaluated by applying the Ritz procedure to the variational expressions (8) and (15). Taking the edge effect into account, the unknown aperture field e(x) in (8) and the unknown charge distribution $\sigma(x)$ in (15) are expanded in terms of the appropriate basis functions as follows:

$$e(x) = \sum_{k=1}^{N_1} A_k^{(1)} f_k^{(1)}(x) + \sum_{k=1}^{N_2} A_k^{(2)} f_k^{(2)}(x) \tag{17-a}$$

$$\sigma(x) = \sum_{k=1}^{N_1} B_k^{(1)} f_k^{(1)}(x) + \sum_{k=1}^{N_2} B_k^{(2)} f_k^{(2)}(x) \tag{17-b}$$

$$f_k^{(i)} = \frac{T_k\left\{\dfrac{2(x-S_i)}{W_i}\right\}}{\sqrt{1-\left\{\dfrac{2(x-S_i)}{W_i}\right\}^2}} \qquad i=1,2 \tag{18}$$

$$S_{\substack{1\\2}} = \pm(a + b_{\substack{1\\2}})/2, \qquad W_{\substack{1\\2}} = b_{\substack{1\\2}} - a$$

where $T_k(y)$ are Chebyshev's polynomials of the first kind, and $A_k^{(i)}$ and $B_k^{(i)}$ are unknown coefficients which can be determined by substituting (17) into (8) and (15), and requiring

$$\frac{\partial}{\partial A_k^{(i)}} C = 0 \tag{19-a}$$

and

$$\frac{\partial}{\partial B_k^{(i)}} \left[\frac{1}{C}\right] = 0 \ , \tag{19-b}$$

respectively.

III. NUMERICAL RESULTS

In this section, the numerical results derived from the application of the Ritz procedure to the variational expressions for the line capacitance are presented. The accuracy of computation depends on the number of basis functions, i.e., N_1 and N_2 in (17). Tables 1 and 2 show the numerical results of the line capacitance of ACPW and ACSL for different values of N_1, N_2. The tables include the results for the special cases with air replacing the substrate material, i.e., for $\epsilon_{||} = \epsilon_{\perp} = 1$. This special case lends itself to an exact analytical solution via the use of a sequence of conformal transformations [Appendix I]. Note that the values of capacitance for ACPW, obtained by using (8), are slightly larger than the exact values (upper bounds) while those of ACSL are slightly smaller than the exact values (lower bounds). The convergence is seen to be very rapid for both the cases and for a wide range of parameters, as evidenced by the fact that very accurate results are obtained with only a small number of basis functions. Note that $N_1 = N_2 = 2$ is sufficient in most cases except for extremely small values of a/W_1 and large values of W_2/W_1. Even so, $N_1 = N_2 = 3$ is sufficient for these extreme cases.

Figures 2 through 5 present numerical examples for the asymmetrical structures of both the ACPW and ACSL types. Figure 2 shows the effective dielectric constant ϵ_{eff} and the characteristic impedance Z_o as a function of the width ratio W_2/W_1, for ACPW with an isotropic dielectric substrate. The values for the symmetrical case ($W_2/W_1 = 1$) [4] are presented only as an indication of the accuracy of computation.

Figure 3 shows the characteristics of ACPW on an anisotropic sapphire substrate. The effective dielectric constant and the characteristic impedance are shown as a function of the inclination of the optical axis γ.

Figure 4 shows the variation in the characteristics of ACSL on an isotropic dielectric substrate with the width ratio W_2/W_1. The effective dielectric constant of ACSL becomes smaller as the width ratio W_2/W_1 becomes larger in the same manner as the ACPW case; however, the characteristic impedance of ACSL becomes smaller as W_2/W_1 becomes larger as opposed to the ACPW case.

TABLE 1

Line capacitance of ACPW C/ε_o

Without substrate $\varepsilon_{||} = \varepsilon_{\perp} = 1$, $h = 0$

a/W_1	W_2/W_1	N_1 / N_2	1 / 1	2 / 2	3 / 3	Conformal mapping
	1		2.199	2.107	2.105	2.105
0.25	2		2.085	1.946	1.940	1.940
	4		2.085	1.858	1.838	1.836
	1		3.520	3.510	3.510	3.510
1.50	2		3.219	3.198	3.198	3.198
	4		3.005	2.956	2.956	2.956

With sapphire substrate

$\varepsilon_{||} = 11.6$, $\varepsilon_{\perp} = 9.4$, $\gamma = \pi/4$, $h/W_1 = 1$

a/W_1	W_2/W_1	N_1 / N_2	1 / 1	2 / 2	3 / 3	4 / 4
	1		10.792	10.574	10.569	10.569
0.25	2		9.463	9.220	9.215	9.212
	4		8.534	8.251	8.246	8.236
	1		13.800	13.797	13.795	13.795
1.50	2		11.755	11.750	11.740	11.740
	4		10.226	10.213	10.179	10.179

TABLE 2

Line capacitance of ACSL C/ε_o

Without substrate $\varepsilon_{||} = \varepsilon_{\perp} = 1$, $h = 0$

a/W_1	W_2/W_1	N_1 / N_2	1 / 1	2 / 2	3 / 3	Conformal mapping
0.25	1		1.819	1.899	1.901	1.901
	2		1.918	2.055	2.062	2.062
	4		1.918	2.153	2.175	2.178
1.50	1		1.136	1.140	1.140	1.140
	2		1.243	1.251	1.251	1.251
	4		1.331	1.353	1.353	1.353

With boron-nitride substrate

$\varepsilon_{||} = 5.12$, $\varepsilon_{\perp} = 3.40$, $\gamma = \pi/4$, $h/W_1 = 2$

a/W_1	W_2/W_1	N_1 / N_2	1 / 1	2 / 2	3 / 3	4 / 4
0.25	1		4.513	4.752	4.757	4.757
	2		4.624	5.057	5.075	5.076
	4		4.383	5.176	5.246	5.252
1.50	1		2.569	2.586	2.586	2.586
	2		2.719	2.759	2.760	2.760
	4		2.778	2.886	2.890	2.890

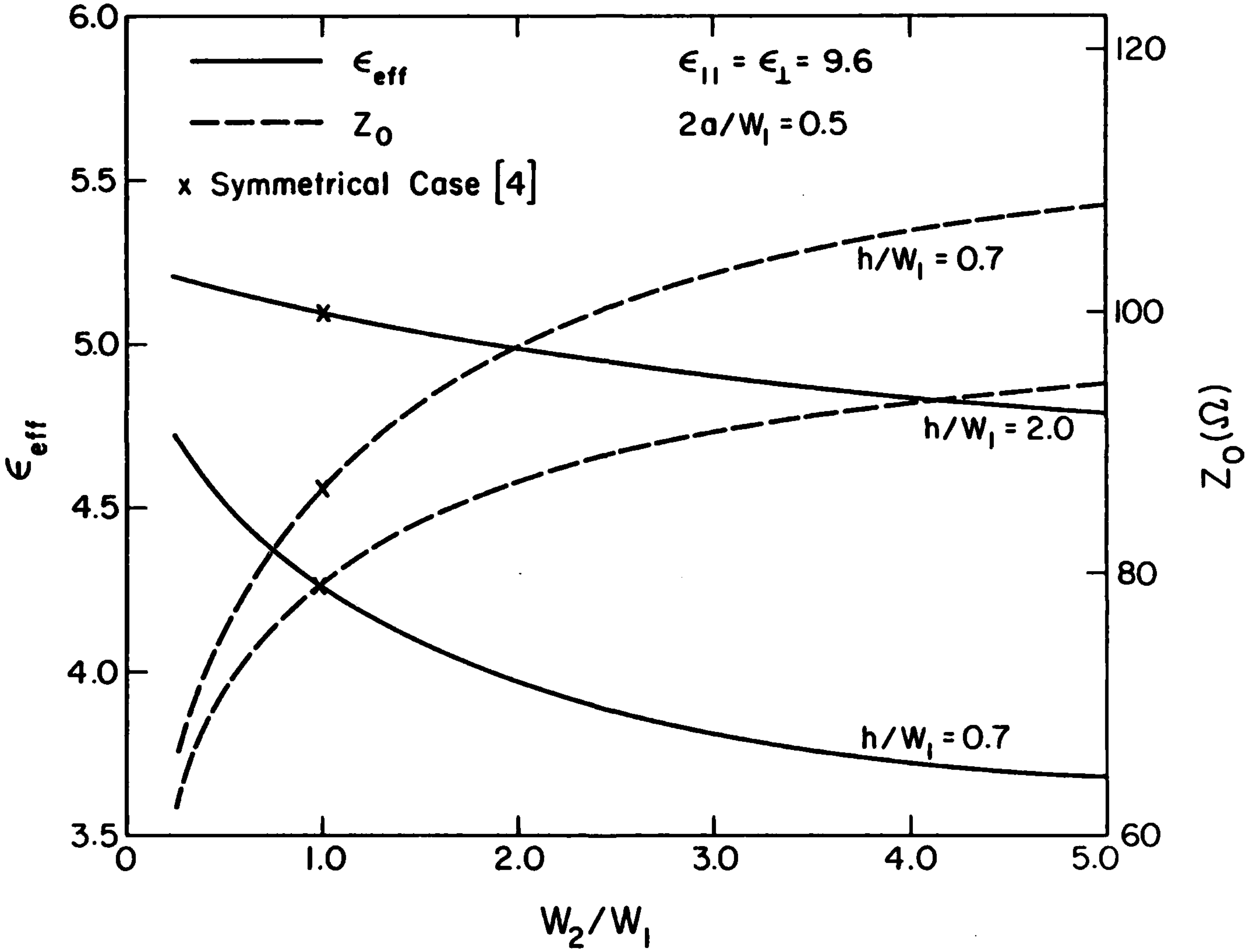


Figure 2. ACPW on isotropic dielectric substrate.

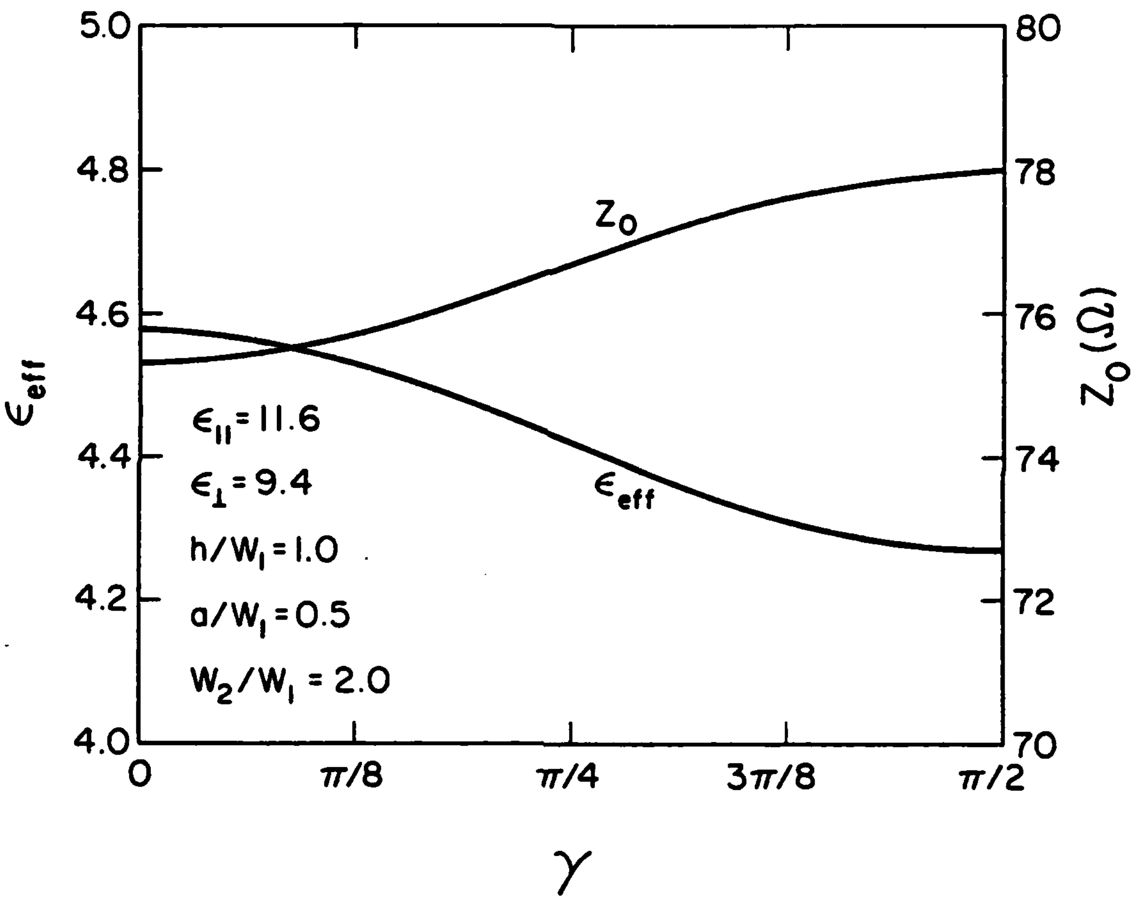


Figure 3. ACPW on anisotropic sapphire substrate.

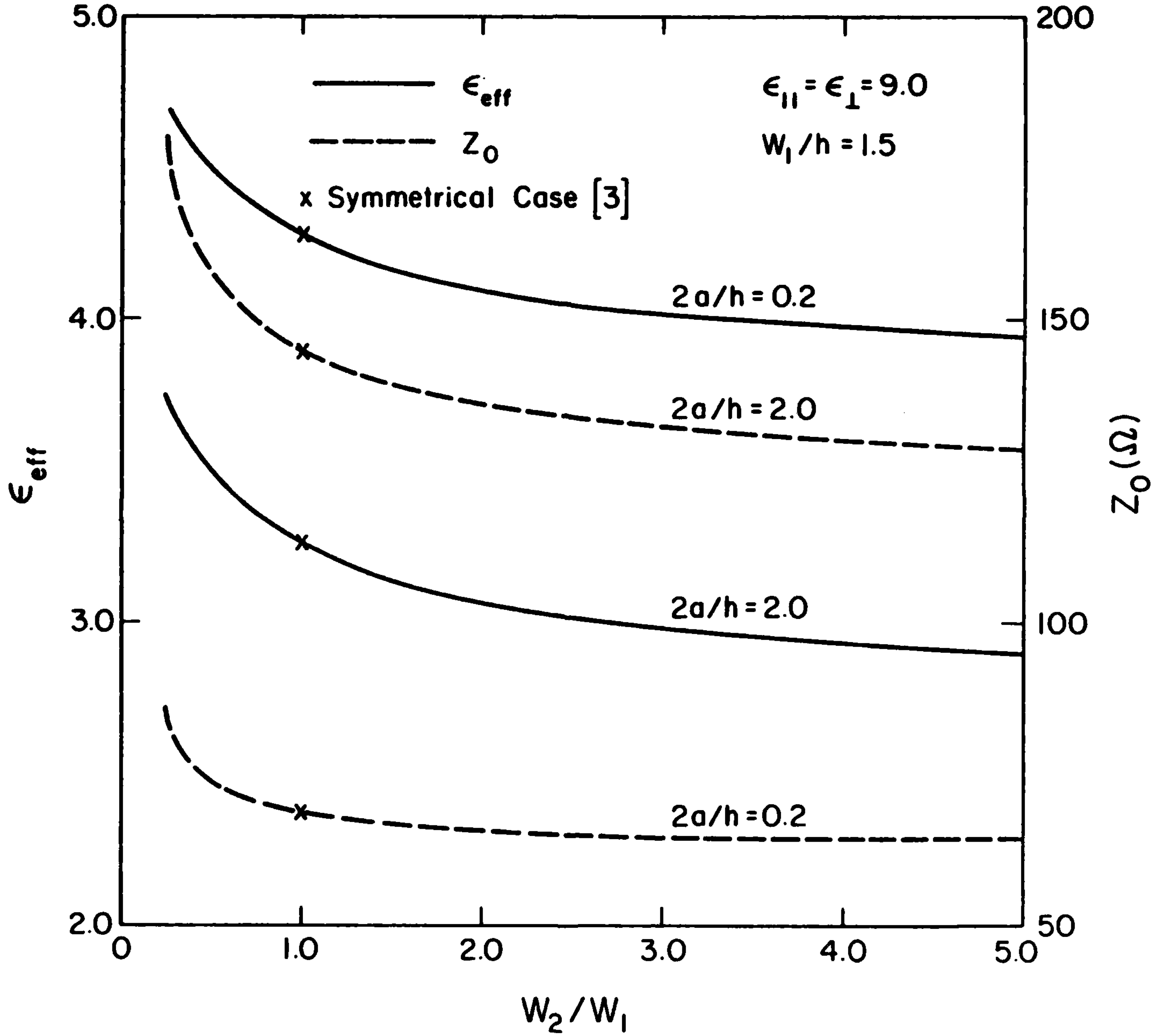


Figure 4. ACSL on isotropic dielectric substrate.

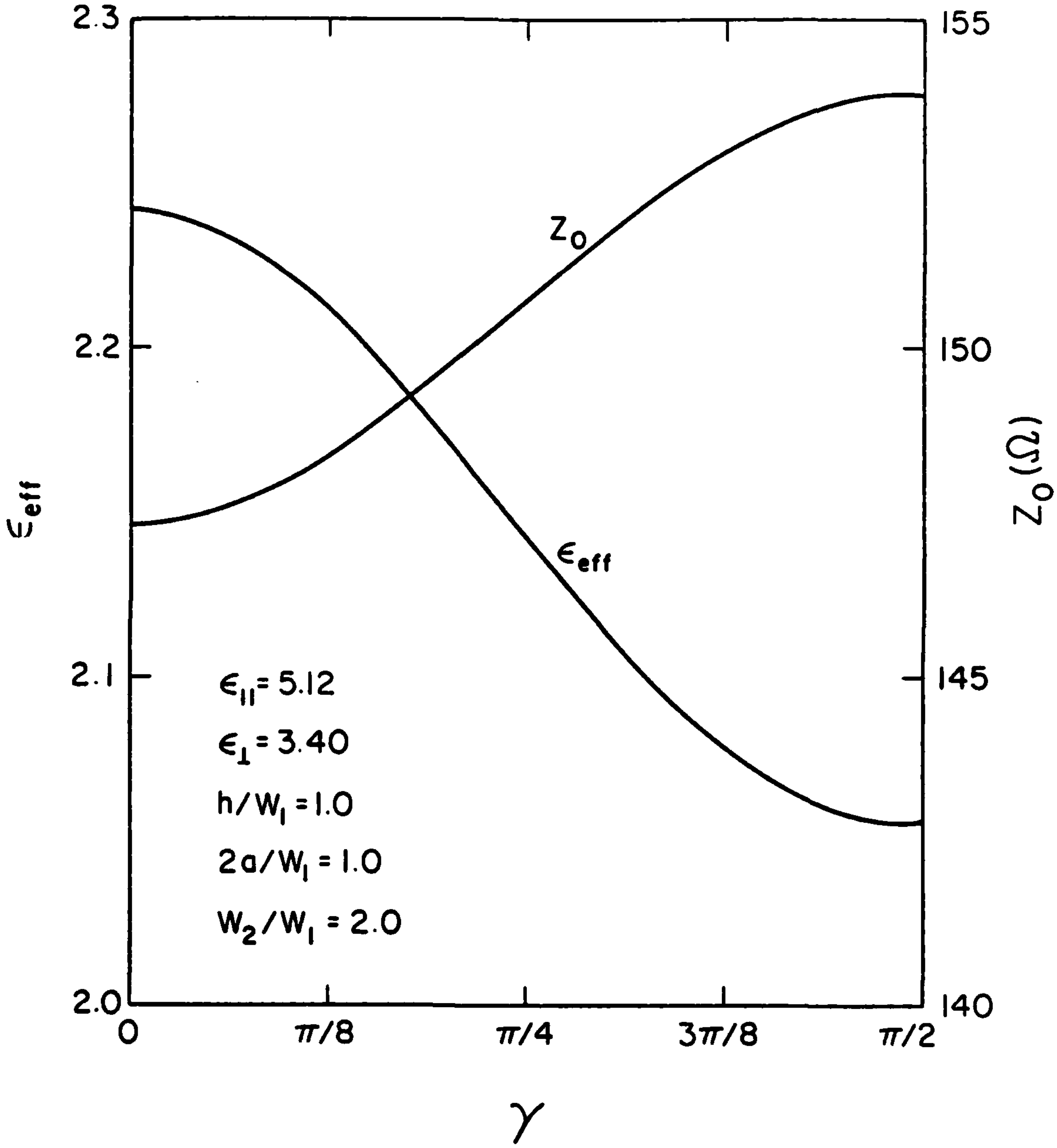


Figure 5. ACSL on anisotropic boron-nitride substrate.

Figure 5 shows the variation in the effective dielectric constant and the characteristic impedance of ACSL on an anisotropic boron-nitride substrate with the width ratio W_2/W_1.

IV. CONCLUSIONS

In this paper, the variational expressions for the line capacitance of the asymmetrical coplanar waveguide (ACPW) and the asymmetrical coplanar strip line (ACSL) have been derived. An efficient computational scheme, based on the Ritz procedure, has been employed for the numerical computations. Numerical results have been compared with the exact analytical solutions for the special case of air as the substrate material and excellent agreement has been found for a wide range of parameters. Also, some numerical data for ACPW and ACSL are shown for both the isotropic and anisotropic substrates.

The analytical approach presented herein is quite general and is easily applicable to other structures, such as asymmetrically coupled microstriplines.

APPENDIX I

ASYMMETRICAL COPLANAR WAVEGUIDE AND ASYMMETRICAL COPLANAR STRIP LINE WITHOUT SUBSTRATE

The line capacitances of the asymmetrical coplanar waveguide (ACPW, Fig. 6(a)) and the asymmetrical coplanar strip line (ACSL, Fig. 7(a)) without substrates can be evaluated analytically by a repeated application of conformal mapping. A series of transformations for ACPW and ACSL are shown in Figs. 6 and 7, respectively. The determinantal equations for the ratio t_3/u_3 and s_3/t_3, which determine k_3, are given in the following:

the determinantal equation of t_3/u_3

$$2\,\frac{K(k_o)}{K'(k_o)} = \frac{K\left(\dfrac{t_3}{u_3}\right)}{K'\left(\dfrac{t_3}{u_3}\right)} \tag{A-1}$$

the determinantal equation of s_3/t_3

for ACPW

$$\frac{F(\arcsin\dfrac{p_o}{q_o},\,k_o)}{K(k_o)} + 1 = 2\,\frac{F(\arcsin\dfrac{s_3}{t_3},\,\dfrac{t_3}{u_3})}{K\left(\dfrac{t_3}{u_3}\right)} \tag{A-2}$$

for ACSL

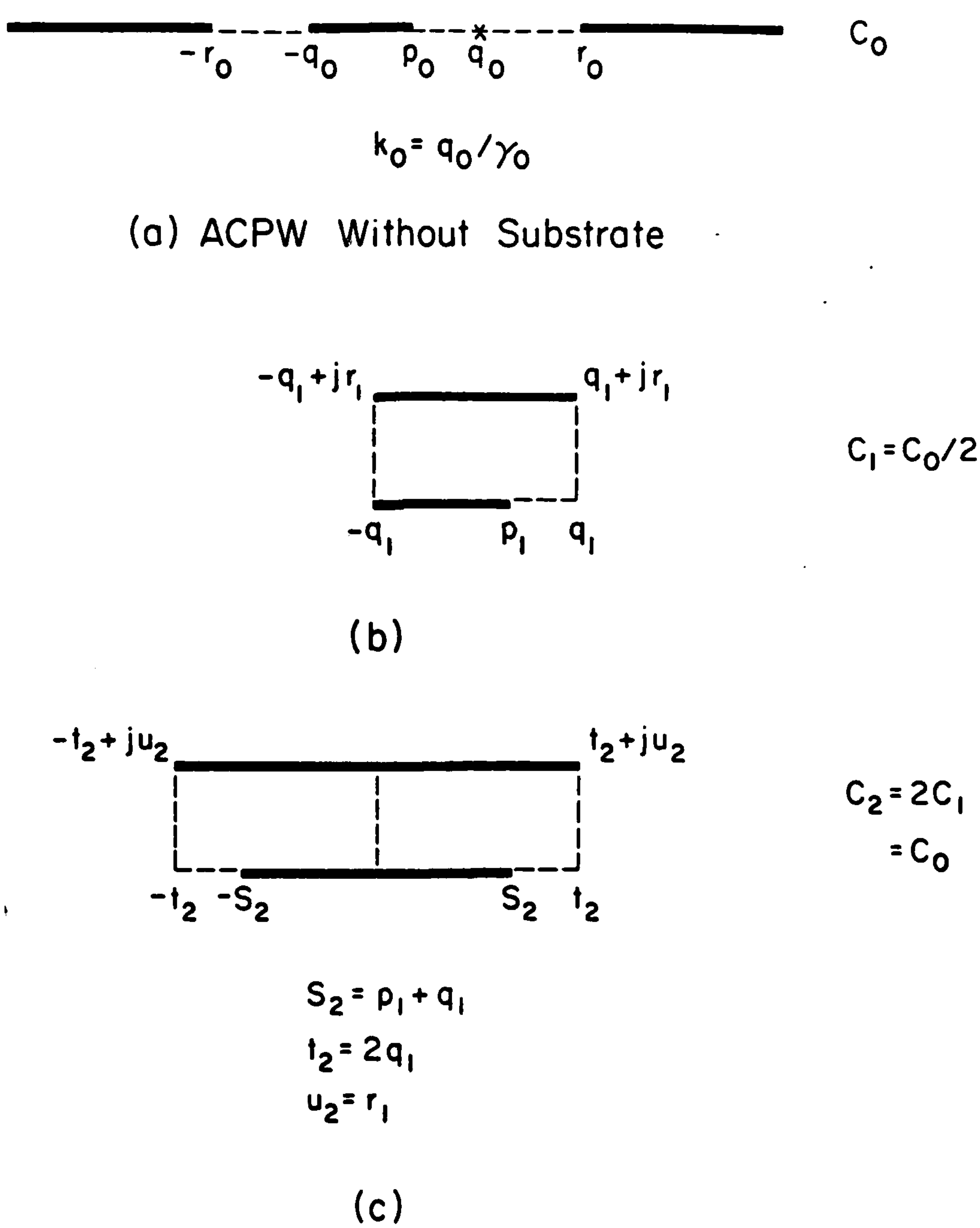


Figure 6. A series of transformations for the asymmetrical coplanar waveguide without substrates.

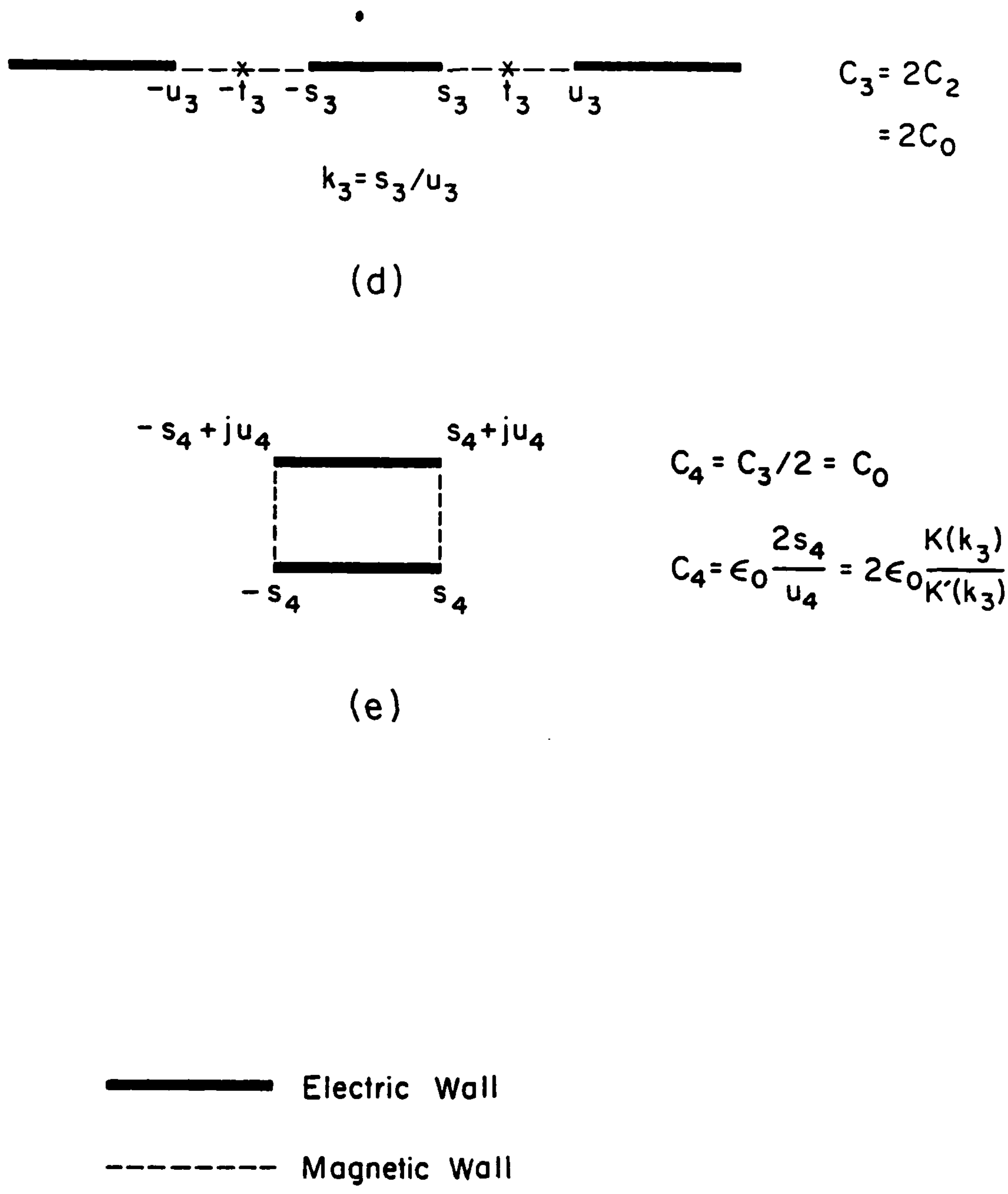


Figure 6. Continued

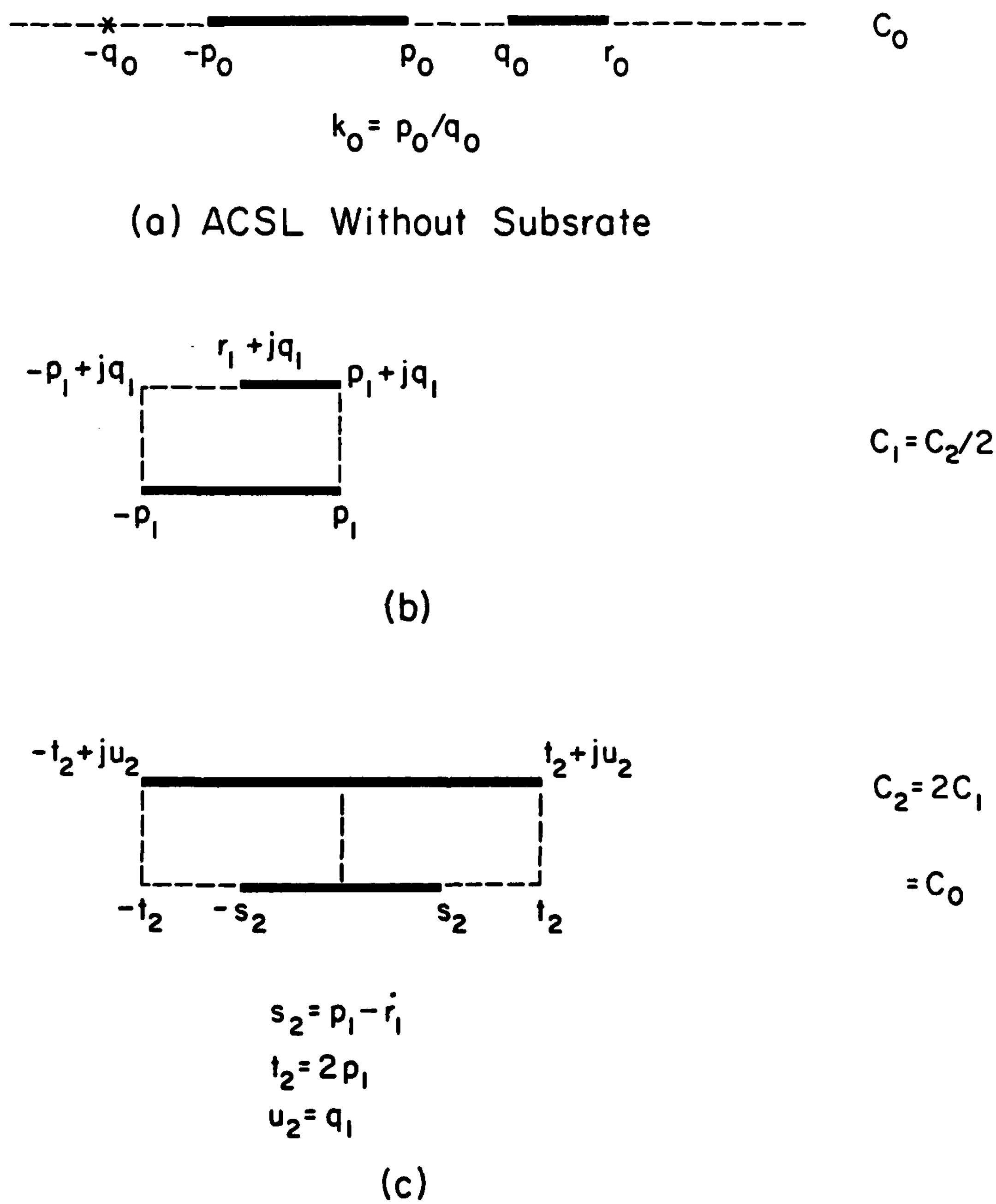
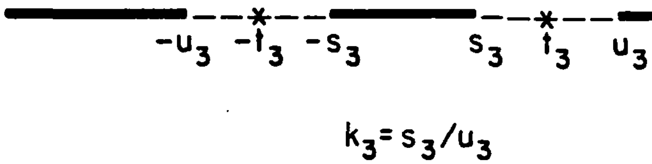
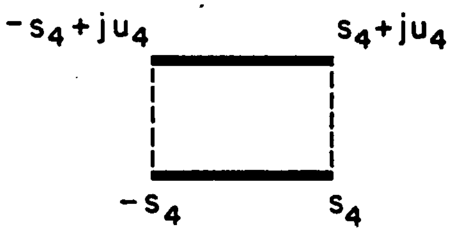


Figure 7. A series of transformations for the asymmetrical coplanar strip line without substrate.



$$C_3 = 2C_2 = 2C_0$$

(d)



$$C_4 = C_3/2 = C_0$$

$$C_4 = \epsilon_0 \frac{2s_4}{u_4} = 2\epsilon_0 \frac{K(k_3)}{K'(k_3)}$$

(e)

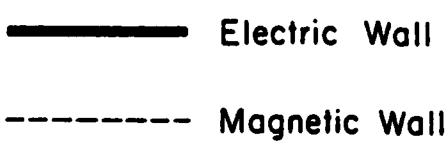


Figure 7. Continued

$$\frac{F(\arcsin \frac{q_o}{r_o}, k_o)}{K(k_o)} - 1 = 2\,\frac{F(\arcsin \frac{s_3}{t_3}, \frac{t_3}{u_3})}{K\left(\frac{t_3}{u_3}\right)} \tag{A-3}$$

where F(a,b) is the elliptic integral of the first kind and K(k) is the complete elliptic integral of the first kind.

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