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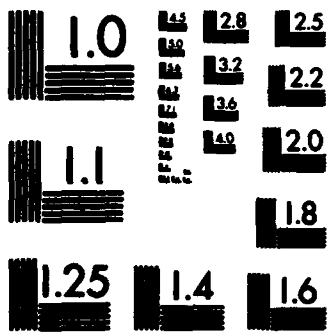
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EFFECT OF INERTIA ON FINITE NEAR-TIP DEFORMATION FOR FAST MODE-III CRACK GROWTH

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Abstract

The combined effects of finite deformation and material inertia have been analyzed for fast crack growth under anti-plane loading conditions. A steady-state dynamic solution has been obtained for the finite strain on the crack line, from the moving crack tip to the moving transition boundary with the zone of small strains. The crack propagates in a material with a response curve in uniform shear that is linear at small strains, and that remains constant once a critical strain has been exceeded. The corresponding quasi-static solution is given in the full zone of large deformation. For the dynamic formulation, an explicit expression for the crack-line strain has been obtained by expanding the displacement in a power series in the distance to the crack line, with coefficients which depend on the distance to the moving crack tip. Substitution in the equation of motion yields a nonlinear ordinary differential equation for the relevant coefficient, which can be solved rigorously. The finite deformation crack-line fields have been matched to appropriate small-strain fields at the transition boundary. The principal result is that the dynamic strain remains bounded at the crack tip, apparently due to the effect of material inertia. The crack-line strain has been plotted for several crack-tip speeds. It decreases with higher crack-tip speed. An explicit expression has been given for the extent of the zone of finite deformation, as a function of the crack tip speed and the far-field loading.

1. Introduction

Several recent investigations have been concerned with the effects of finite deformations on the fields of stress and deformation near a crack tip. Most notable are a series of papers by Knowles and Knowles and Sternberg, in which full nonlinear equilibrium theory of homogeneous and isotropic, incompressible elastic solids has been employed to construct fields near the tip of a crack. A summary has been presented in Ref.[1]. The most complete results have been obtained for a crack in a body subjected to finite anti-plane shear, as reported in Refs.[2] and [3]. Related work for the anti-plane shear case has been carried out by Lo [4], who found that for certain forms of the strain energy density, the formulation of the problem becomes formally identical to those of certain previously studied small strain elastic-plastic problems.

The present paper is also concerned with finite deformations near a crack tip under anti-plane shear loading. The formulation of the governing equations follows by-and-large Refs.[2] and [3]. A notable difference is, however, that we study a growing crack, and that the effects of inertia have been included in the formulation. We consider a strain energy density which yields a response curve for uniform shear that is linear at small strains, but that maintains a constant plateau once a certain critical strain has been exceeded.

The method of solution employs power series expansions in the distance to the crack line, with coefficients which depend on the distance to the crack tip. To analyze small strain crack line fields in elastic perfectly-plastic materials, such expansions have previously been used by Achenbach and Dunayevsky [5] and Achenbach and Li [6] for quasi-static problems, and by Achenbach and Li [7] for dynamic crack growth cases. Substitution of a displacement expansion in the equation of motion yields a nonlinear ordinary differential equation for the relevant coefficient, which can be solved rigorously. The crack-line fields have been matched to appropriate small strain elastic fields at a transition boundary.

The paper also includes a section which shows that for dynamic crack growth the stresses and the displacement gradients are continuous across the moving transition boundary which precedes the crack tip, at least near the crack line. Continuity of these quantitites all around the transition boundary has been shown for the quasi-static case. Furthermore, the full quasi-static field has been obtained in the zone of large deformation. For the elastic material that is considered in this paper, the static fields for a moving and a stationary crack are identical for coordinates centered at the crack tip. The quasi-static crack-tip strain is shown to be singular. An independent application of the expansion method yields the correct crack-line approximation to the exact solution.

The analysis of the crack-line fields for steady-state dynamic crack growth has yielded the most surprising result of this paper. It is shown that the effect of inertia is to remove the strain singularity at the crack tip. Thus, for the dynamic problem both the stress and the strain are bounded at the crack tip. Curves are presented which plot the strain versus distance along the crack line. An expression has been obtained for the length of the zone of large deformation as a function of the crack-tip speed.

2. Governing Equations

The equations which govern finite anti-plane shear deformation in an unbounded body containing a crack have been formulated by Knowles [2] and Knowles and Sternberg [3]. In this Section we briefly review the formulation of [2] and [3] in a slightly different notation, we introduce a different strain-energy density than was used in these papers, and we extend the formulation to the dynamic case.

Let X_i and x_i be the Cartesian coordinates of a typical point in the undeformed and deformed states, respectively. For anti-plane strain we have

$$x_3 = X_3 + U(X_1,X_2,t), \quad x_\alpha = X_\alpha \qquad \alpha = 1,2 . \tag{2.1}$$

The corresponding components of the deformation gradient $\underset{\sim}{F}$ are

$$F_{ij} = \frac{\partial x_i}{\partial X_j} = \begin{pmatrix} \delta_{\alpha\beta} & 0 \\ U_{,\beta} & 1 \end{pmatrix} \tag{2.2}$$

and the first fundamental scalar invariant is

$$I_1 = \mathrm{tr}(F^T F) = 3 + \varepsilon^2, \tag{2.3}$$

where

$$\varepsilon^2 = U_{,\alpha}U_{,\alpha} \quad , \quad U_{,\alpha} = \partial U/\partial X_\alpha \tag{2.4a,b}$$

We consider an incompressible material. Hence the deformation is locally volume preserving, and the Jacobian of the mapping (2.1) is

$$J = \det(\underset{\sim}{F}) = 1 \tag{2.5}$$

Let $\underset{\sim}{\tau}$ be the actual (Cauchy) stress tensor field in the deformed body, and let $\underset{\sim}{S}(X_\alpha,t)$ be the corresponding nominal (Piola) stress field in the undeformed configuration. Then

$$\underset{\sim}{\tau} = \underset{\sim}{S}\,\underset{\sim}{F}^T , \quad \underset{\sim}{S} = \underset{\sim}{\tau}(\underset{\sim}{F}^T)^{-1} \tag{2.6a,b}$$

The analysis of this paper is restricted to the class of incompressible solids for which W depends on I_1 only

$$W = W(I_1) \quad \text{for all } I_1 \geq 3,\ W(3) = 0. \tag{2.7}$$

As discussed by Knowles [1], we then have

$$\tau_{3\alpha} = \tau_{\alpha 3} = S_{3\alpha} = S_{\alpha 3} = 2W'(I_1)U_{,\alpha}, \tag{2.8}$$

$$\tau_{\alpha\beta} = S_{\alpha\beta} = S_{33} = 0\ , \quad \tau_{33} = 2W'(I_1)\varepsilon^2 \tag{2.9}$$

where ε is defined by (2.4a,b), and

$$W'(I_1) = dW/dI_1, \tag{2.10}$$

For dynamic problems these equations are supplemented by

$$S_{3\alpha,\alpha} = \rho_o\ddot{U}\ , \quad (\dot{\ }) = \partial/\partial t\ , \tag{2.11}$$

where ρ_o is the mass density in the reference configuration.

To define the desired relation between $S_{3\alpha}$ and $U_{,\alpha}$, we consider the static problem of an unbounded body, which is subjected to the following condition at infinity

$$U(X_1,X_2) = \kappa X_2 + o(1) \quad \text{as } X_\alpha X_\alpha \to \infty\ . \tag{2.12}$$

The solution then is

$$U(X_1,X_2) = \kappa X_2 \qquad \text{for all } (X_1,X_2) \tag{2.13}$$

and

$$\tau_{23} = \tau_{32} = \tau(\kappa) = 2W'(3+\kappa^2)\kappa, \quad \tau_{33} = \tau(\kappa)\kappa \quad (0 \leq \kappa < \infty) \tag{2.14a,b}$$

The graph of τ versus $\kappa(0 \leq \kappa < \infty)$ is called the response curve in simple shear. In this paper we wish to consider a response curve of the form shown in Fig. 1, which is defined by

$$\tau(\kappa) = \begin{cases} \mu\kappa & \kappa \leq \varepsilon_o & (2.15a) \\ \mu\varepsilon_o & \kappa \geq \varepsilon_o & (2.15b) \end{cases}$$

The corresponding strain energy density for an incompressible elastic solid then follows from (2.14a) as

$$W(I_1) = \int_0^{(I_1-3)^{1/2}} \tau(\kappa)d\kappa \qquad (3 \leq I_1 < \infty) \tag{2.16}$$

Substitution of (2.15a,b) into (2.16) yields

$$W(I_1) = \begin{cases} \frac{1}{2}\mu(I_1-3) & 3 \leq I_1 \leq 3 + \varepsilon_o^2 \quad (2.17a) \\ \mu[(I_1-3)\varepsilon_o^2]^{1/2} - \frac{1}{2}\mu\varepsilon_o^2 & 3 + \varepsilon_o^2 \leq I_1 \quad (2.17b) \end{cases}$$

The stress-strain relations follow from (2.8) and (2.17) as

$$S_{3\alpha} = \mu U_{,\alpha} \qquad \text{for } 0 \leq \varepsilon \leq \varepsilon_o \ , \tag{2.18}$$

$$S_{3\alpha} = S_o U_{,\alpha}/\varepsilon \quad \text{for } \varepsilon \geq \varepsilon_o, \tag{2.19}$$

where

$$S_o = \mu\varepsilon_o \tag{2.20}$$

It is noted that (2.19) implies that for $\varepsilon > \varepsilon_o$ we have

$$S_{31}^2 + S_{32}^2 = S_o^2 \ . \tag{2.21}$$

It is noted that the Cauchy stresses τ_{ij} do not satisfy a relation of analogous simplicity.

3. Discontinuities across a Propagating Transition Boundary

A surface which separates regions where the constitutive behavior is defined by (2.18) and (2.19), respectively, will be called a transition boundary. In this Section we examine the possibility of propagating discontinuities in the stress, $S_{ij}(\underset{\sim}{X},t)$, the displacement gradient, $F_{ij}(\underset{\sim}{X},t)$, and the particle velocity, $\dot{x}_i(\underset{\sim}{X},t)$, across a propagating transition boundary, for the case that the linear relation (2.18) holds ahead of the moving surface, while the nonlinear relation (2.19) applies behind it.

The propagating surface is represented by the relation

$$\Sigma(\underset{\sim}{X},t) = 0 \tag{3.1}$$

The unit normal to the surface is denoted by $\underset{\sim}{N}$. The speed of propagation V_N, which is the speed with which the surface traverses the material, is positive when the surface moves in the direction of $\underset{\sim}{N}$. A discontinuity of a field quantity, say $G(\underset{\sim}{X},t)$, is denoted in the usual manner by

$$[[G]] \equiv G^+ - G^- \tag{3.2}$$

where G^+ and G^- are the limits as $\underset{\sim}{X}$ approaches a point on the surface along paths entirely ahead of and entirely behind it .

The displacement is continuous, and hence displacement gradients in the surface are also continuous

$$[[\partial x_i/\partial \Gamma_\beta]] = 0 \;, \qquad \beta = 1,2 \tag{3.3}$$

where Γ_1 and Γ_2 are two independent coordinates in the surface $\Sigma(X,t) = 0$. The balance of linear momentum yields the well-known relation

$$[[S_{ij}]]N_j = -\rho_o V_N[[\dot{x}_i]] \ , \tag{3.4}$$

where ρ_o is the mass density in the reference configuration, and $\dot{\underset{\sim}{x}}$ denotes the particle velocity. From the balance of energy it follows that the energy dissipated locally as the surface passes, may be written as

$$H = [[W + \frac{1}{2}\rho_o\dot{x}_i\dot{x}_i]]V_N + [[\dot{x}_i S_{ij}]]N_j \ , \tag{3.5}$$

where W(X,t) is the strain energy density. It is assumed that

$$H \geq 0 \ , \tag{3.6}$$

i.e., the energy dissipation across a propagating transition boundary cannot be negative [8].

It can be verified that H may be rewritten as

$$H = [[W - \frac{1}{2}\rho_o\dot{x}_i\dot{x}_i]]V_N + \dot{x}_i^+[[\rho_o\dot{x}_i V_N + S_{ij}N_j]] + [[\dot{x}_i]](\rho_o\dot{x}_i V_N + S_{ij}N_j)^- \tag{3.7}$$

The second term on the right-hand side cancels by virtue of Eq.(3.4). A further rearrangement of H yields

$$H = ([[W]] - \frac{1}{2}\rho_o[[\dot{x}_i]][[\dot{x}_i]])V_N + [[\dot{x}_i]]S_{ij}^-N_j \ , \tag{3.8}$$

or, by the use of (3.4) in the second term,

$$H = [[W]]V_N + \frac{1}{2}[[\dot{x}_i]]([[S_{ij}]]N_j + 2S_{ij}^-N_j)$$

$$= [[W]]V_N + \frac{1}{2}[[\dot{x}_i]](S_{ij}^+ + S_{ij}^-)N_j \tag{3.9}$$

By using the compatibility relation

$$[[\dot{x}_i]] = -[[\partial x_i/\partial N]]V_N \ , \tag{3.10}$$

we finally find

$$H = \{[[W]] - \frac{1}{2}[[\partial x_i/\partial N]](S_{ij}^{+} + S_{ij}^{-})N_j\} V_N \tag{3.11}$$

For anti-plane shear, (3.11) reduces to

$$H = \{[[W]] - \frac{1}{2}[[\partial x_3/\partial N]](S_{3\alpha}^{+} + S_{3\alpha}^{-})N_\alpha\}V_N \tag{3.12}$$

where $\alpha = 1,2$. An expression similar to (3.12) has been presented in [9].

In this paper the interest is focussed on a field $x_3(X_1,X_2,t)$ which is antisymmetric with respect to the X_1 axis, but which has discontinuities across a transition boundary which is symmetric relative to $X_2 = 0$, and on which $V_N > 0$. This case applies, for example, to a transition boundary ahead of a Mode-III crack tip. The condition $H \geq 0$ then becomes

$$\bar{H} = H/V_N = [[W]] - \frac{1}{2}[[\partial x_3/\partial N]](S_{3\alpha}^{+} + S_{3\alpha}^{-})N_\alpha \geq 0 \tag{3.13}$$

Let us first consider the intersection point on $X_2 = 0$, which is defined by $X_2 = 0$, $X_1 = X_o(t)$. Because of antisymmetry we have $x_3(X_1,0,t) \equiv 0$, and hence

$$\partial x_3/\partial X_1 \equiv 0 \quad , \tag{3.14}$$

while (3.3) implies

$$\varepsilon^{-} = \varepsilon^{+} = \partial x_3/\partial X_2 = \varepsilon_o \tag{3.15}$$

where ε is defined by (2.20). By using the expressions for W given by (2.17a) and (2.17b), it then follows that

$$\bar{H} = [[W]] = \frac{1}{2}\mu(\varepsilon^{+})^2 - \mu\varepsilon^{-}\varepsilon_o + \frac{1}{2}\mu\varepsilon_o^2 = 0 \tag{3.16}$$

Next we consider a position (X_o, dX_2). Here we have by (3.13) and (3.16)

$$d\bar{H} \geq 0 \; , \tag{3.17}$$

where

$$d\bar{H} = \mu\varepsilon^{+}d\varepsilon^{+} - \mu\varepsilon_{o}d\varepsilon^{-} - \frac{1}{2}(\varepsilon_N^{+} - \varepsilon_N^{-}) \; (\mu d\varepsilon_N^{+} + \frac{\mu\varepsilon_o}{\varepsilon^{-}} d\varepsilon_N^{-} - \frac{\mu\varepsilon_o\varepsilon_N^{-}}{(\varepsilon^{-})^2} d\varepsilon^{-})$$

$$- \frac{1}{2} \; (d\varepsilon_N^{+} - d\varepsilon_N^{-})(\mu\varepsilon_N^{+} + \mu\varepsilon_o \frac{\varepsilon_N^{-}}{\varepsilon^{-}}) \tag{3.18}$$

In (3.18),

$$\varepsilon_N = \partial x_3/\partial N \tag{3.19}$$

Since $\varepsilon^{+} = \varepsilon^{-} = \varepsilon_o$ and $\varepsilon_N^{+} = \varepsilon_N^{-} = 0$, (3.18) reduces to

$$\mu\varepsilon_o(d\varepsilon^{+} - d\varepsilon^{-}) \geq 0 \tag{3.20}$$

In the domain of small deformation the quantity ε cannot increase with distance from the crack line, hence $d\varepsilon^{+} \leq 0$. On the other hand ε maintains the condition of large deformation, $\varepsilon \geq \varepsilon_o$, behind the transition boundary, and hence $d\varepsilon^{-} \geq 0$. Consequently (3.20) can be satisfied only if

$$d\varepsilon^{+} = d\varepsilon^{-} \equiv 0 \tag{3.21}$$

Equations (3.15) and (3.21) imply

$$[[\varepsilon]] = 0 \tag{3.22}$$

Equation (3.22), together with the constitutive equations, (2.18) and (2.19), give

$$[[S_{3\alpha}]] = \mu[[\partial x_3/\partial X_\alpha]] = \mu N_\alpha[[\partial x_3/\partial N]] \tag{3.23}$$

where (3.3) has also been used. Next (3.4), (3.10) yield

$$[[S_{3\alpha}]]N_\alpha - \rho_o V_N^2[[\partial x_3/\partial N]] = 0 \tag{3.24}$$

Substitution of (3.23) subsequently gives

$$\mu(1 - M_N^2) \quad [[\partial x_3/\partial N]] = 0 \ , \tag{3.25}$$

where $M_N^2 = \rho_o V_N^2/\mu$. It follows that

$$[[\partial x_3/\partial N]] = 0 \tag{3.26}$$

provided that $M_N < 1$. Equation (3.26) in turn implies by (3.10) that $[[\dot{x}_3]] = 0$, and hence

$$[[S_{3\alpha}]] = 0 \tag{3.27}$$

by (3.23). We have thus proven the continuity of the relevant quantities across the transition boundary near the crack line, for dynamic problems.

In (quasi) static problems ($\rho_o = 0$), we can prove the same results all along the transition boundary. Indeed, (3.3) and (3.4) give [10]

$$[[S_{3\alpha}]][[\partial x_3/\partial X_\alpha]] = 0 \ , \tag{3.28}$$

which is written explicitly as

$$I = \{\mu(\partial x_3/\partial X_\alpha)^+ - S_o(\partial x_3/\partial X_\alpha)^-/\varepsilon^-\}\{(\partial x_3/\partial X_\alpha)^+ - (\partial x_3/\partial X_\alpha)^-\} = 0 \tag{3.29}$$

It follows that

$$I = \mu(\varepsilon^+)^2 + S_o\varepsilon^- - S_o(\partial x_3/\partial X_\alpha)^+(\partial x_3/\partial X_\alpha)^-/\varepsilon^- - \mu(\partial x_3/\partial X_\alpha)^+(\partial x_3^-/\partial X_\alpha)^- . \tag{3.30}$$

By using the inequality

$$|x^+_{3,\alpha}x^-_{3,\alpha}| \le (x^+_{3,\alpha}x^+_{3,\alpha})^{1/2}(x^-_{3,\beta}x^-_{3,\beta})^{1/2} \ , \tag{3.31}$$

it follows from (3.30) that

$$\begin{aligned} I \ge \mu(\varepsilon^+)^2 + S_o\varepsilon^- - S_o\varepsilon^+ - \mu\varepsilon^+\varepsilon^- &= \mu[(\varepsilon^+)^2 + \varepsilon_o\varepsilon^- - \varepsilon_o\varepsilon^+ - \varepsilon^+\varepsilon^-] \\ &= \mu(\varepsilon^+ - \varepsilon^-)(\varepsilon^+ - \varepsilon_o) \ge 0 \ , \end{aligned} \tag{3.32}$$

since $\varepsilon^+ \leq \varepsilon^-$ and $\varepsilon^+ \leq \varepsilon_o$. But (3.28) and (3.32) will be in conflict, unless $\varepsilon^+ = \varepsilon_o$ on the transition boundary (note that $\varepsilon^+ = \varepsilon^-$ will lead to $\varepsilon^+ = \varepsilon_o$ also). It can be shown that this result, together with (3.3) and (3.4), yield either $\varepsilon^+ = \varepsilon^-$, or $\underset{\sim}{N} \cdot \underset{\sim}{\nabla} x_3 = 0$ (i.e. x_3 is locally constant) on the transition boundary. As will be seen later, we may exclude the latter possibility for the present application. We thus conclude that $\varepsilon^+ = \varepsilon^- = \varepsilon_o$ on the transition boundary. With this result, we can follow the procedures in Eqs.(3.23) ~ (3.27) to prove continuity of $S_{3\alpha}, \partial x_3/\partial N$ and $\dot{x}_3$.

4. Static Fields Near a Crack Tip

In the immediate vicinity of a crack tip the deformation is large, i.e., $\varepsilon > \varepsilon_o$, and consequently Eqs.(2.19) and (2.21) hold. Outside the region of large deformation, linearized elasticity according to Eq.(2.18) applies. The two regions are separated by a transition boundary. In this section it is assumed that the largest length dimension of the region of large deformation is small as compared to the length of the crack. This assumption implies that the crack may be taken as semi-infinite, and that the displacement outside the region of large deformation may be represented by the well-known square-root solution according to linearized elasticity.

The conditions on the faces of the crack are

$$U_{,2}(X_1, 0^{\pm}) = 0 \qquad -\infty < X_1 < 0 \tag{4.1}$$

The equation which governs the displacement field in the zone of large deformation is obtained by substituting (2.19) into the balance equation for static equilibrium. The result is

$$(U_{,2})^2 U_{,11} - 2U_{,1}U_{,2}U_{,21} + (U_{,1})^2 U_{,22} = 0 \ . \tag{4.2}$$

Equation (4.2) can be rewritten as

$$(1/U_{,1})L\{U_{,1}\} - (1/U_{,2})L\{U_{,2}\} = 0 \ , \tag{4.3}$$

where the differential operator L is defined as

$$L = (\partial/\partial X_1) - (U_{,1}/U_{,2})(\partial/\partial X_2) \tag{4.4}$$

It is now noted that L is a total differential along the characteristic line defined by

$$dX_2/dX_1 = - U_{,1}/U_{,2} \tag{4.5}$$

Along such a line (4.3) can be integrated to yield

$$\ell n(U_{,1}/U_{,2}) = C \quad , \text{ or } \quad U_{,1}/U_{,2} = -D \quad , \tag{4.6a,b}$$

where C and D are constants. By combining (4.5) and (4.6b), we conclude that the characteristics are straight lines:

$$dX_2/dX_1 = D \tag{4.7}$$

On these characteristics the displacement U is constant, because it follows from (4.5) that

$$dU = 0 \tag{4.8}$$

The problem formulation in the zone of large deformation as defined by Eqs.(2.21) and (4.8) is now identical to the one for small strain deformation near a crack in an elastic perfectly-plastic material with the Huber-Mises yield condition. The solution to the latter problem was given first by Hult and McClintock [11]. It was discussed in some detail by Rice [12]. In the present context the Hult-McClintock solution shows that the zone of large deformation is a circle of radius

$$R_o = \frac{1}{2\pi} (K_{III}/S_o)^2 \tag{4.9}$$

which is centered at a point E on the crack line (see Fig. 2), a distance

R_o ahead of the crack tip. In the zone of large deformation we have [11],[12]:

$$U = \left(\frac{R_o}{2\pi}\right)^{\frac{1}{2}} \frac{2}{\mu} K_{III} \sin\theta \tag{4.10}$$

$$S_{31} = -S_o \sin\theta \quad , \quad S_{32} = S_o \cos\theta \tag{4.11a,b}$$

In the region of linearized elasticity we have

$$U = \left(\frac{R}{2\pi}\right)^{\frac{1}{2}} \frac{2}{\mu} K_{III} \sin\tfrac{1}{2}\psi \tag{4.12}$$

$$S_{31} = -\left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_{III} \sin\tfrac{1}{2}\psi \tag{4.13}$$

$$S_{32} = \left(\frac{1}{2\pi R}\right)^{\frac{1}{2}} K_{III} \cos\tfrac{1}{2}\psi \tag{4.14}$$

where (R,ψ) are polar coordinates centered at the point E. Thus, (4.10) - (4.11a,b) hold for $R < R_o$, while (4.12) - (4.14) are valid for $R > R_o$.

We conclude this section by rederiving the field on the crack line in the zone of large deformation, by the use of an expansion method. This method can be used for problems that are not otherwise solvable. The method is shown here to demonstrate its utility, and as a preliminary to its application in the next section to the problem of dynamic crack growth. Following the approach of Achenbach and Dunayevsky [5] and Achenbach and Li [6], we seek a solution to Eq.(4.2) for small values of $X_2 (X_2/X_1 << 1)$, in the form

$$U = U^{(1)}(X_1)X_2 + U^{(3)}(X_1)X_2^3 + O(X_2^5) \tag{4.15}$$

Substitution of (4.15) into (4.2) followed by collection of terms of order X_2 yields

$$U^{(1)}_{,11}U^{(1)} - 2(U^{(1)}_{,1})^2 = 0 \tag{4.16}$$

Equation (4.16) is satisfied by

$$U^{(1)}_{,1} = (1/C)(U^{(1)})^2 \tag{4.17}$$

where C is a constant. The general solution to (4.17) is $U^{(1)} = -C/(X_1+D)$. A singular and multiple-valued solution at the crack tip is obtained by setting $D = 0$. Hence

$$U^{(1)} = \bar{C}/X_1 \tag{4.18}$$

where $\bar{C}$ is a still to be determined constant. At $X_1 = X_o$, $X_2 = 0$, we have $U^{(1)} = \varepsilon_o$, and thus $\bar{C} = X_o\varepsilon_o$ where X_o is the X_1 coordinate of the transition boundary on the crack line. It follows that

$$U = (X_o/X_1)\varepsilon_o X_2 + O(X_2^3) \tag{4.19}$$

The corresponding stresses at the transition boundary follow from (2.19) as

$$S_{31} = -S_o(X_2/X_o) \quad , \quad S_{32} = S_o \tag{4.20a,b}$$

Near the crack line in the zone of linearized elasticity, we now assume solutions of the general form (4.12) - (4.14). Continuity of S_{31} and S_{32} then yields by the use of (4.13) and (4.14)

$$\theta \simeq X_2/X_o = \frac{1}{2}\psi \tag{4.21}$$

which implies that $X_o = 2R_o$. It is now easily verified that for $\theta << 1$, (4.19) and (4.20a,b) do indeed correspond to (4.10), (4.11a,b) with (4.9), respectively.

As a final comment we note that the results derived in this section also apply to the crack line fields for quasi-static crack growth. For a propagating crack it is convenient to define the coordinate system (X_1,X_2) as moving with the crack tip. Stationary coordinates (X,Y) and moving coordinates (X_1,X_2) are related by

$$X_1 = X - a(t) \quad , \quad X_2 = Y \ , \tag{4.22a,b}$$

where $X = a(t)$, $Y = 0$ defines the position of the crack tip in the (X,Y) system. In the moving (X_1,X_2) system, the material time derivative is defined as

$$(\dot{\ }) = (\partial/\partial t) - \dot{a}(\partial/\partial X_1) \quad , \tag{4.23}$$

where $\dot{a} = da/dt$ is the crack-tip speed. Spatial derivatives are the same in the two systems. Since the time derivatives do not appear in the elastic constitutive equations used in this paper, they do not enter the formulation of the quasi-static crack growth problem. Hence, the governing equations and, therefore, the fields are identical for the stationary and the moving crack tip, provided that they are defined relative to a coordinate system that is centered at the (stationary or moving) crack tip.

5. Dynamic Fields near a Moving Crack Tip

It is convenient to formulate the governing equations in the moving coordinates (X_1,X_2) defined by (4.22a,b), with the material time derivative as defined by (4.23). In this paper we will restrict the attention to the steady-state field near a crack tip which moves with a constant speed c. The material time derivatives then reduce to

$$(\dot{\ }) = -c(\partial/\partial X_1), \quad (\ddot{\ }) = c^2(\partial^2/\partial X_1^2) \tag{5.1a,b}$$

Relative to the moving coordinate system, the equation of motion (2.11) becomes

$$S_{3\alpha,\alpha} = \rho_o c^2 U_{,11} , \quad \text{where} \quad \alpha = 1,2 \tag{5.2}$$

Substitution of (2.19) into (5.2) yields in the zone of large deformation

$$(U_{,2})^2 U_{,11} - 2U_{,1}U_{,2}U_{,12} + (U_{,1})^2 U_{,22} = \lambda[(U_{,1})^2+(U_{,2})^2]^{3/2} U_{,11} \tag{5.3}$$

where

$$\lambda = c^2\rho_o/S_o \tag{5.4}$$

A solution in the vicinity of the crack line can be obtained by using the expansion given by Eq.(4.15). By substituting (4.15) into (5.3) and collecting terms of order X_2 we find

$$U^{(1)}_{,11}U^{(1)} - 2(U^{(1)}_{,1})^2 = \lambda(U^{(1)})^2 U^{(1)}_{,11} \tag{5.5}$$

For $\lambda = 0$, (5.5) reduces to (4.16). It is convenient to define

$$V^{(1)} = U^{(1)}_{,1} , \tag{5.6}$$

and to rewrite (5.5) in the form

$$\frac{dU^{(1)}}{U^{(1)}(1-\lambda U^{(1)})} = \frac{dV^{(1)}}{2V^{(1)}} \tag{5.7}$$

This equation can be integrated to yield

$$\left(\frac{U^{(1)}}{U^{(1)}-1/\lambda}\right)^2 = \pm \frac{V^{(1)}}{V_o} \quad , \tag{5.8}$$

where V_o is a positive constant, and the sign is chosen so as to make the right hand side positive. For a few values of λ , $\pm V^{(1)}/V_o$ has been plotted in Fig. 3. Also plotted in Fig. 3 is the corresponding relation for $\lambda \equiv 0$, which follows from (4.19) and (5.6) as

$$\varepsilon_o X_o V_{qs}^{(1)} = -(U_{qs}^{(1)})^2 \tag{5.9}$$

Since we expect that $U^{(1)} \to U_{qs}^{(1)}$ and $V^{(1)} \to V_{qs}^{(1)}$ as $\lambda \to 0$, it can be seen from Fig. 3 that the relevant dynamic solution for $U^{(1)}$ and $V^{(1)}$ should be located in the part of the phase plane defined by

$$0 < U^{(1)} < 1/\lambda \;, \quad V^{(1)} < 0 \tag{5.10a,b}$$

Hence, for $V_o > 0$, the minus sign applies in Eq.(5.8). An important consequence of (5.10a) is that $U^{(1)}$ remains bounded for $\lambda \neq 0$.

We now return to Eq.(5.6), and we write

$$\frac{dX_1}{dU^{(1)}} = \frac{1}{V^{(1)}} = -\frac{(U^{(1)} - 1/\lambda)^2}{V_o(U^{(1)})^2} \tag{5.11}$$

where (5.8) has also been used. Integration yields

$$X_1 = -\frac{1}{V_o}\left\{U^{(1)} - \frac{2}{\lambda}\ln U^{(1)} - \frac{1}{\lambda^2}\cdot\frac{1}{U^{(1)}}\right\} + C \tag{5.12}$$

This expression contains 2 constants, V_o and C, which can be obtained from the conditions at $X_1 = 0$ and $X_1 = X_o$, where X_o is the intersection of the transition zone with the X_1 axis in the moving coordinate system. We have

$$U^{(1)} = \varepsilon_o \quad \text{at } X_1 = X_o \tag{5.13}$$

The only place on the crack line where singularities can be expected is $X_1 = 0$. Since Eq.(5.8) shows that $V^{(1)} \to \infty$ as $U^{(1)} \to 1/\lambda$, we conclude

$$U^{(1)} = 1/\lambda \quad \text{at } X_1 = 0 \tag{5.14}$$

Application of (5.13) and (5.14) to (5.12) yields

$$C = (2/\lambda V_o)\ell n\lambda \tag{5.15}$$

$$V_o = (-M^2 + 2\ell nM^2 + 1/M^2)/\lambda X_o \tag{5.16}$$

where M is the Mach number

$$M^2 = c^2/(\mu/\rho_o) \tag{5.17}$$

Substitution of (5.15) and (5.16) into (5.12) yields

$$\frac{X_1}{X_o} = \frac{1-M^4\bar{U}^2 + 2M^2\bar{U}\ell n(M^2\bar{U})}{(1-M^4 + 2M^2\ell nM^2)\bar{U}} \tag{5.18}$$

Here $\bar{U}$ is the normalized strain

$$\bar{U} = U^{(1)}/\varepsilon_o \tag{5.19}$$

The condition that the crack tip is in the zone of large deformation implies $1/\lambda > \varepsilon_o$, and hence $M^2 < 1$, since $M^2 = \varepsilon_o\lambda$.

Plots of $\bar{U}$ versus X_1/X_o can be obtained from (5.18). Results are shown in Fig.4, for various values of M. For $M > 0$, $\bar{U}$ remains finite (and equal to $1/M^2$) at $X_1 = 0$. In the limit $M \to 0$ we find

$$X_1/X_o = 1/\bar{U} \quad , \quad \text{or} \quad U^{(1)} = (X_o/X_1)\varepsilon_o \quad , \tag{5.20}$$

in agreement with the quasi-static results given by (4.19).

Equation (5.18) is the main result of this paper. It is, however, desirable to determine an explicit expression for X_o in terms of the crack tip speed and the far-field loading, albeit by approximation. In Section 3 it was shown that $\partial x_3/\partial N$ (or $\partial U/\partial N$), and $S_{3\alpha}$ are continuous at the transition boundary. Thus, the fields of large deformation can, in principle, be matched to corresponding linearly elastic fields, to yield an expression for X_o. Under the assumption that the zone of large deformation is small, the small strain fields are taken as the asymptotic near-tip fields for a crack in a linearly elastic solid. For the quasi-static case the solution (4.12)-(4.14) shows that exact matching can be achieved, provided that the center of the small strain field is shifted to a point on the crack line ahead of the actual crack tip. For the dynamic problem we do not expect that the zone of large deformation will be circular, and hence it is unlikely that a simple solution valid along the complete transition boundary can be obtained. In any event we have expressions for the large deformation fields only in the immediate vicinity of the crack line. It is, however, noted that to first order in X_2 the small-strain quasi-static and steady-state dynamic solutions are both of the general form, given by (4.12) - (4.14). Hence it seems reasonable to assume linearly elastic fields of the type (4.12) - (4.14), at least near the crack line, but where K_{III} should now be interpreted as an elastodynamic Mode-III stress intensity factor. For a semi-infinite crack the elastodynamic and quasi-static stress-intensity factor are related by $(K_{III})_{dyn} = (1-M)^{\frac{1}{2}} (K_{III})_{qs}$, see e.g. [13,P.35]. It follows that (4.9) also holds for the dynamic problem, but with the understanding that R_o is the local radius of curvature of the transition boundary at its intersection with the crack line.

Near the crack line continuity of $\partial U/\partial N$ implies continuity of $V^{(1)}$. At the transition boundary we then have

$$S_{31} = \mu X_2 V^{(1)}\Big|_{X=X_o} = -\mu X_2 V_o \left(\frac{\varepsilon_o}{\varepsilon_o - 1/\lambda}\right)^2 \tag{5.21}$$

$$S_{32} = S_o \quad , \tag{5.22}$$

where (5.8) has been used. Continuity of S_{31} and S_{32} yields by the use of (4.13) and (4.14)

$$\mu X_2 V_o \left(\frac{M^2}{1-M^2}\right)^2 / S_o \cong \frac{1}{2}\psi = \frac{1}{2}\frac{X_2}{R_o} \tag{5.23}$$

Substitution of V_o and R_o as given by (5.16) and (4.9) finally yields

$$X_o = \frac{1}{\pi}\left(\frac{K_{III}}{S_o}\right)^2 \frac{1-M^4 + 2M^2 \ell n M^2}{(1-M^2)^2} \tag{5.24}$$

The dimensionless form $(S_o/K_{III})^2 X_o$ has been plotted versus M in Fig. 5. It is noted that the zone of large deformation ahead of the crack tip decreases as M increases.

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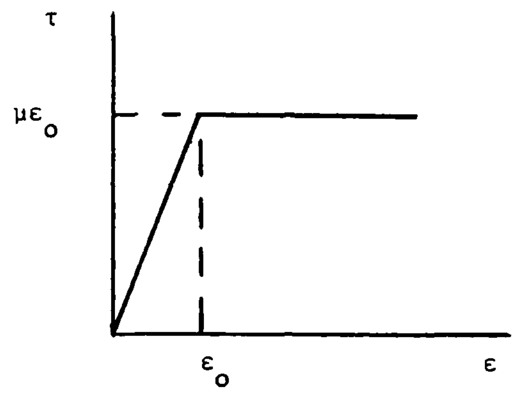

Fig. 1: Response curve in simple shear

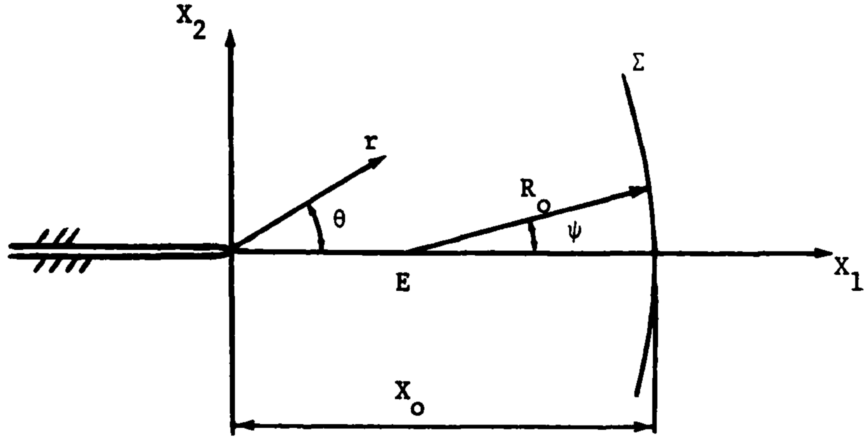

Fig. 2: Geometry of crack tip and transition boundary

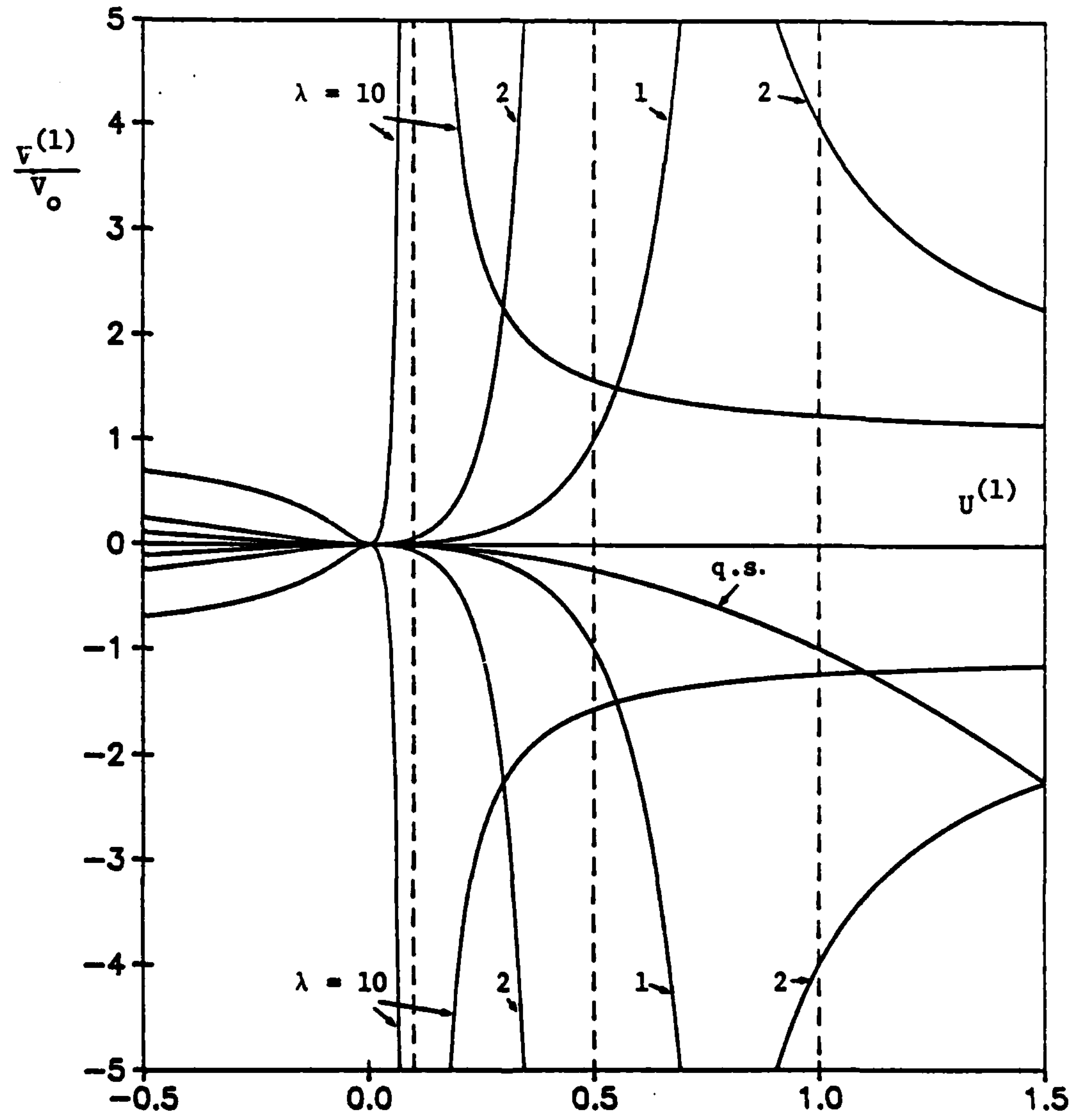

Fig. 3: $V^{(1)}/V_o$ versus $U^{(1)}$ for $\lambda = 10$, $\lambda = 2$ and $\lambda = 1$. The corresponding quasi-static curve of $\epsilon_o X_o V_{qs}^{(1)}$ versus $U_{qs}^{(1)}$ is indicated by q.s.

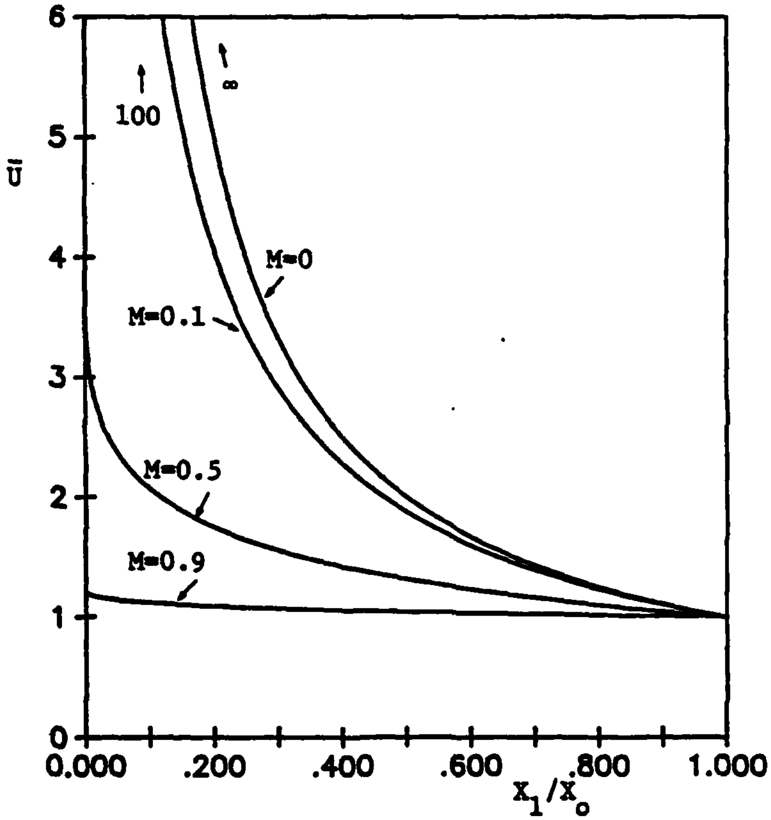

Fig. 4: Normalized crack-line strain in zone of large deformation.

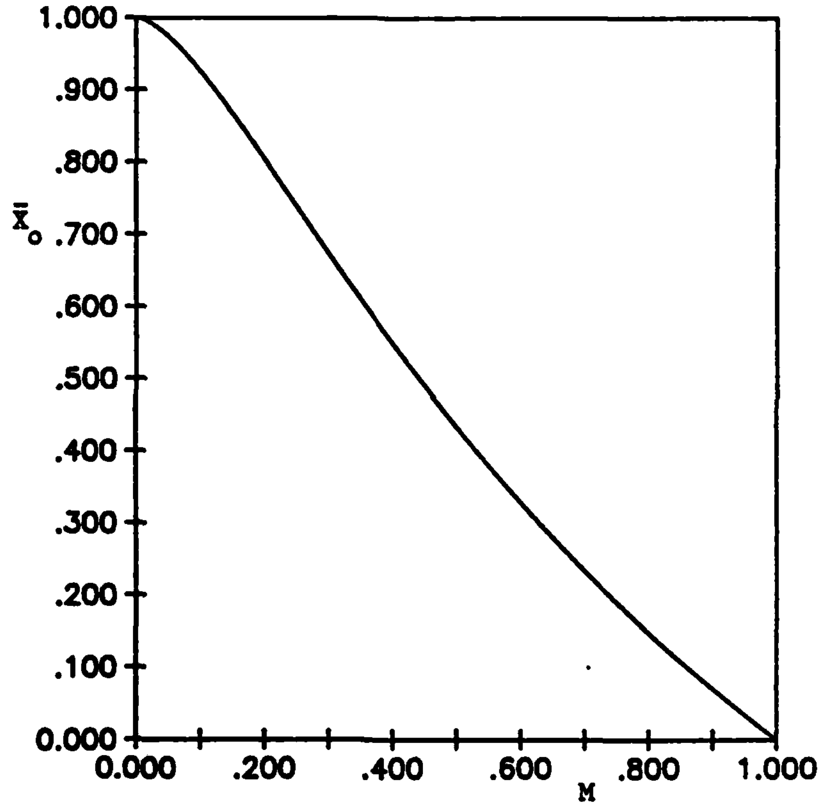

Fig. 5: Dimensionless length of zone of large deformation versus M; $\bar{X}_o = (S_o/K_{III})^2 \pi X_o$.

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The combined effects of finite deformation and material inertia have been analyzed for fast crack growth under anti-plane loading conditions. A steady-state dynamic solution has been obtained for the finite strain on the crack line, from the moving crack tip to the moving transition boundary with the zone of small strains. The crack propagates in a material with a response curve in uniform shear that is linear at small strains, and that remains constant once a critical strain has been exceeded. The corresponding quasi-static solution is given in the full zone of large deformation. For the dynamic formulation, an explicit expression for the crack-line strain has been obtained by expanding the displacement in a power series in the distance to the crack line, with coefficients which depend on the distance to the moving crack tip. Substitution in the equation of motion yields a nonlinear ordinary differential equation for the relevant coefficient, which can be solved rigorously. The finite deformation crack-line fields have been matched to appropriate small-strain fields at the transition boundary. The principal result is that the dynamic strain remains bounded at the crack tip, apparently due to the effect of material inertia. The crack-line strain has been plotted for several crack-tip speeds. It decreases with higher crack-tip speed. An explicit expression has been given for the extent of the zone of finite deformation, as a function of the crack tip speed and the far-field loading.

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