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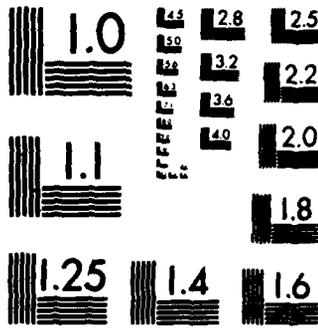
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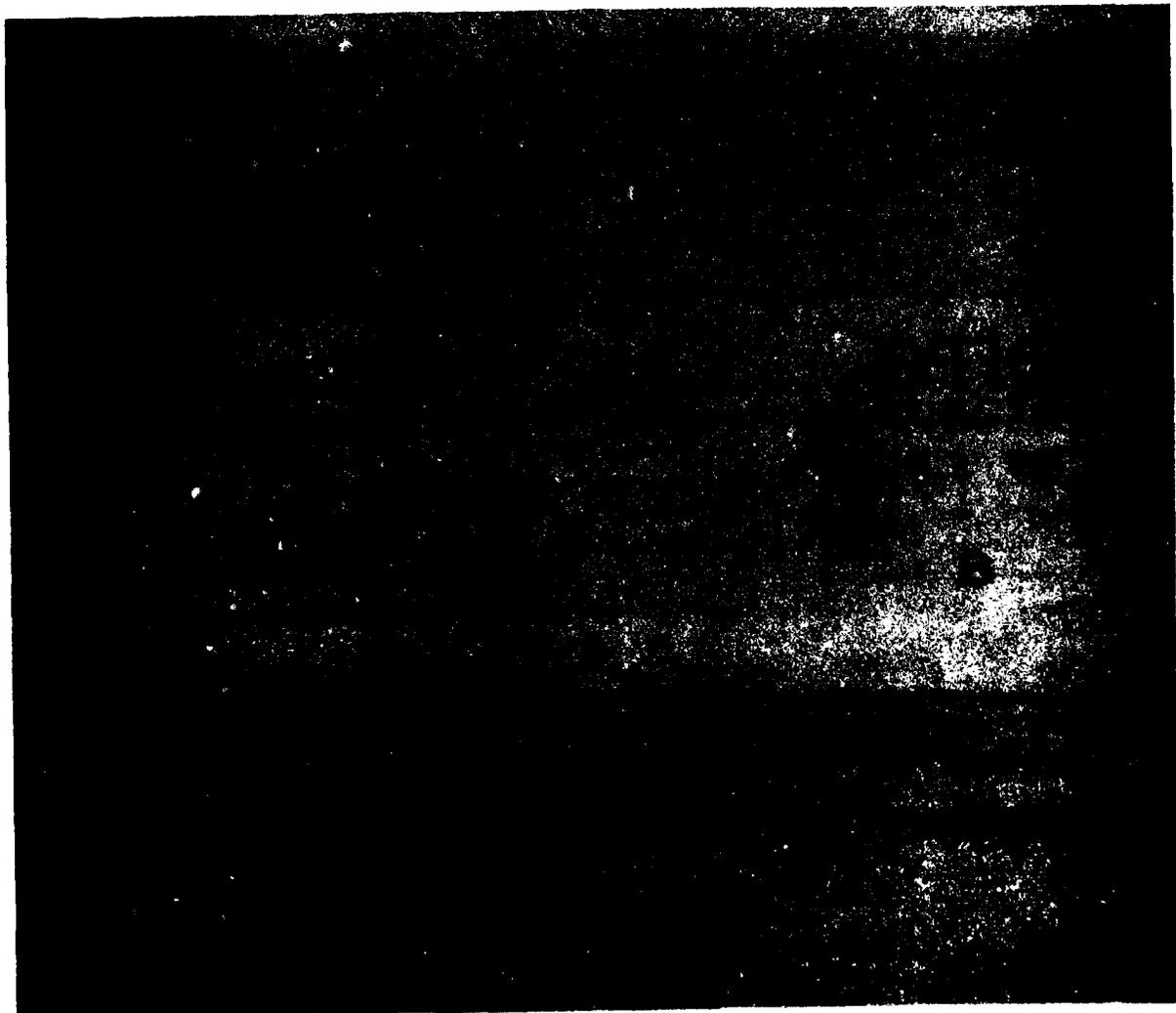
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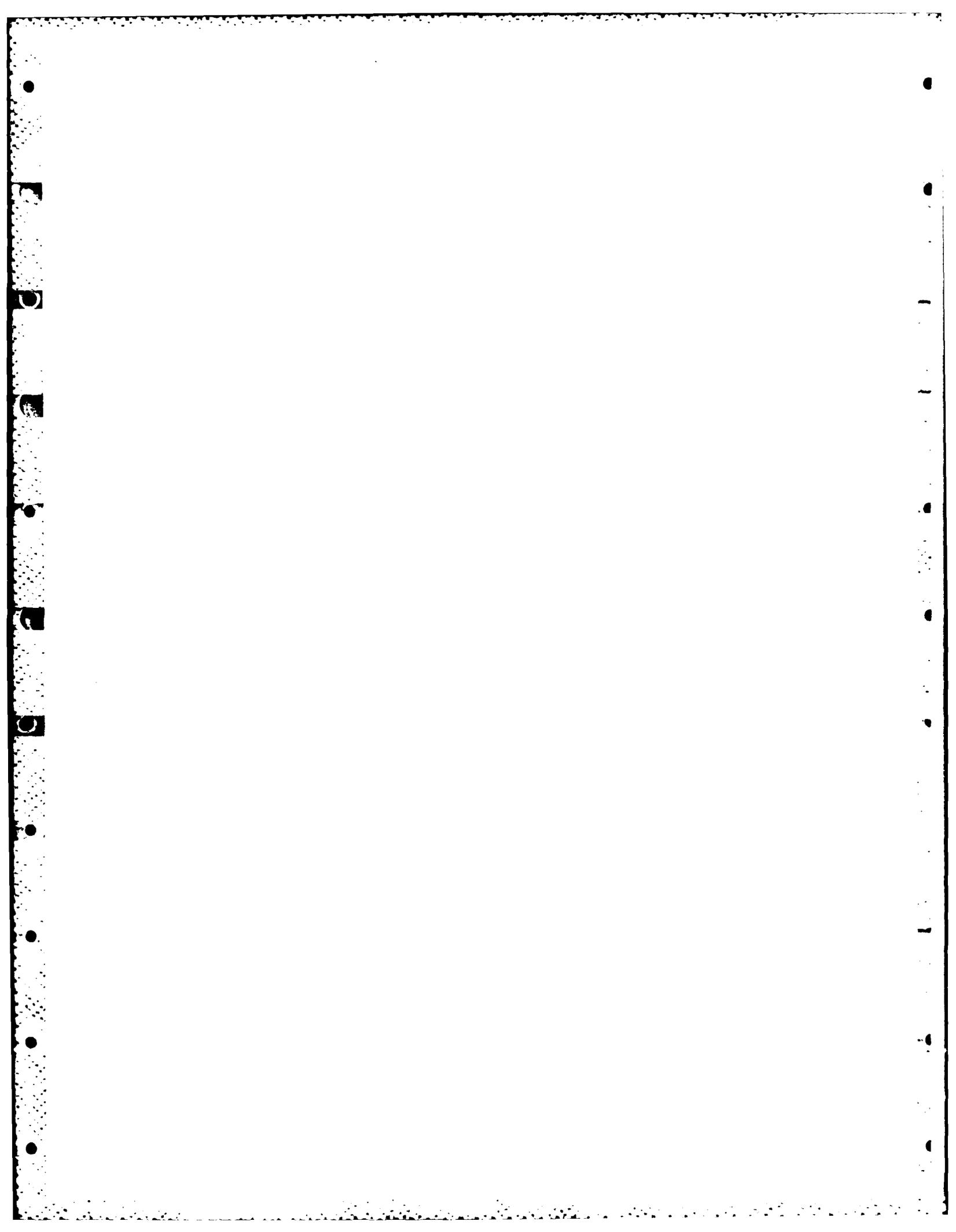
**THE CORRECTNESS OF
TISON'S METHOD FOR
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THE CORRECTNESS OF TISON'S METHOD

FOR GENERATING PRIME IMPLICANTS

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February 1982

Abstract. Tison devised a generalized consensus method to generate the prime implicants of a switching function. We present a complete, rigorous proof of its correctness.

Index terms: Tison's method, generalized consensus, prime implicant, switching function.

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Tison [3,4] developed novel methods for determining all prime implicants and all irredundant sums for a switching function. These methods employ a generalized notion of consensus and do not require the minterms of the function, unlike the well known Quine-McCluskey tabulation procedure. Neither Tison's published paper [4] nor the textbook by Muroga [2] includes a complete proof of correctness. The available correctness proofs are either inaccessible [3] or lengthy [4]. Nevertheless, we refer the reader to these two works for a comprehensive treatment of the methods.

In this note we establish the correctness of Tison's method for prime implicants; the correctness of the method for irredundant sums follows by a similar argument. We review a portion of Tison's paper [4] that shows that every generalized consensus can be formed via a sequence of consensus operations of order 2. The statement and proof of Theorem 2 below are new, however; they enable us to demonstrate that each biform variable can be treated just once. Because this note is self-contained, the reader need not have studied Tison's paper.

Let $S = xS_0$ be a product term (a conjunction of literals without duplication) with a variable x and $T = \bar{x}T_0$ be a product term with \bar{x} . If there is no variable y such that y appears in one of S_0 , T_0 and \bar{y} in the other, then S and T have consensus $S_0 T_0$ with respect to x . (Duplicate occurrences of literals are omitted from the consensus.)

Let (P, Q, \dots, R) be a list of terms. The variable x is monoform in this set if either x or \bar{x} appears among the literals in P, Q, \dots, R , but not both x and \bar{x} . The variable x is biform in this set if both x and \bar{x} appear among P, Q, \dots, R .

Tison's Method

Let $P + Q + \dots + R$ be a sum-of-products expression for a function f . This procedure produces a list of all prime implicants of f .

Step 1. Initially, let L be the list (P, Q, \dots, R) . Throughout the computation, L is a list of implicants of f called the implicant list. At the completion of the computation, L is the list of all prime implicants of f .

Step 2. For each biform variable x in (P, Q, \dots, R) perform Steps 2.1 and 2.2.

Step 2.1. For every pair of terms S, T on L , add to L the consensus of S, T , with respect to x , if such a consensus exists.

Step 2.2. Delete from L all terms S such that S implies another term on L .

Example. Let $\bar{w}x + \bar{y}z + wxy + xy\bar{z} + w\bar{x}yz$ be an expression for $f(w,x,y,z)$.

Initially,

$$L = (\bar{w}x, \bar{y}z, wxy, xy\bar{z}, w\bar{x}yz).$$

We show the terms generated when the variables are processed in the order w, x, y, z .

w : The only consensus with respect to w is xy , which covers wxy and $xy\bar{z}$.

(Recall that a product term P covers another product term Q if and only if every literal in P occurs in Q , equivalently, Q implies P .)

$$L = (\bar{w}x, xy, \bar{y}z, w\bar{x}yz).$$

x : The only consensus with respect to x is wyz , which covers $w\bar{x}yz$.

$$L = (\bar{w}x, xy, \bar{y}z, wyz).$$

y : The consensi with respect to y are xz and wz , and wz covers wyz .

$$L = (\bar{w}x, wz, xy, xz, \bar{y}z).$$

z : There are no consensi with respect to z on this last list L , which is the list of prime implicants of f .

According to the Consensus Theorem, if R and S have consensus T , then $R + S = R + S + T$, and T implies $R + S$. Thus, Tison's Method generates implicants of a function f . We prove that it produces precisely the set of prime implicants of f .

Let T_1, \dots, T_p be product terms. For each i let

B_i = product of literals of T_i that are biform in (T_1, \dots, T_p) ,

X_i = product of literals of T_i that are monoform in (T_1, \dots, T_p) .

By definition, $T_i = B_i X_i$. Set

$$X = X_1 \dots X_p, \quad (1)$$

omitting duplicate occurrences of literals. Call X the generalized consensus of T_1, \dots, T_p if

$$B_1 + \dots + B_p = 1 \quad (\text{irr.}) \quad (2)$$

holds irredundantly. Write $X = \text{GC}(T_1, \dots, T_p)$, and call p the order of X in T_1, \dots, T_p .

Theorem 1. For all product terms X, T_1, \dots, T_p , $X = \text{GC}(T_1, \dots, T_p)$ if and only if

- (i) $X \leq T_1 + \dots + T_p$;
- (ii) the deletion of any literal in X invalidates (i); and
- (iii) the deletion of any T_i invalidates (i) (the sum is irredundant).

Proof. We employ the notations B_i and X_i defined above.

Necessity.

- (i) If $X = 1$, then every $X_i = 1$, hence every $T_i = B_i$, and $T_1 + \dots + T_p = B_1 + \dots + B_p = 1$.
- (ii) Let $X = uY$ for a literal u , and suppose u occurs in T_1, \dots, T_m . We assert that $Y \not\leq T_1 + \dots + T_p$. When $Y = 1$ but $u = 0$,

$$T_1 + \dots + T_p = B_{m+1} + \dots + B_p \neq 1$$

because equation (2) holds irredundantly.

(iii) Suppose, to the contrary, $X \leq T_2 + \dots + T_p$. Then when $X = 1$,

$$T_2 + \dots + T_p = 1$$

implies

$$B_2 + \dots + B_p = 1.$$

Sufficiency. Suppose X satisfies (i), (ii), (iii).

Claim: No variable in X is biform in (T_1, \dots, T_p) . Suppose $X = xY$, $T_1 = \bar{x}T_1^*$ and $T_2 = xT_2^*$. If $X = 1$, then $x = 1$ and $T_1 = 0$. It follows from (i) that

$$X \leq T_2 + \dots + T_p,$$

a contradiction of (iii).

Claim: X has all the monoform variables in (T_1, \dots, T_p) . Suppose that the monoform variable x appears in T_1, \dots, T_m but not in X . Then when $X = 1$ and $x = 0$,

$$T_1 + \dots + T_p = T_{m+1} + \dots + T_p,$$

hence

$$X \leq T_{m+1} + \dots + T_p,$$

again contravening (iii).

By (ii), X has no variables that do not occur among (T_1, \dots, T_p) . Since X has precisely the monoform variables in (T_1, \dots, T_p) , we can write X in the form (1). If $X = 1$, then every $X_i = 1$ and $T_i = B_i$, hence by (i),

$$B_1 + \dots + B_p = 1.$$

The sum in this equation is irredundant, because if

$$B_2 + \dots + B_p = 1,$$

then

$$X \leq T_2 + \dots + T_p,$$

violating (iii). \square

We employ the notations that lead to (1) and (2). Let $X = GC(T_1, \dots, T_p)$. Let x be a biform variable in (T_1, \dots, T_p) . Renumbering terms if necessary, suppose x occurs in B_1, \dots, B_m , literal \bar{x} in B_{n+1}, \dots, B_p , and neither x nor \bar{x} in B_{m+1}, \dots, B_n , where $1 \leq m \leq n \leq p-1$. Rewrite (2):

$$x \sum_{i=1}^m (B_i/x) + \sum_{i=m+1}^n B_i + \bar{x} \sum_{i=n+1}^p (B_i/\bar{x}) = 1 \quad (\text{irr.}), \quad (3)$$

where B_i/x denotes the product of all literals of B_i except x . For $x = 1$ in (3),

$$\sum_{i=1}^m (B_i/x) + \sum_{i=m+1}^n B_i = 1, \quad (4)$$

and for $x = 0$ in (3)

$$\sum_{i=m+1}^n B_i + \sum_{i=n+1}^p (B_i/\bar{x}) = 1.$$

We contend that canceling redundant terms in these two equations yields (with renumbering of terms)

$$\sum_{i=1}^m (B_i/x) + \sum_{i=m'+1}^{n'} B_i = 1 \quad (\text{irr.})$$

$$\sum_{i=m'}^n B_i + \sum_{i=n'+1}^p (B_i/\bar{x}) = 1 \quad (\text{irr.})$$

with $m < m' \leq n'+1$ and $n' \leq n \leq p-1$. For if to the contrary, some B_j/x were redundant in (4), then B_j would be redundant in (2). Furthermore, if to the contrary $m' > n'+1$, then $B_{n'+1}$ would be redundant in (2). Define

$$\begin{aligned} Y &= xX_1 \dots X_{n'} = GC(T_1, \dots, T_{n'}) \\ Z &= \bar{x}X_{m'} \dots X_p = GC(T_{m'}, \dots, T_p) \end{aligned} \quad (5)$$

Both Y and Z are generalized consensi of order at most $p-1$, and $X = GC(Y, Z)$ with respect to x .

From this discussion, it is clear that every generalized consensus can be formed via a sequence of consensus operations of order 2. We must demonstrate that in Step 2 of Tison's Method each biform variable can be treated just once.

Theorem 2. Let T_1, \dots, T_q be implicants of a function and $X = GC(T_1, \dots, T_p)$ for some $p \leq q$. If Tison's Method is started with (T_1, \dots, T_q) , then it generates a product term that covers X .

Proof. During the computation of Tison's Method on (T_1, \dots, T_q) , let x_1, x_2, \dots be the order in which the biform variables of (T_1, \dots, T_q) are used. Call the taking of consensus of order 2 with respect to x_i Stage i of this computation.

Let t be the smallest nonnegative integer for which all biform variables of (T_1, \dots, T_p) are among x_1, \dots, x_t . We prove by induction on t that at the end of Stage t and during all subsequent Stages, the implicant list has a term that covers X . If $t = 0$, then $p = 1$ and $X = T_1$, and the result holds trivially.

Suppose $t \geq 1$. In the preceding discussion we established that $X = GC(Y, Z)$ with respect to $x = x_t$ for product terms $Y = GC(T_1, \dots, T_n)$ and $Z = GC(T_m, \dots, T_p)$ of the form (5). Let r and s be the smallest nonnegative integers such that all biform variables of (T_1, \dots, T_n) are among x_1, \dots, x_r and all biform variables of (T_m, \dots, T_p) are among x_1, \dots, x_s . Because $x = x_t$ is monoform both in (T_1, \dots, T_n) and in (T_m, \dots, T_p) (by definition of m and n), $r < t$ and $s < t$.

By the inductive hypothesis, at the beginning of Stage t , the implicant list has a term Y^* that covers Y and a term Z^* that covers Z .

If Y^* does not contain x_t , then by construction of Y in (5) and X in (1), Y^* covers X . Similarly, if Z^* does not contain \bar{x}_t , then Z^* covers X . If Y^* contains x_t and Z^* contains \bar{x}_t , then by (1) and (5), the consensus $GC(Y^*, Z^*)$ generated during Stage t covers X . Ergo, at the end of Stage t , the implicant

list has a term that covers X . Furthermore, since a term on the implicant list is deleted only when another term that covers it is on the list, after Stage t the implicant list always has a term that covers X . \square

Suppose $f = T_1 + \dots + T_q$ for some implicants T_1, \dots, T_q of f . Let P be a prime implicant of f . In the inequality

$$P \leq T_1 + \dots + T_q,$$

redundant terms on the right side can be removed to form an irredundant sum; renumbering terms if necessary, we find

$$P \leq T_1 + \dots + T_p \quad (\text{irr.})$$

for some $p \leq q$. Because P is prime, no literal in the left side can be omitted from the left side of this inequality. By Theorem 1, $P = \text{GC}(T_1, \dots, T_p)$. By Theorem 2, Tison's Method, when started on (T_1, \dots, T_p) , generates a term that covers P ; since P is prime, it generates P itself. Therefore, all prime implicants are generated. Consequently, every implicant on the list at the completion of the computation must be prime; otherwise, it would have been deleted in Step 2.2. We conclude that Tison's Method generates precisely the set of prime implicants of f .

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