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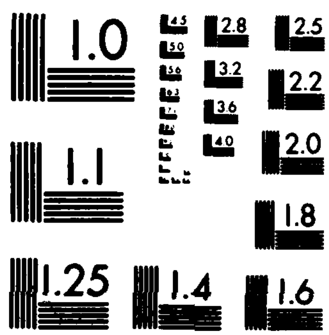

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Research supported under this effort investigated some of the spectral and invertibility properties of band matrices. Spectral theory and classical approximation theory were used to give a new proof of the exponential decay of the entries of the inverses of band matrices. The problem of obtaining spectral information about a given matrix from its submatrices was studied. Another result obtained during the research effort was that for subspaces of the continuous functions on the unit interval that have B-spline like bases there is an interpolating projection whose noun depends only on the condition number of the basis.

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Investigations into the spectral and invertibility properties of band matrices and other classes of sparse matrices were carried out. Mathematical techniques used included methods from classical approximation theory, from the spectral theory of self-adjoint operators, and from the theory of Banach spaces. Computer experiments were used to help formulate problems and to search for counterexamples. The main scientific content will appear in five papers. Three of these have been submitted for publication and accompany this report. The other two will be forwarded to AFOSR on completion. We now give short summaries of each of these papers together with a discussion of other results and observations we made which, while not suitable for publication now, might be useful to the sponsor and other readers of this report.

1.Decay Rates for Inverses of Band Matrices (written with W.F. Moss and P.W. Smith), to appear in Mathematics of Computation, 1984.

Spectral theory and classical approximation theory are used to give a new proof of the exponential decay of the entries of the inverses of band matrices. The rate of decay is bounded in terms of the extrema of the spectrum of the absolute value of the matrix. The rate predicted can be attained in some cases. These results are then used to establish the exponential decay of the eigenvectors for a class of generalized eigenvalue problems, to establish exponential decay of inverses of certain sparse but non-banded matrices, and to prove exponential decay for the Moore-Penrose inverse of full rank banded rectangular matrices.

2.Spectral Inequalities for Principal Submatrices, submitted to Linear Algebra and Its Applications.

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The problem of obtaining spectral information about a given matrix from its submatrices is studied. Inequalities relating the extremal eigenvalues of Hermitian matrices to like quantities of certain principal submatrices are obtained. The spectral norm of a matrix is bounded in terms of the spectral norms of some of its principal submatrices. The Hausdorff distance between the numerical range of a matrix and the convex hull of the numerical ranges of certain of its principal submatrices is bounded.

3. On the Existence of Interpolating Projections onto Spline Spaces, submitted to Journal of Approximation Theory.

It is shown that for subspaces of the continuous functions on the unit interval that have B-spline like bases there is an interpolating projection whose norm depends on only the condition number of the basis. The relation between the norm of the projection and the condition number of the associated B-spline collocation matrix is exploited.

4. Condition Numbers of Rectangular Matrices and Bounds for Generalized Inverses, in preparation.

A notion of intrinsic condition number of a rectangular matrix is defined and shown to enjoy some properties of the classical condition number. For example, the relative distance to the set of all matrices of smaller rank is the reciprocal of the intrinsic condition number. The question of whether a matrix with a small intrinsic condition number must also have a generalized inverse of small norm is then studied. The answer turns out to be norm dependent. In particular, for the maximum norm the answer is "no" while for the sum norm the answer is "yes." These results are consequences of recent deep results in the geometric theory of Banach spaces.

5.Spectral Bounds for $\|A^{-1}\|_\infty$, in preparation.

The problem of bounding $\|A^{-1}\|_\infty$ in terms of the distance of the spectrum of A from zero is considered. It is shown that if A is banded and positive definite then $\|A^{-1}\|_\infty < c\|A^{-1}\|_2^{5/4}$ where c depends on only the bandwidth of A and the spectral norm of A. The argument relies on a Chebyshev series expansion of the inverse of A. We conjecture that the correct exponent is 1. The case of bi-infinite Toeplitz matrices is also considered.

6.Other Findings.

(a).We observed that a theorem of Shmuel Friedland implies that if A is an N by N positive definite matrix, then there is a diagonal matrix D such that the spectral radius of I-DA is zero. This implies that there is an iterative method for Ax=b based on D which converges in N steps and which is readily vectorizable. The catch is that it is virtually impossible to find such a D. However, the gap between current technology and what is theoretically possible is large and might be worth investigating. Friedland's proof is quasi-constuctive in that it is bases on a variational principal. I have discussed this with him but to date no progress has been made.

(b).We briefly considered an abstract marching technique for banded linear systems. Discussions with other mathematicians revealed that the method of parallel shooting was very similar to a concrete realization of our method. Consequently, we did not feel that further development would be an efficient use of time.

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