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NONLOCAL CONTINUUM THEORY FOR DISLOCATION AND FRACTURE
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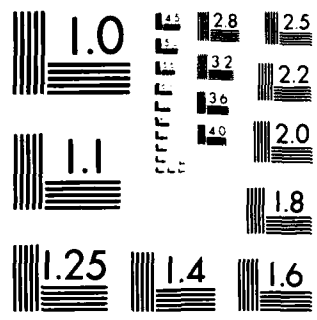
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FOR DISLOCATION AND FRACTURE

A.C. Eringen
PRINCETON UNIVERSITY

Technical Report No. 61
Civil Engng Res. Rep. No. 84-SM-2

PRINCETON UNIVERSITY
Department of Civil Engineering



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Research Sponsored by the
OFFICE OF NAVAL RESEARCH
under
Contract N00014-83-K-0126 Mod 4
Task No. NR 064-410

April 1984

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER #61	PRINCETON UNIVERSITY	2. GOVT ACCESSION NO. AD-A141 538	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) NONLOCAL CONTINUUM THEORY FOR DISLOCATION AND FRACTURE		5. TYPE OF REPORT & PERIOD COVERED technical report	
		6. PERFORMING ORG. REPORT NUMBER 84-SM-2	
7. AUTHOR(s) A. Cemal Eringen		8. CONTRACT OR GRANT NUMBER(s) N00014-83-K-0126 Mod 4	
9. PERFORMING ORGANIZATION NAME AND ADDRESS PRINCETON UNIVERSITY Princeton, NJ 08544		10. PROGRAM ELEMENT PROJECT, TASK AREA & WORK UNIT NUMBERS NR 064-410	
11. CONTROLLING OFFICE NAME AND ADDRESS OFFICE OF NAVAL RESEARCH (Code 471) Arlington, VA 22217		12. REPORT DATE April 1984	
		13. NUMBER OF PAGES 11	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) unclassified	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) APPROVED FOR PUBLIC RELEASE DISTRIBUTION UNLIMITED			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Fracture, Dislocations, Nonlocal Continua			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) By means of linear theory of nonlocal elasticity, solutions are given for some Volterra dislocations situated in an elastic solid. The stress fields are determined for screw and edge dislocations. The stresses and elastic energy are devoid of usual singularities predicted by the classical (local) elasticity. A Theory is developed for continuous distributions of dislocations on the basis			

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ABSTRACT (cont)

of nonlocal incompatible elasticity. Stress fields are given for volume, surface and line distributions of dislocations. Celebrated Peach-Koehler formula is modified to include nonlocal Green's functions. The stress fields for three- and two-dimensional cases and for the anti-plane strain are determined for line distributions. Calculations are carried out for the uniform distributions of edge and screw dislocations along a straight line segment. By means of the maximum stress hypothesis, a fracture criteria is introduced. Calculated theoretical strengths are in good agreement with those based on the atomic models. Reduction of material strength with the presence of dislocation line and the maximum number of dislocations are given.



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Princeton University



NONLOCAL CONTINUUM THEORY FOR DISLOCATION AND FRACTURE

A.C. Eringen

ABSTRACT - By means of linear theory of nonlocal elasticity, solutions are given for some Volterra dislocations situated in an elastic solid. The stress fields are determined for screw and edge dislocations. The stresses and elastic energy are devoid of usual singularities predicted by the classical (local) elasticity. A theory is developed for continuous distributions of dislocations on the basis of nonlocal incompatible elasticity. Stress fields are given for volume, surface and line distributions of dislocations. Celebrated Peach-Koehler formula is modified to include nonlocal Green's functions. The stress fields for three- and two-dimensional cases and for the anti-plane strain are determined for line distributions. Calculations are carried out for the uniform distributions of edge and screw dislocations along a straight line segment. By means of the maximum stress hypothesis, a fracture criteria is introduced. Calculated theoretical strengths are in good agreement with those based on the atomic models. Reduction of material strength with the presence of dislocation line and the maximum number of dislocations are given.

IT IS A WELL-KNOWN FACT that stress fields due to Volterra dislocations, contain singularities at the center of the dislocation so that in a small region around the center (the core region) classical elasticity fails to apply. The radius of this region is estimated, usually by means of atomic models. Because of these stress and energy singularities, several other methods have been devised for the prediction of fracture. Clearly such singularities are non-physical and a proper model should eliminate them.

Phonon dispersion experiments have shown that the phase velocity of plane waves in single crystals depends on the wave length so that dispersion is the rule rather than the exception. Yet classical elasticity predicts constant phase velocities for plane waves, independent of frequency and wave length.

There are many other physical phenomena in the microscopic scale, that cannot be predicted

by means of classical elasticity (linear or nonlinear). Among these, we mention the state of stress at a sharp crack tip, surface tension, atomic inclusions, defects, granular, porous and composite solids.

In several previous papers [cf., 1-4], we have shown that the stress fields, due to dislocations and cracks, predicted by the nonlocal elasticity contains no singularities. In fact, they vanish at the center of dislocations and at the crack tip. Moreover, the maximum stress occurs at a short distance away from these points. By equating the maximum stress to the cohesive stress that holds atomic bonds together, a physically realistic fracture criteria was established^{5,6}. In the classical limit, the celebrated Griffith criterion is obtained with the dividend that the Griffith constant is determined without any additional assumption on the surface energy which could not be measured to within any reasonable accuracy. Estimated errors in such measurements is known to be not less than several hundred percent.

The dispersion curves predicted by the nonlocal theory are nearly in coincidence with those based on the atomic lattice dynamics and observations on phonon dispersions [cf. 4-6].

There exist ample evidence that recently developed theory of nonlocal elasticity⁶⁻⁹ is a proper mathematical model which can eliminate the foregoing difficulties, extending the domain of applicability of continuum mechanics to physical phenomenon with internal characteristic lengths at the molecular, atomic or microstructural scales.

Based on these observations, we expect that a nonlocal theory may bear fruit in dealing self-stress and energies of dislocation loops for both the discrete and continuous distributions of dislocations. Moreover, the onset of fracture and the theoretical strength of solids may be estimated by means of the continuum theory which permit extensions to amorphous solids and composites. The major portion of the present paper stems from these observations.

A.C. Eringen

In section 2, I present a summary of the linear theory of nonlocal isotropic elastic solids. In section 3, the stress field and energy due to a screw dislocation are calculated. Both turned out to be devoid of singularities. The stress field due to an edge dislocation is treated in section 4. In section 5, I develop a theory for the continuous distribution of dislocations. Field equations are obtained for stress functions for two and three-dimensional state of strain.

Green's functions for the infinite solids are obtained in section 6 leading to a generalized Peach-Koehler formula for the stress field due to line distribution of dislocations. Results are gratifying in that they contain no singularities so that calculations can be carried out with uniformity, for surface and line distributions of dislocations throughout core regions. In fact, stress fields due to a uniform distribution of screws along a line segment verifies our predictions. The theoretical strength of a single crystal predicted by the nonlocal theory is in agreement with that known in atomic theory. Also given are the shear strengths reduction due to line distributions and the maximum allowable number of screws within a straight line segment of length $2a$.

2. BASIC EQUATIONS

From the atomic theory of lattice dynamics and experimental observations on phonon dispersions, it is well-known that the stress at a material point x in a body depends not only on strains at x but also on strains at all other points x' of the body. In linear theory of nonlocal elasticity, this is expressed by an integral constitutive equation of the form [cf. 2,4].

$$(2.1) \quad \tau_{ki} = \int_V c_{kilm}(x'-x) e_{mn}(x') dv(x')$$

where c_{kilm} are the material property functions which depend on the vector $x'-x$ and e_{kl} is the linear strain measure defined by

$$(2.2) \quad e_{kl}(x') = \frac{1}{2} \left[\frac{\partial u_k(x')}{\partial x'_l} + \frac{\partial u_l(x')}{\partial x'_k} \right]$$

where $u_k(x,t)$ is the displacement vector. For homogeneous and isotropic solids, (2.1) may be simplified to

$$(2.3) \quad \tau_{ki}(x) = \int_V \sigma(x'-x) \tau_{ki}(x') dv(x')$$

where τ_{ki} is the classical local stress tensor given by Hooke's law:

$$(2.4) \quad \tau_{ki}(x') = \lambda e_{TT}(x') \delta_{ki} + 2\mu e_{ki}(x')$$

in which λ and μ are the usual Lamé constants. In Eq. (2.3), the kernel $\sigma(x'-x)$ is

an influence function representing a function which brings influences of strains at various points x' to x , in different proportions. According to the accompanying neighborhood hypothesis¹⁰, it assumes the maximum value at $x'=x$, sharply decreasing with the distance from \tilde{x} . From Eq. (2.3), it is clear that σ depends on a length scale ϵ . This is an internal characteristic length which may be selected to be proportional to the lattice parameter a for single crystals, i.e.

$$(2.5) \quad \epsilon = e_0 a,$$

average granular distance for amorphous bodies and the average distance for fiber composites, etc. In Eq. (2.5), e_0 is a non-dimensional constant which can be determined by one experiment.

When $\epsilon \rightarrow 0$, Eq. (2.3) must revert to (2.1). This implies that σ is a Dirac delta sequence. Thus, in this limit nonlocal theory reverts to the classical elasticity theory. By discretizing Eq. (2.3), it can be shown that equations of nonlocal elasticity also reduce to those of atomic lattice dynamics¹¹.

In several previous papers^{4,12,13}, I gave special representations for $\sigma(x'-x)$ which lead to excellent predictions in accord with the atomic lattice dynamics. For example, for the two-dimensional case, an appropriate kernel is

$$(2.6) \quad \sigma(x) = (2 - \epsilon^{-2})^{-1} K_0(\epsilon \sqrt{x^2})$$

where K_0 is the modified Bessel's function. For two-dimensional lattices, Eq. (2.6) provides an excellent match between acoustical dispersion curves, based on the nonlocal elasticity and those based on Born-Kármán theory of atomic lattice dynamics. In the entire Brillouin zone, the error is less than 6%. It is also interesting to note that for the infinite medium σ is the Green's function of a linear differential operator. In the case of Eq. (2.6), this means that σ satisfies

$$(2.7) \quad (1 - \epsilon^2 \nabla^2) \sigma = \delta(x'-x)$$

vanishing at infinity.

Under the mild assumptions of vanishing nonlocal effects for the body forces and couples, the momentum balance laws of nonlocal elasticity reduces to Cauchy's laws

$$(2.8) \quad \tau_{ki,k} + \rho f_i - \ddot{u}_i = 0$$

$$(2.9) \quad \tau_{ki} = \tau_{ik}$$

where ρ is the mass density and f_i is the body force density.

As usual, repeated indices denote summation over the range of indices. We also abbreviate partial differentiation with respect to x_k as an index following a comma, and use a superscripted dot to indicate the time derivative, e.g.,

$$\tau_{ki,k} = \frac{\partial \tau_{ki}}{\partial x_k}, \quad \dot{\tau}_{ki} = \frac{\partial \tau_{ki}}{\partial t}$$

The field equations of nonlocal elasticity are obtained by combining (2.1), (2.2) and (2.8).

$$(2.10) \quad - \int_{\partial V} c_{kilmn}(\underline{x}' - \underline{x}_i) e_{mn}(\underline{x}') da'_k + \int_V c_{kilmn}(\underline{x}' - \underline{x}_i) u_{m,nk}(\underline{x}') dv' + c(f_i - \ddot{u}_i) = 0$$

where a superposed prime denotes dependence on \underline{x}' , e.g. $u'_m = u_m(\underline{x}')$, $dv' = dv(\underline{x}')$. In deriving (2.10), we used the identity

$$\begin{aligned} \frac{\partial}{\partial x_k} (c_{kilmn} e'_{mn}) &= - \frac{\partial c_{kilmn}}{\partial x'_k} e'_{mn} \\ &= - \frac{\partial}{\partial x'_k} (c_{kilmn} e'_{mn}) \\ &\quad + c_{kilmn} \frac{\partial e'_{mn}}{\partial x'_k} \end{aligned}$$

and the Green-Gauss theorem to convert the first term to the surface integral over ∂V in (2.10). In (2.10), the first integral represents the surface stresses (e.g. surface tension). Consequently, nonlocal theory accounts for the *surface physics*. This important asset of nonlocal theory is not included in classical field theories.

The integro-partial differential equations (2.10) replaces Navier's equations of classical elasticity. The displacement field u is to be determined by solving (2.10) under appropriate initial and boundary conditions. Boundary and initial conditions on displacement and velocity fields are identical to those of classical elasticity. Boundary conditions on tractions is based on the true stress tensor t_{ki} , not on \bar{t}_{ki} , i.e.

$$(2.11) \quad t_{ki} n_k = t_{(n)i}$$

where $t_{(n)i}$ are prescribed boundary tractions. For mixed boundary-value problems, to avoid a possible overspecification, care is necessary along the boundary of the two surfaces on one of which u_i and on the other $t_{(n)i}$ are prescribed¹⁴.

If we assume that the nonlocal kernel α satisfies (2.7), then for the homogeneous and isotropic solids of infinite extent, integro-differential equations (2.10) can be replaced by *singularly perturbed* partial differential equations. This is achieved by noting

$$(2.12) \quad (1 - \alpha \nabla^2) t_{ki} = \bar{t}_{ki}$$

and using (2.8) in (2.12):

$$(2.13) \quad (\lambda + \mu) u_{k,kk} + \mu u_{i,kk} + (1 - \alpha \nabla^2)(c f_k - \rho \ddot{u}_k) = 0$$

In the static case and vanishing body forces (2.13) reduce to Navier's equations

$$(2.14) \quad (\lambda + \mu) u_{k,kk} + \mu u_{i,kk} = 0$$

However, note that the stress field is determined by solving (2.12).

3. SCREW DISLOCATION

A screw dislocation is introduced by cutting a solid along the plane $x_2 = 0$, $x_1 \geq 0$ and introducing a constant displacement discontinuity b (called Burger's vector) along the x_3 -direction of the rectangular coordinates. In this case, the displacement field has only single component $u_3(x_1, x_2, t)$ which is determined by solving

$$(3.1) \quad \nabla^2 u_3 = 0$$

The stress field is obtained by solving (2.12), where σ_{31} and σ_{32} are the only non-vanishing components of the local stress tensor. These and u_3 are given by¹⁵:

$$(3.2) \quad u_3 = \frac{b}{2\pi} \theta,$$

$$(3.3) \quad \sigma_{31} = -\frac{\mu b}{2\pi r} \sin \theta, \quad \sigma_{32} = \frac{\mu b}{2\pi r} \cos \theta$$

where (r, θ, z) are the cylindrical coordinates, i.e.

$$(3.4) \quad x_1 = r \cos \theta, \quad x_2 = r \sin \theta, \quad x_3 = z$$

To obtain the stress tensor t_{ij} , we determine the solution of (2.12) under the condition that t_{ij} must vanish on a circular cylindrical surface \bar{r} of radius r , $-\pi/2 < \theta < \pi/2$ as $r \rightarrow \infty$. Such a solution was given in a previous paper¹⁶.

$$(3.5) \quad t_{z\theta} = \frac{\mu b}{2\pi r} \left[1 - \frac{r}{\bar{r}} k_1(r/\bar{r}) \right], \quad t_{zz} = 0$$

This is regular for all $0 < r < \infty$. It is valid in the core region. In fact, for $r=0$, $t_{z\theta}$ vanishes so that the hoop stress is zero at the center of the dislocation, a result which is in accord with the physics of the problem.

Classical elasticity solution results in the case of $\alpha=0$, leading to a stress singularity r^{-1} . The strain energy per unit length of x_3 in a region bounded by concentric cylinders of radius r_0 and R , is given by

$$(3.6) \quad \begin{aligned} U &= 2\pi \int_{r_0}^R t_{z\theta} e_{z\theta} r dr \\ &= \frac{\mu b^2}{2\pi} \left[\ln(R/r_0) + k_0 R/\bar{r} - k_0 R/\bar{r} \right] \end{aligned}$$

Contrary to the classical result, this has no singularity for $r_0 = 0$. In fact,

$$(3.7) \quad \Sigma/L \Big|_{r_0=0} = \frac{\mu b^2}{8\pi} [\ln(R/2\epsilon) + K_0(R/\epsilon)]$$

which makes sense on physical grounds. For large R , Eq. (3.7) may be approximated by

$$(3.8) \quad \Sigma/L \Big|_{r_0=0} = \frac{\mu b^2}{8\pi} [\ln(R/2\epsilon) + (\pi\epsilon/2R)^{1/2} \exp(-R/\epsilon)]$$

which exhibits the dependence of the energy on the size of the solid.

4. EDGE DISLOCATION

A straight edge dislocation in a solid causes plane strain with displacement vectors characterized by

$$(4.1) \quad u_1 = u_1(x_1, x_2); \quad u_2 = u_2(x_1, x_2); \quad u_3 = 0$$

where u_i are the rectangular components of the displacement vector. The displacement component u_1 undergoes a constant jump discontinuity b along the half plane $x_1 \geq 0, x_2 = 0$. The classical elasticity solution of this problem is well-known¹⁵. For convenience, we write the classical stress in the form

$$(4.2) \quad \begin{aligned} \sigma_{\theta} &= \sigma_{11} + \sigma_{22} = iBr^{-1}(e^{i\theta} - e^{-i\theta}), \\ \sigma_{\phi} &= \sigma_{22} - \sigma_{11} + 2i\sigma_{12} = iBr^{-1}(e^{-i\theta} + e^{-3i\theta}) \end{aligned}$$

where

$$(4.3) \quad B = \mu b/2\pi(1-\nu)$$

From Eq. (2.12), it follows that the true stress field satisfies

$$(4.4) \quad (1 - \epsilon^2 \nabla^2) \{\mathcal{G}, \mathcal{F}\} = \{\sigma_{\theta}, \sigma_{\phi}\}$$

where \mathcal{G} and \mathcal{F} have the forms

$$(4.5) \quad \mathcal{G} = t_{11} + t_{22}, \quad \mathcal{F} = t_{22} - t_{11} + 2it_{12}$$

Thus, we must find the general solution of

$$(4.6) \quad (1 - \epsilon^2 \nabla^2) F = (\epsilon r)^{-1} e^{in\theta}, \quad n = \pm 1, -3$$

The solution of (4.6) which is regular at $r=0$ and $r=\infty$ is found to be

$$(4.7) \quad F = f_n(\rho) e^{in\theta}$$

where

$$(4.8) \quad f_n(\rho) = \int_0^{\rho} I_n(\rho') K_n(\rho) d\rho' + \int_{\rho}^{\infty} K_n(\rho') I_n(\rho) d\rho',$$

$$\rho = r/\epsilon$$

Here, $I_n(z)$ and $K_n(z)$ are modified Bessel's functions. By superposition, we obtain

$$(4.9) \quad \mathcal{G} = iB\epsilon^{-1} [f_1(\rho) e^{i\theta} - e^{-i\theta}],$$

$$(4.10) \quad \mathcal{F} = iB\epsilon^{-1} [f_1(\rho) e^{-i\theta} + f_3(\rho) e^{-3i\theta}]$$

The integration can be carried out for $n=1$ so that

$$(4.11) \quad f_1(\rho) = \rho^{-1} - K_1(\rho)$$

$$(4.12) \quad f_3(\rho) = \int_0^{\rho} I_3(\rho') K_3(\rho) d\rho' + \int_{\rho}^{\infty} K_3(\rho') I_3(\rho) d\rho'$$

The stress field follows from (4.5)

$$(4.13) \quad \begin{aligned} t_{11} &= -\frac{\mu b}{4(1-\nu)\epsilon} [5f_1(\rho) \sin\theta + f_3(\rho) \sin 3\theta], \\ t_{22} &= -\frac{\mu b\epsilon}{4(1-\nu)\epsilon} [f_1(\rho) \sin\theta - f_3(\rho) \sin 3\theta], \\ t_{12} &= \frac{\mu b\epsilon}{4(1-\nu)\epsilon} [f_1(\rho) \cos\theta - f_3(\rho) \cos 3\theta] \end{aligned}$$

In polar coordinates, components of the stress tensor follows from

$$(4.14) \quad t_{rr} + t_{\theta\theta} = \mathcal{G}, \quad t_{\theta\theta} - t_{rr} + 2it_{r\theta} = \mathcal{F} e^{2i\theta}$$

Hence,

$$(4.15) \quad \begin{aligned} t_{rr} &= -\frac{\mu b}{4\pi(1-\nu)\epsilon} [f_1(\rho) + f_3(\rho)] \sin\theta, \\ t_{\theta\theta} &= -\frac{\mu b}{4\pi(1-\nu)\epsilon} [5f_1(\rho) - f_3(\rho)] \sin\theta, \\ t_{r\theta} &= \frac{\mu b}{4\pi(1-\nu)\epsilon} [f_1(\rho) - f_3(\rho)] \cos\theta \end{aligned}$$

Again, we notice that the stress field vanishes at $r=0$ so that contrary to the classical result, no

stress singularity is present at the center of the dislocation.

It should also be noted that (4.15) do not reduce to the formulas of the classical theory by setting $\epsilon = 0$. This situation is, of course, well-known for singularly perturbed differential equations.

5. CONTINUOUS DISTRIBUTION OF DISLOCATIONS

A small neighborhood $n(x)$ of x in a *distorted body* V , may be relaxed to a small neighborhood $N(X)$ of the image X of x , in an *undistorted* (or *natural*) configuration V , by releasing constraints exerted to $n(x)$ by the rest of the body. A line element dx at $x \in n(x)$ can be expressed in terms of its image $dX \in N(X)$ by

$$(5.1) \quad dx = A dX$$

where $A(X)$ is called the *elastic distortion*. We assume that $A(X)$ is continuously differentiable and possesses a unique inverse, so that

$$(5.2) \quad dX = A^{-1} dx$$

Consider a smooth surface S in V bounded by a closed curve C . The true Burger's vector b of the dislocations piercing through S is defined by

$$(5.3) \quad b = \oint_C dx = \oint_C A^{-1} da = \int_S a_n da$$

where n is the unit normal to S , the positive sense of C being counter-clockwise, when sighting along n . Here, a is called the *true dislocation density*.

$$(5.4) \quad a = \text{curl } A^{-1}, \quad a_{jk} = \epsilon_{kmn} A^{-1}_{j,n,m}$$

For small distortions, we can write

$$(5.5) \quad A^{-1}_{jk} = \delta_{jk} + \alpha_{jk}, \quad A^{-1}_{jk} = \delta_{jk} - \alpha_{jk}$$

so that

$$(5.6) \quad a_{jk} = \epsilon_{kmn} \alpha_{j,n,m}$$

From this, it follows that

$$(5.7) \quad a_{j[k,k]} = 0$$

The linear strain tensor e_{kj} and rotation tensor w_{kj} are given by

$$(5.8) \quad e_{kj} = \frac{1}{2} (\alpha_{kj} + \alpha_{jk})$$

$$(5.9) \quad w_{kj} = \frac{1}{2} (\alpha_{kj} - \alpha_{jk})$$

The strain incompatibility is expressed by

$$(5.10) \quad \epsilon_{ijk} \epsilon_{kmn} e_{i,n,jm} = \tau_{kj}$$

where τ_{kj} is called the *discompatibility tensor* and it is given by

$$(5.11) \quad \tau_{kj} = \frac{1}{2} (\epsilon_{kmn} a_{i,n,m} + \epsilon_{kmn} a_{n,k,m})$$

All of these results are well-known in classical theory (cf. [16]).

Nonlocal theory stipulates that the stress-strain relations is given by (2.1). For homogeneous, isotropic solids, under a mild assumption on the nature of the attenuating kernel, we have (2.12), i.e.

$$(5.12) \quad (1 - \epsilon^2 \nabla^2) t_{kj} = \lambda e_{rr,kk} + 2\mu e_{kj}$$

From this we solve for e_{kj} :

$$(5.13) \quad e_{kj} = \frac{1}{2\mu} (1 - \epsilon^2 \nabla^2) t_{kj} - \frac{\lambda}{1-2\mu} t_{rr,kk}$$

where $\nu = \lambda / (2\lambda + \mu)$ is Poisson's ratio. If we substitute Eq. (5.13) into (5.12), we obtain

$$(5.14) \quad (1 - \epsilon^2 \nabla^2) \left[(1 - \frac{2\nu}{1-\nu}) t_{kj} + \frac{\nu}{1-\nu} (t_{rr,kk} - \nabla^2 t_{rr,kk}) \right] = 2\mu t_{kj}$$

These equations must be solved for t_{kj} which are subject to the equilibrium condition

$$(5.15) \quad t_{k,j,k} = 0$$

Following Kröner's classical approach¹, modifying the Beltrami solution of (5.15), we take

$$(5.16) \quad t_{kj} = \nabla^2 \chi_{kj} - \frac{1}{1-\nu} (\chi_{rr,kk} - \nabla^2 \chi_{rr,kk})$$

where the symmetric stress function χ_{kj} is subject to

$$(5.17) \quad \chi_{k,j,k} = 0$$

Substituting (5.16) into (5.14), we obtain

$$(5.18) \quad (1 - \epsilon^2 \nabla^2) \nabla^2 \chi_{kj} = \tau_{kj}$$

Thus, given the dislocation density function a , through (5.11), we calculate the incompatibility tensor τ_{kj} . The solution of (5.18) will then give χ_{kj} .

As expected, Eq. (5.18) reduces to the classical equation, when $\epsilon = 0$. It is a singularly perturbed partial differential equation. To obtain the solution of (5.18), we must find Green's

function, $G_{k,lmn}(\lambda, \xi)$, which must satisfy

$$(5.19) \quad (1 - \epsilon^2 \nabla^2) \nabla^4 G_{k,lmn} = \delta(x - \xi) \delta_k \delta_l \delta_m$$

When $G_{k,lmn}$ is known, then the solution of (5.18) is given by

$$(5.20) \quad \chi_{k\lambda} = \int_V G_{k,lmn}(x, \xi) \tau_{mn}(\xi) dv(\xi)$$

In the case of the *plane strain*, introducing Airy's stress function $\phi(x_1, x_2)$ by

$$(5.21) \quad \tau_{11} = \phi_{,22}, \quad \tau_{22} = \phi_{,11}, \quad \tau_{12} = -\phi_{,12}$$

we obtain an equation replacing (5.18)

$$(5.22) \quad (1 - \epsilon^2 \nabla^2) \nabla^4 \phi = 2\mu n$$

where

$$(5.23) \quad n = n_{33} = a_{23,1} - a_{13,2}$$

$$(5.24) \quad a_{23} = a_{21,2} - a_{22,1}, \quad a_{13} = a_{11,2} - a_{12,1}$$

depend on x_1 and x_2 only.

In the case of the *anti-plane strain*, equations of equilibrium are satisfied if

$$(5.25) \quad \tau_{13} = \frac{\partial c}{\partial x_2}, \quad \tau_{23} = -\frac{\partial c}{\partial x_1}$$

and we obtain

$$(5.26) \quad (1 - \epsilon^2 \nabla^2) \nabla^2 c = \mu a_{33}$$

where

$$(5.27) \quad a_{33} = a_{31,2} - a_{32,1}$$

6. GREEN'S FUNCTIONS: STATE OF STRESS

The determination of the stress fields arising from the continuous distributions of dislocations requires that we obtain the Green function for the stress functions $\chi_{k\lambda}$, ϕ and c . Here we determine Green's functions for solids of infinite extent.

(i) **THREE-DIMENSIONAL INFINITE SOLID** - The operator ∇^2 is invariant under the rotations of coordinates. For the infinite space, we seek a solution of (5.18) which depends upon $|x - \xi|$ only, i.e.

$$(6.1) \quad (1 - \epsilon^2 \nabla^2) \nabla^4 G = \delta(x - \xi)$$

Since the operators $(1 - \epsilon^2 \nabla^2)$ and ∇^4 commutes, we set

$$(6.2) \quad (1 - \epsilon^2 \nabla^2) G = H, \quad \nabla^4 H = \delta(x - \xi)$$

For the infinite space, H is given by

$$(6.3) \quad H = -\frac{\lambda - \xi}{8\pi} \delta^3$$

In spherical coordinates, using the operator

$$\nabla^2 = r^{-2} \frac{d}{dr} (r^2 \frac{d}{dr})$$

we obtain

$$(6.4) \quad G(|x - \xi|) = \frac{\epsilon^2}{4 - \lambda - \xi} \exp(-|x - \xi|/\epsilon) - \frac{\lambda - \xi}{8\pi}, \quad \epsilon \neq 0$$

$$(6.5) \quad G(|x - \xi|) = -\frac{\lambda - \xi}{8\pi} \delta^3, \quad \epsilon = 0$$

where we also determined constant to render τ_{kk} regular at $x = \xi$.

The solution of (5.18) for the infinite media, is given by

$$(6.6) \quad \chi_{k\lambda} = \int_V G(|x - \xi|) \tau_{k\lambda}(\xi) dv(\xi)$$

which satisfies conditions (5.17) on account of (5.7) and (5.11). Upon substituting from (5.11), this may be expressed as

$$(6.7) \quad \chi_{k\lambda}(x) = \frac{1}{2} \epsilon_{1jk} \int_V a_{1j}(\xi) \frac{\partial G}{\partial x_1} dv(\xi) + \frac{1}{2} \epsilon_{1jk} \int_V a_{jk}(\xi) \frac{\partial G}{\partial x_1} dv(\xi)$$

where we used the Green-Gauss theorem and set a surface term at infinity to zero.

Eq. (6.7) may be used to study various special cases involving surface and line distributions of dislocations. For example, for a line distribution of dislocation along a closed curve C , we obtain

$$(6.8) \quad \chi_{k\lambda} = \frac{1}{2} \epsilon_{1jk} b_j \oint_C \frac{\partial G}{\partial x_1} ds_k + \frac{1}{2} \epsilon_{1jk} b_j \oint_C \frac{\partial G}{\partial x_1} ds_k$$

where b_j is the Burger's vector per unit length of C and ds_k is the element of arc.

The stress field due to a line distribution of dislocation is obtained by substituting (6.8) into (5.16):

$$(6.9) \quad \tau_{kj}/2L = \frac{1}{2} \epsilon_{r1j} b_r \int_C \left[\nabla^2 G_{,1} (\delta_{rk} ds_k + \delta_{rk} ds_k) + \frac{2}{1-\nu} (G_{,k;i} - \nabla^2 G_{,i} \delta_{k,i}) ds_r \right]$$

This result is identical to the Peach-Koehler^{15,16} formula with the modification that here G is the nonlocal Green's function (6.4) with $\epsilon \neq 0$. The most interesting new feature of (6.9) is that it does not exhibit unbounded stress fields and energies at any point on C or elsewhere.

(ii) TWO-DIMENSIONAL INFINITE PLANE - Green's function in this case can be found to be similar to decomposition (6.2) with \bar{r} and H , given by

$$(6.10) \quad \bar{r}^2 = \frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right),$$

$$(6.11) \quad H = -\frac{1}{2} \ln |x-\xi|$$

Hence,

$$(6.12) \quad G(x-\xi) = \frac{1}{2\pi} K_0(|x-\xi|/\epsilon) - \frac{(x-\xi)_i (x-\xi)_i}{8\pi \epsilon^2} \ln(|x-\xi|/\epsilon), \quad \epsilon \neq 0$$

$$(6.13) \quad G(x-\xi) = -\frac{(x-\xi)_i (x-\xi)_i}{8\pi} \ln(|x-\xi|), \quad \epsilon = 0$$

where $K_0(z)$ is the modified Bessel's function.

Airy's stress function is obtained to be

$$(6.14) \quad \phi(x) = 2L \int_S \left[\frac{\partial G}{\partial x_1} a_{23}(\xi) - \frac{\partial G}{\partial x_2} a_{13}(\xi) \right] da$$

where we used the Green-Gauss theorem and set a line integral to zero at infinity. For a line distribution of dislocations in the $x_3 = 0$ -plane, we will have

$$(6.15) \quad \phi(x) = -2L \int_C \left[\frac{\partial G}{\partial x_1} b_2(\xi) d\xi_1 + \frac{\partial G}{\partial x_2} b_1(\xi) d\xi_2 \right]$$

The stress is calculated by using (5.21)

$$(6.16) \quad \begin{aligned} \tau_{11} &= -2L \int_C (G_{,112} b_2 d\xi_1 + G_{,211} b_1 d\xi_2) \\ \tau_{22} &= -2L \int_C (G_{,112} b_2 d\xi_1 + G_{,211} b_1 d\xi_2) \\ \tau_{12} &= 2L \int_C (G_{,112} b_2 d\xi_1 + G_{,211} b_1 d\xi_2) \end{aligned}$$

where indices after a comma denote partial derivatives with respect to x_k , e.g.

$$G_{,122} = \frac{\partial^2 G}{\partial x_1 \partial x_2^2}$$

(iii) ANTI-PLANE STRAIN - The Green function for the differential operator in (5.20) is obtained to be

$$(6.17) \quad G(x-\xi) = -\frac{1}{2\pi} \left[K_0(|x-\xi|/\epsilon) + K_0(|x-\xi|/\epsilon) \right], \quad \epsilon \neq 0$$

$$(6.18) \quad G(x-\xi) = -\frac{1}{2\pi} \ln |x-\xi|, \quad \epsilon = 0$$

The stress function ϕ is given by

$$(6.19) \quad \phi(x) = 2L \int_S \left[G_{,1} a_{23}(\xi) - G_{,2} a_{13}(\xi) \right] da$$

The stress field is found to be

$$(6.20) \quad \begin{aligned} \tau_{13} &= -2L \int_C \frac{\partial G}{\partial x_2} b_2(\xi) d\xi_1 d\xi_2 \\ \tau_{23} &= -2L \int_C \frac{\partial G}{\partial x_1} b_1(\xi) d\xi_1 d\xi_2 \end{aligned}$$

For a line distribution of dislocations on the $x_3 = 0$ -plane, the stress field is given by

$$(6.21) \quad \begin{aligned} \tau_{13} &= -2L \int_C \frac{\partial G}{\partial x_2} b_2(\xi) ds \\ \tau_{23} &= -2L \int_C \frac{\partial G}{\partial x_1} b_1(\xi) ds \end{aligned}$$

Example

In plane polar coordinates (r, θ) , we have

$$(6.22) \quad \begin{aligned} t_{rr} &= \frac{1}{r} \int_0^{\theta} \frac{\partial G}{\partial \theta} b(\xi) ds \\ t_{\theta\theta} &= - \int_0^{\theta} \frac{\partial G}{\partial r} b(\xi) ds \end{aligned}$$

7. STRESS DISTRIBUTIONS

Here, I present some results on the stress field due to continuous distributions of dislocations along a line segment.

(i) EDGE DISLOCATION ALONG A LINE SEGMENT - The stress field due to an edge dislocation can be calculated by using (6.16). In this case, $b_2 = 0$ and we consider $b_1 = \text{const.}$ distributed along a line segment $x_2 = 0$, $-x_1 < i$. Using the fact that $G_{,22} = \partial G / \partial x_2 = - \partial G / \partial x_1$, the integrations in (6.16) are performed readily

$$(7.1) \quad \begin{aligned} t_{11} &= 2ub_1 [G_{,22}(c_1) - G_{,22}(c_2)], \\ t_{22} &= 2ub_1 [G_{,11}(c_1) - G_{,22}(c_2)], \\ t_{12} &= - 2ub_1 [G_{,12}(c_1) - G_{,12}(c_2)] \end{aligned}$$

where

$$(7.2) \quad \begin{aligned} c_1 &= [(x_1 - i)^2 + x_2^2]^{1/2} e, \\ c_2 &= [(x_1 + i)^2 + x_2^2]^{1/2} e \end{aligned}$$

Green's function G is given by (6.12).

(ii) SCREW DISLOCATION - For a uniform distribution of screw dislocations along a line segment $x_2 = 0$, $-x_1 < i$, through (6.21), we find that

$$(7.3) \quad t_{23} = -b[G(c_1) - G(c_2)]$$

where G is given by (6.17) and c_1 and c_2 by (7.2).

If the number of dislocations is N over a distance i and b_0 is the atomic Burger's vector, we have for the macroscopic Burger's vector,

$$(7.4) \quad b = b_0 N i$$

The shear stress given by (7.3) may be expressed in non-dimensional form

$$(7.5) \quad T_2 = t_{23} t_D = \ln \frac{x-1}{x+1} + k_0 \gamma \ln \frac{x-1}{x+1} - k_0 \gamma \lambda^{-1}$$

where

$$(7.6) \quad \begin{aligned} t_D &= ub_0 i = ub_0 N i^2, \\ x &= x_1 / i, \quad \gamma = i / e \end{aligned}$$

(iii) SINGLE SCREW - For a single screw, the shear stresses can be obtained by substituting

$$(7.7) \quad b_1 \delta(x_1) = b_0 \delta(x)$$

into (6.20), where $\delta(x)$ is the Dirac-delta measure. This leads to

$$(7.8) \quad \begin{aligned} t_{13} &= ub_0 \frac{\partial G}{\partial x_1} = - \frac{ub_0}{2\pi} \frac{x_2}{r^2} [1 - \frac{\gamma}{e} k_0 r e], \\ t_{23} &= -ub_0 \frac{\partial G}{\partial x_1} = \frac{ub_0}{2\pi} \frac{x_1}{r^2} [1 - \frac{\gamma}{e} k_0 r e] \end{aligned}$$

where $r = (x_1^2 + x_2^2)^{1/2}$. This result is identical to Eq. (3.5), obtained differently, in polar coordinates.

For a single screw, the shear stress Eq. (3.5) may be expressed in non-dimensional form

$$(7.9) \quad T_2(\rho) = (2\pi ub_0 t_{23} = 2\pi^2 [1 - \frac{\gamma}{e} k_0 r e])$$

where

$$(7.10) \quad \rho = r e.$$

The stress field given by Eq. (7.9) is displayed graphically in Fig. 1. It has no singularity. In fact, contrary to the prediction made in classical elasticity, $T_2(\rho)$ vanishes at $\rho = 0$. The maximum stress occurs at $\rho = 1.1$ and is given by

$$(7.11) \quad t_{23}^{\text{max}} = 0.3993 \frac{b}{2\pi e}$$

The fracture will begin when $t_{23}^{\text{max}} = t_c$ = the cohesive stress that holds atomic bonds together. Therefore, nonlocal theory predicts that

$$(12) \quad \text{Fracture begins at a screw dislocation } \rho = 1.1 \text{ from the center of dislocation.}$$

Note that while this is only atomic distances away for single crystals, nevertheless, it may be a

finite distance for amorphous and composite materials. It depends on the internal characteristic length a (more precisely, $e_0 a$).

(b) Fracture Criterion: The fracture occurs when the maximum shear stress (given by (7.11)) reaches the value of the cohesive stress that holds atomic bonds together.

With this criterion, the maximum stress hypothesis for fracture is restored for microscopic and atomic phenomena as well.

If we write $h = \epsilon/0.3993$, Eq. (7.11) agrees with Frenkel's estimate of the theoretical strength of single crystals, based on atomic considerations (cf. Kelly [16], p. 12). In fact, if we use $\epsilon = e_0 a = 0.59a$, we find for the single aluminum crystal,

$$(7.12) \quad \tau_c/L = 0.12 \quad (\text{Al: } [111] \langle 1\bar{1}0 \rangle)$$

This is very close to the theoretical strength $\tau_{y/L} = 0.11$ based on atomic models.

In the case of line distribution of screws with constant Burgers vector, the non-dimensional shear stress is displayed in Fig. 2 for various values of γ . Here we observe that the shear stress is maximum near the end points of the line segment $x_1 = \pm l$, $x_2 = 0$. It is located slightly outside of the end points and for $\gamma > 3$ it is very close to the end points (see Table 1). Again, contrary to the classical elasticity solution, there is no singularity at $x_1 = \pm l$, $x_2 = 0$. Behavior of T_2 is governed basically by the first term in (7.5) except near $x = \pm l$. At $x = l$, we have

$$(7.13) \quad \tau_{23} = \frac{ub_0 N}{2-\epsilon} \frac{\ln \gamma}{\gamma} = \frac{bB}{2-h}$$

If we write

$$(7.14) \quad B = b_0 N, \quad h = \epsilon \gamma \ln \gamma$$

Eq. (7.13) may be interpreted in terms of the slip of atomic layers of distance h by one macroscopic dislocation of Burger's vector B . The ratio of the cohesive stress τ_d for the line distribution to that of a single screw is given by

Table 1: Maximum Shear Stress and its Location for a Uniform Distribution of Screw Dislocations Along a Straight Line Segment

$\gamma =$	1	1.5	2	3	5	10
$x =$	1.446	1.197	1.103	1.059	1.000	1.000
$T_{23, \max} =$	0.7478	1.0501	1.5008	1.6651	2.3026	2.9957

$$(7.15) \quad \frac{\tau_d}{\tau_c} = \frac{N}{0.3993 \ln \gamma}$$

This gives the shear stress reduction due to the presence of $2N$ dislocations distributed uniformly along a straight line segment of length L . Since $\tau_d \leq \tau_c$, the maximum number of dislocations is given by

$$(7.16) \quad N_{\max} = 0.3993 \frac{L}{\ln \gamma}$$

Of course, this number will have to be modified when the distribution is not uniform.

The stress fields due to a uniform distribution of screw dislocations along a circle are given in another publication.

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ACKNOWLEDGEMENTS

This work was supported by ONR. The author is indebted to Dr. N. Basdekas for his encouragement and enthusiasm. The skillful typing of this manuscript was done by Betty Kaminski.

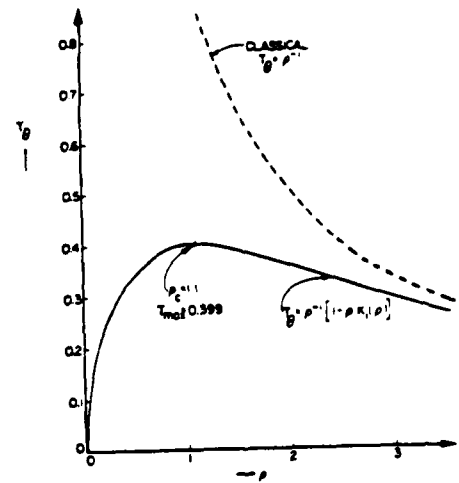


Fig. 1: NON-DIMENSIONAL HOOP STRESS FOR SCREW DISLOCATION

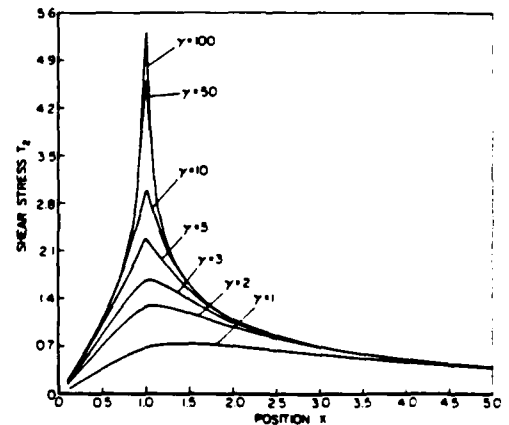


Fig. 2: SHEAR STRESS DISTRIBUTION (Line Segment)

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New York, New York 10027

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The University of Liverpool
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England

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Clarkson College of Technology
Department of Mechanical Engineering
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Texas A&M University
Aerospace Engineering Department
College Station, Texas 77843

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University of Arizona
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Tucson, Arizona 85721

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Carnegie-Mellon University
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Wayne State University
School of Medicine
Detroit, Michigan 48202

Dean B. A. Boley
Northwestern University
Department of Civil Engineering
Evanston, Illinois 60201

Professor P. G. Hodge, Jr.
University of Minnesota
Department of Aerospace Engineering
and Mechanics
Minneapolis, Minnesota 55455

Dr. D. C. Drucker
University of Illinois
Dean of Engineering
Urbana, Illinois 61801

Professor N. M. Newmark
University of Illinois
Department of Civil Engineering
Urbana, Illinois 61803

Professor E. Reissner
University of California, San Diego
Department of Applied Mechanics
La Jolla, California 92037

Professor William A. Nash
University of Massachusetts
Department of Mechanics and
Aerospace Engineering
Amherst, Massachusetts 01002

Universities (Con't)

Professor G. Herrmann
Stanford University
Department of Applied Mechanics
Stanford, California 94305

Professor J. D. Achenbach
Northwest University
Department of Civil Engineering
Evanston, Illinois 60201

Professor S. B. Dong
University of California
Department of Mechanics
Los Angeles, California 90024

Professor Burt Paul
University of Pennsylvania
Towne School of Civil and
Mechanical Engineering
Philadelphia, Pennsylvania 19104

Professor H. W. Liu
Syracuse University
Department of Chemical Engineering
and Metallurgy
Syracuse, New York 13210

Professor S. Bodner
Technion R&D Foundation
Haifa, Israel

Professor Werner Goldsmith
University of California
Department of Mechanical Engineering
Berkeley, California 94720

Professor R. S. Rivlin
Lehigh University
Center for the Application
of Mathematics
Bethlehem, Pennsylvania 18015

Professor F. A. Cozzarelli
State University of New York at
Buffalo
Division of Interdisciplinary Studies
Karr Parker Engineering Building
Chemistry Road
Buffalo, New York 14214

Universities (Con't)

Professor Joseph L. Rose
Drexel University
Department of Mechanical Engineering
and Mechanics
Philadelphia, Pennsylvania 19104

Professor B. K. Donaldson
University of Maryland
Aerospace Engineering Department
College Park, Maryland 20742

Professor Joseph A. Clark
Catholic University of America
Department of Mechanical Engineering
Washington, D.C. 20064

Dr. Samuel B. Batdorf
University of California
School of Engineering
and Applied Science
Los Angeles, California 90024

Professor Isaac Fried
Boston University
Department of Mathematics
Boston, Massachusetts 02215

Professor E. Kreml
Rensselaer Polytechnic Institute
Division of Engineering
Engineering Mechanics
Troy, New York 12181

Dr. Jack R. Vinson
University of Delaware
Department of Mechanical and Aerospace
Engineering and the Center for
Composite Materials
Newark, Delaware 19711

Dr. J. Duffy
Brown University
Division of Engineering
Providence, Rhode Island 02912

Dr. J. L. Swedlow
Carnegie-Mellon University
Department of Mechanical Engineering
Pittsburgh, Pennsylvania 15213

Universities (Con't)

Dr. V. K. Varadan
Ohio State University Research Foundation
Department of Engineering Mechanics
Columbus, Ohio 43210

Dr. Z. Hashin
University of Pennsylvania
Department of Metallurgy and
Materials Science
College of Engineering and
Applied Science
Philadelphia, Pennsylvania 19104

Dr. Jackson C. S. Yang
University of Maryland
Department of Mechanical Engineering
College Park, Maryland 20742

Professor T. Y. Chang
University of Akron
Department of Civil Engineering
Akron, Ohio 44325

Professor Charles W. Bert
University of Oklahoma
School of Aerospace, Mechanical,
and Nuclear Engineering
Norman, Oklahoma 73019

Professor Satya N. Atluri
Georgia Institute of Technology
School of Engineering and
Mechanics
Atlanta, Georgia 30332

Professor Graham F. Carey
University of Texas at Austin
Department of Aerospace Engineering
and Engineering Mechanics
Austin, Texas 78712

Dr. S. S. Wang
University of Illinois
Department of Theoretical and
Applied Mechanics
Urbana, Illinois 61801

Professor J. F. Abel
Cornell University
Department of Theoretical
and Applied Mechanics
Ithaca, New York 14853

Universities (Con't)

Professor V. E. Neubert
Pennsylvania State University
Department of Engineering Science
and Mechanics
University Park, Pennsylvania 16802

Professor A. W. Leissa
Ohio State University
Department of Engineering Mechanics
Columbus, Ohio 43212

Professor C. A. Brebbia
University of California, Irvine
Department of Civil Engineering
School of Engineering
Irvine, California 92717

Dr. George T. Rahn
Vanderbilt University
Mechanical Engineering and
Materials Science
Nashville, Tennessee 37235

Dean Richard H. Gallagher
University of Arizona
College of Engineering
Tucson, Arizona 85721

Professor E. F. Rybicki
The University of Tulsa
Department of Mechanical Engineering
Tulsa, Oklahoma 74104

Dr. R. Haftka
Illinois Institute of Technology
Department of Mechanics and Mechanical
and Aerospace Engineering
Chicago, Illinois 60616

Professor J. G. de Oliveira
Massachusetts Institute of Technology
Department of Ocean Engineering
77 Massachusetts Avenue
Cambridge, Massachusetts 02139

Dr. Bernard W. Shaffer
Polytechnic Institute of New York
Route 110
Farmingdale, New York 11735

Industry and Research Institutes

Dr. Norman Hobbs
Kaman Avidyne
Division of Kaman
Sciences Corporation
Burlington, Massachusetts 01803

Argonne National Laboratory
Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. M. C. Junger
Cambridge Acoustical Associates
54 Rindge Avenue Extension
Cambridge, Massachusetts 02140

Mr. J. H. Torrance
General Dynamics Corporation
Electric Boat Division
Groton, Connecticut 06340

Dr. J. E. Greenspon
J. G. Engineering Research Associates
3831 Menlo Drive
Baltimore, Maryland 21215

Newport News Shipbuilding and
Dry Dock Company
Library
Newport News, Virginia 23607

Dr. W. F. Bozich
McDonnell Douglas Corporation
5301 Bolsa Avenue
Huntington Beach, California 92647

Dr. E. N. Abramson
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Industry and Research Institutes (Con't)

Dr. T. L. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

Mr. William Caywood
Applied Physics Laboratory
Johns Hopkins Road
Laurel, Maryland 20810

Dr. Robert E. Dunham
Pacifica Technology
P.O. Box 148
Del Mar, California 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

Dr. A. A. Hochrein
Daedalean Associates, Inc.
Springlake Research Road
15110 Frederick Road
Woodbine, Maryland 21797

Dr. James W. Jones
Swanson Service Corporation
P.O. Box 5415
Huntington Beach, California 92646

Dr. Robert E. Nickell
Applied Science and Technology
3344 North Torrey Pines Court
Suite 220
La Jolla, California 92037

Dr. Kevin Thomas
Westinghouse Electric Corp.
Advanced Reactors Division
P. O. Box 158
Madison, Pennsylvania 15663

Dr. H. D. Hibbitt
Hibbitt & Karlsson, Inc.
132 George M. Cohan Boulevard
Providence, Rhode Island 02903

Dr. R. D. Mindlin
89 Deer Hill Drive
Ridgefield, Connecticut 06877

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Industry and Research Institutes (Con't)

Dr. Richard E. Dame
Mega Engineering
11961 Tech Road
Silver Spring, Maryland 20904

Mr. G. M. Stanley
Lockheed Palo Alto Research
Laboratory
3251 Hanover Street
Palo Alto, California 94304

Mr. R. L. Cloud
Robert L. Cloud Associates, Inc.
2972 Adeline Street
Berkeley, California 94703