



٠.

MICROCOPY RESOLUTION TEST CHART NATIONAL BUREAU OF STANDARDS (MARA

Division of Engineering Division of Applied Mathematics Brown University Providence, Rhode Island 02912

Elastic-Plastic Deformation of Pulse-Loaded Structures

•

Final Technical Report

Ъy

H. Kolsky P. S. Symonds

APPROVED THE STATES MAGINE

Office of Naval Research Contract Number N00014-82-K-0063 October 1981 - September 1983

÷.,



84 06 20 035

AD-A142 350

April, 1984

FILE COPY



Elastic-Plastic Deformation of Pulse-Loaded Structures

Final Technical Report

Contract Number N00014-82-K-0063

## 1. Main Purpose and Motivation.

The research aimed to provide a simple but realistic approximation technique for the plastic deformation and maximum deflection of a structure subjected to a general pulse loading.

The desirability of simple but reliable estimation techniques for problems of plastic deformations due to impact or high intensity pressure pulse loading needs no argument, despite the fact that many computer programs now are available for dynamic plastic problems, with nonlinear capability. Simpler methods always help to gain a better understanding of the response. They also serve to guide and check more complete numerical solutions, which are still costly and time consuming for most users and are by no means always reliable.

We are considering structures such as beams, frames, plates, and shells; of ductile metal either rate independent or rate sensitive (steel, titanium). Other materials such as reinforced or pre-stressed concrete and fibrereinforced materials, may be treated by extensions of the new methods.

#### 2. Previous Work.

When blast, shock, or impact loads produce plastic deformations, the actual response typically is a complex mixture of elastic and plastic effects. This prevents analytical solutions and makes complete numerical solutions costly. The premise of our approach under development for several years has been that realistic simplification might be obtained by an artificial separation of elastic and plastic responses. This idea has been implemented by a three-stage simplified elastic-plastic scheme (1-7). The first stage is wholly elastic and is terminated when a suitable yield condition is satisfied. The second stage is wholly plastic, i.e. rigid-plastic, and continues until the structure is brought to rest. The third stage involves elastic recovery and vibrations that are eventually damped out by the usual mechanisms.

This approach was first applied with success to impulsively loaded beam and frame structures (1-4). The beams were fully clamped specimens, while the rectangular portal frames were loaded on the transverse member so that deformations were symmetric about the middle line. Effects of large deflections, and strain rate sensitivity, were important, as well as interactions between elastic and plastic responses. Laboratory test results were available for all these cases, with short force pulses produced by close range explosive loading. These were the main source of checks on the method.

Further applications to portal frames (5,6) involved more complex response patterns, including reversals of plastic deformation. The frames were assumed subjected to "impulsive" (vanishingly short duration pulse) load pressure uniformly distributed over one column, causing asymmetric deformations to occur. The application of the simplified elastic-plastic method required imposing a global yield condition for terminating the initial elastic stage. This was defined arbitrarily in terms of an "effective moment" taken as a function of the bending moments at the locations of the plastic hinges which are present in the subsequent rigid-plastic stage. In applications to steel, a strongly strain rate sensitive metal, the critical value of the effective moment was taken as a function of the strain rates at the same locations. Constant dynamic plastic moment magnitudes were then assumed. Transfer of energy and momentum from the elastic to the rigidplastic stage was effected in the same way as in the previous applications. However the rigid-plastic stage was not assumed to be in modal form, and the method therefore furnished estimates of peak and permanent deflection magnitudes at points of interest.

To summarize the previous research to develop a realistic simple approach, this work successfully included elastic interactions with plastic deformations, together with important effects of large deflections and strain rate dependence with fairly complex response patterns; however certain deficiencies are evident. In particular these include the fact that only impulsive loading was considered, comparisons having been made with test results where very short pulses were applied by detonating explosives at short range. Interactions between elastic and plastic deformations are much more critical when (as in most practical cases) the peak loads are not greatly in excess of limit load magnitudes, and large plastic deformations occur because the pulse has long duration.

# 3. Summary of Completed Work.

The work has been concerned with the analysis of deformations due to force pulses of general shape, finite duration, and force magnitudes ranging from near the static collapse value to several multiples of this. The work has involved (a) laboratory experiments, (b) complete finite element numerical solutions by ABAQUS, and (c) the extension of the three-stage simplified elastic-plastic approach to such problems. The procedure outlined above for impulsive loading requires modification, as will be discussed.

(a) Impact experiments were performed, in which the frame fixed at the end of a track was struck by a hammer propelled along the track. Impact was at midheight of the column against a layer of neoprene. The force pulse applied by the hammer during contact with the frame was measured by a foil strain gage on an elastic cylinder at the front of the hammer. This pulse has total duration, impulse, and shape which were varied over considerable ranges by changing the striking mass and velocity. The applied impulse and response of the structure were monitored in various ways, including cine photography using a Fastax camera. Tests of this type have been made of 11 frames of mild steel and of 7 frames of 6061-T6 aluminum alloy. For both materials tests of properties defining plastic behavior were made. The steel is strain rate sensitive, and tension tests at strain rates obtainable in an Instron machine (up to about 0.1 sec<sup>-1</sup>) were made. The 6061-T6 aluminum alloy was

í č

appropriately treated by tempering to give a reproducible flat yield diagram with yield stress of about 45,000 psi. Strain rate tests at Instron speeds showed essentially no strain rate dependence at these rates.

(b) Calculations of the response were made by the finite element program ABAQUS<sup>\*</sup>, using a digitized representation of the force pulse applied to each specimen frame. The computer model of the frame used 20 elements in each column and in the transverse member (60 elements in all). A lumped mass beam element was used which permits shear strain (elastic but not plastic) in the sense of a rotation of the normal to the element relative to the beam axis. An over-all Newton implicit integration scheme is employed in ABAQUS, with the distinctive feature of an automatically controlled time step. Final values of displacements are not computed directly by ABAQUS, since the residual elastic vibration continues indefinitely; our version of ABAQUS has no provision for structural damping. Hence estimates of final displacements were obtained by averaging maximum and minimum displacements of the continuing vibration, which is typically found to have a smooth nearly sinusoidal shape.

The finite element solutions were made and compared with the final deflections of the test frames in order to check both the experiments and the numerical solutions, so that either one could be used to assess the approximation technique.

(c) Extensions of the simplified elastic-plastic technique are necessary when the load pulse is of duration comparable to elastic and plastic response times, and the peak magnitudes are moderately large compared to the static plastic collapse load. When the load is taken as impulsive the three-stage approach is relatively straightforward. To deal with pulses of finite duration, we first tried to use the simplification that has been found to work well in rigid-plastic treatments, namely the replacement of the actual pulse by an equivalent rectangular pulse defined as having the same impulse and as having its centroid time coincide with the centroid time of the given pulse. This replacement has been applied by Youngdahl (7) and others to a variety of pulse shapes and structures, treated as rigid-plastic, showing remarkable agreement with complete solutions obtained by numerical integration. In the present problems elastic effects are important, and when the given pulse was replaced by a rectangular load pulse defined in the same manner as for the rigid-plastic treatment, final deflections were predicted by the simplified elastic-plastic method which were much too low (for example, a third of the deflections predicted by ABAQUS; use of a wholly rigid-plastic treatment gave even smaller predictions in these cases).

The three-stage scheme which we have developed for arbitrary finite pulses is summarized below by reviewing the major questions that arise. The decisions made to answer them are briefly indicated and references given to one or more of the reports where full discussions are given. These questions are listed as follows:

ABAQUS was made available for our research use by Hibbitt, Karlsson and Sorensen, Inc., Providence R.I. 02906

- (i) How should the elastic stage be terminated?
- (ii) How should the subsequent rigid-plastic stage be started?
- (iii) How should deflections in the rigid-plastic stage be calculated?
- (iv) How should plastic strain rate sensitivity be included in the analysis?
- (v) How should maximum (elastic plus plastic) and final permanent displacemnts be determined?

(i) The first elastic stage is terminated at time t1. In previous work a global yield condition involving bending moments at the locations of plastic hinges in the subsequent rigid-plastic stage. This may exaggerate the importance of moments that are small in the early elastic stage. One alternative is to use only the largest moment (or the two largest moments) to define a yield condition. A better alternative is to regard t1 as an arbitrary parameter rather than fixed by a yield condition, and to scan the entire range of possible values from zero to a value near the pulse duration. Plots of the major plastic deflections and of the total plastic work as functions of  $t_1$ are easily obtained. (The elastic solution needs to be run only once, and although the rigid-plastic solution has to be obtained for each choice of  $t_1$ , each such calculation requires minor computing effort.) Such plots typically have a maximum value at some value of  $t_1$ , and this can be taken as a conservative estimate of the quantity sought.

Our preferred function of  $t_1$ , whose maximum furnishes the "best" choice of  $t_1$  to terminate the elastic stage, is suggested by the analysis of a single degree of freedom mass - spring system with elastic-perfectly plastic spring. It consists of the plastic work computed from the rigid-plastic stage plus a work computed from the elastic solution at time  $t_1$ . This is the counterpart in terms of work and energy of the maximum displacement in the single d.o.f. model, which is maximized by the correct value of  $t_1$ . Details are given in Technical Report NR-2.

(ii) The velocity field of the elastic solution at time  $t_1$  must be used to determine the initial velocities of the subsequent rigid-plastic response. The velocity field of the first phase of the rigid-plastic response is described in terms of a number of parameters, depending on how the structure is modelled in this stage. The two velocity fields - elastic and rigid-plastic - have different shapes, so that continuity cannot be imposed. The best values of the initial parameters of the rigid-plastic field are taken as those that minimize the difference  $\Delta_0$ , where  $\Delta_0$  is the squared difference between the two fields weighted by the specific mass and integrated over the structure. Minimization with respect to the parameters of rigidplastic field can be interpreted as scala conservation of momentum equations. The "minimum  $\Delta_0$ " device was first proposed by Martin and Symonds (8). It has been much used since then, and has intrinsic advantages.

(iii) The second stage is rigid-plastic by assumption. The analysis of motion in this stage depends on how the frame is modelled; one allows plastic hinges to occur at the smallest number of locations that is considered adequate. To obtain both the local and the global deformation in the present

problem the five hinge locations shown in Fig. 2 are sufficient. The rigidplastic response normally involves a succession of <u>phases</u>, as in Fig. 2 each phase being defined by a particular hinge configuration. The actual sequence depends on  $t_1$  and on P(t) for  $t > t_1$ . The impulsive load case previously considered (with uniform distribution over the column) required the sequence shown in Fig. 2, where the initial shape (a) changes to (b), then to (c), and ends up in the mode form solution (d). To treat the general case, a computer program is written which changes from one phase to another as governed by the current value of the force P(t) and the velocities and moments of the current configuration. The final shape is always the four-hinge mode solution of Fig. 2d. However this may be a small part of the total response.

(iv) When the material is strongly rate sensitive, as in the case of mild steel, the rigid-plastic analysis must be performed with moment magnitudes at the plastic hinges which correspond to the current strain rate magnitudes. We have used the dynamic "over-stress" expressions for the stress  $\sigma'$  and bending moment M', namely

$$\frac{\sigma'}{\sigma_0} = 1 + \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right)^{\frac{1}{n}}$$
(1)  
$$\frac{M'}{M_0} = 1 + \left(\frac{\dot{\kappa}}{\dot{\kappa}_0}\right)^{\frac{1}{n}}$$
(2)

Where  $\varepsilon$  and  $\kappa$  are strain rate and curvature rate, respectively,  $\sigma_{1}$ ,  $\varepsilon_{0}$ , n are constants of material behavior,  $M_{0} = bH^{2}\sigma_{0}/4$  and  $\kappa_{0} = 2\dot{\varepsilon}_{0}(1 + \frac{1}{2n})^{n}/H$ for a beam of rectangular shape with width b, depth H. For steel, good values are  $\dot{\varepsilon}_{0} = 40 \text{ sec}^{-1}$  and n = 5, with  $\sigma_{0}$  an average yield stress for quasi-static loading. A more convenient form for a beam is obtained in terms of the angular velocity  $\dot{\psi}$  across a plastic hinge. We may put  $\kappa = \dot{\psi}/z_{h}$ , where  $z_{h}$  is the assumed length of the hinge (finite but small compared to main structural dimensions). Then Eq. (1) may be written

$$\frac{M'}{M_o} = \mu = 1 + \left(\frac{\alpha \psi}{\epsilon_o}\right)^{\frac{1}{n}}$$
(3)

where

$$\alpha = \frac{H}{z_{h}} \frac{1}{2(1 + \frac{1}{2n})n} = \frac{H}{3.22z_{h}}$$
(4)

for n = 5. The factor  $\alpha$  is not a constant, but nevertheless a constant value can be used together with an average value of  $\psi$  to give an average moment factor  $\mu$  for a certain interval of time. We have made computer studies with ABAQUS of a basic beam problem, using computed values of  $\mu$  and  $\psi$  to determine  $\alpha$  from Eq. (10a). In an example  $\alpha$  varied from 0.08 to 0.26 while the angular velocity varied from 19.6 sec<sup>-1</sup> to 2.4 sec<sup>-1</sup> across a plastic hinge. The average value 0.16 used with the average angular velocity give a moment factor  $\mu$  = 1.56; the actual moment factor during this response had an average of 1.53. The dynamic moment actually depends on the angular velocity and on the structural and material parameters in an insensitive manner, as indicated by Eq. (10a), so that seemingly crude averages may involve minor errors. Further details are given in Technical Reports NR-2 and NR-3.

(v) The deflections computed at time t1 (at the end of the elastic stage) and the increments during the rigid-plastic stage are added to furnish peak deflection magnitudes. The final permanent displacements are obtained by subtracting displacements corresponding to the amplitude of the elastic vibration which continues after the completion of plastic deformations. This amplitude is determined by the elastic strain energy present in the final stage of the rigid-plastic solution. This is always a mode form solution. In the present frame problem the final mode pattern is one of simple side-sway (Fig. 2d). The determination of final deflections is discussed fully in Technical Report NR-2.

### 4. Technical Reports

The work summarized above is described in several manuscripts that either have appeared or will appear first as Technical Reports and subsequently as published papers. Abstracts and publication details are as follows:

 Technical Report NR-1 (45 pages of text, 33 pages of figures) June, 1984 (expected).

"Experiments, Finite Element Solutions and a Simplified Elastic-Plastic Approach to Impact of Frames," by J. M. Mosquera, H. Kolsky and P. S. Symonds.

Abstract. Experimental and analytical work is described, in further investigation of a simplified method for estimating final and peak deflection amplitudes of a pulse loaded structure. The method includes effects of elastic response interacting with plastic deformation, but gains simplicity by treating them in artificially separated stages: an initial elastic response stage is followed by a wholly plastic (i.e. rigid-plastic) stage, which in turn is followed by elastic vibrations. Experiments were performed in which portal frames of aluminum alloy and mild steel were impacted at the midpoint of one column by masses projected at various speeds. The contact force pulse was recorded and used as input data for analyses. In addition to the simplified elasticplastic (SEP) method, numerical analyses were carried out by a finite element code (furnishing nominally "exact" solutions) and by an approach assuming rigid-plastic behavior for the entire response. It is found that with the relatively long pulses produced in the present impact experiments, elastic-plastic interactions may be critically important. Consideration of strain rate sensitive plastic flow in the steel frames requires special treatment. Reasonably good agreement with results of tests and finite element calculations was given by the SEP method, whereas a rigid-plstic solution grossly underestimated final deflections in certain cases.

(This report is based on Part II of the thesis for Ph.D. of J. M. Mosquera entitled, "The Response of Metallic Structures to Dynamic Loading," awarded by Brown University on May 28, 1984. It will be submitted for journal publication, possibly in a shorter version.)

 Technical Report NR-2 (20 pages of text, 5 pages of figures) December, 1983.

"A Simplified Approach to Elastic-Plastic Response to General Pulse Loading," by P. S. Symonds and J. M. Mosquera.

Abstract. The paper describes a method for estimating permanent and maximum (elastic plus plastic) deflections of a structure subjected to a force pulse loading of arbitrary shape and duration. The concept of artificially separating the response into purely elastic and purely plastic (i.e. rigid-plastic) stages is adopted. Previous applications have been to very short pulses. When the approach is applied to long pulses of only moderately large force compared to limit load magnitudes the interaction between elastic and plastic effects is more critical. The inclusion of plastic rate sensitivity and the determination of the duration of the initial elastic stage require new approaches. Insight is provided by studying a mass-spring (elastic-perfectly plastic) system of one degree of freedom. The duration of the elastic stage is chosen to maximize a specially defined work function. Comparisons are shown for examples of rate independent and rate sensitive frames between deflections given by the simple technique of the paper, final deflections obtained in laboratory tests, and maximum and final displacements furnished by the finite element code ABAQUS.

(Accepted April, 1984 by Journal of Applied Mechanics.)

3. Technical Report NR-3 (in preparation).

"On Elastic-Plastic and Rigid-Plastic Dynamic Response with Strain Rate Sensitivity," by P. S. Symonds, J. M. Mosquera and H. Kolsky.

Abstract. The problem of a cantilever beam whose tip mass is given an impulsive loading was solved by Parkes as a rigid-perfectly plastic (RPP) response, with a fully plastic bending moment increased to take account of rate sensitivity when applied to a steel beam. The "actual" response of a steel beam, including effects of elastic as well as plastic behavior, is obtained by a finite element numerical solution using program ABAQUS, with (EVP) and without (EPP) strain rate sensitivity. These are compared in detail with Parkes's RPP solution and with a simplified elastic-plastic approach. The initial travelling hinge phase of the RPP solution does not appear in the EPP and EVP solutions. The latter also provides useful information about the relation between angular velocity across a plastic hinge, the strain rates in the hinge region, and the increase of the fully plastic moment due to rate sensitivity. 4. Technical Report NR-4, November, 1983.

"Simple Elastic-Plastic Method for Pulse Loading - Comparisons with Experiments and Finite Element Solutions," by P. S. Symonds, H. Kolsky and J. M. Mosquera.

<u>Abstract</u>. Examples are given of a simple approach to determine maximum and final displacements of a structure subjected to pulse loading of arbitrary length. Simplicity as compared with a wholly numerical solution is gained by separating elastic from plastic response, plastic deformations being determined by a rigid-plastic analysis following an elastic stage. Advantageous use is made of experiments and of full elastic-plastic solutions by finite elements, in developing the simple approach.

(Proceedings of Third Oxford Conference on Mechanical Properties of Materials at High Rates of Strain, Oxford, April 9-12, 1984, J. Harding, Ed., in press.)

# References

- Symonds, P.S. and Raphanel, J.L., "Large Deflections of Impulsively Loaded Plane Frames - Extensions of the Mode Approximation Technique," Proc. Conf. on Mechanical Properties of Materials at High Rates of Strain, Institute of Physics Conf. Ser. No. 47 (J. Harding, Ed.) pp. 277-287, London and Bristol, 1979.
- Symonds, P.S., "Elastic, Finite Deflection and Strain Rate Effects in a Mode Approximation Technique for Plastic Deformation of Pulse Loaded Structures," J. Mechanical Engineering Science, Vol. 22, No. 4, pp. 189-197, 1980.
- 3. Symonds, P.S., "Finite Elastic and Plastic Deformations of Pulse Loaded Structures by an Extended Mode Technique," Int. J. Mech. Sci., Vol. 22, pp. 597-605, 1980.
- 4. Symonds, P.S., "Elastic-Plastic Deflections due to Pulse Loading," Proc. of the Second Specialty Conf. on Dynamic Response of Structures: <u>Experimentation, Observation, Prediction, and Control</u>, Gary Hart, Ed., New York, NY, ASCE, 1980, pp. 887-901.
- Symonds, P.S. and Raphanel, J.L., "Rigid-Plastic and Simplified Elastic-Plastic Solutions for Dynamically Side-Loaded Frames," in <u>Mechanics of</u> <u>Material Behavior</u>, (Drucker Anniversary Volume) Eds. G.J. Dvorak and R.T. Shield, Elsevier Scientific Publishing Co., Amsterdam (in press).
- Raphanel, J.L. and Symonds, P.S., "The Estimation of Large Deflections of a Portal Frame under Asymmetric Pulse Loading," Report CEE 80/3, October 1982 of Brown University, Division of Engineering to U.S. National Science Foundation. J. Applied Mechanics (to appear).
- Youngdahl, C.K., "Correlation Parameters for Eliminating the Effect of Pulse Shape on Dynamic Plastic Deformation," J. Applied Mechanics, Vol. 37, pp. 744-752, 1970.
- Martin, J.B. and Symonds, P.S., "Mode Approximations for Impulsively Loaded Rigid-Plastic Structures," J. Eng. Mech. Div., Proc. ASCE, Vol. 92, No. EM5, pp. 43-66, 1966.







(a) Frame used in impact tests; (b),(c) Typical measured force pulses, used as input loads for analyses.







(a)-(d) Typical sequence of velocity fields in rigid-plastic stage; (e) Typical final deflections.