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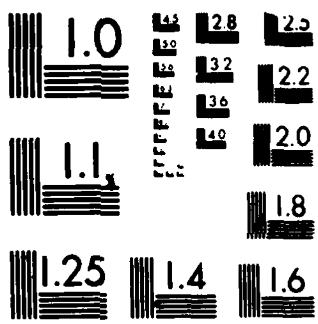


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Under the first period of the grant support, mathematical developments were performed on algebraic, adaptive, suface, and orthogonal transformations. Algebraic transformations were extended by forming Boolean sums of multisurface transformations and, by appropriate compositions, lifting the results to curved 2-D surfaces. Orthogonal trajectories were a major part of the general development of orthogonal transformations under the grant support. Additional parts included field methods. This report summarizes progress made in these areas and lists publications resulting from the research.

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# Investigation of Three Dimensional Mesh Generation with Precise Controls

Interim

[redacted] Report
on
First Grant Period

Principal Investigators: Peter R. Eiseman and C.K. Chu

(May 1, 1982 to September 30, 1983)

Columbia University
New York, NU 10027

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Under the first period of the grant support, mathematical developments were performed on algebraic, adaptive, surface, and orthogonal transformations. Algebraic transformations were extended by forming Boolean sums of multisurface transformations and, by appropriate compositions, lifting the results to curved 2-D surfaces. The basic formulation was reported upon in [1] and [2]. In [1], it is viewed from the perspective of an eventual insertion into the automatic algebraic grid generation code that was developed for NASA. In [2], the mathematical formulation was given for arbitrary 2-D surfaces on which an orthogonal grid was to be constructed by means of orthogonal trajectories.

Orthogonal trajectories were a major part of the general development of orthogonal transformations under the grant support. Additional parts included field methods. As reported in [2], the Boolean sum of multisurface transformations permitted a specification of geometry on all four boundaries and a pointwise distribution on three out of four of them. The desire to arbitrarily put distributions on all four boundaries was the primary reason for field methods. The results, however, for field methods are inconclusive. With orthogonal trajectories a unifying theoretical framework was derived, a new leap-frog method was created, and the earlier method of Graves and McNally was extended with various refinements. Both methods were inserted into the automatic algebraic grid code [1] and tested in planar regions. The details on the theoretical framework and the methods given in [2].

Of the methods, the leap-frog method resulted from an investigation into the orthogonality of a discrete grid as compared to that of a continuous coordinate transformation. The potential utility of each viewpoint depends upon whether or not coordinate derivatives are to be estimated with central finite differences; thereby, giving a metric based only on relative grid point locations rather than analytic evaluations. Since central finite differences are commonly used for the estimation of metric data in order to preserve conservation properties, the leap-frog method is attractive because exact central difference orthogonality is obtained on the grid. Other methods such as the original version of the Graves-McNally approach have been severely criticized for orthogonality errors which ironically were measured with finite differences. The measurement, however, should be based upon the use. If, for example, metric data arises from the continuous coordinate transformation; thereby, approaching the analytic evaluation of derivatives in the formation; then it is more important to use the transformation itself as a basis of reference rather than the specific grid of points that results from a fixed discretization. In contrast, when the metric comes from only the specific grid, the appropriate measure comes from central differences and in that sense the leap-frog method produces an optimal orthogonal grid since orthogonality is exactly given up to machine accuracy at each internal grid point. While smooth exactly orthogonal grids are produced for a significant number of problems, there are occasions where smoothness is sacrificed. This occurs because the leap-frog method produces an even-odd decoupling which can result in trajectories with wiggles. In such instances, there then is a trade off between the desired exact orthogonality and the undesired wiggles. The smoothing of wiggles would destroy the exact orthogonality, the preservation of which lead to the wiggles. If orthogonality is considered to be less important than smoothness, then it would seem reasonable to post-process the exactly orthogonal grid from the leap-frog method in order to supply smoothness on those cases where wiggles appeared in certain locations. In comparison, the refined version of the Graves-McNally approach did not produce wiggles and certainly was not optimal in a finite difference sense. Rather, it was constructed to be nearly optimal in an analytic sense. This was accomplished by using a succession of finer grids than the actual one employed for numerical simulations. The finer resolution gave a substantially better approximation to the derivatives than would have been computed by analytically differentiating the basic coordinate transformation. The theory for both of the above orthogonal trajectory methods was developed to produce orthogonal grids on arbitrary surfaces which were then specialized to planar cases within the automatic algebraic grid generation code. The further application to 2-D surfaces remains to be done and will be useful when coordinate orthogonality is desired at the boundaries of a 3-D region.

The general 2-D surface problem was undertaken simultaneously with the development of adaptive grid strategies. Each strategy was based upon the formation of an abstractly defined surface which contains all of the pertinent solution properties that are in need of resolution for an accurate numerical simulation of the phenomena under study. With the pertinent properties expressed in the form of a single abstract surface over physical space, the primary adaptive objective is to put or push the points into positions which most accurately represent the surface. This same objective also appears when arbitrary 2-D surfaces are used as boundaries for 3-D regions. Although the abstract surfaces for adaptive purposes are defined over physical space, the various adaptive strategies that were developed are readily applicable to the arbitrary surfaces required to bound 3-D regions. The strategies are mean value relaxation [3], alternating direction [4], and triangular [5]-[8]. The development for mean value relaxation [3] was done entirely under this grant. The theoretical basis for the alternating direction approach [4] was established under this grant and continued under NASA sponsorship (NAG1-355) culminating in a code at NASA Langley. The adaptive triangular methodology was developed with my doctoral student, Gordon Erlebacher, for his thesis research. The primary portion of this work was supported by the AFOSR grant with the remainder by grants from NASA and DOE. Upon completion of his doctoral requirements in December 1983, Gordon Erlebacher joined NASA Langley Research Center where joint work is still continuing on the adaptive triangular mesh code.

In the adaptive context, the surface is represented by a grid of points and by local multilinear interpolation in between those points. In all cases grid points are moved into positions which give a better rendition of the surface. Point addition and subtraction is considered only in the triangular mesh case because of its general connectivity structure. With movement as the primary adaptive mechanism, a weight function is established to attract points when it is large.

In the mean value relation technique a geometrically constructed local difference molecule is developed in a manner where weighted surface volumes are used to nonsingularly pull points along surface coordinate curves in a mean value sense. With mean value pulls for each coordinate direction coupled into a multilinear interpolation, the surface grid is moved in a pointwise iterative sense until it relaxes to a final configuration for the given surface. The mean value relaxation procedure was formulated both on the surface and in the parameter space for the surface. When movement is required for parametricaly defined boundaries of 3-D regions, the parameter space formulation is preferred in order to exactly preserve the prescribed boundary geometry which would otherwise suffer somewhat due to successive bilinear interpolation in 3-D.

In the alternating direction algorithm, the weights are used to move points along the coordinate curves of a direction in a cycle which alternates directions. The algorithm converges rapidly, is applied both on the surface and in parameter space, and represents a substantial extension and synthesis of many previous studies.

The adaptive triangular mesh study brings the previous Lagrangian triangular mesh developments typified by the NRL work pioneered by M.J. Fritts and J.P. Boris into the full adaptive context. Principally, adaptive movement is supplied with weighted surface areas to form a center of gravity type of molecule which is then iterated in a pointwise sense.

In all of the techniques, curvature is the primary mechanism which is used to get a better resolution of a surface. The main building block is normal curvature. In the triangular mesh case, mean curvature is used because of the mesh structure. The better resolution comprises both an increased number of points in regions where the surface changes direction and an alignment with those regions. In addition, geodesic curvature is considered for nontrivial surface boundaries.

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