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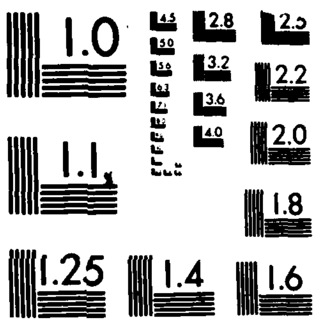

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AN ITERATIVE METHOD FOR RESTORING NOISY BLURRED IMAGES*

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ABSTRACT

This paper introduces a new iterative image restoration method which is capable of restoring noisy, blurred images by incorporating a priori knowledge about the image and noise statistics into the iterative procedure. The iteration equation consists of a prediction part which is based on a nonnausal image model description and an innovation part which is weighted by a gain factor. The gain is computed using a linear MSE optimization procedure and is updated at each step of the iteration. This image restoration scheme can be interpreted as an iterative procedure with a statistical constraint on the image data.

INTRODUCTION

In many practical situations, the image degradation can be adequately modelled by a linear blur (motion, defocussing, atmospheric turbulence) and an additive, white Gaussian noise process [1]. If the observed image is represented by an $M\times N$ array of real picture elements $\{y(i,j): 1\le i\le M, 1\le j\le N\}$, then in the spatially invariant case it can be described by the following two-dimensional (2-D) convolution summation:

$$y(i,j) = \sum_{(m,n)\in W_b}\sum b(m,n)x(i-m,j-n)+w(i,j) \tag{1}$$

where $y(i,j)$ is the degraded image; $x(i,j)$ is the original image; $w(i,j)$ is the additive observation noise, which is assumed to be uncorrelated with the image data; and $b(m,n)$ is the impulse response or point-spread function (*PSF*) of the imaging system that is introducing the blur. It is assumed that the support of the PSF, W_b, is much smaller than the size of the image.

With this model, the problem of image restoration is represented as the problem of operating on the degraded image $y(i,j)$ in order to get an improved image $\hat{x}(i,j)$ which is as close to the original image $x(i,j)$ as possible, subject to a suitable optimality criterion and given some prior knowledge about the PSF of the blur, the image, and the noise statistics.

Inverse filtering techniques, which aim at perfect restoration of the image by using the convolutional inverse of the blur, become poor restoration techniques when noise is present. This can also be the case with the class of iterative restoration algorithms which were introduced in [2]. The iterative techniques, however, do have advantages when compared with the inverse filter, such as the possibility of incorporating physical constraints on the data [2], man-machine interaction [3], and the ability to deal with non-linear or shift-varying blurs [2]. Therefore, considerable effort has been expended in trying to diminish the high noise *sensitivity of the iterative procedures*, while still producing reasonably sharp images. Of the different procedures proposed so far, we mention here the low-pass filtering of the observed data prior to applying a constrained iterative procedure [4], the "reblurring" procedure [2], and the use of some type of stopping rule based on the error residual and the variance of the observation noise [5]. In all of these approaches, however, no attempt has been made to incorporate statistical knowledge of the image and the noise directly into the restoration scheme, as is common practice with Wiener and Kalman filters and is done in the new proposed algorithm.

IMAGE MODELING

We assume that the original, undistorted image can be interpreted as a sample from a discrete, homogeneous random field $\{x(i,j): 1\le i\le M, 1\le j\le N\}$. For convenience we will assume that the mean of the image has been estimated and subtracted. For a discussion of autoregressive models for images with non-zero means see [6]. Under these assumptions the original image $x(i,j)$ can be modeled by the 2-D autoregressive equation

$$x(i,j) = \sum_{(p,q)\in S}\sum a(p,q)x(i-p,j-q)+u(i,j) \tag{2}$$

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where $x(i,j)$ represents the image intensity value at spatial coordinates (i,j) and where $u(i,j)$ can be viewed as either the input process or as the error in generating $x(i,j)$.

The shape of the region S specifies that some samples of $x(i,j)$ must be computed before others. Causality, however, is a time-domain concept which has no meaning when talking about the spatial variables in an image. Imposing an ordering relation on the samples is unnatural and should be avoided if possible. Therefore, we restrict ourselves to the class of non-causal image models for which

$$S = \{(p,q): \ p^2+q^2 \leq \rho^2, \ (p,q) \neq (0,0)\} \qquad (3)$$

In this case, the image can be seen to be a sample from an anisotropic homogeneous random field that satifies a *noncausal stochastic difference* equation [7]. Note that with this description we make no assumptions about the separability or the exponential form of the autocovariance function. Also note that unlike causal minimum variance models, noncausal minimum variance models, are not driven by white noise [7].

There are different approaches that can be followed to estimate the parameters $a(p,q)$ of eq. (2). If we have a noise-free unblurred prototype image the noncausal model could be fitted to the measured autocovariance function (under the assumption of homogeneity), for different values of ρ by using a linear MSE fitting procedure. Which model ultimately serves to describe the image data would depend upon some model quality criterion [8]. In the case we have noisy image data, we could make use of the model parameter identification procedure described by Kaufman et. al., [9].

A NEW ITERATIVE RESTORATION ALGORITHM

Formulation

Given the observation equation (1), representing the noisy blurred image data, and a noncausal image model description of the original undistorted image (2), we propose the following iteration:

$$\hat{x}_0(i,j) = \lambda_0 y(i,j) \qquad (4a)$$

$$\bar{x}_k(i,j) = a(i,j)**\hat{x}_{k-1}(i,j) \qquad (4b)$$

$$\hat{x}_k(i,j)=\bar{x}_k(i,j)+\lambda_k\left[y(i,j)-b(i,j)**\bar{x}_k(i,j)\right] \qquad (4c)$$

Here $a(i,j)$ and $b(i,j)$ result from the image and observation models respectively, k denotes the iteration index, and $**$ denotes 2-D convolution. It should be noted that the predicted signal in (4b) and the filtered signal in (4c) are truncated after each step in the iteration to the size of the observed image pixels.

The proposed iteration seems to be similar in many respects to a Kalman filter. Contrary to the recursive Kalman filter, however, this filter is iterative by nature due to the noncausal image and blur description. In [10] Pu and Kaufman suggest a similar scheme for restoring noisy unblurred images. They report improved performance relative to recursive noise smoothing algorithms.

Filter Gain

To compute the gain λ_k at the kth iteration in (4), we minimize the quantity

$$J(\lambda_k) = E\{(\hat{x}_k(i,j)-x(i,j))^2\} \qquad (5)$$

The resulting optimal value of λ_k can be shown to be equal to

$$\lambda_k = \frac{b(0,0)}{\sum\limits_{(m,n)\in W_b}\sum b^2(m,n)} \cdot \left[1 - \frac{\sigma_w^2}{\frac{1}{N_W}\sum\limits_{(i,j)\in W}\sum \left(y(i,j)-b(i,j)**\bar{x}_k(i,j)\right)^2}\right] \qquad (6)$$

where σ_w^2 is the variance of the observation noise and where N_W is the number of pixels in the support region W [11]. For a homogeneous image the support region can be taken to be the whole image, but for a nonhomogeneous image it would be a subsection. If a window is used to limit the support region, then a new model and a new gain should be computed for every window position. The window size should be larger than the support of the blur but small enough so that over the region covered by the window, the blur is space-invariant.

By considering limiting cases, the innovation gain can be shown to satisfy the bounds

$$0 < \lambda_k < \frac{b(0,0)}{\sum\limits_{(m,n)\in W_b}\sum b^2(m,n)} \qquad (7)$$

For the case of linear motion blur the upper bound will be equal to one, while for a gaussian blur it may be somewhat larger.

Convergence

To investigate the convergence of the iteration in the spatially invariant case, we can rewrite (4) in the frequency domain.

$$\hat{X}_0(\omega_1,\omega_2)=\lambda_0 Y(\omega_1,\omega_2)$$

$$\bar{X}_k(\omega_1,\omega_2)=A(\omega_1,\omega_2)\hat{X}_{k-1}(\omega_1,\omega_2)$$

$$+\lambda_k\left[Y(\omega_1,\omega_2)-B(\omega_1,\omega_2)A(\omega_1,\omega_2)\hat{X}_{k-1}(\omega_1,\omega_2)\right] \quad (8)$$

Here $A(\omega_1,\omega_2)$ is the frequency response of the predictor and $B(\omega_1,\omega_2)$ is the frequency response of the blur. By solving the iteration we see that $\hat{X}_k(\omega_1,\omega_2)$ can be written as

$$\hat{X}_k(\omega_1,\omega_2) = \sum_{m=0}^{k-1} \lambda_m \left\{\prod_{\ell=m+1}^{k} A(\omega_1,\omega_2)\left[1-\lambda_\ell B(\omega_1,\omega_2)\right]\right\}\cdot$$

$$Y(\omega_1,\omega_2) + \lambda_k Y(\omega_1,\omega_2) \quad (9)$$

If we assume that

$$\lambda_k = \lambda_{ss} \quad \text{for all } k > K$$

then the sum in (9) becomes a geometric series which converges if

$$|A(\omega_1,\omega_2)|\cdot|1-\lambda_{ss}B(\omega_1,\omega_2)|<1, \quad -\pi<\omega_1,\omega_2<\pi \quad (10)$$

If both $A(\omega_1,\omega_2)$ and $B(\omega_1,\omega_2)$ are known, equation (10) provides a means for determining acceptable values of λ_{ss}. Further insight into this issue might be gained by considering the following 1-D example.

Example:

An arbitrary image line is modelled by the first-order autoregressive relation

$$x(n) = a\,x(n-1) + a\,x(n+1) + u(n) \quad (11)$$

This image line is then convolved with a simple linear motion blur of the form

$$b(n) = \begin{cases} \frac{1}{L}, & 0\le n\le L-1 \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

Then the necessary condition for convergence is found by substituting into (10) which results in the inequality

$$\sqrt{1 + \frac{\lambda_{ss}^2}{L^2}\frac{\sin^2(\frac{\omega L}{2})}{\sin^2(\frac{\omega}{2})} - 2\,\frac{\lambda_{ss}}{L}\,\frac{\sin(\frac{\omega L}{2})}{\sin(\frac{\omega}{2})}\cos(\frac{L-1}{2}\omega)} < \frac{1}{|2a\cos\omega|} \quad (20)$$

In Fig. 1, the expressions from the left- and right-hand sides of this inequality are plotted for $\lambda_{ss}=1$, $a=.5$, and $L=7,14$. It should be noted that the convergence condition is not satisfied for $L=14$. A way to overcome the convergence problems in the second case is to use the reblurring procedure [2]. According to this procedure the degraded image is first convolved with the point spread function $b^*(-i,-j)$, where * denotes the complex conjugate. While this further degrades the image, it guarantees that the overall blur, $b(i,j)**b^*(-i,-j)$ will have a real, nonnegative transfer [illegible] The resulting equations for the algorithm and the filter gain are very similar to the ones given by equations (4), and (6), [11]. For an 8-neighbor symmetric 2-D noncausal prediction model, however, that was used with a linear motion blur given by equation (12), the condition for convergence is never satisfied, thus reblurring is always necessary. More generally, convergence is assured for every positive real transfer function of the distorting system provided that $|A(\omega_1,\omega_2)|<1$.

Experimental Results

Because of the preliminary nature of this work, only one-dimensional results are presented. Each image line is modelled by the first-order autoregressive relation described by equation (11). The optimal value of the coefficient a is found by minimizing the squared error between the noise-free, blur-free image line and the best output of the prediction model, over a whole line. White Gaussian noise was added to the distorted signal. The performance of the filter was evaluated by measuring the improvement in signal-to-noise ratio (SNR) after k iterations, according to the formula

$$\Delta_{SNR} = 10\log_{10}\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}[y(i,j)-x(i,j)]^2}{\sum_{i=1}^{M}\sum_{j=1}^{N}[\hat{x}(i,j)-x(i,j)]^2}$$

Figure 2 shows the removal of a Gaussian and a linear motion blur for the "girl" image. Reblurring was used for the linear motion case. In both cases the signal to noise ratio was 20dB. The SNR improvements were 1.9dB for the Gaussian blur and 7.2 dB for the motion blur. Better results would be expected if a true 2-D algorithm were used which could exploit the vertical correlation between pixels.

Discussion and Conclusions

As mentioned in the introduction one of the advantages of the iterative procedure is the possibility of incorporating physical (possibly nonlinear) constraints on the restoration. In [2] these constraints were expressed in terms of operators which project a signal onto an allowable subset of signals, and can be interpreted as hard constraints. In the new algorithm the estimation part *can be interpreted as a "soft"* statistical constraint. The similarity in form between the two mentioned iterations suggests the possibility of hybrid algorithms where nonlinear predictors are used which incorporate both physical and statistical information.

The performance of the new stochastic iterative algorithm has been compared with the performance of the deterministic algorithm of Schafer et. al. [2], in the presence of noise [11]. For high SNRs the deterministic algorithm outperforms the stochastic one, while for low SNRs only the stochastic algorithm converges. It should be noted that the reblurring procedure eliminates the noise at high frequencies, resulting in an improved performance of the deterministic algorithm.

Further research will involve experimentation with 2-D causal, semi-causal, non-causal linear prediction models, use of shift-varying models for the blur, simultaneous estimation of the gain and the prediction coefficients at each iteration step as well as identification of the parameters of the distorting system.

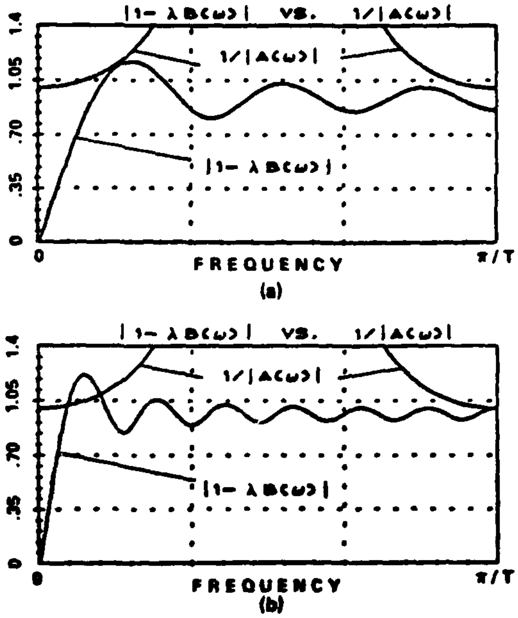

Figure 1 : Convergence analysis for motion blur. (a) L=7, a=.5, λ=1., (b) L=14, a=.5, λ=1.

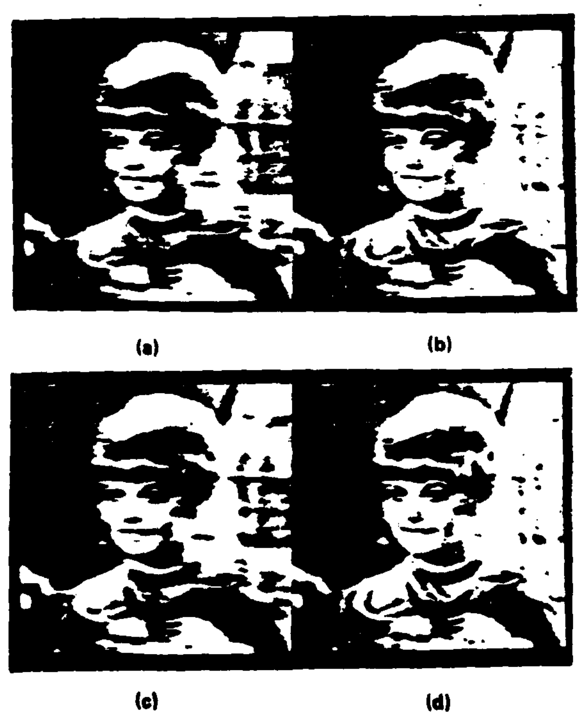

Figure 2 : (a) Noisy blurred image : Gaussian blur,st. dev.=4 samples , SNR=20dB , (b) restored image : improvement in SNR=1.9dB , (c) noisy blurred image : motion blur , L=8 , reblurring , SNR=20dB , (d) restored image : improvement in SNR=7.2dB

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