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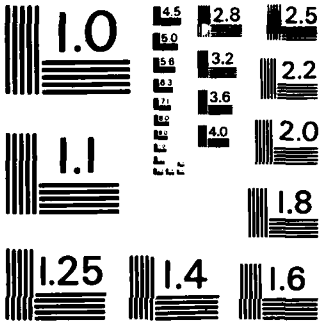

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FOREIGN TECHNOLOGY DIVISION

OSCILLATIONS OF A RESERVOIR WITH A FLUID ON THE FREE
SURFACE OF WHICH A MEMBRANE IS LOCATED

by

L. V. Dokuchayev

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EDITED TRANSLATION

FTD-ID(RS)T-0073-84 31 May 1984

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OSCILLATIONS OF A RESERVOIR WITH A FLUID ON THE FREE SURFACE OF WHICH A MEMBRANE IS LOCATED

By: L. V. Dokuchayev

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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OSCILLATIONS OF A RESERVOIR WITH A FLUID ON THE FREE SURFACE OF WHICH A MEMBRANE IS LOCATED

L. V. Dokuchayev
(Kalingrad, Moscow Oblast')

The creation of large-capacity reservoirs requires the careful analysis of the possible resonance excitation of wave motions of the liquid by wind or seismic pulsed loads. A number of reports are concerned with this problem, e.g., [1, 2]. A rib, a special membrane, or a thick plate that covers the free surface of the liquid can be used to limit the liquid's mobility. The dynamics of this type of system are considered below.

So far, a specific shape of reservoir (sphere, cylinder) has been studied in all of the works on the dynamics of shells with a liquid. This report considers oscillations of a rigid reservoir on an elastic base. The constant structure of the differential equations of the membrane makes it possible to obtain the dynamic characteristics of the "housing - liquid - membrane" system in analytical form for the general case of the configuration of a reservoir.

1. We will consider an arbitrary configuration of a reservoir with a liquid, on the free surface S_0 of which a smooth plate that is fastened on its periphery C is placed. We will designate the remainder of the wet surface of the reservoir by S (Fig. 1).

We will assume that the liquid is perfect and incompressible, and we will consider the case of an isotropic plate.

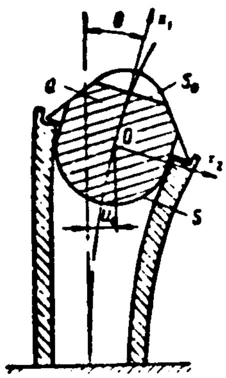

Fig. 1.

Suppose that a reservoir filled with a liquid rotates at an angular velocity $\dot{\vec{\theta}}_0$ relative to the elastic base and moves at speed $\dot{\vec{u}}_0$ under the action of seismic or wind loads. We will designate the displacement of the plate particles on the normal by w, and the displacement potential of the liquid - by χ.

The plate has a constant flexural rigidity of D and is subjected to tensile stresses T in its middle surface.

Then, in the case of small oscillations of the "housing - plate - liquid" system, we obtain the following linear system of equations in partial derivatives:

$$\Delta\chi = 0 \quad (1.1); \quad \partial\chi/\partial\nu\big|_S = (\vec{u}_0, \vec{\nu}) + [\vec{R}\times\vec{\nu}, \vec{\theta}_0] = N; \quad \partial\chi/\partial\nu\big|_S = w + N; \qquad (1.2)$$

$$\rho_п \delta_п \partial^2 w/\partial t^2 + D\nabla^2\nabla^2 w - T\Delta^2 w = p; \qquad (1.3)$$

$$w\big|_{S_0} < \infty; \quad \nabla w\big|_{S_0} < \infty; \quad w\big|_C = \partial w/\partial n\big|_C = 0. \qquad (1.4)$$

Equation (1.1) follows from the condition of the incompressibility of the liquid. Equation (1.2) consists of the conditions under which the liquid does not flow through the surface. Equation (1.3) is the equation of the oscillations of the plate; (1.4) - the conditions of the boundedness of the oscillations and the boundary conditions of the rigid attachment of the plate; $\vec{\nu}$ — орт the external normal to the surface of a liquid occupying volume Q; $\vec{n}$ — орт the external normal to the contour C; the values $\rho_п$ and $\delta_п$ designate the density and thickness of the plate, respectively. The transverse load p to which the plate is subjected from the side of the liquid can be determined with precision down to the constant by using the linearized Lagrange-Cauchy integral according to the formula

$$p = -\rho\,[\partial^2\chi/\partial t^2 - g\,(x_1 + x_2\theta_2 + x_3\theta_3)]. \qquad (1.5)$$

where ρ - the density of the liquid; g - the acceleration of gravity.

We will consider the homogeneous problem of free oscillations of the "flexible plate - liquid" system when the reservoir housing is immobile, and we will use the solution of the following boundary problem on free oscillations of a liquid in a mass force field:

$$\Delta\varphi=0;\quad \partial\varphi/\partial\nu|_S=0;\quad \partial\varphi/\partial\nu|_{S_0}=\varkappa\varphi. \tag{1.6}$$

This problem has an infinite discrete range of eigenvalues $\varkappa_n$ and the eigenfunctions φ_n corresponding to them.

According to report [3], we will isolate the time coordinate, and we will expand the unknown function of into a series according to this system of functions:

$$\chi=e^{i\omega t}\sum_{n=1}^{\infty}B_n\varphi_n(x_1,\ x_2,\ x_3);\ B_n=\frac{1}{N_n^2}\int_{S_0}v\frac{\partial\varphi_n}{\partial\nu}dS;\ \ N_n^2=\int_{S_0}\left(\frac{\partial\varphi_n}{\partial\nu}\right)^2dS. \tag{1.7}$$

We obtain the function v from the solution of the homogeneous boundary problem of elastic oscillations of a plate, which, according to (1.3)-(1.4), has the form:

$$\left.\begin{aligned}&L(v)=D\nabla^2\nabla^2v-T\nabla^2v+(\rho g-\rho_n\delta_n\omega^2)v=\rho\omega^2\sum_{n=1}^{\infty}B_n\varphi_n;\\&v|_{S_0}<\infty;\quad \nabla v|_{S_0}<\infty;\ v|_C=\partial v/\partial n|_C=0.\end{aligned}\right\} \tag{1.8}$$

The general solution of boundary problem (1.8) is represented in the form of the linear combination of the unknown solutions v_i^0 of the homogeneous equation L(v) = 0 and the partial solution v of heterogeneous equation (1.8) in the form of the expansion of problem (1.6) into eigenfunctions.

Using (1.4), and also the second formula in (1.7), we obtain the solution to problem (1.8) in the form of functions that form a complete orthogonal system:

$$v_k = \sum_{l=1}^{2} C_{lk} v_l^0 + \omega_k^2 \sum_{n=}^{\infty} (B_{nk}/\alpha_{nk})\, \partial\varphi_n/\partial\nu = \sum_{n=1}^{\infty} B_{nk}\, \partial\varphi_n/\partial\nu, \tag{1.9}$$

where the eigenvalues ω_k^2 are the roots of the characteristic equation

$$1-\omega^2 \sum_{n=1}^{\infty} \frac{K_n^* B_n}{\alpha_n} + \frac{\omega^2}{L_2 K_1 - L_1 K_2} \left(\Omega_2 \sum_{n=1}^{\infty} \frac{L_n^* B_{1n}^0}{\omega^2-\alpha_n} - \Omega_1 \sum_{n=1}^{\infty} \frac{L_n^* B_{2n}^0}{\omega^2-\alpha_n}\right) = 0. \tag{1.10}$$

The following designations are used here:

$$K_l = v_l^0\big|_C;\ \ L_l = \partial v_l^0/\partial n\,\big|_C;\ \ K_n^* = \varkappa_n \varphi_n\big|_C;\ \ L_n^* = \varkappa_n \partial\varphi_n/\partial n\,\big|_C;$$

$$\alpha_{nk} = \frac{L(\varkappa_n \varphi_n)}{\rho\varphi_n};\ \ B_{ln}^0 = \frac{1}{N_n^2}\int_{S_0} v_l^0 \frac{\partial\varphi_n}{\partial\nu}\, dS;\ \ \Omega_l = K_l - \omega^2 \sum_{n=1}^{\infty} \frac{K_n^* B_{ln}^0}{\omega^2-\alpha_n};$$

$$B_{nk} = \left(L_2 B_{1n}^0 - L_1 B_{2n}^0\right) \alpha_{nk} / \left(\omega_k^2 - \alpha_{nk}\right)(L_2 K_1 - L_1 K_2). \tag{1.11}$$

Using the eigenfunctions v_k, we can construct the solution to the system of equations in partial derivatives (1.1), (1.3) and for the case of an oscillating reservoir. We will represent the displacement potential of the liquid in the form of the sum of terms, the first of which corresponds to the displacements of the liquid in an absolutely rigid shell, while the last corresponds to the elastic deformations of the plate

$$\chi = \sum_{l=2}^{3} x_l\, u_l(t) + \sum_{l=1}^{3} \Psi_l(x_1, x_2, x_3)\,\theta_l(t) + \sum_{n=1}^{\infty} \varphi_n(x_1, x_2, x_3)\, H_n(t). \tag{1.12}$$

Here the functions Ψ_l are the Zhukovskiy potentials that satisfy the following boundary problem:

$$\Delta_0 \Psi_l = 0,\ \ \partial\Psi_l/\partial\nu|_{S=S_0} = (\vec{R}\times\vec{\nu})_l;$$

functions φ_n - the eigenfunctions of boundary problem (1.6). Expansion coefficients (1.12) are determined by the following formula:

$$H_n = \frac{1}{N_n^2}\int_{S_0} w \frac{\partial\varphi_n}{\partial\nu}\, dS.$$

We will represent the unknown function w in the form of an infinite expansion by types of free oscillations of the plate (1.9):

$$w = \sum_{k=1}^{\infty} v_k (x_2, x_3)\, p_k(t). \tag{1.13}$$

Substuting expansion (1.13) in equation (1.3), which makes it possible to perform differentiation by terms and the integration of the series, and also considering the properties of orthogonality, we multiply (1.3) by $v_k dS$ and integrate with respect to S_0. As a result, we obtain the infinite system of equations

$$\mu_k (\ddot{p}_k + \omega_k^2 p_k) + \sum_{l=2}^{3} \lambda_{lk} \ddot{u}_l + \lambda_{01k} \ddot{\theta}_1 + \sum_{l=2}^{3} [\lambda_{0lk} \ddot{\theta}_l + g(-1)^l \lambda_{5-l,k}\, \theta_l] = 0,$$

$k = 1, 2, \ldots$; the dots designate differentiation through time. The square of the plate oscillation frequency ω_k^2 is the k-th eigen number of the characteristic equation (1.10), and the remaining coefficients are determined by the following formulae:

$$\left.\begin{aligned}
\mu_k &= \rho \sum_{n=1}^{\infty} N_n^2 B_{nk}^2 / x_n = \sum_{n=1}^{\infty} B_{nk}^2 \mu_n^{\circ}; \\
\lambda_{ik} &= \rho \int_{S_0} x_i v_k\, dS = \sum_{n=1}^{\infty} B_{nk}\, \rho \int_{S} \varphi_n \nu_i\, dS = \sum_{n=1}^{\infty} B_{nk} \lambda_{in}^{\circ}; \\
\lambda_{0lk} &= \rho \int_{S_0} \Psi_l v_k\, dS = \sum_{n=1}^{\infty} B_{nk}\, \rho \int_{S+S_0} \varphi_n (\vec{R} \times \vec{\nu})_l\, dS = \sum_{n=1}^{\infty} B_{nk} \lambda_{0ln}^{\circ},
\end{aligned}\right\} \tag{1.14}$$

where the coefficients

$$\left.\begin{aligned}
\mu_n^{\circ} &= \rho \int_{S_0} \varphi_n \frac{\partial \varphi_n}{\partial \nu}\, dS; \quad \lambda_{ln}^{\circ} = \rho \int_{S_0} x_l \frac{\partial \varphi_n}{\partial \nu}\, dS = \rho \int_{S} \varphi_n \nu_l\, dS; \\
\lambda_{0ln}^{\circ} &= \rho \int_{S_0} \Psi_l \frac{\partial \varphi_n}{\partial \nu}\, dS = \rho \int_{S+S_0} \varphi_n (\vec{R} \times \vec{\nu})_l\, dS
\end{aligned}\right\} \tag{1.15}$$

are determined by the methods described in report [4], for example. The coefficients B_{nk} are calculated according to formula (1.11).

The forces and moments acting on the body from the side of the liquid and the elastic plate are determined by the pressure p, which is calculated using formula (1.5) with consideration of (1.12). Then the equation of motion of the "reservoir - liquid - plate" system can be written as follows, with the assumption that the axes of the introduced coordinate system are the primary ones,

$$(m^0+m)\ddot{u}_i+(-1)^i(L^0+L)\ddot{\theta}_{5-i}+\sum_{k=1}^{\infty}\lambda_{ik}\ddot{p}_k=P_i,\ i=2,3;$$

$$(I_i^0+I_i)\ddot{\theta}_i-(-1)^i(L^0+L)\ddot{u}_{5-i}-g(L^0+L)\theta_i+\sum_{k=1}^{\infty}\left[\lambda_{0ik}\ddot{p}_k+\right.$$

$$\left.+g(-1)^i\lambda_{5-i,k}p_k\right]=M_i;\quad (I_1^0+I_1)\ddot{\theta}_1+\sum_{k=1}^{\infty}\lambda_{01k}\ddot{p}_k=M_1,$$

where λ_{ik} and λ_{0ik} coincide with coefficients (1.14); $m^0 + m$, $L^0 + L$, $I^0{}_i + I_i$ - the mass, static moment and inertial moment of the system with an absolutely rigid membrane; P_i and M_i - the projections of the actions of the elastic base of the reservoir and the external loads.

2. We can construct an approximate solution for a reservoir with an arbitrary rotational contour for the boundary problem (1.1)-(1.4) from the known range of eigenfrequencies of problem (1.6). This solution precisely satisfies the Laplace equation and the boundary condition on the free surface S_0. It satisfies the boundary condition on the wet surface S on the average, which is described in rather great detail in report [4].

In a cylindrical coordinate system, it has the following form:

$$\varphi_{pm}=\psi_{pm}(x,r)\,e^{im\eta};\ \psi_{pm}(x,r)=X_{pm}(x)\,Y_{pm}(r)\ (p,\ m=1,2\ldots),$$

where

$$X_{pm}=\frac{r_0}{\zeta_{pm}}\,e^{\zeta_{pm}(x-x_0)/r_0};\ Y_{pm}=\frac{J_m(\zeta_{pm}r/r_0)}{J_m(\zeta_{pm})};\quad \zeta_{pm}=\varkappa_{pm}r_0.$$

Here x_0 - the coordinate of the free surface of the liquid; $\varkappa_{pm}$ - the predetermined range of eigenvalues of the boundary problem (1.6); r_0 - the radius of the contour of the free surface C.

Instead of the single-parameter family of subscripts n, here we will introduce the two-parameter family of subscripts p and m corresponding to the radial and circular harmonics.

We will consider an inertialess plate in the absence of a mass force field. In this case, boundary problem (1.8) for a circular flexible plate is written as follows, with the isolation of the circular coordinate:

$$\left.\begin{aligned} D\left[L_0^2(v_m) + L_0(v_m)\gamma^2/r_0^2\right] = \rho\,\omega_m^2 \sum_{p=1}^{\infty}\left[r_0 B_{pm} Y_{pm}(r)/\zeta_{pm}\right]; \\ v_m|_{r=r_0} = 0;\quad \partial v_m/\partial r|_{r=r_0} = 0, \end{aligned}\right\} \tag{2.1}$$

where $$\gamma^2 = T r_0^2/D;\quad L_0 = (d/r\,dr)(r\,d/dr) - m^2/r^2. \tag{2.2}$$

The general solution to this equation is the function

$$v_m = C_1 (r/r_0)^m + C_2 Z_m(r) + \omega_m^2 \sum_{p=1}^{\infty}\left[B_{pm} Y_{pm}(r)/\alpha_{pm}\right],$$

where

$$Z_m(r) = I_m(\gamma r/r_0)/I_m(\gamma);\quad \alpha_{pm} = D\,\zeta_{pm}^3\left(\zeta_{pm}^2 + \gamma^2\right)\rho\,r_0^5.$$

According to formulae (1.11), the coefficients B_{ipm}^0 and B_{pmk} assume the form

$$\left.\begin{aligned} B_{1pm}^0 = \frac{2\left(m - Y'_{pm}\right)}{\zeta_{pm}^2 - m^2 + Y'^2_{pm}};\quad B_{2\,pm}^0 = \frac{2\zeta_{pm}\left(Z'_m - Y'_{pm}\right)}{\left(\zeta_{pm}^2 + \gamma^2\right)\left(\zeta_{pm}^2 - m^2 + Y'^2_{pm}\right)}; \\ B_{pmk} = \left(Z'_m B_{1\,pm}^0 - m B_{2\,pm}^0\right)\alpha_{pm}/\left(\omega_{mk}^2 - \alpha_{pm}\right)\left(Z'_m - m\right), \end{aligned}\right\} \tag{2.3}$$

where $$Y'_{pm} = \zeta_{pm} J'_m(\zeta_{pm})/J_m(\zeta_{pm});\quad Z'_m = \gamma I'_m(\gamma)/I_m(\gamma).$$

Considering the property of orthogonality, we come to the conclusion that only harmonics with the subscript m = 1 can be excited during any movement of the reservoir.

All of the coefficients (1.15) for the different shapes of reservoirs encountered in practice are determined in [4]. Therefore, in order to create a dynamic diagram of the "reservoir - liquid - plate" system, it suffices to calculate the coefficients (2.3) using the Bessel function, and to find the root of characteristic equation (1.10).

3. As an example, we will consider a circular cylinder with

a unit radius r_0 = 1. Then the eigen numbers and eigenfunctions of boundary problem (1.6) will be written as:

$$\beta_n = \xi_{pm}; \quad \varkappa_{pm} = \xi_{pm} \operatorname{th} (\xi_{pm} h);$$
$$\chi_n = e^{im\varphi} J_m (\xi_{pm} r)/J_m (\xi_{pm}). \tag{3.1}$$

where ξ_{pm} - the root p of equation $J'_m(\xi) = 0$.

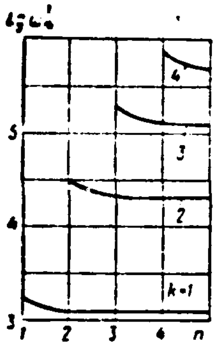

Fig. 2.

The characteristic equation assumes the form:

$$\omega_m^{-2} - \sum_{p=1}^{\infty} [d_{pm} / (\omega_m^2 - a_{pm})] = 0, \tag{3.2}$$

where the following dimensionless values are introduced, with consideration of (2.3):

$$\left. \begin{aligned} a_{pm} = \varkappa_{pm} \xi_{pm}^2 (\xi_{pm}^2 + \gamma^2); \quad d_{pm} = 2m\gamma Z'_m / (\xi_{pm}^2 + \gamma^2)(\xi_{pm}^2 - m^2) Z_{m+1}; \\ B_{pmk} = d_{pm} a_{pm} / (\omega_{mk}^2 - a_{pm}); \quad Z_{m+1} = I_{m+1}(\gamma) / I_m(\gamma); \quad D/\rho r_0^5 = 1. \end{aligned} \right\} \tag{3.3}$$

The dynamic characteristics for the cases $\rho = 1$ assume the form (1.14), where

$$\mu^{\bullet}_{pm} = \frac{\pi (\xi_{pm}^2 - m^2)}{2 \xi_{pm}^3 \operatorname{th} (\xi_{pm} h)}; \quad \lambda^{\bullet}_{p1} = \frac{\pi}{\xi_{p1}^2}; \quad \lambda^{\bullet}_{0p1} = \frac{\pi}{\xi_{p1}^2} \left[h - \frac{2}{\xi_{p1}} \operatorname{th} \left(\frac{\xi_{p1} h}{2} \right) \right]. \tag{3.4}$$

In the limiting case of an absolutely elastic plate, $\gamma \to 0$, i.e., when there are no forces in the middle surface (T = 0), formulae (3.2)-(3.4) remain valid if we use the limiting transition for the Bessel function of an imaginary argument $\gamma Z''_m / Z_{m+1} \to 2m(m+1)$ in them. Formulae (3.2)-(3.4) also remain valid for an absolutely flexible plate, if we use the limiting transition as $\gamma \to \infty$:

$$\omega_{mk}^2 / \gamma^2 \to \sigma_{mk}^2; \quad a_{pm} / \gamma^2 \to \varkappa_{pm} \xi_{pm}^2; \quad Z'_m / \gamma Z_{m+1} \to 1.$$

Now we will consider the case of an absolutely flexible inertialess plate (D = 0) in a field of mass forces with gradient g.

In this case, equation (1.8), written in dimensionless form for a cylindrical cavity, analogously with (2.1) and (3.2), can be represented as follows:

$$L_0(v_m) - \delta^2 v_m = -\sigma_m^2 \sum_{p=1}^{\infty} (B_{pm}/\varkappa_{pm})\, Y_{pm}(r); \quad \delta^2 = \rho g\, r_0^2/T. \tag{3.5}$$

With precision down to normalization, the general solution to equation (3.5) with the boundary condition $v_m(1) = 0$ is the function

$$v_{mk}(r) = -v_m^0(r) + \sigma_{mk}^2 \sum_{p=1}^{\infty} \frac{B_{pmk}}{\alpha_{pm}} Y_{pm}(r); \quad v_m^0 = \frac{I_m(\delta r)}{I_m(\delta)}. \tag{3.6}$$

Here $\alpha_{pm} = \sigma^2_{mk}$ is the k-th root of characteristic equation (3.2), where

$$d_{pm} = \frac{2\xi_{pm}^2 \delta I_m'(\delta)}{(\xi_{pm}^2 + \delta^2)(\xi_{pm}^2 - m^2) I_m(\delta)}; \quad B_{pmk} = \frac{d_{pm}\alpha_{pmk}}{\sigma_{mk}^2 - \alpha_{pmk}}. \tag{3.7}$$

and the dynamic characteristics will be determined using formulae (1.14), (3.6).

When there are no mass forces $(\delta \to 0)$ at the limit, equations (3.7) are transformed into (3.3) as $\gamma \to \infty$.

On the other hand, when the mass forces predominate over the surface tension forces $(\delta \to \infty)$, the coefficient $d_{pm} \to 0$, and the square of the frequency and coefficients (1.14) approach the following values (3.4):

$$\sigma_{mk}^2/\delta^2 \to \varkappa_{km}; \quad \lambda_k \to \mathring{\lambda}_{k1}; \quad \lambda_{0k} \to \mathring{\lambda}_{0k1}; \quad \mu_{km} \to \mathring{\mu}_{km}. \tag{3.8}$$

Infinite series (3.2) and (3.5) rapidly converge. Figure 2 shows the values of the first four roots (k = 1, 2, 3, 4) of equation (3.2) at m = 1 and $\gamma = 0$ for a different number of terms of the sum for subscript p. It is evident that the value of the first root ω^2_{11} already converged with three terms (n = 3); it suffices to take the first five terms of the sum (n = 5) in order to obtain the values of the first three roots ω^2_{1k} .

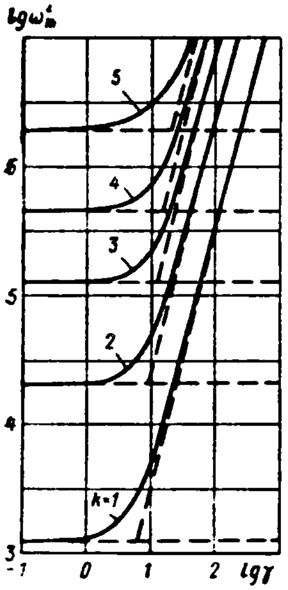

Fig. 3.

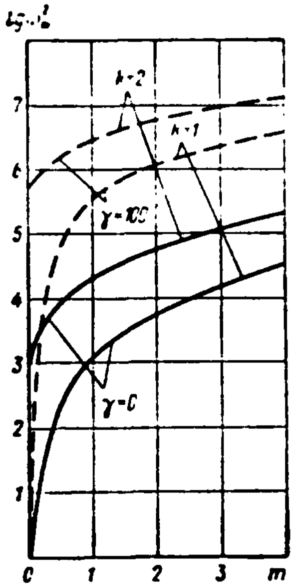

Fig. 4.

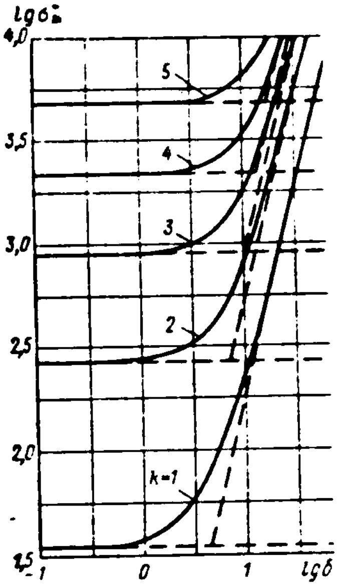

Fig. 5.

Figure 3 shows the dependence of the square of the dimensionless frequency on the dimensionless parameter γ, determined by formula (2.2) for different k, using a logarithmic scale. The horizontal broken lines designate the asymptotes which the curves approach as $\gamma \to 0$; this is the case of the absence of tension in the middle surface of an elastic membrane with flexural rigidity. The sloping broken lines represent the asymptotes that the curves approach as $\gamma \to \infty$, i.e., the flexible membrane is not resistant to bending, and it is subjected to tensile forces. The dependence of the square of the frequency on the value of the number of waves in the peripheral direction m for different k and γ is shown in Fig. 4.

The first three figures show the case of the absence of a mass force field. Figure 5 shows the effect of its gradient, determined by the dimensionless parameter δ using formula (3.5). The broken lines correspond to the asymptotic behavior of the curves at δ approaching zero and infinity.

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