

MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

AD-A141 815

NAVAL POSTGRADUATE SCHOOL  
Monterey, California



DTIC  
ELECTE  
JUN 4 1984  
S B

THESIS

THE EFFECT OF COMPONENT REDUNDANCY  
UPON AIRCRAFT COMBAT SURVIVABILITY

by

Angelo J. Kangelos

December 1983

Thesis Advisor:

R. E. Ball

Approved for public release; distribution unlimited.

DTIC FILE COPY

84 06 04 039

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
AD-A141815		
4. TITLE (and Subtitle) The Effect of Component Redundancy Upon Aircraft Combat Survivability		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis December 1983
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Angelo J. Kangelos		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		12. REPORT DATE December 1983
		13. NUMBER OF PAGES 62
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Aircraft Survivability, Susceptibility Evaluation, Vulnerability Evaluation, Probability of Hit Density Function, Probability of Kill, Cookie Cutter Kill Function, Carleton Kill Function, Redundancy, Transition Matrix, Cumulative Probability of Survival, Cumulative Probability of Kill.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Aircraft combat survivability has developed into a discipline of its own. This thesis presents a brief overview of the two integral parts of survivability, susceptibility and vulnerability. It examines the probability of an aircraft being killed given a single shot fired by the enemy. It further examines the redundancy of critical components and the effect redundancy has on the probability of kill of the aircraft. Several numerical examples are given. The thesis also examines the		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE  
S/N 0102-LF-014-6601

UNCLASSIFIED

1 SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

**UNCLASSIFIED**

**SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)**

Antiaircraft Artillery Simulation Computer Program, and the modifications required in order to introduce component redundancy into it are developed.

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

0210  
COPY  
NUMBERED

**UNCLASSIFIED**

**SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)**

Approved for public release; distribution unlimited.

The Effect of Component Redundancy  
Upon Aircraft Combat Survivability

by

Angelo J. Kangalos  
Lieutenant Commander, United States Navy  
B.S., Youngstown State University, 1970

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL  
December 1983

Author:

Angelo John Kangalos

Approved by:

R. E. Ball

Thesis Advisor

Donald M. Layton

Chairman, Department of Aeronautics

M. Oyer

Dean of Science and Engineering

## ABSTRACT

Aircraft combat survivability has developed into a discipline of its own. This thesis presents a brief overview of the two integral parts of survivability, susceptibility and vulnerability. It examines the probability of an aircraft being killed given a single shot fired by the enemy. It further examines the redundancy of critical components and the effect redundancy has on the probability of kill of the aircraft. Several numerical examples are given. The thesis also examines the Antiaircraft Artillery Simulation Computer Program, and the modifications required in order to introduce component redundancy into it are developed.

TABLE OF CONTENTS

I.	INTRODUCTION . . . . .	8
	A. SUSCEPTIBILITY . . . . .	8
	B. VULNERABILITY . . . . .	9
II.	PROBABILITY OF KILL GIVEN A SINGLE SHOT . . . . .	11
	A. PROBABILITY OF HIT DENSITY FUNCTION . . . . .	11
	B. PROBABILITY OF KILL FUNCTION GIVEN A HIT . . . . .	13
III.	REDUNDANCY AND THE $P_{KSS}$ . . . . .	17
IV.	PROBABILITY OF KILL AND REDUNDANCY . . . . .	25
V.	AFATL P001 COMPUTER PROGRAM AND THE $P_{K/H}$ . . . . .	35
VI.	CONCLUSIONS . . . . .	46
	APPENDIX A. EVALUATION OF EQUATIONS . . . . .	47
	APPENDIX B. $\bar{P}_K$ CALCULATIONS THROUGH THE TRANSITION MATRIX METHOD . . . . .	56
	APPENDIX C. P001 'FIRE' SECTION . . . . .	59
	LIST OF REFERENCES . . . . .	61
	INITIAL DISTRIBUTION LIST . . . . .	62

LIST OF TABLES

1.	CUMULATIVE PROBABILITIES OF KILL FOR EXAMPLE AIRCRAFT . . . . .	32
2.	$\bar{P}_K$ CALCULATIONS THROUGH THE TRANSITION MATRIX METHOD . . . . .	56

LIST OF FIGURES

1.	VULNERABILITY OF TWO SEPARATE REDUNDANT CRITICAL COMPONENTS, COOKIE CUTTER APPROACH . . . . .	19
2.	VULNERABILITY OF TWO SEPARATE REDUNDANT CRITICAL COMPONENTS, CARLETON APPROACH . . . . .	20
3.	AIRCRAFT KILL TREE DIAGRAM, FIRST HIT . . . . .	27
4a.	AIRCRAFT KILL TREE DIAGRAM, SECOND HIT ALONE . . .	27
4b.	AIRCRAFT KILL TREE DIAGRAM, SECOND HIT ALONE . . .	28
4c.	AIRCRAFT KILL TREE DIAGRAM, SECOND HIT ALONE . . .	28
5.	CUMULATIVE PROBABILITY OF KILL VERSUS NUMBER OF HITS . . . . .	33

## I. INTRODUCTION

Since World War I, aircraft combat survivability has become a factor of constantly increasing importance to both pilots and aircraft designers alike. The definition of aircraft combat survivability which will be adopted in this study is "the capability of an aircraft to avoid and/or withstand a man-made, non-nuclear hostile environment" [1].

Given an aircraft flying in a hostile environment, there is a probability that the aircraft will be hit by a damage causing mechanism and a different probability, that the damage causing mechanism will have the catastrophic effect of killing the aircraft. These two probabilities are of paramount importance, and they will be given special attention.

The probability that the aircraft will be hit,  $P_H$ , is usually referred to as the susceptibility of the aircraft, while the probability that the aircraft will be killed, given that it has been hit,  $P_{K/H}$ , is called the vulnerability of the aircraft.

### A. SUSCEPTIBILITY

Assuming that the enemy is alert and possesses weapons, aircraft susceptibility is influenced by the enemy's ability to detect, identify and track the aircraft; launch and

direct threat propagators toward the aircraft; and detonate them at or near the aircraft. If one identifies two phases during the flight of the aircraft, i.e., prelaunch and after launch of the threat propagator, then these abilities can be measured by  $P_T$ , the probability that the aircraft has been detected, identified and tracked, and by  $P_D$ , the probability that a threat propagator is being launched and detonated at or near the aircraft. Thus, susceptibility  $P_H$  is given by

$$P_H = P_T * P_D \quad (1)$$

#### B. VULNERABILITY

Aircraft vulnerability is primarily influenced by the aircraft's design. Any survivability features built into the aircraft, such as redundancy, armor plating, and the ability of critical components to operate after taking a hit, will decrease its vulnerability,  $P_{K/H}$ .

The probability of the aircraft being killed,  $P_K$ , is the product of its susceptibility and vulnerability. Thus

$$P_K = P_H * P_{K/H} \quad (2)$$

$P_K$  is related to the probability of survival of the aircraft,  $P_S$ , by

$$P_S = 1 - P_K \quad (3)$$

This study will examine the computation of the probability of kill of an aircraft and how it is effected by redundancy and the number of hits that the aircraft has taken.

## II. PROBABILITY OF KILL GIVEN A SINGLE SHOT

The probability of an aircraft being killed given a single shot,  $P_{KSS}$ , is based on the assumptions that the aircraft has been detected, and that a threat propagator has been fired or launched. With these assumptions, the  $P_{KSS}$  becomes solely a function of the fire control system's ability to direct the threat propagator in the vicinity of the aircraft and the aircraft's vulnerability. Thus, the expression for the  $P_{KSS}$  in a one-dimensional analysis is

$$P_{KSS} = \int_{-\infty}^{\infty} \rho_H(x) * P_{K/H}(x) dx \quad (4)$$

and in a two-dimensional analysis is

$$P_{KSS} = \int_{-\infty}^{\infty} \int_0^{\infty} \rho_H(x,y) * P_{K/H}(x,y) dx dy \quad (5)$$

where  $\rho_H(x)$  is the probability of hit density function at  $x$ , and  $P_{K/H}(x)$  is the probability that the aircraft will be killed given that it has been hit at  $x$ .

### A. PROBABILITY OF HIT DENSITY FUNCTION

The probability of hit density function is the mathematical expression of the ability of a fire control system

to direct a threat propagator in the vicinity of the aircraft. In most  $P_{KSS}$  assessments, the normal or Gaussian density function is used. If the propagator paths are uniformly distributed around the aircraft, the one-dimensional circular normal is applicable. This function has the form

$$\rho_H(r) = \frac{r}{\sigma^2} \exp \left( -\frac{r^2}{2\sigma^2} \right) \quad 0 \leq r \leq \infty \quad (6)$$

where  $r$  is the propagator miss distance (with respect to  $r = 0$ ), and  $\sigma$  is the standard deviation, which is a measure of the dispersion of the shots about the target. This density function  $\rho_H(r)$  is used to determine the probability that any shot will have a miss distance that lies within a certain band defined by  $r_1$  and  $r_2$ . This probability is given by

$$P_H \Big|_{r_1}^{r_2} = \int_{r_1}^{r_2} \frac{r}{\sigma^2} \exp \left( -\frac{r^2}{2\sigma^2} \right) dr \quad (7)$$

It is important to note that the probability that any shot lies within the band  $r_1 = 0$  and  $r_2 = \infty$  is unity.

In the two-dimensional analysis, the bivariate normal distribution in the x-y intercept plane is used for the

probability of hit density function. This two-dimensional normal can be given in the form

$$\rho_H(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x-\mu_x}{\sigma_x} \right)^2 + \left( \frac{y-\mu_y}{\sigma_y} \right)^2 \right] \right\} \quad (8)$$

where  $\sigma_x$  and  $\sigma_y$  are the standard deviations in the  $x$  and  $y$  directions, respectively, and  $\mu_x$  and  $\mu_y$  are the mean values of the shots about the  $x$  and  $y$  axes, respectively. The mean value represents a bias or aiming error and is given by

$$\mu_x = \frac{1}{N} \sum_{i=1}^N x_i \quad , \quad \mu_y = \frac{1}{N} \sum_{i=1}^N y_i$$

where  $N$  is a large number of shots and  $x_i$ ,  $y_i$  are the  $x$  and  $y$  coordinates of the  $i$ th miss distance.

#### B. PROBABILITY OF KILL FUNCTION GIVEN A HIT

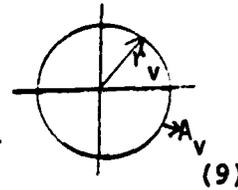
For contact warheads, the aircraft vulnerability can be represented by the vulnerable area  $A_v$ , where any hit within the vulnerable area causes an aircraft kill. In the one-dimensional analysis, the vulnerable area becomes a circle with a radius given by

$$r_v = \sqrt{\frac{A_v}{\pi}}$$

and the  $P_{K/H}$ , or kill function is given by

$$P_{K/H} = 1 \quad \text{for} \quad 0 \leq r \leq r_v$$

$$= 0 \quad \text{for} \quad r > r_v$$



This type of kill function is referred to as a Cookie Cutter kill function.

Another kill function of particular interest has the form



$$P_{K/H} = \exp \left( - \frac{r^2}{r_v^2} \right) \quad (10)$$

and is referred to as a Carleton kill function. Note, that once again, the area under this  $P_{K/H}$  curve is the vulnerable area  $A_v$ .

For the two-dimensional analysis, the circular Carleton kill function becomes

$$P_{K/H}(x,y) = \exp \left\{ - \left[ \left( \frac{x}{r_v} \right)^2 + \left( \frac{y}{r_v} \right)^2 \right] \right\} \quad (11)$$

where  $r^2$  has been replaced by  $x^2 + y^2$  in equation (10).

Substituting equations (6) and (9) into equation (4) and evaluating the integral, the one-dimensional probability of kill given a single shot for the Cookie Cutter kill function becomes

$$P_{KSS} = 1 - \exp \left( - \frac{A_v}{2\pi\sigma^2} \right) \quad (12)$$

When equations (6) and (10) are substituted into equation (4), the one-dimensional probability of kill given a single shot for the Carleton kill function becomes

$$P_{KSS} = \frac{\left(\frac{A_v}{2\pi}\right)}{\left(\frac{A_v}{2\pi}\right) + 2\sigma^2} \quad (13)$$

When equations (8) and (11) are substituted into equation (4), the two-dimensional probability of kill given a single shot for the Carleton kill function becomes

$$P_{KSS} = \frac{\frac{A_v}{2\pi}}{\sqrt{\sigma_x^2 + \frac{A_u}{2\pi}} * \sqrt{\sigma_y^2 + \frac{A_v}{2\pi}}} \exp \left\{ - \frac{1}{2} \left[ \frac{\mu_x^2}{\sigma_x^2 + \frac{A_u}{2\pi}} + \frac{\mu_y^2}{\sigma_y^2 + \frac{A_v}{2\pi}} \right] \right\} \quad (14)$$

If one assumes  $A_v \ll \sigma_x^2, \sigma_y^2$  then

$$P_{KSS} = \frac{A_v}{2\pi\sigma_x\sigma_y} \exp \left[ - \frac{1}{2} \left( \frac{\mu_x^2}{\sigma_x^2} + \frac{\mu_y^2}{\sigma_y^2} \right) \right] \quad (15)$$

Thus, according to equation (15),  $P_{KSS}$  is a linear function of the aircraft's vulnerable area.

Detailed evaluations of equations (12), (13) and (14) are included in Appendix A.

### III. REDUNDANCY AND THE $P_{KSS}$

As it was previously defined, vulnerability is the inability of an aircraft to withstand one or more hits by damage causing mechanisms. The higher the vulnerability of an aircraft, the worse its chances of survival are. Thus, one might say that vulnerability is a disadvantage in an aircraft. In order to reduce the vulnerability and overcome this disadvantage, designers are utilizing six vulnerability reduction concepts [1], one of which is component redundancy.

An aircraft is comprised of a multitude of components, and each one of them contributes to the overall aircraft vulnerability. Some components contribute much more than the others, and for that reason they are called critical components. Thus, a component is categorized as critical, if a hit by a damage causing mechanism on the component leads to an aircraft kill. If an aircraft is comprised of  $N$  non-redundant critical components, then the aircraft will survive if and only if each of the  $N$  critical components survives. Thus, the aircraft's probability of survival after the first hit,  $P_s^{(1)}$ , will be

$$P_s^{(1)} = P_{s_1}^{(1)} * P_{s_2}^{(1)} * P_{s_3}^{(1)} * \dots * P_{s_N}^{(1)} \quad (16)$$

where  $P_{s_1}^{(1)}$ ,  $P_{s_2}^{(1)}$ ,  $P_{s_3}^{(1)} \dots P_{s_N}^{(1)}$  are the probabilities of survival of critical component 1, 2, 3, ..., N, after the first shot. If one takes into consideration equation (3) the probability of aircraft survival could be written as

$$P_s^{(1)} = \prod_{i=1}^N P_{s_i}^{(1)} = \prod_{i=1}^N (1 - P_{k_i}^{(1)}) \quad (17)$$

where  $P_{s_i}^{(1)}$  is the probability of survival of the  $i$ th component after the first shot, and  $P_{k_i}^{(1)}$  is the probability of kill of the  $i$ th component after the first shot.

If there are sets of redundant critical components, the validity of equation (16) breaks down, that is, a hit on a redundant critical component does not constitute an aircraft kill, since another component could sustain the aircraft flight. This can be best illustrated by the following example aircraft, which is comprised of four critical components, the pilot, the fuel tank and the redundant set of engines one and two. (The aircraft is assumed to be capable of flying on only one engine.) The probability that the aircraft will survive the first shot is

$$P_s^{(1)} = P_{s_p}^{(1)} * P_{s_f}^{(1)} * P_{s_{EE}}^{(1)} \quad (18)$$

where  $P_{s_p}^{(1)}$ ,  $P_{s_f}^{(1)}$  and  $P_{s_{EE}}^{(1)}$  are the probabilities of survival of the pilot, the fuel tank, and either one or both engines respectively, since both engines must be killed

simultaneously in order to kill the aircraft. The probability that either engine one or engine two or both engines survive after the first shot is given by

$$P_{s_{EE}}^{(1)} = 1 - P_{k_{E1}}^{(1)} * P_{k_{E2}}^{(1)} \quad (19)$$

where  $P_{k_{E1}}$  and  $P_{k_{E2}}$  are the probability of kill of engines one and two, respectively, and the product of  $P_{k_{E1}}$  and  $P_{k_{E2}}$  is the probability that both engines will be killed by the first shot. This last concept is an interesting one and will now be investigated further.

In the following investigation, the one-dimensional expression for the Cookie Cutter and Carleton kill functions will be utilized along with the one-dimensional probability of hit density function. First, using the Cookie Cutter approach, figure 1 represents the situation for the vulnerability of two separate redundant critical components.

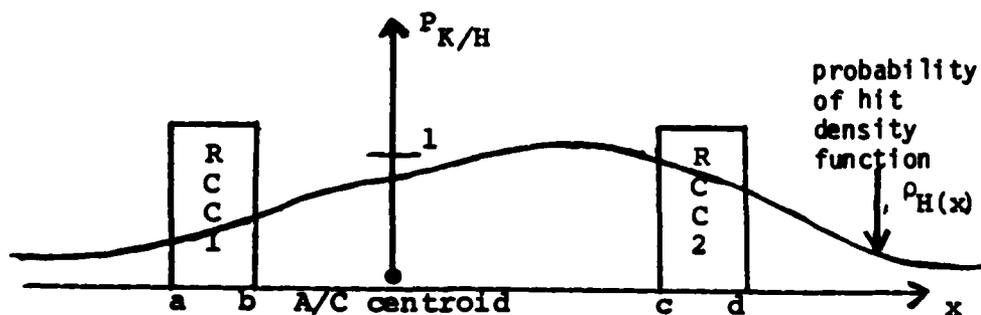


Figure 1. Vulnerability of Two Separate Redundant Critical Components, Cookie Cutter Approach

In figure 1, the x-axis represents the aircraft length (vice area, since one-dimensional analysis is used), a-b is the vulnerable length of redundant component one, and c-d is the vulnerable length of redundant component two. In this approach, the kills of redundant components one and two are mutually exclusive events; if component one is killed on the hit, then component two is still alive and vice versa, since only one vulnerable length could be hit at one time.

With the Carleton kill function shown in figure 2 everything changes. As seen from figure 2, the kills of redundant critical components one and two are not mutually exclusive events.

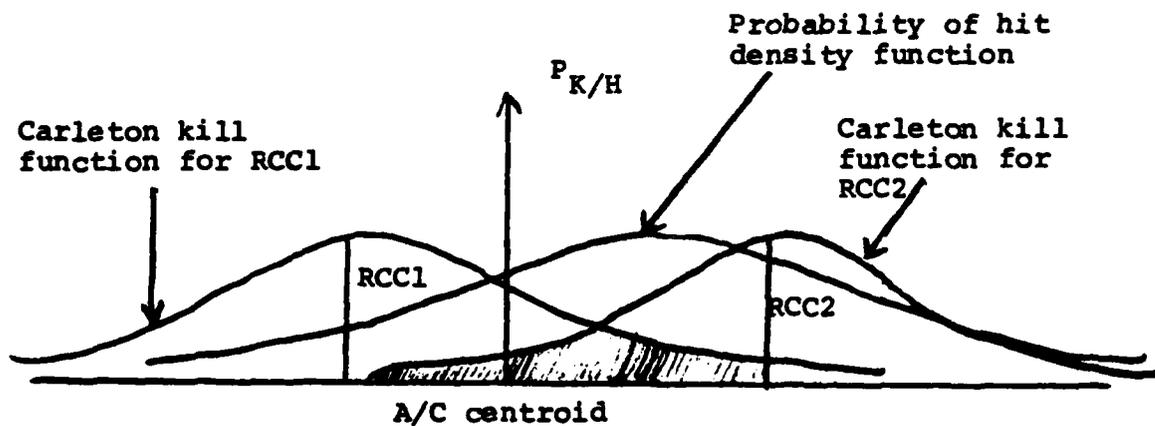


Figure 2. Vulnerability of Two Separate Redundant Critical Components, Carleton Approach

A hit anywhere from  $-\infty$  to  $+\infty$  will have a non-zero probability of killing both redundant critical components. In essence, the Carleton kill function creates an overlap of the components. Thus, one hit can possibly kill the aircraft because both redundant components can be killed by one hit.

Returning back to the example aircraft, equation (18) becomes

$$P_s^{(1)} = (1 - P_{k_p}^{(1)}) * (1 - P_{k_f}^{(1)}) * (1 - P_{k_{E1}}^{(1)}) * P_{k_{E2}}^{(1)} \quad (20)$$

This equation is valid for an aircraft with one set of two redundant critical components where only a kill of both of them will result in an aircraft kill. There are some other combinations of redundant critical components that one might encounter in a kill tree analysis. They are

(1) A set of three-redundant critical components and only a kill of all three will result in an aircraft kill. For this particular combination, the aircraft kill logical expression due to just the redundant set will be

$$A/C \text{ KILL}_{RC} = (RC1.AND.RC2.AND.RC3)$$

while the probability of any one, any two or all three components surviving the first shot, will be given by

$$P_{s_{RC}}^{(1)} = 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} \quad (21)$$

where  $P_{k_{RC}}^{(1)}$ ,  $P_{k_{RC2}}$ ,  $P_{k_{RC3}}$  are the probabilities of kill for redundant critical components one, two and three, respectively.

(2) A set of three redundant critical components and a kill of any two out of three will result in an aircraft kill. The logical expression for this combination is

$$A/C \text{ KILL}_{RC} = (RC1.AND.RC2).OR.(RC2.AND.RC3).OR.(RC1.AND.RC3)$$

while the probability of any two or all three redundant critical components, surviving the first shot will be

$$P_{s_{RC}}^{(1)} = 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} - P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} - P_{k_{RC1}}^{(1)} * P_{k_{RC3}}^{(1)} + P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} \quad (22)$$

(3) A set of four redundant critical components and a kill of any two out of four will result in an aircraft kill. The logical expression for this combination will be

$$A/C \text{ KILL}_{RC} = (RC1.AND.RC2).OR.(RC1.AND.RC3).OR.(RC1.AND.RC4).OR.(RC2.AND.RC3).OR.(RC2.AND.RC4).OR.(RC3.AND.RC4)$$

while the probability of any two, three or all four critical components surviving the first shot will be.

$$\begin{aligned}
 P_{S_{RC}}^{(1)} = & 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} - P_{k_{RC1}}^{(1)} * P_{k_{RC3}}^{(1)} \\
 & - P_{k_{RC1}}^{(1)} * P_{k_{RC4}}^{(1)} - P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} \\
 & - P_{k_{RC2}}^{(1)} * P_{k_{RC4}}^{(1)} - P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \\
 & + P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} + P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC4}}^{(1)} \\
 & + P_{k_{RC1}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} + P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \\
 & + P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \quad (23)
 \end{aligned}$$

(4) A set of four redundant critical components and a kill of any three out of four will result in an aircraft kill. The logical expression for this combination is

$$\begin{aligned}
 A/C \text{ KILL}_{RC} = & (RC1.AND.RC2.AND.RC3).OR.(RC1.AND.RC2.AND.RC4). \\
 & .OR.(RC1.AND.RC3.AND.RC4) \\
 & .OR.(RC2.AND.RC3.AND.RC4)
 \end{aligned}$$

while the probability of any one, two, three or all four redundant components surviving the first shot will be

$$\begin{aligned}
P_{S_{RC}}^{(1)} &= 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} \\
&\quad - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC4}}^{(1)} \\
&\quad - P_{k_{RC1}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \\
&\quad - P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \\
&\quad + P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \quad (24)
\end{aligned}$$

(5) A set of four redundant critical components and a kill of all four only will result in an aircraft kill. The logical expression for this combination will

$$A/C \text{ KILL}_{RC} = RC1.AND.RC2.AND.RC3.AND.RC4$$

while the probability that any one, two, three or all four redundant components will survive the first shot will

$$P_{S_{RC}}^{(1)} = 1 - P_{k_{RC1}}^{(1)} * P_{k_{RC2}}^{(1)} * P_{k_{RC3}}^{(1)} * P_{k_{RC4}}^{(1)} \quad (25)$$

#### IV. P<sub>K/H</sub> AND REDUNDANCY

The analysis done in the previous chapter, though necessary, was limited to the probability that an aircraft is killed due to a single shot. A further and more realistic analysis stems from the fact that in any given combat scenario, an aircraft is likely to take multiple hits. Thus, the total probability of kill of an aircraft that has taken multiple hits will now be examined.

If one assumes that an aircraft consists of N critical components and that it has taken n hits is given by

$$P_{S_i}^{-n} = P_{S_i}^{(1)} * P_{S_i}^{(2)} * P_{S_i}^{(3)} * \dots * P_{S_i}^{(n)} = \prod_{j=1}^n P_{S_i}^{(j)} \quad (26)$$

where  $P_{S_i}^{(1)}$ ,  $P_{S_i}^{(2)}$ ,  $P_{S_i}^{(3)}$ ... $P_{S_i}^{(n)}$  are the probabilities that the ith critical component will survive the first, second, third...nth hit, respectively. Using a parallel which may be inferred from equation (17),

$$P_{S_i}^{-n} = \prod_{j=1}^n (1 - P_{k/h_i}^j) \quad (27)$$

where  $P_{k/h_i}$  is the probability that the ith component will be killed by the jth hit. In a similar manner, the total probability of aircraft survival after n hits can be derived to yield

$$\bar{P}_s^{(n)} = \prod_{j=1}^n (1 - P_{K/H}^j) \quad (28)$$

where  $P_{K/H}^j$  is the probability that the aircraft will be killed by the  $j$ th hit.

In the case of an aircraft consisting of only non-redundant critical components, the  $P_{K/H}$  does not change with each hit because the vulnerable area is constant for each hit. However, in the case of redundant critical components,  $P_{K/H}$  changes considerably due to the increasing probability the redundant critical components are killed and thus, it will be studied separately.

The simplest approach to the  $P_{K/H}$  with redundancy study is through the use of a kill tree diagram. For that, assume an A/C consisting of three non-redundant critical components CC1, CC2 and CC3, with individual probabilities of kill  $P_{K/H_1}$ ,  $P_{K/H_2}$ ,  $P_{K/H_3}$ , respectively, and or set of two redundant critical components RCC1 and RCC2 with individual probabilities of kill  $P_{K/H_{R_1}}$  and  $P_{K/H_{R_2}}$ , respectively. Thus, the kill diagram is depicted on figure 3 for the first hit on the aircraft and the  $P_{K/H}^{-(1)}$  for the Cookie Cutter approach becomes

$$P_K^{-(1)} = P_{K/H_1} + P_{K/H_2} + P_{K/H_3} \quad (29)$$

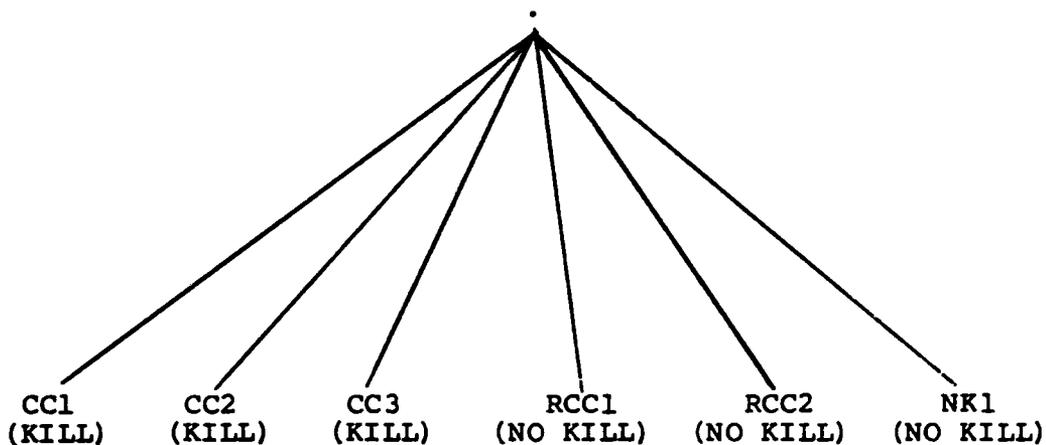


Figure 3. Aircraft Kill Tree Diagram, First Hit

In order for the A/C to be killed by the second hit, one must assume that it survived the first hit, and therefore only the RCC1, RCC2 and NK1 branches will be affected by the second hit. Thus, the kill tree for the second hit alone is depicted on figures 4a, 4b and 4c.

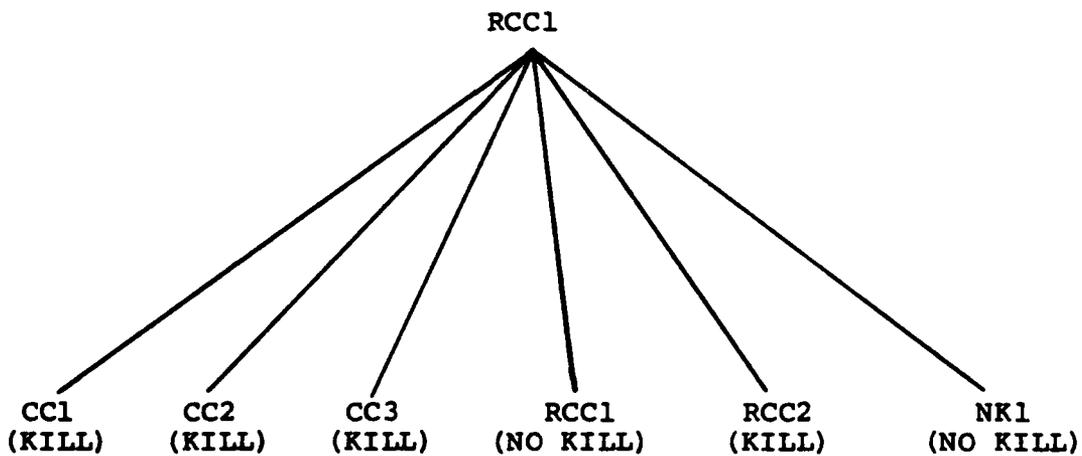


Figure 4a. Aircraft Kill Tree Diagram, Second Hit Alone

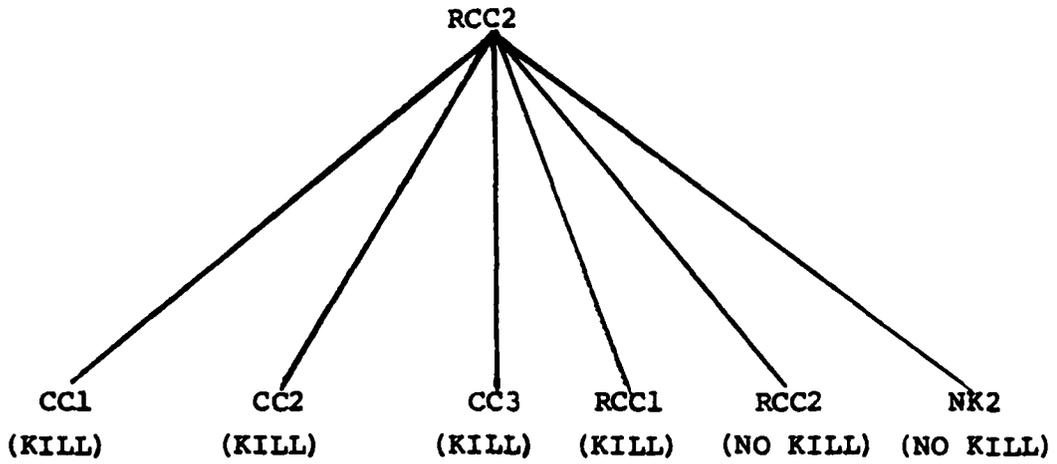


Figure 4b. Aircraft Kill Tree Diagram, Second Hit Alone

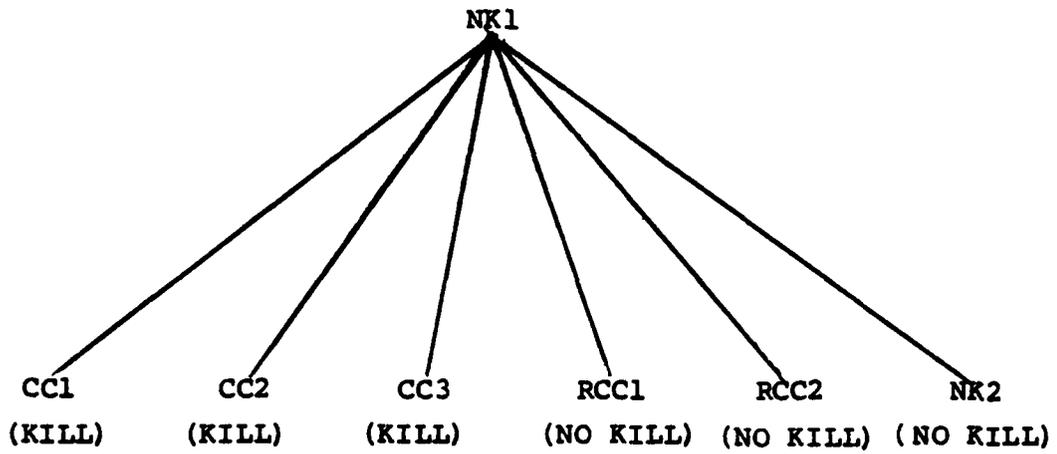


Figure 4c. Aircraft Kill Tree Diagram, Second Hit Alone

Thus, the probability of the A/C being killed by the second hit alone becomes

$$\begin{aligned}
P_K^{(2)} &= P_{k/H_{R1}}^{(1)} * \left( P_{k/H_1}^{(2)} + P_{k/H_2}^{(2)} + P_{k/H_3}^{(2)} + P_{k/H_{R2}}^{(2)} \right) \\
&+ P_{k/H_{R2}}^{(1)} * \left( P_{k/H_1}^{(2)} + P_{k/H_2}^{(2)} + P_{k/H_3}^{(2)} + P_{k/H_{R1}}^{(2)} \right) \\
&+ P_s^{(1)} * \left( P_{k/H_{R1}}^{(2)} * P_{k/H_{R2}}^{(2)} \right)
\end{aligned} \tag{30}$$

where  $P_s^{(1)}$  is the probability that all of the critical components survived the first hit (the NK1 branch). Thus, the total probability of kill for the two hits  $P_{K/H}^{-(2)}$  becomes

$$P_K^{-(2)} = P_K^{-(1)} + P_K^{(2)} \tag{31}$$

One could go on to the third and subsequent hits in a similar manner, and the  $P_{K/H}$  will be increasing as more and more terms are added for the subsequent hits.

Another method of computing the total probability of kill is the transition matrix method [2]. Using that method on the following example aircraft, one can clearly see the effect of redundancy on the  $P_{K/H}$ .

A/C #1 (NO REDUNDANCY)

CRITICAL COMPONENT	$A_{P_i}$ (ft <sup>2</sup> )	$P_{k/h_i}$	$A_{V_i}$ (ft <sup>2</sup> )
Pilot	5	1.0	5
Engine	30	.8	24
Fuel Tank	40	.6	24
Controls	25	.6	15

Total aircraft area present to the threat propagator is assumed to be  $A_p = 500 \text{ ft}^2$ , and  $A_{P_i}$  is the area presented by the  $i$ th component,  $P_{k/h_i}$  is the probability of  $i$ th component given a hit on the  $i$ th component, and  $A_{V_i}$  is the vulnerable area of the  $i$ th component. The logical kill expression for this aircraft will be

A/C KILL = PILOT.OR.ENGINE.OR.FUEL.OR.CONTROLS

A/C #2 (25% REDUNDANT)  $A_p = 500 \text{ ft}^2$

CRITICAL COMPONENT	$A_{P_i}$ (ft <sup>2</sup> )	$P_{k/h_i}$	$A_{V_i}$ (ft <sup>2</sup> )
Pilot	5	1.0	5
Engine	30	.8	24
Fuel Tank	40	.6	24
Controls I	12.5	.6	7.5
Controls II	12.5	.6	7.5

The logical expression for this aircraft is

A/C KILL = PILOT.OR.ENGINE.OR.FUEL.OR.(CONTROLI.AND.CONTROLII)

A/C #3 (46% REDUNDANT)  $A_p = 530 \text{ ft}^2$

CRITICAL COMPONENT	$A_{p_i}$ (ft <sup>2</sup> )	$P_{k/h_i}$	$A_{v_i}$ (ft <sup>2</sup> )
Pilot	5	1.0	5
Engine I	30	.8	24
Engine II	30	.8	24
Fuel Tank	40	.6	24
Controls	25	.6	15

The logical kill expression for this aircraft is

A/C KILL = PILOT.OR.FUEL.OR.CONTROLS.OR.(ENGINEI.AND.  
ENGINEII)

A/C #4 (77% REDUNDANT)  $A_p = 530 \text{ ft}^2$

CRITICAL COMPONENT	$A_{p_i}$ (ft <sup>2</sup> )	$P_{k/h_i}$	$A_{v_i}$ (ft <sup>2</sup> )
Pilot	5	1.0	5
Engine I	30	.8	24
Engine II	30	.8	24
Fuel Tank I	20	.5	10
Fuel Tank II	20	.5	10
Controls	25	.6	15

The logical expression for A/C #4 is

A/C KILL = PILOT.OR.CONTROLS.OR.(ENGINEI.AND.ENGINEII).  
OR.(FUELI.AND.FUELII)

TABLE 1

CUMULATIVE PROBABILITIES OF KILL FOR EXAMPLE AIRCRAFT

	A/C #1	A/C #2	A/C #3	A/C #4
$P_k^{-(1)}$ =	.1360	.1060	.0830	.0377
$P_k^{-(2)}$ =	.2535	.2012	.1632	.0788
$P_k^{-(3)}$ =	.3550	.2866	.2397	.1220
$P_k^{-(4)}$ =	.4227	.3633	.3117	.1662
$P_k^{-(5)}$ =	.5185	.4320	.3788	.2107
$P_k^{-(6)}$ =	.5840	.4934	.4411	.2548
$P_k^{-(7)}$ =	.6406	.5485	.4984	.2981
$P_k^{-(8)}$ =	.6895	.5978	.5508	.3403
$P_k^{-(9)}$ =	.7317	.6418	.5986	.3809
$P_k^{-(10)}$ =	.7682	.6811	.6419	.4199

Figure 5 compares the  $\bar{P}_k$  for the nonredundant aircraft #1 with redundant aircraft #2, 3 and 4.

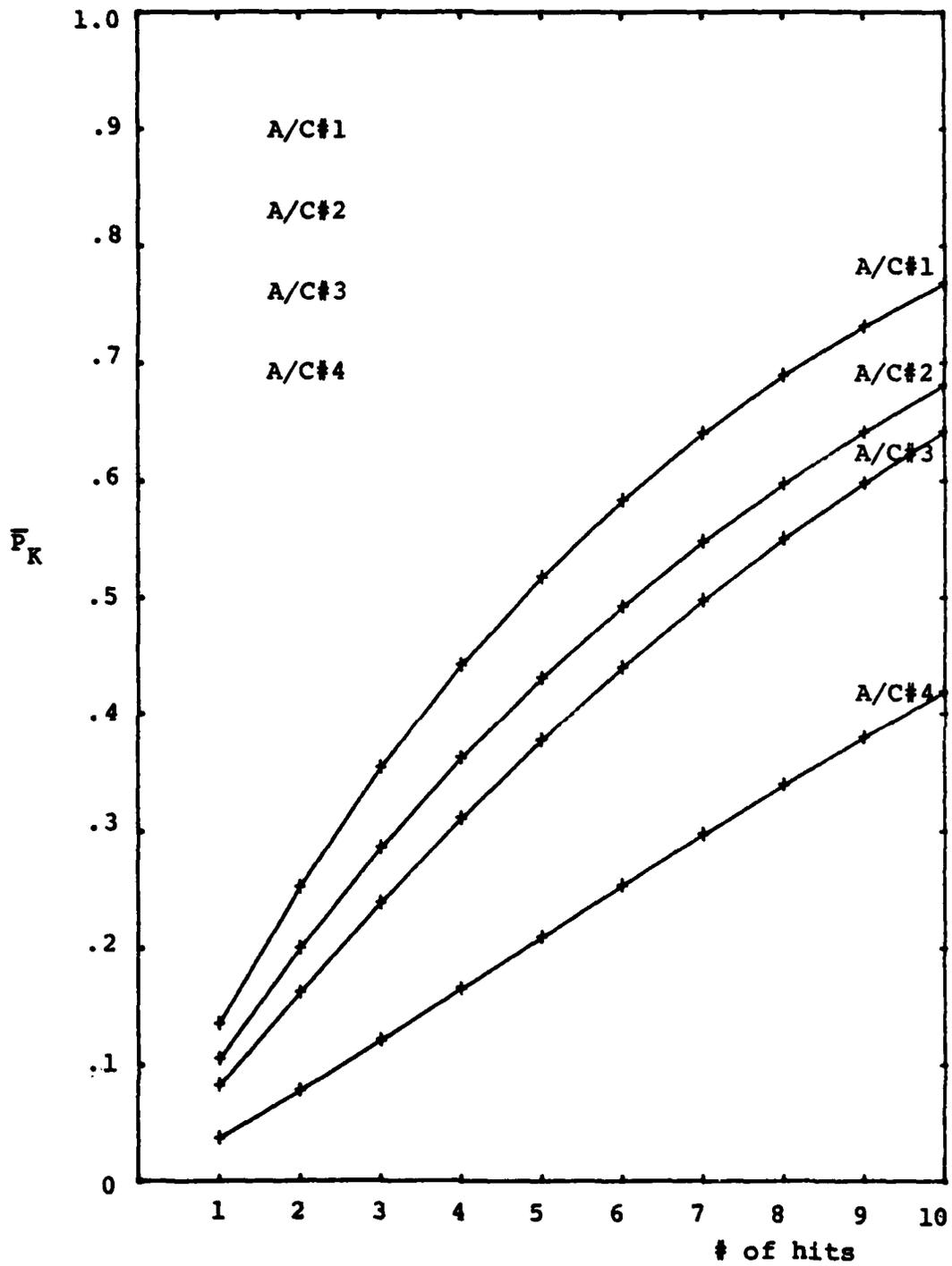


FIGURE 5. CUMULATIVE PROBABILITY OF KILL VERSUS NUMBER OF HITS

Note that the values of  $A_{P_i}$  and  $P_{k/h_i}$  are assumed, while the values of  $A_{V_i}$  are computed through the equation

$$A_{V_i} = A_{P_i} * P_{k/h_i} \quad (32)$$

Through the use of the transition matrix method (see Appendix B), the values of  $\bar{P}_k^{(n)}$  for the example aircraft have been found and are presented in Table 1.

As can be very easily seen from Table 1, the  $\bar{P}_k$  for the same shot decreases, as the redundancy of the example aircraft increases.

## V. AFATL POOL COMPUTER PROGRAM AND THE $P_{K/H}$

As shown by the previous chapters, the determination of the survivability of an aircraft can be a very complex and time consuming task. Thus, the aid of the computer has been employed, and several programs have been developed by both the military and the civilian industry. One that was selected for use by the Joint Technical Coordinating Group for Aircraft Survivability (JTCG/AS) Survivability Assessment Subgroup, is the Antiaircraft Artillery Simulation Computer Program, AFATL Program POOL. This program is currently in use at the Naval Postgraduate School and is the one that this study will be concerned with.

Many modifications have been made to the POOL that was originally developed by the Weapons Systems Analysis Division of the Air Force Armament Laboratory at Eglin Air Force Base, Florida. The latest version of it used by Naval Weapons Center, China Lake, employs an aircraft consisting of twenty (20) critical components and computes the single shot probability of kill of a target (after consideration is given to various errors in predicting an aircraft/projectile intercept point). Computation of the  $P_{KSS}$  is performed throughout the entire flight path and after the accumulation of all shots, the total probability of target

attrition is presented. In the computation of the cumulative  $\bar{P}_K$ , redundancy of the critical components has not been taken into consideration. This is a potential source of error in the final results, since the majority of the present aircraft have some degree of redundancy.

In the latest version of POOL, the computation of the single shot probability of kill is performed in the "FIRE" section of the program. The equation used to compute the probability of killing, at least one critical component by a single round,  $P_K$ , is presented on line 717 of the program listing given in Appendix C, and it states

$$\bar{P}_K = \bar{P}_K + P_{KS(IJ)} - \bar{P}_K * P_{KS(IJ)} = \bar{P}_K + P_{KS(IJ)} * (1 - \bar{P}_K)$$

where the initial value of  $P_K$  is set to zero and  $P_{KS(IJ)}$  is the probability of killing component IJ by a single round. This equation does not account for redundancy because component IJ might be redundant in the aircraft and killing it will not constitute an aircraft kill, as it was discussed in Chapter 3. Thus, in order for the program to account for possible redundancy of components, the following changes and additions to POOL are recommended.

Define:

IJ1 = the total number of non-redundant components

IJ22 = the total number of sets of two redundant components  
of which two must be killed for a subsystem kill

IJ23 = the total number of sets of three redundant components  
of which two must be killed for a subsystem kill

IJ33 = the total number of sets of three redundant components  
of which three must be killed for a subsystem kill

IJ24 = the total number of sets of four redundant components  
of which two must be killed for a subsystem kill

IJ34 = the total number of sets of four redundant components  
of which three must be killed for a subsystem kill

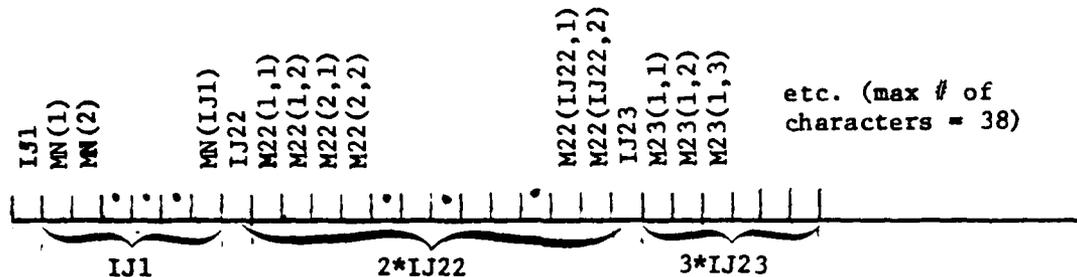
IJ44 = the total number of sets of four redundant components  
of which four must be killed for a subsystem kill

Thus,  $IJ = IJ1 + 2*(IJ22) + 3*(IJ23 + IJ33) + 4*(IJ24 + IJ34 + IJ44) \leq 20$

Define:

- MN(I) = A column matrix containing the numbers of the non-redundant components (I=1,IJ1)
- M22(I,J) = A (IJ22x2) matrix containing the IJ22 pairs of redundant components (I=1,IJ22, J=1,2)
- M23(I,J) = A (IJ23x3) matrix containing the IJ23 sets of three redundant components (I=1,IJ23, J=1,3)
- M33(I,J) = A (IJ33x3) matrix containing the IJ33 sets of three redundant components (I=1,IJ33, J=1,3)
- M24(I,J) = A (IJ24x4) matrix containing the IJ24 sets of four redundant components (I=1,IJ24, J=1,4)
- M34(I,J) = A (IJ34x4) matrix containing the IJ34 sets of four redundant components (I=1,IJ34, J=1,4)
- M44(I,J) = A (IJ44x4) matrix containing the IJ44 sets of four redundant components (I=1,IJ44, J=1,4)

The user must create a file containing the number of the non-redundant components, then redundant components themselves, the number of the IJ22,IJ23,IJ33,IJ24,IJ34,IJ44 sets, and the numbers of the components that belong to each set as shown below:



Once the file is constructed, the MN,M22,M23,M33,M24,M34,M44 matrices can be read, with the following routine:

```

READ(2,*) (TEMP(I), I=1, 38)

I=1

IJ1=TEMP(I)

IF IJ1=0    GO TO 6

DO 5 K=1,IJ1

    MN(K)=TEMP(I+K)

5  CONT

6  I=1+IJ1+1

    IJ22=TEMP(I)

    IF IJ22=0    GO TO 10

```

```

DO 15 K=1, IJ22
    M22(K,1)=TEMP(I+K)
    M22(K,2)=TEMP(I+K+1)
    I=I+2
15 CONT
10 I=2+IJ1+2*IJ22+1
    IJ23=TEMP(I)
    IF IJ23=0 GO TO 20
    DO 25 K=1,IJ23
        M23(K,1)=TEMP(I+K)
        M23(K,2)=TEMP(I+K+1)
        M23(K,3)=TEMP(I+K+2)
        I=I+3
25 CONT
20 I=3+IJ1+2*IJ22+3*IJ23+1
    IJ33=TEMP(I)
    IF IJ33 = 0 GO TO 30
    DO 35 K=1,IJ33
        M33(K,1)=TEMP(I+K)
        M33(K,2)=TEMP(I+K+1)
        M33(K,3)=TEMP(I+K+2)
        I=I+3
35 CONT

```

```

30 I=4+IJ1+2*IJ22+3*IJ23+3*IJ33+1
   IJ24=TEMP(I)
   IF IJ24=0 GO TO 40
   DO 45 K=1,IJ24
       M24(K,1)=TEMP(I+K)
       M24(K,2)=TEMP(I+K+1)
       M24(K,3)=TEMP(I+K+2)
       M24(K,4)=TEMP(I+K+3)
       I=I+4
45 CONT
40 I=5+IJ1+2*IJ22+3*IJ23+3*IJ33+4*IJ24+1
   IJ34=TEMP(I)
   IF IJ34=0 GO TO 50
   DO 55 K=1,IJ34
       M34(K,1)=TEMP(I+K)
       M34(K,2)=TEMP(I+K+1)
       M34(K,3)=TEMP(I+K+2)
       M34(K,4)=TEMP(I+K+3)
       I=I+4
55 CONT
50 I=6+IJ1+2*IJ22+3*IJ23+3*IJ33+4*IJ24+4*IJ34+1
   IJ44=TEMP(I)
   IF IJ44=0 GO TO 60

```

```
DO 65 K=1,IJ44
    M44(K,1)=TEMP(I+K)
    M44(K,2)=TEMP(I+K+1)
    M44(K,3)=TEMP(I+K+2)
    M44(K,4)=TEMP(I+K+3)
    I=I+4
60 STOP
END
```

Once the matrices have been constructed, the following subroutine could be used to compute the cumulative probability of aircraft survival. The cumulative probability of kill of each of the components (CPKS(IJ)) is computed at line 722. This probability is computed for all of the components in the DO to 460 loop. Immediately after the 460 loop (before line 72) a call to the following subroutine should be made. The parameter computed by this subroutine is the probability the aircraft has survived the encounter up to the current time, PS , the cumulative probability of survival. Note that the PK and PS computed at lines 717, 727, and 732 are no longer valid when redundant components are introduced.

PS=1.

IF IJ1=0 GO TO 10

DO 5 K=1,IJ1

L=MN(K)

PS=PS\*(1.-CPKS(L))

CPKS is the cumulative probability of survival of the component

5 CONTINUE

10 IF IJ22=0 GO TO 20

DO 15 K=1,IJ22

L=M22(K,1)

M=M22(K,2)

PS=PS\*(1.-CPKS(L)\*CPKS(M))

15 CONTINUE

20 IF IJ23=0 GO TO 30

DO 25 K=1,IJ23

L=M23(K,1)

M=M23(K,2)

N=M23(K,3)

PS=PS\*(1.-CPKS(L)\*CPKS(M)-CPKS(L)\*CPKS(N)

-CPKS(M)\*CPKS(N)+CPKS(L)\*CPKS(M)\*CPKS(N))

25 CONTINUE

```

30 IF IJ33=0 GO TO 40
DO 35 K=1,IJ33
    L=M33(K,1)
    M=M33(K,2)
    N=M33(K,3)
    PS=PS*(1.-CPKS(L)*CPKS(M)*CPKS(N))

35 CONTINUE
40 IF IJ24=0 GO TO 50
DO 45 K=1,IJ24
    L=M24(K,1)
    M=M24(K,2)
    N=M24(K,3)
    I=M24(K,4)
    PS=PS*(1.-CPKS(L)*CPKS(M)-CPKS(L)*CPKS(N)-CPKS(L)
*       *CPKS(I)-CPKS(M)*CPKS(N)-CPKS(M)*CPKS(I)
*       -CPKS(N)*CPKS(I)+CPKS(L)*CPKS(M)*CPKS(N)
*       +CPKS(L)*CPKS(M)*CPKS(I)+CPKS(L)*CPKS(N)
*       *CPKS(I)+CPKS(M)*CPKS(N)*CPKS(I)+CPKS(L)*CPKS(M)
*       *CPKS(N)*CPKS(I))

45 CONTINUE
50 IF IJ34=0 GO TO 60
DO 55 K=1,IJ34
    L=M34(K,1)
    M=M34(K,2)
    N=M34(K,3)
    I=M34(K,4)

```

```
PS=PS*(1.-CPKS(L)*CPKS(M)*CPKS(N)-CPKS(L)*CPKS(M)
*CPKS(I)-CPKS(L)*CPKS(N)*CPKS(I)-CPKS(M)
*CPKS(N)*CPKS(I)+CPKS(L)*CPKS(M)*CPKS(N)
*CPKS(I))
```

```
55 CONTINUE
```

```
60 IF IJ44=0 GO TO 70
```

```
DO 65 K=1,IJ44
```

```
L=M44(K,1)
```

```
M=M44(K,2)
```

```
N=M44(K,3)
```

```
I=M44(K,4)
```

```
PS=PS*(1.-CPKS(L)*CPKS(M)*CPKS(N)*CPKS(I))
```

```
65 CONTINUE
```

```
70 WRITE (6,100) PS
```

```
100 FORMAT ('1, 'THE CUMULATIVE PROBABILITY OF SURVIVAL IS,  
F9.7) .
```

```
RETURN
```

```
END
```

## VI. CONCLUSIONS

The probability of kill of an aircraft given a single shot is a linear function of the aircraft's vulnerable area. Redundancy of critical components greatly enhances the probability of survival and in the Cookie Cutter approach the kills of redundant critical components are mutually exclusive events, where as in the Carleton approach they are not. For an aircraft, hit by multiple shots the cumulative probability of kill for the same shot decreases as the redundancy built into the aircraft increases. The modifications proposed in Chapter V should provide a significant improvement in the overall efficiency of the Antiaircraft Artillery Simulation Computer Program, since it will provide for aircraft redundancy.

APPENDIX A

EVALUATION OF EQUATIONS

(1) Evaluation of equation (12).

$$P_{KSS} = \int_0^{r_v} (1) * \frac{r}{\sigma} \exp\left(-\frac{r^2}{2\sigma^2}\right) dr \quad (1)$$

Using integration by parts, let  $-\frac{r^2}{2\sigma^2} = u$ .

Differentiating results in  $-\frac{2r}{2\sigma^2} dr = du$

Therefore

$$P_{KSS} = \int -\exp(u) du = -\exp\left(-\frac{r^2}{2\sigma^2}\right) \Big|_0^{r_v} =$$
$$-\exp\left(-\frac{r_v^2}{2\sigma^2}\right) - (-1) = 1 - \exp\left(-\frac{r_v^2}{2\sigma^2}\right)$$

$$\text{Since } A_v = \pi r_v^2 \rightarrow P_{KSS} = 1 - \exp\left(-\frac{A_v}{2\pi\sigma^2}\right)$$

(2) Evaluation of equation (13).

$$P_{KSS} = \int_0^{\infty} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) * \exp\left(-\frac{r^2}{r_v^2}\right) dr$$
$$= \int_0^{\infty} \frac{r}{\sigma^2} \exp\left[-r^2 * \left(\frac{1}{2\sigma^2} + \frac{1}{r_v^2}\right)\right] dr$$

Using integration by parts, let  $-r^2 \left( \frac{1}{2\sigma^2} + \frac{1}{r_v^2} \right) = u$ .

Differentiation yields  $-2r \left( \frac{1}{2\sigma^2} + \frac{1}{r_v^2} \right) dr = du$

therefore

$$P_{KSS} = \frac{1}{2\sigma^2 \left( \frac{1}{2\sigma^2} + \frac{1}{r_v^2} \right)} \int_0^{r_v} \exp(u) du =$$

or

$$\begin{aligned} P_{KSS} &= \frac{1}{2\sigma^2 \left( \frac{1}{2\sigma^2} + \frac{1}{r_v^2} \right)} (-\exp u) \Big|_0^{r_v} \\ &= \frac{1}{2\sigma^2 \left( \frac{1}{2\sigma^2} + \frac{1}{r_v^2} \right)} * \left[ -\exp \left[ -r^2 \left( \frac{1}{2\sigma^2} + \frac{1}{r_v^2} \right) \right] \right] \Big|_0^{r_v} \\ &= \frac{1}{1 + \frac{2\sigma^2}{r_v^2}} * [-0 + 1] = \frac{r_v^2}{r_v^2 + 2\sigma^2} \quad \text{and for} \end{aligned}$$

$$A_v = \pi r_v^2$$

$$= \frac{A_v/2\pi}{\frac{A_v}{2\pi} + 2\sigma^2}$$

(3) Evaluation of equation (14).

$$P_{KSS} = \int_{-\infty}^{\infty} P_H(x,y) * P_{K/H}(x,y) dx dy$$

$$P_H = \frac{1}{2\pi\sigma_{x_2}\sigma_{y_2}} \exp \left\{ -\frac{1}{2} \left[ \frac{(x-\mu_x)^2}{\sigma_{x_2}^2} + \frac{(y-\mu_y)^2}{\sigma_{y_2}^2} \right] \right\}$$

$$P_{K/H} = \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x}{\sigma_{x_1}} \right)^2 + \left( \frac{y}{\sigma_{y_1}} \right)^2 \right] \right\}$$

Thus

$$P_{KSS} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_{x_2}\sigma_{y_2}} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x-\mu_x}{\sigma_{x_2}} \right)^2 + \left( \frac{y-\mu_y}{\sigma_{y_2}} \right)^2 \right] \right\} \\ \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x}{\sigma_{x_1}} \right)^2 + \left( \frac{y}{\sigma_{y_1}} \right)^2 \right] \right\}$$

Expanding and combining terms yields,

$$P_{KSS} = \frac{\exp \left\{ -\frac{1}{2} \left[ \left( \frac{\mu_x}{\sigma_{x_2}} \right)^2 + \left( \frac{\mu_y}{\sigma_{y_2}} \right)^2 \right] \right\}}{2\pi\sigma_{x_2}\sigma_{y_2}}$$

$$* \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right) x^2 - \left( \frac{2\mu_x}{\sigma_{x_2}^2} \right) x \right. \right.$$

$$+ \left( \frac{1}{\sigma_{Y_1}^2} + \frac{1}{\sigma_{Y_2}^2} \right) y^2 - \left( \frac{2\mu_Y}{\sigma_{Y_2}^2} \right) y \Big] dx dy$$

Separating variables and making substitutions yields,

$$P_{KSS} = \frac{\exp \left\{ -\frac{1}{2} \left[ \left( \frac{\mu_X}{\sigma_{X_2}} \right)^2 + \left( \frac{\mu_Y}{\sigma_{Y_2}} \right)^2 \right] \right\}}{2\pi\sigma_{X_2}\sigma_{Y_2}}$$

$$* \int_{-\infty}^{\infty} \exp[-p_x^2 x^2 + q_x x] dx \int_{-\infty}^{\infty} \exp[-p_y^2 y^2 + q_y y] dy \quad (2)$$

where

$$p_x^2 = \frac{1}{2} \left( \frac{1}{\sigma_{X_1}^2} + \frac{1}{\sigma_{X_2}^2} \right)$$

$$p_y^2 = \frac{1}{2} \left( \frac{1}{\sigma_{Y_1}^2} + \frac{1}{\sigma_{Y_2}^2} \right)$$

$$q_x = \frac{\mu_X}{\sigma_{Y_2}}$$

$$q_y = \frac{\mu_Y}{\sigma_{Y_2}}$$

Integrating each integrand  $I_x$  and  $I_y$  yields

$$\begin{aligned}
 I_x &= \int_{-\infty}^{\infty} \exp\{-p_x^2 x^2 + q_x x\} dx = \int_{-\infty}^{\infty} \exp\left\{-\left(p_x^2 x^2 - q_x x\right.\right. \\
 &\quad \left.\left. + \frac{q_x^2}{4p_x^2} - \frac{q_x^2}{4p_x^2}\right\} dx \\
 &= \int_{-\infty}^{\infty} \exp\left\{-\left(p_x x - \frac{q_x}{2p_x}\right)^2 + \frac{q_x^2}{4p_x^2}\right\} dx \\
 &= \exp\left(\frac{q_x^2}{4p_x^2}\right) \int_{-\infty}^{\infty} \exp\left\{-\left(p_x x - \frac{q_x}{2p_x}\right)^2\right\} dx
 \end{aligned}$$

Let  $u = p_x x - \frac{q_x}{2p_x}$  then  $du = p_x dx$

$$\text{then } I_x = \exp\left(\frac{q_x^2}{4p_x^2}\right) * \frac{1}{p_x} \int_{-\infty}^{\infty} \exp(-u^2) du$$

But in general,  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2} z^2\right\} dz = 1$  or

$$\int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2} z^2\right\} dz = \sqrt{2\pi}$$

Let  $u^2 = \frac{1}{2} z^2$  then  $du = \frac{1}{\sqrt{2}} dz$  and

$$\sqrt{2} \int_{-\infty}^{\infty} \exp \{-u^2\} du = \sqrt{2\pi} \quad \text{or}$$

$$\int_{-\infty}^{\infty} \exp \{-u^2\} du = \sqrt{\pi}$$

therefore  $I_x = \exp \frac{\mu_x^2}{4\sigma_x^2} * \frac{1}{\sigma_x} * \sqrt{\pi}$  or

$$I_x = \frac{\sqrt{\pi}}{\sqrt{\frac{1}{2} \left( \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right)}} \exp \left\{ \frac{\frac{\mu_x^2}{\sigma_{x_2}^2}}{4 \left[ \frac{1}{2} \left( \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right) \right]} \right\}$$

$$I_x = \frac{\sqrt{\pi}}{\sqrt{\frac{1}{2} \left( \frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2} \right)}} \exp \left\{ \frac{\mu_x^2}{2\sigma_{x_2}^2 \left( \frac{\sigma_{x_2}^2 + \sigma_{x_1}^2}{\sigma_{x_1}^2 \sigma_{x_2}^2} \right)} \right\} \quad \text{or}$$

$$I_x = \frac{\sqrt{2\pi} \sigma_{x_1} \sigma_{x_2}}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}} \exp \left\{ \frac{\mu_x^2 \sigma_{x_1}^2}{2\sigma_{x_2}^2 (\sigma_{x_1}^2 + \sigma_{x_2}^2)} \right\}$$

similarly

$$I_Y = \frac{\sqrt{2\pi} \sigma_{Y_1} \sigma_{Y_2}}{\sqrt{\sigma_{Y_1}^2 + \sigma_{Y_2}^2}} \exp \left\{ \frac{\mu_Y^2 \sigma_{Y_1}^2}{2\sigma_{Y_2}^2 (\sigma_{Y_1}^2 + \sigma_{Y_2}^2)} \right\}$$

Substituting  $I_X$  and  $I_Y$  into equation (2) yields

$$P_{KSS} = \frac{\exp \left\{ -\frac{1}{2} \left[ \left( \frac{\mu_X}{\sigma_{X_2}} \right)^2 + \left( \frac{\mu_Y}{\sigma_{Y_2}} \right)^2 \right] \right\}}{\cancel{2\pi\sigma_{X_2}\sigma_{Y_2}}} * \frac{\sqrt{2\pi} \sigma_{X_1} \sigma_{X_2}}{\sqrt{\sigma_{X_1}^2 + \sigma_{X_2}^2}}$$

$$* \frac{\sqrt{2\pi} \sigma_{Y_1} \sigma_{Y_2}}{\sqrt{\sigma_{Y_1}^2 + \sigma_{Y_2}^2}} * \exp \left\{ \frac{\mu_X^2 \sigma_{X_1}^2}{2\sigma_{X_2}^2 (\sigma_{X_1}^2 + \sigma_{X_2}^2)} \right\}$$

$$* \exp \left\{ \frac{\mu_Y^2 \sigma_{Y_1}^2}{2\sigma_{Y_2}^2 (\sigma_{Y_1}^2 + \sigma_{Y_2}^2)} \right\}$$

$$\text{or } P_{KSS} = \frac{\sigma_{X_1} \sigma_{Y_1}}{\left( \sqrt{\sigma_{X_1}^2 + \sigma_{X_2}^2} \right) * \left( \sqrt{\sigma_{Y_1}^2 + \sigma_{Y_2}^2} \right)}$$

$$\exp \left\{ -\frac{1}{2} \left[ \frac{\mu_X^2}{\sigma_{X_1}^2} - \frac{\mu_X^2 \sigma_{X_1}^2}{(\sigma_{X_1}^2 + \sigma_{X_2}^2) \sigma_{X_2}^2} \right] \right\}$$

$$* \exp \left\{ -\frac{1}{2} \left[ \frac{\mu_Y^2}{\sigma_{Y_2}^2} - \frac{\mu_Y^2 \sigma_{Y_1}^2}{(\sigma_{Y_1}^2 + \sigma_{Y_2}^2) \sigma_{Y_2}^2} \right] \right\}$$

or

$$P_{KSS} = \frac{\sigma_{x_1} \sigma_{y_1}}{(\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2})(\sqrt{\sigma_{y_1}^2 + \sigma_{y_2}^2})} * \exp \left\{ -\frac{1}{2} \left[ \frac{\mu_x^2}{\sigma_{x_1}^2 + \sigma_{x_2}^2} + \frac{\mu_y^2}{\sigma_{y_1}^2 + \sigma_{y_2}^2} \right] \right\}$$

If we assume that  $\sigma_{x_1} = \sigma_{y_1}$  and  $A_V = 2\pi\sigma_{x_1}\sigma_{y_1}$  the  $P_{KSS}$  becomes

$$P_{KSS} = \frac{A_V/2\pi}{\sqrt{\sigma_{x_2}^2 + \frac{A_V}{2\pi}} \sqrt{\sigma_{y_2}^2 + \frac{A_V}{2\pi}}} * \exp \left\{ -\frac{1}{2} \left[ \frac{\mu_x^2}{\sigma_{x_2}^2 + \frac{A_V}{2\pi}} + \frac{\mu_y^2}{\sigma_{y_2}^2 + \frac{A_V}{2\pi}} \right] \right\}$$

or

$$P_{KSS} = \frac{A_V/2\pi}{\sqrt{\sigma_x^2 + \frac{A_V}{2\pi}} \sqrt{\sigma_y^2 + \frac{A_V}{2\pi}}}$$

$$* \exp \left\{ -\frac{1}{2} \left[ \frac{\mu_x^2}{\sigma_x^2 + \frac{A_V}{2\pi}} + \frac{\mu_y^2}{\sigma_y^2 + \frac{A_V}{2\pi}} \right] \right\}$$

APPENDIX B

P<sub>K</sub> CALCULATIONS THROUGH THE TRANSITION MATRIX METHOD

TABLE 2

Computation of the Transition Matrix (T) for Example A/C #3  
Probability of Transitioning from this State to this State

	Knrc	Krc1	Krc2	Krc	Nk	
T <sub>3</sub> =	1	$\frac{5+24+15}{530}$	$\frac{5+24+15}{530}$	0	$\frac{5+24+15}{530}$	Knrc
	0	$\frac{24+438}{530}$	0	0	24/530	Krc1
	0	0	$\frac{24+438}{530}$	0	24/530	Krc2
	0	24/530	24/530	1	0	Krc
	0	0	0	0	438/530	Nk

(Note that the sum of each column is unity.)

Now, let the probability that the aircraft exists in each of of the five possible states after the jth hit be expressed by a vector  $\{S\}^{(j)}$ , where

$$\{S^{(j)}\} = \begin{Bmatrix} Knrc \\ Krc1 \\ Krc2 \\ Krc \\ Nk \end{Bmatrix}^{(j)}$$

The probability that the aircraft is in each of the five states after the (j+1)th hit is

$$\{s\}^{(j+1)} = [T]\{s\}^{(j)} \quad (B-1)$$

that is, the aircraft transitions from  $\{s\}^{(j)}$  to  $\{s\}^{(j+1)}$  to [T]. Thus, the cumulative probability the aircraft is killed after j+1 hit,  $\bar{P}_K^{(j+1)}$ , is given by

$$\bar{P}_K^{(j+1)} = K_{nrc}^{(j+1)} + K_{rc}^{(j+1)}$$

Prior to the first hit, the aircraft is entirely in the  $N_k$  state. Thus, according to equation (B-1),

$$\{s\}^{(1)} = [T]\{s\}^{(0)} = [T] \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \begin{matrix} (0) \\ \\ \\ \\ \leftarrow N_k \end{matrix}$$

and hence,

$$\{s\}^{(1)} = \begin{pmatrix} 44/530 \\ 24/530 \\ 24/530 \\ 0 \\ 438/530 \end{pmatrix} \begin{matrix} (1) \\ \\ \\ \\ \end{matrix} = \begin{pmatrix} .0830 \\ .0452 \\ .0452 \\ 0 \\ .8264 \end{pmatrix} \begin{matrix} (1) \leftarrow K_{nrc} \\ \\ \\ \leftarrow K_{rc} \end{matrix}$$

Thus  $\bar{P}_{K_1}^{(1)} = .0830$ .

Similarly, for the second hit,

(1)

$$\{s\}^2 = [T]\{s\}^1 = [T] \begin{Bmatrix} .0830 \\ .0452 \\ .0452 \\ 0 \\ .8264 \end{Bmatrix}$$

$$\{s\}^{(2)} = \frac{1}{530^2} \begin{Bmatrix} 530*44+44*24+44*24+44*438 \\ 462*24+24*438 \\ 462*24+24*438 \\ 24*24+24*24 \\ 438*438 \end{Bmatrix}$$

$$\{s\}^{(2)} = \begin{Bmatrix} .1591 \\ .0769 \\ .0769 \\ .0041 \\ .6829 \end{Bmatrix} \begin{matrix} \leftarrow \text{Knrc} \\ \\ \\ \leftarrow \text{Krc} \end{matrix}$$

Thus, after the second hit

$$\bar{P}_K^{(2)} = .1591 + .0041$$

Similarly, the values for  $\bar{P}_K^{(3)}$ ,  $\bar{P}_K^{(4)}$ , etc., are obtained to yield Table 1.

APPENDIX C

P001 'FIRE' SECTION

703 C  
704 C COMPUTE SURVIVAL AND KILL PROBABILITIES  
705 C  
706 STUFF=DIVIDE(BXF2,(SXF2+AVTPI))+DIVIDE  
(BYF2,(SYF2+AVTPI))  
707 C  
708 CC PKS(IJ) IS THE PROBABILITY OF KILLING COMPONENT IJ  
BY A SINGLE ROUND  
709 CC (ONE BARREL)  
710 C  
711 PKS(IJ)=AMIN1(1.0, DIVIDE(EXP(-.5\*STUFF)\*AVTPI,  
712 SQRT((SXF2+1AVTPI)\*(SYF2+AVTPI))))  
713 C  
714 CC PK IS THE PROBABILITY OF KILLING AT LEAST ONE  
COMPONENT BY A SINGLE  
715 CC ROUND (ONE BARREL)  
716 C  
717 PK=PK+PKS(IJ)-PK\*PKS(IJ)  
718 C  
719 CC CPKS(IJ) IS THE CUMULATIVE PROBABILITY OF KILLING  
COMPONENT IJ

720 CC BY THIS GUN LOCATION FOR THE ENTIRE AIRCRAFT FLIGHT  
PATH

721 C

722 460  $CPKS(IJ) = CPKS(IJ) + (1 - (1 - PKS(IJ))^{**ISB}) - (CPKS(IJ) * (1 - (1 - PKS(IJ))^{**ISB}))$

724 C

725 CC PS IS THE PROBABILITY OF ALL COMPONENTS SURVIVING  
ISB BARRELS FIRING

726 C

727  $PS = (1.0 - PK)^{**ISB}$

728 C

729 CC PK IS THE PROBABILITY OF KILLING AT LEAST ONE  
COMPONENT BY ISB

730 CC BARRELS FIRING

731 C

732  $PK = 1.0 - PS$

733  $TI = TIME + T$

734 C

LIST OF REFERENCES

1. Ball, R. E., The Fundamentals of Aircraft Combat Survivability Analysis and Design, Naval Postgraduate School, 1983.
2. Ferrell, D. R., A Review and Analysis of Aircraft Vulnerability, M. S. Thesis, Naval Postgraduate School, 1983.

INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Technical Information Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0142 Naval Postgraduate School Monterey, California 93943	2
3. Department Chairman, Code 67 Department of Aeronautics Naval Postgraduate School Monterey, California 93943	1
4. Professor Robert E. Ball, Code 67Bp Department of Aeronautics Naval Postgraduate School Monterey, California 93943	1
5. LCDR Angelo J. Kangalos, USN USS Kitty Hawk, CV-63 FPO San Francisco, California 96606	2
6. Commander Naval Weapons Center Attn: Mr. John Morrow, Code 338 China Lake, California 93555	1
7. Mr. Dale Atkinson Air-5164 Naval Air Systems Command Washington, DC 20361	1

