AD-A141 800 A STABLE EXPLICIT SCHEME FOR THE OCEAN ACOUSTIC WAVE EQUATION(U) YALE UNIV NEW HAVEN CT DEPT OF COMPUTER SCIENCE T F CHAN ET AL. 1984 YALEU/DC/TR-315 1/1

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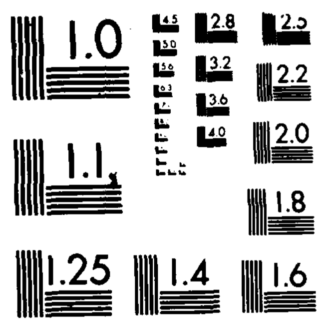

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Technical Report YALEU/DC/TR-315

This work was supported in part by the Department of Energy under contract DE-AC02-81ER10996, by Army Research Office under contract DAAG-83-0177, by Office of Naval Research Grant N00014-82-K-0184, and by Naval Underwater Systems Center Independent Research Project A65020 DAAG29-83-K-0177

*This work was performed while the author was a visiting scholar from Beijing University, China during 1983-1985

ABSTRACT

A class of ocean acoustic wave propagation problems is represented by a parabolic equation of the Schrodinger type. Using conventional explicit finite difference schemes, e.g., the Euler scheme, to solve the parabolic wave equation is unstable. Thus, important advantages of explicit schemes are completely missing. This paper presents a conditionally stable explicit scheme by introducing an extra dissipative term. This new explicit scheme is then applied to solve the ocean acoustic parabolic wave equation fully utilizing the advantages of explicit schemes. The theoretical development, the computational aspects, and the advantages are discussed. Application of the scheme to a realistic ocean acoustic problem is included. The solution obtained is compared with the unconditionally stable Crank-Nicolson solution.

INTRODUCTION

A parabolic equation of the Schrodinger type arises in the application of ocean acoustic wave propagation. In the published literatures, this ocean acoustic parabolic wave equation has been solved by three different methods: the Split-step Fourier algorithm[1], the numerical ordinary-differential-equation method[2,3,4], and an implicit finite-difference method[5,6]. The implicit finite-difference method is, by far, the most general purpose, stable method for solving the parabolic wave equation; in addition, it has a variety of useful capabilities. However, when 3-dimensional, as well as high frequency problems arise, one needs a more effective method to ease the requirement of memory storage, to gain computational speed, and to be implemented easily into modern pipe-line computers. It is known that explicit schemes have these advantages. Way back to 1978, Lee and Papadakis[2] analyzed applicable explicit schemes for such application and found that the explicit scheme such as the Euler scheme was unstable. It is the main result of this paper to introduce a stable explicit scheme, newly developed by Chan, Lee, and Shen[7] for solving the parabolic equation of the Schrodinger type. This new explicit scheme is developed to be conditionally stable by adding a dissipative term, and possesses most advantages any explicit scheme must have. Prior to the discussion of the formulation and the stability of the new explicit scheme, two sections are presented. One is the introduction of the problem background, the other is the discussion of the solution background. An application to a realistic ocean acoustic problem is presented along with a comparison with the solution obtained by the Crank-Nicolson method on accuracy and speed.

PROBLEM BACKGROUND

In order to simplify complicated ocean acoustic wave propagation problems, Tappert[8] introduced the Parabolic Equation approximation (PE) to solve the elliptic wave equation in 2 dimensions by a parabolic equation involving the depth variable z and the range variable r. This PE approximation produces the parabolic wave equation in the form

$$u_r = \frac{i}{2} k_0 (n^2(r,z)-1)\, u + \frac{i}{2k_0} u_{zz} \tag{1}$$

where $u(r,z)$ is the pressure field, k_0 is a reference wave number, and the $n(r,z)$ is the index of refraction which is equal to $c_0/c(r,z)$ (reference sound speed/sound speed). Eq. (1) is regarded as the standard parabolic wave equation first introduced by Tappert[8].

The parabolic wave equation (1) can be obtained in a couple of different ways. We give an outline of our development below.

The elliptic wave equation in cylindrical coordinates takes the form

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} + k_0^2\, n^2(r,z)\, \phi = 0 \tag{2}$$

where $\phi(r,z)$ is the acoustic pressure field.

Expressing

$$\phi(r,z) = u(r,z)\, v(r) \tag{3}$$

where v(r) has strong dependence on the range variable r while u(r,z) is weakly dependent on r. We shall derive that the u(r,z) here satisfied Eq. (1).

Substituting expression (3) into Eq. (2), we obtain

$$\left[v_{rr} + \frac{1}{r} v_r\right] u + \left[u_{rr} + \left(\frac{1}{r} + \frac{2}{v} v_r\right) u_r + u_{zz} + k_0^2 n^2(r,z) u\right] v = 0 \qquad (4)$$

Setting the terms in the first [ ] of Eq. (4) equal to "$-k_0^2 v$" and setting the terms in the second [ ] of Eq. (4) to "$k_0^2 u$," we find

$$v_{rr} + \frac{1}{r} v_r + k_0^2 v = 0 \quad , \qquad (5)$$

and

$$u_{rr} + \left(\frac{1}{r} + \frac{2}{v} v_r\right) u_r + u_{zz} + k_0^2(n^2(r,z)-1)u = 0 \qquad (6)$$

Considering only the outgoing wave in the range direction, it is easily seen that the solution of Eq. (5) is the zeroth order Hankel function of the first kind. If we apply the far-field approximation, $k_0 r >> 1$, to the argument of the Hankel function, we obtain

$$v(r) = H_0^{(1)} (k_0 r) \approx \sqrt{\frac{2}{\pi k_0 r}}\, e^{i(k_0 r - \frac{\pi}{4})} \quad . \qquad (7)$$

Expression (7) is used to simplify the coefficient $(\frac{1}{r} + \frac{2}{v} v_r)$ of Eq. (6) which becomes

$$u_{rr} + 2ik_0 u_r + u_{zz} + k_0^2(n^2(r,z)-1)\, u = 0 \quad . \qquad (8)$$

Making use of the property of n(r,z) being slowly varying in r, and neglecting the scattering in both directions, Eq. (8) can be rewritten in an operator form

$$\left(\frac{\partial}{\partial r} + ik_o - ik_o\sqrt{1 + (n^2-1) + \frac{1}{k_o^2}\frac{\partial^2}{\partial z^2}}\right)\left(\frac{\partial}{\partial r} + ik_o + ik_o\sqrt{1 + (n^2-1) + \frac{1}{k_o^2}\frac{\partial^2}{\partial z^2}}\right)u = 0 \quad . \tag{9}$$

Since a one-way outgoing wave is considered, we can deal with the solution of the out-going wave equation

$$\left(\frac{\partial}{\partial r} + ik_o - ik_o\sqrt{1 + (n^2(r,z)-1) + \frac{1}{k_o^2}\frac{\partial^2}{\partial z^2}}\right)u = 0 \quad . \tag{10}$$

Applying a low order approximation to the square-root operator in Eq. (10), we obtain

$$\sqrt{1 + (n^2-1) + \frac{1}{k_o^2}\frac{\partial^2}{\partial z^2}} \approx 1 + \frac{1}{2}\left[(n^2-1) + \frac{1}{k_o^2}\frac{\partial^2}{\partial z^2}\right] \quad . \tag{11}$$

Substituting (11) into Eq. (10) and simplifying, the following equation is obtained

$$u_r = \frac{i}{2}k_o(n^2(r,z)-1)\ u + \frac{i}{2k_o}u_{zz}$$

which is exactly the same as Eq. (1).

It is understood that if a solution is sought for the elliptic wave equation, Eq. (2), one must solve a purely boundary value problem in a region as described below in Figure 1.

where $\phi(r_0,z)$ is the initial boundary condition,
$\phi(r,z_0)$ is the surface boundary condition,
$\phi(r,z_B)$ is the bottom boundary condition, and
$\phi(r_w,z)$ is the wall boundary condition.

On the other hand, if the elliptic problem was solved by the parabolic equation, Eq. (1), we deal with a solution of initial boundary value problem as shown by Figure 2 where we need only to know $u(r_0,z)$ the initial condition, $u(r,z_0)$ the surface boundary condition, and $u(r,z_B)$ the bottom boundary condition while the wall boundary can be ignored completely.

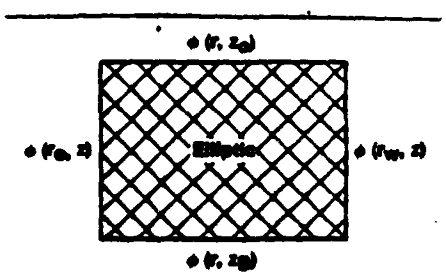

Figure 1. Region of Elliptic Problems

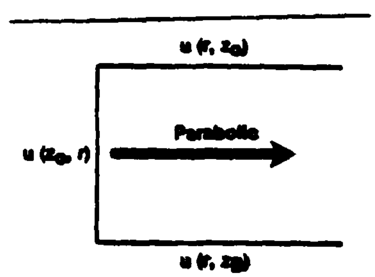

Figure 2. Region of Parabolic Problems

Immediately, we see the advantage that the parabolic problem requires one less boundary condition on the wall which is usually difficult to specify. In addition, the parabolic problem can be solved by a marching process stepping forward in range. It would be more advantageous if the parabolic problem can be solved very efficiently. This is the major result we discuss in this paper.

SOLUTION BACKGROUND

Three different methods exist to solve the parabolic wave equation, Eq. (1). These methods are: Split-step algorithm[1], numerical ordinary-differential-equation methods[2,3,4], and an Implicit Finite Difference (IFD) method[5,6]. Ths split-step Fourier algorithm is effective for deep water problems where the pressure field vanishes at both surface and bottom boundaries. The numerical ordinary-differential-equation solution applies the Generalized Adams method[3], recently much improved by Lee, Jackson, and Preiser[4]. The method is effective and general purpose and it can handle arbitrary boundary conditions. Most frequently used in the application of ocean acoustic propagation for medium to low frequency problems is the implicit finite difference (IFD)[6] solution. This implicit finite difference (IFD) solution is not only general purpose, unconditionally stable, but also has wider angle capabilities than other existing methods. Trying to take advantage of requiring less storage, and the ease of implementation into pipe-line computers, Lee and Papadakis[2] made an attempt to apply explicit schemes such as the Euler scheme to solve the parabolic wave equation. Their analyses showed that using the Euler explicit scheme to solve Eq. (1), the scheme is unstable. Then, Lee, Botseas, and Papadakis applied the IFD, which uses the Crank-Nicolson scheme to solve Eq. (1) because of its favorable unconditional stability. Even though the stability is maintained by the IFD scheme, the desirable advantage of explicit schemes are completely missing. Recently, Chan, Lee, and Shen[7] developed a group of explicit schemes for solving the equations of the Schrodinger type. A new explicit scheme to be introduced in the next section is a member of that group which is formulated to be conditionally stable while other important advantages of the explicit scheme are all retained.

A CONDITIONALLY STABLE EXPLICIT SCHEME

Compared to implicit schemes, explicit schemes are generally easier to implement and demand less storage. These advantages are especially pronounced for multi-dimensional problems. Moreover, another important advantage is that the explicit scheme is often easily vectorized on the many pipeline-oriented computers available today such as Crays, Cyber 205 and FPS164. Taking these advantages our method of attack is to introduce an appropriate dissipative term to derive stable explicit schemes.

We begin by considering the parabolic equation of Schrodinger type in the general form below

$$u_r = ia(r,z)\, u_{zz} + b(r,z)\, u_z + c(r,z)\, u + f(r,z) \tag{12}$$

Here the function $a(r,z)$ is a real-value function, however, functions $b(r,z)$, $c(r,z)$, and $f(r,z)$ may be complex-valued. The effects of adding different dissipative terms have been discussed in Ref. 7. In this paper we discuss one particular dissipative term which leads to the least restrictive stability condition. We use k and h to denote the range and depth increments respectively. u_j^n means $u(r^n, z_j)$.

The simplest explicit scheme for a simple Schrodinger equation,

$$u_r = iu_{zz} \quad , \tag{13}$$

is the Euler scheme,

$$\frac{u_j^{n+1} - u_j^n}{k} = i\left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{k^2}\right) \tag{14}$$

with an initial truncation error $O(k,h^2)$. Scheme (14) is known to be unstable [2]. We introduce a dissipative term R to Eq. (13) to obtain

$$u_r = iu_{zz} + R \tag{15}$$

where the R in this paper is

$$R = (\alpha+i\beta)\ h^2\ u_{zzzz} \tag{16}$$

where α and β are real scalars. With the dissipative term R, the corresponding explicit scheme for Eq. (13) is:

$$\frac{u_j^{n+1} - u_j^n}{k} = i\left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2}\right) + (\alpha+i\beta)\ h^2\left(\frac{u_{j+2}^n - 4u_{j+1}^n + 6u_j^n - 4u_{j-1}^n + u_{j-2}^n}{h^4}\right) \tag{17}$$

and the least restrictive stability constraint is

$$\frac{k}{h^2} \leq \frac{1}{2} \tag{18}$$

which is obtained when $\alpha = -\frac{1}{4}$, $\beta = \frac{1}{4}$.

To show that scheme (17) is stable when it is used to solve Eq. (13), we quote a theorem below which has been proved in Ref. 7.

THEOREM: The scheme (17) is stable if and only if $\alpha < 0$ and

$$\frac{k}{h^2} \leq \min\left(-2\alpha, \frac{-2\alpha}{16\alpha^2 + (4\beta-1)^2}\right) \tag{19}$$

The definition of stability used here is the notion of <u>practical stability</u> as discussed by Richtmeyer-Morton [8] and Chan [9] which requires the discrete solution to have a nongrowing norm.

Now we extend scheme (17) to solve the more general equation, Eq. (12). It is easily seen that the extended stable explicit scheme takes the expression

$$\begin{aligned} \frac{u_j^{n+1} - u_j^n}{k} &= ia_j^n \left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2}\right) \\ &+ \frac{1}{4}\left(-\left|a_j^n\right| + ia_j^n\right) h^2 \left(\frac{u_{j+2}^n - 4u_{j+1}^n + 6u_j^n - 4u_{j-1}^n + u_{j-2}^n}{h^4}\right) \\ &+ b_j^n \left(\frac{u_{j+1}^n - u_{j-1}^n}{2h}\right) + c_j^n u_j^n + f_j^n \quad . \end{aligned} \tag{20}$$

Under a slightly weaker stability definition, the stability of this extended scheme is also given by (18).

We have, thus, introduced a conditionally stable explicit scheme with least restrictive stability condition. We, now, proceed to show its advantages. We begin by applying this scheme to a real problem and comparing its results with the results produced by the Crank-Nicolson scheme in the section to follow.

AN APPLICATION

We apply the explicit scheme (20) to solve a wave propagation problem in the region of the Mediterranean Sea. The representative wave equation is Eq. (1). This is a propagation problem under a shallow water environment where the water depth is 100 m. Under such environment, an isovelocity sound speed is considered. The bottom is characterized by a slightly different sound speed, and a different water density is specified; in addition, bottom attenuation is required due to the bottom energy loss.

Both the source and the receiver are placed at the same depth in the middle of the region. The source propagates at a low frequency. We predict propagation loss up to the range of 25 kilometers. This problem has been solved by three different methods - a normal mode method (SNAP [11]), a split-step Fourier algorithm (PAREQ [11]), and the implicit finite difference method (IFD [6]). The solutions produced by the above three methods agree exactly[5]. The SNAP normal mode solution was used as a benchmark reference solution for comparison. The PAREQ and IFD solutions are all performed in a marching process. In order to satisfy the Fourier requirements, the PAREQ used 512 points as a depth partition, then marched with a range step of 1/2 meter. The IFD used exactly the same step sizes in order to make a point-to-point comparison. Results are all satisfactorily accurate, however, the IFD is two-thirds faster than the PAREQ. For the discussion of the application of this new stable explicit scheme, we choose to compare with the finite-difference (IFD) solution in terms of accuracy, speed, and its implementation effort.

In solving this problem, the following input parameters were used:

source depth	= 50 m
source frequency	= 25 Hz
bottom depth	= 100 m
receiver depth	= 50 m
sound speed (water)	= 1500 m/s
sound speed (bottom)	= 1550 m/s
density (water column)	= 1 g/cm^3
density (at bottom)	= 1.2 g/cm^3
attenuation	= 1 dB/wavelength
maximum range	= 40 km
initial range	= zero m

The result obtained is Propagation Loss (PL) and is measured in "decibels," abbreviated "dB." Propagation Loss in underwater acoustics terminology quantitatively describes the weakening of sound as it travels through the sea. The conventional formula for the propagation loss is PL = $-20 \log_{10}(|u|)$. A graph of PL vs Range is plotted, as shown in Figure 3 where both IFD and explicit numerical results are identical to one another. The explicit scheme produced satisfactorily results, however, the computation speed of the explicit scheme is a little faster than the IFD method (114 min IFD vs 90 min explicit scheme). The largest allowable range step size required by the explicit scheme for stability was ~ 0.5 m, which agrees with the theoretical estimate (18).

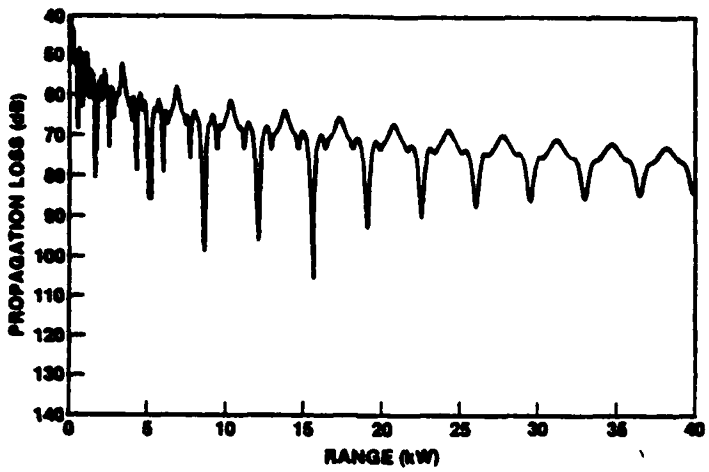

Figure 3. Solutions Comparison

CONCLUSIONS

A conditionally stable explicit finite difference scheme which can be applied to shallow water, low frequency, long range wave propagation problems in the ocean has been introduced. Comparison showed the advantage of computational speed over the Crank-Nicolson solution because the explicit scheme required no solution of a system of equations, but scalar operations. This advantage automatically indicates the relaxation of memory storage. It also indicates an easy vectorization of the explicit scheme on pipeline-oriented computers.

The scheme reported in this article is a member of a group of stable explicit schemes. These schemes are primarily developed to solve the equation of the Schrodinger type in a general nature. It is expected that this group of explicit schemes will have equally efficient applications of problems in other fields such as plasma physics, quantum mechanics, and seismology due to their basic advantages - fast, requiring less memory storage, easy to implement, and easy to vectorize on pipeline-oriented computers.

ACKNOWLEDGMENTS

The authors would like to thank Professor Martin H. Schultz of Yale University for his many valuable discussions as well as his encouragement to apply the scheme for real applications.

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