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Analysis of Nonuniform Dielectric Waveguides

for Millimeter-Wave Applications

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## Analysis of Nonuniform Dielectric Waveguides for Millimeter-Wave Applications

## Introduction

The development of millimeter-wave systems currently under way requires new circuit components, electronic devices and antenna structures, particularly at the high-frequency end of the spectrum. It has been recognized [1] that dielectric structures for use as waveguides and antennas are very attractive for the design of millimeter-wave systems, because they can be made compatible with semiconductor devices and can be fabricated by the use of integrated-circuit approaches. Uniform dielectric waveguides for use as individual circuit components have been extensively studied in recent years [2,3]. To develop a functional millimeter-wave system, it is necessary that various devices and components be interconnected so that electromagnetic signals can be transmitted from one part to another in a circuit. The interconnections usually result in discontinuities in the waveguide system. The present undertaking represents a systematic study of wave phenomena at discontinuities in open dielectric waveguides, so that the circuit components, devices and antennas structures can be optimally interconnected to form a millimeter-wave system.

As contracted, this is a three-year research promgram on the scattering of guided waves by open nonuniform dielectric waveguides, with a particular emphasis on those structures of practical interest. The main objectives of this research program are: (1) to identify all possible physical processes and to estimate their magnitudes, (2) to understand the physical processes and to assess their practical implications, and (3) to determine parametric dependences of important physical processes and to devise methods for controlling them. To accomplish these objectives, we approach the problem of nonuniform dielectric waveguides from different viewpoints and by different methods, so that the wave phenomena can be understood in scope as well as in depth. Therefore, we divide the research program into three phases, each employing a different method of analysis:

- (1) the method of staircase approximation,
- (2) the method of generalized transmission-line equations,
- (3) the exact analysis in terms of a curvilinear coordinate system.

These three phases were planned to be carried out consecutively in the order stated above and each phase was planned for above one year, with some overlap between two phases for achieveing the effectiveness and. internal consistency of the overall program.

We observed during the course of the investigation that it is possible to develop a rigorous proof of the convergence of the iteration process as applied to the generalized transmission-line equations. As explained in the proposal, the first two methods, staircase approximation and generalized transmission-line equations, though very different in appearance, had been shown to be mathematically equivalent to each other in a limiting process. Therefore, if a rigorous proof of the convergence of the iteration process can be achieved, the first two phases will be on a very sound mathematical foundation on which the present research program can be built with confidence. It was then decided that we should pursue first such a fundamental and important problem right at the outset of this research program. In the first six months of the contract period, we had successfully carried out a rigorous proof, among other aspects of the program under investigation. The results have been presented at the National Radio Science Meeting [4]. A summary of the proof is reported here.

## Sufficient Condition for Convergence

It had been shown [5] that the mode amplitudes in a nonuniform

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waveguide are governed by infinite systems of first order differential equations. In general, no exact solution can be expected, and one should be satisfied with an approximate solution with known accuracy. In practice, an infinite system of differential equations has to be truncated to a finite system for an approximate analysis. In the present case of differential equations with variable coefficients, however, the truncation alone will not permit an accurate analysis afterwards, unless it is truncated to a single mode approximation or unless the effect of mode coupling is totally neglected. We have observed that, by converting the differential equations into integral equations, various orders of approximate solutions can be obtained by the iteration procedure. A proof of a sufficient condition for the convergence of the iteration process is described below.

The systems of first order differential equations governing the mode amplitudes can be converted into systems of integral equations, which may take many different forms. After a careful study, we have determined that for the proof of convergence, it is necessary to cast the integral equations in the following form:

$$\underline{\mathbf{a}}(\mathbf{x}) = e^{-\mathbf{j}\mathbf{p}(\mathbf{x})} \left\{ \underline{\mathbf{a}}(0) - \int_{0}^{\mathbf{x}} e^{\mathbf{j}\mathbf{p}(\mathbf{u})} [S(\mathbf{u})\underline{\mathbf{a}}(\mathbf{u}) + T(\mathbf{u})\underline{\mathbf{b}}(\mathbf{u})] d\mathbf{u} \right\}$$
(1)  

$$\underline{\mathbf{b}}(\mathbf{x}) = e^{+\mathbf{j}\mathbf{p}(\mathbf{x})} \left\{ e^{-\mathbf{j}\mathbf{p}(\mathbf{L})} \underline{\mathbf{b}}(\mathbf{L}) - \int_{\mathbf{X}}^{\mathbf{L}} e^{-\mathbf{j}\mathbf{p}(\mathbf{u})} [S(\mathbf{u})\underline{\mathbf{a}}(\mathbf{u}) + T(\mathbf{u})\underline{\mathbf{b}}(\mathbf{u})] d\mathbf{u} \right\}$$
(2)

where <u>a</u> and <u>b</u> are column vectors, S and T are known matrices with the general elements  $S_{mn}$  and  $T_{mn}$  defined by inner products of the mode functions of the waveguide and p is a diagonal matrix with  $p_m$  at the m-th diagonal position. It is important to note that the ranges of integration for <u>a</u> and <u>b</u> in the last two equations are complementary to each other within the nonuniform region,  $0 \le x \le L$ . Physically, <u>a(x)</u> represents the forward traveling waves, and the contribution to <u>a(x)</u> must be from the entrance end to the field point x. On the other hand, <u>b(x)</u> represents the backward traveling waves due to the reflections from the exit end and the contribution to <u>b(x)</u> must be

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from the exit end to the field point x. Therefore, the basic physical processes of the interaction of the electromagnetic waves with the nonuniformity of the structure have been included in the basic equations from which the field quantities are determined. It turns out that the particular form of the integral equations, (1) and (2), is the key step for the convergence proof. In other words, it provides not only a natural physical interpretation, but also a mathematical basis for the convergence of the solutions, as explained next.

By the iteration procedure, we may start the zero-th order approximate solutions of (1) and (2):

$$\underline{a}^{(0)}(\mathbf{x}) = e^{-\mathbf{j}\mathbf{p}(\mathbf{x})}\underline{a}(0) \tag{3}$$

$$\underline{b}^{(0)}(x) = e^{-j[p(L) - p(x)]} \underline{b}(L)$$
(4)

and then generate the n-th order approximate solutions:

$$\underline{a}^{(n)}(x) = e^{-jp(x)} \left\{ \underline{a}^{(0)} - \int_{0}^{x} e^{jp(u)} [S(u)\underline{a}^{(n-1)}(u) + T(u)\underline{b}^{(n-1)}(u)] du \right\}$$
(5)  
$$\underline{b}^{(n)}(x) = e^{-jp(x)} \left\{ e^{-jp(L)}\underline{b}^{(L)} - \int_{x}^{L} e^{jp(u)} [S(u)\underline{b}^{(n-1)}(u) + T(u)\underline{a}^{(n-1)}(u)] du \right\}$$
(6)

Taking the difference of two successive approximations, we obtain from the last two equations:

$$\underline{a}^{(n)}(x) - \underline{a}^{(n+1)}(x) = -e^{-jp(x)} \int_{0}^{x} e^{jp(u)} \{S(u)[\underline{a}^{(n-1)}(u) - \underline{a}^{(n)}(u)] + T(u)[\underline{b}^{(n+1)}(u) - \underline{b}^{(n)}(u)] \} du$$
(7)  
$$\underline{b}^{(n)}(x) - \underline{b}^{(n+1)}(x) = -e^{jp(x)} \{L_{a}^{-jp(u)} \{S(u)[b^{(n-1)}(u)\} \} du$$
(7)

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$$- \underline{b}^{(n)}(u) + T(u)[\underline{a}^{(n-1)}(u) - \underline{a}^{(n)}(u)] du \qquad (8)$$

By the Schwartz inequality, we then obtain, in the norms of the vectors and matrices:

$$\underline{a}^{(n)}(x) - \underline{a}^{(n+1)}(x) < S(x) | \underline{a}^{(n-1)}(x) - \underline{a}^{(n)}(x) | + | T(x) | | \underline{b}^{(n-1)}(x) - \underline{b}^{(n)}(x) |$$
(9)  
$$\underline{b}^{(n)}(x) - \underline{b}^{(n+1)}(x) | < S(x) | | \underline{b}^{(n-1)}(x) - \underline{b}^{(n)}(x) | + | T(x) | | \underline{a}^{(n-1)}(x) - \underline{a}^{(n)}(x) |$$
(10)

Based on these inequalities, the iteration procedure as described by (3)-(6) converges, if the matrices S and T satisfy the following condition:

$$S(X) + T(X) = q < 1$$
 (11)

Thus, we have established a sufficient condition for the convergence of the iterative solutions, (3)-(6). For a given nonuniform structure, S and T can be readily determined and we are now in the process of examining this sufficient condition for various nonuniform waveguides of practical interest. As an example, in the limit of uniform waveguide, we have: S = 0 and T = 0. In such an extreme case, we obtain from (11): q = 0, and the zero-th order approximate solutions (3) and (4) are, in fact, the exact solutions, as should be expected.

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