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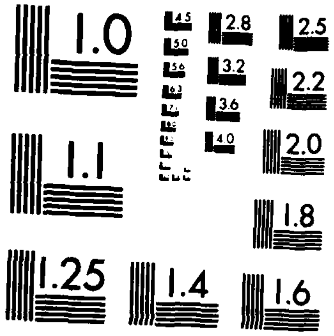

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Research Report CCS 455

A NONLINEAR CONGESTION NETWORK MODEL FOR PLANNING INTERNAL MOVEMENT IN THE HAJJ

by

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ABSTRACT

This paper develops a non-linear dynamic capacitated network model for planning the movements of pilgrims in the Hajj, one of the world's largest mass movements, according to religious ritual, which would assist in minimizing traffic congestion and the overcrowding of the holy sites. A new non-linear representation of congestion with convenient mathematical properties is made. The model is effective in producing quantitative and qualitative background for general policy decisions on the Hajj transportation.

KEY WORDS

Non-linear dynamic capacitated network model
Goal programming
Hajj Pilgrimage
Nonlinear congestion
Policy analysis

A NONLINEAR CONGESTION NETWORK MODEL FOR PLANNING INTERNAL MOVEMENTS IN THE HAJJ

by

A. Charnes, S. Duffuaa, and A. Yafi

1.0 Introduction

The purpose of this paper is to identify issues in the Hajj internal transportation network and model the situation in order to draw general policy decisions in management of this unique situation. The most important constraints in this case are the religious ones, since they define the transportation and the movement of the people from one place to another. Any model must reflect these constraints in order to be implementable. These religious constraints are represented as time and space constraints. A temporal capacitated network is developed to represent the situation, with the objective to minimize congestion of traffic on roads and overcrowding of the holy places. This objective is chosen to enable the visitors to perform their duties as easily as possible.

In section 2 we describe the Hajj situation briefly, and in section 3 we identify issues to be considered in the modeling process. In section 4 we develop a linear network model, and in section 5 we present a numerical example for the Hajj situation. In section 6 we extend the model to a non-linear model to properly account for congestion via a non-linear function and we draw conclusions.

2.0 Description of the Hajj

The Hajj is an annual meeting for about two million Muslims to achieve one of the five pillars of Islam. The Hajj involves several visitations to several holy sites and it occurs every 354 days, since it is dated by the Islamic Lunar year. The Hajj process in this paper will be divided into three phases. Phase one mainly consists of air and sea transportation of foreign population to Jeddah, and also inland transportation of foreign and native population. The duration of this phase is up to the eighth of the last month of the Lunar year, by that time all the pilgrims have gathered in one of the following towns: Jeddah, Makkah, Muna or Medina.

Phase two starts from the morning of the eighth of the last month in the Lunar year up to the end of the Hajj and the arrival of the pilgrims at the ports to leave for home. This phase includes part of the in-land transportation from Jeddah and Medina to Makkah and all the movement between the holy sites up to the completion of the Hajj. Its duration is roughly up to the nineteenth or the twentieth of the month. Phase three consists partly of the transportation between Makkah and the ports. It also includes the departure of all prilgrims for home. In this paper we deal with phase two, the "internal transportation network of the intramovement of the pilgrims", which constitutes the logistics of the religious process.

2.1 Phase Two of the Hajj

In general, the Hajj season begins at the start of the tenth month of the Muslim lunar calendar year and ends about the twentieth of the Dihu'L-Hijjah (the twelfth and final month of the lunar year). Most of the pilgrims arrive in the last fifteen days of the season. According the our model, phase two starts the morning of the eighth day of the last month of the lunar calendar year. By this time all pilgrims have already arrived in Saudi Arabia and they are either at Makkah, Muna, Jeddah or Medina (see dagram in Figure I).

It is a ritual that before arriving at Makkah every pilgrim has to wear the garment of iharm (restriction), is forbidden to hunt, argue, cut his hair, clip his nails or engage in any sexual activities. Also on arrival at

Makkah (45 miles east of Jeddah) each pilgrim has to make the greeting tawaf, the prescribed seven counterclockwise circumambulations of the Kaaba (a black room in the center of the holy mosque built by the prophet Ibraham). With the tawaf they perform the Sa'y--making seven trips between the hills of Safa nad Marwah. The path of Sa'y is enclosed in a long gallery which is part of the holy mosque.

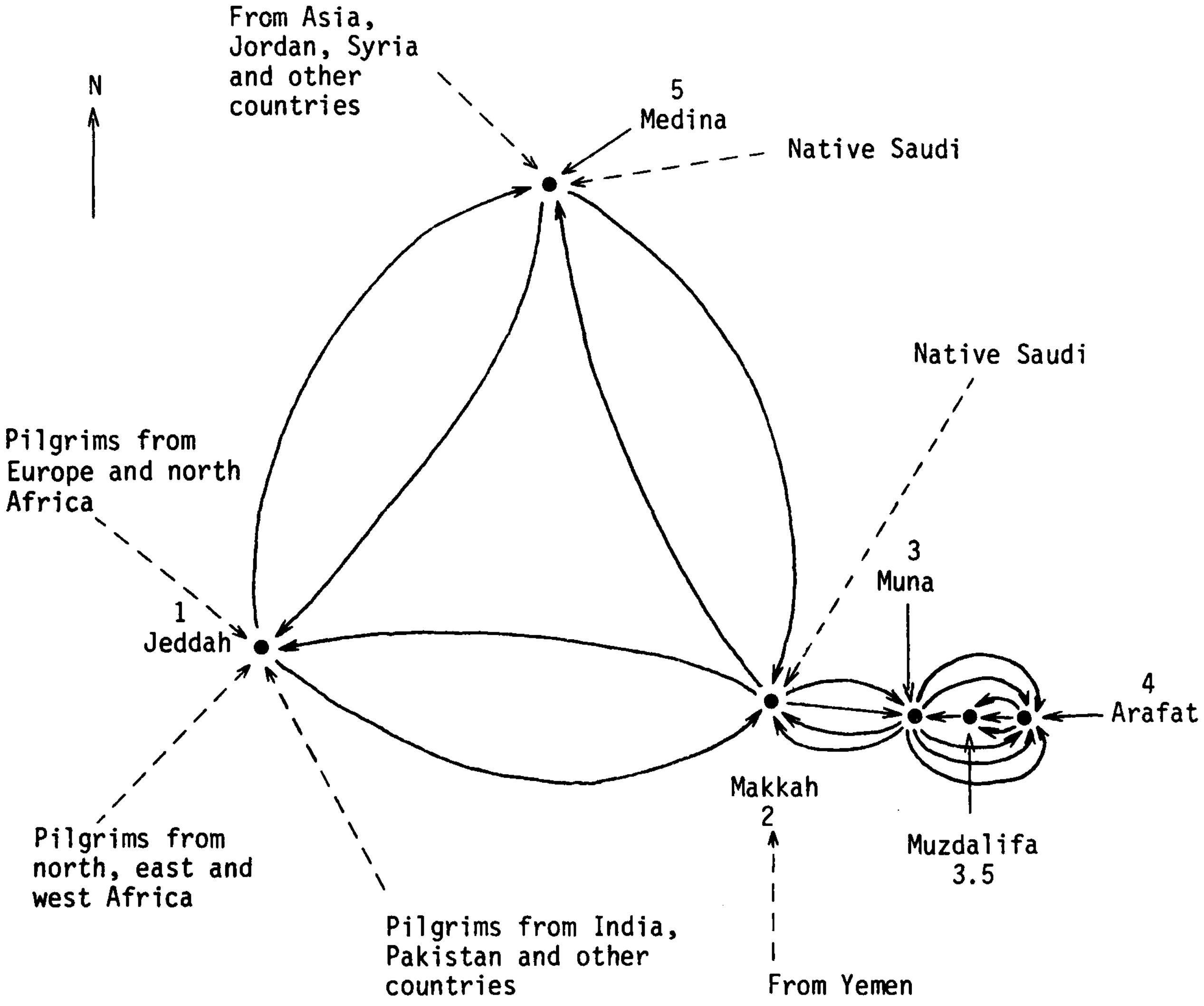
Also on the eighth day, those pilgrims who are not already in Muna, four miles east of Makkah, move through Makkah to Muan for the essential final days of the Hajj. The next day (on the ninth) everybody has to be in Arafat, eight miles east of Muna, to perform the Standing, the central ritual of the Hajj. The duration of the Standing is from noon of the ninth until sunset.

On the eve of the tenth, the pilgrims leave Arafat and stop at Muzdalifa, a place between Muna and Arafat. At Muzdalifa they collect pebbles to throw at the three "Satan's stoning points" in Muna during the following days. These points symbolize the force of evil. Those who leave for Makkah start doing one of the pillars of the Hajj Tawaf El ifada, the post-Arafat tawaf done in the same manner as the greeting tawaf. After finishing this tawaf pilgrims can put off iharm (restriction) and return to Muna to finish the Stoning of the devil. After finishing the Stoning, the Hajj is complete.

All pilgrims after the Hajj perform a farewell tawaf and some of them leave for Medina to visit the prophet's grave. Visiting Medina can be done before the Hajj starts and those who did it before the Hajj leave via Jeddah or by inland routes for home. Figure 1 shows all the holy sites and the routes of the pilgrims' movements.

Figure 1

A Diagram for the Islamic Holy Sites and the Flow of Pilgrims



———→ The solid lines represent the internal movement

- - - - - - -> The broken lines represent the external movement for coming to Hajj and leaving

3.0 Issues to be Considered in the Model

The most important elements here are the timing and place of each of the holy practices. This determines the flow of people from one place to another. We notice from section two that there is some flexibility in the timing of most of the pillars of the Hajj. The model must be able to take full advantage of that in order to make the Hajj as smooth as possible.

When the pilgrims travel to Makkah they are in large numbers and there is overcrowding of the roads which lead to Jakkah. On arrival at Makkah they perform the greeting tawaf. There they need some scheduling to ease overcrowding at the Kaaba.

The central ritual of the Hajj is the Standing at Arafat. This is a large place and can take all the pilgrims, but on the way to and from Arafat the big crunch occurs. Then the people shuttle between Makkah and Muna until they finish their Hajj.

In scheduling events in the Hajj, the following must be noted. First, the greeting tawaf can be done any time before the morning of the ninth of the Hajj month. The post-Arafat tawaf can be performed at any time from the eve of the tenth to the evening of the twelfth of the Hajj month, but most people like to do it earlier to finish their Hajj. Secondly, the Stoning of the Satan is flexible within two or three days, i.e., from the tenth up to the twelfth of the Hajj month. Third, the Sacrifice of the sheep can be done any time from the morning of the tenth until the evening of the twelfth, but most people do it on the tenth. Fourth, most people catch the 'Id prayer at the Sacred Mosque on the morning of the tenth. Fifth, the farewell tawaf can be done any time before leaving Makkah. Sixth, the visitation to Medina can be done before or after the Hajj and there are no time constraints on it.

The model should take advantage of the flexibility in the timing of the Hajj rituals, but also should allow room for pilgrims' preferences in choosing the time to do rituals if the models' specifications are to have any chance of being implemented.

4.0 Statement of the Model

4.1. The Constraints Set

The following constraints set of the Hajj internal movement is shown in Figure 2. Figure 2 represents movements of pilgrims by half-days from the eighth of the Hajj month till the nineteenth. Each half a day is considered as a period, so the model consists of twenty-four periods. We think half a day is a reasonable period for general policy decisions, but for close monitoring of the pilgrims' movements the model can be represented in terms of shorter periods, i.e., in terms of one hour period. This will expand the model, but since the model is of network type, it can be solved easily.

Figure 2 gives a complete representation of the Hajj internal movements. There are two sets of arcs in Figure 2, one set represents movement of pilgrims within periods, the other set represents pilgrims who are staying in the same town for the next period.

The upper bounds on the arcs in the model should be considered very closely and the following considerations would be very useful:

(1) the time of the period evening or morning,

(2) the lodging capacity of each town,

(3) the food and public facilities of each location,

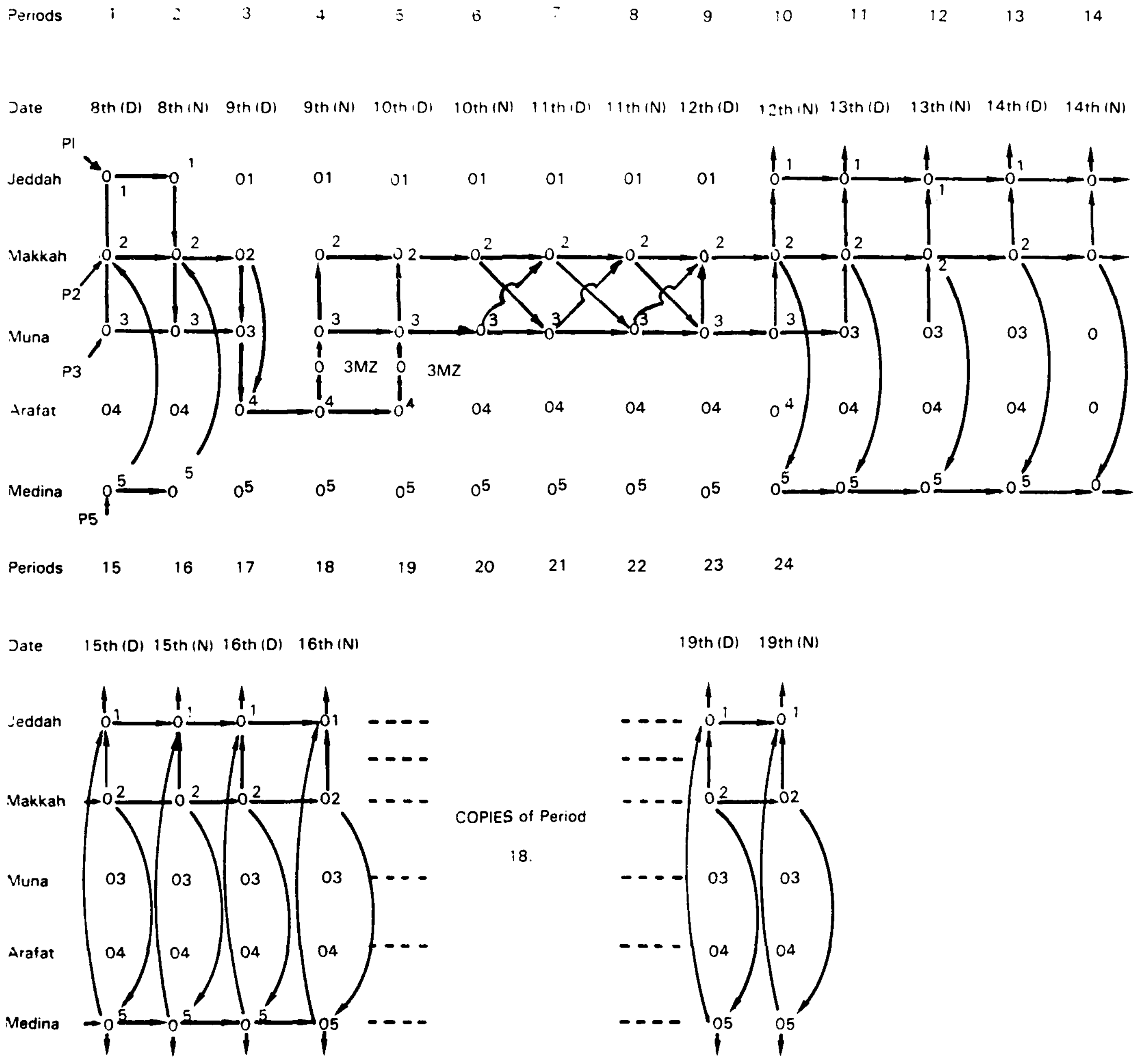
(4) the health facilities,

(5) the availability of buses, cars, between any two towns on each route, and

(6) the timing of each ritual.

FIGURE 2

NETWORK REPRESENTATION For THE HAJJ
Internal MOVEMENT
Each Half A day is A period



1 ≡ represent Jeddah, 2 ≡ represent Makkah, 3 ≡ represent Muna, 4 ≡ represent Arafat,

5 ≡ represent Medina, 3MZ ≡ represent Mozdalifa

D ≡ Day, N ≡ Night

Let

$x_{ij}(t)$ = number of pilgrims going from location i to location j in period t;

$y_{ij}(t,t+1)$ = number of pilgrims going from location i to location j during period t;

$y_{ii}(t,t+1)$ = number of pilgrims staying at location i during period t;

[In this case we do not consider in detail cross travel between periods from one site to another, since one-half day is more than sufficient travel time between any two sites; therefore, flexibility in the timing of rituals enables the model to accommodate cross travel.]

K_{ij} = the maximum number of pilgrims who can travel easily at day times from location i to location j, i.e., this is the capacity of route (i,j)

L_{ij} = the maximum number of pilgrims who can travel easily at night time on route (i,j)

L_i = lodging capacity of town i,

K_i = food capacity of town i,

b_i = the minimum of the public facilities' capacities at town i.

Let 1 be Jeddah, 2 be Makkah, 3 be Muna, 3.5 be Muzdalifa , 4 be Arafat, and 5 be Medina.

P_i = number of pilgrims at town i at the beginning of the first period.

The following are the equations defined by the network in Figure 2. The numbers to the left of the equations represent the period and the node, i.e., 1.5 represent the equation for period one at node 5, or the equation for period one at Medina and so on. Note that the equations $y_{ii}(t,t+1)$ is written as $y_i(t,t+1)$.

	$x_{12}(1)$	$x_{23}(1)$	$x_{52}(1)$	$y_1(1,2)$	$y_2(1,2)$	$y_3(1,2)$	$y_5(1,2)$	$x_{12}(2)$	$x_{23}(2)$	$x_{52}(2)$	$y_1(2,3)$	$y_2(2,3)$	$y_3(2,3)$	$y_5(2,3)$	
1.1	1			1											$= P_1$
1.2	-1	1	-1		1										$= P_2$
1.3		-1				1									$= P_3$
1.4															$= P_4$
1.5			1				1								$= P_5$
2.1				-1				1			1				= 0
2.2					-1			-1	1	-1		1			= 0
2.3						-1			-1				1		= 0
2.4															= 0
2.5							-1			-1				1	= 0

Part of period 3 is on the next page.

	$x_{12}(1)$	$x_{23}(1)$	$x_{52}(1)$	$y_1(1,2)$	$y_2(1,2)$	$y_3(1,2)$	$y_5(1,2)$	$x_{12}(2)$	$x_{23}(2)$	$x_{52}(2)$	$y_1(2,3)$	$y_2(2,3)$	$y_3(2,3)$	$y_5(2,3)$	
3.1											-1				→
3.2												-1			→
3.3													-1		→
3.4															→
3.5														-1	→

The arrow means the equations are completed on the next page.

Equations for period 3, 4 and part of period 5. Notice period 3 has some variables on the previous period

	$x_{23}(3)$	$x_{24}(3)$	$x_{34}(3)$	$y_2(3,4)$	$y_3(3,4)$	$y_4(3,4)$	$x_{32}(4)$	$x_{4\ 3.5}(4)$	$x_{3.5\ 3}(4)$	$y_2(4,5)$	$y_3(4,5)$	$y_{3.5}(4,5)$	$y_4(4,5)$	
3.1														= 0
3.2	1	1		1										= 0
3.3	-1		1		1									= 0
3.3.5														= 0
3.4		-1	-1			1								= 0
3.5														= 0
4.1														= 0
4.2				-1			-1			1				= 0
4.3					-1		1		-1		1			= 0
4.3.5								-1	1			1		= 0
4.4						-1		1					1	= 0
4.5														

Part of the equations of period 5 and completion of these equations is on the next page.

	$x_{23}(3)$	$x_{24}(3)$	$x_{34}(3)$	$y_2(3,4)$	$y_3(3,4)$	$y_4(3,4)$	$x_{32}(4)$	$x_{4\ 3.5}(4)$	$x_{3.5\ 3}(4)$	$y_2(4,5)$	$y_3(4,5)$	$y_{3.5}(4,5)$	$y_4(4,5)$	
5.1														
5.2										-1				→
5.3											-1			→
5.3.5												-1		→
5.4													-1	→
5.5														→

Constraints for period 5, 6 and part of period 7:

	$x_{32}(5)$	$x_{4\ 3.5}(5)$	$x_{3.5\ 3}(5)$	$x_{32}(5)$	$y_2(5,6)$	$y_3(5,6)$	$y_{23}(6,7)$	$y_{32}(6,7)$	$y_2(6,7)$	$y_3(6,7)$	
5.1											= 0
5.2	-1			-1	1						= 0
5.3	1		-1	1		1					= 0
5.3.5		-1	1								= 0
5.4		1									= 0
5.5											= 0
6.1											= 0
6.2					-1		1		1		= 0
6.3						-1		1			= 0
6.4											= 0
6.5											= 0

Part of the constraints for period 7 and its completion is on the next page.

	$x_{32}(5)$	$x_{4\ 3.5}(5)$	$x_{3.5\ 3}(5)$	$x_{32}(5)$	$y_2(5,6)$	$y_3(5,6)$	$y_{23}(6,7)$	$y_{32}(6,7)$	$y_2(6,7)$	$y_3(6,7)$	
7.1											→
7.2								-1	-1		→
7.3							-1			-1	→
7.4											→
7.5											→

Constraints set for period 7, 8 and part of period 9:

	$y_{23}(7,8)$	$y_{32}(7,8)$	$y_2(7,8)$	$y_3(7,8)$	$y_{23}(8,9)$	$y_{32}(8,9)$	$y_2(8,9)$	$y_3(8,9)$	
7.1									= 0
7.2	1		1						= 0
7.3		1		1					= 0
7.4									= 0
7.5									= 0
8.1									= 0
8.2		-1	-1		1		1		= 0
8.3	-1			-1		1		1	= 0
8.4									= 0
8.5									= 0
9.1									→
9.2						-1	-1		→
9.3					-1			-1	→
9.4									→
9.5									→

Part of the constraints set for period 7 and its completion is on the next page.

Constraints set for period 9, 10 and part of period 11:

	$x_{32}(9)$	$y_2(9,10)$	$y_3(9,10)$	$x_{21}(10)$	$x_{32}(10)$	$x_{25}(10)$	$y_1(10,11)$	$y_2(10,11)$	$y_3(10,11)$	$y_5(10,11)$	
9.1											= 0
9.2	-1	1									= 0
9.3	1		1								= 0
9.4											= 0
9.5											= 0
10.1				-1			1				= 0
10.2		-1		1	-1	1		1			= 0
10.3			-1		1				1		= 0
10.4											= 0
10.5						-1				1	= 0
11.1							-1				→
11.2								-1			→
11.3									-1		→
11.4											→
11.5										-1	→

Part of the constraint set for period 11 and its completion is on the next page.

Equations for period 11, 12 and part of those for period 13:

	$x_{21}(11)$	$x_{25}(11)$	$x_{32}(11)$	$y_1(11,12)$	$y_2(11,12)$	$y_3(11,12)$	$y_5(11,12)$	$x_{21}(12)$	$x_{25}(12)$	$x_{32}(12)$	$y_1(12,13)$	$y_2(12,13)$	$y_3(12,13)$	$y_5(12,13)$	
11.1	-1			1											= 0
11.2	1	1	-1		1										= 0
11.3			1			1									= 0
11.4															= 0
11.5		-1					1								= 0
12.1				-1				-1			1				= 0
12.2					-1			1	1	-1		1			= 0
12.3						-1				1			1		= 0
12.4															= 0
12.5							1		1					1	= 0

Completion of the equations for period 13 is on the next page.

13.1											-1				→
13.2												-1			→
13.3													-1		→
13.4															→
13.5														-1	→

Equations for periods 13, 14 and part of those for period 15:

	$x_{21}(13)$	$x_{25}(13)$	$y_1(13,14)$	$y_2(13,14)$	$y_5(13,14)$	$x_{21}(14)$	$x_{25}(14)$	$y_1(14,15)$	$y_2(14,15)$	$y_5(14,15)$	
13.1	-1		1								= 0
13.2	1	1		1							= 0
13.3											= 0
13.4											= 0
13.5		-1			1						= 0
14.1			-1			-1		1			= 0
14.2				-1		1	1		1		= 0
14.3											= 0
14.4											= 0
14.5					-1		-1			1	= 0

Completion of the equations for period 15 is on the next page.

15.1								-1			→
15.2									-1		→
15.3											→
15.4											→
15.5										-1	→

Equations for period 15, 16 and part of those for period 17:

	$x_{21}(15)$	$x_{25}(15)$	$y_1(15,16)$	$y_2(15,16)$	$y_5(15,16)$	$x_{21}(16)$	$x_{25}(16)$	$x_{52}(16)$	$x_{51}(16)$	$y_1(16,17)$	$y_2(16,17)$	$y_5(16,17)$	
15.1	-1		1										= 0
15.2	1	1		1									= 0
15.3													= 0
15.4													= 0
15.5		-1			1								= 0
16.1			-1			-1			-1	1			= 0
16.2				-1		1	1	-1			1		= 0
16.3													= 0
16.4													= 0
16.5					-1		-1	1	1			1	= 0

Completion of the equations for period 17 is on the next page.

	$x_{21}(15)$	$x_{25}(15)$	$y_1(15,16)$	$y_2(15,16)$	$y_5(15,16)$	$x_{21}(16)$	$x_{25}(16)$	$x_{52}(16)$	$x_{51}(16)$	$y_1(16,17)$	$y_2(16,17)$	$y_5(16,17)$	
17.1										-1			→
17.2											-1		→
17.3													→
17.4													→
17.5												-1	→

Equations for period 17 and 18:

	$x_{21}(17)$	$x_{25}(17)$	$x_{52}(17)$	$x_{51}(17)$	$y_1(17,18)$	$y_2(17,18)$	$y_5(17,18)$	$x_{21}(18)$	$x_{25}(18)$	$x_{51}(18)$	$x_{52}(18)$	$y_1(18,19)$	$y_2(18,19)$	$y_5(18,19)$	
17.1	-1			-1	1										= 0
17.2	1	1	-1			1									= 0
17.3															= 0
17.4															= 0
17.5		-1	1	1			1								= 0
18.1					-1					-1		1			= 0
18.2						-1		-1			-1		1		= 0
18.3												1			= 0
18.4															= 0
18.5							-1		-1	1	1				= 0

The following periods 19, 20 up to 24 have the same pattern or structure as 18 so we will have just copies of the equations in 18. The following are the capacity constraints on the routes and the holy places.

$0 \leqslant x_{ij}(t) \leqslant K_{ij}$ if t is odd

$0 \leqslant x_{ij}(t) \leqslant L_{ij}$ if t is even

$0 \leqslant y_{ij}(t,t+1) \leqslant L_{ij}$

4.2. Objective Function

One of the objectives to be considered is space and routes utilization. The objective is of goal type to ease crowding as much as possible, i.e.

$$\text{(4.2.1)} \quad \text{Max} \sum_{i} |\min(K_i, L_i, b_i) - y_{ii}(t,t+1)| + \sum_{\substack{t \\ \text{odd}}} \sum_{ij} |K_{ij} - x_{ij}(t)| + \sum_{\substack{t \\ \text{even}}} \sum_{ij} |L_{ij} - x_{ij}(t)| + \sum_{t} \sum_{ij} |L_{ij} - y_{ij}(t,t+1)|$$

Maximizing this function alone will allow for overcrowding of holy sites and we will be maximizing a convex function, but with the capacity constraints

$$x_{ij}(t) \leqslant L_{ij} \quad , \quad x_{ij}(t) \leqslant K_{ij} \quad \text{and}$$

$$y_{ii}(t,t+1) \leqslant \min(K_i, L_i, b_i) \ , \ y_{ij}(t,t+1) \leqslant L_{ij}$$

that will not happen and the objective function will be transformed into the following equivalent linear function.

$$\text{(4.2.2)} \quad \text{Min} \sum_{t} \sum_{i} y_{ii}(t,t+1) + \sum_{t} \sum_{ij} x_{ij}(t) + \sum_{t} \sum_{ij} \sum_{t} y_{ij}(t,t+1)$$

Hence the model for the Hajj internal movement will minimize the linear function in (4.2.2) subject to all the constraints in section 4.1.

The above model could be updated if conditions on the routes and the holy sites are changed. It could be formulated in terms of shorter periods, for example on an hourly basis, if detailed monitoring of the Hajj internal movement is needed.

Another objective function to be considered is

$$\text{Min} \sum_{t} \sum_{i=1}^{n} | \min(K_i, L_i, b_i) - y_i(t,t+1)| + \sum_{\substack{t \\ \text{odd}}} \sum_{ij} | K_{ij} - x_{ij}(t)|$$

$$+ \sum_{\substack{t \\ \text{even}}} \sum_{ij} | L_{ij} - x_{ij}(t)| + \sum_{t} \sum_{ij} | L_{ij} - y_{ij}(t,t+1)|$$

This is another type of a goal type objective and this with the constraints in section 4.1 can be transformed into a larger pure network.

Other objectives and constraints may also need consideration, e.g., the role of individual contractors ("Mutwafeen") may need more explicit consideration.

5.0 Examples and Extensions

The following example is a reasonable representation for the Hajj situation in the year 1980. The example was constructed using [4], a study done by the Hajj Research Center at Jeddah. Using this study and some personal judgement, the following data on routes and town capacities were specified.

Routes	Day	Night
Jeddah to Makkah	0 to 200,000	0 to 150,000
Makkah to Muna	0 to 500,000	0 to 400,000
Makkah to Medina	0 to 200,000	0 to 150,000
Makkah to Arafat	0 to 900,000	0 to 700,000
Muna to Arafat	0 to 800,000	0 to 700,000
Muna to Makkah	0 to 250,000	0 to 200,000
Medina to Makkah	0 to 100,000*	0 to 150,000

*when 2-way traffic

Capacity Of	No. of Pilgrims
Jeddah	300,000
Makkah	1,000,000
Muna	600,000
Arafat	2,000,000
Medina	1,000,000

$P_1 = 200{,}000$; $P_2 = 800{,}000$; $P_3 = 400{,}000$; $P_5 = 200{,}000$; $P_4 = 0$

The above example, with some variations to allow for pilgrim increase, was solved in order to evaluate the model and its representation of the real situation. To achieve this evaluation, the dual variables of the model were utilized. The dual variables give an evaluation of the model response to traffic volumes and routes capacities.

In all three examples the dual evaluations point to the route Mina to Mecca at period four as pre-eminent in relief of congestion since it was the most negative dual variable. An increase in the capacity of this route by one unit will result in 127 units reduction in the objective function (which increases congestion) since the value of its dual variable is -127. The examples further show that major concern should be given to the Hajj internal movement in period four and five. The complete solution of this example is in Table 1 in the Appendix.

The other negative dual variables occur for the routes Jeddah to the Sink and Medina to the Sink. The most negative dual variable is associated with the capacity of the route Jeddah to the Sink of value -14. That means an improvement in Jeddah port capacity of one unit will result in a reduction of 14 units in the objective function. Using the magnitude of the dual variables, the ports can be ranked in terms of their importance in reducing traffic congestion. The Jeddah port is the most important to the Hajj internal movement and Medina is next.

The second and the third examples give directions where improvement should be made in case the number of pilgrims increase. Both examples show that the Jeddah-Mecca route must be improved to increase its capacity to handle the anticipated increase in pilgrims in the future.

These examples give some insight into decision making for Hajj scheduling but the simple objective function does not reflect very well all the possible effects of levels of congestion on different routes. Hence, they give scheduling plans which are insensitive to routes and to changes in capacity. A more appropriate measure of congestion, which will result in better distributions of traffic between the routes, is needed.

Such a measure of congestion will be presented in the next section. It results in a non-linear objective function network. We solve it by piecewise linearization and shall compare its results with those of this model.

6.0 Non-Linear Network for the Hajj Internal Movement

In the previous section we examined a linear model for the Hajj Internal Movement. The linear objective is not an appropriate measure of congestion, since it does not correctly reflect the fact that rate of congestion increases with increases in closeness to capacity on a route segment. In this section we introduce a non-linear measure of congestion for each route which depends on the capacity of the route segment and the density of traffic on it. In the next section we will elaborate on the properties of this measure.

Let $R_{ij}(x_{ij}(t))$ and $\overline{R}_{ij}(y_{ij}(t,t+1))$ be convex non-linear functions which measure the congestion on arcs $x_{ij}(t)$ and $y_{ij}(t,t+1)$. The model in section 4.2 is now altered (only in the objective function) to be

$$\text{(6.0.1)} \quad \text{Min} \sum_{t}^{24} \sum_{ij} R_{ij}(x_{ij}(t)) + \sum_{t}^{24} \sum_{ij} \overline{R}_{ij}(y_{ij}(t,t+1))$$

$$\text{subject to} \quad \sum_{j=1}^{5} \varepsilon_{ij} x_{ij}(t) + \sum_{j} \varepsilon_{ij}(t) y_{ij}(t,t+1) = 0$$

$$i = 1,2,\dots,5 \; ; \; t = 1,2,\dots,24 \; ; \; x_{ij}(t),\ y_{i}(t,t+1) \geqslant 0$$

where the incidence numbers ε_{ij} are 1, -1, or zero. The model in (6.0.1) can be replaced by the following capacitated linear model wherein we have approximated the $R_{ij}(x_{ij}(t))$ and $R_{ij}(y_{ij}(t,t+1))$ functions by two linear segments. In the next section we shall develop a method for choosing the two line segments. Employing such an approximation (6.0.1) becomes

$$\text{(6.0.2)} \quad \text{Min} \sum_{t}^{24} \sum_{i}^{5} \sum_{j}^{5} \sum_{k=1}^{2} R_{ijk} x_{ijk}(t) + \sum_{t} \sum_{i} \sum_{j} \sum_{k=1}^{2} \overline{R}_{ijk} y_{ijk}(t)$$

$$\text{subject to} \sum_{j}^{24} \sum_{k}^{2} \varepsilon_{ij} x_{ijk}(t) + \sum_{j}^{5} \sum_{k}^{2} \varepsilon_{ij}(t) y_{ijk}(t,t+1) = 0$$

$$i = 1,2,\dots,5 \; ; \quad 0 \leqslant x_{ijk}(t) \leqslant \Delta_{ijk} \; ; \quad 0 \leqslant y_{ijk}(t) \leqslant \Delta_{ijk}$$

The dual of (6.0.2) is

$$\text{(6.0.3)} \quad \text{Max} \sum_{t=1}^{24} \sum_{i}^{5} \phi_{i}(t) \cdot 0 - \sum_{t=1}^{24} \sum_{i}^{5} \sum_{j}^{5} \sum_{k}^{2} \Psi_{ijk}(t) \Delta_{ijk} - \sum_{t=1}^{24} \sum_{i}^{5} \sum_{j}^{5} \sum_{k}^{2} w_{ijk}(t) \overline{\Delta}_{ijk}$$

subject to

$$\sum_i \varepsilon_{ij}\Phi_i(t) - \Psi_{ijk}(t) \leqslant R_{ijk}$$

$$\sum \varepsilon_{ij}\Phi_i(t) - w_{ijk} \leqslant R_{ijk} \ \forall \ j, t, \text{ where } \Phi_i \text{ unrestricted, } w_{ijk}, \Psi_{ijk} \geqslant 0.$$

In an optimal solution to (6.0.2) the dual variables Φ^*_i, w^*_{ijk} and Ψ^*_{ijk} provide evaluators for the initial points at which changes in "resistance" to traffic volumes occurs. Hence a means is provided for evaluating possible changes associated with those features of the network in terms of their effects on the resulting traffic volumes at the times associated with the movements.

To develop (6.0.2) we nedd the R_{ijk}, $\bar{\Delta}_{ijk}$ for the model. The next section will elaborate on the congestion measure we have chosen and on how the R_{ijk}, $\bar{\Delta}_{ijk}$ and Δ_{ijk} are determined.

6.1 A Convex Non-linear Measure of Traffic Congestion

The measure of Congestion of traffic should:

1) increase with increase in traffic flow,

2) have an increasing rate of increase with traffic flow increase (i.e. be a convex function),

3) be zero or very close to zero at zero flow,

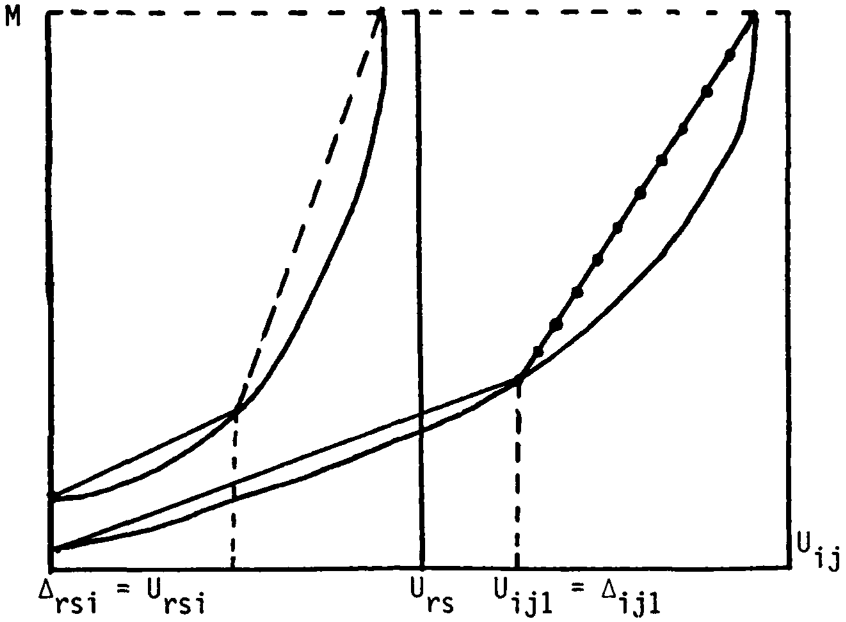
4) approach infinity when flow approaches the capacity of the route.

One such measure of traffic congestion is

$$(6.1.1) \qquad f(x_{ij}) = \frac{1}{U_{ij} - x_{ij}}$$

where U_{ij} is the capacity of the route movement (i,j,). To apply (6.0.2) we need to find R_{ijk} and Δ_{ijk} and what the relationships are between these Δ_{ijk} and R_{ijk}. Thus we break the non-linear function into two line segments. In order to exhibit further relationships, in figure 6-1 we exhibit simultaneously two examples.

Figure 6.1



For any given two routes (i,j) and (r,s), they have the measures of congestion $R_{ij}(x_{ij})$ and $R_{rs}(x_{rs})$. We break the function $R_{ij}(x_{ij})$ into two line segments. The first line segment is the line that joins the two points $\left(0, \frac{1}{U_{ij}}\right)$ and $\left(\frac{1}{U_{ij1}}, \frac{1}{U_{ij}-U_{ij1}}\right)$. We now show how from determination of the breakpoint for one route (i,j), we can simultaneously determine the breakpoints for all other routes (r,s). Using the concept that congestion will be the same if traffic density relative to route capacity is the same, we consider

$$\frac{x}{U_{ij}} = \frac{y}{U_{rs}} \tag{6.1.2}$$

where x is the flow in route (i,j) and y is the traffic flow in route (r,s). Using (6.1.2) we have

(6.1.3) $$\frac{U_{rs1}}{U_{rs}} = \frac{U_{ij1}}{U_{ij}}$$

then

(6.1.4) $$U_{rs1} = \left(\frac{U_{rs}}{U_{ij}}\right) U_{ij1}$$

Thus the breakpoints are determined immediately by the route capacity and so are the slopes for all the routes.

(6.1.5) The $R_{rs1} = \left(\frac{1}{U_{rs}-U_{rs1}} - \frac{1}{U_{rs}}\right) \div U_{rs1}$

<u>6.1.1 THEOREM</u>

If R_{ij1} is the slope of the first line segment for $R_{ij}(x_{ij})$, then

$$R_{rs1} = \frac{U^2_{ij}}{U^2_{rs}} \cdot R_{ij1}$$

for all (r,s).

<u>Proof</u>: $$R_{rs}(x_{rs}) = \frac{1}{U_{rs}-x_{rs}}$$

R_{rs1} is the slope of the line joining $\left(0, \frac{1}{U_{rs}}\right)$ and $\left(U_{rs1}, \frac{1}{U_{rs}-U_{rs1}}\right)$

$$R_{rs1} = \left(\frac{1}{U_{rs}-U_{rs1}} - \frac{1}{U_{rs}}\right) \div U_{rs1}$$

$$= \frac{U_{rs} - U_{rs} + U_{rs1}}{U_{rs1}\left(U_{rs}-U_{rs1}\right)U_{rs}} = \frac{1}{U_{rs}\left(U_{rs} - U_{rs1}\right)}$$

But we have
$$U_{rs1} = \frac{U_{rs}}{U_{ij}} U_{ij1}.$$

Hence
$$R_{rs1} = \frac{1}{U_{rs}\left(U_{rs} - \frac{U_{rs}}{U_{ij}} U_{ij1}\right)} = \frac{1}{\frac{U^2_{rs}}{U_{ij}}\left(U_{ij} - U_{ij1}\right)}$$

$$= \left(\frac{U^2_{ij}}{U^2_{rs}}\right) \frac{1}{U_{ij}(U_{ij} - U_{ij1})} = \frac{U^2_{ij}}{U^2_{rs}} R_{ij1}$$

Q•E•D

To determine the second line segment slope, let

$$M = \frac{1}{U_{ij} - x^*_{ij}} \qquad \text{where}$$

$U_{ij} - x^*_{ij} = n$ for all (i,j). Then

$$R_{ij2} = \frac{M - \frac{1}{U_{ij} - U_{ij1}}}{x^*_{ij} - U_{ij1}}$$

Computing R_{ijk}, Δ_{ijk} for all functions on the routes, putting them and their associated variables into (6.0.2), we abtain a linear network. This model has been applied to the Hajj Internal Movement using the same data as in the linear case, and solved.

6.2 Results and Conclusions

The non-linear measure of traffic congestion is applied to the same example previously solved for the linear case. All the necessary piece-wise linearization is performed as described in section 6.1. The optimal solution for the non-linear case depicts the Hajj situation in more realistic terms by picking up the required movements between Muna and Makkah in periods 5, 6, and 7 without imposing lower bounds on traffic movements on routes.

Moreover we see that the evaluation given by the dual variables for the non-linear case give a clear indication of routes which might ease traffic congestion.

The model shows that the time interval of concern for the Hajj Internal Movement is from period four to period seven. To comprehend the ways of easing the traffic congestion there we need to concentrate on the Hajj Internal Movement from period three to period eight and use the results of this model as a guide for traffic improvements.

The model evaluators clearly signal the following routes as the key routes in easing the traffic congestion. The route with the most negative dual variable is Muna to Makkah in period four. The second route is Muna to Makkah at period 5, followed by Muna to Muna between periods five and six.

The model shows that the pilgrims' time of departure depends strictly on the capacity of the ports. Upgrading the capacities of the ports will help in smoothing the Hajj process. Another important way to ease the traffic congestion is to divide the pilgrims into two groups: one to visit Medina prior to the Hajj, the other after the Hajj. Then the group which visits Medina after the Hajj departs from Medina. This will ease the traffic after the Hajj.

Complete optimal solutions for the linear and the non-linear cases are shown in Tables 1 and 2 in the appendix. The optimal solution includes the optimal traffic flow, the upper bound on each arc, the "cost" of each arc in terms of resistance to traffic and the dual evaluators.

TABLE 1

Optimal Solution for the Linear Example

```
PROBLEM TITLE      HAJJ INTERNAL MOVEMENT

OPTIMAL SOLUTION

SOURCES=    77   SINKS=      0   TOTAL NODES=    77   ARCS=    169

OBJ FCN=            35510000   NO. ITER=    85


     ARC      FROM      TO             UNIT    UPPER    LOWER                ARC     DUAL    BASIS
  NUMBER      NODE      NODE           COST    BOUND    BOUND      FLOW     COST      VAR   STATUS

       1      SOUR      JID1              0   200000   200000    200000        0        1
       2      SOUR      MEC1              0   800000   800000    800000        0        0
       3      SOUR      MIN1              0   400000   400000    400000        0       -1
       4      SOUR      MED1              0   200000   200000    200000        0        1
       5      JID1      MEC1              1   200000        0    200000   200000        0      BND
       6      JID1      JID2              1   300000        0         0        0        0      BSC
       7      MEC1      MIN1              1   500000        0    300000   300000        0      BSC
       8      MEC1      MEC2              1  1000000        0    900000   900000        0      BSC
       9      MIN1      MIN2              1   700000        0    700000   700000        0      BSC
      10      MED1      MEC1              1   200000        0    200000   200000        0      BSC
      11      MED1      MED2              1  1000000        0         0        0        0      BSC
      12      JID2      MEC2              1   150000        0         0        0        0
      13      MEC2      MIN2              1   400000        0         0        0        0
      14      MEC2      MEC3              1  1000000        0    900000   900000        0      BSC
      15      MIN2      MIN3              1   700000        0    700000   700000        0      BSC
      16      MED2      MEC2              1   200000        0         0        0        0
      17      MEC3      MIN3              1   500000        0         0        0        0
      18      MEC3      ARF3              1   900000        0    900000   900000       -1      BND
      19      MIN3      ARF3              1   700000        0    700000   700000        0      BSC
      20      ARF3      ARF4              1  2000000        0   1600000  1600000        0      BSC
      21      MEC4      MEC5              1  1000000        0    400000   400000        0      BSC
      22      MIN4      MEC4              1   400000        0    400000   400000     -127      BND
      23      MIN4      MIN5              1   700000        0    500000   500000        0      BSC
      24      ARF4      MZD4              1   900000        0    900000   900000        0      BND
      25      ARF4      ARF5              1   700000        0    700000   700000        0      BSC
      26      MZD4      MIN4              1  2000000        0    900000   900000        0      BSC
      27      MEC5      MEC6              1  1000000        0    900000   900000        0      BSC
      28      MIN5      MEC5              1   500000        0    500000   500000     -127      BND
      29      MIN5      MIN6              1   700000        0    700000   700000     -127      BND
      30      ARF5      MZD5              1   900000        0    700000   700000        0      BSC
      31      MZD5      MIN5              1   700000        0    700000   700000        0      BSC
      32      MEC6      MIN6              1   200000        0         0        0        2
      33      MEC6      MEC7              1  1000000        0    900000   900000        0      BSC
      34      MIN6      MEC6              1   200000        0         0        0        0      BSC
      35      MIN6      MIN7              1   700000        0    700000   700000        0      BSC
```

```
PROBLEM TITLE       MAJJ INTERNAL MOVEMENT

    ARC      FROM      TO              UNIT    UPPER     LOWER                ARC      DUAL     BASIS
  NUMBER     NODE      NODE            COST    BOUND     BOUND      FLOW      COST      VAR     STATUS

      36     MEC7      MIN7               1    250000        0         0         0        2
      37     MEC7      MEC8               1   1000000        0    900000    900000        0       BSC
      38     MIN7      MEC7               1    250000        0         0         0        0
      39     MIN7      MIN8               1    700000        0    700000    700000        0       BSC
      40     MEC8      MIN8               1    200000        0         0         0        2
      41     MEC8      MEC9               1   1000000        0    900000    900000        0       BSC
      42     MIN8      MEC8               1    200000        0         0         0        0
      43     MIN8      MIN9               1    700000        0    700000    700000        0       BSC
      44     MEC9      MEC10              1   1000000        0    900000    900000        0       BSC
      45     MIN9      MEC9               1    500000        0         0         0        0
      46     MIN9      MIN10              1    700000        0    700000    700000        0       BND
      47     JID10     SINK               0     60000        0     60000         0      -14       BND
      48     JID10     JID11              1    300000        0     90000     90000        0       BSC
      49     MEC10     JID10              1    150000        0    150000    150000        0       BND
      50     MEC10     MED10              1    150000        0    150000    150000        0       BND
      51     MEC10     MEC11              1   1000000        0   1000000   1000000        0       BND
      52     MIN10     MEC10              1    400000        0    400000    400000        0       BSC
      53     MIN10     MIN11              1    700000        0    300000    300000        0       BSC
      54     MED10     MED11              1   1000000        0    150000    150000        0       BSC
      55     JID11     SINK               0     80000        0     80000         0      -13       BND
      56     JID11     JID12              1    300000        0    210000    210000        0       BSC
      57     MEC11     JID11              1    200000        0    200000    200000        0       BND
      58     MEC11     MED11              1    200000        0    200000    200000        0       BND
      59     MEC11     MEC12              1   1000000        0    900000    900000        0       BSC
      60     MIN11     MEC11              1    500000        0    300000    300000        0       BSC
      61     MIN11     MIN12              1    700000        0         0         0        0
      62     MED11     MED12              1   1000000        0    350000    350000        0       BSC
      63     JID12     SINK               0     60000        0     60000         0      -12       BND
      64     JID12     JID13              1    300000        0    300000    300000        0       BND
      65     MEC12     JID12              1    150000        0    150000    150000        0       BSC
      66     MEC12     MED12              1    150000        0    150000    150000        0       BND
      67     MEC12     MEC13              1   1000000        0    600000    600000        0       BSC
      68     MIN12     MEC12              1    400000        0         0         0        0       BSC
      69     MED12     MED13              1   1000000        0    500000    500000        0       BSC
      70     JID13     SINK               0     80000        0     80000         0      -11       BND
      71     JID13     JID14              1    300000        0    300000    300000        0       BND
      72     MEC13     JID13              1    200000        0     80000     80000        0       BSC
      73     MEC13     MED13              1    200000        0    100000    100000        0       BSC
      74     MEC13     MEC14              1   1000000        0    420000    420000        0       BSC
      75     MED13     MED14              1   1000000        0    600000    600000        0       BSC
      76     JID14     SINK               0     60000        0     60000         0      -10       BND
      77     JID14     JID15              1    300000        0    300000    300000        0       BND
      78     MEC14     JID14              1    150000        0     60000     60000        0       BSC
      79     MEC14     MED14              1    150000        0         0         0        0
      80     MEC14     MEC15              1   1000000        0    360000    360000        0       BSC
      81     MED14     MED15              1   1000000        0    600000    600000        0       BSC
      82     JID15     SINK               0     80000        0     80000         0       -9       BND
      83     JID15     JID16              1    300000        0    300000    300000        0       BND
      84     MEC15     JID15              1    200000        0     80000     80000        0       BSC
      85     MEC15     MED15              1    100000        0         0         0        0
```

TABLE 1, Cont.

PROBLEM TITLE HAJJ INTERNAL MOVEMENT

ARC NUMBER	FROM NODE	TO NODE	UNIT COST	UPPER BOUND	LOWER BOUND	FLOW	ARC COST	DUAL VAR	BASIS STATUS
86	MEC15	MEC16	1	1000000	0	280000	280000	0	BSC
87	MED15	MEC15	1	100000	0	0	0	2	
88	MED15	JID15	1	100000	0	0	0	1	
89	MED15	MED16	1	1000000	0	530000	530000	0	BSC
90	MED15	SINK	0	70000	0	70000	0	-9	BND
91	JID16	SINK	0	60000	0	60000	0	-8	BND
92	JID16	JID17	1	300000	0	300000	300000	0	BND
93	MEC16	JID16	1	150000	0	60000	60000	0	BSC
94	MEC16	MED16	1	75000	0	0	0	0	
95	MEC16	MEC17	1	1000000	0	220000	220000	0	BSC
96	MED16	MEC16	1	75000	0	0	0	2	
97	MED16	JID16	1	75000	0	0	0	1	
98	MED16	MED17	1	1000000	0	480000	480000	0	BSC
99	MED16	SINK	0	50000	0	50000	0	-8	BND
100	JID17	SINK	0	80000	0	80000	0	-7	BND
101	JID17	JID18	1	300000	0	300000	300000	0	BND
102	MEC17	JID17	1	200000	0	80000	80000	0	BSC
103	MEC17	MED17	1	100000	0	0	0	0	
104	MEC17	MEC18	1	1000000	0	140000	140000	0	BSC
105	MED17	MEC17	1	100000	0	0	0	2	
106	MED17	JID17	1	100000	0	0	0	1	
107	MED17	MED18	1	1000000	0	410000	410000	0	BSC
108	MED17	SINK	0	70000	0	70000	0	-7	BND
109	JID18	SINK	0	60000	0	60000	0	-6	BND
110	JID18	JID19	1	300000	0	300000	300000	0	BND
111	MEC18	JID18	1	150000	0	60000	60000	0	BSC
112	MEC18	MED18	1	75000	0	0	0	0	
113	MEC18	MEC19	1	1000000	0	80000	80000	0	BSC
114	MED18	MEC18	1	75000	0	0	0	2	
115	MED18	JID18	1	75000	0	0	0	1	
116	MED18	MED19	1	1000000	0	360000	360000	0	BSC
117	MED18	SINK	0	50000	0	50000	0	-6	BND
118	JID19	SINK	0	80000	0	80000	0	-5	BND
119	JID19	JID20	1	300000	0	300000	300000	0	BND
120	MEC19	JID19	1	200000	0	80000	80000	0	BSC
121	MEC19	MED19	1	100000	0	0	0	0	
122	MEC19	MEC20	1	1000000	0	0	0	0	
123	MED19	MEC19	1	100000	0	0	0	2	
124	MED19	JID19	1	100000	0	0	0	1	
125	MED19	MED20	1	1000000	0	290000	290000	0	BSC
126	MED19	SINK	0	70000	0	70000	0	-5	BND
127	JID20	SINK	0	60000	0	60000	0	-4	BND
128	JID20	JID21	1	300000	0	240000	240000	0	BSC
129	MEC20	JID20	1	150000	0	0	0	0	BSC
130	MEC20	MED20	1	75000	0	0	0	0	
131	MEC20	MEC21	1	1000000	0	0	0	0	
132	MED20	MEC20	1	75000	0	0	0	2	
133	MED20	JID20	1	75000	0	0	0	1	
134	MED20	MED21	1	1000000	0	240000	240000	0	BSC
135	MED20	SINK	0	50000	0	50000	0	-4	BND

TABLE 1, Cont.

PROBLEM TITLE HAJJ INTERNAL MOVEMENT

ARC NUMBER	FROM NODE	TO NODE	UNIT COST	UPPER BOUND	LOWER BOUND	FLOW	ARC COST	DUAL VAR	BASIS STATUS
136	JID21	SINK	0	80000	0	80000	0	-3	BND
137	JID21	JID22	1	300000	0	160000	160000	0	BSC
138	MEC21	JID21	1	200000	0	0	0	0	BSC
139	MEC21	MED21	1	100000	0	0	0	0	
140	MEC21	MEC22	1	1000000	0	0	0	0	
141	MED21	MEC21	1	100000	0	0	0	2	
142	MED21	JID21	1	100000	0	0	0	1	
143	MED21	MED22	1	1000000	0	170000	170000	0	BSC
144	MED21	SINK	0	70000	0	70000	0	-3	BND
145	JID22	SINK	0	40000	0	40000	0	-2	BND
146	JID22	JID23	1	300000	0	120000	120000	0	BSC
147	MEC22	JID22	1	150000	0	0	0	0	BSC
148	MEC22	MED22	1	75000	0	0	0	0	
149	MEC22	MEC23	1	1000000	0	0	0	0	
150	MED22	MEC22	1	75000	0	0	0	2	
151	MED22	JID22	1	75000	0	0	0	1	
152	MED22	MED23	1	1000000	0	120000	120000	0	BSC
153	MED22	SINK	0	50000	0	50000	0	-2	BND
154	JID23	SINK	0	80000	0	80000	0	-1	BND
155	JID23	JID24	1	300000	0	40000	40000	0	BSC
156	MEC23	JID23	1	200000	0	0	0	0	BSC
157	MEC23	MED23	1	100000	0	0	0	0	
158	MEC23	MEC24	1	1000000	0	0	0	0	
159	MED23	MEC23	1	100000	0	0	0	2	
160	MED23	JID23	1	100000	0	0	0	1	
161	MED23	MED24	1	1000000	0	50000	50000	0	BSC
162	MED23	SINK	0	70000	0	70000	0	-1	BND
163	JID24	SINK	0	40000	0	40000	0	0	BSC
164	MEC24	JID24	1	150000	0	0	0	0	
165	MEC24	MED24	1	75000	0	0	0	0	BSC
166	MED24	MEC24	1	75000	0	0	0	2	
167	MED24	JID24	1	75000	0	0	0	1	
168	MED24	SINK	0	50000	0	50000	0	0	BSC
169	SINK	SOUR	0	1600000	1600000	1600000	0	156	

TABLE 2

Optimal Solution for the Non-linear Example

```
PROBLEM TITLE      HAJJ INTERNAL MOVEMENT NON-EX

OPTIMAL SOLUTION

SOURCES=    78   SINKS=     0   TOTAL NODES=    78   ARCS=    308

OBJ FCN=            26301875   NO. ITER=   329


     ARC      FROM      TO             UNIT    UPPER    LOWER                ARC      DUAL    BASIS
   NUMBER     NODE      NODE           COST    BOUND    BOUND      FLOW      COST      VAR    STATUS

       1      SOUR      JID1              0      200      200       200         0   168796
       2      SOUR      MEC1              0      800      800       800         0   159616
       3      SOUR      MIN1              0      400      400       400         0   160000
       4      SOUR      MED1              0      200      200       200         0   160016
       5      JID1      MEC1            400      100        0       100     40000    -8780     BND
       6      JID1      MEC1          10000      100        0         0         0      820
       7      JID1      JID2            180      150        0       100     18000        0     BSC
       8      JID1      JID2           6000      150        0         0         0     5820
       9      MEC1      MIN1             64      250        0         0         0      448
      10      MEC1      MIN1           4000      250        0         0         0     4384
      11      MEC1      MEC2             16      500        0       500      8000    -1984     BND
      12      MEC1      MEC2           2000      500        0       500   1000000        0     BSC
      13      MIN1      MIN2             33      350        0       350     11550    -2967     BND
      14      MIN1      MIN2           3000      350        0        50    150000        0     BSC
      15      MED1      MEC1            400      100        0       100     40000        0     BSC
      16      MED1      MEC1          10000      100        0         0         0     9600
      17      MED1      MED2             16      500        0       100      1600        0     BSC
      18      MED1      MED2           2000      500        0         0         0     1984
      19      JID2      MEC2            711       75        0        75     53325   -10289     BND
      20      JID2      MEC2          11000       75        0        25    275000        0     BSC
      21      MEC2      MIN2            100      200        0       200     20000     -516     BND
      22      MEC2      MIN2           5000      200        0         0         0     4384
      23      MEC2      MEC3             16      500        0       500      8000    -3536     BND
      24      MEC2      MEC3           2000      500        0       500   1000000    -1552     BND
      25      MIN2      MIN3             33      350        0       350     11550    -2967     BND
      26      MIN2      MIN3           3000      350        0       250    750000        0     BSC
      27      MED2      MEC2            400      100        0       100     40000    -1984     BND
      28      MED2      MEC2          10000      100        0         0         0     7616
      29      MEC3      MIN3             64      250        0       100      6400        0     BSC
      30      MEC3      MIN3           4000      250        0         0         0     3936
      31      MEC3      ARF3             20      450        0       450      9000    -3044     BND
      32      MEC3      ARF3           2200      450        0       450    990000     -864     BND
      33      MIN3      ARF3             33      350        0       350     11550    -2967     BND
      34      MIN3      ARF3           3000      350        0       350   1050000        0     BSC
      35      ARF3      ARF4              4     1000        0      1000      4000     -996     BND
```

TABLE 2, Cont.

PROBLEM TITLE   HAJJ INTERNAL MOVEMENT NON-EX

ARC NUMBER	FROM NODE	TO NODE	UNIT COST	UPPER BOUND	LOWER BOUND	FLOW	ARC COST	DUAL VAR	BASIS STATUS
86	MIN9	MEC9	4000	250	0	0	0	4400	
87	MIN9	MIN10	33	350	0	350	11550	-2967	BND
88	MIN9	MIN10	3000	350	0	250	750000	0	BSC
89	JID10	SINK	0	60	0	60	0	-2316	BND
90	JID10	JID11	180	150	0	80	14400	0	BSC
91	JID10	JID11	6000	150	0	0	0	5820	
92	MEC10	JID10	711	75	0	75	53325	-10289	BND
93	MEC10	JID10	11000	75	0	65	715000	0	BSC
94	MEC10	MED10	711	75	0	75	53325	-10289	BND
95	MEC10	MED10	11000	75	0	35	385000	0	BSC
96	MEC10	MEC11	16	500	0	500	8000	-1984	BND
97	MEC10	MEC11	2000	500	0	450	900000	0	BSC
98	MIN10	MEC10	100	200	0	200	20000	-964	BND
99	MIN10	MEC10	5000	200	0	0	0	3936	
100	MIN10	MIN11	33	350	0	350	11550	-2967	BND
101	MIN10	MIN11	3000	350	0	50	150000	0	BSC
102	MED10	MED11	16	500	0	110	1760	0	BSC
103	MED10	MED11	2000	500	0	0	0	1984	
104	JID11	SINK	0	80	0	80	0	-2136	BND
105	JID11	JID12	180	150	0	100	18000	0	BSC
106	JID11	JID12	6000	150	0	0	0	5820	
107	MEC11	JID11	400	100	0	100	40000	-8780	BND
108	MEC11	JID11	10000	100	0	0	0	820	
109	MEC11	MED11	400	100	0	100	40000	-8616	BND
110	MEC11	MED11	10000	100	0	0	0	984	
111	MEC11	MEC12	16	500	0	500	8000	-1984	BND
112	MEC11	MEC12	2000	500	0	450	900000	0	BSC
113	MIN11	MEC11	64	250	0	200	12800	0	BSC
114	MIN11	MEC11	4000	250	0	0	0	3936	
115	MIN11	MIN12	33	350	0	200	6600	0	BSC
116	MIN11	MIN12	3000	350	0	0	0	2967	
117	MED11	MED12	16	500	0	210	3360	0	BSC
118	MED11	MED12	2000	500	0	0	0	1984	
119	JID12	SINK	0	60	0	60	0	-1956	BND
120	JID12	JID13	180	150	0	115	20700	0	BSC
121	JID12	JID13	6000	150	0	0	0	5820	
122	MEC12	JID12	711	75	0	75	53325	-6649	BND
123	MEC12	JID12	11000	75	0	0	0	3640	
124	MEC12	MED12	711	75	0	75	53325	-6321	BND
125	MEC12	MED12	11000	75	0	0	0	3968	
126	MEC12	MEC13	16	500	0	500	8000	-3064	BND
127	MEC12	MEC13	2000	500	0	500	1000000	-1080	BND
128	MIN12	MEC12	100	200	0	200	20000	-1931	BND
129	MIN12	MEC12	5000	200	0	0	0	2969	
130	MED12	MED13	16	500	0	285	4560	0	BSC
131	MED12	MED13	2000	500	0	0	0	1984	
132	JID13	SINK	0	80	0	80	0	-1776	BND
133	JID13	JID14	180	150	0	135	24300	0	BSC
134	JID13	JID14	6000	150	0	0	0	5820	
135	MEC13	JID13	400	100	0	100	40000	-4060	BND

TABLE 2, Cont.

PROBLEM TITLE HAJJ INTERNAL MOVEMENT NON-EX

ARC NUMBER	FROM NODE	TO NODE	UNIT COST	UPPER BOUND	LOWER BOUND	FLOW	ARC COST	DUAL VAR	BASIS STATUS
36	ARF3	ARF4	1000	1000	0	600	600000	0	BSC
37	MEC4	MEC5	16	500	0	400	6400	0	BSC
38	MEC4	MEC5	2000	500	0	0	0	1984	
39	MIN4	MEC4	100	200	0	200	20000	-118904	BND
40	MIN4	MEC4	5000	200	0	200	1000000	-114004	BND
41	MIN4	MIN5	33	350	0	350	11550	-2967	BND
42	MIN4	MIN5	3000	350	0	150	450000	0	BSC
43	ARF4	MZD4	20	450	0	450	9000	-5176	BND
44	ARF4	MZD4	2200	450	0	450	990000	-2996	BND
45	ARF4	ARF5	33	350	0	350	11550	-2967	BND
46	ARF4	ARF5	3000	350	0	350	1050000	0	BSC
47	MZD4	MIN4	4	1000	0	900	3600	0	BSC
48	MZD4	MIN4	1000	1000	0	0	0	996	
49	MEC5	MEC6	16	500	0	500	8000	-1984	BND
50	MEC5	MEC6	2000	500	0	400	800000	0	BSC
51	MIN5	MEC5	64	250	0	250	16000	-115956	BND
52	MIN5	MEC5	4000	250	0	250	1000000	-112020	BND
53	MIN5	MIN6	33	350	0	350	11550	-115387	BND
54	MIN5	MIN6	3000	350	0	350	1050000	-112420	BND
55	ARF5	MZD5	20	450	0	450	9000	-2180	BND
56	ARF5	MZD5	2200	450	0	250	550000	0	BSC
57	MZD5	MIN5	33	350	0	350	11550	-2967	BND
58	MZD5	MIN5	3000	350	0	350	1050000	0	BSC
59	MEC6	MIN7	400	100	0	25	10000	0	BSC
60	MEC6	MIN7	10000	100	0	0	0	9600	
61	MEC6	MEC7	16	500	0	500	8000	-1984	BND
62	MEC6	MEC7	2000	500	0	375	750000	0	BSC
63	MIN6	MEC7	400	100	0	100	40000	-4200	BND
64	MIN6	MEC7	10000	100	0	0	0	5400	
65	MIN6	MIN7	33	350	0	350	11550	-2967	BND
66	MIN6	MIN7	3000	350	0	250	750000	0	BSC
67	MEC7	MIN8	711	100	0	100	71100	-689	BND
68	MEC7	MIN8	11000	50	0	0	0	9600	
69	MEC7	MEC8	16	500	0	500	8000	-1984	BND
70	MEC7	MEC8	2000	500	0	375	750000	0	BSC
71	MIN7	MEC8	324	125	0	125	40500	-3276	BND
72	MIN7	MEC8	8000	125	0	0	0	4400	
73	MIN7	MIN8	33	350	0	350	11550	-2967	BND
74	MIN7	MIN8	3000	350	0	150	450000	0	BSC
75	MEC8	MIN9	711	100	0	100	71100	-1689	BND
76	MEC8	MIN9	11000	50	0	0	0	8600	
77	MEC8	MEC9	16	500	0	500	8000	-1984	BND
78	MEC8	MEC9	2000	500	0	400	800000	0	BSC
79	MIN8	MEC9	400	100	0	100	40000	-2200	BND
80	MIN8	MEC9	10000	100	0	0	0	7400	
81	MIN8	MIN9	33	350	0	350	11550	-2967	BND
82	MIN8	MIN9	3000	350	0	150	450000	0	BSC
83	MEC9	MEC10	16	500	0	500	8000	-4448	BND
84	MEC9	MEC10	2000	500	0	500	1000000	-2464	BND
85	MIN9	MEC9	64	250	0	0	0	464	

TABLE 2, Cont.

PROBLEM TITLE HAJJ INTERNAL MOVEMENT NON-EX

ARC NUMBER	FROM NODE	TO NODE	UNIT COST	UPPER BOUND	LOWER BOUND	FLOW	ARC COST	DUAL VAR	BASIS STATUS
136	MEC13	JID13	10000	100	0	0	0	5540	
137	MEC13	MED13	400	100	0	100	40000	-3568	BND
138	MEC13	MED13	10000	100	0	0	0	6032	
139	MEC13	MEC14	16	500	0	500	8000	-1984	BND
140	MEC13	MEC14	2000	500	0	300	600000	0	BSC
141	MED13	MED14	16	500	0	385	6160	0	BSC
142	MED13	MED14	2000	500	0	0	0	1984	
143	JID14	SINK	0	60	0	60	0	-1596	BND
144	JID14	JID15	180	150	0	150	27000	-780	BND
145	JID14	JID15	6000	150	0	0	0	5040	
146	MEC14	JID14	711	75	0	75	53325	-1929	BND
147	MEC14	JID14	11000	75	0	0	0	8360	
148	MEC14	MED14	711	75	0	75	53325	-1273	BND
149	MEC14	MED14	11000	75	0	0	0	9016	
150	MEC14	MEC15	16	500	0	500	8000	-1053	BND
151	MEC14	MED15	2000	500	0	150	300000	0	BSC
152	MED14	MED15	16	500	0	460	7360	0	BSC
153	MED14	MED15	2000	500	0	0	0	1984	
154	JID15	SINK	0	80	0	80	0	-636	BND
155	JID15	JID16	180	150	0	80	14400	0	BSC
156	JID15	JID16	6000	150	0	0	0	5820	
157	MES15	JID15	400	100	0	0	0	0	BSC
158	MEC15	JID15	10000	100	0	0	0	7469	
159	MEC15	MED15	1600	50	0	0	0	669	
160	MEC15	MED15	40000	50	0	0	0	39069	
161	MEC15	MEC16	16	500	0	500	8000	-1984	BND
162	MEC15	MEC16	2000	500	0	0	0	0	BSC
163	MED15	MEC15	1600	50	0	0	0	2531	
164	MED15	MEC15	40000	50	0	0	0	40931	
165	MED15	JID15	1600	50	0	10	16000	0	BSC
166	MED15	JID15	40000	50	0	0	0	38400	
167	MED15	MED16	16	500	0	500	8000	-1984	BND
168	MED15	MED16	2000	500	0	30	60000	0	BSC
169	MED15	SINK	0	70	0	70	0	-2236	BND
170	JID16	SINK	0	60	0	60	0	-456	BND
171	JID16	JID17	180	150	0	20	3600	0	BSC
172	JID16	JID17	6000	150	0	0	0	5820	
173	MEC16	JID16	711	75	0	0	0	0	BSC
174	MEC16	JID16	11000	75	0	0	0	10289	
175	MEC16	MED16	1600	50	0	0	0	669	
176	MEC16	MED16	40000	50	0	0	0	39069	
177	MEC16	MEC17	16	500	0	500	8000	-328	BND
178	MEC16	MEC17	2000	500	0	0	0	1656	
179	MED16	MEC16	1600	50	0	0	0	2531	
180	MED16	MEC16	40000	50	0	0	0	40931	
181	MED16	JID16	1600	50	0	0	0	1820	
182	MED16	JID16	40000	50	0	0	0	40220	
183	MED16	MED17	16	500	0	480	7680	0	BSC
184	MED16	MED17	2000	500	0	0	0	1984	
185	MED16	SINK	0	50	0	50	0	-236	BND

TABLE 2, Cont.

PROBLEM TITLE HAJJ INTERNAL MOVEMENT NON-EX

ARC NUMBER	FROM NODE	TO NODE	UNIT COST	UPPER BOUND	LOWER BOUND	FLOW	ARC COST	DUAL VAR	BASIS STATUS
186	JID17	SINK	0	80	0	80	0	-276	BND
187	JID17	JID18	180	150	0	40	7200	0	BSC
188	JID17	JID18	6000	150	0	0	0	5820	
189	MEC17	JID17	400	100	0	100	40000	-147	BND
190	MEC17	JID17	10000	100	0	0	0	9453	
191	MEC17	MED17	1600	50	0	0	0	997	
192	MEC17	MED17	40000	50	0	0	0	39397	
193	MEC17	MEC18	16	500	0	400	6400	0	BSC
194	MEC17	MEC18	2000	500	0	0	0	1984	
195	MED17	MEC17	1600	50	0	0	0	2203	
196	MED17	MEC17	40000	50	0	0	0	40603	
197	MED17	JID17	1600	50	0	0	0	1656	
198	MED17	JID17	40000	50	0	0	0	40056	
199	MED17	MED18	16	500	0	410	6560	0	BSC
200	MED17	MED18	2000	500	0	0	0	1984	
201	MED17	SINK	0	70	0	70	0	-220	BND
202	JID18	SINK	0	60	0	60	0	-96	BND
203	JID18	JID19	180	150	0	0	0	328	
204	JID18	JID19	6000	150	0	0	0	6148	
205	MEC18	JID18	711	75	0	20	14220	0	BSC
206	MEC18	JID18	11000	75	0	0	0	10289	
207	MEC18	MED18	1600	50	0	0	0	997	
208	MEC18	MED18	40000	50	0	0	0	39397	
209	MEC18	MEC19	16	500	0	380	6080	0	BSC
210	MEC18	MEC19	2000	500	0	0	0	1984	
211	MED18	MEC18	1600	50	0	0	0	2203	
212	MED18	MEC18	40000	50	0	0	0	40603	
213	MED18	JID18	1600	50	0	0	0	1492	
214	MED18	JID18	40000	50	0	0	0	39892	
215	MED18	MED19	16	500	0	360	5760	0	BSC
216	MED18	MED19	2000	500	0	0	0	1984	
217	MED18	SINK	0	50	0	50	0	-204	BND
218	JID19	SINK	0	80	0	80	0	-244	BND
219	JID19	JID20	180	150	0	20	3600	0	BSC
220	JID19	JID20	6000	150	0	0	0	5820	
221	MEC19	JID19	400	100	0	100	40000	-147	BND
222	MEC19	JID19	10000	100	0	0	0	9453	
223	MEC19	MED19	1600	50	0	0	0	997	
224	MEC19	MED19	40000	50	0	0	0	39397	
225	MEC19	MEC20	16	500	0	280	4480	0	BSC
226	MEC19	MEC20	2000	500	0	0	0	1984	
227	MED19	MEC19	1600	50	0	0	0	2203	
228	MED19	MEC19	40000	50	0	0	0	40603	
229	MED19	JID19	1600	50	0	0	0	1656	
230	MED19	JID19	40000	50	0	0	0	40056	
231	MED19	MED20	16	500	0	290	4640	0	BSC
232	MED19	MED20	2000	500	0	0	0	1984	
233	MED19	SINK	0	70	0	70	0	-188	BND
234	JID20	SINK	0	60	0	60	0	-64	BND
235	JID20	JID21	180	150	0	0	0	328	

TABLE 2, Cont.

PROBLEM TITLE HAJJ INTERNAL MOVEMENT NON-EX

ARC NUMBER	FROM NODE	TO NODE	UNIT COST	UPPER BOUND	LOWER BOUND	FLOW	ARC COST	DUAL VAR	BASIS STATUS
236	JID20	JID21	6000	150	0	0	0	6148	
237	MEC20	JID20	711	75	0	40	28440	0	BSC
238	MEC20	JID20	11000	75	0	0	0	10289	
239	MEC20	MED20	1600	50	0	0	0	997	
240	MEC20	MED20	40000	50	0	0	0	39397	
241	MEC20	MEC21	16	500	0	240	3840	0	BSC
242	MEC20	MEC21	2000	500	0	0	0	1984	
243	MED20	MEC20	1600	50	0	0	0	2203	
244	MED20	MEC20	40000	50	0	0	0	40603	
245	MED20	JID20	1600	50	0	0	0	1492	
246	MED20	JID20	40000	50	0	0	0	39892	
247	MED20	MED21	16	500	0	240	3840	0	BSC
248	MED20	MED21	2000	500	0	0	0	1984	
249	MED20	SINK	0	50	0	50	0	-172	BND
250	JID21	SINK	0	80	0	80	0	-212	BND
251	JID21	JID22	180	150	0	20	3600	0	BSC
252	JID21	JID22	6000	150	0	0	0	5820	
253	MEC21	JID21	400	100	0	100	40000	-147	BND
254	MEC21	JID21	10000	100	0	0	0	9453	
255	MEC21	MED21	1600	50	0	0	0	997	
256	MEC21	MED21	40000	50	0	0	0	39397	
257	MEC21	MEC22	16	500	0	140	2240	0	BSC
258	MEC21	MEC22	2000	500	0	0	0	1984	
259	MED21	MEC21	1600	50	0	0	0	2203	
260	MED21	MEC21	40000	50	0	0	0	40603	
261	MED21	JID21	1600	50	0	0	0	1656	
262	MED21	JID21	40000	50	0	0	0	40056	
263	MED21	MED22	16	500	0	170	2720	0	BSC
264	MED21	MED22	2000	500	0	0	0	1984	
265	MED21	SINK	0	70	0	70	0	-156	BND
266	JID22	SINK	0	40	0	40	0	-32	BND
267	JID22	JID23	180	150	0	0	0	328	
268	JID22	JID23	6000	150	0	0	0	6148	
269	MEC22	JID22	711	75	0	20	14220	0	BSC
270	MEC22	JID22	11000	75	0	0	0	10289	
271	MEC22	MED22	1600	50	0	0	0	997	
272	MEC22	MED22	40000	50	0	0	0	39397	
273	MEC22	MEC23	16	500	0	120	1920	0	BSC
274	MEC22	MEC23	2000	500	0	0	0	1984	
275	MED22	MEC22	1600	50	0	0	0	2203	
276	MED22	MEC22	40000	50	0	0	0	40603	
277	MED22	JID22	1600	50	0	0	0	1492	
278	MED22	JID22	40000	50	0	0	0	39892	
279	MED22	MED23	16	500	0	120	1920	0	BSC
280	MED22	MED23	2000	500	0	0	0	1984	
281	MED22	SINK	0	50	0	50	0	-140	BND
282	JID23	SINK	0	80	0	80	0	-180	BND
283	JID23	JID24	180	150	0	20	3600	0	BSC
284	JID23	JID24	6000	150	0	0	0	5820	
285	MEC23	JID23	400	100	0	100	40000	-147	BND

TABLE 2, Cont.

PROBLEM TITLE HAJJ INTERNAL MOVEMENT NON-EX

ARC NUMBER	FROM NODE	TO NODE	UNIT COST	UPPER BOUND	LOWER BOUND	FLOW	ARC COST	DUAL VAR	BASIS STATUS
286	MEL23	JID23	10000	100	0	0	0	9453	
287	MEC23	MED23	1600	50	0	0	0	997	
288	MEC23	MED23	40000	50	0	0	0	39397	
289	MEL23	MEL24	16	500	0	20	320	0	BSC
290	MEL23	MEC24	2000	500	0	0	0	1984	
291	MED23	MEC23	1600	50	0	0	0	2203	
292	MED23	MEC23	40000	50	0	0	0	40603	
293	MED23	JID23	1600	50	0	0	0	1656	
294	MED23	JID23	40000	50	0	0	0	40056	
295	MED23	MED24	16	500	0	50	800	0	BSC
296	MED23	MED24	2000	500	0	0	0	1984	
297	MED23	SINK	0	70	0	70	0	-124	BND
298	JID24	SINK	0	40	0	40	0	0	BSC
299	MEL24	JID24	711	75	0	20	14220	0	BSC
300	MEL24	JID24	11000	75	0	0	0	10289	
301	MEL24	MED24	1600	50	0	0	0	997	
302	MEL24	MED24	40000	50	0	0	0	39397	
303	MED24	MEC24	1600	50	0	0	0	2203	
304	MED24	MEL24	40000	50	0	0	0	40603	
305	MED24	JID24	1600	50	0	0	0	1492	
306	MED24	JID24	40000	50	0	0	0	39892	
307	MED24	SINK	0	50	0	50	0	-108	BND
308	SINK	SOUR	0	1600	0	1600	0	0	BSC

REFERENCES

[1] Abdul-Rauf, M., "Pilgrimage to Makkah," National Geographic, Vol. 154, No. 5, November 1978, pp. 581-560.

[2] Charnes, A. and Cooper, W.W., Management Models and Industrial Application of Linear Programming, Vol. 1-11, New York: John Wiley and Sons Inc., 1961.

[3] Elfghi, M., Hajj in Islam "El Hajj Fee El Islam", series of Islamic Research, November, 1972.

[4] Hajj Research Center, Holy Areas Vehicle Cordon 1395 A.H., King Abdulaziz University, Jeddah, Kingdom of Saudi Arabia.

[5] Hussien, M., El Hajj, the Library of Anglo Egyption.

[6] Sabig, S., Fagh El Suna, the House of Arabic Books, Beurit, Lebenan, Vol. 1, Sections 1, 2, 3, 4, 5.

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This paper develops a non-linear dynamic capacitated network model for planning the movements of pilgrims in the Hajj, one of the world's largest mass movements, according to religious ritual, which would assist in minimizing traffic congestion and the overcrowding of the holy sites. A new non-linear representation of congestion with convenient mathematical properties is made. The model is effective in producing quantitative and qualitative background for general policy decisions on the Hajj transportation.

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