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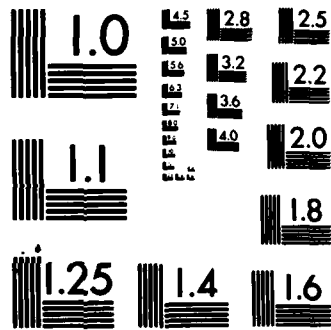
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Department of Engineering Sciences and Applied Mathematics
Northwestern University
Evanston, IL 60201

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Edward L. Reiss
Principal Investigator

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The main thrust of our research program has been the development and applications of asymptotic and perturbation methods for analyzing: the stability and dynamics of elastic structures, fluid flow, and other nonlinear problems; and for problems of scattering of acoustic, electromagnetic and other waves. The work is summarized in the following papers which have been published, accepted for publication, submitted for publication, or are in preparation for publication.

1. Dellwo, D., Keller, H. B., Matkowsky, B. J., and Reiss, E. L., On the Birth of Isolas, SIAM J. Appl. Math., 42 (1982), pp. 956-963.

Isolas are isolated, closed curves of solution branches of nonlinear problems. They have been observed to occur in the buckling of elastic shells, the equilibrium states of chemical reactors, and other problems. In this paper, we present a theory to analytically describe the structure of a class of isolas. Specifically, we consider isolas that shrink to a point as a parameter k of the problem, approaches a critical value k_0 . The point is referred to as an **isola center**. Equations that characterize the isola centers are given. Then solutions are constructed in a neighborhood of the isola centers, by a perturbation expansion in a small parameter ϵ , that is proportional to $(k - k_0)^\alpha$, with α appropriately chosen. The theory is applied to problems in chemical reactor theory, and nonlinear oscillations.

2. Grotberg, J. B., and Reiss, E. L., A Subsonic Flutter Anomaly, J. Sound and Vibration, 80 (1982), pp. 444-446.

Experiments on the flutter of cylindrical tubes conveying subsonic flows reveal that the tubes first partially collapse into a static; nearly flat state at a critical flow velocity. Then at a second and larger critical flow velocity the tube flutters about this flat state. These results are at variance with existing theoretical studies which show that the nearly flat state loses stability by divergence. In these theories the fluid is assumed to be inviscid and incompressible. We have resolved this anomaly by including fluid damping in the model. Then the nearly flat state loses stability by flutter. It is low frequency flutter for small damping.

3. Strumolo, G., and Reiss, E. L., Poiseuille Channel Flow with Driven Walls, J. Fluid Mech., submitted.

The effects of a prescribed wall motion on the nonlinear stability of Poiseuille channel flow are studied by an asymptotic method. The motion represents a traveling wave in the upper wall of the channel. It can be considered either as a disturbance to the flow that results from experimental imperfections, or as an externally imposed motion. The frequency of this disturbance depends on the Reynolds number of the flow. In the classical Poiseuille channel flow problem, the walls are assumed to be rigid. Then a periodic solution bifurcates from the laminar, Poiseuille flow at the critical Reynolds number, R_c . In the resonance

case, the wall motion destroys the bifurcation. The transition from the laminar state then occurs by jumping at new critical Reynolds numbers. These Reynolds numbers either exceed, or are below R_c , depending on the variation of the wall motion frequency with the Reynolds number. Thus the wall motions can stabilize or destabilize the laminar flow, and hence they can be used to control the transition to turbulence. In the non-resonance cases, the bifurcation is preserved and the critical Reynolds number is slightly perturbed.

4. Reiss, E. L., Cascading Bifurcation, SIAM J. Appl. Math., 43 (1983), pp.57-65.

Sequential bifurcation of solutions of nonlinear equations, as the bifurcation parameter increases, is called cascading bifurcation. It has been proposed as a mechanism to describe the transition from laminar to turbulent fluid flows, and as a mechanism for chemical and biochemical morphogenesis and pattern formation. The creation of cascading bifurcation by the splitting of multiple primary bifurcation points is described. A perturbation method is employed in the analysis. Primary, secondary, tertiary, etc. bifurcation points and states are determined for a specific nonlinear boundary value problem.

5. Sinay, L. R., and Reiss, E. L., Secondary Transitions in Panel Flutter: A Simple Model, to be submitted.

A simple, two degree of freedom mechanical model of panel flutter is presented. A perturbation method is employed to determine the secondary bifurcation of flutter states from divergence states, and divergence states from flutter states. The latter suggests a new method for controlling flutter by allowing small amplitude flutter and then increasing the flow velocity until secondary bifurcation into a divergence state occurs.

6. Grotberg, J. B., and Reiss, E. L., Subsonic Flapping Flutter, J. Sound and Vibration, in press.

A mathematical model of two-dimensional flow through a flexible channel is analyzed for its stability characteristics. Linear theory shows that fluid viscosity, modelled by a Darcy friction factor, induces flutter instability when the dimensionless fluid speed, S , attains a critical flutter speed, S_0 . This is in qualitative agreement with experimental results, and it is at variance with previous analytical studies where fluid viscosity was neglected and divergence instability was predicted. The critical flutter speed and the associated critical flutter frequency depend on three other dimensionless parameters: the ratio of fluid to wall damping; the ratio of wall to fluid mass; and the ratio of wall bending resistance to elastance. Non-linear theory predicts stable, finite amplitude flutter for $S > S_0$, which increases in frequency and amplitude as S increases. Both symmetric and antisymmetric modes of deformation are discussed.

7. Magnan, J., and Reiss, E. L., Double-Diffusive Convection and λ -Bifurcation, submitted, Physical Review A.

We consider convection in a rectangular box where two "substances" such as temperature and a solute are diffusing. The solutions of the Boussinesq theory depend on the thermal and solute Rayleigh numbers R_T and R_S in addition to other geometrical and other fluid parameters. The conduction state is unstable with respect to steady (periodic) convection states if R_S is sufficiently small (large). The boundary between steady and periodic convection occurs at a critical value $R_S = \bar{R}_S$. The linearized theory at $R_S = \bar{R}_S$ is characterized by the frequency $\omega = 0$ appearing as a root of algebraic multiplicity two and geometrical multiplicity one. Asymptotic approximations of the solutions are obtained for R_S near \bar{R}_S by the Poincare-Linstedt method. It is found that a periodic (steady) solution bifurcates supercritically (subcritically) from the conduction state at $R_T = R_T^D(R_S^S)$, where $R_T^D < R_T^S$. The periodic branch joins the steady branch with an "infinite period bifurcation" at $R_T = R_b$, where $R_T^D < R_b < R_T^S$. The shape of the resulting bifurcation diagram suggests the term, bifurcation. The infinite periodic bifurcation corresponds to a heteroclinic orbit in the appropriate amplitude phase plane. The periodic (steady) convection states are stable (unstable), as we demonstrate by solving the initial value problem employing the multi-scale method.

8. Erneux, T., and Reiss, E. L., Brussellator Isolates, SIAM J. Appl. Math., 43 (1983), pp. 1240-1246.

The Brussellator is a simple chemical model describing pattern formation by bifurcation of solutions. For a one-dimensional system, the bifurcation parameter is related to the ratio of the square of the size of the system to a diffusion coefficient. It has been observed from numerical computations, that there are closed branches of steady state solutions, which are called isolas, that connect neighboring bifurcation points. In addition, these isolas depend on a parameter B . As B approaches a critical value B^0 the neighboring bifurcation points coalesce, so that the isola shrinks to a point. We employ a perturbation method to obtain asymptotic expansions of the isolas for B near B^0 . Implications of the results for pattern formation are discussed.

9. Erneux, T., and Reiss, E. L., Splitting of Steady Multiple Eigenvalues May Lead to Periodic Bifurcation, SIAM J. Appl. Math., 43 (1983), pp. 613-624.

A general bifurcation problem is considered that depends on two parameters in addition to the bifurcation parameter λ . It is assumed that all primary bifurcation states correspond to steady solutions and that they branch supercritically. Then it is shown that for a range of system parameters, and near a triple primary bifurcation point the following cascade of bifurcations from the minimum primary bifurcation state is possible. As λ increases there is secondary and then tertiary bifurcation to steady states, and finally Hopf bifurcation at a quarternary bifurcation point. Related transitions have been observed experimentally in thermal convection and other hydrodynamic stability problems. In addition, we show that Hopf bifurcation near a double primary bifurcation point is not possible when both primary states near the double point bifurcate supercritically. However, it is possible near such a double bifurcation point if imperfections are included in the formulation, as we demonstrate.

10. Kriegsmann, G. A., Exploiting the Limiting Amplitude Principle to Numerically Solve Scattering Problems, Wave Motion 4 (1982), pp. 371-380.

A numerical method for solving reduced wave equations is presented. The technique is basically a relaxation scheme which exploits the limiting amplitude principle. A modified radiation condition at "infinity" is also given. The method is tested on two model problems: the scattering of plane shallow water waves off shoals and the scattering of plane acoustic waves off a sound-soft cylinder imbedded between two homogeneous but different half spaces. The numerical solutions exhibit correct refractive and diffractive effects at moderate frequencies.

11. Magnan, J. F., and Goldstein, R. A., Perturbed Bifurcation of Stationary Striations in a Contaminated, Nonuniform Plasma, SIAM J. Appl. Math., 43 (1983), pp. 16-30.

A cylindrical, weakly ionized and collision dominated neon plasma can be described by a system of nonlinear, parabolic reaction-diffusion equations for the electron and metastable atom axial densities. The equations exhibit a bifurcation from a uniform to a striated state at a critical length of the plasma column. The sharp transition between states predicted by the theory is in contrast with the smooth transition observed in experiments. We apply the theory of singular perturbation of bifurcations to show that small inhomogeneities in the plasma, such as those caused by nonuniform heating and contamination, are sufficient to qualitatively explain the experimental results. We observe that a steady, axial magnetic field in the plasma can also produce a smooth transition.

12. Grotberg, J. B., Louhi, B., and Reiss, E. L., Secondary Bifurcation of Quasiperiodic Solutions Can Lead to Period Multiplication, submitted.

It is shown using perturbation and asymptotic methods that secondary bifurcation of quasi-periodic solutions from periodic solutions occurs for a model problem. The model is a coupled system of two van der Pol-Duffing oscillators. For special values of the detuning parameters the secondary states are periodic. Then periodic multiplication of solutions can occur at the secondary bifurcation point.

13. Kriegsmann, G. A., and Reiss, E. L., Low Frequency Scattering by Local Inhomogeneities, SIAM J. Appl. Math., 43 (1983), pp. 923-934.

An asymptotic expansion which is uniformly valid in space is obtained for the low frequency scattering of a plane wave incident on a localized inhomogeneity. The scattering region, which may be simply or multiply (collection of scatterers) connected, has a characteristic length which is small compared with the wave length of the incident wave. The index of refraction n is unity outside the scattering region and it is arbitrary inside the region. The method of matched asymptotic expansions is used in the analysis. The Born approximation is shown to agree with

the uniform expansion in the far and the near fields. The leading term in the uniform expansion is a linear functional of $1 - n^2$. Thus, statistics of the scattered field are easily evaluated from the statistics of n , when n is a random process. The method is then applied to the low frequency scattering of an acoustic plane wave by localized inhomogeneities in the density and the index of refraction. Finally, the scattering by a plane which is acoustically hard except for a small impedance spot, is analyzed by the same method.

14. Morawetz, C. A., and Kriegsmann, G. A., The Calculations of an Inverse Potential Problem, SIAM J. Appl. Math., 43 (1983).

A method is developed for solving an inverse problem for the wave equation with potential where the object is to find the potential given Cauchy data on a time-like surface. The computation is carried out with one space variable by an iterative procedure. The point of this method is that it can be extended to higher dimensions in principle. A coarse mesh finite difference scheme is used which yields fair accuracy.

15. Kriegsmann, G. A., and Reiss, E. L., Acoustic Propagation in Wall Shear Flows and the Formation of Caustics, J. Acoust. Soc. Amer., 74 (1983), pp. 1869-1879.

The propagation of acoustic waves from a high frequency line source in a two dimensional parallel shear flow adjacent to a rigid wall is analyzed by a ray method. The leading term in the resulting expansion is equivalent to the geometrical acoustics theory of classical wave propagation. It is shown that energy from the source is radiated either directly to the far field, or by first reflecting from the wall. In addition, energy is trapped in a channel adjacent to the wall and downstream from the source. The rays in this channel form an infinite sequence of caustics progressing downstream. Since the geometrical acoustics approximation is invalid on and near caustics, a boundary layer method is employed to determine the acoustics field near the caustics. It is shown that the amplitude of the field on and near the caustics is $k^{2/3}$ larger than the geometrical acoustics field for large k . Here K is a dimensionless wave number of the source. Moreover, the vorticity of the acoustics field in the caustic regions is $k^{7/6}$ larger than the geometrical acoustics field. The possible significance of these results for vehicle self-noise and the formation of turbulent spots in the sub-layer of a turbulent boundary layer is discussed.

16. Erneux, T. and Reiss, E. L., Singular Secondary Bifurcation, SIAM J. Math., in press.

A bifurcation problem is analyzed for a Brussellator boundary value problem, which is a typical reaction-diffusion system. The bifurcation parameter λ is proportional to the length of the system. We employ previously developed perturbation methods to analyze the secondary bifurcation of steady solutions that arise from the splitting of multiple primary bifurcation points. The resulting bifurcation equations are nonlinear algebraic equations to determine the amplitudes of the solutions. The coefficients in these equations depend upon the system parameters in the Brussellator problem, such as the two prescribed constant reactants A and B. For critical values of these parameters the solutions of the algebraic system are singular, and hence the perturbation method is invalid for parameter values at and near these critical values. A new perturbation method is employed yielding new branches of steady solutions and possible tertiary bifurcations to time-periodic solutions. Thus the analysis of singularities in the bifurcation equation reveals new mechanisms for the occurrence of steady and time-periodic solutions. Some implications of the results for chemical morphogenesis are discussed.

17. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., Acoustic Scattering by Baffled Membranes, J. Acoust. Soc. of America, in press.

A flexible membrane is set in an infinite plane baffle. The plane separates an acoustic fluid from a vacuum. A time harmonic wave is incident from the fluid on the membrane. When the frequency of the incident wave is not close to an in-vacuo resonant frequency of the membrane, the reaction of the fluid on the membrane is small. However near a resonant frequency the fluid-membrane coupling is significant. We use the method of matched asymptotic expansions to obtain an asymptotic expansion of the scattered field. It is uniformly valid in the incident frequency. The expansion parameter $\epsilon \ll 1$ is the ratio of the fluid and membrane densities. The outer expansion, valid away from resonance, is $O(\epsilon)$. The inner expansion valid near resonance is of order unity. The fluid loading is shown to have the effect of decreasing the resonant frequencies from those of the in-vacuo membrane. Simple and double resonant frequencies are analyzed. However, the method is applied to normal incidence of a plane wave on a circular membrane.

18. Erneux, T., Matkowsky, B. J. and Reiss, E. L., Singular Bifurcation in Reaction-Diffusion Systems, Comm. at the conference on Modelling of Patterns in Space and Time, Heidelberg, Germany, July 1983. To appear in Lecture Notes in Biomathematics.

Bifurcation Theory is a study of the branching of solutions of equations as a parameter λ , called the bifurcation parameter, is varied. The branching, or bifurcation points, are singular points of the solutions. In Bifurcation Theory solutions are analyzed near bifurcation points. In this theory there is usually a distinguished solution which is called the basic state. Typically, it exists for all values of λ and it is usually determined by the physics, or biology that the bifurcation problem models. The bifurcation points of the basic state are the primary bifurcation points and the solutions branching from them, other than the basic state, are the primary bifurcation states. These states are determined analytically by asymptotic methods, such as the Poincare-Linstedt or other equivalent methods. The results of these asymptotic analyses yield a system of equations, simpler than the original bifurcation problem, to determine the amplitudes of the primary bifurcation states. They are called the primary amplitude equations. The primary bifurcation states are usually more complex spatially and/or temporally than the basic state, thus initiating a pattern formation. Similarly, solutions of the bifurcation problem, other than the basic state, that branch from the primary bifurcation states are secondary bifurcation states and the corresponding branching points are secondary bifurcation points. The secondary bifurcation states are frequently more complex than the primary bifurcation states, thus continuing the pattern formation. More generally, there may be a sequence of bifurcations whose corresponding solutions are increasingly more complex spatially and temporally. We have referred to this elsewhere as cascading bifurcation. The idea of cascading bifurcation has been proposed as a possible explanation of the transition from laminar to turbulent flows in certain hydrodynamic stability problems as a flow parameter λ , such as the Reynolds number, increases. Similarly, cascading bifurcations have been offered as a theoretical explanation of biological and biochemical morphogenesis. In many problems the primary bifurcation points, which we denote by $\lambda_1(p), \lambda_2(p), \dots$, depend on a vector p of auxiliary parameters that occur in the original bifurcation (reaction-diffusion) problem. It is customary to analyze secondary and cascading bifurcation by considering the additional singular situation when two or more primary bifurcation points coincide as the vector $p \rightarrow p_0$ to produce a multiple primary bifurcation point. Then asymptotic and other methods are employed to determine the cascading bifurcation of solutions, where the small parameter in the asymptotic analysis is a monotone function of $|p-p_0|$. The results of the analysis yield amplitude equations for the secondary and higher order states. It has recently been recognized in certain reaction-diffusion problems that the primary or secondary amplitude equations are singular i.e. they are not uniquely solvable as p approaches special values p_1 . We refer to this situation as singular bifurcation (primary or secondary) or more generally singular cascading bifurcation. Then the asymptotic expansions that have been obtained are not uniformly valid as $p \rightarrow p_1$.

New asymptotic expansions are then obtained that are uniformly valid in $|p-p_1|$. An analysis of the resulting amplitude equations reveals new mechanisms for the appearance of additional steady and periodic solutions of the original bifurcation problem. We demonstrate this phenomenon of singular bifurcation by briefly describing two examples of reaction-diffusion problems. The problems we consider are a Brusselator boundary value problem and a model reaction-diffusion problem whose solutions possess propagating fronts. The study of the latter model was motivated by investigations of flame front propagation in combustible fluids.

19. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., An Optical Theorem for Acoustic Scattering by Baffled Flexible Surfaces, J. Sound and Vib., in press.

The classical optical theorem for scattering by compact obstacles is a forward scattering theorem. That is, the total cross-section of the obstacle is proportional to the imaginary part of the far field directivity factor evaluated in the forward scattering direction. We derive an analogous theorem for the scattering of acoustic waves by baffled membranes and plates. In this "optical" theorem the directivity factor is evaluated in the direction of the specularly reflected wave, so that it is a reflected scattering theorem.

20. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., Scattering by Penetrable Acoustic Targets, Wave Motion, in press.

An acoustic target of constant density ρ_t and variable index of refraction is imbedded in a surrounding acoustic fluid of constant density ρ_a . A time harmonic wave propagating in the surrounding fluid is incident on the target. We consider two limiting cases of the target where the parameter $\epsilon \equiv \rho_a/\rho_t \rightarrow 0$ (the nearly rigid target) or $\epsilon \rightarrow \infty$ (the nearly soft target). When the frequency of the incident wave is bounded away from the "in-vacuo" resonant frequencies of the target, the resulting scattered field is essentially the field scattered by the rigid target for $\epsilon = 0$ or the soft target if $\epsilon \rightarrow \infty$. However, when the frequency of the incident wave is near a resonant frequency, the target oscillates and its interaction with the surrounding fluid produces peaks in the scattered field amplitude. In this paper we obtain asymptotic expansions of the solutions of the scattering problems for the nearly rigid and the nearly soft targets as $\epsilon \rightarrow 0$ or $\epsilon \rightarrow \infty$, respectively, that are uniformly valid in the incident frequency. The method of matched asymptotic expansions is used in the analysis. The outer and inner expansions correspond to the incident frequencies being far or near to the resonant frequencies, respectively. We have applied the method only to simple resonant frequencies, but it can be extended to multiple resonant frequencies. The method is applied to the incidence of a plane wave on a nearly rigid sphere of constant index of refraction. The far field expressions for the scattered fields, including the total scattering cross-sections, that are obtained from the asymptotic method and from the partial wave expansion of the solution are in close agreement for sufficiently small values of ϵ .

21. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., An Asymptotic Analysis of Acoustic Scattering by Nearly Rigid or Soft Objects, Proc. of Symp. on Wave Phenomena (1983), in press.
22. Kriegsmann, G. A., Norris, A. N. and Reiss, E. L., Can Acoustic Focusing Generate Turbulent Spots?, Proc. of Symp. on Wave Phenomena (1983), in press.
23. Proter, M. B. and Reiss, E. L., A Numerical Method for Acoustic Normal Modes for Shear Flows, J. Sound and Vib., submitted.

The normal modes and their propagation numbers for acoustic propagation in wave guides with flow are the eigenvectors and eigenvalues of a boundary value problem for a non-standard Sturm Liouville problem. It is non-standard because it depends nonlinearly on the eigenvalue parameter. (In the classical problem for ducts with no flow, the problem depends linearly on the eigenvalue problem). In this paper we present a method for the fast numerical solution of this problem. It is a generalization of a method that was developed for the classical problem. A finite difference method is employed that combines well known numerical techniques and a generalization of the Sturm sequence method to solve the resulting algebraic eigenvalue problem. Then a modified Richardson extrapolation method is used that dramatically increases the accuracy of the computed eigenvalues. The method is then applied to two problems. They correspond to acoustic propagation in the ocean in the presence of a current, and to acoustic propagation in shear layers over flat plates.

24. Magnan, J. and Reiss, E. L., Rotating Thermal Convection: Quasi-Periodicity and Escape, in preparation.
25. Erneux, T., Mandel, P. and Magnan, J. F., Quasi-Periodicity in Lasers with Saturable Absorbers, Phys. Rev. (A), in press.

In this paper, we consider the mean-field equations for the laser with a saturable absorber (LSA) and concentrate on the low-intensity solutions. We show that the LSA equations may admit two successive bifurcations: the first bifurcation corresponds to the transition from the zero-intensity state to time-periodic intensities and is a Hopf bifurcation. The second bifurcation corresponds to the transition from these time-periodic intensities to quasi-periodic intensities which are characterized by two incommensurable frequencies. In order to describe these transitions, we investigate a particular limit of the parameters and propose a new perturbation method for solving the LSA equations. We give analytical conditions for the existence of both the primary and secondary bifurcations.

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