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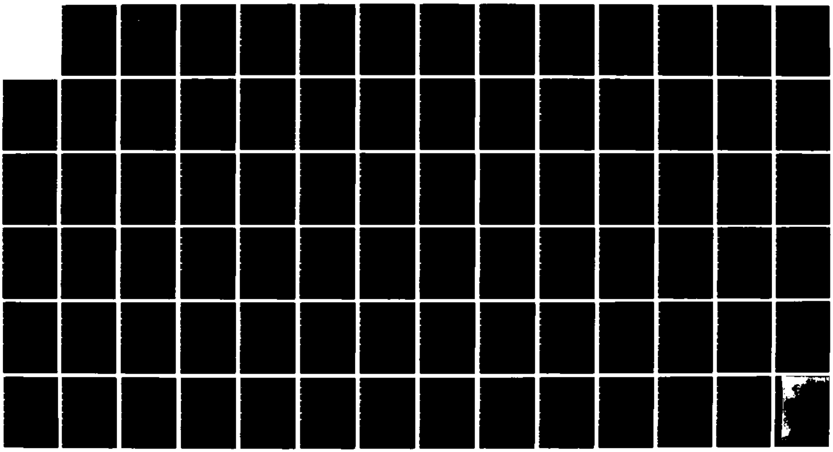

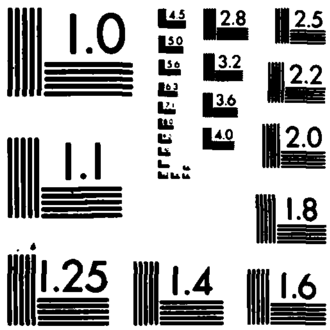

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NAVAL CIVIL ENGINEERING LABORATORY
Port Hueneme, California

Sponsored by
NAVAL FACILITIES ENGINEERING COMMAND

A MATHEMATICAL MODEL FOR CALCULATING CURRENT-INDUCED LOADS ON MOORED VESSELS USING FREE-STREAMLINE AND STRIP THEORIES

February 1984

An Investigation Conducted by
ALTA MAGNA TECH INC.
Pasadena, California

N62474-82-C-8271

METRIC CONVERSION FACTORS

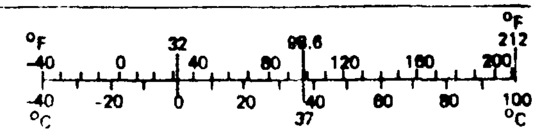
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2,000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1,000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report details a mathematical model for calculating the steady forces and yaw moment on a moored vessel induced by an incident current from any oblique angle. For oblique flows the lateral forces and yaw moment are modelled using a free-streamline strip theory which accommodates arbitrary mono-hull shapes, end effects, finite water depths, horizontally and

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vertically sheared current profiles, and wave radiation effects. For near head-on and beam-on flows the longitudinal force is modelled using conventional ship resistance and low aspect ratio airfoil relationships.

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EXECUTIVE SUMMARY

This report presents the theoretical basis of a mathematical model which is capable of characterizing and quantifying the hydrodynamic forces and yaw moment induced by a current on a moored ship.

In Part I, the behavior of the flow induced by a current on a floating vessel (any monohull shape) has been investigated under the assumptions that (i) the fluid is inviscid and incompressible, (ii) deep water conditions are assumed, and (iii) free surface effects (such as wave making and scattering of possible incident waves) are neglected. All possible current angles relative to the ship centerplane are included. On this premise the necessary mathematical formulas are provided herein for predicting the longitudinal and transverse forces and yaw moment due to an incident current for any given hull and any incident angle. The theoretical model is constructed with application and further development of free-streamline theory, strip theory and low-aspect-ratio wing theory for the overall solution. Various physical mechanisms have been elucidated for their relative contributions to the resultant forces and yaw moment under different operating conditions including in particular the different ranges of incidence angle of the current.

A synopsis of this idealized theoretical model is given in the Appendix of Part I.

In Part II several extensions to that theory are presented to encompass more general conditions that may prevail in practice, such as sheared flow, finite water depth, and wave-making resistance. These extensions include: the first order correction for the more general case when the free stream velocity has a (weakly) sheared distribution in both the vertical and horizontal directions; the contribution and effect of the rudder force; wave radiation; and the effects of shallow water.

A summary of the complete mathematical model is presented in the Conclusions and Discussion section of Part II.

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INTRODUCTION

This report presents new research results achieved under the contract N62474-82-C-8271 sponsored by the Naval Civil Engineering Laboratory (NCEL). The primary objective of the study is to assist NCEL in the development of a general purpose mooring analysis capability. This capability is based primarily on computer models which provide solutions for the static and dynamic responses of an arbitrarily moored surface vessel subject to given wind, wave, and current loads.

For the design of mooring system and dynamic positioning (for localized deep sea operation, for instance) of a ship, it is fundamentally important to attain a valid estimate of the external forces and moments induced on the vessel by the motion of the fluids surrounding it. There are three kinds of fluid mechanical forces and moment induced by the motion of the surrounding fluids--namely, those due to wind, wave and current loadings. The methodology acquired by Owens and Palo (1982) at NCEL for estimating the wind load has proven satisfactory. The so-called 'drift force' induced by incident waves is generally rather insignificant under the circumstances of NCEL's specified needs. In sharp contrast, the current load is known to be of primary importance, particularly in some regions where surface currents up to 3 to 5 knots (or even greater) may prevail. But the existing knowledge about current loads and the very few experimental results available (see Remery et al., 1973; English and Wise, 1975, and as reviewed in a survey article by Palo and Owens, 1982) are insufficient to allow them to be reliably applied to wide classes of ships. We agree with the view held by the

NCEL research team that the few crude methods available for estimating the current-induced loads are inadequate for several reasons. First, for those methods based on small-scale experimental results, the information is insufficient for users to make reliable applications. Second, these methods have questionable assumptions that appear in conflict with physically expected scaling behavior. Further, until recently there had been no full-scale tests designed for validating these methods. A series of full-scale tests was carried out by the NCEL research team (as has been discussed by Palo, 1982) forming the commencing phase of an important task (devoted to full-scale tests) which deserves strong support for further development. In view of the urgent need of conducting theoretical investigations in parallel to the NCEL experimental program, the present project has been undertaken with efforts concentrated on the characterization and prediction of the current loads under a set of specified operating conditions.

Current force is a hydrodynamic force which depends on the magnitude and incidence angle of the water stream relative to the moored ship, the characteristic Reynolds number and Froude number, as well as the geometric shape of the ship hull. The present investigation has been pursued with the primary objective of providing a mathematical model and methodology of general validity and practical value.

The distribution of hydrodynamic forces induced by a current on the vessel can be divided into several components:

(i) the pressure distribution associated with the wake-flow of a beam-on current leading to a strong transverse form drag which in turn results in lift and drag forces and a yaw moment,

(ii) the pressure distribution due to a vortex-sheet shedding flow of a circulatory nature when the incidence angle of the current is sufficiently small,

(iii) the frictional force, of a viscous origin, along the hull surface,

(iv) the drag contributed by the various appendages such as propellers, rudder, and bulbous bow and stern.

Of these components the longitudinal frictional drag is generally small compared with the lateral force component at finite incidence angles. While the forces contributed by various appendages can be considerable, they are not emphasized here for to do so further efforts will be required.

EQUATIONS OF MOTION

A ship, which is free to move as a rigid body except for its being constrained to a one-point mooring, has six degrees of freedom, with three in translation along its three principal axes of motion and three in rotation about these three axes. It is convenient to employ two cartesian coordinate systems as shown in Figure 1. The (x_o, y_o, z_o) system is fixed in an absolute inertial frame of reference K_o, with the origin O_o situated at the point of mooring (assumed to be fixed in frame K_o), and the z_o-axis pointing vertically upward. The (x,y,z) system is attached to the body frame of reference K, with the origin fixed at the center of gravity of the ship. The current velocity in frame K_o is given, denoted by $\underset{\sim}{U}_c$, and is assumed (at this stage) to be uniform over the length scale of the ship, steadily moving parallel to the horizontal plane. The prospects that the current velocity $\underset{\sim}{U}_c$ may have a spatial gradient over the length of ship dimension, especially when it forms a sheared profile in vertical and horizontal directions under the influence of shallow water and markedly three-dimensional bathymetry can be admitted by further modification of the method presented here. Such new physical effects can be significant in inducing new types of forces and moments; but they will be deferred for a future study. Nevertheless, temporal variations of the current velocity can be admitted on the assumption that they are slowly varying when compared with the characteristic time of the ship's natural oscillation and hence can be regarded as quasi-steady.

The motion of the ship is described by the velocity $\underset{\sim}{U}_b$ of its center of gravity and the angular velocity $\Omega(t) = (0,0,d\gamma/dt)$ about the vertical axis through the center of gravity. The translational velocity of the ship relative to the current is therefore

$$\underset{\sim}{U}_r = \underset{\sim}{U}_b - \underset{\sim}{U}_c \tag{1}$$

Its x- and y-components are

$$u_r = u - u_c \quad , \quad v_r = v - v_c \tag{2a}$$

respectively, where

$$u = U_b \cos \Theta, \quad v = U_b \sin \Theta, \quad u_c = U_c \cos \alpha, \quad v_c = U_c \sin \alpha, \tag{2b}$$

Θ being the angle of inclination of the velocity vector $\underset{\sim}{U}_b$ with respect to the x-axis and α the angle of incidence of the current velocity $\underset{\sim}{U}_c$ relative to the ship's centerplane (see Figure 1).

The equations of motion for the moored ship are conveniently expressed in terms of the body-fixed system of coordinates because the symmetry of body geometry can then be fully exploited. The slow oscillation of a moored ship can be described as consisting of three modes of motion, namely surge (along the longitudinal x-axis), sway (along the transverse y-axis) and yaw (rotation about the vertical z-axis), since the remaining three modes of heaving, pitching and rolling are usually associated with wave excitation and are relatively insignificant in the present case. Accordingly, the equations for the horizontal motion can be derived from Kirchhoff's equations (see, e.g., Lamb, 1932, Sect. 124) together with proper modification to account for the effects of the prevailing current. We have for

surge: $$(m_s + m_{11})\frac{du}{dt} - (m_s + m_{22})\Omega v = F_x + T_x - (m_{22} - m_{11})\Omega v_c, \quad (3)$$

sway: $$(m_s + m_{22})\frac{dv}{dt} + (m_s + m_{11})\Omega u = F_y + T_y - (m_{22} - m_{11})\Omega u_c, \quad (4)$$

yaw: $$(I_{sz} + I_{33})\frac{d\Omega}{dt} + (m_{22} - m_{11})(u - u_c)(v - v_c) = M_z + \ell_c T_y, \quad (5)$$

where m_s - total mass of the ship,

m_{11}, m_{22} - x- and y-components of the principal added masses of the ship,

I_{sz}, I_{33} - moment of inertia and added moment of inertia (about the z-axis) of the ship,

F_x, F_y - the x- and y-hydrodynamic forces acting on the ship (in addition to those due to the added mass and added moment of inertia of the ship)

M_z - hydrodynamic yaw moment about the z-axis,

T_x, T_y - x- and y-components of the mooring line tension,

ℓ_c - distance between c.g. and the mooring hinge of the ship.

For dealing with the present problem of slow oscillations of ships moored to a single point, in the absence of any incident waves, the mass, added masses, the moment of inertia and the added moment of inertia can all be assumed to be constant and given. We proceed next to discuss the hydrodynamic forces and their origins that are of primary importance to the dynamics of moored ships.

LONGITUDINAL CURRENT FORCE

From practical experience as well as on physical grounds it is known that the current force in the longitudinal direction is generally quite small compared with that in the transverse direction except when the current runs nearly parallel to the ship length. The basic reason is because the typical ship form is slender. Flow separation induced by the longitudinal flow would occur only in the stern region whereas viscous separation of the transverse flow usually occurs along the entire length of the leading keel line, resulting in an extensive wake formation on the downstream lateral side and hence a very large form-drag coefficient.

The longitudinal current force can therefore be adequately estimated by the conventional viscous resistance formula:

$$F_{cx} = \frac{1}{2} \rho_w u_r^2 (1 + \kappa) C_F S , \tag{6}$$

where ρ_w is the density of anbient water,

κ is the form factor (in professional use for given ship form class),

C_F is the plank viscous resistance coefficient,

S is the wetted surface area.

For all practical purposes of applications to hulls under commonly used conditions (the surface being not overly eroded or rough) we propose to adopt the 1957 ITTC line of C_F (see Figure 2), namely

$$C_F = 0.075(\log_{10} Re - 2)^{-2}, \tag{7}$$

where $Re = u_r \ell/\nu$ is the Reynolds number based on ship length ℓ and the kinematic viscosity ν of the ambient water. This viscous resistance line coincides with the 1956 ATTC (American Towing Tank Conference) line for $Re > 10^7$ and has a steeper slope than the latter for $Re < 10^7$. Thus our choice of the ITTC line should be conservative for prediction of the mooring force on models to be tested and for extrapolation to the prototype. While there is no universally accepted tables for the form factor κ for various types of ship geometry (including the generalized kind that allows κ to depend on both the Reynolds and Froude numbers), the constant form factors κ which have been adopted in current professional use for nearly all categories of ship forms should be adequate for the present purpose. It is more important to establish a method capable of estimating the transverse current load with comparable accuracy because the latter is generally much greater in magnitude.

TRANSVERSE CURRENT FORCE AT MODERATE INCIDENCES

The flow field around the ship generated by a steady current at sufficiently high incidence angles (with respect to the ship's center-plane) resulting in a viscous wake formation is very complex to analyze. There is no fully adequate theory available for analytical calculation or numerical computation of the flow for Reynolds numbers lying in the range encountered in general practice. For a best-estimate engineering solution, we recommend the application of free-streamline theory to evaluate the wake shape and form drag when a wake is formed at high enough angles of incidence. At smaller incidences, the oblique wake on the transverse side will cease to exist and we adopt slender-body theory to determine the hydrodynamic forces. It is therefore convenient to consider the two regimes of (i) small incidence angles, without transverse wake formation, and (ii) large incidence angles, with a transverse wake manifested.

The case of large incidence angles can be most effectively treated by means of strip theory of free-streamline flow. The wetted hull is divided into an appropriate number of strips perpendicular to the center-plane of the vessel. The flow in each strip (except for the end sections, which will be discussed later) is taken as the lower half of the two-dimensional flow of a double body which is formed by continuation of the flow field by mirror reflection into the originally undisturbed water surface (see Figure 3). The flow, after encountering the leading side of the hull which forms a blunt surface, is assumed to detach (the term used here to represent approximately the viscous separation) from the body surface. The detached streamlines, called free-streamlines, will envelop a near wake in which the fluid has a low kinetic energy and can be approximated by a region of constant pressure $p = p_c = p_{180°}$ (at 180° point of the base, see Figure 3). The force so determined for each strip can then be combined to yield the total force and yaw moment acting on the vessel.

To solve the two-dimensional free-streamline problem in each strip, we adopt the general theory of Wu and Wang (1964) which provides an analytical solution in terms of a set of nonlinear functional equations for computing flow variables of the wake flow past a two-dimensional body of arbitrary shape. The cross-sectional shape function of the 'double body' may be prescribed in parametric form as

$$y = \hat{y}(s), \qquad z = \hat{z}(s) \qquad (0 \leq s \leq S), \tag{8}$$

where s is the arc length measured along the wetted portion of the hull surface (now the lower half of the double body) in the cross section of a fixed x. Application of the theory of Wu and Wang to the present case, however, will still require some extension. The detailed procedure is given as follows.

The set of nonlinear functional equations can be expressed as

$$s(\zeta)/A_0 = \mathcal{E}[s(\zeta), \beta(s(\zeta)); \zeta_0], \tag{9}$$

$$\zeta_0 = (U_{yc}/q_c)\mathcal{F}[s(\zeta), \beta(s(\zeta)); \zeta_0], \tag{10a}$$

where (see Eq. 2) $$U_{yc} = -V_r = V_c - V, \tag{10b}$$

$$\mathcal{E}[s(\zeta), \beta(s); \zeta_0] = \int_{-1}^{\zeta} \exp\left\{\frac{(1-\zeta^2)}{\pi}\int_{-1}^{1}\frac{\beta(\tau)-\beta(\zeta)}{(\tau-\zeta)(\tau\zeta-1)}d\tau\right\}\nu_*(\zeta;\zeta_0)\,d\zeta, \tag{11a}$$

$$\nu_*(\zeta;\zeta_0) = -\zeta(\zeta^2-1)\,\frac{2\zeta^2-(\zeta_0+\bar{\zeta}_0)(1+\frac{1}{\zeta_0\bar{\zeta}_0})\zeta-2}{\left[(\zeta-\zeta_0)(\zeta-\bar{\zeta}_0)(\zeta-\frac{1}{\zeta_0})(\zeta-\frac{1}{\bar{\zeta}_0})\right]^2}, \tag{11b}$$

$$\mathcal{F}[s(\zeta), \beta(s); \zeta_0] = -i\exp\left\{\frac{(1-\zeta_0^2)}{\pi}\int_{-1}^{1}\frac{\beta(\tau)\,d\tau}{(\tau-\zeta_0)(\tau\zeta_0-1)}\right\}. \tag{12}$$

In these equations $\beta(s)$ denotes the angle of inclination of the tangent to the solid boundary with respect to the coordinate axes, defined by

$$\beta(s) = \tan^{-1}\left\{ \frac{d\hat{z}(s)}{ds} \Big/ \frac{d\hat{z}(s)}{ds} \right\}, \tag{13}$$

here expressed in terms of the arc length s along the boundary. The variable ζ is a parametric complex variable in which plane the original flow region outside the near-wake in the physical plane is mapped conformally into a lower semi-circle of unit radius, the point ζ_o (with $|\zeta| < 1$) being the image of the point of infinity in the physical plane. The quantity A is a real constant depending on the geometric scale of the body configuration. The transformation (9) and (10) maps $s(\zeta)$ onto itself, i.e., $s(\zeta)$ is a fixed point of the transformation.

This nonlinear functional transformation further depends on the end conditions in regard to the point(s) at which the free-streamline detaches from the solid surface. There are two categories of detachment, (i) fixed detachment and (ii) free detachment points.

(i) Fixed detachment -- The detachment points of this category are sharp corners on the body surface, from which free-streamlines can separate from the body to yield a valid solution. One example of this case is the cross flow past a ship hull equipped with bilge keel plates. In this case both detachment points (on the double body) are known, and equations (9) - (13) can be used to compute the solution as represented by the quantities:

$$P\left[\beta(\zeta);\ \zeta_o;\ A_o\right]. \tag{14}$$

An iterative algorithm has been developed for numerical computation of the above parametric solution. In nearly all the cases investigated, the theoretical results given by this model have been very satisfactory in comparison with carefully conducted experiments, such as the lift and drag of a cavitating hydrofoil with a uniform camber of 16° as shown in Figures 4 and 5.

(ii) Free detachment flows -- By free detachment we mean those flows in which either one or both of the two branches of free streamlines will separate from the body surface at a point where the surface curvature is finite and continuous and the point(s) of detachment is unknown a priori and must be determined as a part of the solution. In order to determine the detachment point(s), we must impose an additional condition for each free detachment point. The classical condition is

$$\frac{d}{d\zeta}\left\{\zeta \exp\left[\frac{(\zeta^2-1)}{\pi}\int_{-1}^{1}\frac{\beta(\tau)d\tau}{(\tau-\zeta)(\tau\zeta-1)}\right]\right\} \longrightarrow 0 \qquad \begin{matrix} \text{as} & \zeta \longrightarrow -1, \\ \text{and/or} & \zeta \longrightarrow 1, \end{matrix} \tag{15}$$

as the case may be. The complexity of satisfying this end condition in the numerical solution is well known since the problem is highly nonlinear and its solution will require a more generalized algorithm and a new numerical code. Several possible approximate methods have been pursued under the present program, including some empirical approaches that may effectively aid our prediction of the transverse current load for the case when flow separates from a smooth keel shoulder. These methods will be further explored when the development of a numerical code is carried out.

STRIP THEORY AND END EFFECTS

The free-streamline theory described above will enable us to determine, for given type of ship form (i.e., beam/draft ratio, block coefficient, bilge form, etc.), the local sectional drag coefficient, $C_{Dc}(x;\sigma)$ say, due to a beam-on current (i.e., at sufficiently large angles of incidence). This drag coefficient will vary with the longitudinal station x because of the varying sectional hull shape and will depend on a nondimensional parameter: i.e., the wake-underpressure coefficient,

$$\sigma = (p_\infty - p_{180^\circ}) / \tfrac{1}{2}\rho_w U_{yc}^2 \tag{16}$$

Here p_∞ is the pressure at upstream infinity and p_{180° the wake underpressure at the near-wake base (see Figure 3), both with the hydrostatic part deleted (or cancelled), and $U_{yc} = -v_r = v_c - v$ is the speed of the beamwise component of the current relative to the local ship surface (see Equation 2). In engineering applications this parameter is usually determined by experimental means due to lack of theoretical recourse. Some empirical relations will be sought for σ in the future stage when the computer code will be developed.

The end effect on wake flow past the ends of a ship-like slender body is an unsolved problem. The complexities of the problem lie partly in its fully three-dimensional nature and partly in the highly variant curvature of the hull near the bow and stern. It makes quite a marked difference between a V-shaped and a U-shaped bow, and stern, to the extent that separates the fixed and free detachment categories discussed earlier. For the present purpose we propose to apply the same basic free-streamline method to the two end sections. More specifically, each end section will be subdivided into an optimum number of horizontal strips together with a suitable double-body construction in order to apply the two-dimensional free-streamline theory for each strip of the end section. It should be qualified, however, that the validity of this new method for correcting the end effects on a slender wake flow will require further research, with both theoretical and experimental endeavors.

The total transverse current force can then be obtained by direct integration over the strips, taking into account the variations of local current relative velocity due to possible yawing of the ship. Thus, at moderate angles of incidence the current will give rise to a transverse force which can be expressed as

$$F_{yc} = \frac{1}{2} \rho_w \int_{-\ell_s}^{\ell_b} C_{Dc}(x;\sigma)(U_{yc} - \Omega x)^2 h(x)\, dx , \tag{17}$$

where h(x) is the sectional draft of the ship, ℓ_s and ℓ_b are the distances from the wetted stern end and bow end, respectively, to the center of gravity of the ship. If deemed necessary, the magnitude U_{yc} of the transverse current velocity may be allowed to vary with x. Such a more general case can arise due to local variations of bathymetry under some extreme circumstances. In the above equation (17), the transverse sectional drag coefficient $C_{Dc}(x;\sigma)$ also symbolically represents the end sections for which alternative options with further subdivision into horizontal strips will be explored and the best methodology ascertained in future stages of this program.

Based on the same assumptions, the yawing moment acting on the ship about the vertical axis fixed at the center of gravity induced by the oblique current in this range of incidence is

$$M_c = \frac{1}{2} \rho_w \int_{-\ell_s}^{\ell_b} C_{Dc}(x;\sigma)(U_{yc} - \Omega x)^2 h(x)\, x\, dx . \tag{18}$$

TRANSVERSE CURRENT FORCE AT SMALL INCIDENCES

When the ship encounters a streaming current at sufficiently small incidence, the model of current-induced wake flow and the strip method presented above will very likely cease to be a valid model. The reason for this skepticism is the argument that, on physical grounds, such a current will induce vortex sheets to be shed from the stern (or the bow, whichever is downstream) to trail behind the ship, so that the ship will behave more like the lower half of a low-aspect-ratio wing rather than a stalled slender wing as delineated in the previous case (with a transverse wake). We therefore assume that there is another kind of hydrodynamic force arising from a circulatory flow origin when the current approaches the ship at a small angle of incidence. In this operating condition, we shall regard the vertical centerplane of the double-body as a lifting surface of low aspect ratio. A general slender-body theory has been developed by following the approach of a previous theory by Wu (1971) for analyzing arbitrary motion of a deformable slender body (such as a swimming fish), equipped with side fins from which vortex sheets are shed as a result of body movement. Now with the contribution from an oblique current included, the theory is applicable to the moored ship problem at hand, especially when longitudinal vortices are shed from bilge keel plates over the ship length. As these vortex sheets are convected downstream and finally enter the stern region, they can have a significant interaction with the rudder, generating lift and drag forces in a manner quite unlike those of the wake-flow origin as we have just evaluated. According to this theory, the lift and moment on the ship depends on whether or not there are vortex sheets being shed from the hull surface upstream of the stern-rudder section. This circulation originated flow is depicted in Fig. 6.

In the case when there are vortex sheets being shed from the bilge keels, we shall assume that the shedding starts from the section at $x = \ell_m$, physically corresponding to the section of maximum transverse dimension, and continues along the keel plates downstream of that section where the keel plates become trailing edges. In the presence of these trailing vortex sheets, the total lift in the direction perpendicular to the incoming current (which is assumed to be quasi-steady) is

$$L_{cir} = \frac{1}{2}\rho_w U_{xc} A(\ell_s) V(\ell_s, t) - \frac{1}{2}\rho_w U_{xc} \int_{\ell_m}^{\ell_f} V(x,t)\, \frac{dA}{dx}\, dx\,, \tag{19}$$

where

$$U_{xc} = u_c - u\,, \qquad V(x,t) = U_{yc} - \Omega x\,, \tag{20}$$

(u_c, u being given by Eq. 2b, U_{yc} by Eq. 10b) $x = \ell_s$ at the stern-end rise, $x = \ell_f$ is the station where vortex sheets ceases to be shed from the hull, and $\rho_w A(x)$ is the added mass of the transverse segment of the ship per unit length at x, for relative motions in the y-direction. A(x) can be [illegible] calculated for given ship form. We note that the added mass of the ship is half the added mass of the double body for the sway mode. The induced drag associated with the lift of this circulatory flow has been derived from the theory of Wu (1971) by assuming quasi-steady approximation and taking the drag to be the negative of the forward thrust, [illegible]

$$D_{cir} = \frac{1}{4}\rho_w A(\ell_s)(U_{yc}^2 - \Omega^2 \ell_s^2) - \frac{1}{4}\rho_w \int_{\ell_m}^{\ell_f} (U_{yc}^2 - \Omega^2 x^2)\frac{dA}{dx}\, dx \quad [illegible]$$

Transforming the force system (L_{cir}, D_{cir}) into the x- and y-components, we obtain

$$F_{xc} = L_{cir} \sin\alpha - D_{cir} \cos\alpha, \tag{22}$$

$$F_{yc} = L_{cir} \cos\alpha + D_{cir} \sin\alpha. \tag{23}$$

The corresponding moment of force about the vertical axis at the origin is

$$M_{cir} = \frac{1}{2}\rho_W U_{xc}\int_{-\ell_b}^{\ell_s} x \frac{\partial}{\partial x}\left[A(x)V(x,t)\right]dx - \frac{1}{2}\rho_W U_{xc}\int_{\ell_m}^{\ell_f} x V(x,t)\frac{dA}{dx}dx. \tag{24}$$

In these formulas, the stern end section at $x = \ell_s$ can be regarded either as the ship's stern with the contributions from such appendages as rudder and propeller units superimposed on the lift and drag forces or as an extension to the trailing edge of the rudder with proper account taken of the effects of the gap between stern and rudder. The details of these accounts, however, will be curtailed here but can be accomplished when the development of a computer code is undertaken.

A direct comparison between the hydrodynamic forces and yaw moment in the two operating ranges of moderate and small incidences (see equations 17, 22, and 23) shows that when $\Omega\ell \ll U_c$, the transverse force F_{yc} of the wake-flow origin behaves like $\sin^2\alpha$ whereas the F_{yc} of circulatory origin varies linearly in $\sin\alpha$ as $\alpha \to 0$. In order to achieve a smooth transition between the two theories for the two ranges of current incidences, it may be assumed that the circulatory flow predominates in a range of small $|\sin\alpha|$ until its contribution to F_y becomes equal to that given by the wake-flow theory and drops out beyond that transition incidence in favor of the latter. This assumption is brought forth as a new physical concept, which seems to be well founded but should be verified by experiments for its full validity.

DISCUSSION

The theoretical basis for a mathematical model has been developed which can be applied to calculate the hydrodynamic forces and moment on a surface ship due to a current at an arbitrary angle with respect to the ship. Every attempt has been made to adopt techniques which not only represent the best theoretical approach available, but also can be implemented with a reasonable computational effort. It is believed that the methodology presented herein can be used to obtain the current loads on a ship moored at a single point and in turn to deduce its resulting motion subject to the assumptions of (i) deep water, (ii) inviscid and incompressible fluid, and (iii) no free surface waves. Even though much of the difficulties due to the highly nonlinear features of the problem have been addressed in the methodology, there are still a few remaining physical factors not yet included. These additional issues, which are beyond the scope of the present project, include the following:

(i) shallow water effects. The present methodology ignores the effects due to the presence of the ocean bottom. In practice, for large ships or shallow mooring sites, the influence of the bottom can be significant. The current flow under these circumstances would tend to be accelerated in passing under the confined space below the keel, resulting in larger transverse loads.

(ii) spatial variations of the ambient current. The ambient current has been treated herein as a steady uniform one. In the ocean, there can be shear currents, i.e., current velocity changing vertically (with depth) and horizontally (location along the ship).

(iii) drifting force due to wave scattering. The effect of surface waves have been neglected in this investigation. The water surface is treated as a plane. In reality, the presence of the ship tends to interact with the prevailing ocean waves. Scattering of surface waves would result in a drifting force on the ship.

(iv) slow oscillations of a moored ship. It has been known in the profession that slow oscillations of a tanker (fishtailing) in single-point mooring (SPM) generally induce a large influence on the mooring load. However, investigations of this problem have been so far limited to practically only tankers, yielding little understanding of the basic mechanism involved. Whether the current load is the most effective when compared with wind and wave drift forces, in inducing slow oscillations of all types of ships remains to be answered.

(v) scaling rules for model-prototype comparisons. Small scale laboratory models can behave quite differently from prototype ships since the various physical phenomena vary with the characteristic Reynolds number and Froude number and take on different relative importance in determining the current load. Due to the large expense of field tests, they will undoubtedly remain few in number. However, they are indispensable. Reliable methods for extrapolating model test results to prototype predictions are invaluable.

RECOMMENDATIONS

The methodology described in this report for calculating the forces and moment due to a current on a ship consists of several separate formulations each applicable for portions of the ship hull or some range of the parameters such as angle of incidence of the current. The overall application involves utilizing all these formulations and integrating the results appropriately. It is not possible, a priori, to accurately define the exact points where one formulation should be chosen in favor of another. This must await the actual calculations on specific hulls and comparison with available experimental data. It is therefore recommended that the methods described herein be implemented to permit the numerical evaluation of the forces and moment induced by a current on typical ship hulls. Computer codes should first be developed which can be used to calculate the forces and yaw moment acting on arbitrary hull shapes. A set of representative candidate hulls should also be chosen including ships of different basic designs such as tankers, destroyers, cruisers and so on. The results of computations for these hulls under a variety of conditions should then be critically examined not only for purposes of model validation and comparison with experimental data, but also for insight and guidance in delineating the zones of applicability of the various separate formulations so that a unified theory can be achieved.

In addition to the numerical implementation of the mathematical model recommended above, effort should also be made to evaluate the effects due to the various physical factors discussed previously which have not been incorporated yet in the present formulations. Some of these, such as the shallow water effects, can be of great practical importance. To assure successful applications of the methodology, it is also essential to secure valid scaling relations between small scale model test results and the prototype behavior.

ACKNOWLEDGMENTS

The effort detailed in this report benefited a great deal from the numerous stimulating discussions the investigators have had with a number of individuals. Particular thanks are due to Mr. Paul Palo who made available not only his time for discussions but also results of field experiments which provided much guidance in the formulations.

The financial support of the Naval Civil Engineering Laboratory is gratefully acknowledged.

NOMENCLATURE

A_o	A real constant representing the geometric scale of the wetted body surface, used in Eq. 9
$A(x)$	$A(x)$ times the water density ρ_w gives the added mass of the transverse segment of the ship hull per unit length at x
b	Beam of the ship
C_F	Plank viscous resistance coefficient
C_{Dc}	The transverse sectional drag coefficient
D_{cir}	Induced drag associated with the circulatory flow for small incident current angle
F_{cx}	Longitudinal current force
F_x, F_y	The x- and y- hydrodynamic forces acting on the ship (in addition to those due to the added masses and added moment of inertia of the ship

F_{xc}, F_{yc}	The x- and y- hydrodynamic forces on the ship due to the drag and lift of the circulatory flow induced by the current at small incidence angles to the ship
$h(x)$	Sectional draft of the ship
I_{sz}, I_{33}	Moment of inertia and added moment of inertia (about the z axis) of the ship
κ	The form factor of a given ship hull
L_{cir}	Total lift in the direction perpendicular to the mean incoming current due to the sheeding of vortex sheets (see Eq. 19)
ℓ	Ship length
ℓ_c	Distance between the mooring hinge and the center of gravity of the ship
ℓ_b, ℓ_s	Distance in x direction representing the locations of bow and stern ends to the center of gravity of the ship
ℓ_m	Location (in x-direction) of the maximum transverse dimension of the ship
ℓ_f	Location in x-direction where vortex sheets cease to be shed from the hull
m_s	Total mass of the ship
m_{11}, m_{22}	x- and y- components of the principal added masses of the ship
M_z	Hydrodynamic yaw moment about the z-axis
M_c	Yaw moment acting on the ship about the vertical z-axis (see Eq. 18) due to the wake forming oblique current
M_{cir}	Moment of forces about the vertical z-axis due to the sheeding of vortex sheets
p_∞	Pressure at upstream infinity
$p_{180°}$	Wake underpressure at the near-wake base
Re1	Reynolds number defined as $U_r \cdot \ell/\nu$
Re2	Reynolds number defined as $U_r \cdot b/\nu$
S	Wetted surface area of the ship

s	Arc length used in prescribing the cross sectional shape function of the "double body"
T_x, T_y	x- and y- components of the mooring line tension
$\underset{\sim}{U}_b(t) = (u,v)$	The x- and y- velocity components of the ship's center of gravity
$\underset{\sim}{U}_c = (U_c \cos\alpha, U_c \sin\alpha) = (u_c, v_c)$	the x- and y- components of the current velocity
$u_r = u - u_c$, $v_r = v - v_c$	The x- and y- velocity components of the ship relative to the current
$U_{yc} = v_c - v = -v_r$	The beamwise velocity component of the current relative to the local ship surface
$U_{xc} = u_c - u$	The longitudinal velocity component of the current relative to the local ship surface
α	Incidence angle of the current with respect to the ship centerplane
$\beta(s)$	Angle of inclination of the boundary slope of the hull surface in x = constant plane
$\mathcal{E}[\beta]$, $\mathcal{F}[\beta]$	Two nonlinear functionals defined in Eqs. (11) and (12)
ρ	A complex variable forming a parametric plane
ν_*	A function defined in Eq. (11b)
ν	Kinematic viscosity of the ambient water
ρ_w	Density of the ambient water
σ	The wake-underpressure coefficient, or the cavitation number
$\Omega(t)$	The angular velocity of the ship about the z-axis through the ship's center of gravity

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APPENDIX

SYNOPSIS OF THE PROPOSED THEORETICAL MODEL

In order to assist the principal engineers concerned with the continued planning and accomplishment of the project, and to aid the future users of the theoretical model which is being developed here, we present in this appendix a brief orderly outline affording a quick general view of the physical background, simplifying assumptions, conditional qualifications and estimated range of validity of the proposed theoretical model.

1. Equations of motion

As specified in the text, the free stream approaching the moored ship is assumed to be uniform, but the current velocity can be slowly varying with time, both in magnitude and in direction. Typical departures from this ideal condition can be admitted in the continued investigation to include such important effects as vertically and horizontally sheared velocity profiles of the current.

The equations of motion (3) - (5) are for the calculation of surge, sway and rolling modes of motion, all being assumed free from having high-frequency forcing (such as that due to incident waves with frequencies falling in that range) so that the modes of heaving, pitching and rolling can be neglected in this stage of work. Under this assumption, only the x- and y-components of the principal added mass coefficients (m_{11} and m_{22}, given in universal definition) of the ship and only the z-component of the moment of inertia (I_{zz}) and of the added moment of inertia (I_{33}) of the ship arise in the equations of motion.

These equations are expressed with respect to the body frame of reference (fixed with the moored ship); they are derived especially for the present case in which we have a current of arbitrary magnitude and incidence angle passing about the moored ship.

Under the same premise, the added mass coefficients m_{11}, m_{22} and I_{33} can, quite accurately, assume their values calculated from the "double-body model," meaning the extended flow obtained by reflection of the original into the initially undisturbed water surface. By this approximation, m_{11}, m_{22} and I_{33} attain values which are one-half of their corresponding coefficients of the double-body flow, which is unbounded in extent and symmetric about the mean water surface. Clearly, the values of these coefficients depend only on the hull shape, but not on the time history or the frequency of body motion.

2. Longitudinal current force

For the determination of the longitudinal viscous resistance induced by a prevailing current, the formula (6) is proposed for the following reasons. First, when the current runs nearly parallel to the ship length, the problem becomes that of a ship moving under tow or by self-propulsion, in which case the formula is well based on the abundant towing-tank test results and prototype experiments acquired in the profession of naval architecture. While it is true that in the case of an oblique current past a ship no assessment of the formula (6) has been made available (to our knowledge), the existing experimental data, though very scanty, have confirmed our expectation that in such cases the longitudinal viscous resistance is quite insignificant in comparison with the transverse current force when the flow has a considerable length of separation along the hull.

3. Transverse current force at moderate incidences

In this case large values of transverse current force coefficient (of order unity) are known to exist. To determine this coefficient by numerical methods using a fully viscous fluid model (e.g. the Navier-Stokes equations or a turbulent boundary layer model) is still beyond the present state of the art. For engineering applications, such as for our problem at hand, the free-streamline theory proposed here is considered to be the most effective to provide a two-parameter family of solutions. This method is different from the singularity method that employs mass sources, doublets and vortices, the expansion method that uses orthogonal functions, or the boundary integral equation method based on Green's integral formula. The great power of this method is fully exploited in the case of two-dimensional plane flow by means of generalized conformal transformation (involving mathematical branch points) and complex function theory. Application of the method to the present problem of three-dimensional ship motion is effected by the concept of strip theory which appears to be applicable to hull shapes of conventional form. The idea of introducing the strip approximation for the bow and stern end sections is rather innovative and should require further studies for its full verification.

It should again be noted that the two parameters the solution will depend upon are (i) the wake under-pressure coefficient, and (ii) the location of the detachment point from which the free streamline will separate from the hull surface to generate a cross-flow wake formation. Although a closure condition has been proposed, in terms of equation (15) of the text, for the determination of parameter (ii), various alternative closure conditions for the two parameters have been explored in the present investigation, including several empirical approaches, with intent to improve the accuracy of the solution. Furthermore, extensive efforts have been spent during this study in examining the prospects of convergence of the numerical iteration of the nonlinear integral equations (eqs. 9 - 12 in the text). This concentrated effort was deemed necessary for the present application (to hull forms whose cross-sectional shape may be regarded as blunt and with highly curved corners) because our experience of the previous success of the theory was limited to thin bodies with sharp corners.

4. Transverse current force at small incidences

The slender-lifting-body theory proposed here is believed to be particularly well suited for the case when the incidence held by the approaching current is small. In this regime the cross flow is expected to become so weak that the cross-flow wake formation should be evanescing, but with vortex sheets continually being shed, only now confined more to the stern region. Since the original theory cited in the text is best applicable to thin bodies, such as thin ships of the Wigley form, further modifications of the theory have been explored in order to extend its validity to cover hull forms of conventional categories. The optimum modification remains to be identified by numerical calculations to be carried out later.

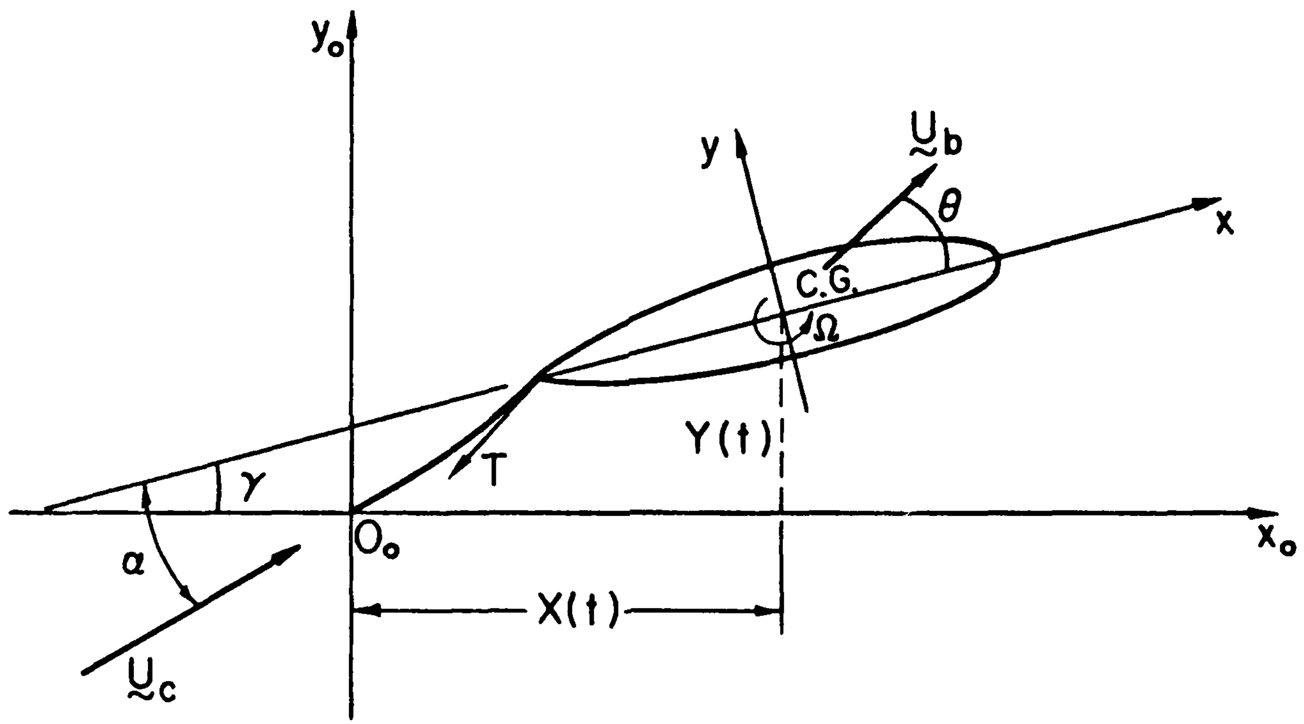

Figure 1. The absolute coordinate system (x_o, y_o, z_o) and the body-fixed coordinate system (x, y, z).

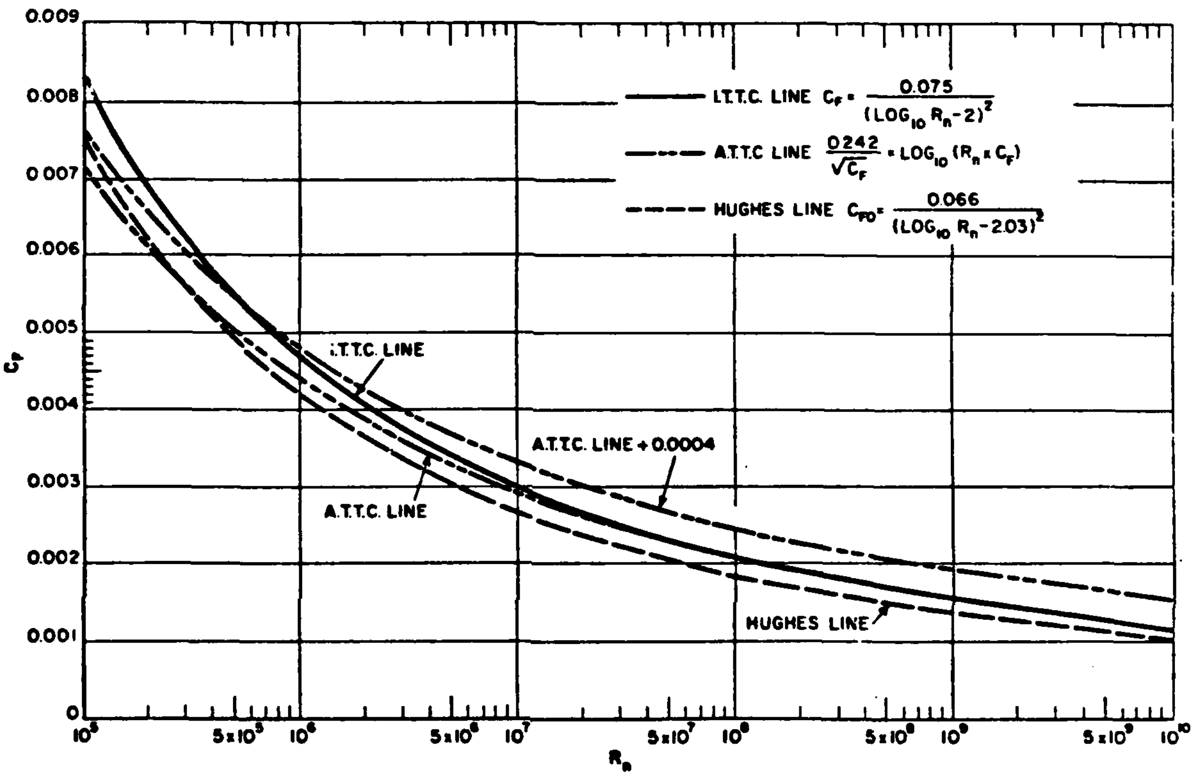

Figure 2. 1957 ITTC line of plank viscous resistance coefficient.

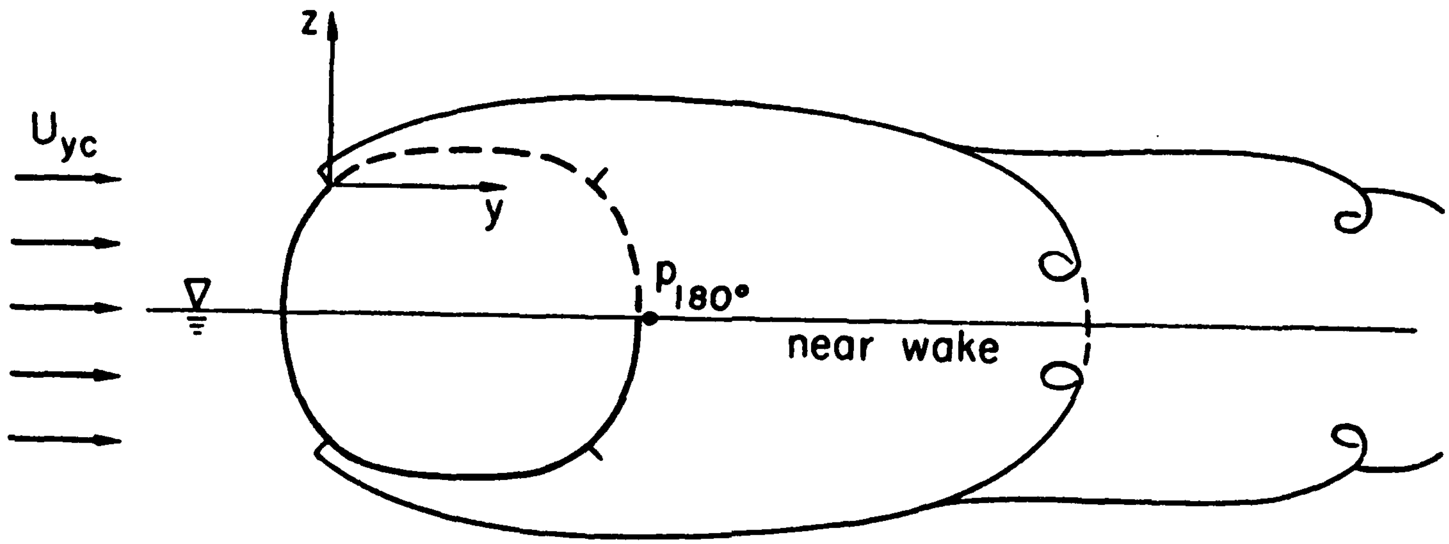

Figure 3. A strip representation of the transverse wake flow.

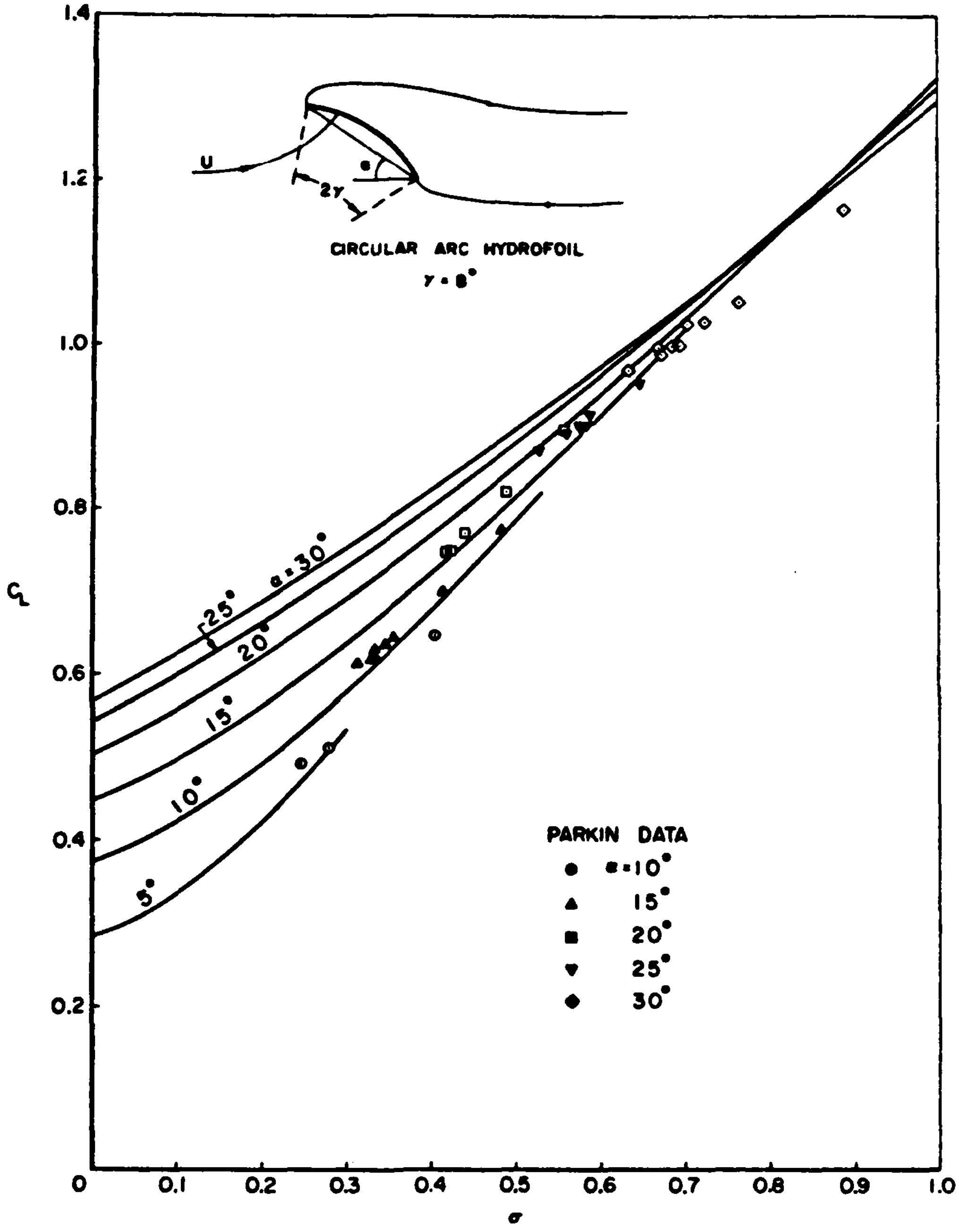

Figure 4. Comparison between theory (Wu and Wang, 1964) and experiment (Parkin, 1958, J. Ship Research 1, 34-56) for the lift coefficient C_L of a cavitating circular arc hydrofoil (of semiarc angle $\gamma = 8°$) and its variation with cavitation number (or wake underpressure coefficient) σ.

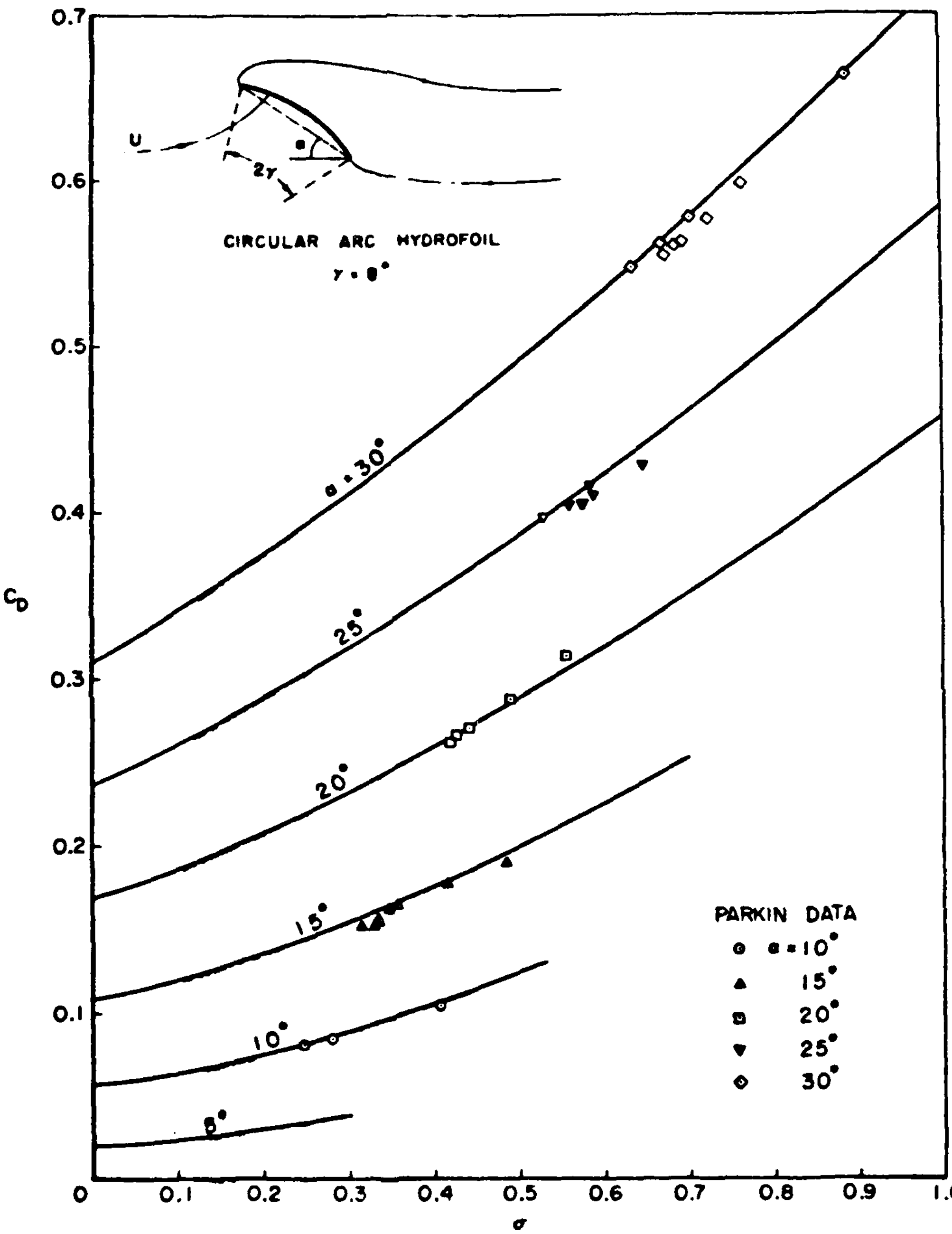

Figure 5. Comparison between the same theory and experiment as listed in the caption of figure 4 for the drag coefficient C_D of the circular arc hydrofoil.

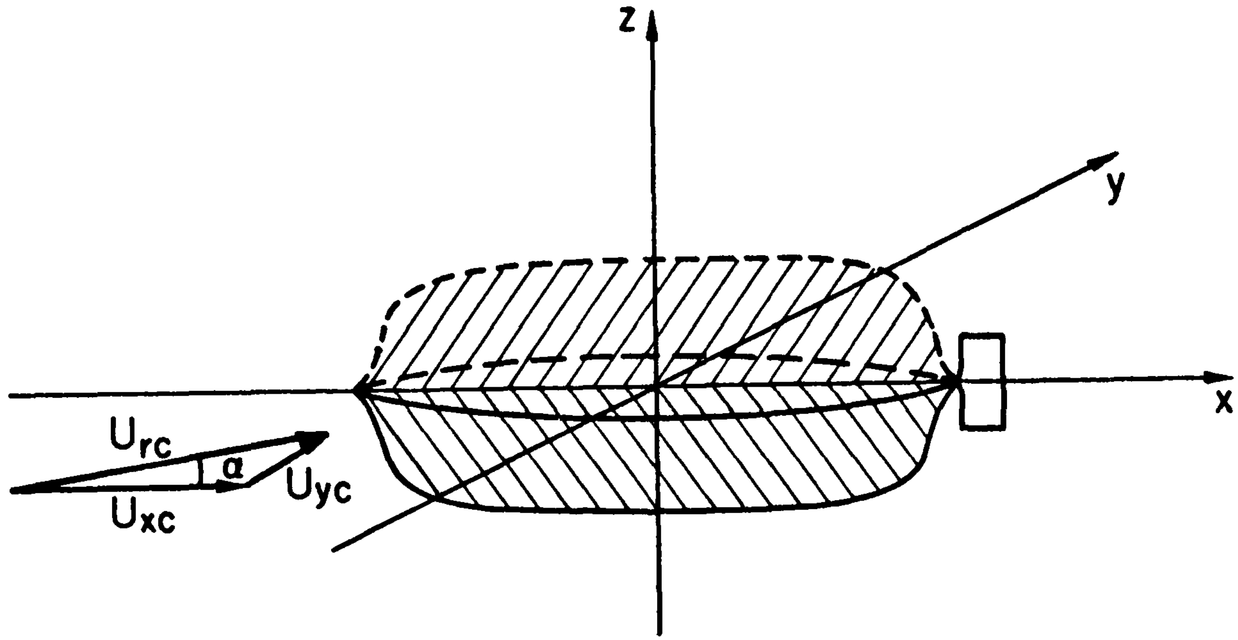

Figure 6. Circulating flow contribution to the hydrodynamic forces at small incidence angle α. The centerplane of the double-body (shaded flat plate) is assumed to act like a low aspect-ratio wing.

PART II

1. Introduction

In Report AMT R-101 entitled "Investigation of Current Loads on Moored Vessels", a methodology was presented with a proposed theoretical model for calculation of hydrodynamic forces on a moored vessel in the presence of a uniform current in deep water without considering the effect of surface waves. The theoretical model was constructed with application and further development of free-streamline theory, strip theory and low-aspect-ratio wing theory for the overall solution.

In this present sequel to the first original report (AMT R-101), several extensions to that theory are presented to encompass more general conditions that may prevail in practice, such as sheared flow, finite water depth, and wave-making resistance.

In Sections 2 and 3, a methodology is formulated in detail to obtain the first order correction to the original freestreamline - strip theory model for the more general case when the free stream velocity has a sheared distribution in both the vertical and horizontal directions. It is based on a small perturbation approach and is applicable to the case of weak shear. Section 4 presents results for hydrodynamic forces and moments on the moored ship as a result of a current. The particular contribution and effect of the rudder force is evaluated in Section 5. Wave radiation from a ship moored in a steady current while not a major contributor to the hydrodynamic forces, is analyzed in Section 6. Finally, Section 7 discusses several aspects of the effects of shallow water.

2. Ship moored in a sheared current

In coastal waters, prevailing current generally varies spatially in both magnitude and direction and may also undergo temporal changes. For practical applications to ship mooring problems, however, it is quite sufficient to consider those cases in which temporal changes are so slow compared with the characteristic time of ship motion that they can be regarded as quasi-steady, and the spatial variations in magnitude and direction may also be assumed to be weak, i. e.,

$$u_c = u_c(x, z, t) \quad , \qquad v_c = v_c(x, z, t) \qquad (1a)$$

$$|\nabla u_c|, \quad |\nabla v_c| \ll U_{c\ max}/\ell \qquad (1b)$$

where $U_{c\ max}$ is a maximum measure of the current speed and ℓ is the ship length. Such typical variations can generally be ascribed to local topographical distribution of water depth and changes in forcing disturbances such as tides and distant storms. Variations of this weak category can usually be approximated by its Taylor series about the mean water suface (at $z = 0$),

$$u_c = u_c(x, o, t) + z\left(\frac{\partial u_c}{\partial z}\right)_o \quad , \qquad v_c = v_c(x, 0, t) + z\left(\frac{\partial v_c}{\partial z}\right)_o \quad . \qquad (2)$$

Vertical variations of the current velocity beyond the terms linear in z are less important and can be neglected when the local water depth h_o is large compared to the ship draft d (say $h_o > 2d$). In very shallow water, a steadily running current has its velocity varying with depth according to a one-seventh power distribution typical of turbulent flow over a smooth bottom. If the clearance beneath the ship is of order of the ship's draft or shallower, the effective current velocity experienced by the ship may require terms nonlinear in z to be also included in the expression in (2), but such exceptional cases must be treated separately since the shallow-water effects then becomes of primary importance, dominating over the effects due to nonuniformities in the current. This extreme case will be analyzed in a later section (see Section 7).

The first terms of the expansions in (2) represent the velocity components of the surface current, which may vary with time, but only very slowly, and may vary with x along the ship, but also very gradually. Further expansion of these surface current velocity is unnecessary since in strip theory for slender ship motions the weak variation of current velocity with x and t can be taken as being parametrical in nature. Consequently, the strip theory developed earlier in this report can be directly applied without modification. We now proceed to consider the problem of the transverse wake flow in a vertically sheared stream for each strip of the ship.

3. Two-dimensional wake flow in a vertically sheared stream

We consider the wake flow of a uniformly sheared free stream past a two-dimensional blunt body of arbitrary form, with the stagnation streamlines separating from body surface to become free streamlines enveloping a finite cavity (or wake) of constant pressure, p_c. Viscosity is neglected* so that the effects of the vorticity field alone on free-streamline wakes are considered. The basic flow has the velocity components $(0, U(1+\gamma z), 0)$ in the Cartesian coordinates shown in Fig. 1 where U and γ are constant, and hence the flow has a uniform vorticity

$$\omega_o = -\partial U(1+\gamma z)/\partial z = -U\gamma . \qquad (3)$$

* For justification of this assumption, please see Section 8.

It follows from Helmholtz's vorticity theorem that the resulting velocity field $\boldsymbol{u}(\boldsymbol{x})$ of the wake flow will be characterized by the same uniform vorticity distribution throughout,

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{u} = \omega_0 \boldsymbol{e}_x = - U\gamma\, \boldsymbol{e}_x \ , \tag{4}$$

where $\boldsymbol{e}_x$ is the unit vector pointing out of the paper and $\boldsymbol{\omega}$ is the vorticity vector. This is an immediate consequence of the Helmholtz vorticity equation,

$$\frac{d\boldsymbol{\omega}}{dt} = \left(\frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla\right) \boldsymbol{\omega} = 0 \ , \tag{5}$$

which holds for two-dimensional motion of an incompressible, inviscid fluid as is the case at hand.

Accordingly, the resulting velocity field $\boldsymbol{u} = (0, v, w)$ will consist of two parts, one retaining the original uniform vorticity throughout and the other being irrotational, hence possessing a scalar velocity potential $\phi(y, z; \gamma)$ which depends on the vorticity strength γ as a parameter. Thus the velocity components can be expressed as

$$u = U\gamma z + \frac{\partial \phi}{\partial y} = \frac{\partial \Psi}{\partial z} \ , \qquad w = \frac{\partial \phi}{\partial z} = - \frac{\partial \Psi}{\partial y} \tag{6}$$

where Ψ is the stream function. From the continuity equation, $v_y + w_z = 0$, and the vorticity distribution (4) required of the flow it follows that

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{7}$$

and

$$\frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = U\gamma \ . \tag{8}$$

In addition to these equations for the kinematic variables, we have, for the dynamics, the Bernoulli equation

$$p + \frac{1}{2}\rho(u^2 + w^2) = H(\Psi) \tag{9}$$

where $p(\underset{\sim}{x})$ is the pressure, ρ the (constant) fluid density, and $H(\Psi)$ represents a constant along each stream line. In terms of the kinematic and dynamic variables we can prescribe the boundary conditions of the problem as follows

$$v\frac{\partial F_1}{\partial y} + w\frac{\partial F_1}{\partial z} = 0 \qquad \text{on} \quad F_1(y, z) = 0 \ , \tag{10}$$

and

$$p = p_c = \text{const.} \qquad \text{on} \quad F_2(y, z) = 0 \ , \tag{11}$$

where $F_1(y, z) = 0$ denotes the wetted body surface and $F_2(y, z) = 0$, the free streamlines that form the wake boundary, but are unknown á priori. The constant pressure p_c inside the wake, called the wake underpressure ($p_c < p_\infty$, the pressure at infinity, in general), is assumed to be given. In order to make possible our consideration of the general case of $p_c < p_\infty$, a theoretical model of finite wake is required, for which we again adopt the 'open-wake model' as in the previous case (of uniform free stream treated in Report AMT-R-101). Finally, we have the following boundary conditions at infinity:

$$v \to U, \quad w \to 0, \qquad p \to p_\infty \quad \text{as} \ y^2 + z^2 \to \infty \ . \tag{12}$$

3.1 Small perturbation theory

To render this nonlinear problem tractable to complex functional analysis, we shall assume that the primary flow is only weakly sheared,

i. e., $\gamma \ell \ll 1$ as measured with respect to the body dimension ℓ. Thus we seek the solution in the form of an asymptotic expansion for small $\gamma \ell$, or in complex notation

$$\chi = y + iz , \qquad \bar{\chi} = y - iz , \tag{13}$$

$$W(\chi, \bar{\chi}; \gamma) = v - iw = W_o(\chi) + U\gamma z + \gamma W_1(\chi) + O(\gamma^2) , \tag{14}$$

$$W_n(\chi) = df_n(\chi)/d\chi = v_n - iw_n , \qquad (n=0, 1, 2, \ldots) \tag{15}$$

where $f_n(\chi) = \phi_n(y, z) + i\psi_n(y, z)$ $(n = 0, 1, 2, \ldots)$ are analytic functions of χ, regular everywhere in the interior of the flow field, but may have branch-point type singularities at isolated points on the flow boundary. The wetted body surface will be given parametrically by

$$y = \hat{y}(s), \qquad z = \hat{z}(s) \qquad (0 < s < S) , \tag{16}$$

where s is the arc length along the body surface, running from point A (where $s = 0$) to point B (where $s = S$, see Fig. 1). To facilitate our functional analysis, we continue to employ the same parametric $\zeta = \xi + i\eta$ plane as in the case of uniform current, such that the flow region is mapped conformally into the lower half of the ζ-plane, with the wetted body surface mapped onto $(-1 < \xi < 1, \ \eta = 0)$, the stagnation point provisionally onto the origin $\zeta = 0$, and the cavity boundary onto $(|\xi| > 1, \ \eta = 0)$. Substituting the expansion (14) in (10) and using $(\partial F_1/\partial y)/(\partial F_1/\partial z) = -\hat{z}'(s)/\hat{y}'(s)$, we obtain

$$(v_o + \gamma Uz + \gamma v_1 + \ldots)\hat{z}'(s) = (w_o + \gamma w_1 + \ldots)\hat{y}'(s) \qquad (|\xi| < 1, \ \eta=0),$$

where the primes denote differentiation, $\hat{y}'(s) = d\hat{y}/ds$, $\hat{z}'(s) = d\hat{z}/ds$. Since γ is otherwise arbitrary, we must have for $|\xi| < 1$, $\eta = 0$,

$$w_o/v_o = \hat{z}'(s)/\hat{y}'(s) \tag{17a}$$

$$\frac{w_1}{Uz + v_1} = \frac{\hat{z}'(s)}{\hat{y}'(s)} \tag{17b}$$

and

$$w_n/v_n = \hat{z}'/\hat{y}' \qquad (n = 2, 3 \ldots), \tag{17c}$$

for the higher order terms. In addition, the pressure condition imposed on the free streamline can be written

$$W_o \overline{W}_o + 2\gamma [v_o(Uz + v_1) + w_o w_1] = U^2(1 + \sigma + 2\gamma z_\infty) + O(\gamma^2)$$

where

$$\sigma = (p_\infty - p_c)/(\tfrac{1}{2}\rho U^2) \tag{18}$$

is the wake-underpressure coefficient and z_∞ is the asymptotic value of z of the stagnation streamline at upstream infinity. This condition therefore requires that to the first two orders

$$|W_o| = q_c = U(1 + \sigma)^{1/2} \qquad (|\xi| > 1, \quad \eta = 0), \tag{19a}$$

$$v_o(Uz + v_1) + w_o w_1 = U^2 z_\infty \qquad (|\xi| > 1, \quad \eta = 0), \tag{19b}$$

Similar conditions can be derived for v_n and $w_n (n = 2, 3, \ldots)$. The conditions at infinity, Eq. (12), now become

$$W_o \to U, \quad W_n \to 0 \qquad (n = 1, 2, \ldots) \text{ as } \zeta \to \zeta_o, \tag{20}$$

where $\zeta = \zeta_o$ is the image of the point $\chi = \infty$.

The first order theory has been developed to provide solutions as already presented in AM Tech Rept AMT-R-101. For the convenience of applications in the sequel we delineate the main steps of the analysis as follows. First, we represent the finite wake by the generalized open-wide model introduced by Wu & Wang (1964a), according to which the conformal transformation between the leading order complex potential $f_o = \phi_o + i\psi_o$ and the parametric variable $\zeta = \xi + i\eta$ (as depicted in Fig. 2) is given by the Schwarz-Christoffel mapping.

$$\frac{df_o}{d\zeta} = A \frac{\zeta(\zeta-\xi_c)}{(\zeta-\zeta_o)^2(\zeta-\bar{\zeta}_o)^2} , \tag{21}$$

where A is a real constant, ζ_o is the image point of $\chi = \infty$ (also $f_o = \infty$), $\bar{\zeta}_o$ its complex conjugate, ξ_c is a real constant marking the rear end of the near wake at which the constant wake pressure region terminates.

To best effect satisfaction of the kinematic and dynamic conditions we introduce the logarithmic hodograph variable

$$\Omega_o = \log \frac{q_c}{W_o} = \tau_o + i\theta_o, \quad \tau_o = \log \frac{q_c}{|W_o|}, \quad \theta_o = \tan^{-1} w_o/v_o , \tag{22}$$

q_c being the constant flow speed, to leading order, on the wake boundary. Here θ_o is the leading-order angle subtended by the flow velocity to the y-axis. In terms of $\Omega_o(\zeta) = \tau_o(\xi,\eta) + i\theta_o(\xi,\eta)$ conditions (17a) and (19a) become

$$\theta_o^-(\xi) \equiv \theta_o(\xi, 0^-) = \pi\mathcal{H}(-\xi) + \hat{\beta}(s(\xi)) \qquad (|\xi| < 1) , \tag{23}$$

$$\tau_o^-(\xi) \equiv \tau_o(\xi, 0^-) = 0 \qquad (|\xi| > 1), \qquad (24)$$

where $\mathcal{H}(-\xi)$ denotes Heaviside's step function

$$\mathcal{H}(-\xi) = 1 \quad \text{for } \xi < 0 , \quad \mathcal{H}(-\xi) = 0 \quad \text{for } \xi > 0 ,$$

and

$$\hat{\beta}(s) = \tan^{-1} [\hat{z}'(s)/\hat{y}'(s)] . \qquad (25)$$

This Riemann-Hilbert problem can be resolved as follows. We first continue analytically the flow field in the lower half ζ-plane into the upper half ζ-plane by the Schwarz reflection

$$\Omega_o(\overline{\zeta}) = -\overline{\Omega_o(\zeta)} , \qquad (26)$$

where the overhead bar denotes taking the complex conjugate, or equivalently,

$$\tau_o(\xi, -\eta) = -\tau_o(\xi, \eta) , \qquad \theta_o(\xi, -\eta) = \theta_o(\xi, \eta) . \qquad (27)$$

Then, noting that

$$\Omega_o^{\pm}(\xi) = \tau_o^{\pm}(\xi) + i\,\theta^{\pm}(\xi) = \pm\, \tau_o^{+}(\xi) + i\,\theta_o^{+}(\xi) , \qquad (28)$$

in which the last step follows from (27), conditions (23) and (24) can be written as

$$\Omega_o^+ + \Omega_o^- = 2i\theta_o^-(\xi) \qquad (\text{given for } |\xi| < 1) , \qquad (29)$$

$$\Omega_o^+ - \Omega_o^- = 2\tau_o^+(\xi) = 0 \qquad (|\xi| > 1) . \qquad (30)$$

The associated homogeneous problem (corresponding to (29) and (30), is defined by

$$H^+(\xi) + H^-(\xi) = 0 \qquad (|\xi| < 1) , \tag{31}$$

$$H^+(\xi) - H^-(\xi) = 0 \qquad (|\xi| > 1) , \tag{32}$$

where $H(\zeta)$ is required to be holomorphic everywhere in the ζ-plane except possibly at $\zeta = \pm 1$ and $\zeta = \infty$; it has infinitely many solutions:

$$H(\zeta) = (\zeta^2 - 1)^{1/2} R(\zeta) , \tag{33}$$

where $R(\zeta)$ is any suitable rational function of ζ. Here the function $(\zeta^2 - 1)^{1/2}$ is defined with a branch cut in the ζ-plane from $\zeta = -1$ to 1 along $\eta = 0$ so that it is single-valued in the entire cut plane of ζ and tends to ζ as $|\zeta| \to \infty$ for all arg ζ. We next introduce the new function

$$F(\zeta) = \Omega_o(\zeta)/H(\zeta) . \tag{34}$$

From (29)-(32) we see that $F(\zeta)$ has a jump across the ξ-axis by

$$F^+(\xi) - F^-(\xi) = (\Omega_o^+ + \Omega_o^-)/H^+(\xi) = 2i\theta_o^-(\xi)/H^+(\xi) \qquad (|\xi| < 1), \tag{35}$$

$$= (\Omega_o^+ - \Omega_o^-)/H^+(\xi) = 0 \qquad (|\xi| > 1) . \tag{36}$$

A particular solution can therefore be obtained for $F(\zeta)$ by applying Plemelj's formula (see, e.g. Muskhelishvile 1953, §17)

$$F(\zeta) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{F^+(t) - F^-(t)}{t - \zeta} dt , \tag{37}$$

from which we deduce

$$\Omega_o(\zeta) = F(\zeta)H(\zeta) = \frac{1}{\pi} H(\zeta) \int_{-1}^{1} \frac{\theta_o^-(t)}{H^+(t)(t-\zeta)} \, dt \quad . \tag{38a}$$

Under the required condition that $\Omega_o(\zeta)$ be analytic and regular everywhere in the cut ζ-plane, in particular at $\zeta = \pm 1$ and $\zeta = \infty$, the only possible form for $H(\zeta)$ is

$$H(\zeta) = (\zeta^2 - 1)^{1/2} \quad , \tag{38b}$$

and then (38a) gives the unique solution of $\Omega_o(\zeta)$. Using the boundary value (23) for $\theta_o^-(t)$ in (38) and carrying out some integration finally yields

$$\Omega_o(\zeta) = \log \frac{1+i(\zeta^2-1)^{1/2}}{\zeta} + \frac{1}{\pi i} \int_{-1}^{1} \left(\frac{\zeta^2-1}{1-t^2}\right)^{1/2} \frac{\beta(t)dt}{t-\zeta} \tag{39}$$

where

$$\beta(\xi) = \hat{\beta}(s(\xi)) \qquad (|\xi| < 1) \quad . \tag{40}$$

The solution is thus determined to leading order in parametric representation, $f_o = f_o(\zeta)$, $\Omega_o = \Omega_o(\zeta)$, given by (21) and (39). The physical plane is then given to leading order by quadrature

$$z_o(\zeta) = \int \frac{1}{W_o(\zeta)} \frac{df_o}{d\zeta} d\zeta \quad , \tag{41}$$

The arc length $s_o(\xi)$, measured along the body surface from $\xi = -1$, is

$$s_o(\xi) = \frac{A}{q_c} \int_{-1}^{\xi} e^{\Gamma_o(\xi)} \nu(\xi) d\xi \quad . \qquad (-1 < \xi < 1) \ , \tag{42}$$

$$\Gamma_o(\xi) = -\frac{1}{\pi}\int_{-1}^{1}\left(\frac{1-\xi^2}{1-t^2}\right)^{1/2}\frac{\beta_o(t)dt}{t-\xi} , \tag{43}$$

$$\nu(\xi) = [1+(1-\xi^2)^{1/2}]\frac{\xi-\xi_c}{(\xi-\zeta_o)^2(\xi-\bar{\zeta}_o)^2} \tag{44}$$

The integral in Eq. (43) assumes its Cauchy principal value. The total wetted arc length is specified by

$$S = s_o(1) . \tag{45}$$

Moreover, the velocity condition at $\chi = \infty$, or $\zeta = \zeta_o$, requires that

$$\Omega_o(\zeta_o) = \log\frac{q_c}{U} = \frac{1}{2}\log(1+\sigma) . \tag{46}$$

This completes the formal solution to leading order for the wake flow past a ship section with a uniform incident current. The foregoing analysis provides a set of nonlinear functional equations (42) and (46) which we write symbolically as

$$s_o(\xi)/(A/q_c) = \mathcal{E}[s_o(\xi), \hat{\beta}(s_o(\xi)); \zeta_o] , \tag{47}$$

$$\zeta_o = (U/q_c)\mathcal{F}[s_o(\xi), \hat{\beta}(s_o(\xi)); \zeta_o] , \tag{48}$$

where

$$\mathcal{E}[s_o(\xi), \hat{\beta}(s_o(\xi)); \zeta_o] = \int_{-1}^{\xi} e^{\Gamma_o(\xi)}\nu(\xi;\zeta_o)d\xi , \tag{49}$$

$$\mathfrak{F}[s_o(\xi), \hat{\beta}(s_o(\xi));\zeta_o] = [1+i(\zeta_o^2-1)^{1/2}]\exp\{\frac{1}{\pi i}\int_{-1}^{1}(\frac{\zeta_o^2-1}{1-t^2})^{1/2}\frac{\beta_o(t)dt}{t-\zeta_o}\}. \tag{50}$$

Solution of the above set of nonlinear functional equations can be obtained by numerical methods. Further development of a workable numerical method notwithstanding, we proceed to discuss below the first-order theory by which the effect due to a weak vertical shear of the incident current can be evaluated.

3.2 The first-order perturbation theory for a weakly sheared current

To first order in the shear gradient γ, condition (17b) - after eliminating $\hat{z}'/\hat{y}'$ in it by making use of (17a) - and condition (19b) become

$$v_o w_1 - w_o v_1 = U w_o z \qquad (|\xi| < 1, \ \eta = 0^-), \tag{51}$$

$$v_o v_1 + w_o w_1 = U(U z_\infty - v_o z) \qquad (|\xi| > 1, \ \eta = 0^-). \tag{52}$$

These two equations can be recast as

$$\mathrm{Im}\, G_1 = -U w_o z_o \qquad (|\xi| < 1, \ \eta = 0^-), \tag{53}$$

$$\mathrm{Re}\, G_1 = U(U z_\infty - v_o z_o) \qquad (|\xi| > 1, \ \eta = 0^-), \tag{54}$$

where $$G_1(\zeta) = \overline{W}_o W_1 = (v_o + i w_o)(v_1 - i w_1) \tag{55}$$

In (51) and (52) we have replaced the unknown function $z(\xi,\eta)$ by its leading-order solution (41). Now with the function $G_1(\zeta)$ analytically continued into the upper half ζ-plane by

$$G_1(\bar{\zeta}) = -\overline{G_1(\zeta)} \quad , \tag{56}$$

we can rewrite (53) and (54) as

$$G_1^+(\xi) + G_1^-(\xi) = -2iU\, w_o z_o \qquad (|\xi| < 1) \; , \tag{57}$$

$$G_1^+(\xi) - G_1^-(\xi) = -2U(U\, z_\infty - v_o z_o) \qquad (|\xi| > 1) \; , \tag{58}$$

in which the known quantities v_o, w_o and z_o are understood to assume their values in the limit as $\eta \to 0^-$. This boundary value problem can be solved by employing the same method as that for Ω_o prescribed by (29) and (30) to give

$$G_1(\zeta) = -\frac{U}{i\pi} H(\zeta)\{(\int_{-\infty}^{-1} + \int_1^{\infty}) \frac{v_o(t)z_o(t) - Uz_\infty}{H^+(t)(t-\zeta)} dt + i\int_{-1}^{1} \frac{w_o(t)z_o(t)dt}{H^+(t)(t-\zeta)}\} . \tag{59}$$

in which $H(\zeta)$ is given by (38b). Then $W_1(\zeta)$ immediately follows from (55).

Within the framework of harmonic function theory, we next modify the complex potential function from $f_o(\zeta)$ for the leading order theory to $f^{(1)}$ such that

$$W_o(\zeta) + \gamma W_1(\zeta) = \frac{df^{(1)}}{d\chi} = \frac{df^{(1)}}{d\zeta} \frac{d\zeta}{d\chi} \quad . \tag{60}$$

It should be pointed out that in

$$f^{(1)}(\zeta) = \varphi^{(1)}(\xi, \eta) + i\psi^{(1)}(\xi, \eta) \quad , \tag{61}$$

$\psi^{(1)} = \text{const}$ does not represent a streamline (to first order for small γ), but rather it takes constant values of

$$\Psi = \frac{1}{2} U\gamma z^2 + \psi^{(1)} = \psi_o + \frac{1}{2} U\gamma z^2 + \gamma\psi_1 \quad . \tag{62}$$

We further note that the stagnation point on the body surface is no longer necessarily at $\zeta = 0$ but is generally shifted to $\zeta = \xi_*$ on the real ζ-axis due to the contribution from W_1, i.e.,

$$W_o(\xi_*) + \gamma W_1(\xi_*) = 0 \quad , \qquad (63)$$

ξ_* being clearly of order γ. This in turn requires that the transformation (21) be modified to the following form

$$\frac{df^{(1)}}{d\zeta} = A \frac{(\zeta - \xi_*)(\zeta - \xi_c)}{(\zeta - \zeta_o)^2(\zeta - \bar{\zeta}_o)^2} \quad , \qquad (64)$$

in which the relocated zero of $df^{(1)}/d\zeta$ at $\zeta = \xi_*$ is required to make the new stagnation point a branch point of the stagnation streamline. With this up-graded $f^{(1)}(\zeta)$, the physical plane can be calculated anew by integrating

$$df^{(1)}/d\chi = W_o(\zeta) + \gamma W_1(\zeta) \quad , \qquad (65)$$

giving

$$\chi^{(1)}(\zeta) = \int_{-1}^{\zeta} \frac{1}{W_o(\zeta) + \gamma W_1(\zeta)} \frac{df^{(1)}}{d\zeta} d\zeta \qquad (66)$$

Finally, the same conditions as (42) and (46) for the leading order solution can be applied to $\chi^{(1)}$ of (66) and Ψ of (62); and the remaining task is to develop a numerical method for computing the solution.

4. The hydrodynamic forces and moments

The hydrodynamic forces and moments of significance during the motion of a moored ship are primarily the sway force F_y and yaw moment M_z. To facilitate possibly accounting for large excursions in the horizontal plane we employ a coordinate system (x, y, z) fixed with respect to the ship. The remaining three modes of rigid-body motion, namely the heave, pitch and roll can be assumed to be unimportant in moored vessel problems, as is generally the case, and are accordingly ignored.

The motion of a moored ship, even for the case of a single-point mooring, is relatively slow, as compared to vertical ship motions in waves, due to the predominant inertial effects. Thus it is reasonable to neglect the wave effects associated with the wave making by the current, which can be evaluated separately later. For the same reason we assume that the transverse flow with sway and yaw can be analyzed on the basis of the low-frequency limit of the free-surface conditions. In linear theory the two free-surface conditions (when expressed with respect to a coordinate system moving with the current) for small free surface elevation $\eta(x, y, t)$ and velocity potential $\phi(x, y, z, t)$ read

$$\frac{\partial \phi}{\partial t} + g\eta = 0 \qquad (z = 0) \, , \tag{67}$$

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} \qquad (z = 0) \, . \tag{68}$$

For time harmonic motions, with the factor $e^{i\omega t}$, the above two conditions combine to give

$$\frac{\partial \phi}{\partial z} - \frac{\omega^2}{g} \phi = 0 \, . \qquad (z = 0) \tag{69}$$

In the low-frequency limit, $\omega^2 \ell / g \rightarrow 0$, the above condition reduces to

$$\frac{\partial \phi}{\partial z} = 0 \qquad (z = 0) \quad , \tag{70}$$

that is, the free surface becomes in this limit a rigid horizontal plane. With this approximation the flow field can be continued into the upper half space $z > 0$ by mirror image, and the problem becomes one of irrotational flow of an inviscid fluid past a rigid double body, symmetrical with respect to the plane $z = 0$. Thus the wetted portion of the hull is merely the lower half of the double body.

In our analysis of the transverse hydrodynamic force associated with the total transverse current velocity

$$V(x, z, t) = U_c(x, t)(1+\gamma_o z) - U_G(t) - x\,\Omega(t) = U_{yc} - x\,\Omega \, , \tag{71}$$

which now includes slowly varying spatial and temporal variations, we distinguish between two fundamentally different flow regimes. One is characterized by a viscous wake formation that occurs at sufficiently high incidence angles of the current; the other takes place at low incidence angles when the lateral wake swings backwards to merge with any existing vortex sheet shed from the keel into a trailing stern wake. For practical purposes a pragmatic continuous transition between the two regimes can be sought by numerical matching.

For the case of lateral wake flow problem we first evaluate for the local strip (at station x and at time t) the two-dimensional wake flow in a vertically sheared current following the analysis and solution method presented in the previous section. The total y-component velocity given by (71) can be recombined as

$$V(x, z, t) = U_2(x, t)\,[1 + \gamma z] \quad , \tag{72}$$

in which the specific shear rate γ may be taken as general as a slowly varying function of x and t. By integrating the resulting pressure over the wetted hull of the double body with respect to the wake underpressure as datum, we obtain the y-component of the hydrodynamic differential force as

$$dF_{yc} = \frac{1}{2}\rho_w C_{DC}(x, t;\sigma) U_2^2(x, t) h(x) dx \quad , \tag{73}$$

where $h(x)$ is the sectional draft of the ship (see Fig. 3). This result is now applicable to cases more general than (17) of Part I, since both spatial and temporal variations in the current velocity have been taken into account, provided that these variations are slowly varying and the water depth is not much shallower than twice the ship's draft. The total transverse current force and the yaw moment can then be obtained by integration from the ship's bow end at $x = -\ell_b$ to the stern at $x = \ell_s$ as

$$F_{yc}(t) = \frac{1}{2}\rho_w \int_{-\ell_b}^{\ell_s} C_{DC}(x, t;\sigma) U_2^2(x, t) h(x) dx \quad , \tag{74}$$

$$M_c(t) = \frac{1}{2}\rho_w \int_{-\ell_b}^{\ell_s} C_{DC}(x, t;\sigma) U_2^2(x, t) h(x) x dx \quad . \tag{75}$$

We remark again that the end strips at the stern end, $x = \ell_s$, and at the bow end, $x = -\ell_b$, can be handled by employing the refined scheme described below Eq. (17), Part I.

For the case of small incidences of the incoming current with respect to the ship's centerplane, the general theory of Wu (1971) developed for unsteady slender-body flow remains valid even when the current has spatial and temporal variations in velocity provided they are slowly varying in units of the body's transverse dimension and its characteristic time of motion. Thus the local transverse force (called the differential lift due to circulation) acting on a body strip of unit thickness (see Wu 1971) is given by $\mathcal{L}_{cir} = dL_{cir}/dx$, where

$$\mathcal{L}_{cir} = (\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x})[V(x,t)m_{22}(x)] \quad \text{(without side vortex)} \; , \qquad (76a)$$

or

$$= m_{22}(x)(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x})V(x,t) \qquad \text{(with side vortex)} \; , \qquad (76b)$$

where $U_1(x,t)$ is the local longitudinal velocity component with respect to the body section and $m_{22}(x)$ is the sectional, frequency-independent added-mass coefficient of the lower half of the double body for the sway mode. Strictly speaking, the above formulas for the differential lift holds only for currents having no vertical gradient of shear, that is, with $\gamma = 0$ in (72). When $\gamma \neq 0$, attempts may be made to derive an equivalent added-mass coefficient $m_{22}^{e}(x)$ to replace $m_{22}(x)$ in (76), but such a correction for the vertical shear effect would require considerable amount of new developments, analysis and calculation.

On the same premise the differential induced drag associated with the local lift of this circulatory flow can be obtained by applying the general slender-body theory of Wu (1971) to yield

$$\mathcal{D}_{cir} = dD_{cir}/dx = \frac{1}{2}(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x})[(U_{yc}^2 - x^2\Omega^2)m_{22}(x)] \; , \qquad (77a)$$

or

$$= \frac{1}{2} m_{22}(x)(\frac{\partial}{\partial t} + u_1 \frac{\partial}{\partial x})(U_{yc}^2 - x^2\Omega^2) \; , \qquad (77b)$$

according as there is, (77a), or there is not, (77b), a local vortex sheet being shed from the keel of the hull surface. The total longitudinal and transverse (i. e., the x- and y-component) hydrodynamic forces are therefore given by the integrals

$$F_{xc} = \int_{-\ell_b}^{\ell_s} [\mathcal{L}_{cir}(x,t) \sin \alpha - \mathcal{D}_{cir}(x,t) \cos \alpha] dx \quad , \tag{78}$$

$$F_{yc} = \int_{-\ell_b}^{\ell_s} [\mathcal{L}_{cir}(x,t) \cos \alpha + \mathcal{D}_{cir}(x,t) \sin \alpha] dx \quad , \tag{79}$$

where the local incidence angle α may also be a slowly varying function of x and t. Finally, we can express the yaw moment about the origin as

$$M_{cir} = \int_{-\ell_b}^{\ell_s} \mathcal{L}_{cir}(x,t)\, x \;dx \quad . \tag{80}$$

5. <u>The rudder force</u>

The general slender-body theory can further be applied to predict the rudder force on the assumption that the hull-rudder system as represented by its double model is a single low-aspect-ratio lifting surface with prescribed surface displacement. As the contribution from the transverse current velocities to sway force and yaw moment have already been determined for both cases of finite and small values of the incidence angle, we need onlyconsider the action of the rudder as a deflection of the stern part of the slender hull, which is the rudder itself and its double model image, from its stretched-straight position by the amount

$$y = -\Theta_R(t)(x - \ell_R) \qquad (\ell_R < x < \ell_s,\ h_{1R} < |z| < h_{2R})$$
$$= 0 \qquad (\text{elsewhere}) \qquad (81)$$

where the above specified region in x and $|z|$ denotes the rudder planform, $x = \ell_R$ is the coordinate of the rudder axis (assumed vertical) and $\Theta_R(t)$ is the angle of rudder deflection. This rudder displacement gives rise to an equivalent transverse flow velocity (relative to body)

$$V(x,t) = (\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x})\Theta_R(t)(x - \ell_R) = + \dot{\Theta}_R(t)(x - \ell_R) + U_1 \Theta_R . \qquad (82)$$

Substituting this relation for V in (76) and integrating the resulting lift over the rudder section gives the rudder force

$$F_{yR} = +[\frac{1}{2}\ddot{\Theta}_R C_R^2 + 2U_1 C_R \dot{\Theta}_R + U_1^2 \Theta_R]\, m_R \quad , \qquad (83)$$

where $C_R = \ell_s - \ell_R$ is the constant chord of the rudder and m_R is the two-dimensional added mass of the rudder which is assumed to be a constant along its chord. The yaw moment due to rudder action is simply

$$M_R = F_{yR}\ell_R \quad . \qquad (84)$$

This contribution from rudder to the sway force and yaw moment can be combined with that from other sources in assembling the equations of motion for the moored ship.

6. Wave radiation from a ship moored in steady current

The problem of interaction between the ship hull and a steady current has been resolved in the preceding sections for the low frequency limit, that is, with the assumption that the gravity effect on wave making by the ship held fixed in the current is of secondary importance and was therefore neglected in order to facilitate a full account of the primary effect of wake formation. (To give a rough estimate for comparison, we note that for the Froude number of a typical ship of length ℓ to be $Fr = U/(g\ell)^{1/2} < 0.2$, the wave resistance coefficient is < 0.1 as a rule whereas the pressure drag coefficient associated with a transverse wake due to a cross current can be as large as 1 or somewhat greater.)

We now proceed to investigate this gravity effect and give a separate account of the energy loss due to wave generation caused by interaction between the hull and a steady current. We shall assume that the incoming stream is a steady, at least a quasi-steady current, without carrying any incident gravity wave, and that the incidence angle α (between the current velocity and the longitudinal x-axis of the ship) is arbitrary ($0 < \alpha < 360^\circ$). The last assumption distinguishes the present problem from the classical theory of wave-making resistance of ships; in fact, we note that the vast literature is very much limited to the case of $\alpha = 0$ as only this case is required for consideration of ship propulsion.

To effect a viable engineering solution to the present problem, we shall assume that when the transverse wake is manifest, the shape of which is supposed to be already determined by the method proposed, the wake boundary plus the front part of the wetted hull outside the wake will be taken as the new 'floating-body' surface that is responsible for wave generation. This assumption is made on physical ground that the wake boundary (assumed stationary in the mean) is a stream surface, but its enforcement will make the new body no longer as thin or slender as the original ship hull. Consequently the classical thin-ship theory and slender-ship theory should both become nonapplicable for the present case, but the method of Green's function can still be useful. To render the problem tractable, however, we shall further assume that the resulting waves are small in amplitude so that the free-surface boundary conditions can be linearized. Thus we have the following formulation.

There exists an additional velocity potential $\phi(x, y, z)$ which is solely responsible for wave generation. This $\phi(\underset{\sim}{x})$ must satisfy, in the linearized flow region $0 > z > -h_o = \text{const.}$, the following equations

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \qquad (-h < z < 0) , \tag{85}$$

$$\phi_{xx} + K\phi_z = 0 \qquad (z = 0, \ \ K = g/U_c^2) , \tag{86}$$

$$\frac{\partial\phi}{\partial n} = -U_c n_1 \qquad (\underset{\sim}{x} \text{ on } S_{BW}) , \tag{87}$$

$$\phi = o((x^2 + y^2)^{-1/2}) \qquad \text{as } x^2 + y^2 \to \infty, \quad x < 0 , \tag{88}$$

$$\phi_z = 0 \qquad (z = -h_o) . \tag{89}$$

Here, we have adopted a new rectangular coordinate system with the x-axis pointing in the direction of the current velocity $\underset{\sim}{U}_c$, the z-axis remaining the same as before so that the $z = 0$ plane coincides with the undisturbed water surface S_f, the $z = -h_o$ plane is the horizontal floor of the water layer (including $h_o = \infty$ as the deep water limit), g is the gravitational acceleration. Equation (86) is the linearized free-surface condition which is the stationary version of (69). In (87), $\partial\phi/\partial n = \underset{\sim}{n} \cdot \nabla\phi$, and n_1 is the x-component of the outward (from the fluid) normal $\underset{\sim}{n}$ to the surface $S_{BW} = S_B + S_W$ which is the union of the wetted body and wake boundary. The order limitation (88) is the Sommerfeld radiation condition for three-dimensional water waves. The free surface elevation $Z(x, y)$ is determined by

$$Z(x, y) = \frac{U_c}{g} \phi_x(x, y, 0) \quad . \tag{90}$$

The hydrodynamic force acting on the hull due to the effects of wave making is

$$\underset{\sim}{F} = -\rho U \int_{S_B} \phi_x \underset{\sim}{n} \, dS + \int_{S_W} (p_w - p_c) \underset{\sim}{n} \, dS \quad , \tag{91}$$

where $\underset{\sim}{n}$ is the unit normal pointing into the hull and the wake, and $p = p_w$ on the wake boundary enclosing the near wake with a specified base pressure p_c.

The desired Green's function may assume the form

$$G(\underset{\sim}{x} ; \underset{\sim}{\xi}) = r^{-1} + G^*(\underset{\sim}{x};\underset{\sim}{\xi}) \quad , \tag{92}$$

where

$$r = |\underset{\sim}{x} - \underset{\sim}{\xi}| = [(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2]^{1/2} \tag{93}$$

is the distance between two field points $\underset{\sim}{x}$ and $\underset{\sim}{\xi}$, G^* is harmonic and regular in the flow region $0 > z > -h_o$,

$$\nabla^2 G^* = 0 \qquad (0 > z > -h_o) . \tag{94}$$

The function G satisfies the following boundary conditions

$$G_{\xi\xi}(\underset{\sim}{x};\xi,\eta,0) + KG_\eta(\underset{\sim}{x};\xi,\eta,0) = 0 \tag{95}$$

$$G_\eta(\underset{\sim}{x};\xi,\eta,-h_o) = 0 \tag{96}$$

and

$$G = 0(|\xi^2+\eta^2|^{-1/2}) \quad \text{as } \xi^2+\eta^2 \to \infty, \quad \xi < 0. \tag{97}$$

We now apply Green's third formula to the region bounded by the plane $S_f(\zeta = 0)$, the bottom $S_h(\zeta = -h_o)$, a large control surface Σ, $(\xi^2+\eta^2)^{1/2} = R$, the wetted hull surface S_B and the wake boundary S_w. Then we have

$$4\pi\varphi(\underset{\sim}{x}) = \int_S \left[\frac{\partial\varphi(\underset{\sim}{\xi})}{\partial n} G(\underset{\sim}{x};\xi) - \varphi(\underset{\sim}{\xi}) \frac{\partial G}{\partial n}\right] dS_\xi \tag{98}$$

where $S = S_f + S_h + \Sigma + S_B + S_w$, and dS_ξ signifies a surface element at $\underset{\sim}{\xi}$ on S. By assuming for ϕ and G the following asymptotic representations (taking the real component for physical interpretation)

$$\begin{pmatrix}\phi\\ G\end{pmatrix} = \begin{pmatrix}A\\ B\end{pmatrix} \frac{1}{r^{1/2}} e^{+iKrf(\theta)} + 0\left(\frac{1}{r}\right) \qquad \text{as } r \to \infty, \quad \xi > 0 \tag{99}$$

where $f(\theta)$ stands for a function of $\theta = \arctan\,(\eta/\xi)$ only, as can be verified á posteriori, it is readily shown that the integral over the control surface Σ vanishes as $R \to \infty$. After using the boundary conditions for ϕ and G, we obtain

$$4\pi\phi(\underset{\sim}{x}) = \int_{S_B + S_w} [-Un_1(\underset{\sim}{\xi})G(\underset{\sim}{x};\underset{\sim}{\xi}) - \varphi(\underset{\sim}{\xi})\frac{\partial G}{\partial n}]\, dS + \frac{1}{K}\int_{S_f} \frac{\partial}{\partial \xi}(\varphi G_\xi - \varphi_\xi G)d\xi d\eta \,.$$

The last integral over the free-surface S_f may be intregrated by parts to give

$$4\pi\phi(\underset{\sim}{x}) = -\int_{S_B+S_w} [Un_1(\underset{\sim}{\xi})G(\underset{\sim}{x};\underset{\sim}{\xi}) + \varphi(\underset{\sim}{\xi})\frac{\partial G}{\partial n}]dS + \frac{1}{K}\oint_{C_{BF}} (\varphi_\xi G - \varphi G_\xi)d\eta \,, \tag{100}$$

where the line integral is taken counterclockwise around the contour C_{BF} in the ξ-η plane which is the intersection of S_B and S_f. (The line integral around the intersection of S_f and Σ vanishes as $R \to \infty$.) Now if in the surface integral we let a field point $\underset{\sim}{x} \to \underset{\sim}{x}_o$ on $S_B + S_w$, then by a well known theorem (a lemma of Green's theorems) the integral with $\partial G/\partial n$ converges to

$$\lim_{\underset{\sim}{x} \to \underset{\sim}{x}_o} \int_{S_{Bw}} \varphi(\underset{\sim}{\xi})G_n(\underset{\sim}{x};\underset{\sim}{\xi})dS_\xi = -2\pi\varphi(\underset{\sim}{x}_o) + \int_{S_{Bw}} \varphi(\underset{\sim}{\xi})G_n(\underset{\sim}{x}_o;\underset{\sim}{\xi})dS_\xi \,.$$

We then have the following integral equation for $\varphi(\underset{\sim}{x})$, where $\underset{\sim}{x}$ is a point of S_{Bw}:

$$2\pi\varphi(\underset{\sim}{x}) + \int_{S_{Bw}} \varphi(\underset{\sim}{\xi})G_n(\underset{\sim}{x};\underset{\sim}{\xi})dS_\xi = -U\int_{S_{Bw}} n_1(\underset{\sim}{\xi})G(\underset{\sim}{x};\underset{\sim}{\xi})dS_\xi + \frac{1}{K}\oint_{C_{BF}} (\phi_\xi G - \varphi G_\xi)d\eta. \tag{101}$$

When this equation can be solved for ϕ for $\underset{\sim}{x}$ on S_{Bw} by numerical methods, then ϕ is determined everywhere by (100). However, for the general case of a blunt body in transverse current, no numerical method is known to exist and it may require development. The Green's function satisfying (94)-(97) for infinite depth ($h_o = \infty$) can be given as follows:

$$G(x;\xi) = r^{-1} - r_1^{-1} \tag{102}$$

$$- \frac{4}{\pi} K \int_o^{\pi/2} \sec^2\theta d\theta \oint_o^{\infty} \frac{e^{k(z+\zeta)}}{k - K\sec^2\theta} \cos k_1(x-\xi) \cos k_2(y-\eta) dk$$

$$- 4K \int_o^{\pi/2} e^{K(z+\zeta)\sec^2\theta} \sec^2\theta \sin[K_1(x-\xi)\sec^2\theta] \cos[K_1(y-\eta)\sec^2\theta] d\theta$$

where

$$r_1 = [(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2]^{1/2}, \tag{102b}$$

$$k_1 = k\cos\theta, \quad k_2 = k\sin\theta, \quad K_1 = K\cos\theta, \quad K_2 = K\sin\theta. \tag{102c}$$

7. The shallow water effects

In investigating the shallow water effects on the hydrodynamic characteristics of a ship moored in an incident current we consider the possible resulting changes in

(i) the transverse wake formation and the associated pressure drag (which is a quasi-steady force);

(ii) the added mass coefficients for (unsteady) sway and yaw modes of motion, especially in the presence of a transverse wake;

(iii) the energy loss due to wave radiation and the resistance due to wave making.

Before we proceed with the formulation of these problems, it is worth noting that the main problem at hand distinguishes itself from the more classical problem of propulsion and maneuvering of a ship in shallow water by having a transverse wake as a drastic new feature for a ship moored in current. The nonlinear effects that arise from the transverse wake formation now combine with the nonlinear shallow water effects to make the problem extremely difficult and there is practically no literature available that are related to the problem we now seek to resolve. The complexity is somewhat alleviated by the operating condition that the depth Froude number of the primary current usually lies in a low sub-critical range (less than 0.5 say). Consequently, the nonlinear boundary conditions on the water surface can be linearized, and this will be the basis of our subsequent analysis of the problem.

7.1 The transverse wake and its pressure drag

Strip theory can be applied to study of the shallow water effects on the transverse wake formation and its corresponding pressure drag. We shall however assume, as explained above, that the Froude number based on the water depth, F_h, of the primary current of velocity U_c, lies in a low subcritical range, i.e.,

$$F_h = U_c/(gh_o)^{1/2} < 0.5 \,, \tag{103}$$

where h_o is the (uniform) water depth. Adopting strip theory for evaluating the local two-dimensional flow, as in section 3, and restricting our attention to the low-frequency limit, the free-surface condition can be approximated by the rigid-surface condition $\partial\phi/\partial z = 0$ on $z = 0$, while the same condition also applies on the bottom $x = -h_o$. Thus the wake flow becomes equivalent to one past the double model placed symmetrically in a closed tunnel of spacing $2h_o$ — which is a pure-drag problem.

The wall effects in the pure-drag case of plane wake flow past an arbitrary body held in a closed tunnel have been investigated by Wu et al. (1971), both theoretically and experimentally. It has been shown there that for wake flows past fairly blunt bodies, such as the double model section of a typical mid-ship hull, the wall effects are generally very small and can be neglected in predicting the pressure drag coefficient C_D if the comparison is based on the <u>same</u> wake underpressure coefficient σ. For example, when the water depth is no less than 1.5 times the ship's draft, the drag coefficient C_D will have only a small reduction from its deep-water limit, by a fraction no greater than 3 % for σ up to as large as 2. Such predictions have been strongly confirmed by experiments. Since σ will be employed in the present model as an empirical parameter, we may therefore assume the shallow water effects on the transverse wake pressure drag to be negligible.

7.2 The added-mass coefficients

As already explained, the primary effects of shallow water on the unsteady hydrodynamic characteristics are reflected in the resulting added-mass coefficients for sway and yaw motions. The leading-order results for these motions can be constructed from a low-frequency strip-theory analysis such that the sectional two-dimensional added-mass coefficient $m_{22}(x)$ is evaluated for the double model placed symmetrically in a channel of total depth $2h_o$. When the transverse flow has no wake (that is, when flow separation is negligible) $m_{22}(x)$ can be determined by classical methods, such as the Green's function method, the boundary integral equation method or conformal mappings. For given hull section profiles, $m_{22}(x)$ can be evaluated by these numerical methods. The general trend of increase in m_{22} due to the bottom confinement can be exemplified by the flow past a circle of radius a placed midway between two plane walls at distance $2h_o$ apart, for which we can show by perturbation theory that

$$m_{22} = \rho\pi a^2[1 + \frac{\pi^2}{12}\frac{a^2}{h_o^2} + 0(\frac{a}{h_o})^4] \quad , \tag{104a}$$

for $a/h_o \ll 1$. If the circle is replaced by a flat plate of breadth 2a, held broadwise to the bounded stream, we have

$$m_{22} = \rho\pi a^2[1 + \frac{\pi^2 a^2}{24\, h_o^2} + 0(\frac{a}{h_o})^4] \ . \tag{104b}$$

These results show that the sway and yaw added-mass coefficients are both increased by proximity to the bottom (i. e., as water depth is decreased) and that the increment seems to depend more sensitively on hull shape than the leading-order quantity.

The above approach, however, is further complicated in practice by the presence of a transverse wake. On physical ground we should expect that the value of $m_{22}(x)$ will be reduced by the lateral wake because the motion in the wake is invariably one of low 'total head' (hence one of nearly uniform pressure) and consequently its boundary becomes less effective than a wetted hull surface in imparting kinetic energy to the fluid surrounding it. Nevertheless, theoretical calculation of the added-mass coefficients of unsteady wake flow remains a most difficult task as the subject still needs major development. Of the limited literature on the subject reference can be made to Wang & Wu (1963) who analyzed the small-time (or corresponding to high-frequency limit) wake flow due to sudden acceleration of a flat plate, giving for the normal force on a broadwise plate of breath 2a the result:

$$N = N_o + N_1 + tN_2 + o(t^2) \quad , \tag{105a}$$

$$N_o = 0.884\,(1+\sigma)(\rho u_{yc}^2\, a) \quad , \tag{105b}$$

$$N_1 = m^{(c)}{}_{22}\,\dot{U}_{yc} \quad , \qquad m^{(c)}{}_{22} = 0.5377\,(\pi\rho a^2) \quad , \tag{105c}$$

where N_o is the normal force of the basic flow, and σ is the wake-under-pressure coefficient. This result shows that the added mass of the plate in wake flow is 0.5377 of its value of a fully wetted plate, and is very insensitive to σ.

For an engineering solution to the problem in hand, we recommend that attempt be made to explore an empirical approach, based on the results of (105 a, b, c) as a guide, towards a solution for moderate and low frequencies.

6.3 Wave making in shallow water

Our discussion here will be limited to the range of (103) for the depth Froude number, namely $F_h < 0.5$. This range should be sufficient for all practical purposes except in unusual operations. We shall further assume that the undisturbed water depth, h_o, is uniform.

One method to calculate the wave field of a ship moving in shallow water is by applying slender-body theory and employing the long-wave approximation. Adopting this method Tuck (1966) obtained the solution for both subcritical $(F_h < 1)$ and supercritical $(F_h > 1)$ cases; his result for the drag force for $F_h < 1$ is

$$F_x = \frac{\rho U^2}{2\pi h_o(1-F_h^2)^{1/2}} \int_o^{\ell}\int_o^{\ell} S'(x)S'(\xi)\log |x-\xi| \, dx d\xi \,, \qquad (106)$$

where $S(x)$ is the cross-sectional area of the double model, $S'(x) = dS/dx$. For the present application, however, Tuck's theory has several drawbacks in that it over simplifies the dispersion effects in the subcritical range and the nonlinear effects in the transcritical range and in that it will require separate account of the transverse current velocity and the lateral wake.

An alternative method is by applying Green's theorem as delineated in Section 6. This latter method is expected to produce results with higher accuracies over Tuck's theory since it only neglects the nonlinear effects which are expected to be insignificant except for the case of transcritical motions. It is therefore recommended that the required Green's function be first determined which will also satisfy the kinematic condition at the bottom $z = -h_o$; a numerical scheme should be then developed, based on the analysis.

8. Conclusions and discussion

The present report is Part II of a series of two reports wherein a mathematical model is formulated for estimating the loads on a moored vessel in an ocean current. In achieving this objective, it is necessary to assemble a number of different methodologies which, in total, constitute the model. We believe that the material developed herein, and in several areas (such as the treatment on the effect of shear in separated flow) extends the present state-of-the-art on the subject. Because of this, it is not possible to forecast precisely the errors committed without actually comparing model predictions with laboratory and field data.

The numerous formulations and methodologies presented in the two reports are needed to cover the various cases, each of which demands a different treatment. At first, they may appear somewhat disjointed. To facilitate easy reference, a chart is prepared and shown in Table 1 which gives a roadmap of the overall structure and network of the methodology.

Referring to Table I, the methods can be divided into two main groups in terms of the angle of incidence of the current with respect to the axis of the vessel. This is mandated by the completely different fluid mechanical behavior of the flow field when there is a significant region of separated flow such as one that would certainly occur for beam-on current. We believe the combination of free streamline and strip theories recommended here for high incidence angles to be the most rational approach. While it is not possible to precisely delineate its region of applicability without comparing predictions with experimental observations, we believe it would probably remain reasonably accurate even at incidence angles as low as, say, 30°. For small angles of incidence, low aspect ratio wing theory should be used instead. We recommend computing the forces

TABLE I

A matrix network showing the methodologies for hydrodynamic loads on moored vessels

	Current force		Wave radiation force	
	Uniform current	Vertically and horizontally sheared current	Deep water	Shallow water $F_h = U_c / \sqrt{gh_o} < 0.5$
small incidence angles	A. longitudinal force: -use Part I, eqs. (6), (7) augmented by I- (22) B. transverse force -Part I, eqs. (19)-(23) C. yaw moment -Part I, eq. (24)	A. longitudinal force -Part I, eqs. (6), (7) augmented by II - (78) B. transverse force Part II, Section 4 eqs. (74)(76)(77)(79) C. yaw moment Part II - eqs. (75)(76)(80)	Recommended method for solution: boundary-integral-equation method Part II. eqs. (91)-(102) the integral over the near wake boundary S_w may be neglected	A. longitudinal force due to wave-making Part II - eq. (106) B. transverse force —use double-model added-mass coefficients (frequency independent)-by Green's function method
large incidence angles	A. longitudinal force -Part I, eqs. (6) (7) augmented by I - (22) B. transverse force (i) strip theory Part I, eqs. (8)-(13) (ii) integration Part I, (17) augmented by I- (23) C. yaw moment Part I - eq. (18) augmented by I - (24)	A. longitudinal force -Part I, eq. (6) (7) augmented by II - (78) B. transverse force (i) strip theory Part II, entire Sect 3, II - eqs. (38)-(66) (ii) integration Part II-eq. (74) augmented by II - (79) C. yaw moment Part II - eq. (75) augmented by II - (80)	Recommended to use the same method as above (the case of small incidence angles), but the integral over the wake boundary S_w should be taken into account	A. longitudinal and transverse D'Alembert force —added-mass coef. needs correction for wake effects (by following the approach of Wang/Wu (1963)-II-eq. (105) B. Wave-making resistance Beyond the present state of the art.

using both methods for any angle less than, say 30^{o}, and reserving judgement as to which one to adopt pending experimental verification.

Among the various formulations in the two parts of the report, those detailed in Part I constitute first order models which are expected to provide quite good estimates of the forces and moments in practice. The formulations in Part II consists of higher order models whose practical effects would be a modification to those forces and moments predicted by the methods in Part I. By their nature as higher order models, they tend to be more complex and thus more difficult to implement. Their contributions to the augmented or improved forces and moments are not expected to be large.

It is recommended that the methodologies developed be implemented numerically in a computational model which can then be used to calculate the forces and moments on a ship moored in a current. Rather than constructing a grand model incorporating all the formulations in these reports, it would be much more cost effective to proceed in a stepwise fashion. We recommend first the implementation of the methods in Part I in a computational model. It should then be used to predict the forces and moments on a number of generic ship hullshapes such as destroyers, tankers, and cruisers. The predictions can then be examined in light of experimental findings, both from the laboratory and the field. A rational decision can then be reached as to the choice of further implementation.

The ultimate practical engineering goal is the estimation of current loads on moored vessels. It would be highly desirable to be able to obtain rapid and reasonably accurate estimates on the basis of a relatively small number of parameters characterizing the ship (such as length, beam, draft, block coefficient, ship type, etc.) rather than detailed shiphull cross sections along the full length of the ship. We foresee the possibility that results from calculations using the detailed model and generic ship hull-shapes, when properly interpreted in conjunction with laboratory and field observations, might be utilized to form the basis of an engineering model.

A justification of the simplifying assumption of neglecting the viscous effects in treating the wake flow in a sheared stream can be provided as follows. There are two roles the viscous effects can play in this case. One is giving rise to a boundary layer over the wetted part of the hull (upstream of the wake region), which is thin compared to the ship dimension. The other is to increase transverse diffusion of the vorticity at the rate of $(\nu t)^{1/2}$, where ν is the kinematic viscosity of water and t is the time. This length of diffusion would amount to less than 1 cm in the time when a current of 1 knot has traversed the ship's beam of 15m. The impact of viscous diffusion on altering the velocity of a rotational flow is therefore insignificant and can be neglected.

A final comment is in order concerning the applicability of the present model for the case of a random wave field. Throughout the development of both the original model and the present extensions, no incident wave field is considered since it is not part of the present contract work statement. The wave radiation resistance consideration described in section 6 pertains only to the wave making as a result of the interaction of the current with the ship and cannot be extened to apply to the effect of an ambient wave field, whether it is regular or random. The latter problem is one of wave scattering. Techniques are available for calculating the flow field due to incident waves scattered and diffracted by a ship either stationary or self-propelled in the forward direction, but in either case without a transverse current. In such cases the incident waves are usually taken to be monochromatic and applications to the case of a random sea are made by means of linear superposition based on a known wave energy spectrum. However, in asmuch as the present problem is involved with the possible presence of a transverse wake, the wave scattering problem would require new techniques for its solution. At present, there is a virtually complete lack of information for this case, in either theoretical analyses, or experimental observations.

Since the present problem is aimed at a moored ship and hence in a rather sheltered water, incident waves, if any, are not expected to be large and their forces would be small compared to the other hydrodynamic forces considered herein. In the unexpected circumstance of a sudden storm resulting in large incident waves, such as when the peak orbital velocity of the water particles is equal to the mean current speed, the combined forces will be so strongly time dependent that the quasi-steady theory will no longer be adequate.

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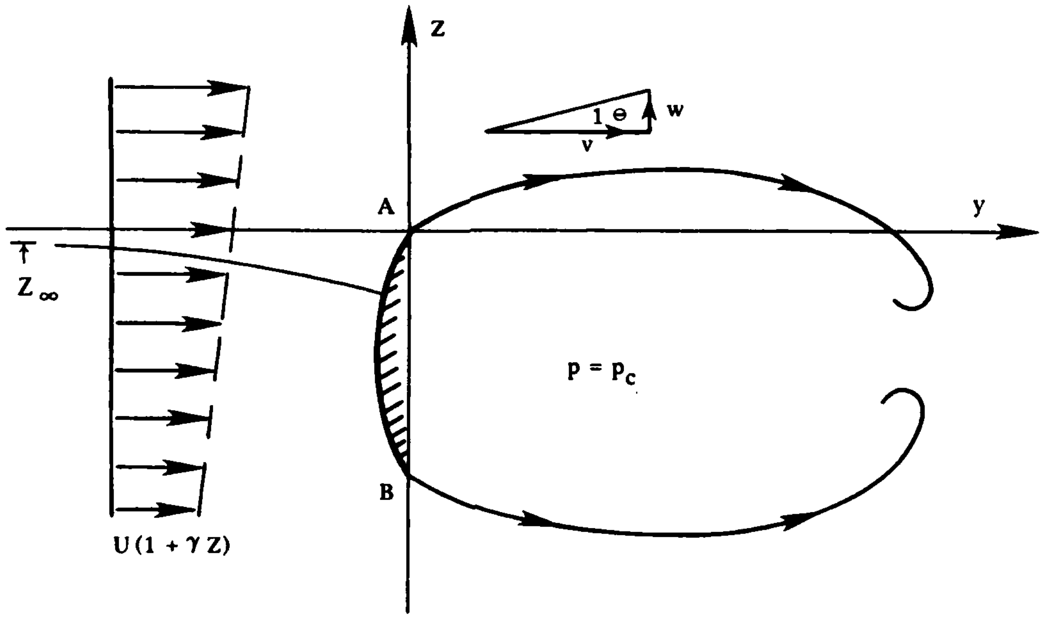

Figure 1. A strip double-model representation of the transverse wake flow with a sheared free stream. Points A and B represent points of separation of the free streamlines characterized by constant pressure p_c.

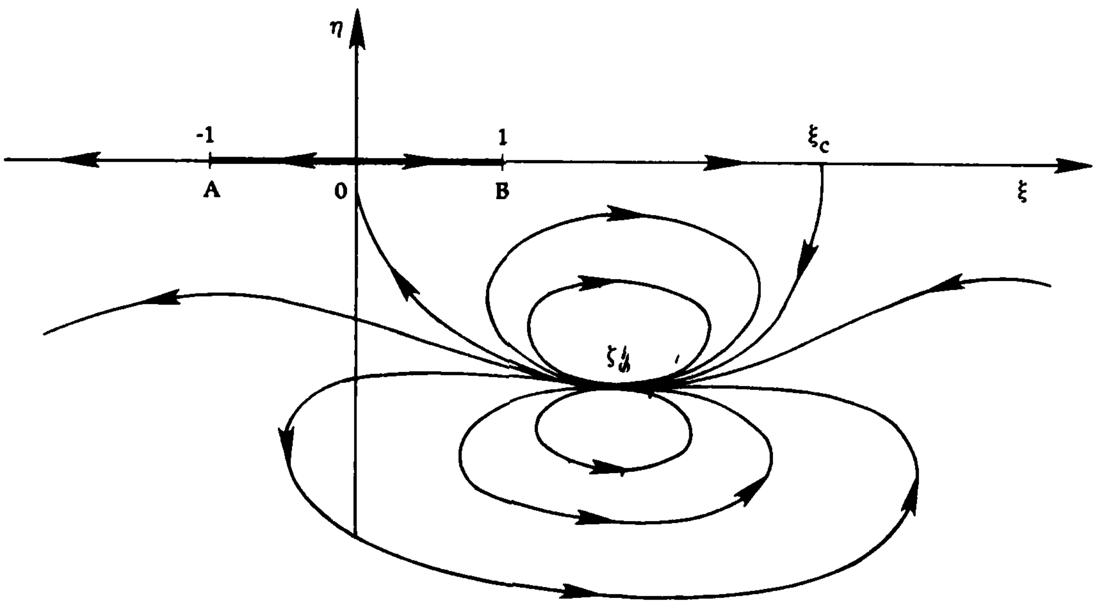

Figure 2. A sketch of streamlines in the parametric $\zeta = \xi + i\eta$ plane as the images mapped by conformal transformation of the physical plane shown in Figure 1.

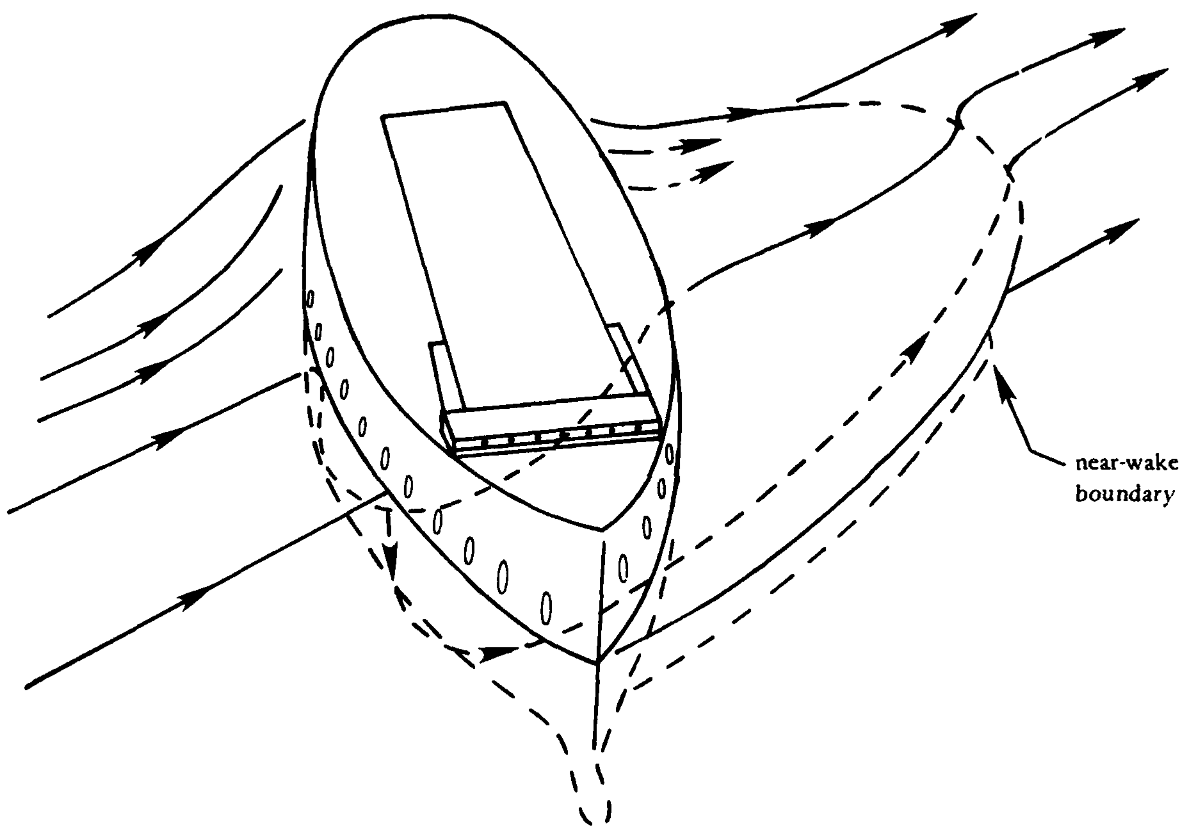

Figure 3. An oblique current and the transverse wake flow.

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