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# Theory of Beam Channel Hydrodynamics

J. M. PICONE, J. P. BORIS, J. H. GARDNER, J. R. GREIG,\* M. RALEIGH,\* M. LAMPE,\*\* R. E. PECHACEK,\* AND R. F. FERNSLER<sup>†</sup>

Laboratory for Computational Physics

Experimental Plasma Physics Branch Plasma Physics Division

> \*Plasma Theory Branch Plasma Physics Division

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#### THEORY OF BEAM CHANNEL HYDRODYNAMICS

#### Introduction

The propagation of an intense beam through a background gas can result in the deposition of significant energy along the beam path, producing a hot, reduced-density channel. The properties of the channel will, in turn, affect subsequent beam pulses which travel along the same path. In recent experiments, Greig <u>et al</u>. [1-3] have studied the evolution of channels produced by lasers and electric discharges in air or nitrogen at approximately 1 atm. From those data a general picture of the dynamics of beam channels has emerged, including time scales characterizing channel evolution.

In this paper, we will consider pulses which deposit energy on time scales which are short compared to those of hydrodynamic motions. A pulse will then produce a hot channel which is initially at increased pressure. The experiments of Greig <u>et al</u>. show that, over time scales  $\tau \approx 2(R_1-R_0)/c_s$ , the hot channel will expand almost adiabatically to reach pressure equilibrium with the ambient gas. Here  $R_0$  is the initial channel radius;  $R_1 \equiv R(\tau)$  is the channel radius when equilibrium is reached; and  $c_s$  is the ambient speed of sound. Energy losses and radiation should be less than 20%.

Following pressure equilibration, the channels cool and expand according to the equation

$$R^{2}(t) = R_{1}^{2} + 4a(t-t), \qquad (1)$$

where a is the effective thermal diffusivity and t is the time elapsed since energy was deposited. The experimental data show that Manuscript approved November 16, 1963.

$$\alpha = c_R_1/k$$
, (2)

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in which  $k = 66\pm 6$ . This gives values for a which are 3-4 orders of magnitude larger than that due to nonturbulent thermal conduction in ambient air. At the same time, channel turbulence grows to a fully developed state. These observations suggest that cooling occurs through the entrainment of cooler ambient gas by the turbulent flow field.

Recent theoretical analysis and numerical simulations have provided both a viable mechanism for the generation of turbulence in hot channels and a useful framework for analyzing and predicting the properties of experimental channels. In the following sections, we review recent progress in the theory of turbulent channel cooling. We show that any deviations from cylindrical symmetry when a pulse deposits energy in the surrounding gas will produce considerable large scale vorticity. This vorticity persists and smaller scale structure develops as ambient gas mixes with the channel. We identify basic asymmetry classes and state equations for the vortex strength and mixing time scale. From the vortex strength, we then obtain the effective thermal diffusivity for comparison to experimental data. The final section presents data from numerical simulations of the various types of asymmetry.

## Theory of Convective Cooling

As a hot channel relaxes to pressure equilibrium, any deviations from cylindrical symmetry will lead to asymmetries between the local gradients of pressure and density. This will cause vorticity to be generated according to the following equation:

$$\frac{d\xi}{dt} + \xi \nabla \cdot \nabla = \xi \cdot \nabla \nabla + (\nabla \rho \times \nabla P) / \rho^2, \qquad (3)$$

where  $\xi \equiv \sqrt[n]{x \ y}$  is the vorticity, y is the fluid velocity,  $\rho$  is the density, and P is the pressure. All variables are functions of the position  $\chi$  and the time t. The three generic types of asymmetry are as follows: (1) noncollinearity of successive pulses, (2) pulses with concentric, noncircular (e.g., elliptical) energy contours, and (3) curvature or kinking of the hot channel. In case (1), the shock wave produced by the second pulse propagates through the channel produced by the first. Because the pulses are not aligned, the source term in Eq. (3) is nonzero. An important extension of this case is the situation in which the interior of a pulse or discharge is nonuniform; for example, the pulse or discharge may contain "hot spots" or might have a filamentary structure. Several subregions will contain significant vorticity, and mixing will be increased over case (2), in which the contours by definition obey a similarity transformation.

We have found that the residual vorticity for the three generic asymmetry classes may be represented in terms of one or more vortex pairs [2,4] of strength or circulation  $\pm \kappa_{\alpha'}$  where  $\alpha = r$ ,  $\theta$ , or s. If we align the z-axis with the average position of the beam path, we find that  $\kappa_{z}$  is the only nonzero cylindrical component for the two-dimensional asymmetry classes (1 and 2) and that  $\kappa_{g}$  and  $\kappa_{\theta}$  are important for curved or kinked channels in class (3).

To discuss the transport of energy by the residual rotational flow field, we consider one vortex pair, aligning the y-axis with the centers of the vortices and placing the x-axis midway between them. The fluid velocity along the x-axis is

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$$\nabla_{\mu}(x,0) = \kappa_{\mu} \delta / \pi (x^2 + \delta^2)$$
, (4)

where 20 is the displacement of the vortices. We have found that 0 is of the same order as the length scale (radius)  $S_0$  of the local density gradient (channel). We define a mixing time scale  $\tau_{mix}$  as the time required for a fluid element to move from  $(x,y) = (-S_0,0)$  to  $(x,y) = (S_0,0)$ . Integrating Eq. (4), we have

$$\tau_{mix} = 2\pi S_0 \left( S_0^2 / 3 + \delta^2 \right) / |\kappa_z| \delta .$$
 (5)

We have computed the vortex strength  $\kappa_{g}$  for the three generic asymmetry classes and  $\kappa_{A}$  for class (3). All have the same basic form:

$$\kappa = (U_{2}^{2} \tau/2) \ln (\rho_{p}/\rho_{p}) f , \qquad (6)$$

where  $U_{\rm m}$  is a characteristic velocity of the expanding channel boundary  $(U_{\rm m} \sim c_{\rm s})$ ,  $\rho_{\rm m}$  is the ambient density,  $\rho_{\rm 0}$  is the minimum density in the channel, and T has been defined as  $2(R_1 - R_0)/c_{\rm s}$ . The quantity f is a form factor, which is < 1, the nominal range being 0.15-0.33.

We may now derive an approximate equation relating the effective thermal diffusivity  $\alpha$  and the vortex strength  $\kappa$ . We consider a pair of oppositely oriented vortex tubes which are contained in a cylindrical region of radius  $S_0$ , so that the separation of the vortex axes is  $2\delta \sim 2S_0$ . The z-axis is parallel to the axes of the vortices and the (y,z) plane contains the axes. Using Eq. (4), we find that the mass flow of ambient gas into a length L of the boundary cylinder (and between the vortices) is

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$$L/2 \quad \delta/2$$

$$\rho_{\infty} \int dz \int dy \, \mathbf{v} \sim \rho_{\infty} \, \mathbf{L} \, |\kappa| / \pi \, . \tag{7}$$

$$-L/2 - \delta/2 \quad \mathbf{X} \qquad (7)$$

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We integrate over only half the separation of the vortices to account for their finite radii (~  $\delta/2$ , as indicated by the numerical simulations). From Eq. (1), we compute the mass flow rate due to this entrainment to be

$$\frac{dm}{dt} = \rho_{\omega} L \frac{R^2(t) - R_1^2}{t - T} = 4\alpha\rho_{\omega} L .$$
(8)

From Eqs. (7) and (8), we then have

$$\mathbf{a} \sim |\mathbf{k}| / 4\mathbf{T} \quad . \tag{9}$$

Substituting for  $\kappa$  and  $\tau$ , we have

$$a \sim c_{\mu} (R_1 - R_0) \ln(\rho_{\mu} / \rho_0) |f| / 4\pi$$
 (10)

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For an electric discharge depositing 300 - 600 J/m in air, we use adiabatic expansion to estimate  $R_1 \simeq 1.4$  cm  $\simeq 3$   $R_0$ , and  $\rho_{ab}/\rho_0 \simeq 10$ . Comparing Eq. (10) with the experimental data, Eq. (2), gives an empirical form factor  $|f| \approx 0.12$ , which lies at the low end of the range which we would expect. Similarly for a CO<sub>2</sub> laser depositing approximately 9 J/m in nitrogen doped with SF<sub>6</sub>, we estimate  $R_1 \sim 1.4 R_0 \sim 0.5 \text{ cm}$ ,  $\rho_{\perp}/\rho_0 \sim 2$ . Equations (2) and (10) predict a value of | f = 0.96. This high value occurs because the multimode structure of the laser pulse produces several hot spots, each pair of which interact to produce vortex filaments. To treat the problem properly, we would have to propose a distribution of hot spots which satisfies the constraints of total energy deposition and marginal detectability by laser burn patterns. We could then analyze all pairwise interactions and compute a vortex strength for the channel. For pulses containing a few (N  $\sim$  5) hot spots, we might expect an enhancement in diffusivity (or effective total form factor) of ~ N or so, depending on the average separations, overpresssures, and spatial scales of the hot spots.

#### Numerical Simulations

Using the FAST2D and FAST3D computer codes [5-7] we have performed numerical simulations of vorticity generation in the generic asymmetry classes and the case of a laser pulse with hot spots. Descriptions of algorithms, grids, and initial conditions for the simulations appear elsewhere [4,7-8]. In all cases, energy deposition was instantaneous; the ambient gas was air;

and the initial conditions were quite similar to those in the corresponding experiments of Greig <u>et al.</u> [1-3]. The figures below show only the fine grid surrounding the hot channel, and time t = 0 corresponds to the instant at which energy is first deposited. Due to limited resolution, we model only the large scale vorticity and cannot follow a cascade to smaller scales.

Figure 1 illustrates the case of sequential noncollinear pulses [4]. The first pulse deposits energy in the center at time t = 0.00 ms, while the second deposits energy to the right of center with a displacement of one pulse radius at t = 1.00 ms. The peak overpressures at the instants of energy deposition are 4.7 and 2.4 atm, respectively. Figures 1(a) and (b) show density contours and velocity vectors, respectively, at 1.26 ms. The noncollinearity of the pulses has produced a hot, oblong channel, and after the shock wave from the second pulse has propagated away, a vortex filament pair remains. The separation is  $2^{\delta} = 1.1$  cm  $\sim R_1$  and the enhanced flow between the vortices covers a vertical distance of approximately  $\delta$ , as assumed in the derivation of Eq. (9). The vortex strength falls within 20% of the value given by Eq. (6).

To illustrate asymmetry class (2), we study a pulse with elliptical contours [4]. Symmetry considerations permit us to simulate only the upper half of the channel. Figure 2 shows density contours at t = (a) 36 µm and (b) 7.8 ms. In Fig. 2(a), we see the shock wave propagating away from a hot elliptical channel having an equilibrium area of ~ 0.40 cm<sup>2</sup>. Not visible are two oppositely directed vortex filament pairs, one on either side of the major axis. Fig. 2(b) shows that the flow between each pair and the slow,

horizontal, outward movement of the pairs have pulled fluid into the channel along the major axis and out of the channel along the minor axis. The value of a coming from the calculation is a factor of more than 6 too low when compared to experiment [3], primarily because the interior structure of laboratory pulses was not smooth [8].

Figure 3 shows vorticity contours computed by FAST3D at t = 192µs for a half-wavelength section of a sinusoidally curved electric discharge, which oscillates (spatially) about the z-axis [7]. Figure 3(a) shows the vorticity in a given x-y plane (corresponding to  $\xi_z$ ), and Fig. 3(b) shows the x-z symmetry plane (corresponding to  $\xi_y$ ). The magnitude of a agrees well with the experimental value.

In the case of a laser pulse with a circular envelope and smooth energy contours, we would expect little vorticity to be produced by the mechanism proposed in this paper. In fact, experimental data on laser pulses with roughly circular envelopes have given a value of  $\alpha \sim 250 \text{ cm}^2/\text{s}$  [3]. We have, therefore, investigated the possibility that the pulses contain nonuniformities or hot spots due to the existence of nonaxial modes [8]. In Fig. 4, we show (a) pressure contours at t=0.00 ms and density contours at t=(b) 0.06 ms, (c) 0.41 ms and (d) 1.24 ms from a numerical simulation in which the laser pulse is assumed to contain seven hot spots. Each hot spot produces a shock wave which sweeps through the shocks and the density minima resulting from the other hot spots. These interactions produce vortex pairs which reside between the six outer density minima which are left after the shocks have propagated away. At later times, the outward movement of the vortex pairs and the enhanced flows between each pair distort the channel boundaries and eventually pull the channel apart.

Our numerical simulation yields  $\alpha \sim 100 \text{ cm}^2/\text{s}$ , which is within a factor of 2.5 of the experimental value. More realistic initial conditions should produce even better agreement. In addition, Greig <u>et al</u>. have studied a roughly circular laser pulse containing only axial modes and have found a significantly reduced diffusivity ( $\alpha$ ) of 3 cm<sup>2</sup>/s, as the theory would predict.

#### Conclusions

We have reviewed a general theory of convective cooling of hot channels produced by a beam propagating through a gas. The theory provides good agreement with experimental data on laser and electric discharge channels; the range of energy deposition covered by the data is roughly two orders of magnitude. We predict that any deviation from cylindrical symmetry by beam pulses, including noncollinearity, will produce long-lived, local vorticity. Pulses having interior fluctuations in energy will exhibit enhanced cooling over those with smoothly distorted energy contours. A goal of future work is to develop better models of energy deposition and the turbulent cascade to permit more realistic simulation of the entire process of channel production and cooling.

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Vortex pair generation by sequential, noncollinear pulses of energy. The diagrams show (a) density contours and (b) velocity vectors after the shock wave from the second pulse has propagated far away from the channel.



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(b)



Density contours (a) just after an elliptical pulse has deposited energy and (b) several milliseconds later. The two vortex filament pairs propagate horizontally outward, distorting the channel significantly.



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Contours of vorticity produced by a numerically simulated electric discharge with a sinusoidally curved axis. The diagrams show typical vorticity distributions in (a) transverse and (b) longitudinal planes.



**0.41 ms** 

1.24 ms (d)

# Figure 4

(a) Pressure contours and (b)-(d) density contours showing energy deposition by a simulated laser pulse with interior hot spots. The resulting vortex filament pairs propagate outward, pulling the channel apart.

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