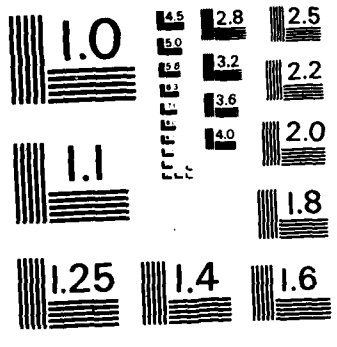MICROCOPY RESOLUTION TEST CHART

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THE EFFICIENT USE OF SPECTRAL AND FINITE DIFFERENCE METHODS

FOR THE SOLUTION OF CERTAIN HYPERBOLIC AND PARABOLIC PROBLEMS

Final Technical Report

by

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I.   SPECTRAL METHODS

During the period of the grant we concentrated on three areas of research in spectral methods. The first area is the theoretical study of spectral methods for hyperbolic and parabolic equations. Appendix A contains a summary of what is known in the field. This appendix is a review and also contains new results found under the research grant. In particular, we have found a stability proof for the Chebyshev Method for hyperbolic equations using the standard collocation points. The behavior of the eigenvalues of the second derivative Chebyshev operator have been explored. The modified equation for collocation and Galerkin Methods have been established. This shows the relationship of the Galerkin Method with a collocation scheme with a particular set of nodes. In addition, we prove the stability of the Chebyshev parabolic problem with Neumann boundary conditions.

The second area of research has been to investigate the use of time dependent equations to reach a steady state using spectral methods. For Fourier Methods we have analyzed a residual smoothing technique and have shown that it leads to an unconditionally stable scheme. In practice this allows a larger time step. However, when the time step is too large then the method is inefficient. This method is currently being used to study transonic flow about an airfoil.

The third area of investigation is the study of the time discretization for time dependent hyperbolic equations. In particular nonstiff equations have been studies where the time accuracy is as important as the space accuracy. This work has been done by Tal-Ezer who was supported by the grant. This work is part of Tal-Ezer's dissertation under the supervision of D. Gottlieb. He constructed a spectral method in time and space and showed that it is superior to a finite difference approach from both an accuracy point of view as well as efficiency. This method had been successfully applied to the Schroedinger Equation. He also has shown that if one uses finite differences in time then one must choose a very small time step, much less than the stability condition, in order to preserve the accuracy of the scheme.

II.   FINITE DIFFERENCE ALGORITHMS   ,   also presented

Within finite differences the emphasis has been on ways to reach a steady state as fast as possible. Implicit methods have been analyzed for both hyperbolic and parabolic equations.

In the hyperbolic case unconditional stability of approximate factorization type methods is analyzed in both two and three space dimensions. The unconditional instability of standard methods is

shown for the three dimensional wave equation and also for the inviscid equations of fluid dynamics. A modified scheme is proposed which is unconditionally stable for the scalar wave equation. Analytical results are verified by numerical experimentation.

For the parabolic case the rate of convergence to a steady state is analyzed in terms of the L_2 norms of the residuals. The analysis allows one to predict the number of iterations necessary for convergence as a function of the Courant Number. In addition, a simple modification to existing ADI Codes is presented. This modification improves the convergence rate substantially and renders the scheme insensitive to the Courant Number.

In addition, a review paper has been written which surveys the present state of the art in computational physics. All aspects of a large scale code are described including the space and time discretizations, initial and boundary conditions.

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The following papers have been published which are results of research supported by the grant.

1. Progress in Computational Physics, E. Turkel, Computers and Fluids, Vol. 11, pp. 121-144 (1983).

2. Spectral Methods for Time Dependent Partial Differential Equations, D. Gottlieb and E. Turkel, Proc. C.I.M.E. Conference Numerical Methods in Fluid Dynamics, Como, Italy, July 1983.

3. Improving the Convergence Rate of Parabolic ADI Methods, S.S. Abarbanel, D.L. Dwoyer, D. Gottlieb, To appear Journal Computational Physics.

4. Stable Implicit Finite Difference Methods for Three Dimensional Hyperbolic Systems, S.S. Abarbanel, D.L. Dwoyer, D. Gottlieb, To appear Journal Computational Physics.

5. Spectral Methods in Time for Hyperbolic Equations, H. Tal-Ezer, Submitted SIAM Journal Numerical Analysis.

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