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ACTIVE IONOSPHERIC GENERATION ON ELF/VLF WAVES(U)

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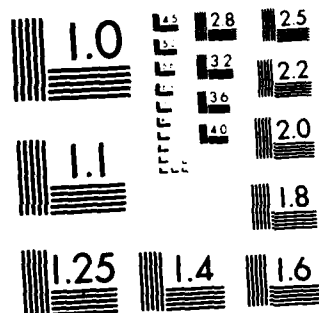
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FINAL REPORT NO. SAI-84/1029

K. Papadopoulos

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Plasma Physics Division
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Prepared by
K. Papadopoulos
Science Applications, Inc.
1710 Goodridge Drive
McLean, VA 22102

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SCIENCE APPLICATIONS, INCORPORATED

1710 Goodridge Drive, McLean, Virginia 22102 (703) 734-5840

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- (i) The first code named CMPower required as input a model of the ionospheric electron density and magnetic field, and the transmitter characteristics. It's output is the ELF/VLF signal as a function of the modulation frequency and distance from the HF facility.
- (ii) The second named TWC (three wave coupling) is a nonlinear code, which includes dissipation and finite pump size effects.

SUMMARY

Our work during the past year on active ELF/VLF generation has produced major breakthroughs in the understanding of nonlinear interactions in a dissipating medium such as the ionosphere for the purpose of downconverting H.F. to ELF/VLF waves. A key result was the fact that contrary to the conventional understanding of such processes, the presence of a dissipative medium lowers the threshold for complete decay and increases the downconversion efficiency over the one expected on the basis of the Manley Rowe relations.

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I. INTRODUCTION

Generating VLF/ELF waves by utilizing the ionosphere as an active medium is an exciting prospect, since it avoids many of the difficulties connected with large and inefficient ground based VLF and ELF transmitters. By adjusting the HF beam geometry, the low frequency waves can be made to propagate upwards in the magnetosphere for use in active magnetospheric stimulation experiments, or downwards into the earth ionosphere waveguide for communication purposes. Two basic downconversion mechanisms have been proposed:

- (i) Modulation of the auroral or electrojet current by large HF heaters¹⁻⁶.
- (ii) Stimulated nonlinear coupling of two HF wavepackets with frequency difference equal to the desired VLF/ELF frequency^{4,7,8}.

The first mechanism has a long history, dating from the "Luxembourg" effect. The mechanism is based on the possibility of changing the ionospheric electron temperature by HF radiowave heating. The electron neutral collision frequency ν_{en} depends almost linearly on the electron temperature T_e ,

$$\nu_{en} = \nu_{eno} \left(\frac{T_e}{T_o} \right) \quad (1)$$

where the "o" subscript denotes properties of the ambient ionosphere. Consequently, the electron conductivity of the lower ionosphere can be made to vary in time at a frequency ω

$$\sigma(\omega) = \frac{ne^2 v_{en}}{m(\omega^2 + v_{en}^2)} \quad (2)$$

as a result of the temperature changes caused by the amplitude variation of the HF waves. The "Luxembourg" effect arises because the time dependent conductivity of the lower ionosphere causes a time dependent attenuation of HF radio-waves propagating through it. At the receiving station, this appears as an amplitude modulation of the HF waves. The VLF/ELF generation by current modulation is based on the formation of time dependent current loops in the lower ionosphere by modulating the electron temperature in regions of naturally occurring geophysical currents, such as the auroral and the equatorial electrojets. Note that current loops, rather than electric dipoles are expected at VLF/ELF frequencies due to the large conductivity of the ionosphere (i.e., $\sigma \gg \omega/4\pi$). This approach has been very successfully applied in the Soviet Union³ and more recently to the MPE experiments at Tromso, Norway^{1,2} and by the Pennsylvania State University group at Aracibo⁹. More details concerning this approach can be found in the review given by Stubbe¹⁰.

The second mechanism is more novel^{7,8} and generates the nonlinear current sources without relying on pre-existing ionospheric currents. It mixes two HF wavepackets in the ionosphere to produce VLF/ELF signals. However, unless the process occurs in a stimulated (i.e., lasing) fashion, the downconversion efficiencies are trivial. Stimulated down-conversion occurs for pump power above a threshold value that depends on the ionospheric parameters, the HF and VLF/ELF

frequencies and mainly on the angle of HF beam to the ionospheric density gradient. Recent theoretical calculations indicate fairly high downconversion efficiencies. An important variation of the scheme using only one HF pump can amplify pre-existing ionospheric VLF/ELF wavepackets by tens of db's. This mechanism has not yet been experimentally proven, although evidence of the amplifier operation has fortuitously been seen in Aracibo¹⁹.

Our work in the past year emphasized the second. In the section we review some of the fundamental physical notions involved in collisionless interactions. Section III discusses some recent results concerning the semi-collisional regime, where the frequency of the VLF/ELF wave is below the electron neutral collision frequency. This regime is of interest for ELF generation in the E-region. The final section applies our theoretical results to the ionospheric VLF/ELF generation. Appendix I discusses the basic derivation of the equations, while Appendices II, III include recent publications.

II. MAXIMUM ENERGY TRANSFER IN COLLISIONLESS THREE WAVE INTERACTIONS

The equations describing the generation of a lower frequency wave $(\omega_3, \underline{k}_3)$, from the interaction of two high frequency wave packets $(\omega_1, \underline{k}_1)$ and $(\omega_2, \underline{k}_2)$ satisfying the resonance conditions

$$\begin{aligned}\omega_1 - \omega_2 &= \omega_3 \\ \underline{k}_1 - \underline{k}_2 &= \underline{k}_3\end{aligned}\tag{3}$$

are

$$\left(\frac{\partial}{\partial t} + U_{1x} \frac{\partial}{\partial x} + \Gamma_1\right)a_1 = \kappa a_2 a_3 \quad (4a)$$

$$\left(\frac{\partial}{\partial t} + U_{2x} \frac{\partial}{\partial x} + \Gamma_2\right)a_2 = \kappa a_1 a_3^* \quad (4b)$$

$$\left(\frac{\partial}{\partial t} + U_{3x} \frac{\partial}{\partial x} + \Gamma_3\right)a_3 = \kappa a_1 a_2^* \quad (4c)$$

where U_j are the group velocities, Γ_j the damping rates, and a_j the slowly varying complex amplitudes of wave-packets, normalized to action density^{13,14}, so that

$$a_j^2 = \frac{|E_j|^2}{16\pi} \left| \frac{\partial D_j}{\partial \omega_j} \right|. \quad (5)$$

E_j is the electric field amplitude of the wave and D_j is its dielectric function. In deriving Equations (4), we have assumed positive energy waves in a collisionless ($\omega_1 > \omega_2 > \omega_3 \gg \Gamma_1, \Gamma_2, \Gamma_3$), homogeneous medium, with the finite size of the pump ($\omega_1, \underline{k}_1$) being the only source of inhomogeneity (Figure 1). The value of κ for any conservative interaction of interest can be found by following the method of electrodynamics of weakly non-linear media or the ponderomotive force approach^{11,13,18}. An example of such a calculation can be found in Ref. 18.

The important issue in the downconversion problem is the maximum efficiency of power transfer from the pump to the lower frequency wave and the conditions under which it can be achieved. For the collisionless case described here, the maximum power transfer from the pump P_1 to the low frequency wave P_3 can be found from the conservation of the system's

action and energy¹² and is given by the well known Manley-Rowe relations as

$$\frac{P_3}{P_1} < \frac{\omega_3}{\omega_1} \equiv \eta_0. \quad (6)$$

An important practical issue is how close to the maximum theoretical efficiency given by (6) can one reach and under what conditions. An extremely important result given by Reiman¹⁸ by solving numerically the system of Equations (4), is that if we define a length

$$L_c \equiv \frac{|U_{2x} U_{3x}|^{1/2}}{|\kappa a_1(t=0)|}, \quad (7)$$

maximum efficiency (i.e., complete pump decay) can be achieved if

$$U_{2x} U_{3x} < 0 \quad (8a)$$

$$\frac{L}{L_c} > \frac{\pi}{2} \quad (8b)$$

where L is the size of the pump. Equations (8) are strictly valid for a square pump, but they have been found to vary by less than a factor of two for other pump profiles. Figure 2 shows the efficiency η achieved as a function of L/L_c compared to the theoretical efficiency η_0 given by Equation (6) as found in Reiman¹⁴, demonstrating that $\eta/\eta_0 \approx 0.7 - 0.8\eta_0$. Note that the numerical results are consistent with analytical results based on the method of inverse scattering^{14,16}, which, however, is applicable only for non-overlapping initial wavepackets.

An observation with profound consequences, is that the conditions for complete decay given by Equations (8) coincide with the conditions for an absolute instability¹⁷ of the system of linear equations

$$\begin{aligned} \left(\frac{\partial}{\partial t} + U_{2x} \frac{\partial}{\partial x} \right) a_2 &= \gamma_0(x) a_3^* \\ \left(\frac{\partial}{\partial t} + U_{3x} \frac{\partial}{\partial x} \right) a_3 &= \gamma_0(x) a_2^* . \end{aligned} \quad (9)$$

Equations (9) can be found from Equations (4) by assuming the pump energy (i.e., $|a_1|^2$) as constant.

We found above the conditions for pump depletion for the case of one dimensional space-time evolution. Of more interest for our application is the steady state three dimensional boundary value problem. If the group velocities of the waves are coplanar, the steady state equations reduce to

$$\begin{aligned} (U_{1z} \frac{\partial}{\partial z} + U_{1x} \frac{\partial}{\partial x}) a_1 &= \kappa a_2 a_3 \\ (U_{2z} \frac{\partial}{\partial z} + U_{2x} \frac{\partial}{\partial x}) a_2 &= \kappa a_1 a_3^* \\ (U_{3z} \frac{\partial}{\partial z} + U_{3x} \frac{\partial}{\partial x}) a_3 &= \kappa a_1 a_2^* . \end{aligned} \quad (10)$$

It is easy to see that the results of the previous section are directly transferable to Equations (10) if we make the transformation

$$\hat{a}_j = a_j \hat{U}_{jz}^{1/2}, \quad U_j = \frac{U_{jx}}{U_{jz}}, \quad \hat{\kappa} = \frac{\kappa}{(U_{1z} U_{2z} U_{3z})^{1/2}}, \quad (11)$$

Equations (10) then become

$$\begin{aligned}
 \left(\frac{\partial}{\partial z} + \hat{U}_1 \frac{\partial}{\partial x} \right) \hat{a}_1 &= \hat{\kappa} \hat{a}_2 \hat{a}_3 \\
 \left(\frac{\partial}{\partial z} + \hat{U}_2 \frac{\partial}{\partial x} \right) \hat{a}_2 &= \hat{\kappa} \hat{a}_1 \hat{a}_3^* \\
 \left(\frac{\partial}{\partial z} + \hat{U}_3 \frac{\partial}{\partial x} \right) \hat{a}_3 &= \hat{\kappa} \hat{a}_1 \hat{a}_2^* .
 \end{aligned} \tag{12}$$

Note that the role of the time variable is now played by z , so that the pump depletes while it propagates. Therefore the steady state interaction corresponds to an amplifier in space for a_3 , while the absolute instability found before would correspond to an oscillator in time. The conditions for complete decay in the steady state interaction would be

$$\hat{U}_2 \hat{U}_3 < 0 \tag{13a}$$

$$\frac{L}{\hat{L}_c} > \frac{\pi}{2} \tag{13b}$$

where

$$\hat{L}_c \equiv \frac{|\hat{U}_2 \hat{U}_3|^{1/2}}{|\hat{\kappa} \hat{a}_1(z=0)|} . \tag{14}$$

For $U_{1z}, U_{2z}, U_{3z} > 0$, as is our case, (13a) coincides with (8a) and is automatically satisfied. Using the definitions given by Equations (11), it is easy to see that the threshold condition (13b) is identical to (8b), but with the boundary condition $a_1(z=0)$ replacing the initial condition $a_1(t=0)$.

In summary, the results presented above indicate that downconversion efficiencies of the order of 70 to 80% of the theoretical maximum, i.e.,

$$\eta = \frac{P_3}{P_1} \approx (.7-.8) \frac{\omega_3}{\omega_1} \quad (15)$$

can be achieved for interactions whose daughter waves satisfy (8a) and pump power and size satisfy (8b). Note that the threshold computations involve only the group velocities of the daughter waves and the characteristics of the pump (i.e., P_1 and antenna gain).

III. MAXIMUM ENERGY TRANSFER IN THE SEMI-COLLISIONAL REGIME

The analysis presented above is valid in the collisionless regime. In examining the generation of ELF waves in the lower ionosphere by HF pumps, we notice that the low frequency ω_3 can be much smaller than the electron neutral collision frequency ν_{en} which controls the damping of the HF waves. The damping of ELF waves is determined by the ion neutral collision where $\omega_3 \gg \nu_{in}$. We call the regime

$$\omega_1, \omega_2 \gg \nu_{en} \gg \omega_3 \gg \nu_{in}$$

the semi-collisional regime. In this regime the dominant part of the low frequency nonlinear force is due to the thermal nonlinearity rather than the usual ponderomotive force. This leads to nonsymmetric coupling coefficients in Equations (4) with rather profound consequences. The coupled equations for the semi-collisional regime, whose derivation is given in Ref. 18 are

$$(\frac{\partial}{\partial t} + \underline{U}_1 \cdot \frac{\partial}{\partial \underline{x}} + \Gamma_1) a_1 = \kappa (1 - i \frac{\nu}{\omega_2}) a_2 a_3 \quad (16a)$$

$$(\frac{\partial}{\partial t} + \underline{U}_2 \cdot \frac{\partial}{\partial \underline{x}} + \Gamma_2) a_2 = \kappa (1 - i \frac{\nu}{\omega_1}) a_1 a_3^* \quad (16b)$$

$$(\frac{\partial}{\partial t} + \underline{U}_3 \cdot \frac{\partial}{\partial \underline{x}} + \Gamma_3) a_3 = \kappa (1 + i \frac{4}{3} \frac{\nu}{\omega_3}) a_1 a_2^* \quad (16c)$$

where κ is the coupling coefficient of the collisionless interaction and $\nu \equiv \nu_{en}$. For $\nu \rightarrow 0$, we recover Equations (4).

The conditions for absolute instability can be found as previously by assuming the pump a_1 constant and solving the eigenvalue problem given by Equations (16b) and (16c) with the appropriate boundary conditions. We find that Equations (8) are now modified to

$$U_{2x} U_{3x} < 0 \quad (17a)$$

$$\frac{L}{\lambda_c} > 1.8 \quad (17b)$$

where

$$\lambda_c \equiv \frac{L_c}{|\Lambda|^{1/2}} \quad (17c)$$

$$\Lambda \equiv 1 - i \frac{4}{3} \frac{\nu}{\omega_3} . \quad (17d)$$

For $\nu \ll \omega_3$, (17b) would reduce to (8b). For $\nu \gg \omega_3$, however, the pump power threshold is reduced by a factor of the order ν/ω_3 . Since the system of Equations (16) is not conservative, we cannot apply the inverse scattering method to the three wave system. To determine the downconversion efficiency when the conditions (17) are satisfied, we solved Equations (16) numerically. For simplicity, we considered a model problem of a one dimensional space-time evolution with $U_{1x} = 0$, $U_{2x} = -U_{3x}$, $\Gamma_j = 0$, $\omega_1 = 10.1$, $\omega_2 = 10.0$, $\nu = 1.2$ and $\omega_3 = .1$; $|a_1| = 1$, $|a_2| = .01$, $|a_3| = 0..$ The results are shown in Figure 3. The interaction leads to pump depletion. More importantly, the downconversion efficiently is increased by a factor of the order ν/ω_3 , over the value given by (6). The preferential transfer of action to a_3 is understandable since for $\nu \gg \omega_3$, the coupling coefficient of the low frequency equation is ν/ω_3 larger than that of the high frequency sideband. Physically it means that additional ω_3 photons are generated at a large rate via a temperature perturbation, with a net loss of energy to the medium. As a comparison, we show also the results of the collisionless interaction. In order to have the proper threshold normalization for a given pump width L , we have used $|a_1| = 1.0$ for the collisionless case and $|a_1| = |\Lambda|^{-1/2} = .25$ for the semi-collisional case so that the respective normalized widths L/L_C and L/ℓ_C have the same value. From Figure 3, we see that pump depletion is seen to occur in both interactions. For the collisionless case, the emergence of symmetric pulses identifiable as solitons is as predicted by the IST solutions, and $|a_3| \approx |a_2|$ which implies downconversion efficiency of the order of ω_3/ω_1 consistent with the Manley-Rowe relations. In the collisional case, however, $|a_3| \approx (\nu/\omega_3)^{1/2} |a_2|$ indicating a major improvement in the efficiency.

IV. APPLICATION TO IONOSPHERIC VLF/ELF GENERATION

In this section we discuss the HF antenna requirements in order to achieve efficient downconversion from HF to VLF/ELF in the ionosphere. Let us first summarize the highlights of the previous sections.

- (i) Our study was confined to interactions produced by two HF pumps propagating transverse to the ionospheric density gradient. Such interactions minimize the required threshold for complete pump decay by avoiding the phase mismatch caused by the density gradient. The practical implications of this to system design are:
 - (a) Beam steerability such that grazing HF incidence can be achieved at ionospheric altitudes of optimum interaction.
 - (b) Antenna gain such that the beam width L in the direction of the density gradient, is at most equal to the density gradient scalelength L_N (i.e., $L < L_N$).
 - (c) Ability to radiate several frequencies simultaneously.
- (ii) For collisionless interactions ($\omega_3 > \nu_{en}$), complete pump decay can be achieved for

$$\frac{L}{L_c} > \frac{\pi}{2}$$

in which case downconversion efficiency is

$$\frac{P_3}{P_1} \approx (.7-.8) \frac{\omega_3}{\omega_1} .$$

(iii) For semi-collisional interactions ($\omega_1, \omega_2 > v_{en} > \omega_3$), complete pump decay can be achieved for

$$\frac{L}{\lambda_c} > 1.8$$

in which case the downconversion efficiency is

$$\frac{P_3}{P_1} \approx (.7-.8) \frac{\omega_3}{\omega_1} \frac{v_{en}}{\omega_3} .$$

We have applied the collisionless interaction to VLF generation at an altitude of 150 Km and the semi-collisional interaction of ELF generation at a lower altitude, ~100 Km, where electron neutral collisions are important. In both cases, we considered a 5 MHz HF pump incident perpendicular to the ionospheric density gradient and almost parallel to the magnetic field (angle between the wavevector $\underline{k}_1$ and $\underline{B}_0$ is 5°) with a beamwidth L equal to the local density scalelength L_N . We have taken L_N to be 20 Km at 150 Km and 10 Km at 100 Km. The threshold electric field required to achieve pump decay was computed from Equations (8b) and (17b) respectively. For the coupling coefficient, we used the following expression from Ref. 18,

$$\kappa = \frac{-ie}{4\sqrt{2}M_e} \frac{n_3 \omega_3^{1/2} \omega_{pi}}{\omega_1} \frac{\alpha}{\beta^{1/2}} \quad (18a)$$

$$\text{where } \alpha = 1 + \frac{\omega_3}{\omega_{ce}} \frac{\epsilon_3^{xy}}{\epsilon_3^{xx} - n_{3z}^2} \quad (18b)$$

$$\beta = \frac{M_e}{M_i} \left[1 - \frac{\omega_{ce}^2}{\omega_3^2} (1 - \alpha) \alpha \right] . \quad (18c)$$

We have defined the low frequency refractive index as $n_3 = k_3 c / \omega_3$ and ϵ_3^{xx} , ϵ_3^{xy} are the elements of the cold plasma dielectric tensor.²⁰ Equations (18a) was derived under the assumption that the pump and its sideband are unmagnetized ($\omega_{1,2} \gg \omega_{ce}$, the electron cyclotron frequency) while the low frequency wave dispersion relation is kept general but with $\omega_3 < \omega_{pi}$, the ion plasma frequency. In the ELF range, (ω_3, k_3) is the compressional Alfvén branch whereas in the VLF regime, it corresponds to the whistler mode. The calculations were performed with plasma parameters pertinent to the particular altitude under daytime conditions.

The results for VLF generation by collisionless interaction are rather disappointing due to the large powers required for complete decay. Figure 4 shows that extremely high electric fields are needed in the interaction region to overcome the decay threshold. If no resonant enhancement of the field occurs in the interaction region, the effective radiative power (ERP) required will be of the order of 10^6 - 10^7 MW. Such a peak power requirement of 1-10 msec duration is beyond the capability of any present pulsed power systems. This leaves the possibility of utilizing the near field of space-born antenna systems. If the threshold power can be achieved, the VLF power generated will be of the order of hundreds of KW.

The ELF wave generation in the semi-collisional regime gives more interesting results. This is due to the lower group velocity of the ELF wave and the collisional reduction of the length l_c as given in (17c). Figure 5 shows a much lower threshold field which lies in the range of 10-15 V/M for a frequency of around 100 Hz. The HF ERP in this case is of the order of 10^4 MW and for times less than 1 sec, this peak power requirement is well within the state of the art. The threshold can be further reduced if the increase in v_{en} due to the local electron HF heating is included. The downconverted ELF power will be of the order of a few hundred watts if we assume an antenna gain of 40 db. It is to be emphasized that the above values are for the specific case of HF incidence perpendicular to the density gradient so that a key requirement for a proof of principle experiment will be the steerability of the HF antenna.

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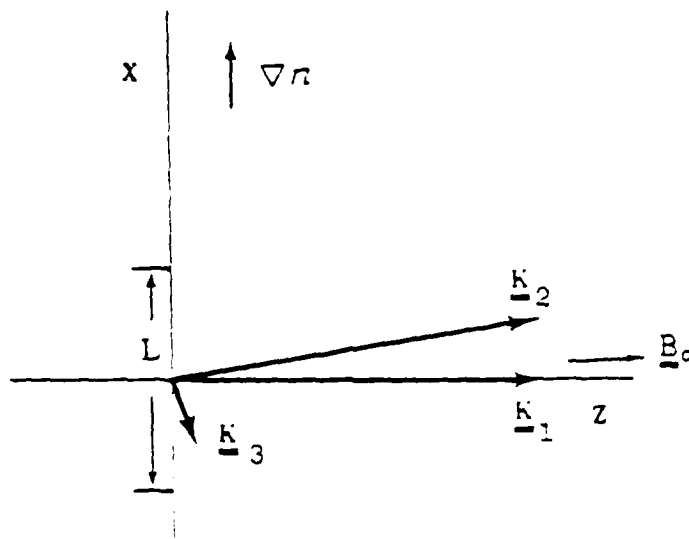


Figure 1. Geometry of parametric interaction

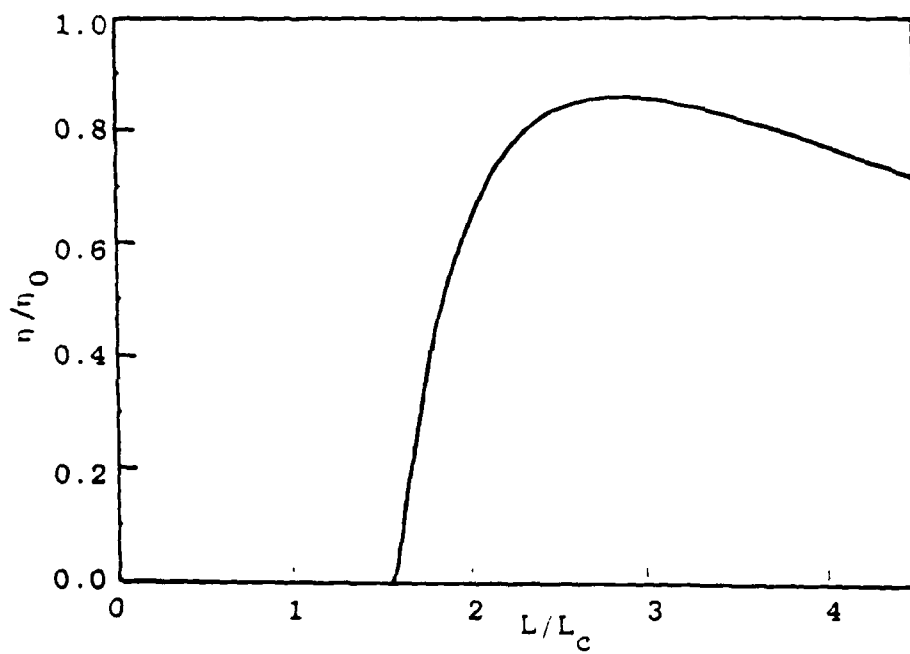


Figure 2. Fractional energy transfer, as a function of pump width, for a rectangular pump in a homogeneous medium.

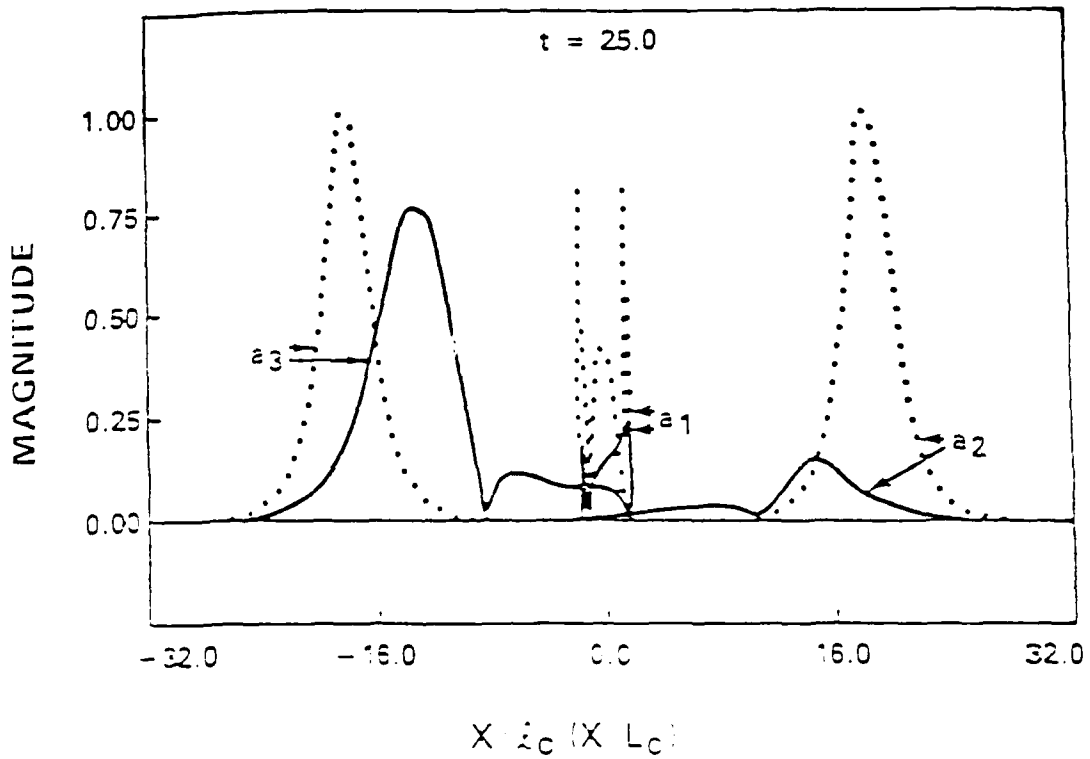


Figure 3. Wave profiles at $t = 25.0$ for the collisional (solid) and collisionless (dotted) interactions.

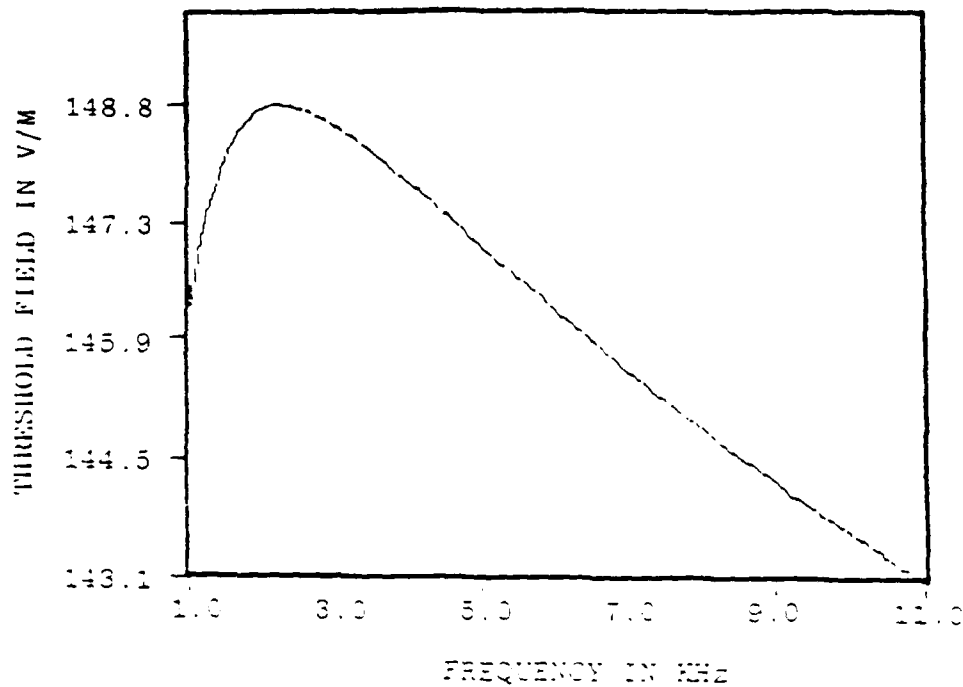


Figure 4. Threshold electric field versus frequency for VLF generation by collisionless interaction at an altitude of 150 Km.

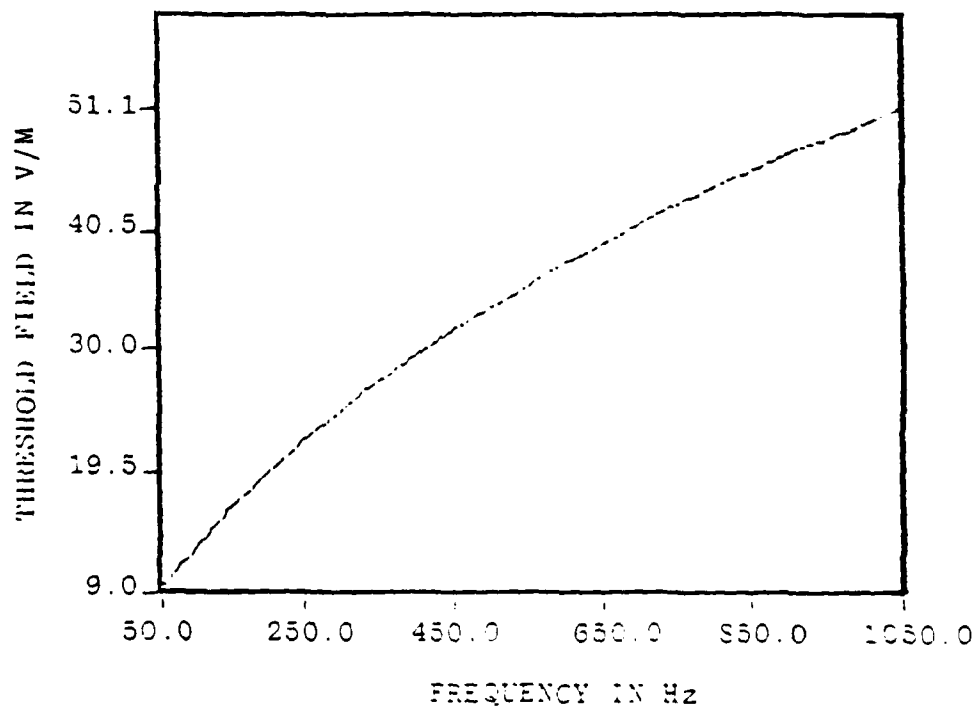


Figure 5. Threshold electric field versus frequency for ELF generation by semi-collisional interaction at an altitude of 100 Km.

APPENDIX I

Efficient Parametric Decay in Dissipative Media

K. Papadopoulos,^(a) K. Ko, and V. Tripathi^(a)
Science Applications, Inc., McLean, Virginia 22102
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Three-wave-coupling interactions have been examined for a dissipative medium. It is shown that in the semicollisional regime, where the frequency of the low-frequency wave (ω_3) is smaller than the collision frequency ν that affects the weakly damped high-frequency waves (ω_1, ω_2), i.e., $\omega_1, \omega_2 \gg \nu > \omega_3$, the thermal nonlinearity predominates and decay efficiencies in excess of that expected from the Manley-Rowe relations can be achieved. In addition, the pump threshold power for complete decay is substantially reduced.

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The interaction of two high-frequency wave packets, $(\omega_1, \vec{k}_1)$ and $(\omega_2, \vec{k}_2)$, with a low-frequency one $(\omega_3, \vec{k}_3)$ is fundamental in the study of nonlinear processes in dispersive media. An often desired goal is the efficient down-conversion of high-frequency power to low frequencies. Such is the case for rf heating of plasmas,¹ and for the development of wireless ionospheric communication schemes.² For interactions in nondissipative media, the maximum energy transfer to the decay products, as described by the Manley-Rowe relations,³ is proportional to their frequencies, ω_3/ω_1 . In this Letter we demonstrate that a channel of decay exists in a semicollisional regime with down-conversion efficiency in excess of that predicted by the Manley-Rowe relations. It is also shown that there is a significant reduction in the threshold required for complete decay⁴ (i.e., pump depletion). The scaling in the semicollisional regime is

$$\omega_1, \omega_2 \gg \nu \gg \omega_3, \quad (1)$$

where ν is the collision frequency which controls the damping of the high-frequency waves. In this regime, the dominant part of the low-frequency nonlinear force on the electrons is due to the thermal nonlinearity (i.e., the same nonlinearity that produces ionospheric self-focusing⁵ or entropy modes⁶) rather than the usual ponderomotive nonlinearity. This leads to nonsymmetric coupling coefficients in the three-wave interaction, with very profound consequences. Although our results are general, for concreteness we focus on the excitation of a compressional Alfvén wave by a high-frequency pump in the lower ionosphere, where the required scaling (1) is satisfied. This particular interaction has important applications to wireless communications at extremely low frequencies.

Consider the interaction in the semicollisional regime described by (1), with ν given by electron-

neutral collisions. For the high-frequency pump and its sideband we assume $\omega_1, \omega_2 \gg \omega_e, \Omega_e$, where ω_e and Ω_e are the electron plasma and cyclotron frequency. The low-frequency wave is taken to be the compressional Alfvén branch. Following standard procedures,⁷ we find that in a homogeneous medium, the slowly varying envelopes of the wave packets $E_j(\vec{x}, t) = a_j a_j(\vec{x}, t) \times \exp[-i(\omega_j t - \vec{k}_j \cdot \vec{x})]$, subject to the resonant conditions,

$$\omega_1 = \omega_2 + \omega_3, \quad \vec{k}_1 = \vec{k}_2 + \vec{k}_3, \quad (2)$$

satisfy the system of equations

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \vec{u}_1 \cdot \frac{\partial}{\partial \vec{x}} + \Gamma_1 \right) a_1 &= K \left(1 - i \frac{\nu}{\omega_2} \right) a_2 a_3, \\ \left(\frac{\partial}{\partial t} + \vec{u}_2 \cdot \frac{\partial}{\partial \vec{x}} + \Gamma_2 \right) a_2 &= -K \left(1 - i \frac{\nu}{\omega_1} \right) a_1 a_3^*, \\ \left(\frac{\partial}{\partial t} + \vec{u}_3 \cdot \frac{\partial}{\partial \vec{x}} + \Gamma_3 \right) a_3 &= -K \left(1 + i \frac{4}{3} \frac{\nu}{\omega_3} \right) a_1 a_2^*, \end{aligned} \quad (3)$$

where $\vec{u}_j$ are the group velocities and Γ_j are the damping rates. The normalization factors are $\alpha_{1,2} = c/V_A$ and $\alpha_3 = (\omega_3/\omega_1)^{1/2}$. In a two-dimensional geometry (x, z) with the ambient magnetic field $\vec{B}_0 = B_0 \hat{z}$, K is given by

$$K = i \frac{e}{4m} \frac{k_{3z}}{(\omega_1 \omega_3)^{1/2}} \frac{\omega_e^2}{\omega_1 \Omega_e}, \quad (4)$$

which is the symmetric coupling coefficient for the collisionless interaction ($\nu = 0$). This expression for K is valid when $\omega_3 \ll \Omega_e$, the ion cyclotron frequency. In deriving the low-frequency equation in (3), we have included in addition to the familiar ponderomotive nonlinearity, the nonlinearity due to Ohmic heating of the electrons,⁸ which produces a temperature perturbation T_3 at ω_3 . Using the electron heating equation⁹ we obtain

$$T_3 \approx i \frac{4}{3} \frac{\nu}{\omega_3} \frac{e^2}{m} \frac{\vec{E}_1 \cdot \vec{E}_2^*}{\omega_1 \omega_2}, \quad (5)$$

where the electron quiver velocities have been approximated by

$$\tilde{v}_{1,2} = -\frac{ie\tilde{E}_{1,2}}{m\omega_{1,2}} \left(1 - \frac{i\nu}{\omega_{1,2}}\right). \quad (6)$$

The total low-frequency nonlinear force on the electrons is the sum of the thermal force due to (5), $-n_0\nabla T_3$, and the usual ponderomotive force,

$$\begin{aligned} \tilde{F}_p = & -mm_0(\tilde{v}_1 \cdot \nabla \tilde{v}_2^* + \tilde{v}_2^* \cdot \nabla \tilde{v}_1) \\ & - \frac{en_0}{c}(\tilde{v}_1 \times \tilde{B}_2^* + \tilde{v}_2^* \times \tilde{B}_1), \end{aligned} \quad (7)$$

where n_0 is the ambient plasma density. If we let $\nabla = i\mathbf{k}$ and use Faraday's law for $\tilde{B}_{1,2}$ in (7), the resulting nonlinear electric field is given by

$$\tilde{E}_3^{NL} \approx i\mathbf{k} \cdot \left(1 + i\frac{4}{3}\frac{\nu}{\omega_3}\right) \frac{e}{m} \frac{\tilde{E}_1 \cdot \tilde{E}_2^*}{\omega_1\omega_2}, \quad (8)$$

which then drives a nonlinear current to generate a_3 . It is clear from (8) that the thermal nonlinearity dominates when $\nu/\omega_3 \gg 1$,⁹ introducing an equivalent enhancement factor in $\tilde{E}_3^{NL}$ as well as a $\pi/2$ phase difference. Coupling of a_3 to the high-frequency waves is accomplished by the beating of $\tilde{v}_1, \tilde{v}_2$ with the low-frequency electron density perturbation

$$n_3 \approx \frac{i\mathbf{k} \cdot \tilde{E}_3^*}{4\pi e} \cdot \tilde{E}_3, \quad (9)$$

$\tilde{E}_3^*$ being the electron contribution to the cold-plasma dielectric tensor.

In the collisionless limit ($\nu \rightarrow 0$), one recovers from (3) the standard three-wave interactions which have been investigated by the inverse scattering transform (IST) and by numerical simulation.^{4,10} For the case of space-time evolution in one dimension, it was found that the threshold for pump depletion is equivalent to the pump value a_1 required to achieve an absolute instability within a width L . The eigenvalue problem which determines the conditions for absolute instability in a rectangular pump is well known¹¹ (i.e., the backward wave oscillator). For pump inhomogeneity along x , the instability is absolute if

$$u_{2x}u_{3x} < 0, \quad (10a)$$

$$L > \frac{1}{2}\pi L_c, \quad (10b)$$

where $L_c = |u_{2x}u_{3x}|^{1/2}/\gamma_0$ is the critical width, while $\gamma_0 = Ka_1$ is the uniform medium growth rate. Nonlinear saturation of the absolute instability by pump depletion as suggested by soliton solutions from both IST and numerical simulation limits the fractional energy transfer to the low-frequency

wave to ω_3/ω_1 .

Our analysis shows that these conclusions are seriously modified when one considers the three-wave interaction in the semicollisional regime. From (3), we obtain the linear equations for the eigenvalue problem that determines absolute instability:

$$\left(\frac{\partial}{\partial t} + u_{2x}\frac{\partial}{\partial x} + \Gamma_2\right)a_2 = -\gamma_0(x)a_3^*, \quad (11a)$$

$$\left(\frac{\partial}{\partial t} + u_{3x}\frac{\partial}{\partial x} + \Gamma_3\right)a_3 = -\gamma_0(x)a_2^*\Lambda, \quad (11b)$$

where γ_0 now contains the pump inhomogeneity and Λ is defined by

$$\Lambda = 1 + i\frac{4}{3}\nu/\omega_3. \quad (12)$$

For the threshold calculation, the small collisional correction on the right-hand side of (11a) has been ignored. If we use Laplace transformation in time and make the substitution

$$(a_2^*, a_3) = \exp\left[-\frac{1}{2}\left(\frac{\rho + \Gamma_2}{u_{2x}} + \frac{\rho + \Gamma_3}{u_{3x}}\right)x\right](A_2, A_3),$$

the conjugates of (11a) and (11b) become

$$\left(\frac{\partial}{\partial x} + \eta\right)A_2 = \frac{-\gamma_0(x)}{u_{2x}}A_3, \quad (13)$$

$$\left(\frac{\partial}{\partial x} - \eta\right)A_3 = \frac{-\gamma_0(x)}{u_{3x}}A_2\Lambda,$$

where

$$\eta = \frac{1}{2}\left(\frac{\rho + \Gamma_2}{u_{2x}} - \frac{\rho + \Gamma_3}{u_{3x}}\right). \quad (14)$$

The eigenvalue or growth rate ρ may in general be complex. For a rectangular pump, (13) readily admits solutions in terms of trigonometric functions and for $u_{2x} > 0$, $u_{3x} < 0$, the boundary conditions are that a_2 is zero at $-L/2$ while a_3 is zero at $L/2$. One then finds the dispersion relation

$$(e^{i\theta} - \eta^2 l_c^2)^{1/2} \cot(e^{i\theta} - \eta^2 l_c^2)^{1/2}(L/l_c) + \eta l_c = 0, \quad (15)$$

where

$$\theta = \tan^{-1}(\frac{4}{3}\nu/\omega_3), \quad l_c = L_c |\Lambda|^{1/2}. \quad (16)$$

We have solved (15) numerically in the limit $\nu/\omega_3 \gg 1$ so that $\theta = \pi/2$ and $|\Lambda| = \nu/\omega_3$. The real part of the normalized growth rate ηl_c for the first four growing modes is shown in Fig. 1 as a function of the normalized width, L/l_c . The critical width threshold now becomes

$$L \approx 1.8 l_c. \quad (17)$$

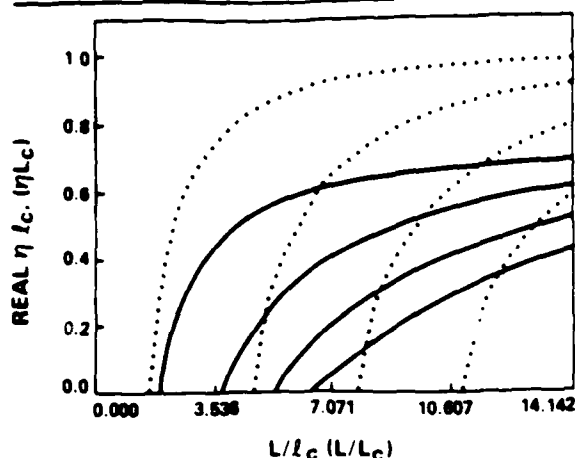


FIG. 1. Real part of ηl_c (ηL_c) vs L/l_c (L/L_c) for the collisional (collisionless) interaction. The dotted lines refer to the collisionless case.

For comparison, the results for the collisionless case ($\nu=0$; $\theta=0$, $\Lambda=1$, $l_c=L_c$) are also plotted. For a given pump width L , the value of a_1 required to achieve absolute instability in the semi-collisional regime is smaller by the factor $l_c/L_c = |\Lambda|^{-1/2}$. This implies a significant reduction in pump-power requirement $P_1 \sim |a_1|^2$, i.e., of the order $(\nu/\omega_3)^{-1}$. For the lower ionosphere, this is more than two orders of magnitude reduction.

Although the threshold reduction is of great practical interest, an even more important physics issue arises from the effect of nonsymmetric coupling coefficients on the down-conversion efficiency. To address this, the fully nonlinear evolution of (3) is required so that pump depletion is taken into account. It can be easily shown that, even in the absence of damping $\Gamma=0$, the Manley-Rowe relations no longer hold for this interaction.¹² The sums of the actions are not time independent and the total energy is no longer an invariant of the system, so that

$$\partial \mathcal{S}_T / \partial t \approx \frac{4}{3} \nu \Delta, \quad (18)$$

where

$$\mathcal{S}_T = \sum_{j=1}^3 \mathcal{S}_j, \quad \mathcal{S}_j = \int_{-\infty}^{\infty} dx \omega_j |a_j|^2. \quad (19)$$

Consequently, (3) is not amenable to analytical solutions by IST for lack of conservation laws. Instead, we resort to numerical solutions. For simplicity, we consider the model problem of a one-dimensional space-time evolution with $u_{1x}=0$, $u_{2x}=-u_{3x}$, and $\Gamma=0$. We choose $\omega_1=10.1$, $\omega_2=10.0$, $\nu=1.2$, and $\omega_3=0.1$ in accordance with (1). As a comparison, the collisionless interaction

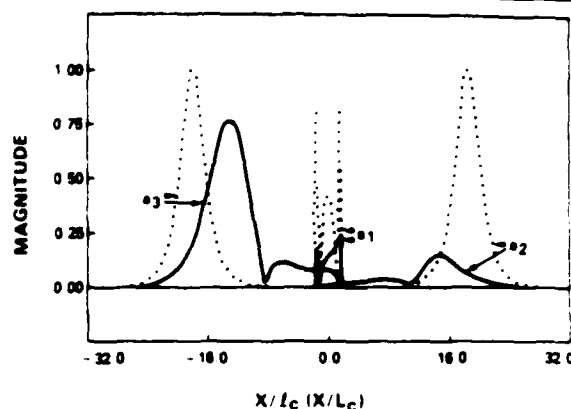


FIG. 2. Wave profiles at $t=25.0$ for the collisional (solid lines) and collisionless (dotted lines) interactions.

($\nu=0$) is also examined. The initial wave packets are rectangular with $|a_2|/|a_1|=0.01$, $|a_3|=0.0$. For a given pump width L , we use $|a_1|=1.0$ for the collisionless case and $|a_1|=|\Lambda|^{-1/2}=0.25$ for the collisional case so that the respective normalized widths L/L_c and L/l_c have the same value; i.e., $L/L_c=L/l_c=3.2$. This ensures from Fig. 1 that there is only one initial growing mode in either interaction.

The time-asymptotic nonlinear behavior, at $t=25.0$, is shown in Fig. 2 where pump depletion is seen to occur in both interactions. In the collisionless case, the emergence of symmetric pulses identifiable as solitons is as predicted by the IST solutions.⁴ The collisional interaction, on the other hand, exhibits nonsymmetric pulses with $a_3 \gg a_2$ which suggests a different action-transfer picture than that described by the Manley-Rowe relations. The preferential transfer of action to a_3 is understandable in view of the factor $\nu/\omega_3 \gg 1$ in the low-frequency equation as compared to the small corrections ν/ω_2 and ν/ω_1 in the equations for a_1 and a_2 , respectively. Physically, it means that additional ω_3 photons are generated at a large rate via the temperature perturbation with a net loss of energy to the medium. We have calculated the down-conversion efficiency $\mathcal{S}_3(t)/\mathcal{S}_1(t=0)$ for this interaction and found an order-of-magnitude enhancement (~ 20) over the collisionless case. The energy ratio $\mathcal{S}_3(t)/\mathcal{S}_2(t)$, which for the collisionless interaction equals the frequency ratio $\omega_1/\omega_2=0.01$, has a much larger value (0.8) for the collisional case. The above considerations are valid for pump powers such that the nonlinear interaction time is faster than the electron heating time.

In our analysis, we have neglected the role of parallel heat conduction by assuming in (5) $\omega_3 \gg k_{\perp}^2 \chi$, where χ is the heat-conduction coefficient. Heat conduction can be included by simply replacing ω_3 with $\omega_3 + ik_{\perp}^2 \chi$ in (3), (5), and (8) and the definition of Λ in (12). An analytic generalization of the analysis to the case of diffusive modes⁹ where $k_{\perp}^2 \chi \gg \omega_3$ is possible by rescaling the equations because the coupling coefficients in (3) become predominantly real. The modified Manley-Rowe relations for this case have the approximate form

$$\frac{\dot{\delta}_1}{\delta_1} = \frac{\dot{\delta}_2}{\delta_2} = \frac{\omega_1}{\omega_1} \left(1 + \frac{4}{3} \frac{\nu}{k_{\perp}^2 \chi} \right). \quad (20)$$

In summary, we have shown that much lower pump power threshold for complete decay is required, and substantial increase in down-conversion efficiency can be achieved in three-wave interactions in the semicollisional regime ($\nu \gg \omega_3$). This result can have very profound consequences for schemes requiring efficient high-frequency power down-conversion, such as wireless ionospheric communications, low-frequency rf plasma heating, etc. Preliminary estimates indicate that pump power of a few megawatts can efficiently down-convert high-frequency power to extremely low-frequency power while satisfying the theoretical constraints. These will be discussed in future publications.

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^(*)Permanent address: University of Maryland, College Park, Md. 20742.

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