TWO-FLUID COUETTE FLOW BETWEEN CONCENTRIC CYLINDERS(U)
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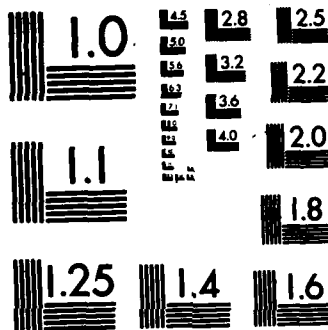F/G 20/4

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TWO-FLUID COUETTE FLOW BETWEEN
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TWO-FLUID COUETTE FLOW BETWEEN CONCENTRIC CYLINDERS

Yuriko Renardy and Daniel D. Joseph*

ABSTRACT

This document

~~We~~ consider the flow of two immiscible fluids lying between concentric
cylinders when the outer cylinder is fixed and the inner one rotates. The
interface is assumed to be concentric with the cylinders and gravitational
The authors
effects are neglected. ~~We~~ present a numerical study of the effect of
different viscosities, different densities and surface tension on the linear
Their
stability of the Couette flow. ~~Our~~ results indicate that with surface
tension, a thin layer of the less viscous fluid next to either cylinder is
linearly stable and that it is possible to have stability with the less dense
fluid lying outside. The stable configuration with the less viscous fluid
next to the inner cylinder is more stable than the one with the less viscous
fluid next to the outer cylinder. The onset of Taylor instability for one-
fluid flow may be delayed by the addition of a thin layer of less viscous
fluid on the inner wall and promoted by a layer of more viscous fluid on the
inner wall.

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SIGNIFICANCE AND EXPLANATION

Steady two-fluid flows of immiscible density-matched fluids with different viscosities arise in applications such as the pipeline transport of oil with the addition of water, the formation of bicomponent fibers such as nylons and modelling of the Earth's mantle. Such flows are typically nonunique, even when the speeds involved are slow. However, experiments usually result in very stable unique arrangements. In order to get an idea of which interface positions are allowed for the 'Taylor Problem' we study the linear stability of the concentric arrangement. We find that, in the absence of gravity, a thin lubrication layer of the less viscous fluid, lying next to either cylinder, is linearly stable. This agrees with the experimental observations. The fact that the less viscous liquid tends to shield the more viscous fluid from shearing suggests that the use of two such fluids in lubrication may be economical. We also study numerically the effect of different densities and surface tension and find that the stabilizing effect of this viscosity stratification can even overcome a destabilizing density difference: the arrangement with the less dense fluid outside can be stable if it is also the less viscous fluid and if this outer layer is thin.

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TWO-FLUID COUETTE FLOW BETWEEN CONCENTRIC CYLINDERS

Yuriko Renardy and Daniel D. Joseph*

Introduction

We consider linear stability of the flow of two immiscible fluids separated by an interface, lying between concentric rotating cylinders. In each fluid, the Navier-Stokes equations for steady flow are assumed to hold. If we prescribe the ratio of the total volume occupied by each fluid, then the interface is an unknown, across which the velocity and normal and shear stresses are to be continuous. If the fluids have equal or nearly equal densities, then a continuum of interface positions are allowed (Joseph, Renardy and Renardy, 1983). However, this non-uniqueness is not borne out by the experiments of Joseph, Nguyen and Beavers (1983) who use water and various oils as the two fluids in an apparatus with the outer cylinder fixed. When the inner cylinder is rotated at even moderate speeds, gravity effects appear negligible and a pattern consisting of two types of cells is usually observed. One type consists mostly of oil rollers stuck to the inner cylinder and rotating almost like a solid body, lubricated by a thin layer of water at the outer cylinder. The second type consists mainly of water cells undergoing Taylor vortex motions. These cells extend from the inner to the outer cylinder but, in some experiments, are covered by a thin layer of oil at the outer cylinder. The two types of cells alternate along the length of the cylinder. This flow is one of many steady bicomponent flows where a study of the selection mechanism for the arrangement of the fluids must be made. One way to study selection is to study stability and in this paper we study stability by computing eigenvalues for the spectral problem associated with the linear theory.

The equations for our numerical computations are given in Part I. Some asymptotic results for short waves are presented in Part II following the ideas of Hooper and Boyd

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(1983). They consider unbounded Couette flow but their method of analysis applies locally at any interface with a viscosity jump. Hooper and Boyd showed that in the absence of surface tension, the flow is unstable to sufficiently short waves which have wave-vectors parallel to the basic flow. The growth rates of these disturbances, however, tend to zero as the waves get shorter. They found that these short-wave instabilities are not suppressed by viscosity as they are in one-fluid flows, but by surface tension. A density difference can stabilize or destabilize them but not as effectively as surface tension. In Part II, we have given a similar analysis for disturbances whose wave-vectors are perpendicular to the basic flow. Our results include surface tension, density differences and centrifugal effects. We have correlated our numerical results with the asymptotic formulas.

When surface tension is effective, the longer waves can cause instability and if periodic boundary conditions are imposed, then this yields a familiar type of instability in which the interaction of a finite number of modes determines what type of solutions bifurcate from the unstable one (M. Renardy and Joseph, 1983). However, when surface tension is not effective, then we have an unusual instability in which the flow is unstable to all short waves below a certain critical size. This type of instability may play a role in the formation of emulsions.

In Part III, we give numerical results for two situations: low Reynolds numbers and Taylor numbers near a critical one. We find that in the preferred configurations, a thin layer of the less viscous fluid may lie next to either cylinder. Our results contradict the selection principle based on minimizing the viscous dissipation in the restricted class of annular layers of two fluids which do not vary along the axis of the cylinder. The solution of this minimization problem (Joseph, Nguyen and Beavers, 1983) consists of the less viscous fluid lying on the inner cylinder, no matter which cylinder rotates. In fact, our numerical results indicate that a narrow stable layer of the less viscous fluid on the inner cylinder is "more" stable than that on the outer cylinder. It is also of interest that the instability leading to Taylor vortices in one-fluid flows may be

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nullified by adding a lubrication layer of the less viscous fluid at the inner cylinder and that instability may be created by adding a thin layer of more viscous fluid at the inner cylinder.

Part I. Stability equations and numerical solution.

We use cylindrical coordinates (r,θ,z) where the z-axis is the axis of cylinders of radii R_1 and R_2. We consider the stability of a circular Couette flow of two fluids, lying between the cylinders. The azimuthal velocity field is given by $V_i(r) = A_i r + B_i/r$, $i = 1,2$, where $i = 1$ refers to the 'inner' fluid, occupying $R_1 < r < D$ and $i = 2$ refers to the 'outer' fluid, occupying $D < r < R_2$. The unperturbed interface is at radius D. The angular velocities of the cylinder are Ω_i, the viscosities are μ_i and the densities are ρ_i.

$$A_1 = (\Omega_1 \mu_1 (\frac{1}{R_2^2} - \frac{1}{D^2}) + \mu_2 (\frac{\Omega_1}{D^2} - \frac{\Omega_2}{R_1^2}))/q$$

$$B_1 = (\Omega_2 - \Omega_1)\mu_2/q$$

$$A_2 = (\mu_2(\frac{\Omega_1}{R_2^2} - \frac{\Omega_2}{D^2}) + \mu_1 \Omega_2 (\frac{1}{D^2} - \frac{1}{R_1^2}))/q$$

$$B_2 = (\Omega_2 - \Omega_1)\mu_1/q$$

$$q = \mu_1(\frac{1}{R_2^2} - \frac{1}{D^2}) + \mu_2(\frac{1}{D^2} - \frac{1}{R_1^2}) \ .$$

We superimpose an infinitesimal disturbance

$$(u(r),v(r),w(r);p(r)) \exp(-i \sigma t + i\alpha z + in\theta)$$

so that in each fluid, the Navier-Stokes equations yield:

$$i(- \sigma + \frac{Vn}{r})w = - \frac{i\alpha p}{\rho} + \nu(\frac{1}{r}(rw')' - \frac{n^2}{r^2}w - \alpha^2 w) \qquad (1)$$

$$i(- \sigma + \frac{Vn}{r})u - \frac{2Vv}{r} = - \frac{p'}{r} + \nu(\frac{1}{r}(ru')' - (n^2+1)\frac{u}{r^2} - \alpha^2 u - \frac{2inv}{r^2}) \qquad (2)$$

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$$i(-\sigma + \frac{Vn}{r})v + 2Au = -\frac{inp}{\rho r} + v(\frac{1}{r}(rv')' - (n^2+1)\frac{v}{r^2} - \alpha^2 v + \frac{2inu}{r^2}) \qquad (3)$$

where $v = \mu/\rho$. Incompressibility yields

$$(ru)' + inv + ri\alpha w = 0 \qquad (4)$$

The interface is also perturbed and the following conditions hold at $r = D$ (see Nguyen and Joseph, 1983). [.] denotes the difference $._1 - ._2$ across $r = D$.

(i) Continuity of velocity: $[u] = 0$, $[w] = 0$, $i(-\sigma + \frac{Vn}{D})[v] = \frac{2u[B]}{D^2}$.

(ii) Continuity of stresses:

$$i\alpha u [\mu] + [\mu\frac{\partial w}{\partial r}] = 0$$

$$[\mu(\frac{v}{r})'] + \frac{inu}{D^2}[\mu] = 0$$

$$i(-\sigma + \frac{Vn}{D})([p] - 2[\mu u']) + u[P'] - Tu(\frac{n^2-1}{D^2} + \alpha^2) = 0$$

where T is the surface tension and $P' = \rho\frac{V^2(D)}{r}$. Boundary conditions at the solid are:

$$u = 0, \ v = 0, \ w = 0 \quad \text{at} \quad r = R_1 \text{ and } R_2.$$

We now describe our discretization scheme. Equations (1) and (4) are used to eliminate w and p from equations (2) and (3). A Chebyschev polynomial expansion (Orszag, 1971) is used for u,v,w and p. If n+1 and n Chebyschev polynomials are used for u and v respectively, then the total number of unknowns is 4n+2. Equation (2) can then be truncated after the n-4th degree because of the presence of r^4u^{iv} and r^3v'''. Equation (3) should be truncated after the n-3rd degree because of the presence of r^3u''' and r^2v''. The resulting system of linear equations for σ were solved with an IMSL routine on VAX/VMS 11/780 in complex double precision. The computations were checked against Table 2 in Krueger, Gross and DiPrima (1966), Hooper and Boyd's asymptotics for large n and the asymptotics in Part II for large α.

When the two fluids are identical, the presence of the interface introduces a neutrally stable eigenvalue for each n and α (called the 'interfacial' eigenvalue by Yih (1967)) in addition to the eigenvalues for one-fluid flow (called 'Taylor' eigenvalues in

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Part III). In Part III, we track the behavior of the interfacial and Taylor eigenvalues as the viscosities, densities, surface tension and volume ratio are changed.

Part II. Asymptotic analysis for short wave disturbances.

Hooper and Boyd (1983) restrict their asymptotic analysis to two dimensions with coordinates (x,y) where x is the direction of the stream. They consider disturbances in normal modes proportional to $\exp(i\alpha x)$ for large α or short waves, and expand the stream function and the interfacial eigenvalue in powers of $1/\alpha^2$. The perturbation problems which arise from this procedure are uniquely solvable. Since our stream is in the azimuthal direction, we must replace α by n/r. The results of Hooper and Boyd apply when n is large and centrifugal effects are neglected. Centrifugal effects considered here, however, play the same role as gravity in their analysis. (We note that a factor α should multiply the gravity term below (16) in their paper). For large n, we find that

$$\sigma \sim \frac{V(D)n}{D} + i(\frac{2B_1}{D})^2 \frac{m}{v_1 n^2 2(1+m)} \left[f - \frac{(1-m)}{(1+m)} (-1 + \frac{n^2}{r}) \right]$$

where

$$f = \frac{nF(r-1)}{bm} - (\frac{n}{b})^3 s$$

$$b = \sqrt{\frac{2B_1}{v_1}}$$

$$F = \frac{D^2 V^2(D)}{2bB_1 v_2}$$

$$s = \frac{DbT}{2B_1 \mu_1}$$

$$m = \mu_1/\mu_2, \quad r = \rho_1/\rho_2 .$$

We can also do short-wave asymptotics for disturbances perpendicular to the (r,θ) plane. We consider axisymmetric $(n = 0)$ disturbances proportional to $\exp(i\alpha z)$ and introduce the following dimensionless variables: $R = \alpha(r-D)$, $T = 2B_1 t/D^2$, $Z = bz/D$. Assuming now that the disturbance is proportional to $\exp(i\alpha_1 CT + i\alpha_1 Z)$ where

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$\alpha_1 b/D = \alpha$, this transformation yields the following six conditions at $R = 0$ as $\alpha_1 \to \infty$:

(1) $[u] = 0$; (2) $[\partial u/\partial R] = 0$; (3) $-i\alpha_1 C[v] = u(1-m)$

(4) $u[\mu] + [\mu \partial^2 u/\partial R^2] = 0$; (5) $[\mu \partial v/\partial R] = 0$;

(6) $[\mu(\partial^3 u/\partial R^3 - 3 \ \partial u/\partial R)] + \dfrac{\alpha_1 b D^2}{4\overline{V}(D)B_1(1-m)} \ [\nu L^2 u]\left(\dfrac{\overline{V}^2(D)}{D} \ [\rho] - \dfrac{T\alpha_1^2 b^2}{D^2}\right) = 0$

where $L \equiv \partial^2/\partial R^2 - 1$.

The equations of motion and continuity with $\partial/\partial \theta = 0$ are now expanded about $R = 0$ for large α_1 and, as in the analysis of Hooper and Boyd,

$$u \sim u_0 + \frac{u_1}{\alpha_1^2} + \ldots$$

$$\alpha_1 C \sim c_0 + \frac{c_1}{\alpha_1^2} + \ldots$$

where the zero[th]-order velocity satisfies $L^3 u_0 = 0$ in each fluid. We find that c_1 is determined by u_0 where

$$u_0 = \begin{cases} e^R(a_0 + a_1 R + a_2 R^2) & \text{for } R < 0 \\ e^{-R}(b_0 + b_1 R + b_2 R^2) & \text{for } R > 0 \ . \end{cases}$$

To leading order, $v \sim \dfrac{\alpha_1^2 b^2 \nu L^2 u_0}{2\overline{V}(D)D}$ and $[p] \sim \dfrac{\alpha b}{D} \ [\mu(\partial^3 u/\partial R^3 - \partial u/\partial R] + \dfrac{2B_1 i\alpha_1 C}{Dab} \ \dfrac{\partial u}{\partial R} \ [\rho]$. Five of the coefficients in u_0 can be found in terms of the sixth by using the interface conditions (1), (2), (4), (5) and (6). Condition (3) yields $c_0 = 0$ and an equation for c_1:

$$-i c_1 \ \frac{b^2}{2\overline{V}(D)D} \ [\nu L^2 u_0] = u_0(1-m) \ .$$

From this we find that

$$c_1 = \frac{i(1-m)}{4(1+m)} \left[\frac{3V(D)D}{2B_1}\frac{(r-m)m}{(m+1)r} + \frac{2\bar{f}m}{(1-m)}\right]$$

where $\bar{f} = \frac{\alpha_1 F(r-1)}{m} - S\alpha_1^3$.

Part III. Numerical results.

We compute the growth and decay rates, $im(\sigma)$, for the stability of Couette flow of two fluids. We consider two flow regimes. The first, treated under (a) and (b) below, is flow at small Reynolds numbers. Here, if either fluid filled the flow, the one-fluid flow would be linearly stable. The only mode which can become linearly unstable for the two-fluid flow is the interfacial mode. The second, treated under (c) below, is flow at higher Reynolds numbers where, if the outer fluid filled the flow, the one-fluid flow would be at a critical Taylor number where linear stability is lost. Here, in addition to the interfacial eigenvalues, the eigenvalues associated with the one-fluid flows can become unstable. This type of loss of stability leads to bifurcation and, finally, to the tesselation of stable (highly viscous) and unstable (Taylor cells in the low-viscosity liquid) regions observed in the experiments of Joseph, Nguyen and Beavers. For each flow regime, we determine which arrangement of the components is stable and the volume ratios of the stable configurations.

(a) Stability of Couette flow of two fluids for low Reynolds numbers neglecting surface tension and density difference.

We compute the growth rates for the following range of variables:
$\Omega_1 = 1$, $\Omega_2 = 0$, $\mu_2 = 1$, $m = 0.2$ to 6, $R_1 = 1$, $R_2 = 2$, $\rho_1 = \rho_2 = 1$, the Reynolds number $Re = V(R_1)(R_2 - R_1)/\nu_1$ ranges from 0.5 to 5, α ranges from 0.01 to 50 and n from 0 to 50. Under these conditions, we find that the configuration with a sufficiently thin layer of the less viscous fluid, situated next to either cylinder, is stable.

The response to long waves (small α and low n) is as follows. The axisymmetric mode becomes insignificant as $\alpha \to 0$ since in that limit, there is no disturbance. For

$\alpha < 0.1$ and small Reynolds numbers, the growth rate $im(\sigma)$ is proportional to $\alpha^2 Re$ when $n = 0$ and to Re when $n \neq 0$. The growth rates in this asymptotic range are shown in figures 1 and 2. In figure 1, the less viscous fluid is situated next to the inner cylinder and hence the modes displayed are stable $(im(\sigma) < 0)$ if the interface is close enough to $R_1 = 1$. The situation is reversed in figure 2. In both figures, modes 10 and 20 show the short-wave asymptotic behavior in which the stable range of interface positions, as well as the maximum growth rates, diminish with n (or α). Disturbances of the stable configurations with the less viscous fluid inside ha   such larger decay rates than that with less viscous fluid outside.

Trends similar to those exhibited in figures 1 and 2 are   ited in figures 3 and 4 for $\alpha = 1.0$. The stable range of interface radii is slightly,   not greatly, reduced. Figure 4 clearly shows that, for $m = 2$, the dependence of $im(\sigma)$ on n at modes 20 and 40 scales with $1/n^2$ over most of the interface positions. In both figures, the relative errors of the asymptotic values at $D = 1.5$ fall from about 50% at mode 9 to 8% at mode 20.

Figures 5 and 6 give growth rates for $\alpha = 10$. For fixed small values of n and large α, we find that

$$c_1 \sim \frac{13}{8} \frac{(1-m)^2}{(1+m)^2} m^2 (1 - \frac{D^2}{R_2^2}) \text{ and}$$

$$im(\sigma) \sim (\frac{2B_1}{D^2})^2 \frac{c_1}{\mu_1} \frac{1}{\alpha^2} + 0(\alpha^{-4}) .$$

In figures 5 and 6, modes $n = 1$ to 3 lie in between modes 0 and 4 and the growth rates of all the modes between 0 and 4 are numerically close. These figures display some qualitative features of large α asymptotics, but $\alpha = 10$ is not high enough to be in the short-wave asymptotic range for $m = 0.4$ or 2. In addition, the lar   the viscosity difference m, the lower is the value of α at which this asymptotic range is attained. For example, at $m = 6$, the relative errors range from 30% at $\alpha = 10$, 16% at $\alpha = 20$ and 8% at $\alpha = 40$, whereas at $m = 2$ (figure 6), these errors are doubled and at $m = 0.4$

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(figure 5), they are quadrupled. As noted by Hooper and Boyd, the short-wave asymptotics breaks down when the interface is too close to R_1 or R_2.

From our computations, we conclude that the largest growth and decay rates arise in the order-one range of α for medium n and for small m. For example, mode 9 at $\alpha = 1$, m = 0.2, attains triple the growth rate attained at m = 0.4 (figure 3) and, in turn, that mode at m = 0.4 attains a larger magnitude than at m = 2 (figure 4). We may also conclude from a comparison of decay rates that the stable flows with thin fluid inside are 'more' stable than those with thin fluid outside.

(b) <u>Stability of Couette flow of two fluids for low Reynolds numbers</u>. <u>The influence of surface tension and density differences</u>.

Surface tension stabilizes short wave interfacial disturbances and destabilizes longer waves. Centrifugal forces, in the absence of surface tension, will produce stability if the more dense fluid is outside. However, with surface tension, it is possible to achieve stability when the denser fluid is inside. This can, of course, only happen if the centrifugal force is not too large and gravity is neglected. Under these conditions, if surface tension is large enough to stabilize the short waves but not so large that the long waves are unstable, then stability is possible at all α and n with the denser fluid inside. One example is $T = 1$, $r = 2$, $m = 2$, $D = 1.9$, $\mu_2 = 1$, $\rho_2 = 1$. Figure 7 shows a graph of $-im(\sigma)$ versus α, showing stability. Figure 8 shows a graph of $im(\sigma)$ versus α at zero surface tension, showing that modes become unstable for large α.

(c) <u>Stability of Couette flow of two fluids near a critical Taylor number</u>. <u>Zero surface tension and density difference</u>.

We study the onset of Taylor instability associated with modes n = 0 and 1. We also study the higher modes (n > 1) for which the Taylor modes are stable. In the classicial Taylor problem for one fluid, the most unstable mode is the axisymmetric (n = 0) one.

The Taylor number is defined as $T_a = 4A\Omega_1 (R_2 - R_1)^4/\nu$ where $A = (\Omega_1 R_1^2 - \Omega_2 R_2^2)/(R_1^2 - R_2^2)$. When $\Omega_2 = 0$, $\Omega_1 = 1$, $R_1 = 1$, $R_2 = 2$, $\rho_1 = \rho_2 = 1$, the

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axisymmetric mode becomes unstable for T_a close to T_{cr} where $T_{cr}(R_1/R_2)^2 = 1549.59$ and $\alpha = 3.16$ (Krueger, Gross and DiPrima, 1966). We fix $\nu_2 = 0.0146$ and vary ν_1 so that if the outer fluid occupied the whole annulus, (i.e. $D = R_1$) the growth rate for the axisymmetric Taylor mode is near criticality. We note that this choice of ν_1 implies that the $im(\sigma)$ for the axisymmetric mode passes through zero when $D = R_1$ as in figures 9-12. With this choice of parameters, we study how the growth rates vary with D.

We consider two situations: the less viscous fluid is at the inner ($m < 1$, figures 9 and 10) or at the outer ($m > 1$, figures 11 and 12) cylinder. Intuition would suggest that when $m > 1$, the flow will become more and more stable as D increases because of the presence of increasingly larger amounts of more viscous (stable) fluid. This expectation is not realized. Figures 11 and 12 show that various modes become unstable as thick fluid is added. Similarly, intuition would suggest that when $m < 1$, we should have instability for increasing D because more and more thin fluid replaces thick fluid. Figures 9 and 10 show that we actually stabilize the flow by adding less viscous fluid near the inner wall. This stabilization near $D = 1$ is associated with the stability of narrow layers near the inner cylinder and could be called 'lubrication' stabilization associated with the layer of thin fluid on the inner cylinder.

A new feature close to or above a critical Taylor number is that the $im(\sigma)$ for the interfacial eigenvalue need not be single-valued. That is, the graph of $im(\sigma)$ versus D for an interfacial eigenvalue which begins at $D = R_1$ with $im(\sigma) = 0$ can proceed to match to a Taylor eigenvalue at $D = R_2$ and a second branch satisfying $im(\sigma) = 0$ at $D = R_2$ can match to a Taylor eigenvalue at $D = R_1$. Figures 10 and 11 show mode 1 to have such branches. In figure 10, the Taylor eigenvalue for mode 1 is unstable at $D = R_2$ and in figure 11, it is stable at $D = R_2$. The behavior of the higher ($n > 1$) modes, for which the Taylor modes are very stable, is as described in part (a).

For $n = 0$, the equations yield a real-valued problem for $i\sigma$, ip, iu, iv and iw. Hence, $i\sigma$ is either a real number or appears in complex conjugate pairs. In the latter case, the two eigenvalues have equal imaginary parts. This behavior is shown in figures 9

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to 12. For example, in figure 9, for $R_1 < D < 1.3$, the $n = 0$ eigenvalues are in conjugate pairs. Near $D = 1.3$, the $im(\sigma)$ splits. One branch becomes increasingly unstable and at $D = R_2$ is the unstable Taylor eigenvalue for one-fluid flow with $\nu = 0.0132$. The second branch becomes stable for $1.3 < D < R_2$ and since $im(\sigma) = 0$ at $D = R_2$ this branch is the interfacial eigenvalue.
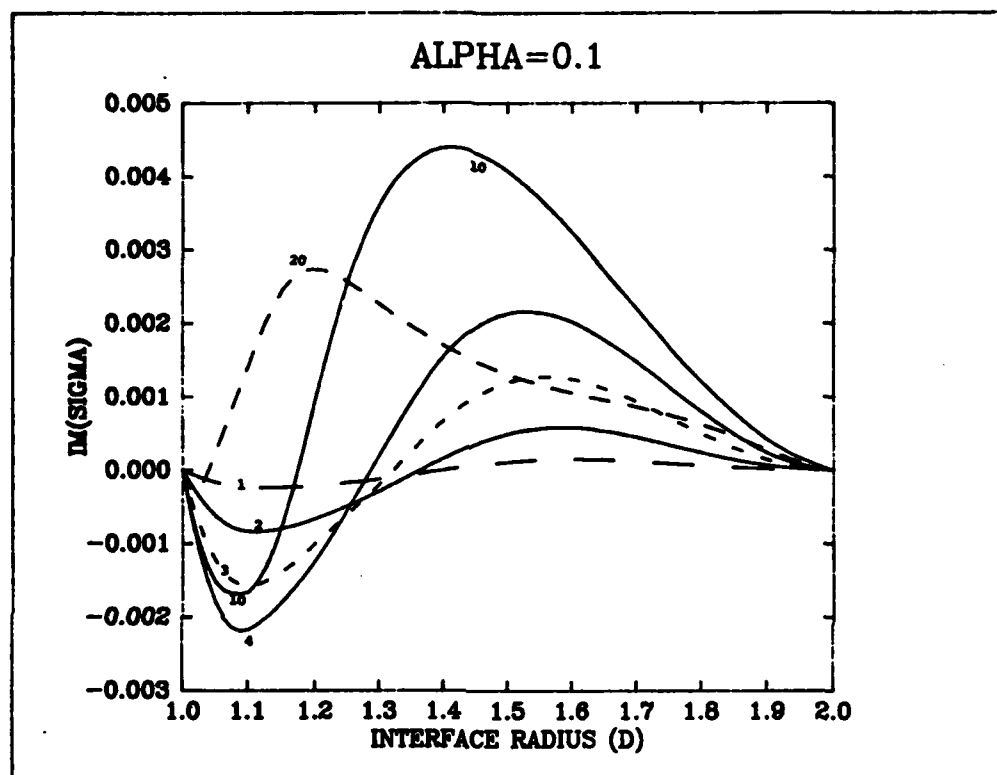
ALPHA=0.1

Figure 1

Growth rate versus interface position with azimuthal wave number (n) as a parameter.
T = 0, r = 1, m = 0.4. The less viscous fluid is on the inner cylinder. Negative
im (σ) corresponds to stability. When surface tension is absent, the flow is unstable
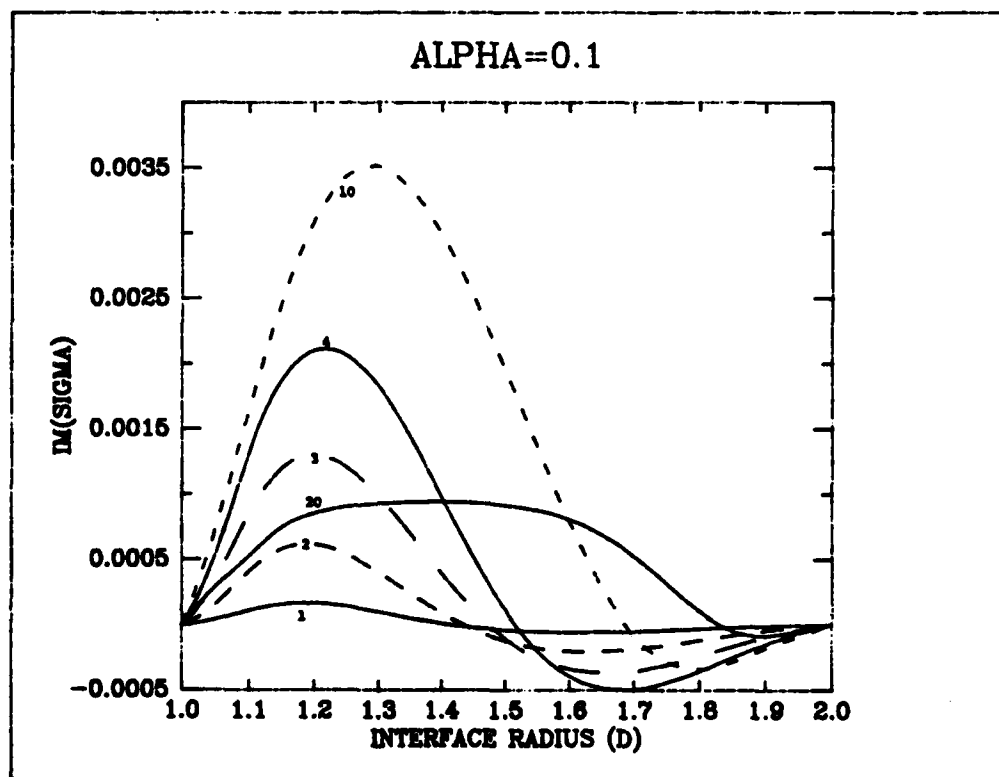at any D (≠ R_1 or R_2) if n is large enough. Mode 0 is insignificant under graph
scales.

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ALPHA=0.1

Figure 2

Growth rate versus interface position when the less viscous fluid is on the outer cylinder. $T = 0$, $r = 1$, $m = 2$. The stable modes near the outer cylinder have less stability than the stable modes near the inner cylinder (cf figure 1) because the decay rates of stable disturbances are much smaller.
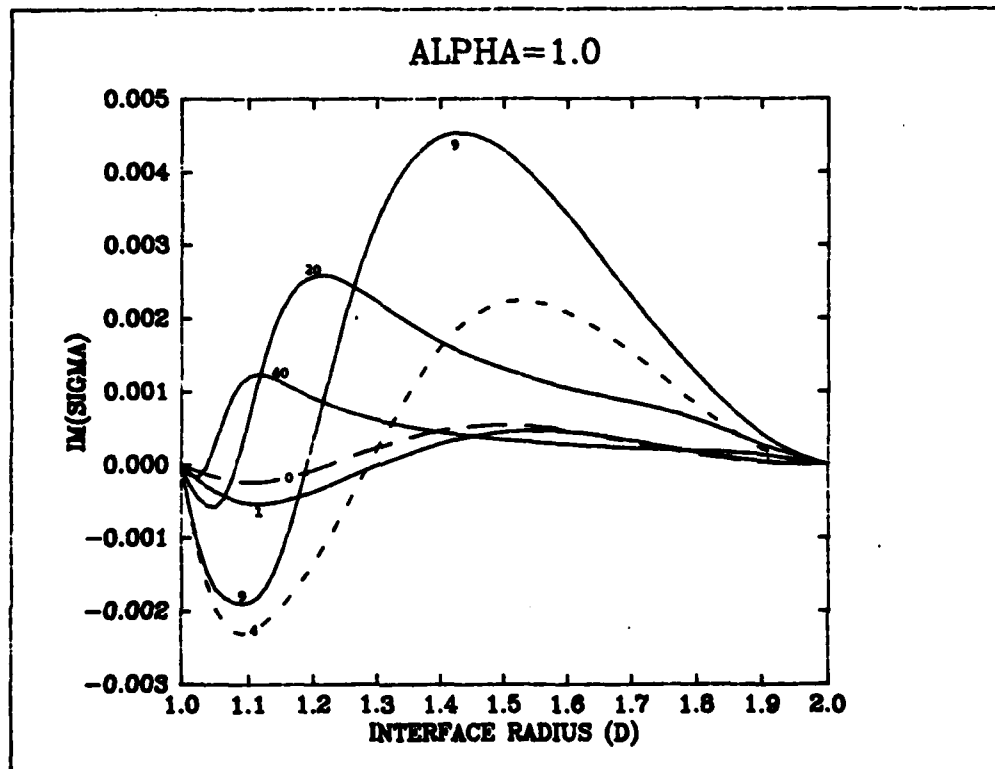
Figure 3

Growth rate curves when the less viscous fluid is inside. $T = 0$, $r = 1$, $m = 0.4$.
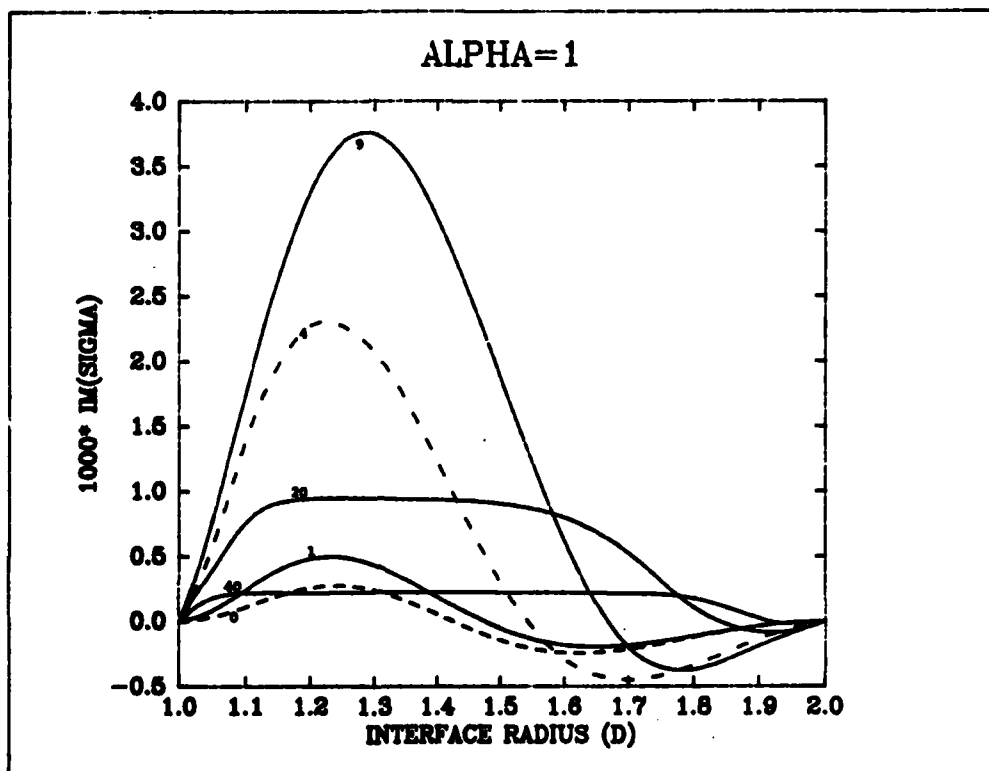
ALPHA=1

Figure 4

Growth rate curves when the less viscous fluid is outside. $T = 0$, $r = 1$, $m = 2$. The decay rates of stable disturbances are an order of magnitude smaller than in figure 3.
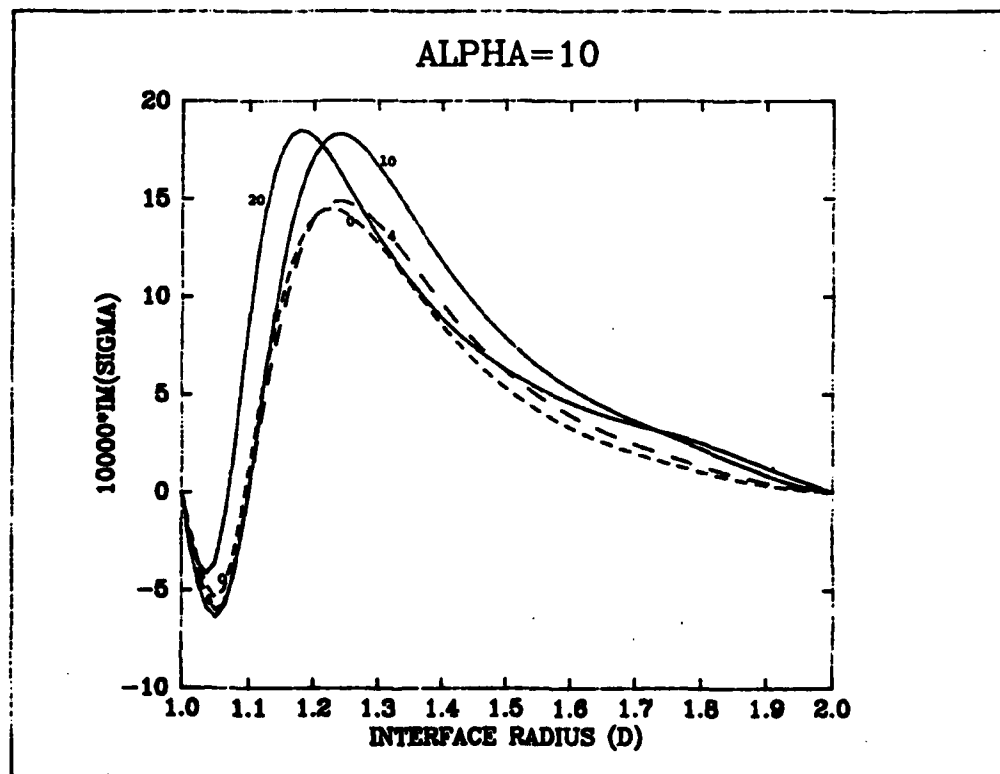
-15-

Figure 5

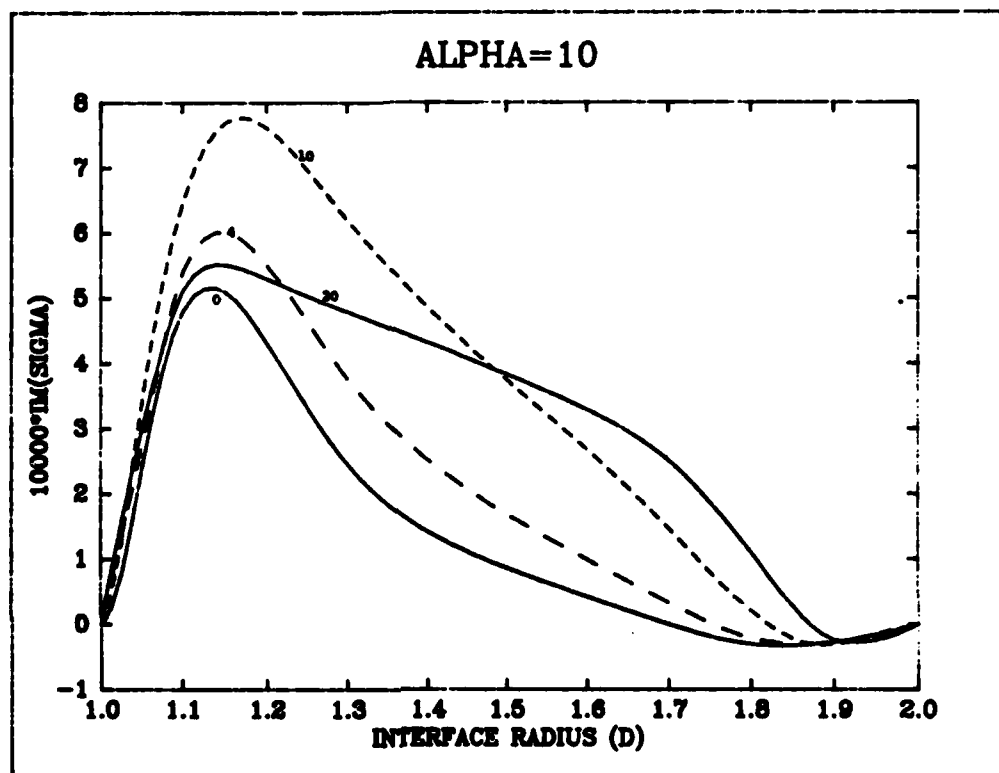Growth rates when less viscous fluid is inside. $T = 0$, $r = 1$, $m = 0.4$.

ALPHA=10

Figure 6

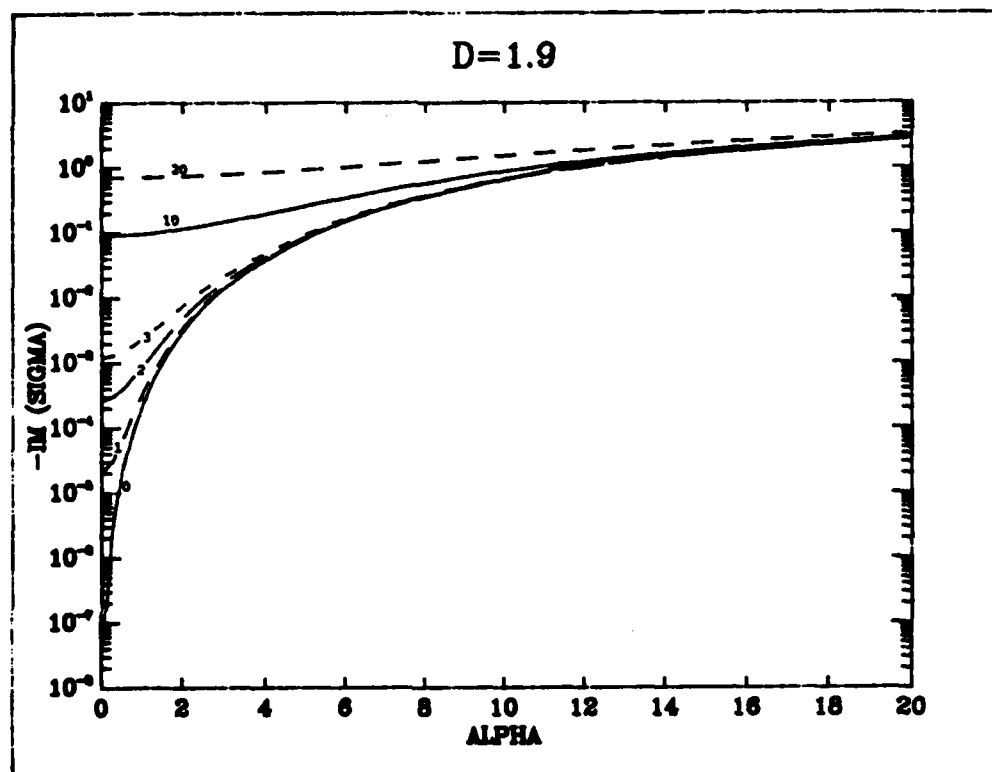Growth rates when less viscous fluid is outside. $T = 0$, $r = 1$, $m = 2$.

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D=1.9

Figure 7

Decay rates for the situation with the more dense fluid in the region between R_1 and

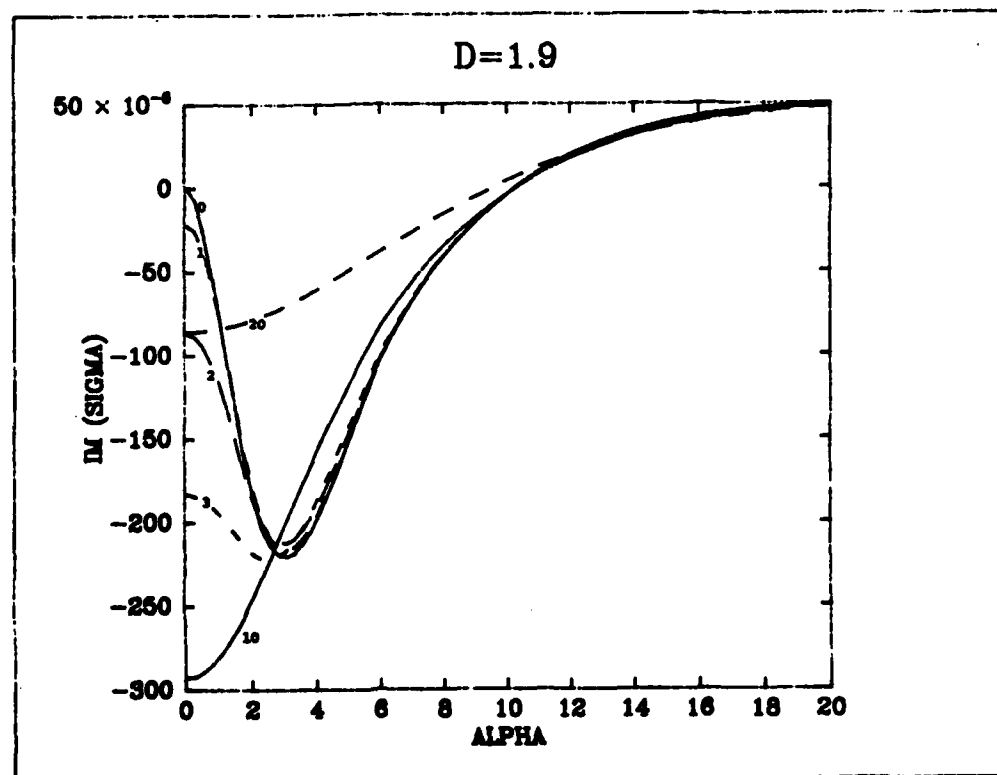$D = 1.9$. $T = 1$, $r = 2$, $m = 2$. All modes are stable.

Figure 8

Growth rates for flows shown in figure 7 when surface tension is zero. $T = 0$, $r = 2$, $m = 2$. All modes are unstable to sufficiently small (large α) disturbances.
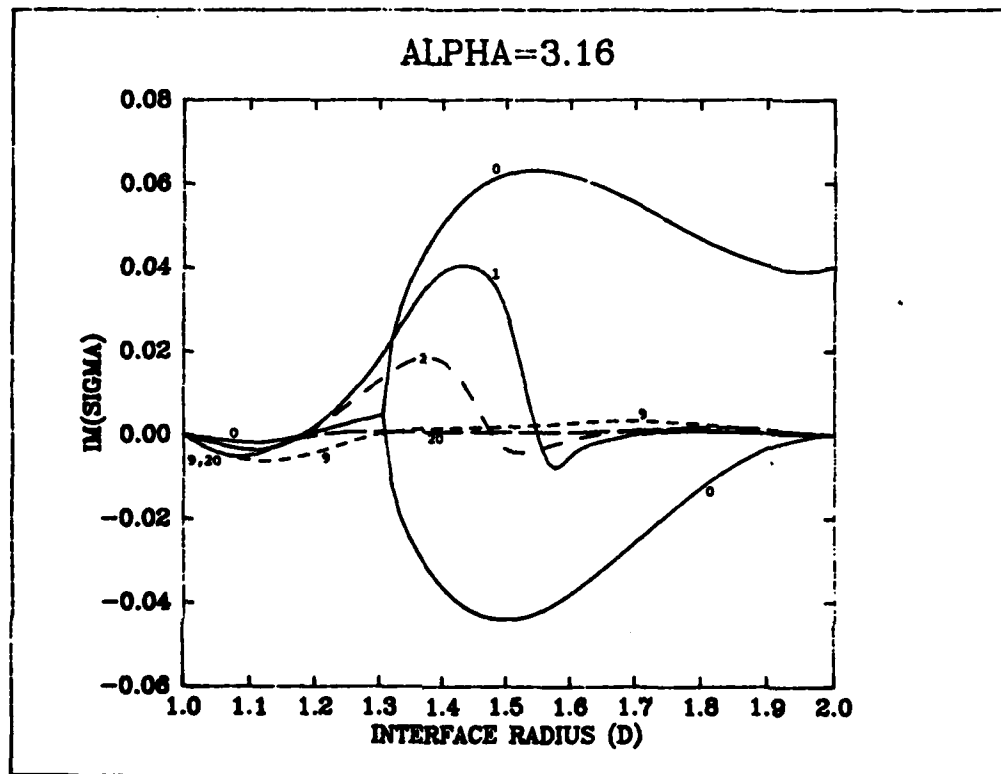
ALPHA=3.16

Figure 9

Growth rates for various modes when the parameters are close to critical for Taylor instability. $T = 0$, $r = 1$, $m = 0.9$. Thick fluid lies next to the outer cylinder. The addition of thin fluid at the inner cylinder surprisingly stabilizes the flow.
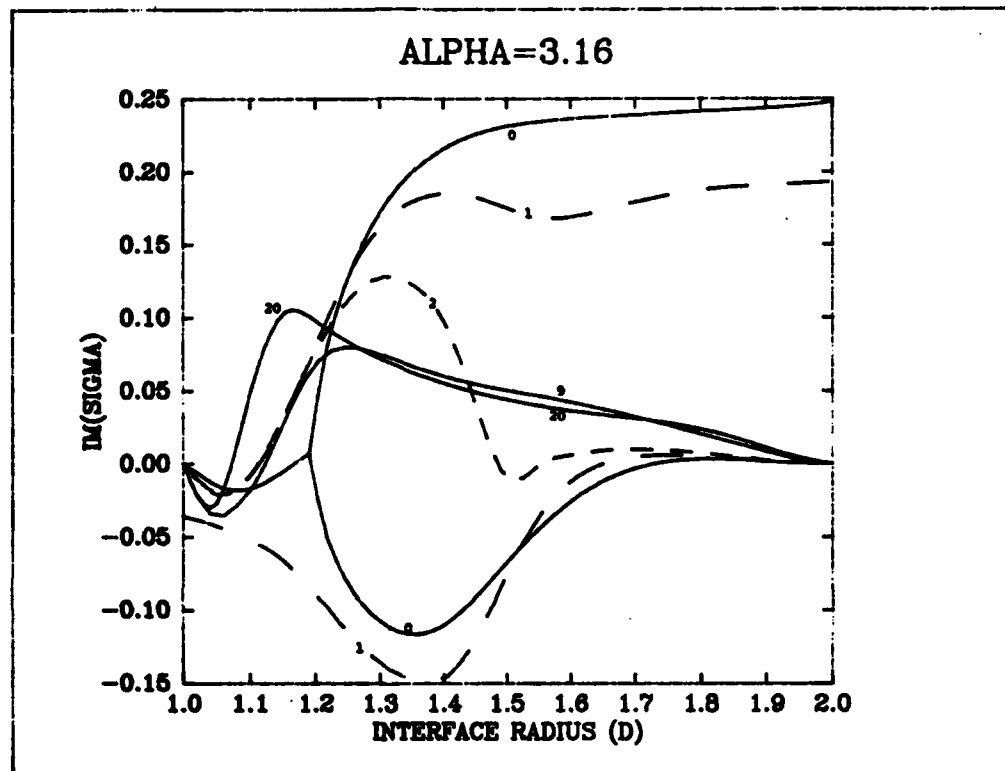
-20-

ALPHA=3.16

Figure 10

Growth rates with thick fluid outside. $T = 0$, $r = 1$, $m = 0.4$. Note the stabilization by a 'lubrication' effect associated with putting a thin layer of less viscous fluid on the inner cylinder. At $D = R_1$, mode 0 is at a critical Taylor number and mode 1 is slightly below.
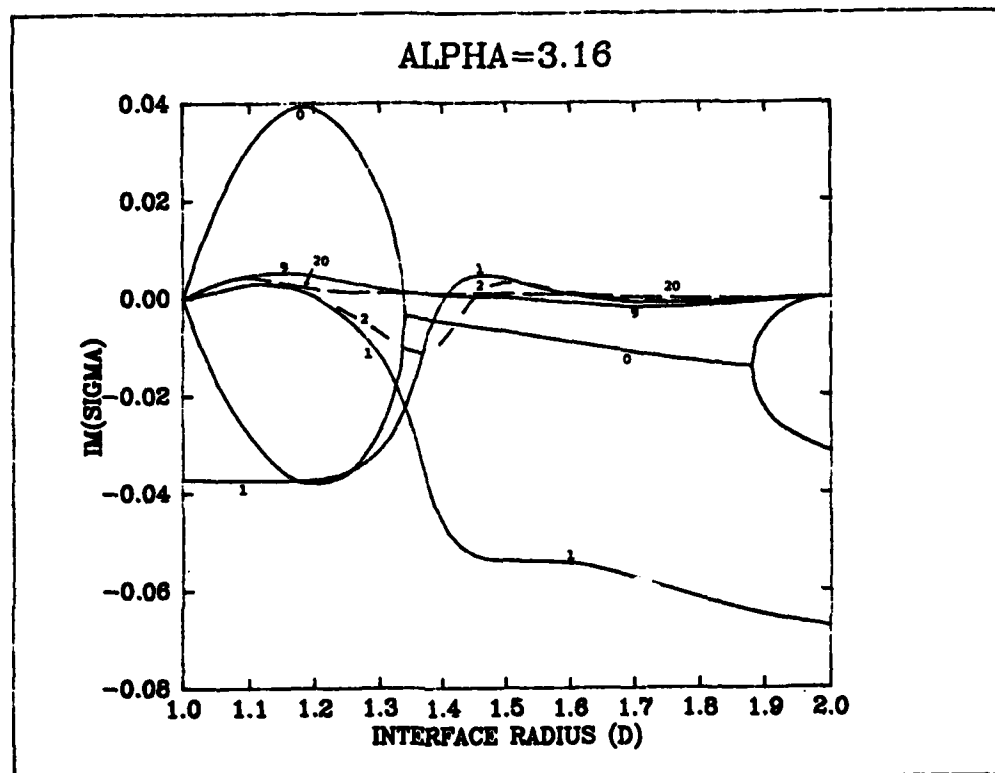
-21-

ALPHA=3.16

Figure 11

Growth rates when the less viscous fluid lies next to the outer cylinder. $T = 0$, $r = 1$, $m = 1.08$. The amount of less viscous fluid decreases as D increases but various modes are unstable except when most of the gap is occupied by the more viscous fluid. At $D = R_1$, mode 0 is at a critical Taylor number and mode 1 is slightly below.

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ALPHA=3.16

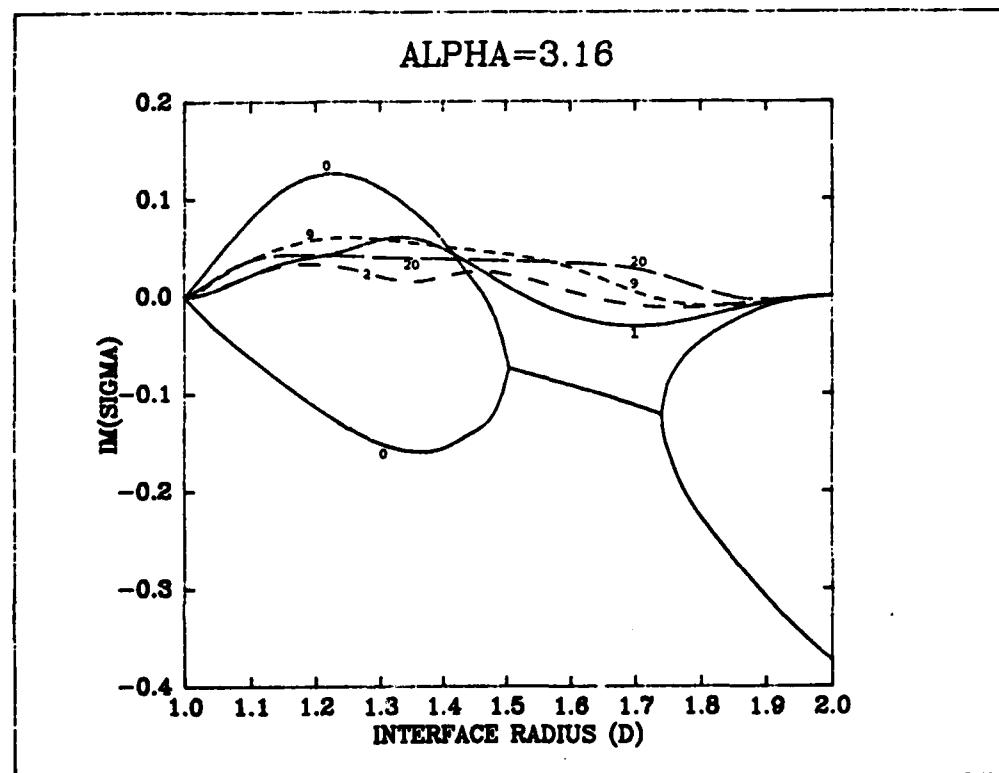**Figure 12**

Growth rates when the thin fluid is outside. $T = 0$, $r = 1$, $m = 2$. The flow is stable when the thick fluid fills the annulus ($D = R_2 = 2$) and is at criticality when thin fluid fills it ($D = R_1 = 1$). However, the addition of thick fluid at the inner cylinder can actually destabilize the flow unless the thick fluid occupies most of the annulus.

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We consider the flow of two immiscible fluids lying between concentric cylinders when the outer cylinder is fixed and the inner one rotates. The interface is assumed to be concentric with the cylinders and gravitational effects are neglected. We present a numerical study of the effect of different viscosities, different densities and surface tension on the linear stability

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20. ABSTRACT - cont'd.

of the Couette flow. Our results indicate that with surface tension, a thin layer of the less viscous fluid next to either cylinder is linearly stable and that it is possible to have stability with the less dense fluid lying outside. The stable configuration with the less viscous fluid next to the inner cylinder is more stable than the one with the less viscous fluid next to the outer cylinder. The onset of Taylor instability for one-fluid flow may be delayed by the addition of a thin layer of less viscous fluid on the inner wall and promoted by a layer of more viscous fluid on the inner wall.

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