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A STUDY OF THE MOTION AND STABILITY OF TORPEDOES IN 3 DEGREES OF FREEDOM

E.H. van Leeuwen

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The dynamic equations of motion for a torpedo represented by a prolate spheroid of finesse ratio ℓ/d are studied in a non-rolling coordinate frame. This model considers the motion to be decoupled in the horizontal and vertical planes and with each plane spanned by three degrees of freedom. The resulting differential equations can be linearized and criteria for stability established.

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C O N T E N T S

A STUDY OF THE MOTION AND STABILITY OF TORPEDOES IN 3 DEGREES OF FREEDOM

1. INTRODUCTION

In this paper, the dynamical elements which are necessary for the analytical development of motion and stability of a torpedo, or slender body are presented.

From classical mechanics it is well known that the translational motion of a body is described by the equation of linear momentum while the rotational motion is given by the equation of angular momentum. In general the translational and rotational motion of a torpedo, or slender body are coupled and need to be considered simultaneously. The general treatment of this problem is complicated since the motion is described by six degrees of freedom. It is possible however to obtain criteria for stability for example, in the horizontal plane by considering just three degrees of freedom, that is, by considering the motion of the centre of gravity, parallel and orthogonal to the longitudinal axis of the torpedo and rotational motion about a horizontal transverse axis through the centre of gravity. A similar procedure can also be used to derive the equations of motion in the vertical plane. These equations are all highly non-linear. However, on linearizing and decoupling the motion, a set of ordinary differential equations for the pitch and yaw angles are found. The Laplace transform of these differential equations yield criteria for stability in the vertical and horizontal planes.

In the case of a torpedo moving through fluid at rest at infinity, the linearized equations can also yield details of the trajectory. Analysis of the torpedo trajectory will be discussed elsewhere [1].

2. EQUATIONS OF MOTION IN THE VERTICAL PLANE

The equations of motion discussed in this section describe the dynamical behaviour of a torpedo in the vertical plane, moving with rectilinear motion through a stationary ideal fluid of infinite extent. The question of dynamic stability and the derivation of the equations of motion will be addressed under the following assumptions:

(a) the thrust, velocity and weight of the torpedo are constant, with the thrust tangential to the trajectory,

(b) the shape of the torpedo is taken to be a prolate spheroid of finesse ratio ℓ/d ($\ell > d$), where ℓ is the length of the torpedo and d its diameter; and

(c) cavitation and free surface effects are assumed to be negligible.

The reason for assuming the torpedo is a prolate spheroid is to enable the mass accession (or added mass) due to the motion of the torpedo through the fluid to be incorporated into the dynamical equations of motion. Moreover when a body accelerates in a fluid, then not only does the torpedo accelerate but the fluid around the torpedo generally also moves. This means that the torpedo has to do work to increase the kinetic energy of both the torpedo and the fluid.

The nomenclature to be used throughout this paper in writing equations of motion is that the translational velocities will be denoted by components (u,v,w) and the rotational velocities by ($\dot{\gamma},\dot{\alpha},\dot{\phi}$). ($\dot{}$) denotes differentiation with respect to time.

A torpedo in rectilinear flight is subject to a number of hydrodynamic and hydrostatic forces as a consequence of the water pressure acting externally on the torpedo. A force-moment diagram is shown in Figure 2.1, for a torpedo on a steady course at set depth in the vertical plane. In this diagram T is the thrust generated by the propulsion unit in newtons (N) and is directed along the longitudinal axis with no resultant torque or misalignment. L and D are the resultant lift and drag forces in newtons respectively, acting parallel and orthogonal to the velocity V (m/s) in the (x,z) plane. B is the buoyancy (N) and represents the nett force of the hydrostatic pressures. W is the weight (N), and M_o the measured moment (N.m). $N\dot{\alpha}$ represents the damping moment which is proportional to the angular velocity and $F\dot{\alpha}$ is the transverse force associated with the damping moment in the vertical plane. α is the orientation angle in pitch, β is the attack angle in pitch and δ is the elevator angle in the vertical plane.

In order to find the equations of motion for a torpedo moving through an incompressible fluid, we firstly derive the kinetic energy of the fluid. When the velocity $\underline{u}$ of the fluid is irrotational, we may introduce a velocity potential ϕ^* satisfying:

$$\underline{u} = \nabla\phi^*,$$

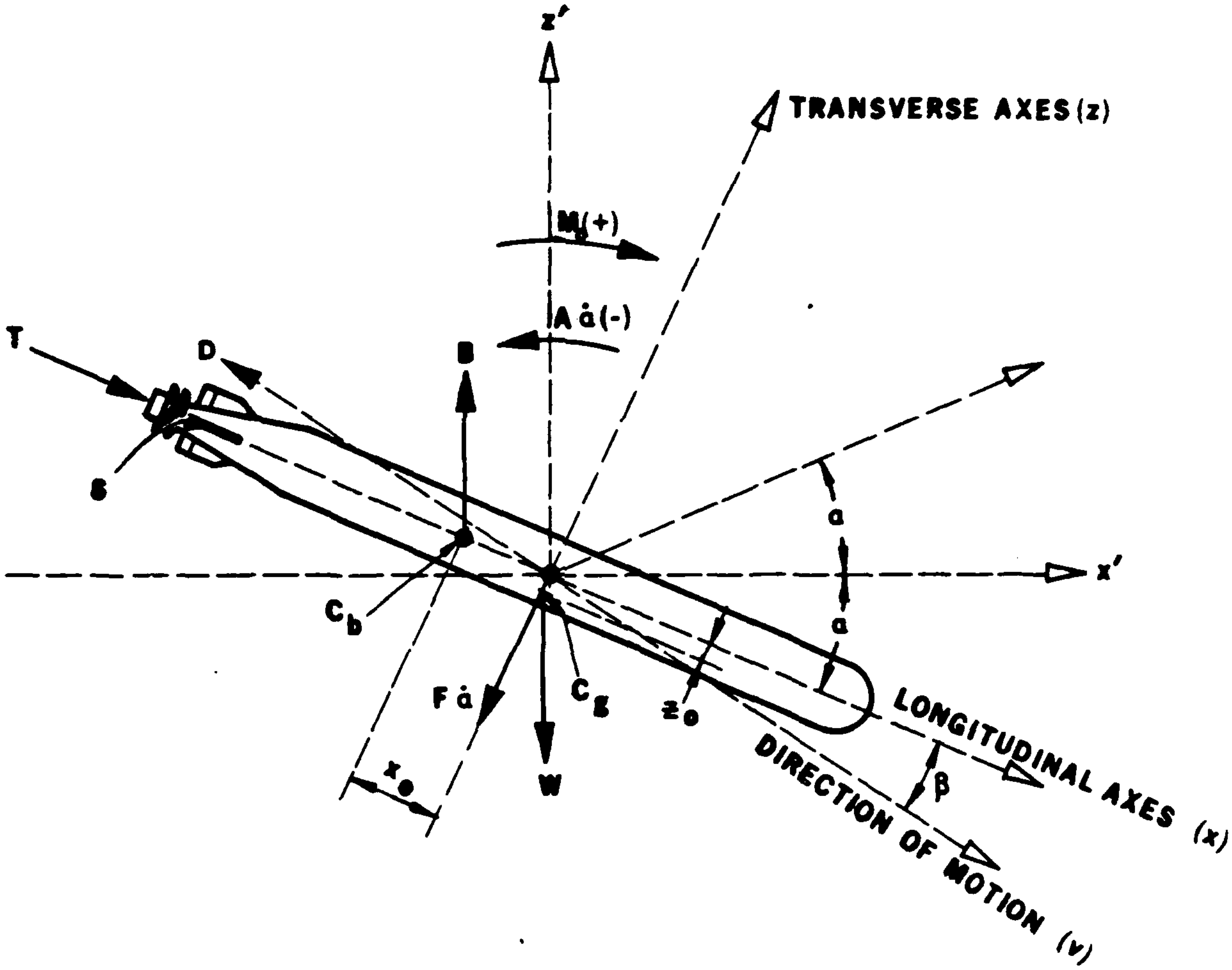

FIGURE 2-1 Hydrodynamic and hydrostatic forces acting or torpedo

(i) x,y,z Body Axes

(ii) x',y',z' Absolute Reference Frame

and because, for incompressible flow, ($\nabla . \underline{u} = 0$) it follows that

$$\nabla^2 \phi^* = 0.$$

For an irrotational flow characterized by the potential ϕ^*, the kinetic energy E_F in the fluid domain Λ is given by [2]:

$$E_F = \frac{\rho}{2} \int_{\Lambda} \phi^*_\mu \; \phi^*_\nu \, d\Lambda, \quad (\mu = 1,2,3)$$

where ϕ_μ denotes differentiation of ϕ with respect to μ . Equivalently:

$$E_F = - \frac{\rho}{2} \int_{S} \phi^* \partial_n \phi^* dS, \tag{2.1}$$

where $\underline{n}$ is a unit normal to the surface S of a slender body and ρ the density of the fluid. Let the translational velocity components of the torpedo or slender body be given by:

$$(u_1, u_2, u_3) = (u, v, w),$$

and the rotational motion by:

$$(u_4, u_5, u_6) = (\dot{\gamma}, \dot{\alpha}, \dot{\phi}).$$

For each component of the motion there exists a corresponding velocity potential for the fluid. That potential, per unit u_ν, will denoted by ϕ_ν. Hence

$$\phi^* = u_\nu \phi_\nu ; \; (\nu = 1, \ldots, 6) \quad , \tag{2.2}$$

i.e.,

$$\phi^* = \phi_1 u + \phi_2 v + \phi_3 w + \phi_4 \dot{\gamma} + \phi_5 \dot{\alpha} + \phi_6 \dot{\phi}. \tag{2.3}$$

Substitution of (2.2) into (2.1) yields:

$$E_F = \tfrac{1}{2} A_{\mu\nu} u_\mu u_\nu \quad ; \quad (\mu,\nu = 1,\ldots,6) \quad , \tag{2.4}$$

where the cartesian tensor $A_{\mu\nu}$ is defined by:

$$A_{\mu\nu} = -\rho \int_S \phi_\mu \partial_n \phi_\nu \, dS,$$

and satisfies the symmetry condition:

$$A_{[\mu\nu]} = 0 \; .$$

We call the elements of $A_{\mu\nu}$ the components of the added-mass tensor [2].

When there is only motion in the vertical plane (i.e., the (x,z) plane) the velocity potential is spanned by just three degrees of freedom and may be written in the absence of torpedo roll (non-rolling coordinate system) as:

$$\phi^* = \phi_1 u + \phi_3 w + \phi_5 \dot{\alpha} \; . \tag{2.5}$$

It follows from (2.4) that the quadratic form of the kinetic energy of the fluid in this coordinate system is:

$$E_F = \tfrac{1}{2}\{A_{11}u^2 + A_{33}w^2 + 2A_{35}w\dot{\alpha} + A_{55}\dot{\alpha}^2\}, \tag{2.6}$$

with all other components of $A_{\mu\nu}$ zero when ϕ^* is given by (2.5). This results from the symmetry of the torpedo ie., replacing u by -u should leave energy invariant; so $A_{13} = A_{15} = 0$.

The kinetic energy E_T of the torpedo can similarly be expressed as a quadratic form, i.e.,

$$E_T = \tfrac{1}{2} B_{\mu\nu} u_\mu u_\nu \; ; \quad B_{[\mu\nu]} = 0,$$

where

$$B_{\mu\nu} = \begin{bmatrix} M & 0 & 0 & 0 & Mz_o & -My_o \\ 0 & M & 0 & -Mz_o & 0 & Mx_o \\ 0 & 0 & M & My_o & -Mx_o & 0 \\ 0 & Mz_o & -My_o & I_{11} & -I_{12} & -I_{13} \\ -Mz_o & 0 & Mx_o & -I_{21} & I_{22} & -I_{23} \\ My_o & -Mx_o & 0 & -I_{31} & -I_{31} & I_{33} \end{bmatrix} \tag{2.7}$$

In (2.7), I_{ij} are the components of the moment of inertia tensor, x_o, y_o and z_o the coordinates of the centre of mass and M the mass of the torpedo. For motion in the vertical plane with the centre of gravity of the torpedo located directly below the longitudinal axis (a distance z_o) the kinetic energy of the torpedo is given by:

$$E_T = \tfrac{1}{2}\{Mu^2 + Mw^2 + I_{22}\dot{\alpha}^2 + 2Mz_o u\dot{\alpha}\}, \tag{2.8}$$

and hence the total energy by:

$$E = \tfrac{1}{2}\{(A_{11}+ M)u^2 + (A_{33}+ M)w^2 + (I_{22}+ A_{55})\dot{\alpha}^2 + 2A_{35}w\dot{\alpha} + 2Mz_o u\dot{\alpha}\}. \tag{2.9}$$

If $\underline{G}$ and $\underline{H}$ are the linear momentum and total angular momentum respectively then:

$$\underline{G} = \underline{e}_j \partial_{u_j} E = \{(M + A_{11})u + Mz_o\dot{\alpha}, 0, (M + A_{33})w + A_{35}\dot{\alpha}\} \ ; \ j=1,2,3, \tag{2.10}$$

and

$$\underline{H} = \underline{e}_k \partial_{u_k} E = \{0, (I_{22} + A_{55})\dot{\alpha} + Mz_o u + A_{35}w, 0\} \ ; \ k=4,5,6 \ . \tag{2.11}$$

Furthermore if $\underline{F}^*$ and $\underline{L}^*$ are the resultant external force and momentum respectively of both the torpedo and the fluid and if an inertial reference frame is fixed to the torpedo moving with velocity $\underline{u} = (u,o,w)$ and angular velocity $\underline{\omega} = (o,\dot{\alpha},o)$ then:

$$\underline{F}^* = \dot{\underline{G}} + \underline{\omega} X \underline{G}, \tag{2.12a}$$

and

$$\underline{L}^* = \dot{\underline{H}} + \underline{\omega}X\underline{H} + \underline{u}X\underline{G} \quad , \tag{2.12b}$$

where X is the vector product. More explicitly:

$$\underline{F}^*=\{(M + A_{11})\dot{u} + Mz_o\ddot{\alpha} + (M + A_{33})w\dot{\alpha} + A_{35}\ \dot{\alpha}^2, 0, (M + A_{33})\dot{w} + A_{35}\ \ddot{\alpha} -(M+A_{11})u\dot{\alpha} - Mz_o\dot{\alpha}^2\} \tag{2.13}$$

and

$$\underline{L}^* = \{0, (I_{22} + A_{55})\ddot{\alpha} + Mz_o\dot{u} + A_{35}\dot{w} - uw(A_{33} - A_{11}) - A_{35}\dot{u}\alpha + Mz_o w\dot{\alpha},\ 0\}. \tag{2.14}$$

Resolution of the forces in Figure 2.1 acting parallel and orthogonal to the direction of motion yields:

$$\underline{F}^* = \{T - D\cos\beta + (B-W)\sin\alpha - L\sin\beta, 0, -D\sin\beta - (B-W)\cos\alpha - L\cos\beta + F\dot{\alpha}\}. \tag{2.15}$$

Similarly, the resolution of torques acting about the centre of gravity yields:

$$\underline{L}^* = \{0, M_o - A\dot{\alpha} + Bx_o\cos\alpha - Wz_o\sin\alpha, 0\}. \tag{2.16}$$

Equating (2.13) and (2.15), (2.14) and (2.16) the equations governing the motion of the torpedo in the (x,z) plane can be expressed as:

$$T - D\cos\beta + (B-W)\sin\alpha - L\sin\beta = (M + A_{11})\dot{u} + Mz_o\ddot{\alpha} + (M + A_{33})w\dot{\alpha}, \tag{2.17a}$$

$$-D\sin\beta - (B-W)\cos\alpha - L\cos\beta + F\dot{\alpha} = (M + A_{33})\dot{w} + A_{35}\ddot{\alpha} - (M + A_{11})u\dot{\alpha} - Mz_o\dot{\alpha}^2, \tag{2.17b}$$

$$M_o - A\dot{\alpha} + Bx_o\cos\alpha - Wz_o\sin\alpha = (I_{22} + A_{55})\ddot{\alpha} + Mz_o\dot{u} + A_{35}\dot{w} - uw(A_{11} - A_{33}) - A_{35}u\dot{\alpha} + Mz_o w\dot{\alpha}, \tag{2.17c}$$

where (u,o,w) = (Vcosβ,0,Vsinβ), and V is defined in Figure 2.1.

3. LINEARIZED EQUATIONS OF MOTION IN THE VERTICAL PLANE

In general, analytic solutions to (2.17) are not possible, however upon linearizing these equations, solutions can be found. To do this we assume that the hydrodynamic forces and moments are approximately linear in attack angle and turning rate.

The forces and moments appearing in (2.17) are both functions of β and δ which can be taken to be independent and small (less than 10°). Expanding L and M_o in a Maclaurin series and neglecting all but first order terms yields:

$$S(\beta,\delta) \simeq S(0,0) + S_\delta \delta + S_\beta \beta,$$

where S_δ for example denotes partial differentiation of S with respect to δ. If the torpedo is highly symmetrical we can assume without loss of generality that $S(0,0) = 0$. Furthermore if α is also small then we have:

$$\cos\alpha = \cos\beta \simeq 1,$$

$$\sin\alpha \simeq \alpha, \quad \sin\beta \simeq \beta \ .$$

Incorporation of the above approximations into the equations of motion yield the following set of linear equations

$$(T - D) + (B - W)\alpha = Mz_o\ddot{\alpha} \ , \tag{3.1a}$$

$$-(D + L_\beta)\beta - A_{35}\ddot{\alpha} - (B - W) - L_\delta\delta + F\dot{\alpha} = -M_T V\dot{\alpha} + M_L V\dot{\beta}, \tag{3.1b}$$

$$M_\delta\delta + M_1\beta + M_2\dot{\alpha} + Bx_o - Wz_o\alpha - A_{35} V\dot{\beta} = \Sigma\ddot{\alpha} \ , \tag{3.1c}$$

where

$$m_T := M + A_{11}, \tag{3.2a}$$

$$m_L := M + A_{33}, \tag{3.2b}$$

$$M_1 := M_\beta - V^2(A_{11} - A_{33}), \tag{3.2c}$$

$$M_2 := -(A - VA_{35}), \tag{3.2d}$$

$$\Sigma := I_{22} + A_{55}. \tag{3.2e}$$

In terms of the coefficients of inertia for a prolate spheroid [3,4] (3.2 a,b,c,e) can be expressed as:

$$m_T = M + k_1 M_f,$$

$$m_L = M + k_2 M_f,$$

$$M_1 = M_\beta - V^2 M_f (k_1 - k_2),$$

$$\Sigma = I_{22} + k' M_f,$$

where M_f is the displaced fluid mass and k_1, k_2 and k' the coefficients of inertia:

$$k_1 = \alpha_o (2-\alpha_o)^{-1},$$

$$k_2 = \beta_o (2-\beta_o)^{-1},$$

$$k' = (a^2-b^2)^2(\beta_o-\alpha_o)\ \{(a^2+b^2)[2(a^2-b^2) - (a^2+b^2)(\beta_o-\alpha_o)]\}^{-1},$$

where a is the semi-major axis and b the semi-minor axis of the prolate spheroid. Furthermore:

$$\alpha_o = \frac{1-e^2}{e^3}\left(\ln\frac{(1 + e)}{(1 - e)} - 2e\right),$$

$$\beta_o = \frac{1-e^2}{e^3}\left(\frac{e}{1 - e^3} - \tfrac{1}{2}\ln\frac{(1 + e)}{(1 - e)}\right),$$

where e is the eccentricity.

If ρ is the density of the fluid, and A, ℓ and V the cross-sectional area, the length and velocity of the torpedo respectively, then (3.1b) and (3.1c) can be expressed in terms of the hydrodynamic coefficients which are defined below on dividing (3.1b) by:

$$A_1 := \tfrac{1}{2}\rho A V^2, \tag{3.3a}$$

and (3.1c) by

$$A_2 := \tfrac{1}{2}\rho A V^2 \ell, \tag{3.3b}$$

together with a change in the independent time variable t ie., the time variable t is changed to the dimensionless arc length parameter ξ,proportional to t and defined by:

$$\xi := t t_c^{-1},$$

where t_c is called the *characteristic time*. On substitution of (3.3a) and (3.3b) into (3.1b) and (3.1c) respectively it becomes apparent that t_c is best chosen as ℓV^{-1}, where ℓ is the length of the torpedo. This yields:

$$- A_{35}^{*}\alpha'' + (C_{F_t} + m_L^{*})\alpha' - m_T^{*}\beta' - (C_D + C_{L_\beta})\beta + C_{L_\delta}\delta - B^{*} = 0, \tag{3.4a}$$

and

$$\Sigma^{*}\alpha'' - C_{M_d}\alpha' + A_{35}^{*}\beta' - C_M\beta - C_{M_\delta}\delta - B^{**} + W^{**}\alpha Z_o = 0, \tag{3.4b}$$

where ()' denotes differention with respect to ξ. In the vertical plane the hydrodynamic coefficients, corrected for the effects of entrained water are:

$C_D := D A_1^{-1}$,	is the drag coefficient,
$C_{L_\beta} := L_\beta A_1^{-1}$,	the lift coefficient,
$C_{F_t} := F(\tfrac{1}{2}\rho A V \ell)^{-1}$,	the transverse force coefficient,
$C_M := M_1 A_2^{-1}$,	the moment coefficient
$C_{M_d} := M_2(\tfrac{1}{2}\rho A V \ell^2)^{-1}$,	the damping moment coefficient
$C_{L_\delta} := L_\delta A_1^{-1}$,	the elevator lift coefficient,
$C_{M_\delta} := M_\delta A_2^{-1}$,	the elevator moment coefficient.

Other terms appearing in (3.4) are defined as follows:

$$A_{35}^* := A_{35}(\tfrac{1}{2}\rho A \ell^2)^{-1}; \quad m_L^* = M_L(\tfrac{1}{2}\rho A\ell)^{-1}; \quad m_T^* = m_T(\tfrac{1}{2}\rho A\ell)^{-1};$$

$$B^* := (B-W)A_1^{-1}\ ; \quad \Sigma^* := \Sigma(\tfrac{1}{2}\rho A\ell^3)^{-1}; \quad B^{**} := x_o B A_2^{-1}\ ; \quad W^{**} := W A_2^{-1}.$$

4. EQUATIONS OF MOTION IN THE HORIZONTAL PLANE

The equations describing the motion in the horizontal plane can be derived following a similar procedure to the one given for deriving the equations in the vertical plane. A number of minor changes are all that are necessary so not all the details are given here. For example, in the horizontal plane the gravity terms, weight and buoyacy can be equated to zero in both the force and moment equation.

Take δ to be the rudder angle, ϕ the orientation angle in the horizontal plane (i.e. yaw angle) and Ψ the angle of attack in yaw. Furthermore $\underline{\omega} = (0,0,\dot{\phi})$ and $\underline{u} = (u,v,o)$. With no displacement of the centre of gravity in the (x,y) plane, the total kinetic energy of both the fluid and torpedo can be expressed as:

$$E = \tfrac{1}{2}\{(M + A_{11})u^2 + (M + A_{22})v^2 + (A_{66} + I_{33})\dot{\phi}^2 + 2A_{26}v\dot{\phi}\}. \qquad (4.1)$$

From (4.1) the linear momentum and total angular momentum have the form:

$$\underline{G} = \{(M + A_{11})u, (M + A_{22})v + A_{26}\dot{\phi},\ 0\},$$

and

$$\underline{H} = \{0,0,(A_{66} + I_{33})\dot{\phi} + A_{26}v\},$$

respectively. From (2.12) it further follows that the external force $\underline{F}^*$ and momentum $\underline{L}^*$ are given by:

$$\underline{F}^* = \{(M + A_{11})\dot{u} - (M + A_{22})v\dot{\phi} - A_{26}\dot{\phi}^2,\ (M + A_{22})\dot{v} + A_{26}\ddot{\phi} + (M + A_{11})u\dot{\phi},\ 0\}, \qquad (4.2a$$

and

$$\underline{L}^* = \{0,0,(A_{66} + I_{33})\ \ddot{\phi} + A_{26}(\dot{v} + \ddot{\phi}u) + uv(A_{22} - A_{11})\},\qquad (4.2b)$$

respectively. Expanding the left side of (4.2) in a Maclaurin series, neglecting all but first order terms gives:

$$\underline{F}^* = (F^1(u,\dot{\phi},\delta),\ F^2(v,\dot{\phi},\delta),0) \simeq (F^1_u u + F^1_{\dot{\phi}}\dot{\phi} + F^1_\delta \delta,\ F^2_v v + F^2_{\dot{\phi}}\dot{\phi} + F^2_\delta \delta,0),\qquad (4.3a)$$

and

$$\underline{L}^* = (0,0,L^3(v,\dot{\phi},\delta)) \simeq (0,0,L^3_v\ v + L^3_{\dot{\phi}}\dot{\phi} + L^3_\delta \delta)\ .\qquad (4.3b)$$

Equating (4.2) and (4.3) yields:

$$F^1_u u + F^1_{\dot{\phi}}\dot{\psi} + F^1_\delta \delta \simeq m_L\dot{u} - m_T v\dot{\phi} - A_{26}\dot{\phi}^2,\qquad (4.4a)$$

$$F^2_v v + F^2_{\dot{\phi}}\dot{\phi} + F^2_\delta \delta \simeq m_T\dot{v} + A_{26}\ddot{\phi} + m_L u\dot{\phi}\ ,\qquad (4.4b)$$

$$L^3_v v + L^3_{\dot{\phi}}\dot{\phi} + L^3_\delta \delta \simeq \Theta\ddot{\phi} + A_{26}(\dot{v} + \dot{\phi}u) + uv(A_{22} - A_{11})\ ,\qquad (4.4c)$$

where

$$m_L = M + A_{11}\ ,$$

$$m_T = M + A_{22}\ ,$$

$$\Theta = I_{33} + A_{66}\ ,$$

and

$$(u,v,o) = (V\cos\Psi,- V\sin\Psi,0),\quad V = \text{constant}.$$

5. LINEARIZED EQUATIONS OF MOTION - HORIZONTAL PLANE

Following the method of Section 3, equations (4.4b) and (4.4c) can be expressed as:

$$-C_{L_\Psi}\Psi + (C_{F_t} - m_L^*)\phi' + C_{L_\delta}\delta - A_{26}^*\phi'' + m_T^*\Psi' \simeq 0, \tag{5.1a}$$

and

$$C_M\Psi + C_{M_d}\phi' + A_{26}^*\Psi' + C_{M_\delta}\delta - \Theta^*\phi'' \simeq 0. \tag{5.1b}$$

In the (x,y) plane the hydrodynamic coefficients corrected for the effects of entrained water are defined by:

$C_{L_\Psi} := \dfrac{F_v^2\, V}{\tfrac{1}{2}\rho A V^2}$,	the lift coefficient,
$C_{F_t} := F_{\dot\phi}^2\, (\tfrac{1}{2}\rho A V \ell)^{-1}$,	transverse force coefficient,
$C_{L_\delta} := F_\delta^2\, A_1^{-1}$,	rudder lift coefficient,
$C_{M_\delta} := L_\delta^3 A_2^{-1}$,	rudder moment coefficient,
$C_{M_d} := L_2 (\tfrac{1}{2}\rho A V \ell^2)^{-1}$,	damping moment coefficient,
$C_M := L_1 (\tfrac{1}{2}\rho A V^2 \ell)^{-1}$,	moment coefficient,

where $L_1 := VL_v^3 - V^2(A_{22} - A_{11})$, $L_2 := L_{\dot\phi}^3 - A_{26}V$, $A_{26}^* := A_{26}(\tfrac{1}{2}\rho A \ell^2)^{-1}$ and $\Theta^* := \Theta(\tfrac{1}{2}\rho A \ell^3)^{-1}$. The coefficients m_T^* and m_L^* have the same algebraic form as given in Section 3.

6. STABILITY CRITERIA FOR MOTION IN THE HORIZONTAL AND VERTICAL PLANES

The problem of determining stability of a linear system is one of finding roots of the characteristic equation, found on taking the Laplace transform (L-transform) of the equation governing the motion. In general if the characteristic equation of the transformed equation is given by [5]:

$$\alpha_o s^n + \alpha_1 s^{n-1} + \ldots + \alpha_{n-1} s + \alpha_n , \tag{6.1}$$

then the criteria for stability are that the coefficients of the characteristic function (6.1) have roots that lie in the left half of the s-plane. This is both a necessary and sufficient condition and is equivalent to saying all the determinants:

$$D_1 = \alpha_1, \quad D_2 = \begin{vmatrix} \alpha_1 & \alpha_o \\ \alpha_3 & \alpha_2 \end{vmatrix}, \quad D_3 = \begin{vmatrix} \alpha_1 & \alpha_o & o \\ \alpha_3 & \alpha_2 & \alpha_1 \\ \alpha_5 & \alpha_4 & \alpha_3 \end{vmatrix}, \ldots,$$

must be positive, i.e.,

$$D_1 > 0, \quad D_2 > 0, \quad D_3 > 0, \ldots .$$

If any coefficient of any power of s is either negative or zero, then there is at least one root in the right half of the s-plane or on the imaginary axis of the s-plane in which case the system is unstable. A more complete description of the motion is unnecessary since all that is required is a knowledge of whether or not the system is stable, i.e., whether a response to a perturbation remains bounded or grows as t (or ξ) $\to \infty$.

(a) <u>Conditions for stable motion in the horizontal plane</u>

On differentiation of (5.1) it is not difficult to show that the decoupled equation for yaw angle ϕ in the ξ domain can be expressed as:

$$\alpha_4\phi''' + \alpha_3\phi'' + \alpha_2\phi' + \alpha_1\delta' + \alpha_o\delta = 0, \tag{6.2}$$

where

$$\begin{aligned}
\alpha_4 &:= m_T^* \Theta^* - A_{26}^{*2}, \\
\alpha_3 &:= -m_T^* C_{M_d} + A_{26}^*(C_{F_t} - m_L^*) - C_{L_\psi}\Theta^* - C_M A_{26}^*, \\
\alpha_2 &:= -(m_L^* - C_{F_t})C_M + C_{L_\psi} C_{M_d}, \\
\alpha_1 &:= A_{26}^* C_{L_\delta} - m_T^* C_{M_\delta}, \\
\alpha_0 &:= C_M C_{L_\delta} + C_{L_\psi} C_{M_\delta}.
\end{aligned} \tag{6.3}$$

Without loss of generality we assume that the desired heading is $\phi = 0$, then (6.2) represents the error in heading. For stability it is necessary that $\phi \to 0$.

The L-transform of (6.2) yields:

$$\bar{\phi}(\alpha_4 s^3 + \alpha_3 s^2 + \alpha_2 s) + \bar{\delta}(\alpha_1 s + \alpha_o) = (s^2\phi_o\alpha_4 + s(\alpha_4\phi_o' + \alpha_3\phi_o) + \alpha_4\phi_o''$$

$$+ \alpha_3\phi_o' + \alpha_2\phi_o + \alpha_1\delta_o) \ , \qquad (6.4)$$

where s is the transform variable and ϕ_o, ϕ_o', ϕ_o'' δ_o, initial conditions.

The stability of the torpedo in the horizontal plane, can be inferred provided the rudder response δ is known. For example if c_1 is a real constant ie., $\delta = c_1\phi$, then (6.4) becomes

$$\bar{\phi}(s) = \frac{s^2\phi_o\alpha_4 + s(\alpha_4\phi_o' + \alpha_3\phi_o) + \alpha_4\phi_o'' + \alpha_3\phi_o' + \alpha_2\phi_o + \alpha_1\delta_o}{\alpha_4 s^3 + \alpha_3 s^2 + (\alpha_2 + c_1\alpha_1)s + \alpha_o c_1} = : \frac{P(s)}{Q(s)} \ .$$

For stability the degree of the polynomial $Q(s)$ must be greater than $P(s)$ and the roots of $Q(s)$ in the complex s-plane should lie in the left half. Furthermore the determinants:

$$D_1 = \alpha_3 \ ; \qquad D_2 = \begin{vmatrix} \alpha_3 & \alpha_4 \\ \alpha_o c_1 & \alpha_2 + c_1\alpha_1 \end{vmatrix} \ ; \qquad D_3 = D_1 D_2 \ ,$$

should all be positive.

If the torpedo has neutral rudder response, that is $\delta = 0$ and further if the origin of the coordinates is at the centre of the prolate spheroid, (i.e., $A_{26} = 0$ as well as $c_1 = 0$) then the above determinants reduce to:

$$\alpha_3 > 0 \quad \text{and} \quad \alpha_2 > 0 \ . \qquad (6.5)$$

For small angles of attack the stability criteria for a torpedo without controls can be expressed in terms of the hydrodynamic coefficients as:

$$C_{L_\gamma}\theta^* + m_T^* C_{M_d} < 0 \quad , \qquad (6.6a)$$

and

$$C_{M_d} C_{L_\psi} - (m_L^* - C_{F_t}) C_M > 0 \quad , \tag{6.6b}$$

The general solution of (6.2) for neutral rudder response and $A_{26}^* = 0$ can be expressed as:

$$\phi(\zeta) = Ae^{a_+\zeta} + Be^{a_-\zeta} \quad ,$$

where

$$a_\pm = -\frac{\alpha_3 \pm (\alpha_3^2 - 4\alpha_2\alpha_4)^{1/2}}{2\alpha_4} \quad ,$$

are the roots of the auxiliary equation of (6.2). For dynamic stability $\phi(\zeta)\to 0$, which means the roots $a_\pm$ must have negative real parts, as well as $\alpha_3^2 < 4\alpha_2\alpha_4$. This will be the case only if (6.5) is satisfied, since α_4 is always positive.

A second case of interest refers to the motion of a torpedo with zero attack angle in yaw ($\psi=0$) and no controls. In such cases the above stability analysis yields $C_{M_d} > 0$ for the transverse damping moment coefficient.

(b) <u>Condition for stable motion in the vertical plane</u>

Following a method similar to the one described for stability in the vertical plane, on decoupling (3.4) the equation for the pitch angle α is:

$$\beta_6\alpha''' + \beta_5\alpha'' + \beta_4\alpha' + \beta_3\alpha + \beta_2\delta' + \beta_1\delta + \beta_o = 0 \quad , \tag{6.7}$$

where

$$\beta_6 := A_{35}^{*2} + m_T^*\Sigma^* \quad ,$$

$$\beta_5 := A_{35}^*(C_{F_t} + m_L^*) - m_T^* C_{M_d} - A_{35}^* C_M + \Sigma^*(C_D + C_{L_\beta}) \quad ,$$

$$\beta_4 := m_T^* W^{**} z_o - C_M(C_{F_t} + m_L^*) - C_{M_d}(C_D + C_{L_\beta}) \quad ,$$

$$\beta_3 := W^{**} z_o (C_D + C_{L_\beta})$$

$$\beta_2 := A^*_{35} C_{L_\delta} - m^*_T C_{M_\delta} ,$$

$$\beta_1 := -C_M C_{L_\delta} - C_{M_\delta}(C_D + C_{L_\beta}) ,$$

$$\beta_0 := B^* C_M - B^{**}(C_D + C_{L_\beta}) .$$

As in case (a), stability criteria can be found if the response δ is known. For example if $\delta = c_2\alpha$, the L-transform of (6.7) yields:

$$\bar{\alpha} = \frac{s^3\alpha_o\beta_6 + s^2(\alpha_o\beta_5 + \alpha'_o\beta_6) + s(\alpha''_o\beta_6 + \alpha'_o\beta_5 + \alpha_o\beta_4 + c_2\alpha_o\beta_2) - \beta_o}{s\{\beta_6 s^3 + \beta_5 s^2 + s(\beta_4 + c_2\beta_2) + \beta_3 + \beta_1 c_2\}} ,$$

and a necessary condition for stability is that all the determinants

$$D_1 = \beta_5 ,$$

$$D_2 = \begin{vmatrix} \beta_5 & \beta_6 \\ \beta_3 + c_2\beta_1 & \beta_4 + c_2\beta_2 \end{vmatrix}$$

$$D_3 = D_1 D_2 ,$$

be positive.

7. DISCUSSION

The model described above considers some basic aspects of torpedo motion and stability in both the vertical and horizontal planes. The question of accessibility of the stability criteria depends on a knowledge of the hydrodynamic coefficients for a particular torpedo body. In the foregoing discussion no explicit studies were made of torpedo stability due to lack of data relating to hydrodynamic coefficients of the torpedo to which this report was directed. Therefore this study should be considered as a first step only in developing a knowledge of motion for submerged bodies.

8. ACKNOWLEDGEMENTS

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