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INST OF TECH CAMBRIDGE LAB FOR COMPUTER SCIENCE
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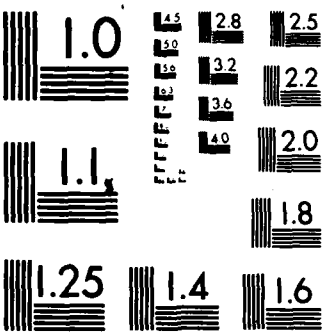

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Professor Shunro Watanabe of Japan has been visiting the group since March 1982. In the past year he has been able, with the support of the group, to develop a major new program for solving in closed form a wide variety of ordinary differential equations.

The MACSYMA system uses two large packages for solving ODEs. The first package was written by Jeffrey Golden of the group. It solves equations by using a few well known techniques (e.g., separation of variables). It is quite weak in dealing with second order equations, such as hypergeometric functions. The second package, due to Ed Lafferty of the MITRE Corporation, incorporates many more techniques, but is still not a very general solver of ODEs.

Professor Watanabe's approach is to convert most second order ODEs into (CONTINUED)

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ITEM #19, ABSTRACT, CONTINUED: instances of the so-called P-function. These functions, originally due to Riemann, have been extensively studied by the Japanese mathematician Fukuhara (also spelled Hukuhara). For the past year, Watanabe has implemented a package that transforms a large class of equations into P-functions. He has used Kame's table as a test bed.

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PROGRESS REPORT 1982/83

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Professor Watanabe's approach is to convert most second order ODEs into instances of the so-called P-function. These functions, originally due to Riemann, have been extensively studied by the Japanese mathematician Fukuhara (also spelled Hukuhara). For the past year, Watanabe has implemented a package that transforms a large class of equations into P-functions. He has used Kamke's table as a test bed. Several examples from Kamke using this package, are given in Appendix I.

A

```
(D2)                                  [DSK, SWATAN]

(C3) showtime:true$
Time= 5 msec.

(C4) loadfile(pmain,fasl);

PMAIN FASL DSK SWATAN being loaded
Loading done
Time= 411 msec.
(D4)                                  DONE

(C5) batch(exampl,2);

(C6) k378a:x*(x-1)*(x+1)^2*'diff(y,x,2)+2*x*(x+1)*(x-3)*'diff(y,x)-2*(x-1)*y=0;
Time= 57 msec.
                                 2
                             2 d Y                                dY
(D6)  (X - 1) X (X + 1)  --- + 2 (X - 3) X (X + 1) -- - 2 (X - 1) Y = 0
                                 2                                dX
                               dX

(C7) lode2(k378a);
                               dY
          2    (2 X - 6) --
         d Y              dX        2 Y
we solve --- + ------------ - ------------- = 0
           2       2           3      2
         dX       X  - 1      X  + 2 X  + X

SOLVE FASL DSK MACSYM being loaded
Loading done

PREMAP FASL DSK SWATAN being loaded
Loading done
              U
we use Y = --------
                  2
           (X + 1)
                               dU
                             2 --     2
               2 U           dX    d U
the result is ------ - ----- + --- = 0
               2           X - 1     2
              X  - X                dX
                 dY
          2    2 --
         d Y     dX      2 Y
we solve --- - ----- + ------ = 0
           2   X - 1     2
         dX              X  - X

PEQP1 FASL DSK SWATAN being loaded
Loading done

PHYPGM FASL DSK SWATAN being loaded
Loading done
```

Kamke example 378a

AIR FORCE OFFICE OF ...
... division

```
(C3) showtime:true$
Time= 4 msec.

(C4) batch(exampl,7);

(C5) K344:X^4*'DIFF(Y,X,2)+(EXP(2/X)-V^2)*Y=0;
Time= 65 msec.
                                  2
                               4 d Y       2/X    2
(D5)                          X  --- + (%E     - V ) Y = 0
                                   2
                                 dX

(C6) loadfile(pmain,fasl);

PMAIN FASL DSK SWATAN being loaded
Loading done
Time= 413 msec.
(D6)                                    DONE

(C7) lode2(k344);
          2        2/X    2
         d Y    (%E     - V ) Y
we solve --- + -------------- = 0
           2           4
         dX           X

PIVTR FASL DSK SWATAN being loaded
Loading done

PTRINV FASL DSK SWATAN being loaded
Loading done
           1
we use T = -
           X
                        dY
               2     2 --
              d Y      dT       2 T    2
the result is --- + ---- + (%E     - V ) Y = 0
                2     T
              dT
                  dY
          2     2 --
         d Y      dX       2 X    2
we solve --- + ---- + (%E     - V ) Y = 0
           2     X
         dX

PIVTR2 FASL DSK SWATAN being loaded
Loading done

PTREXP FASL DSK SWATAN being loaded
Loading done

PDVTR FASL DSK SWATAN being loaded
```

```
the type is hypergeometric
the solution may be written by Riemann's P-functions as follows
    [ 0   1   INF ]
    [             ]
y=P [ 0   0   - 2 ] (x)
    [             ]
    [ 1   3   - 1 ]

PNOPM FASL DSK SWATAN being loaded
Loading done
we solve
    [ 0   1   INF ]
    [             ]
y=P [ 1   0   - 1 ] (x)
    [             ]
    [ 0   3   - 2 ]
       /          2
       [ (X - 1)
y= (K2 I -------- dX + K1) X
       ]     2
       /    X
continue? type y or n
y;

SIN FASL DSK MACSYM being loaded
Loading done

SININT FASL DSK MACSYM being loaded
Loading done

SCHATC FASL DSK MACSYM being loaded
Loading done
Time= 21253 msec.
                                                    2
(D7)                - 2 K2 X LOG(X) + K2 X  + K1 X - K2

Time= 23672 msec.
(D8)                                BATCH DONE

(C9) closefile(buffer,save);
```

```
Loading done

SOLVE FASL DSK MACSYM being loaded
Loading done
            U
we use Y = -
            X
                                    2
                    2 X     2     d U
the result is U (%E     - V ) + --- = 0
                                     2
                                   dX
          2
         d Y       2 X     2
we solve --- + (%E     - V ) Y = 0
           2
         dX
             X
we use T = %E
                     dY
                2    --      2    2
               d Y   dT   (V  - T ) Y
the result is --- + -- - ----------- = 0
                2   T          2
               dT              T
                dY
          2     --      2    2
         d Y    dX   (X  - V ) Y
we solve --- + -- + ----------- = 0
           2    X          2
         dX               X

PCONFL FASL DSK SWATAN being loaded
Loading done
the equation is confluent type

PCNEQ0 FASL DSK SWATAN being loaded
Loading done
the solution may be representable by Fukuhara's P-function.
    [ INF   INF   0  ]
    [                ]
    [        1       ]
    [ - %I   -   - V ]
y=P [        2       ] (x)
    [                ]
    [        1       ]
    [  %I    -    V  ]
    [        2       ]

PCFPTM FASL DSK SWATAN being loaded
Loading done
y= Y         (X)
    B, ABS(V)
Time= 15604 msec.
(D7)                                Y         (X)
                                     B, ABS(V)
```

```
(D2)                                  [DSK, SWATAN]

(C3) showtime:true$
Time= 5 msec.

(C4) batch(exampl,12);
```

Kamke example 406

```
(C5) k406:16*(X^3-1)^2*'DIFF(Y,X,2)+27*X*Y=0;
Time= 41 msec.
                                          2
                                 3     2 d Y
(D5)                        16 (X  - 1)  --- + 27 X Y = 0
                                           2
                                         dX

(C6) k406t:48*x^2*(x-1)^2*'diff(y,x,2)+32*x*(x-1)^2*'diff(y,x)+9*x*y=0;
Time= 50 msec.
                                   2
                            2  2 d Y                2   dY
(D6)              48 (X - 1)  X  --- + 32 (X - 1)  X -- + 9 X Y = 0
                                   2                   dX
                                 dX

(C7) loadfile(pmain,fasl);

PMAIN FASL DSK SWATAN being loaded
Loading done
Time= 426 msec.
(D7)                                  DONE

(C8) lode2(k406t);
                   dY
          2     2 --
         d Y      dX            3 Y
we solve --- + ---- + -------------------- = 0
           2    3 X         3        2
         dX            16 X  - 32 X  + 16 X

SOLVE FASL DSK MACSYM being loaded
Loading done

PHYPGM FASL DSK SWATAN being loaded
Loading done
the type is hypergeometric
the solution may be written by Riemann's P-functions as follows
    [ 0  1  INF ]
    [           ]
    [ 1  3    1 ]
    [ -  -  - - ]
y=P [ 3  4    3 ] (x)
    [           ]
    [    1      ]
    [ 0  -   0  ]
    [    4      ]

PHGHP FASL DSK SWATAN being loaded
Loading done
```

```
PALGS FASL DSK SWATAN being loaded
Loading done
we solve
                  [ 0   1   INF  ]
                  [              ]
                  [ 1   1     1  ]
            1/4   [ -   -   - --  ]
y= (X - 1)     P  [ 3   2     12 ] (x)
                  [              ]
                  [          1   ]
                  [ 0   0    -   ]
                  [          4   ]

PALG4 FASL DSK SWATAN being loaded
Loading done

POHAS2 FASL DSK SWATAN being loaded
Loading done
       1/4          2                                  1/4        1/4
   K1 T    (2 SQRT(T  + T + 1) + SQRT(3) (T + 1))    (X - 1)
y= ---------------------------------------------------------------
                                 3     1/12
                               (T  - 1)

       1/4                                 2          1/4        1/4
   K2 T    (SQRT(3) (T + 1) - 2 SQRT(T  + T + 1))    (X - 1)
 + ---------------------------------------------------------------
                                 3     1/12
                               (T  - 1)

where t=x^(1/3)
Time= 13449 msec.
         1/4          2                                  1/4        1/4
     K1 T    (2 SQRT(T  + T + 1) + SQRT(3) (T + 1))    (X - 1)
(D8) ---------------------------------------------------------------
                                 3     1/12
                               (T  - 1)

                      1/4                                 2          1/4        1/4
                  K2 T    (SQRT(3) (T + 1) - 2 SQRT(T  + T + 1))    (X - 1)
                + ---------------------------------------------------------------
                                                 3     1/12
                                               (T  - 1)

Time= 15278 msec.
(D9)                                BATCH DONE

(C10) closefile(buffer,save);
```

```
(D2)                                  [DSK, SWATAN]

(C3) showtime:true$
Time= 5 msec.
```

Kamke example 180

```
(C4) batch(exampl,13);

(C5) K180:X^2*'DIFF(Y,X,2)+3*X*'DIFF(Y,X,1)+(X^2+1-V^2)*Y=F(X);
Time= 47 msec.
                          2
                       2 d Y        dY      2     2
(D5)                  X  --- + 3 X -- + (X  - V  + 1) Y = F(X)
                          2        dX
                         dX

(C6) loadfile(pmain,fasl);

PMAIN FASL DSK SWATAN being loaded
Loading done
Time= 413 msec.
(D6)                                    DONE

(C7) lode2(k180);
                    dY
           2     3 --      2     2
          d Y      dX   (X  - V  + 1) Y   F(X)
we solve --- + ---- + --------------- = ----
            2     X             2            2
          dX                   X            X
                         dY
                  2    3 --      2     2
                 d Y      dX   (X  - V  + 1) Y
first we solve --- + ---- + --------------- = 0
                   2    X             2
                 dX                  X

SOLVE FASL DSK MACSYM being loaded
Loading done

PCONFL FASL DSK SWATAN being loaded
Loading done
the equation is confluent type

PCNEQ0 FASL DSK SWATAN being loaded
Loading done
the solution may be representable by Fukuhara's P-function.
    [ INF   INF     0    ]
    [                    ]
    [        3           ]
    [ - %I   -   - V - 1 ]
y=P [        2           ] (x)
    [                    ]
    [        3           ]
    [  %I    -    V - 1  ]
    [        2           ]

PCFPTM FASL DSK SWATAN being loaded
```

```
Loading done
   Y          (X)
    B. ABS(V)
y= -------------
         X
                                    Y          (X)
                                     B. ABS(V)
the solution of the homog. eq. is -------------
                                          X

PNONH FASL DSK SWATAN being loaded
Loading done
a special sol. for the nonhom. eq. is
 X
/
[
I  (%PI F(T) J       (T) Y       (X) - %PI F(T) Y       (T) J       (X)) dT
]              ABS(V)      ABS(V)                 ABS(V)      ABS(V)
/
 0
-----------------------------------------------------------------------
                                    2 X
continue? type y or n
n;
Time= 9943 msec.
      X
     /
     [
     I  (%PI F(T) J       (T) Y       (X) - %PI F(T) Y       (T) J       (X)) dT
     ]              ABS(V)      ABS(V)                 ABS(V)      ABS(V)
     /
      0
(D7) ---------------------------------------------------------------------
                                        2 X

                                                                  Y          (X)
                                                                   B. ABS(V)
                                                                + -------------
                                                                        X

Time= 12131 msec.
(D8)                                BATCH DONE

(C9) closefile(buffer,save);
```

END

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