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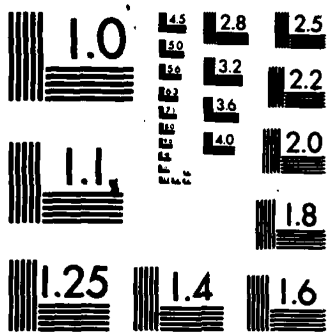

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… R. L. Hewitt
National Aeronautical Establishment

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OTTAWA
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COMPARISON OF THE STRESS CONCENTRATION FACTOR OF A HOLE NEAR A CIRCULAR EDGE NOTCH WITH AN ENGINEERING ESTIMATE

COMPARAISON DU FACTEUR DE CONCENTRATION DE CONTRAINTE D'UN TROU À PROXIMITÉ D'UNE ENTAILLE DE BORD CIRCULAIRE AVEC UNE VALEUR APPROXIMATIVE

by/par

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SUMMARY

The stress concentration factor for a hole near a circular edge notch has been determined using Nisitani's body force method and compared with some engineering approximations. For a hole close to the notch the error in the approximation was about 6% but decreased to near zero as the hole was placed further away.

The stress concentration at the root of the edge notch was found to be decreased in the presence of a hole close to the notch because of the interaction between the hole and the edge of the plate.

RÉSUMÉ

On a évalué le facteur de concentration de contrainte pour un trou à proximité d'une entaille de bord circulaire, au moyen de la méthode des forces de Nisitani, et on a comparé le résultat avec certaines approximations pratiques. Pour un trou voisin de l'entaille, l'erreur de l'approximation est de 6% environ, mais elle diminue jusqu'à devenir presque nulle à mesure que le trou est éloigné de l'entaille.

On a remarqué que la concentration de contrainte à la racine de l'entaille de bord diminue lorsqu'il y a un trou à proximité de l'entaille en raison de l'interaction entre le trou et le bord de la plaque.

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ILLUSTRATIONS (Cont'd)

COMPARISON OF THE STRESS CONCENTRATION FACTOR OF A HOLE NEAR A CIRCULAR EDGE NOTCH WITH AN ENGINEERING ESTIMATE

1.0 INTRODUCTION

In a recent note, Schijve[1] has estimated the stress concentration factor for a small hole in the neighbourhood of a larger notch by comparing with other geometries for which the solutions are known or can be reliably estimated. Although he was unable to give a firm indication of the accuracy of the estimate, he expected it to be better than 5%. Since we have recently implemented a computer technique for solving this kind of problem[2, 3] it seemed appropriate to use it to determine the stress concentration factors for the configurations considered by Schijve to see how accurate the estimates were.

2.0 ANALYSIS

2.1 Body Force Method

The method of analysis used was Nisitani's body force method[4]. In this method a plate with a hole is treated as a plate without a hole but with an infinite number of point forces distributed around the edge of an imaginary hole. Approximating this by a finite number of point forces, the solution is obtained by making use of the equations for the stress at an arbitrary point in a plate due to a point force. The density of the point forces is obtained from the boundary conditions, ie. that the edge of the hole is free of stress.

2.2 Notch Geometries

Schijve studied the problem of two circular edge notches in a finite width plate with a small hole near each edge notch as shown in Figure 1(a). He first approximated this by a single edge notch and hole in a semi-infinite plate as shown in Figure 1(b). This approximation was felt by Schijve to have a negligible effect since the plate was quite wide and this is the geometry that will be studied here. To obtain the solution Schijve made one other approximation which he considered to have a negligible effect and that was that the circular edge notch can be considered as a semi-elliptical notch of the same root radius. Since these were minor variations in geometry in terms of programming, these two geometries were therefore studied also.

2.3 Solution Technique

The solution technique for the hole next to an edge notch as shown in Figure 2 is as follows:

(1) The circular arc DCE is divided into M1 equal intervals while the hole is divided into M2 equal intervals.

(2) The influence coefficients are calculated for each interval, ie. the stresses at each interval on the hole and edge notch are calculated for unit body forces in both the x and y directions at every interval of the hole and edge notch. These are calculated from the stress fields for a point force in a semi-infinite plate[4].

(3) The conditions that the midpoint of each interval be free of stress is used to give a system of 2(M1 + M2) linear equations in the unknown body forces ρ_{xN} and ρ_{yN}. These may then be determined.

(4) The stresses at an arbitrary point are found by determining the influence coefficients for that point (ie. the stresses at that point due to unit forces at each interval of the hole and edge notch) and multiplying by the appropriate body forces ρ_{xN} and ρ_{yN}.

3.0 RESULTS

3.1 Semi-Elliptical Edge Notch

Figure 3(a) shows the dimensions for the semi-elliptical edge notch. The stress concentration factors at the root of the notch, C, and at a distance of 2.25 mm from the notch, A, are shown in Table I as a function of the number of intervals, M1. As Nisitani has shown[4], the errors caused by the finite number of divisions are nearly proportional to 1/M1, and the true value of K_T can be obtained by extrapolation to a value of M1 corresponding to infinity. The extrapolated values are 1.7350 at the notch root, C, and 1.2955 at A.

TABLE I

STRESS CONCENTRATION FACTORS FOR SEMI-ELLIPSE

No. of Divisions M1	Stress Concentration Factor K_{TC}	Stress Concentration Factor K_{TA}
16	1.734081	1.295110
24	1.734349	1.295243
32	1.734532	1.295337
∞	1.7350	1.2955

3.2 Circular Edge Notch

Figure 3(b) shows the dimensions of the circular edge notch, and the stress concentration factors at C and A are shown in Table II as a function of the number of intervals. The extrapolated values at C and A are 1.7036 and 1.2883 respectively. The value at C may be compared with a value of 1.707 given by Green and Zerna[5] for a circular edge notch with a subtended angle of 60°, rather than the 60.07° for the present case.

TABLE II

STRESS CONCENTRATION FACTORS FOR CIRCULAR EDGE NOTCH

No. of Divisions M1	Stress Concentration Factor K_{TC}	Stress Concentration Factor K_{TA}
16	1.712143	1.296008
24	1.709221	1.293408
32	1.707889	1.292183
∞	1.7036	1.2883

3.3 Hole Near Circular Edge Notch

Nisitani et al[6] have published results for a hole near a semi-circular notch in a semi-infinite plate. These provided a method of checking the program. The configuration studied by Nisitani et al is shown in Figure 4 and the stress concentration factors at the root of the notch, C, and the edge of the hole, A, determined in the present investigation are given in Table III for various values of the number of intervals M1 and M2. The extrapolated values are 3.5534 at C and 4.2801 at A as compared with the values of 3.561 and 4.283 given by Nisitani.

The stress concentration factors at the edge of a hole near a circular edge notch, Figure 1(b), are shown in Table IV for the three hole separation distances, s, studied by Schijve, and for various numbers of intervals M1 and M2. The extrapolated values for holes at distances of 2.25, 3.0 and 4.5 mm from the edge of the notch are 3.8136, 3.5399 and 3.2511 respectively. For the equivalent notches Schijve predicted values of 3.58, 3.39 and 3.24, and therefore underestimated the stress concentration factors by 6.4, 4.4, and 0.3%, the largest error being for the hole closest to the notch.

TABLE III

STRESS CONCENTRATION FACTOR FOR HOLE NEAR SEMI-CIRCULAR EDGE NOTCH (FIG. 4)

No. of Divisions		Stress Concentration Factor	
M1	M2	K_{TA}	K_{TC}
8	8	4.340597	3.694438
16	16	4.309273	3.614692
32	32	4.294697	3.584046
∞	∞	4.2801	3.5534

TABLE IV

STRESS CONCENTRATION FACTOR FOR HOLE NEAR CIRCULAR EDGE NOTCH FOR VARIOUS HOLE SEPARATION DISTANCES

No. of Divisions	Stress Concentration Factor K_{TA}		
M1 and M2	s = 2.25 mm	s = 3.0 mm	s = 4.5 mm
8	4.010007	3.735499	3.405721
12	3.949801	3.670475	3.353424
16	3.913948	3.637558	3.327908
24	3.877074	3.605106	3.302993
∞	3.8136	3.5399	3.2511

3.4 Stress Distribution Near the Hole

Having obtained the body force densities to calculate the stress concentration factors, it is a simple matter to calculate the total stress distribution around the hole. This is shown in Figure 5 for the hole placed 2.25 mm from the edge of the notch.

4.0 DISCUSSION

The stress concentration factor for the circular edge notch is about 1.8% lower than that for the semi-ellipse (1.7036 compared with 1.7350). However, as Schijve expected, the difference in stress at a point removed from the notch a distance of 2.25 mm or more is less than 0.6% (1.2883 compared with 1.2955). The difference in stress concentration factor for the semi-ellipse in a semi-infinite plate (1.7350) and an ellipse in an infinite plate ($K_T = 1 + 2a/b = 1.727$) is less than 0.5% as expected. Thus the first two approximations of Schijve, ie. from a circular edge notch to a semi-ellipse of the same root radius and then from a semi-ellipse in a semi-infinite plate to an ellipse in an infinite plate, were quite accurate. The only real inaccuracies stem from his last approximation, where he took the solution for a small circular hole next to a larger circular hole in an infinite plate and applied a correction factor to this to get a solution for a circular hole next to an ellipse. This was done by comparing the stresses near to a circle and an ellipse alone in an infinite plate and assuming that the ratio of the stresses in these two cases would remain the same in the presence of the additional small holes. As expected, the approximation is most accurate for the hole placed furthest away (4.5 mm) from the edge notch since interaction effects are least for this case and the error is only 0.3%. For the hole closest to the notch (2.25 mm) the error is 6.4%. However, this is not much larger than Schijve expected (5%) and indicates that good engineering judgement can produce quite acceptable approximations.

The stress distribution around the hole shown in Figure 5 is similar to that anticipated by Schijve except in the region very close to the edge notch. The stress concentration factor at the notch root is slightly lower in the presence of the hole than without it (1.6224 compared with 1.7036), and there is an inflection in the stress-distance plot about 0.4 mm away from the notch. This was surprising at first sight since Nisitani's solution for the hole next to a semi-circular notch (which had been duplicated using the current program) indicated a higher stress concentration factor in the presence of a hole than without it (3.561 compared with 3.065). However, for a hole near the edge of a semi-infinite plate, the stress concentration factor at the edge of the plate is always less than one[7], and so this case was examined.

The stress distribution for a 1.5 mm diameter hole placed 2.25 mm from the edge of a semi-infinite plate calculated using the present technique is shown in Figure 6. The stress concentration factors agree with those given in Reference 7. Also shown in this figure are the stress distributions for three cases with circular notches at the edge of the plate in addition to the hole, with the root of the notch in each case being 2.25 mm from the hole and 2 mm from the edge of the plate. For a notch radius of 45 mm, the distribution approaches that of the case without the notch, and the stress concentration at the root of the notch is 1.2649 compared to 1.3967 without the hole, ie. it is reduced by the presence of the hole. For a notch radius of 2 mm (ie. a full semicircle) which is similar to the problem of Nisitani, the stress concentration at the root is 3.1231 compared to 3.0653, ie. it has been increased by the presence of the hole. Thus for sharp notches the stress concentration factor is increased by the presence of the hole, while for obtuse notches it is decreased. The stress distribution shown in Figure 5 is therefore quite reasonable and the inflection in the curve is a consequence of the interaction of the hole with the edge of the plate.

CONCLUSIONS

The stress concentration factor for a hole near a circular edge notch was determined using Nisitani's body force method and compared with some engineering approximations presented by Schijve. For a hole close to the notch the error in the approximation was 6.4% but decreased to near zero as the hole was placed further away. These errors are very close to the error limits expected by Schijve, and confirm the usefulness of these kinds of approximations.

The stress distribution around the hole was calculated and the stress concentration at the root of the edge notch was found to be decreased in the presence of a hole close to the notch. This was explained in terms of the interaction between the hole and the edge of the plate.

6.0 ACKNOWLEDGEMENTS

The authors would like to thank Dr. G.R. Cowper of this laboratory for useful discussions and advice during the course of this work.

7.0 REFERENCES

1. Schijve, J. — *Estimation of a K_T-Value by Comparing Different Notch Geometries.* Memorandum M-469, Delft University of Technology, May 1983.

2. Fraga, W.E., Hewitt, R.L. — *Implementation of Nisitani's Body Force Method for the Solution of Notch Problems.* NRC, Aeronautical Note, NAE AN-17, National Research Council Canada, Ottawa, November 1983.

3. Fraga, W.E., Hewitt, R.L. — *Computer Programs for Solving Notch Problems Using Nisitani's Body Force Method.* NRC, Laboratory Technical Report, LTR-ST-1479, National Research Council Canada, Ottawa, November 1983.

4. Nisitani, H. — *Solutions of Notch Problems by Body Force Method.* Mechanics of Fracture, Vol. 5, ed. G.C. Sih, Noordhoff International Publishing, The Netherlands, 1978.

5. Green, A.E., Zerna, W. — *Theoretical Elasticity.* Oxford University Press, London, 1954, p. 320.

6. Nisitani, H., Saito, K., Hara, N. — *Stress Concentration Due to an Elliptic Hole or Crack Existing Near a Notch Under Tension or Longitudinal Shear.* Transactions of the Japan Society of Mechanical Engineers, Vol. 39, No. 324, 1973, (in Japanese), pp. 2312-2322.

7. Peterson, R.E. — *Stress Concentration Factors.* John Wiley & Sons, New York, 1974, p. 151.

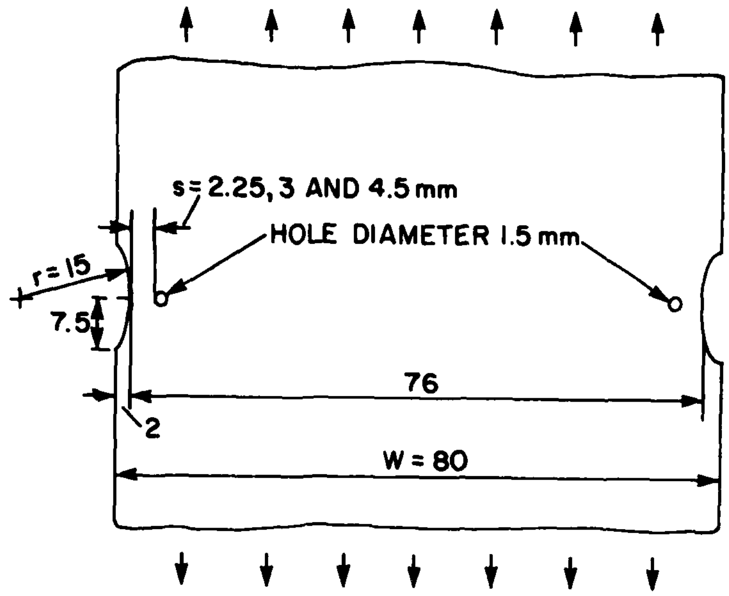

FIG. 1(a): SMALL HOLE NEAR EDGE NOTCH IN FINITE-WIDTH PLATE

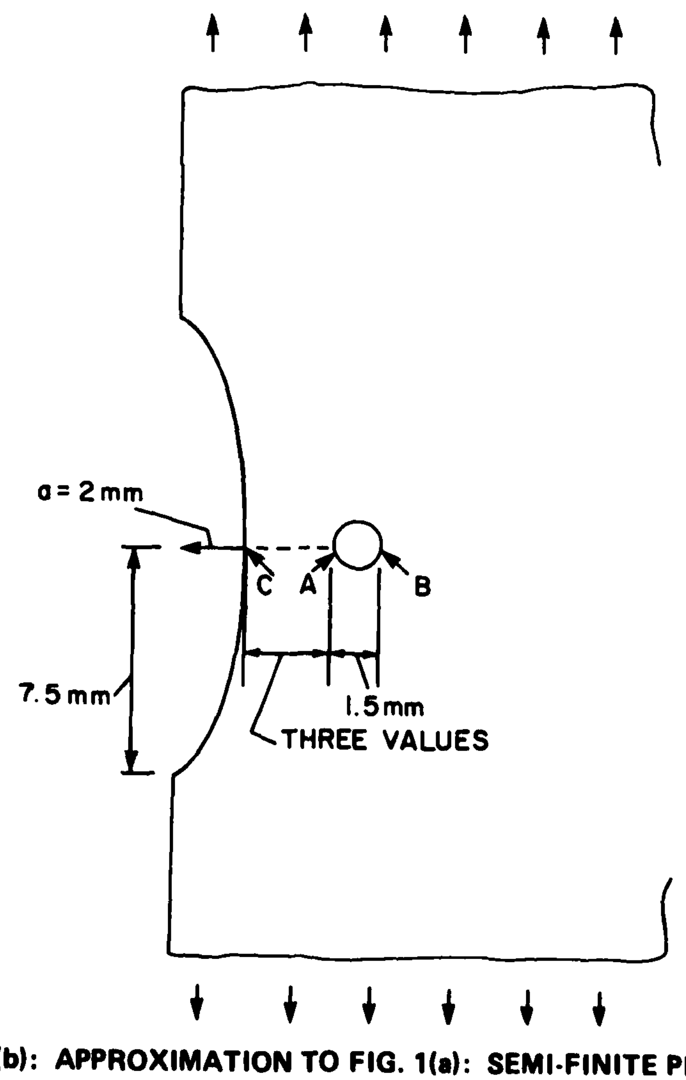

FIG. 1(b): APPROXIMATION TO FIG. 1(a): SEMI-FINITE PLATE

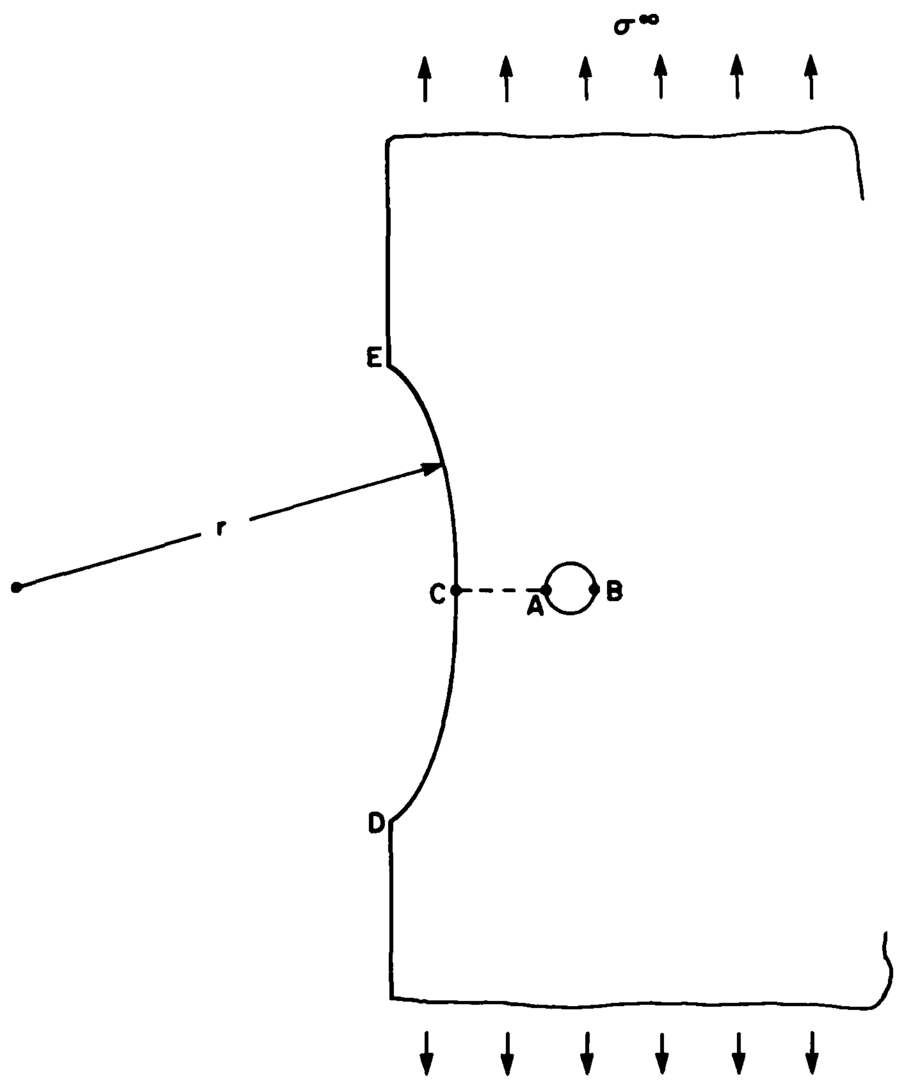

FIG. 2: HOLE NEAR EDGE NOTCH IN SEMI-INFINITE PLATE

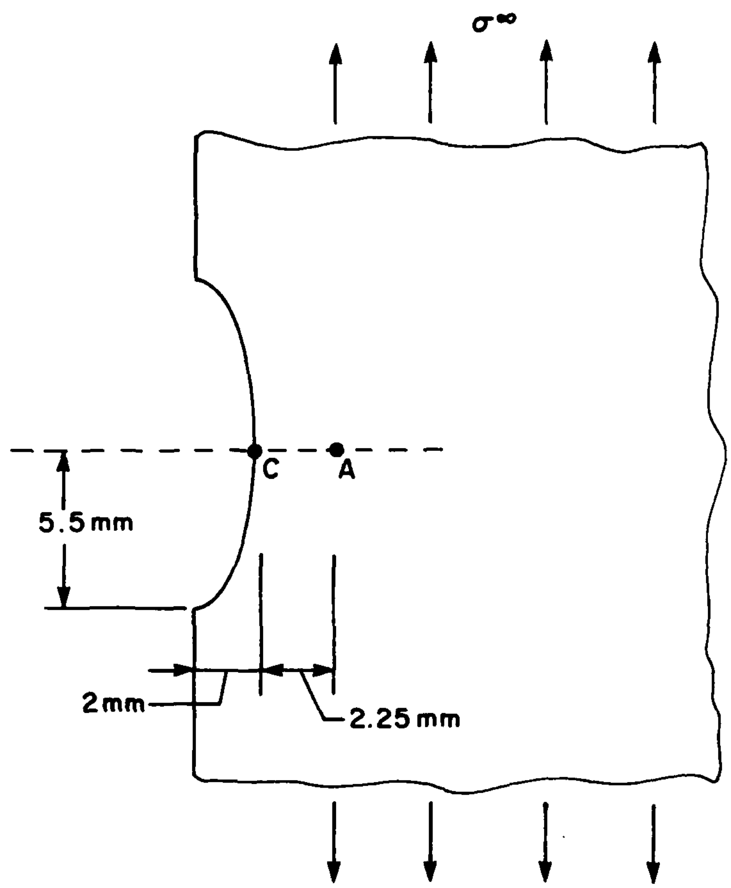

FIG. 3(a): DETAILS OF SEMI-ELLIPTICAL NOTCH CONFIGURATION

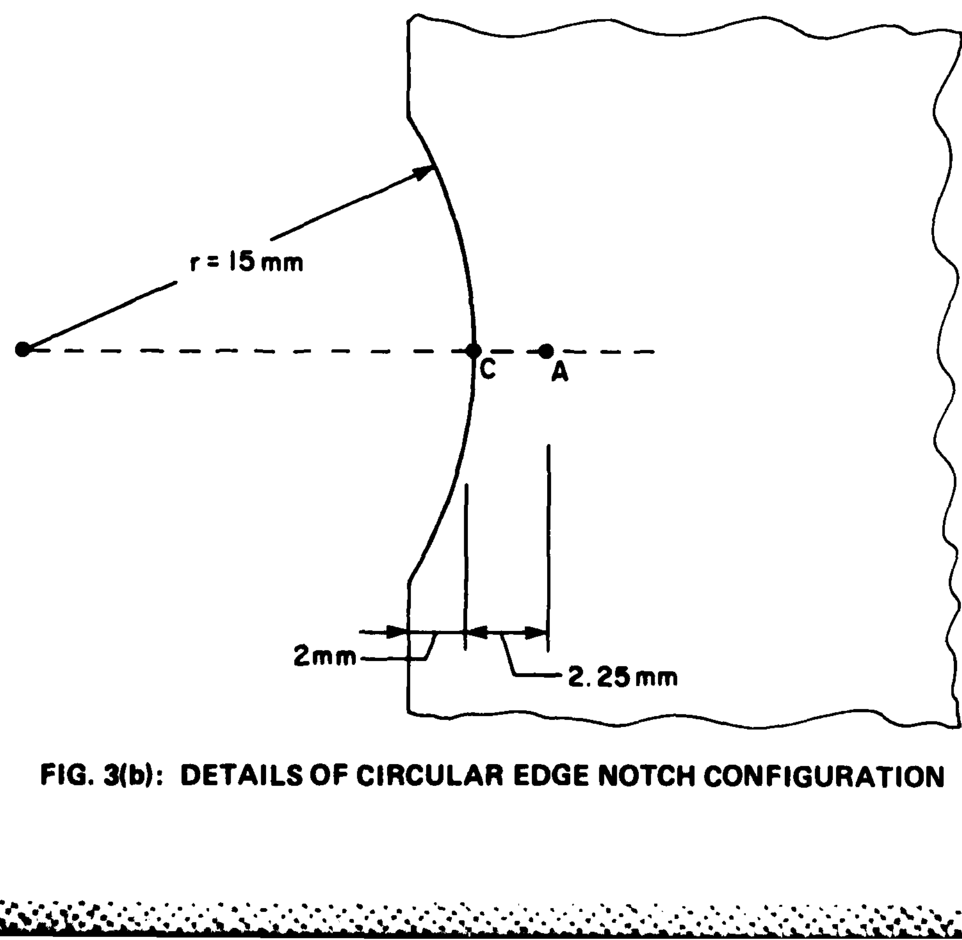

FIG. 3(b): DETAILS OF CIRCULAR EDGE NOTCH CONFIGURATION

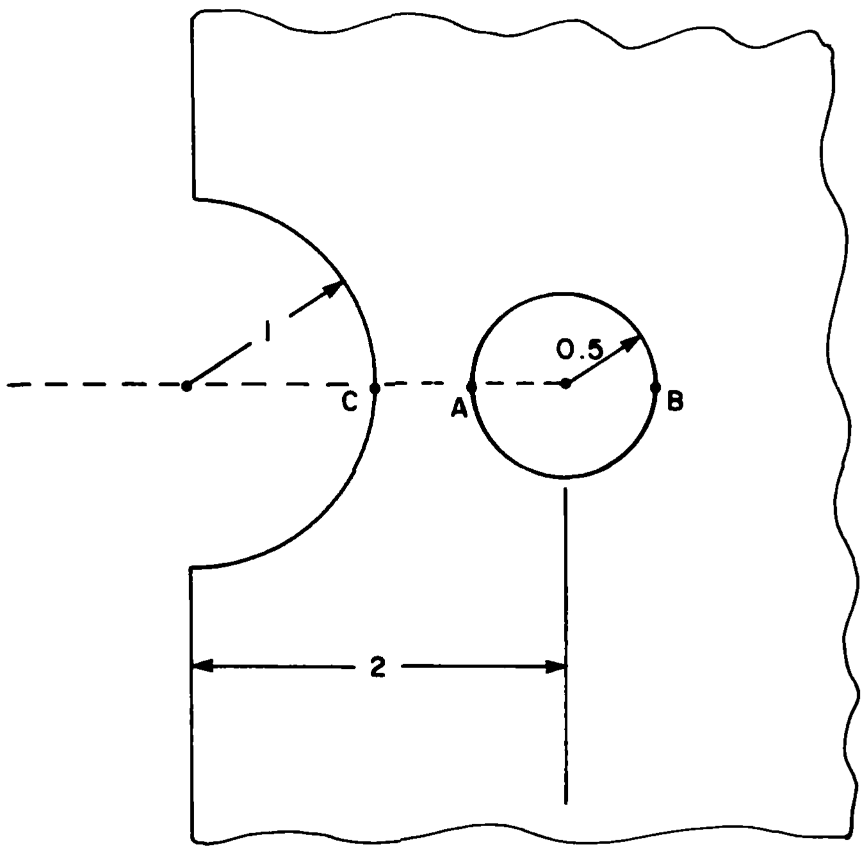

FIG. 4: SEMI-CIRCULAR NOTCH AND HOLE GEOMETRY
STUDIED BY NISITANI ET AL [5]

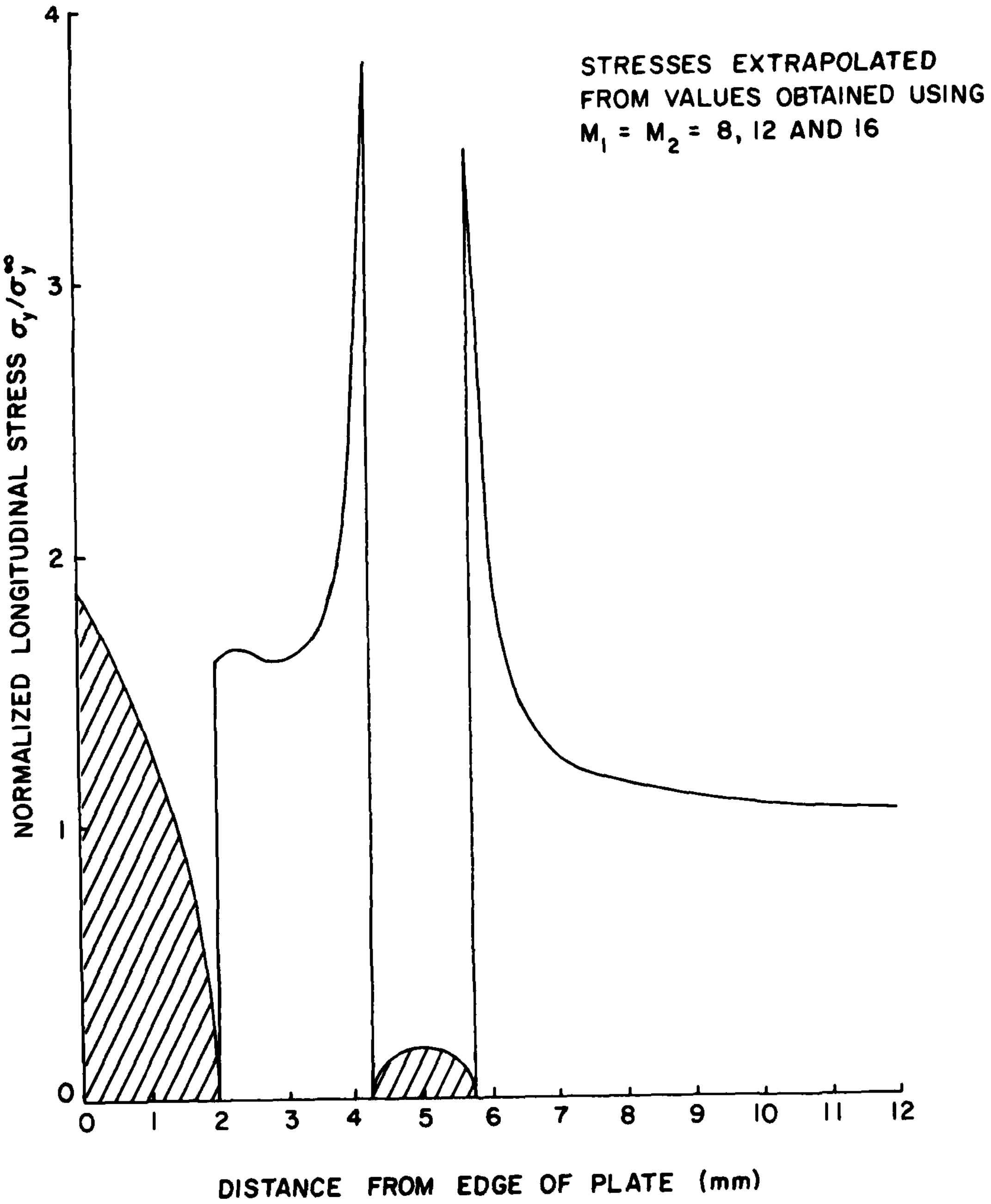

FIG. 5: STRESS DISTRIBUTION AROUND HOLE

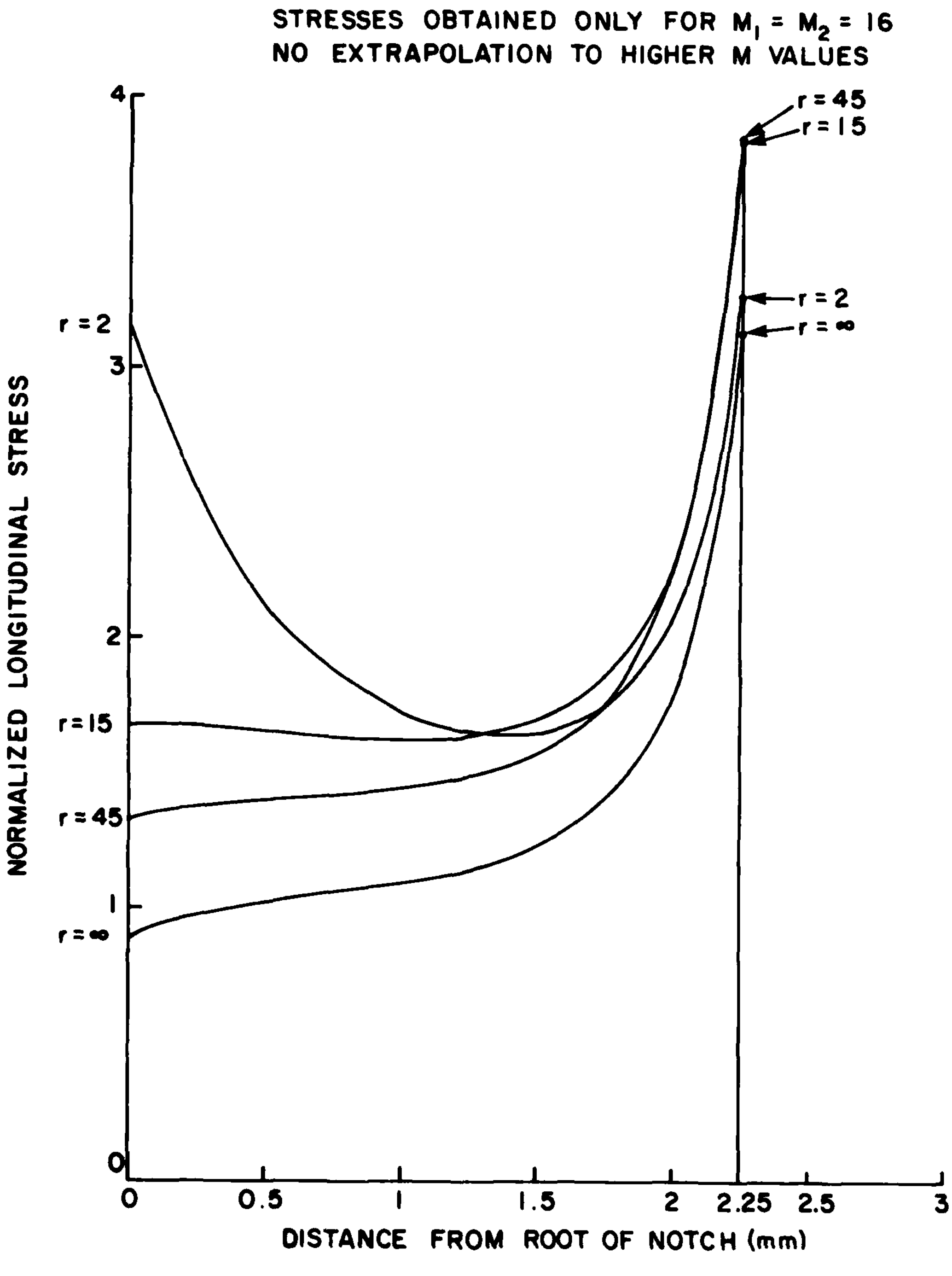

FIG. 6: APPROXIMATE STRESS DISTRIBUTION BETWEEN 1.5 mm DIA HOLE AND CIRCULAR EDGE NOTCHES OF VARIOUS RADII (FIG. 3(b))

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DESCRIPTORS (KEY WORDS)/MOTS-CLÉS

1. Stress concentration
2. Boundary integral method

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SUMMARY/SOMMAIRE

The stress concentration factor for a hole near a circular edge notch has been determined using Nisitani's body force method and compared with some engineering approximations. For a hole close to the notch the error in the approximation was about 6% but decreased to near zero as the hole was placed further away.

The stress concentration at the root of the edge notch was found to be decreased in the presence of a hole close to the notch because of the interaction between the hole and the edge of the plate.

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