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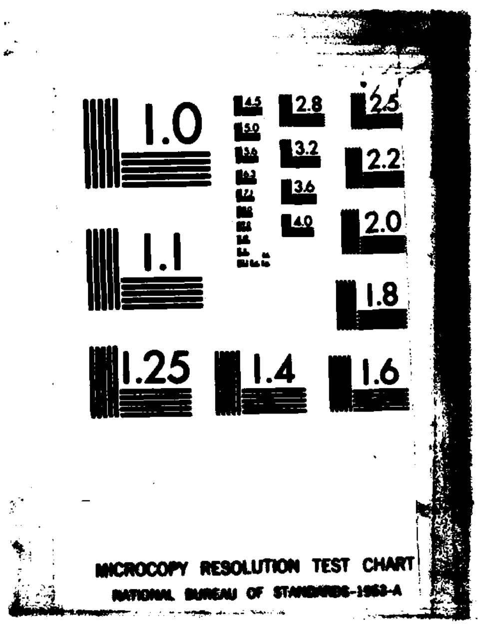
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PERFECTION AND ENTRY: AN EXAMPLE

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Perfection and Entry: an Example

~~In~~ this ~~note we shall~~, in the context of a simplified example, show that circumstances exist under which an incumbent firm can deter a potential entrant even where there is room for both firms in the industry, but that such deterrence may or may not form part of a perfect equilibrium.

This example uses the discounted supergame structure, and the perfectness concept being used is that of subgame perfection. At the end of the discussion ~~we shall~~ briefly describe the impact of another perfectness notion. The example is also of some historical significance as a counter-example to the conjecture that the requirement of perfectness does not reduce the set of equilibrium outcomes in the discounted game[1].

The essential features of the market situation required to produce the phenomenon are: first, that both the incumbent and entrant can make positive profits in the industry and; second, that the incumbent has available to it a Pareto-inferior "punishment strategy." For the sake of simplicity in exposing the underlying events, we shall asume that there are two firms, an incumbent (I) and an entrant (E). The incumbent's strategy is $p \in [0,1]$, while that of the entrant is $a \in [0,1]$. The profit functions for the two players are:

$$(1) \qquad h_I(p,a) = (1-a) + fp \qquad\qquad h_E(p,a) = a + gp \qquad\qquad f,g >> 1$$

There are several interpretations we can have for these strategies. The most direct interpretation is that both the incumbent and the entrant are active in the market. The entrant can expand his share of the market by choosing a level of advertising (a). This does not affect the total size of the market. The incumbent can engage in promotion (p) which does affect the size of the market; it affects the profits of both firms in the same direction but not to the same extent. In this interpretation there is no question of entry or exit.

An alternative interpretation in terms of entry and exit is also possible. In this view, the strategy of the entrant (a) is the market share it achieves but the strategy of the incumbent is more complex. As the payoff functions are written, it is possible for the incumbent to punish the entrant by playing $p < 1$ even when the entrant is not in the market. One way in which this could happen is if the two firms were duopolists in another sector, and the "entrant" was in fact matching the "incumbent's" diversification. However, in all the equilibrium situations with which we shall be concerned, the incumbent will never have cause to play $p < 1$ unless the entrant is playing $a > 0$ or has played $a > 0$ at some previous stage of the game, in which case $p < 1$ will be played as soon and for as short a time as possible. What we assume is that entry involves the formation of some fixed capital which is at least partially specific to the industry. If the entrant is active in the industry, then p corresponds to the price level prevailing in that sector. If the entrant leaves the industry, then $p = 1$ reflects a situation where the incumbent firm absorbs this fixed capital from its defeated rival at a mutually advantageous price. On the other hand, $p < 1$ reflects the ability of the incumbent firm to "punish" both the former entrant and itself by refusing to absorb this fixed capital. This punishes both because the incumbent must now meet its needs for continuing investment at the opportunity cost of uncommitted capital.

No claim is made for the quantitative realism of these specifications. All that is necessary to produce the phenomena we shall examine is captured in the qualitative description: the entrant has a strategy that affects his and the incumbent's payoffs in opposite directions, while the incumbent's strategy affects his and the entrant's payoffs in the same direction. The simple form we have chosen, however, vastly simplifies the exposition.

The game as written has a single equilibrium at [p,a] = [1,1]. This corresponds either to a situation in which the entrant captures the "contestable" part of the market while the incumbent enlarges the market as much as possible, or to a situation in which the entrant enters the market and the incumbent acquiesces in this entry, reaching some reference position (Cournot duopoly, collusive duopoly, etc.) which is less favorable for the incumbent than entry on a smaller scale or deterred entry.

The discounted supergame uses the present discounted value of the stream of payoffs, normalized to lie in the convex hull of the original payoffs. Thus, if the actions taken by the agents form the infinite stream

$$[p,a] = [(p(1),a(1)),\dots,(p(t),a(t)),\dots]$$

where p(t) [a(t)] is the incumbent's [entrant's] action at date t, and if the incumbent and entrant use the discount factors ϑ and ξ respectively, the payoffs associated with the stream [p,a] are:

$$(2)\qquad \begin{aligned} H_I([p,a]) &= (1-\vartheta)\sum_{t=1}^{\infty} h_I(p(t),a(t))\,\vartheta^{t-1} \\ H_E([p,a]) &= (1-\xi)\sum_{t=1}^{\infty} h_E(p(t),a(t))\,\xi^{t-1} \end{aligned}$$

The stationary equilibrium outcomes of such a game are those strategy pairs (p^*,a^*) such that:

$$(3)\qquad \begin{aligned} h_I(p^*,a^*) &\geq (1-\vartheta)\,h_I(1,a^*) + \vartheta\,h_I(1,1) \\ h_E(p^*,a^*) &\geq (1-\xi)\,h_E(p^*,1) + \xi\,h_E(0,1) \end{aligned}$$

since either player's best defection is to play a or p = 1, and the minmax (security) levels of the incumbent and entrant are achieved at (1,1) and (0,1) respectively.

These outcomes were obtained by using the "grim strategies" that threaten to punish any defection from the agreed-upon play of (p^*,a^*) forever; if the incumbent defects, the entrant threatens to play a = 1 ("in") forever, while a defection by the entrant is met with continued play of p = 0 ("punish") by the incumbent.

We can represent these outcomes in strategy space by writing the above conditions as:

$$(4) \qquad p^* \geq \max \ \{(f - \vartheta(1-a^*)) \div f,\ (1-a^*) \div (\xi g)\}$$

This can be translated into payoff space as follows, letting I (resp. E) be the incumbent's (resp., entrant's) payoff:

$$(5) \qquad \frac{(E - 1)(f + \xi g)}{(1 - \xi)g} \geq I \geq \frac{f(f + g - E + 1 - \vartheta)}{f + \vartheta g}$$

To find the subgame perfect equilibria we must see what punishments the players could actually be induced to carry out. For the sake of simplicity, let us restrict our attention to Pareto Optimal outcomes; since we are playing a discounted game, such sequences must involve Pareto Optimal play at each stage: along the equilibrium path the incumbent plays his dominant strategy p = 1 at each stage. This in turn means that we only have to concern ourselves with preventing defections from the equilibrium path by the entrant, although the incumbent may well have an incentive to defect from the specified punishment.

At this point, we introduce one further assumption. We assume that the incumbent is less myopic than the entrant:

$$(6) \qquad \vartheta \geq \xi$$

This implies that the incumbent firm possesses a financial advantage. This may take the form of loyal customers, or it may reflect informational imperfections in the capital market. If the cost of the struggle is substantial, it may reflect the incumbent's access to funds available at inframarginal opportunity cost; the entrant would by assumption have to borrow funds at the marginal rate from skeptical bankers. In addition, the outside sources may require speedy evidence of the likely success of the entry bid, since they have not sunk the capital. Thus, even if the entrant has sunk his investment, he may inherit his creditor's myopia.

The practical effect of this assumption is to ensure that the most cost-effective punishment is the swiftest one. To see this, suppose that the incumbent wishes to punish the entrant using the stream $p' = [p(1),\ldots,p(t),\ldots]$. The total punishment inflicted on the entrant is:

$$(7) \qquad G = g(1-\xi) \sum_{t=1}^{\infty} (1-p(t))\xi^{t-1}$$

and the total cost borne by the incumbent is:

$$(8) \qquad F = f(1-\vartheta) \sum_{t=1}^{\infty} (1-p(t))\vartheta^{t-1}$$

from which it is easily seen that the greatest ratio of G to F is obtained by punishing immediately. We shall therefore restrict attention to strategies where the incumbent reacts to defection immediately with a punishment p' that is sufficiently small (severe) to have prevented defection in the first place.

For Pareto Optimal equilibrium payoffs condition (5) reduces to:

$$\xi g \geq I$$

If the pair (1,a*) is to be sustained in equilibrium, and a defection is to be met by a punishment sequence with payoffs (as of the first date of punishment) (F,G), the condition for equilibrium is:

$$a^* \geq 1 + \xi[G - g - 1]$$

This punishment can be accomplished in one move provided that:

$$a^* \geq [1 - \xi g] \div [1 - \xi]$$

Otherwise, more than one period may be necessary: in general the punishment sequence will involve the incumbent playing p = 0 for T periods; playing p' on the T+1st period, then returning to cooperative play. However, we can deal with this punishment as if it occurred during a single period by allowing the incumbent to play p < 0. In other words, if the punishment sequence is T,p' as described above, this is equivalent to a one-period "virtual punishment" play of:

$$(9) \qquad p = p'\xi^T - \frac{[\xi(1 - \xi^T)(g + a^*)]}{g(1 - \xi)}$$

The condition for such a p to be effective in deterring a defection is:

$$(10) \qquad p \leq 1 - \frac{1 - a^*}{\xi g}$$

However, a question arises as to what action the entrant is to take during the punishment period. In principle, we would like to stipulate that the entrant cooperate with his punishment, since this would reduce the "social" cost of the punishment. However, then there may be an incentive for the entrant to defect during the punishment phase, necessitating further punishment...

Suppose that the equilibrium calls for stationary play of $(p^*,a^*) = (1,a^*)$. If the entrant defects by playing $a' \neq a^*$, the strategy will call for a "punishment play" of (p,α), where:

$$(11) \qquad p \leqq \frac{(1 + \xi)(a^* + g) - g - a' - \xi\alpha}{\xi g}$$

The cheapest such punishment occurs when the entrant is to play $\alpha = 0$, but this also gives the entrant the greatest incentive for a second defection.

Therefore, if a punishment round is to be played with (virtual) moves (p,α) but the entrant plays a', the strategy will specify a second punishment round of (p'',α''), where:

$$(12) \qquad p'' \leqq \frac{\alpha - a' + \xi[a^* + g - \alpha'']}{\xi g}$$

It will be noted that this condition is independant of p, and can therefore be applied to subsequent defections if necessary.

These conditions provide a means of constructing an equilibrium strategy, but they do not say anything about its perfection; whether the incumbent would actually be willing to carry out the prescribed punishment. We secure the cooperation of a player who is supposed to carry out a costly punishment by threatening to punish that player for failing to discharge his obligation. In this game, this is simple to evaluate, since the entrant profits by punishing the incumbent. This means that we can invoke the "grim" punishment of $a = 1$ forever without fear that the entrant will fail to carry it out. The interpretation of this secondary punishment is also reasonable; if the incumbent fails to make good on his threat, it collapses and the game returns to it one-shot equilibrium forever (entry occurs). The condition that the incumbent be willing to play the punishment p under this threat is independant of the degree of cooperation of the entrant:

$$(13) \qquad p \geq 1 - \frac{\vartheta(1 - a^*)}{(1 - \vartheta)f}$$

To see whether there exists a perfect equilibrium with stationary outcome (1,a*), it is thus sufficient to see whether there exist punishments satisfying (11), (12), and (13).

First, suppose that the punishment sequences specify maximal cooperation by the entrant, so that he always plays $\alpha = 0$ during punishment. In this case, there will exist a sufficient "first punishment" satisfying (13) iff:

$$(14) \qquad 1 - \frac{\vartheta(1 - a^*)}{(1 - \vartheta)f} \leq \frac{(1 + \xi)(a^* + g) - g - 1}{\xi g}$$

The condition for "second punishments" is in general more stringent:

$$(15) \qquad 1 - \frac{\vartheta(1 - a^*)}{(1 - \vartheta)f} \leq \frac{\xi(a^* + g) - 1}{\xi g}$$

unless $a^* = 0$, in which case they are identical. Therefore, we can say that (1,a*) is a perfect equilibrium stationary outcome if (15) is satisfied, and that it is not unless (16) is satisfied.

In general, we may have to specify less-than-full cooperation by the entrant during punishment rounds, choosing α and α'' so that:

$$(16) \qquad \xi\alpha'' + (1 - \xi)\alpha = a^*$$

in which case conditions (11) and (12) will be identical. This means that we can restrict attention to the choice of, say, α, restricted by individual rationality. Since these are virtual punishments, what is important is that the payoff in each round give the entrant at least as much as he could secure by continued defection. To evaluate this, we must return to the actual strategies (rather than the virtual ones). If the entrant cooperates, his payoff will be as calculated with the virtual punishments. If he continues to defect, he will in effect be punished forever by continued play of $p = 0$. This forms part of a perfect equilibrium since the incumbent never expects to continue playing $p = 0$ forever, but only for a sufficiently long time to punish the entrant, which he is by assumption willing to do. The individual rationality condition for the entrant is thus:

$$(17) \qquad a^* \geq 1 - \xi g$$

Finally, by feasibility α is nonnegative, so that the condition for existence of a perfect equilibrium stationary outcome of $(1,a^*)$ is condition (14) above.

In particular, entry at any positive level can be credibly forestalled iff:

$$(18) \qquad \frac{g}{f} \geq \frac{(1 - \vartheta)}{\iota\xi}$$

This is particularly interesting in that it implies a comparison of both the vulnerabilities (f,g) and the degrees of myopia (discount factors) of the two participants, and as we might expect, the incumbent does better if he is less vulnerable or less myopic.

As a final remark, it is worthwhile to remember that the perfectness concept used here corresponds to what Selten [1975] has called "subgame perfectness". There are other perfectness concepts that are frequently used; among the most common of these is what is often called "trembling-hand" perfectness. This is a normal-form concept which means that the equilibrium in question is a limit of "ε-perfect equilibria" as $\varepsilon \to 0$. An

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