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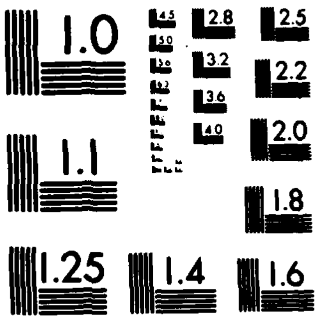

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TEST OF POISSON FAILURE ASSUMPTION

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CONTENTS

TEST OF POISSON FAILURE ASSUMPTION

Chapter 1. INTRODUCTION

1.1 Background.

In stockage models currently in use, the assumption is made that replacement of repair parts follows the Poisson process. A Poisson process is loosely described as a completely random sequence of events; an event can occur at any time, independent of when the previous event occurred, but subject to the restriction that the mean rate of events is constant. Throughout this report a Poisson process will mean a homogeneous Poisson process.

A necessary[1] condition for a Poisson process is that the number of events (failures) occurring within an interval of length L is Poisson distributed with mean λL, where λ is the constant mean rate of events (failure rate, hazard rate) of the process. This property is the basis of various statistical tests used in this study to detect non-Poisson processes. Rejection of a Poisson distribution or rejection of a constant failure rate leads to rejection of a Poisson process.

A Poisson process is retained over superposition and separation. If individual item failures were Poisson, the aggregate of failures over an inventory of items would also be Poisson. In a multi-echelon supply system, if demand at the lowest level is Poisson, and a fixed fraction of each demand is sent to the next higher supply level, then the resulting net demand streams at both levels are Poisson. Under given conditions, a Poisson stream should be preserved up and down the supply system and across levels of item aggregation. A mathematical justification of these ideas can be found in Chapter 9 of [10]. These properties of the Poisson process make it convenient for modelling inventory systems.

1.2 Previous Studies.

Empirical studies have indicated that demand, particularly at the support level, is not always Poisson distributed, and that real world supply systems are not so mathematically well behaved as the theory would imply. Johnson and McCoy [8] examined arrivals of failed aircraft parts at the

[1]This is a sufficient condition if it is true for any subset B of R+ which is the union of a finite number of disjoint intervals whose lengths sum to L [1].

depot. They point out that these might not necessarily reflect failure data; batching requirements to the depot could make the depot demand more lumpy than the original failures.

Metzner [13] looked at lower level maintenance actions for aircraft spares, and he suggests that peaks at the unit might be smoothed out by the time they reach the depot. This would imply an effect opposite to batching.

Proschan [15] saw decreasing failure rates for pooled air conditioning failures on a fleet of airplanes. He presents a theoretical explanation for this, based on the aggregation of failure patterns for individual airplanes with constant, but different, failure rates.

An Air Force study [14] tested almost 10,000 aircraft parts (mostly electronic) using the VMR test used here, and concluded that "the Poisson distribution provides a reasonable fit to almost all items' arrival patterns." The study applies the VMR test to each item individually (see section 3.4 of this report), and almost all of their items had zero or one demand during an observation period.

Factors such as vehicle age, usage rate, and accumulated usage have been considered as explanations for non-Poisson demand, in that failure rates were shown non-constant over time. Rosenman and Hoekstra [17] found that demand is age dependent. Gotwals and Hutchison [7] studied all three factors and found the functional relationship between replacement rate and odometer to be proportional to the square root of the odometer. Galliher and Wilson [6] recommend consideration of age and usage in demand forecasting.

1.3 Intention.

Ideally we would like to test the hypothesis that individual part failures are generated by a Poisson process at the lowest supply level, because the models currently in use for Army systems, for example SESAME (Selected Essential Item Stockage for Availability Method) [9], and Combat PLL/ASL [3], make this assumption. This hypothesis could not be approached directly due to the nature of the data available. However, we can draw some implications about individual part failures from the results of looking at end item failures, aggregated over a fleet of items. As a result, we can not hope to validate the models, but hope that this exploratory data analysis might provide useful insights.

The possible relationships between end item and part failure processes are as follows:

(a) If part failures are Poisson, then end item failures will be Poisson unless there is a correlation between failures of different parts occuring at different times. For example, failure of part A induces failure of part B at some future date. This is not considered likely since if such a relationship were expected both parts would probably be replaced together.

(b) If part failures are not Poisson, the aggregate failures of a fleet could be Poisson distributed. Suppose the part failures constitute independent renewal processes. This means that the times between successive failures of a particular part are independent identically distributed random variables. Drenick [5] finds that in this case the aggregate failure process will approach the Poisson.

(c) If neither part failures nor end item failures are Poisson distributed, then the aggregated part failures would not appear Poisson; tests based on this data will detect the non-Poisson failure character of the parts.

Another intent of the study of end item failures is to determine which "time" to base failure rates on. One of the critical properties of the Poisson process is that the failure rate is constant over time. If calendar time is used, then intuitively the failure rate will vary between time of use and non-use, and hence generate a non-Poisson process. Observations taken over calendar time confirmed intuition. A constant failure rate is more likely if usage (miles driven, rounds fired, or hours operated) is used as the time measurement. A constant failure rate in this context implies that failures are independent of the aging process during the time that observations were taken.

1.4 Overview.

Maintenance data for several end items as recorded in the field were tested for Poisson distribution using graphic and statistical techniques. Out of curiosity, calendar time results were computed at the tactical unit level to see if failures at a unit would exhibit different variance than failures for an end item over all units.

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Chapter 2. DATA

2.1 Source.

Data used were from Sample Data Collection Plans developed under provisions of Army Regulation 750-37, Sample Data Collection - The Army Maintenance Management System (SDC-TAMMS). The data were received from the US Army DARCOM Materiel Readiness Support Activity (MRSA). A plan for any one weapon system included from one to three years of requests for maintenance and maintenance accomplishments at organization and support levels.

2.2 Weapon Systems and Supporting Units.

Data tapes were received for the following plans:

SDC Plan	Item/System	Years	(Ending)
01R	Generator Set, 60 KW, 60HZ	1	(1973)
03R	Generator Set, 60 KW, MEP115A	1	(1977)
09E	Truck, Trac., 5-Ton Commercial	2	(1975)
02E	Truck, 1-1/4 Ton M561 (Gamma Goat)	3	(1975)
02C	Sheridan M551	2	(1974)
01C	Carrier M114A1E1	1	(1973)
01E	Truck, Utility 1/4 Ton M151A1/A2	2	(1974)
03E	Truck, 5 Ton M809 Series	1	(1973)
04E	Truck, 2 1/2 & 5 Ton Avg Use Life	5	(1978)
05E	Goer, M520, M553, 6M559	2	(1976)

Of the ten weapon systems available only four were chosen to study. The first five listed consist of only a single end item type each, and were chosen for processing only because of simplicity. The remaining plans include multiple models. Limiting our work to single end item plans saved a decision on whether to aggregate data over different models of the same system, and avoided dealing with different usage measures for multiple end items on one system. The 01R Generator Set did not have much data (44 failures for 103 generators), thus results for this plan are not included. The remaining data are available if more extensive processing and analysis are deemed worthwhile.

The 03R Generator Set plan included 142 end items supported at five units:

WD27, USAREUR, 29 items

WH15, USAREUR, 32 items

WH17, Korea, 22 items

WH18, USAREUR, 32 items

WH1C, Ft. Bliss, 27 items

These figures are from SDC Plan Analysis Report published by MRSA. Removal actions at the organization level were reported on 137 items, the other five having only higher level removals. (See Section 2.4, Data Selection)

The 09E Truck plan included 107 trucks supported at two units:

WCNM, 1st Trans. Co., 37th Trans. Grp USAREUR

WCN6, 89th Trans. Co., 37th Trans. Grp USAREUR

The 02E Gamma Goat plan included approximately 600 items at 24 units. This plan did not have a consistent set of items over the plan duration. Three of the units which appeared to have consistent data throughout the reporting period were chosen to compute calendar time results by tactical unit:

WAA5, 1st Sqdn, 17th Cav (Air), 82nd Abn Div, Ft. Bragg,NC

WABG, 307th Eng Bn (Abn), 82nd Abn Div. Ft. Bragg, NC

WABJ, 1st Bn, 319 FA (Abn),82nd Abn Div Arty, Ft. Bragg,NC

Results in usage, for which a consistent set of vehicles over time is not necessary, did include data from all 24 units. (See Section 3.3, Usage Intervals)

The 02C Sheridan plan comprised about 350 tanks at 13 units. Again, the plan did not cover a consistent set of tanks, and was treated as the 02E with regard to calendar time. The units chosen to study were:

WAJH, 4th Sqdn, 7th ACR, 2d Inf Div, Tokkae, Korea

WAZC, 4th Bn, 68th Armor Reg, Ft. Bragg, NC

WG2G, 1st Sqdn, 2d Armd Cav Reg, Bindlach, FRG

WG2H, 2nd Sqdn, 2d Armd Cav Reg, Bamberg, FRG

WG2J, 3rd Sqdn, 2d Armd Cav Reg, Amberg, FRG

W1L4, 1st Bn, 1st Bde, USATC-AR, Ft. Knox, KY

Usage data for the Sheridan is in rounds fired as well as miles travelled. To include this plan in the usage results, assumptions would have to be made concerning the applications of the various parts. Due to the fact that a part might be affected by rounds, or miles, or both, we did not include the plan in the usage results.

2.3 Units.

We sought to investigate the data at the lowest possible supply level, i.e. the company. Many battalions, however, do all maintenance at headquarters. Moreover, not all companies which do perform maintenance have distinct unit identification codes, in which case they use the battalion code. To maintain consistency, battalion was the lowest unit level we could consider.

Each tactical unit holds stock and does some of its own repair. When the tactical unit is viewed as part of the Supply System it is called the organization (ORG). Each division level is supported by a supply unit called the direct support unit (DSU), which holds stock in support of the units in its division and performs maintenance beyond the capacity of the tactical unit. In reference to the Supply System we will refer to ORG and DSU.

The data were examined by unit to check for obvious gaps, redundancies and other anomalies, by comparing these figures with those in the analysis reports published by MRSA. As mentioned previously, only those units which reported removals throughout the duration of the plan were included in the results for Poisson in time. All units were included in the usage results.

2.4 Data Selection.

Removal actions were selected from all the maintenance data. Actions were limited to those occurring at the organization level (ORG) units. ORG removals which are sent to the DSU for repair were not lost as they are still coded as removals at the ORG.

Only unscheduled removals were selected as it was felt that these best represent item failures. Scheduled maintenance should occur on a regular basis and should not contribute any variance to the demand process. Examination of the data for the 03R Generator Set reveals unscheduled actions at quite regular usage intervals, for example five actions at regular 100 hour intervals. It is possible that the nature of a generator set precipitates a regular failure pattern; it is also possible that the coding of scheduled vs unscheduled does not reflect what we would expect.

Data elements selected for each action were the end item serial number, the date of the removal, the accumulated usage of the end item (at removal), and the unit where the action occurred.

The accumulated usage is an odometer reading (for the vehicles) and does not necessarily reflect the usage from when the item was new. Usage per vehicle is the additional usage beyond the accumulated usage recorded at the first removal action in the plan for that vehicle, i.e. usage for a vehicle was computed as the difference between accumulated usage measures recorded at the first and last removal actions.

2.5 Formation of Time Series.

The data were converted into time series of removals, one series for each end item and one for each unit supporting the item. Each time series is a chronological list of removal dates.

These series were screened to eliminate multiple part removals. Several removals from the same serial number with the same date were reduced to one, as they were considered the result of one failure. Examination of the part data showed instances of parts which were always removed as a group, e.g. a nut, bolt, and washer. Distributions of the number of parts per action are included in Appendix A. Quantities, e.g. removal of six screws, were not considered.

2.6 Formation of Usage Series.

The same data were selected as for the time series, and, again, based on the removal date, multiple part removals were reduced to one removal action. In this case each series is a list of the accumulated usage recorded at the time of each removal action, rather than a list of the removal dates.

Whereas there was a time series for each unit (and one for the System over all when the data were complete), there is a separate usage series for each individual vehicle (serial number). Aggregation over vehicles was not done at this stage in the processing because tests were performed on various subsets of the items based on available durations of usage. This will be explained in Section 3.3, Usage Intervals.

Chapter 3. METHODOLOGY

3.1 Background.

To appreciate the techniques used, some characteristics of the Poisson distribution should be understood. Brief descriptions will be included here and more detailed explanations can be found in a text on probability and statistics, e.g. [16].

There exists a relationship between a Poisson process and the exponential distribution. When events in time are Poisson, the time between events is distributed exponentially. (In general the independent variable is called "time".)

Let λ be the mean number of events per unit time. The actual number of events in a time interval of length t is a random variable with a Poisson distribution. The probability of k events occuring in t is given by

$$P(k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!} \quad (k = 0,1,2\ldots).$$

The probability density function of the inter-event time is given by $f(t) = \lambda e^{-\lambda t} (t \geq 0)$; hence, the probability that an inter-event time is greater than t is given by $G(t) = e^{-\lambda t}$.

The variable t is not restricted to calendar time. We also consider removals in usage, in which case t is an interval of usage, e.g. a mile. The "unit" refers to the smallest usage (or time) interval considered, and need not equal one.

The rate of Poisson events is constant; hence, an event occurs in each unit interval with equal probability. This feature facilitates the computations required for the variance to mean ratio (VMR) test. (See Section 3.4, The VMR Test.) Moreover, the variance of a Poisson distribution equals its mean; hence, the VMR equals 1.

3.2 Calendar Time.

To test for the Poisson distribution, the series of dates described in Chapter II was used to calculate a distribution of the number of actions in a fixed time (or usage) interval. A two-day time interval was initially considered since that value is a typical ORG order and ship time. For items with three years of data, or even one year, the number of two-day intervals proved cumbersome. Results presented are for intervals of one week, except for the Gamma Goat, for which two weeks was used. In some instances, specified in the results, the last few available time intervals were not included. A lack of observations lead us to believe that reporting did not last the supposed duration of the plan.

3.3 Usage Intervals.

Poisson in usage means that the number of events per fixed interval of usage is Poisson distributed. If a vehicle travels 2000 miles and we choose a fixed interval of 200 miles, we have 10 intervals to consider. We look at the number of removal actions in each of the 10 intervals.

Each vehicle (serial number) has a different total usage. "Vehicle" and "miles" are used as generic names in this report and could mean "generator set" and "hours." If the removal actions are to be accumulated over all the vehicles, total usage considered must be the same for each vehicle. For example, suppose we look at intervals of 200 miles each, and the number of removals in each interval.

Vehicle 1:	3 2 1 3 1	total usage = 1000 miles
Vehicle 2:	3 2 2 3 1 1 3 4 2 1	total usage = 2000 miles
Accumulated:	6 4 3 6 2 1 3 4 2 1	

The usage intervals in the accumulated case are not equally represented.

To solve this problem we could exclude vehicles with less than a prescribed total usage, or could limit the usage period investigated to the minimum amount achieved by all the vehicles. What we did was to test a range of possibilities between these two extremes.

For example, the 09E plan included 107 trucks; 102 of these went 2000 miles, and only 7 went 60000 miles. The 7 are included with the 102, but in results for the 7, only actions in the first 2000 of the available 60000 miles were considered.

Within a set of vehicles with a prescribed total usage, results for various interval sizes were compared. For the 102 trucks which travelled 2000 miles, the mileage could be divided into 10 intervals of 200 miles each, or 100 intervals of 20 miles each. Again, results were computed for a range of possibilities.

In all cases, the first tenth of the usage intervals was not included in the VMR statistics, as the first cell, which started with the initial action, always had at least one action. The first 20 of 200 cells were excluded to be consistent with excluding the first 1 of 10.

3.4 The VMR Test.

The VMR (variance to mean ratio) test is also known as the Poisson distrbution test [14], and the Poisson variance test [2,18]. The VMR is related to a χ^2 distribution, but the test is not subject to the large sample restriction applied to χ^2 tests in general, and is more sensitive than the χ^2 goodness of fit test to the alternative hypothesis that observations follow independent Poisson distributions, but with different means [2].

Count the number of removals in the ith interval.

Let

f_i = frequency in the ith interval

k = number of intervals

$m = \Sigma f_i/k$ (the mean)

Then

$$\chi^2 = \frac{\sum_{i=1}^{k} (f_i-m)^2}{m} \quad \text{with k-1 d.f.}$$

The numerator is (k-1) times the sample variance of the distribution; the denominator is the observed mean. Hence χ^2 = (k-1) VMR.

As mentioned in the introduction, the Poisson is a one parameter distribution with variance equal to the mean. As a result, the VMR for a true Poisson equals 1, and our assessment of a series is based on how close to 1 the VMR is.

The relationship of the VMR to the χ^2 was introduced to formalize the test and put confidence limits on the VMR statistic. VMR = $\chi^2/(k-1)$ so we can look up the VMR in a χ^2 table normalized by the d.f. (k-1) to get limits for an acceptable range for the VMR. Limits based on a 95 percent two-sided confidence interval are included with the results. We will reject a Poisson distribution with probability 0.05, 0.025 on each side.

3.5 The Kolmogorov Goodness of Fit Test.

To test the relationship between a sample distribution and a specified theoretical distribution, the Kolmogorov test applies to the vertical distance between the empirical and theoretical cumulative distribution functions. The Kolmogorov test was chosen when we were looking for a way to quantify the graphic results which tested for exponential inter-removal times. The test is simple, so its use was extended to testing directly for Poisson;

conservative for discrete distributions, and as such provides a check on the VMR results.

The test Kolmogorov statistic, T, is the largest vertical distance between two graphs S(x) and P(x), where S(x) is the empirical distribution function and P(x) is the hypothesized distribution, in this case the Poisson. Large values of T as determined by a table lead to rejection fo P(x) as a reasonable approximation to the data, S(x).

This is a two-sided test with null hypothesis H : S(x) = P(x) for all x from $-\infty$ to $+\infty$. It requires us to reject the Poisson regardless of whether the empirical distribution is above or below the theoretical. When parameters of the distribution are estimated from the sample, as was done here, the rejection region for the null hypothesis is larger than for tests where the parameters are specified a priori. More details on this can be found in [2,11,12]. For hypothesis tests with a priori specified parameters, if T > Maximum Rejection Region (MRR), reject with 95 percent confidence, i.e. if T = 0.35 and MRR = 0.19 we can reject the Poisson. We can assume that we will make a mistake, i.e. reject a series which is Poisson distributed, with probability 0.05 or less. For a sample size *N greater than* 40, the maximum rejection region at the 95 percent confidence level is approximated by $1.36/\sqrt{N}$, which is the value included with the results. Thus our results are more conservative than if the parameters had been specified a priori.

Chapter 4. RESULTS

4.1 Results in Calendar Time.

Table 1 contains the results of testing for Poisson in calendar time. For each test set, the table includes number of time periods, VMR, mean, and Kolmogorov Test Statistic. The acceptable range for the VMR and the maximum rejection region (MRR) for the Kolmogorov Test Statistic (for each of the applicable number of time periods) are included with the results. These regions are based on $\alpha = 0.05$, where α is the probability of rejecting a true hypothesis. In each case the degrees of freedom is the number of time periods minus 1.

For the generator set and truck, with the exception of unit WH17, unit results were less variable than overall weapon system results. Recall that the overall results were computed by combining actions for all units, not by combining the individually computed statistics. An average of the five individual generator set VMR, weighted by their means, equals 3.4, less than the overall VMR of 5.

4.2 Results in Usage.

Results are presented for a number of item sets, with total usage of each set determined by the minimum usage vehicle in the set (see Section 3.3, Usage Intervals). For each set, a range of results is included, based on varying the size of the usage interval. The acceptable ranges for the VMR are included with the results, and are based on $\alpha = 0.05$ as for the results in time. For the usage results, however, df = (9/10)(no. of intervals) -1.

Table 2 presents results for three subsets of the generators. The Generator Set (03R) data include 2666 removal actions on a total of 137 serial numbers (see Appendix A). Of the 137 generators, 56 ran fewer than 1000 hours and are not included in these results. Each of the remaining 81 ran at least 1000 hours, and these constitute the first subset of items. Of the 81 that ran at least 1000 hours, 50 ran at least 2000 hours, and 4 of these ran for 4000 hours. Tables 3 and 4 present similar subsets for the truck and Gamma Goat. Within each set the VMR corresponding to the interval size closest to the average 2-day usage (about 13 hours for the Generator Set) is circled.

4.3 Observations.

From Table 1 (Results in Calendar Time), we can see that of all the units, only two (supporting the generator set) have VMR and Kolmogorov statistic within the acceptable regions. For all others we can reject the hypothesis that the data are Poisson. We were pleased, at least, that in all cases both the VMR and the Kolmogorov tests showed the same result.

The following points may be made from the results in Tables 2, 3 and 4 (Results in Usage):

a. For sets with a large number of items and short usage duration we cannot reject the Poisson distribution. This applies to all the weapon systems.

b. For the Truck, for a few (7) serial numbers with extensive (60,000 mile) usage, we cannot reject the Poisson.

c. The Generator Set has some low VMRs.

d. The Truck and Gamma Goat have some high VMRs when extensive usage periods are considered for a smaller number of items.

It should be noted that what we are calling "high" VMRs in usage are higher than the upper bound specified in the χ^2 table, but are much lower than those for calendar time, and are lower than observed in previous studies.

4.4 Summary of Results.

As pointed out in Chapter 1, we hoped to provide insights into the actual failure process rather than to prove or disprove a Poisson assumption. Several points of interest are raised by the statistics in the tables.

We did not expect our results to vary with the interval size. The question of what length to make the interval was not given serious thought until working with usage intervals showed that the test results vary depending on the interval. The results in time were so far from Poisson that additional analysis for alternative time intervals was not considered worthwhile. Usage results do not support the hypothesis that increasing the number of intervals brings the VMR closer to 1.

As we expected, failures are less variable over usage than over calendar time, and failures at each unit are less variable than those for an entire fleet across units. While not all the data pass the VMR test for a Poisson distribution in usage, few of the VMR were larger than 2.

As discussed in Chapter 1, the methodology used here, while powerful in detecting certain kinds of deviation from the Poisson, will not detect the presence of non-Poisson renewal processes in highly aggregated systems. For higher density systems, aggregation of individual system failures will make the total unit level system failure process approach Poisson, attenuating the impact of the individual non-Poisson renewal processes.

For higher density systems such as those examined here, most inventory models would be concerned with the aggregate failure process at the unit, not the individual system, so that if aggregation is causing Poisson-like behavior this is a step in the right direction.

TABLE 1. RESULTS IN CALENDAR ITEM

Unit	# Time Periods	VMR	Mean	Kolomogorov Test Statistic
03R Generator Set				
WD27	48	1.0	8.9	0.08
WH15	48	1.3	10.1	0.13
WH17	52	6.2	11.6	0.35
WH18	48	5.0	20.8	0.31
WH2C	52	3.3	2.8	0.21
Overall	52	5.0	51.3	
VMR Acceptable range:	(48) 0.64 - 1.44			MRR* (48) 0.20
VMR Acceptable range:	(52) 0.65 - 1.42			MRR (52) 0.19
09E Truck				
WCNM	104	3.0	12.1	0.19
WCN6	104	3.1	5.6	0.21
Overall		3.2	18.5	
VMR Acceptable range:	(104) 0.75 - 1.29			MRR (104) 0.13
02E GAMMA Goat				
WAA5	78	5.0	4.0	0.26
WABG	78	3.8	4.8	0.23
WABJ	78	8.3	3.4	0.35
Overall Not Computed (see Section 2.2, Weapons Systems & Supporting Units)				
VMR Acceptable range:	(78) 0.71 - 1.34			MRR (78) 0.15
02C Sheridan Tank				
WAJH	52	4.0		0.21
WAZC	52	5.4		0.21
WG2G	52	8.6		0.31
WG2H	52	17.1		0.36
WG2J	52	16.7		0.39
W1L4	52	5.1		0.26
Overall Not Computed (see Section 2.2, Weapons Systems & Supporting Units)				
VMR Acceptable range:	(52) 0.65 - 1.42			MRR (52) 0.19

*Maximum Rejection Region

TABLE 2. RESULTS IN USAGE FOR THE GENERATOR SET

	No. Of Intervals	Interval Size	VMR	Mean
81 Items With 1000 Hours (776 Removals)				
	10	100	0.38	77.556
	20	50	0.54	38.778
	40	25	1.1	19.389
	100	10	1.1	7.756
50 Items With 2000 Hours (1004 Removals)				
	10	200	0.58	100.444
	20	100	0.40	50.222
	40	50	0.68	25.111
	100	20	0.70	10.044
4 Items With 4000 Hours (124 Removals)				
	10	400	5.3	12.444
	20	200	3.1	6.222
	40	100	2.0	3.111
	100	40	1.2	1.244

VMR Acceptable range (10)	0.3 - 2.2
VMR Acceptable range (20)	0.4 - 1.8
VMR Acceptable range (40)	0.6 - 1.5
VMR Acceptable range (100)	0.7 - 1.3

Within each set the VMR corresponding to the interval size closest to the average 2-day usage (about 13 hours for the Generator Set) is circled.

TABLE 3. RESULTS IN USAGE FOR THE TRUCK

	No. Of. Intervals	Interval Size	VMR	Mean
102 Items With 2000 Miles (69 Removals)				
	10	200	0.85	6.889
	20	100	0.62	3.444
	40	50	1.3	1.722
	100	20	1.2	0.689
95 Items With 10,000 Miles (396 Removals)				
	10	1000	3.5	39.556
	20	5000	4.9	19.778
	40	250	3.5	9.889
	100	100	2.0	3.956
	200	50	1.6	1.978
	400	25	1.4	0.989
	1000	10	1.1	0.395
35 Items With 40,000 Miles (756 Removals)				
	10	4000	1.4	75.556
	20	2000	1.7	37.778
	40	1000	1.8	18.889
	200	200	1.5	3.778
	400	100	1.2	0.378
7 Items With 60,000 Miles (204 Removals)				
	10	6000	0.33	20.444
	20	3000	0.64	10.222
	40	1500	0.76	5.111
	100	600	0.98	2.044
	200	300	1.00	1.022
	400	150	0.97	0.511

VMR Acceptable range (10)	0.3 - 2.2
VMR Acceptable range (20)	0.4 - 1.8
VMR Acceptable range (40)	0.6 - 1.5
VMR Acceptable range (100)	0.7 - 1.3
VMR Acceptable range (200)	0.8 - 1.2
VMR Acceptable range (400)	0.8 - 1.2
VMR Acceptable range (1000)	0.9 - 1.1

Within each set the VMR corresponding to the interval size closest to the average 2-day usage is circled.

TABLE 4. RESULTS IN USAGE FOR THE GAMMA GOAT

	No. Of Intervals	Interval Size	VMR	MEAN
383 Items With 500 Miles (404 Removals)				
	10	50	0.49	40.444
	20	25	0.74	20.222
	100	5	1.3	4.044
272 Items With 1000 Miles (477 Removals)				
	10	100	1.5	47.667
	20	50	1.4	23.833
	40	25	1.7	11.917
	100	10	1.3	4.767
	200	5	1.4	2.383
141 Items With 2000 Miles (381 Removals)				
	10	200	2.3	38.111
	20	100	1.9	19.056
	40	50	1.7	9.528
	100	20	1.6	3.811
	200	10	1.7	1.906
40 Items With 4000 Miles (187 Removals)				
	20	200	2.2	9.333
	40	100	2.1	4.667
	100	40	1.5	1.867
	200	20	1.3	0.933

VMR Acceptable range (10)	0.3 - 2.2
VMR Acceptable range (20)	0.4 - 1.8
VMR Acceptable range (40)	0.6 - 1.5
VMR Acceptable range (100)	0.7 - 1.3
VMR Acceptable range (200)	0.8 - 1.2

Within each set the VMR corresponding to the interval size closest to the average 2-day usage is circled.

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APPENDIX A

DISTRIBUTIONS FROM DATA PROCESSING

	Gen Set 01R	Gen Set 03R	Truck 09E	Gamma 02E	Sheridan 02C
Usage	hours	hours	miles	miles	miles, rnds
Actions read	3291	14365	19959	12001	44733
Removals	147	5757	3539	4920	13275
ORG Removals	95	5588	3251	4744	11185
# Actions	62	2666	1899	2971	5901
Avg # Parts/Action	1.5	2.1	1.7	1.6	1.9
Distribution of # Parts Per Action					
% With 1	53	51	67	64	61
% With 2	40	21	13	19	19
% With More	7	28	20	17	20
# Ser Nos (SN)	28	137	107	(600)	351
Avg # Actions/SN	2.2	19.5	17.7	(5.0)	16.8

APPENDIX B

GRAPHIC RESULTS

When failures in time are Poisson distributed the time between failures is distributed exponentially. A distribution of inter-failure times was easily calculated from the series of dates described in Chapter II. The probability that an inter-failure time is greater than t is the fraction of these inter-failure times which is greater than t and should equal $e^{-\lambda t}$. Let this fraction = G(t). Then $G(t) = e^{-\lambda t}$, and $\log G(t) = -\lambda t$. Therefore, when G(t) is plotted against t on semi-log paper, the result should be a straight line with slope $-\lambda$.

For each unit the empirical probability that an inter-failure time is greater than t was plotted against t. Results are included for the 03R Generator Set and the 09E truck.

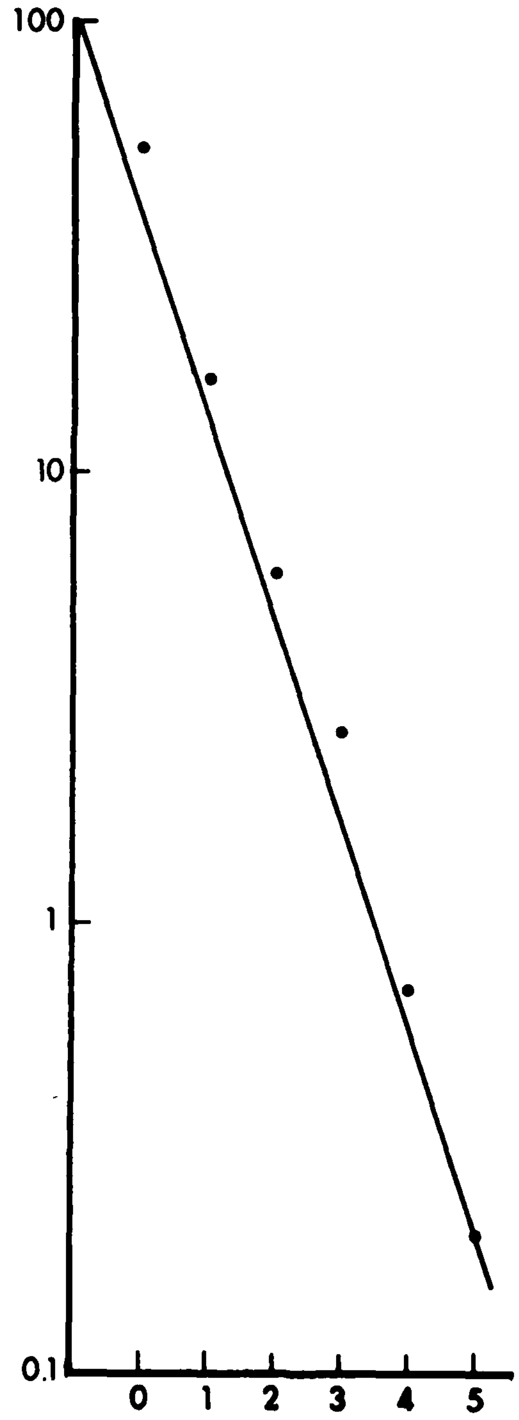
Generator Set, 5 Units: WD27, WH15, WH17, WH18, WH1C

Truck, 2 Units: WCNM, WCN6

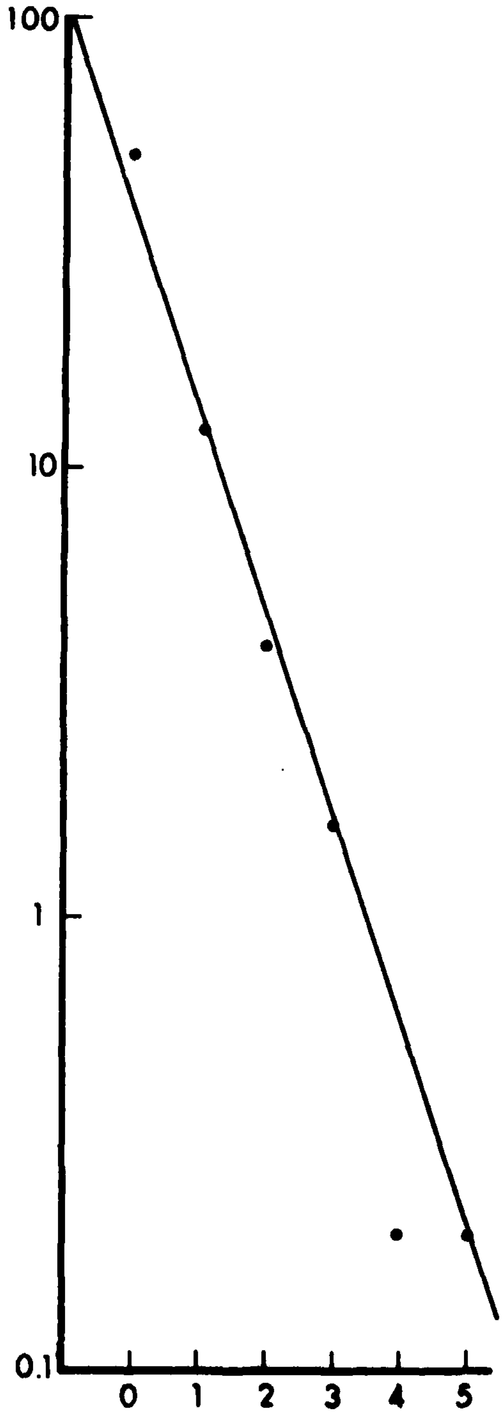
A common problem with the graphic technique is that data include days with more than one failure, resulting in inter-failure times of zero. Data are rarely recorded on more than a daily basis, but even if they were, accumulating removal actions over several items, it would be difficult to find a time interval small enough to preclude more than one action. Strictly speaking, no two Poisson events occur at the same time. Other authors choose to ignore this problem [6,15].

Another drawback of the graphic method was our wish for quantitative results; we were dissatisfied with eyeballing the graphs to determine if a straight line fit the points. In most cases the graphs look better than we would expect from the VMRs in Table 1, but our assessment of the graphs is purely subjective.

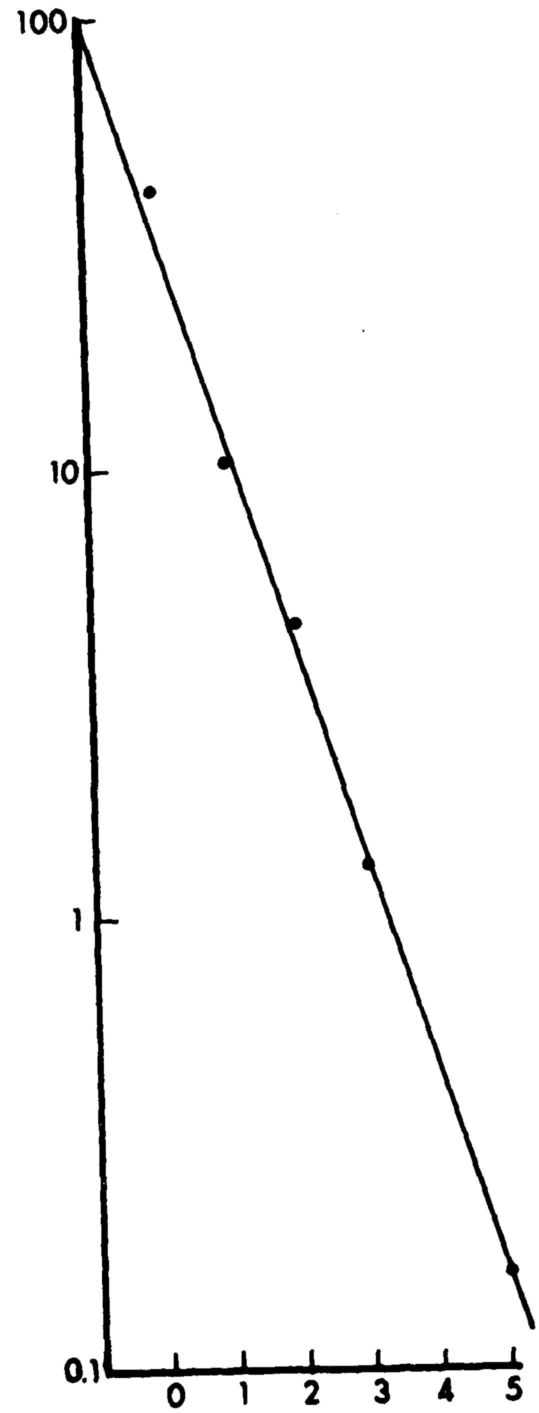
The problems with the graphic method were avoided by testing for Poisson distribution directly.



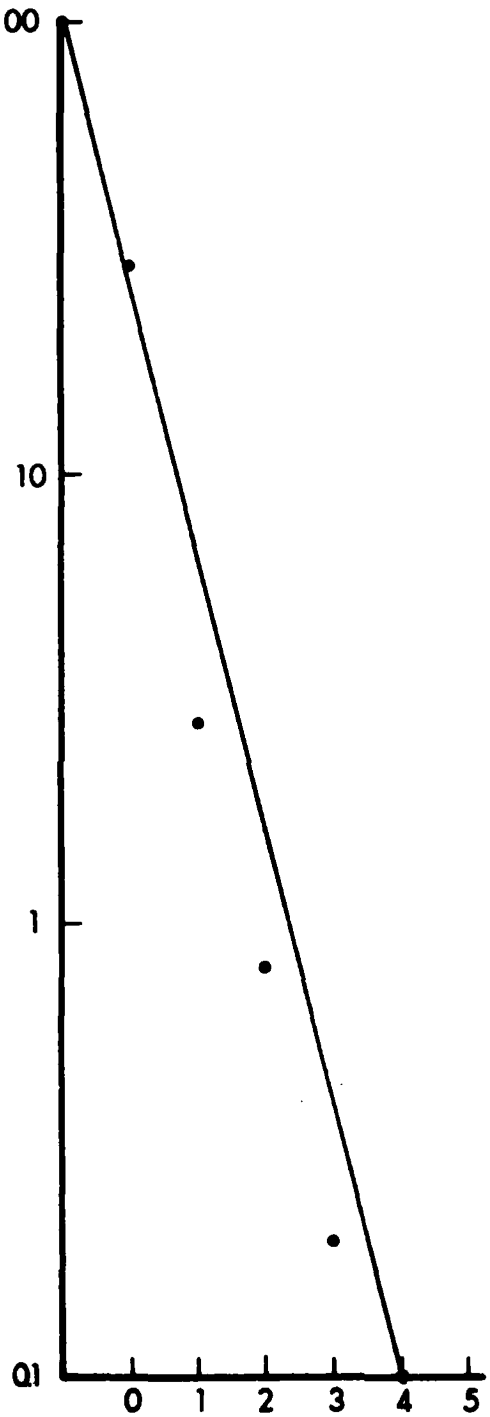
Percent Inter-Removal Times Greater than t Days for Generator Set, Unit WD27



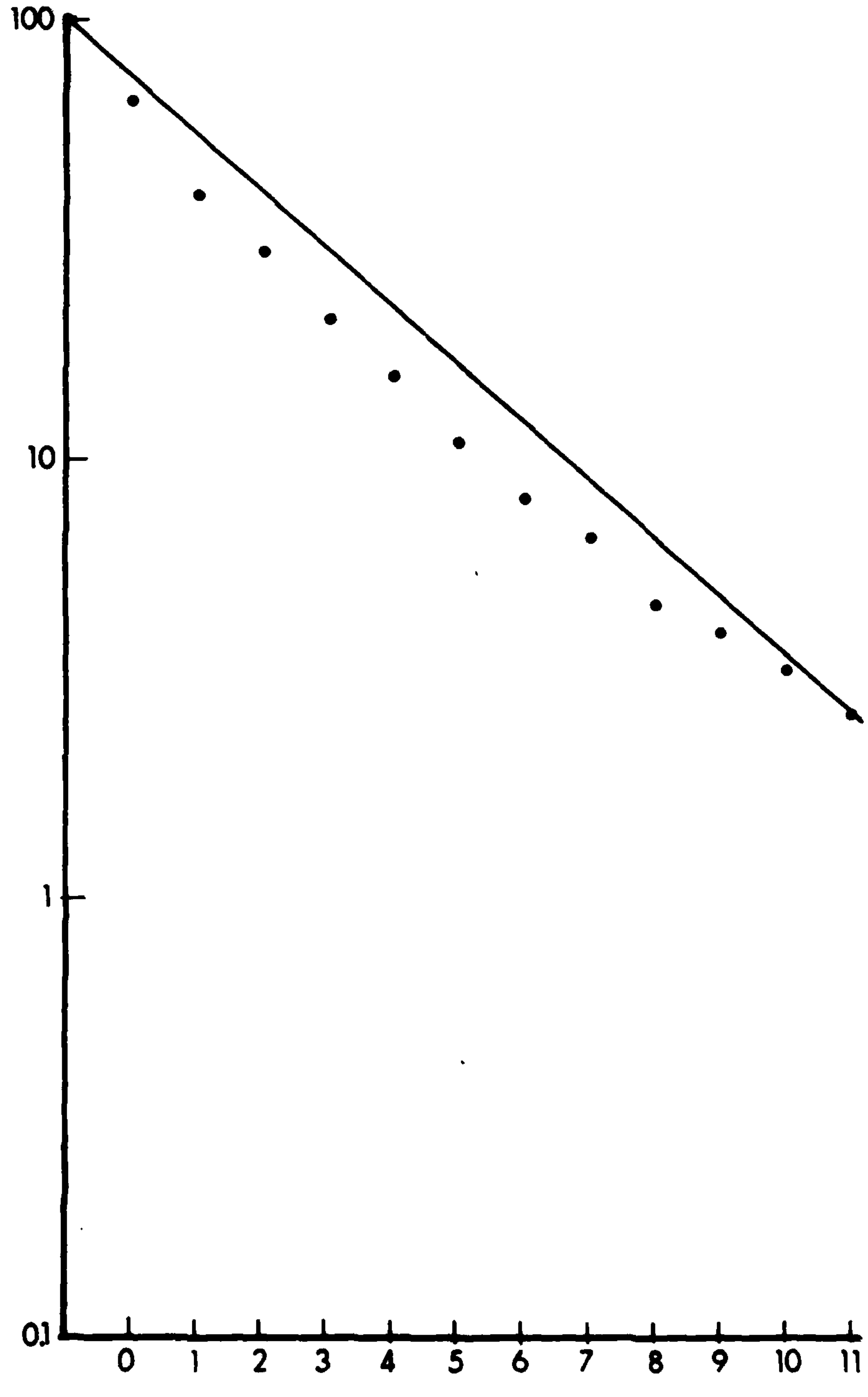
Percent Inter-Removal Times Greater than t Days for Generator Set, Unit WH15.



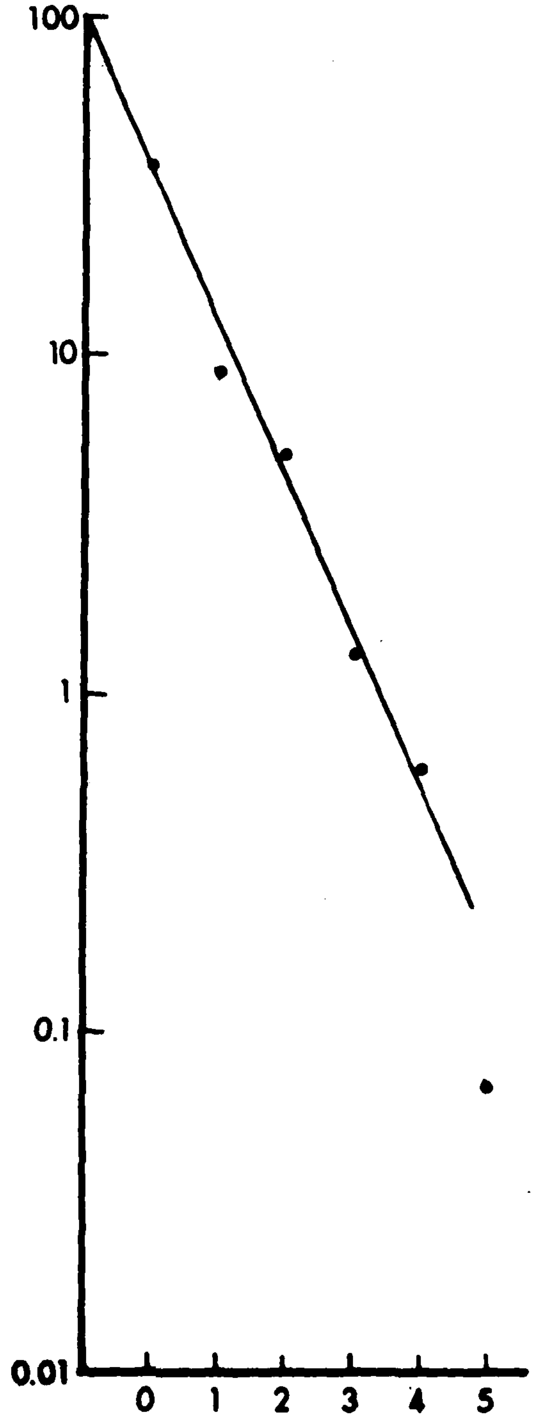
Percent Inter-Removal Times Greater than t Days for Generator Set, Unit WH17.



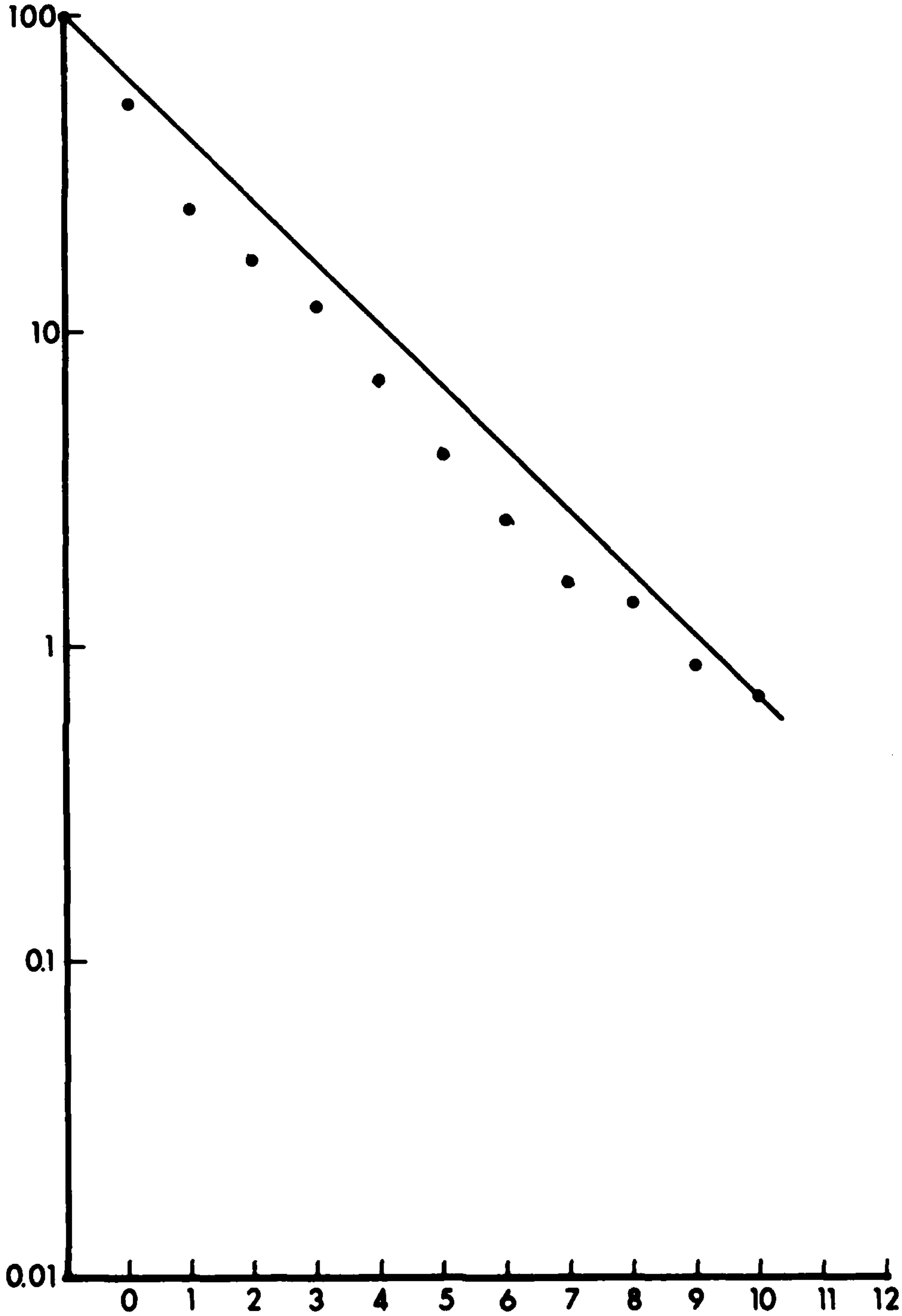
Percent Inter-Removal Times Greater than t Days for Generator Set, Unit WH18.



Percent Inter-Removal Times Greater than t Days for Generator Set, Unit WH2C.



Percent Inter-Removal Times Greater than t Days for Truck, Unit WCNM.



Percent Inter-Removal Times Greater than t Days for Truck, Unit WCN6.

APPENDIX C

FAILURE RATE OVER USAGE

A crucial assumption of the Poisson is that the failure rate is constant. To get a rough idea of the failure rate over usage, we divided the last 9 of 10 equal usage intervals into groups and looked at the number of failures in each group.

Below are the figures for the first 4 and last 4 intervals and the figures for the 3 groups when the 9 intervals are grouped into 3 groups of 3 each. Data for each system are overlapping.

These results are very encouraging as the number of failures appear close for most groups. Underlined are the item sets with the most number of failures. Some of these sets correspond to the item sets in the results.

Interval	# Items	1st 4	Last 4	3	3	3
03R Generator Set						
20	124	132	97	99	91	97
50	104	214	182	166	138	138
100	81	310	313	229	232	237
200	50	394	410	295	289	320
02E Gamma Goat						
10	474	93	61	74	44	42
50	383	163	162	126	116	122
100	272	197	198	155	126	148
200	141	148	145	113	116	114
09E Truck						
300	102	36	45	31	34	27
1000	95	164	154	109	111	136
4000	35	297	299	217	233	230
6000	7	86	76	65	66	53

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