





MICROCOPY RESOLUTION TEST CHART  
 NATIONAL BUREAU OF STANDARDS-1963-A

(R)

"PAIRWISE BALANCED" LATIN SQUARES SHOULD ALWAYS BE  
USED FOR WITHIN-SUBJECTS DESIGNS

AD A 137283

Thomas M. Ostrom  
Ohio State University

Paul D. Isaac  
Ohio State University

and  
C. Douglas McCann  
York University

Technical Report Number TR/ONR-10  
December, 1983

Social Psychology Bulletin-83-2,

DTIC  
ELECTRONIC  
S JAN 27 1984  
A

DTIC FILE COPY

Reproduction in whole or in part is permitted for any purpose of the United States Government. This report was supported by contracts on the Organizational Effectiveness Research Program, Office of Naval Research United States Navy (Code 452) under control No. N00014-81-K-0112, NR 170-927.

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

84 01 27 059

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TR/ONR-10	2. GOVT ACCESSION NO. 40-7137 883	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) "Pairwise balanced" Latin Square should always be used for within-subjects designs	5. TYPE OF REPORT & PERIOD COVERED Technical Report	
	6. PERFORMING ORG. REPORT NUMBER RF 762498/713444	
7. AUTHOR(s) Thomas M. Ostrom, Paul D. Isaac, and C. Douglas McCann	8. CONTRACT OR GRANT NUMBER(s) NO0014-31-K-112	
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Ohio State University Research Foundation 1314 Kinnear Road Columbus, Ohio 43212	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 170-927/10-15-82 (440)	
11. CONTROLLING OFFICE NAME AND ADDRESS Organizational Effectiveness Research Programs Office of Naval Research (Code 452) Arlington, VA 22217	12. REPORT DATE December 20, 1983	
	13. NUMBER OF PAGES 29	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified	
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Research design, repeated measures, Latin Squares, within subjects designs		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  The use of repeated measures designs in many areas of psychological research has prompted concern for the potential confounds inherent in the interpretation of treatments that have been included as within-subject variables. Of the solutions proposed for this problem, the most commonly adopted strategy is the use of Latin Square counterbalancing orders for treatment presentation. Traditional Latin Square designs ensure that each of the experimental treatments included as part of the within-subject factor(s) is		

DD FORM 1473  
1 JAN 73

EDITION OF 1 NOV 68 IS OBSOLETE  
S/N 0102-LF-014-6601

Unclassified  
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Block 20 (Abstract) - Continued

administered in each serial position of the treatment sequence. The present paper presents a discussion of a novel technique for the generation of a subset of Latin Squares that control for two additional features that are seen to be important in many research situations, i.e., pairwise priority and distance. Such Latin Squares are referred to as 'pairwise balanced' Latin Squares. The relative advantages of using such Latin Squares in repeated measures designs are discussed.

S-N 0102-LF-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

**'Pairwise Balanced' Latin Squares Should Always  
Be Used for Within-Subject Designs\***



**Running Head: Pairwise Balanced Latin Squares**

Administrative stamp area containing a grid with a checkmark and handwritten text "A1".

\*This work was supported by Contract N00014-81-K-0112, NR 170-927, Organizational Effectiveness Research Program, Office of Naval Research. The authors would like to thank Steven Breckler, Patricia Devine, Lisa Herron, David Kenny, John Lingle, Anthony Pratkanis, Thomas Pusateri, and Tamara Smith, for their comments on an earlier draft of this paper.

## Abstract

The use of repeated measures designs in many areas of psychological research has prompted concern for the potential confounds inherent in the interpretation of within-subjects effects. Of the solutions proposed for this problem, the most commonly adopted strategy is the use of Latin Square counterbalancing orders for treatment presentation. Traditional Latin Square designs ensure that each of the experimental treatments appear equally often in each serial position of the treatment sequence. The present paper presents a technique for generating a subset of Latin Squares that control for two additional characteristics of treatment sequence. Pairwise priority refers to the proportion of times that for any given treatment pair,  $x$  and  $y$ , Treatment  $x$  precedes Treatment  $y$ . A subset of Latin Squares exists for which this proportion is .5 for all treatment pairs. Pairwise distance refers to the number of other treatments that come between treatment pair  $x$  and  $y$  in the treatment sequence. A subset of Latin Squares exists that partially controls for the distribution of distances across all treatment pairs. The subset of Latin Squares that controls for both pairwise priority and pairwise distance are referred to as 'pairwise balanced' Latin Squares.

Within-subject designs are being used with increasing frequency in psychological research. For example, Poulton (1982) compared the types of experimental designs employed in research reported in the Journal of Experimental Psychology in September 1972 and 1979 and found that the ratio of within-subject to between-subjects designs had increased from 1.7: 1 to 7.3: 1. This increase has been accompanied by commentary and analysis concerning the adequacy of such within-subject designs to provide unambiguous tests of experimental hypotheses (e.g., Greenwald, 1976; Poulton, 1973, 1974, 1982; Rothstein, 1974). The central concerns embodied in these commentaries relate both to matters of experimental procedure and the proper interpretation of experimental results. Of course, the two are interrelated in that improvements in procedure often serve to lessen interpretive cautions.

The most efficient procedure for dealing with the interpretive problems of within-subject designs involves the use of Latin Square counter-balanced orders for treatment presentation (e.g., Lindman, 1974; Myers, 1979; Winer, 1972). The purpose of the present paper is to address the adequacy of traditional Latin Square selection criteria. The traditional criteria focus exclusively on guaranteeing that all treatments appear equally often in all serial order positions of the treatment presentation sequence. In this paper, we argue that two additional criteria should always be invoked when selecting a Latin Square, namely the criteria of "pairwise priority" and "pairwise distance".



Within-subject Treatments and Latin Square Designs

In within-subject designs, each subject is exposed to all of the experimental treatments. This type of design is often preferred because: a) it allows for greater economy in subject utilization, b) it often serves to increase the statistical power of hypothesis tests, and c) it is often a more ecologically valid way of examining specific research hypotheses (e.g., Greenwald, 1976, but see Poulton, 1982).

Although preferred for these reasons, within-subject designs are also encumbered by procedural weaknesses that often leave the research open to plausible alternative explanations for the obtained experimental results. Chief among these potential confounds are those associated with order or sequence effects, practice and/or fatigue effects, and the residual effects (also referred to as transfer, carry-over or range effects) of treatments (e.g., Campbell & Stanley, 1966; Carlsmith, Ellsworth & Aronson, 1976; Christensen, 1980; Crano & Brewer, 1973; D'Amato, 1970; Greenwald, 1976; Poulton, 1973, 1974, 1982). Several general types of solutions have been suggested in attempts to take such potential sequence effects into account. The most commonly adopted procedure involves the use of Latin Square counterbalancing of treatment orders.

Traditional criteria for Latin Square selection.

Latin Squares control serial position effects by ensuring that each treatment appears equally often in each order position.

Traditional selection criteria focus on random selection from the population of all possible squares. Consider the guidelines outlined by Winer (1972). He suggests that one first randomly select a standard square from such sources as Fisher & Yates (1953) or Cochran & Cox (1957). Next, the columns and rows are randomly reordered. Winer provides an example for the 4X4 Latin Square case. Starting with the square on the left of Table 1, Winer reordered the columns and rows according to the random number sequences 2, 4, 1, 3 and 3, 4, 1, 2 producing the square on the

---

Insert Table 1 about here

---

right of Table 1. In Latin Square designs such as this, the columns refer to the serial order of treatments (a within-subjects factor) and the rows refer to subject types (a between-subjects factor).

In practice, many investigators bypass the recommended procedure and generate their own square in the simplest manner possible. This can be done by randomly assigning treatments to positions in the first row of a square and then cyclically permuting each subsequent row. To do this, one simply takes the last condition of the first row and puts it in the first position of the second row. All other treatments are then shifted accordingly one position to the right. By coincidence, the recommended square produced by Winer's randomization procedure (see Table 1) yielded such a cyclical square. This can be seen most easily by transposing rows

two and three in Winer's recommended square. Even Cochran and Cox (1957, p. 145-146) revert to the use of cyclical squares when  $n$  is greater than 6. Such cyclical squares, unfortunately, always introduce pairwise biases, and therefore should always be avoided for counterbalancing in repeated measures designs.

#### New Criteria for Latin Square Selection

It is clear that the cyclically generated squares, as well as those generated in the traditional manner, all satisfy the selection criterion of ensuring that each treatment appears in each of the four treatment serial positions. These commonly used squares, however, fail to explicitly control for two other features of treatment sequence that can affect the interpretation of within-subject treatment differences, i.e., pairwise priority and pairwise distance.

Pairwise priority refers to the proportion of times (across all subject types) that "treatment  $x$ " precedes "treatment  $y$ ". When that proportion is exactly .5, this means that  $x$  precedes  $y$  as often as  $y$  precedes  $x$ . For example, note that in the condition pair of 0, 1 in the recommended square of Table 1 the proportion is exactly .5 (or 2/4), whereas for the pair 0, 2 the proportion is .75 (or 3/4). A subset of squares exists in which all pairs have exactly a .5 probability. Such squares are considered to be balanced for pairwise priority.

Pairwise distance refers to the number of other treatments (counting forward or backward from the numerically smaller member

of the treatment pair to the larger) occurring between a particular pair of treatments,  $x$  and  $y$ . For example, in the first line of the recommended square of Table 1, there is a distance of two units between the 1, 3 condition pair. Ideally, one would want to exactly control the distribution of distances over the entire design for all condition pairs. Unfortunately no Latin Squares exist that provide such a control.

There are two features of the distributions of pairwise distances that can be controlled within a single square. The first is the proportion of pairs that are contiguous (i.e., the proportion of times, over all subject types, that a particular pair has a distance of zero). Note in the recommended square of Table 1 the pair 0, 2 are contiguous three of four times, whereas the pair 0, 1 are never contiguous. Of the six pairs in this square, four have at least one contiguous occurrence and two (0, 1 and 2, 3) have no contiguous subject types. A subset of Latin Squares exists in which all condition pairs have exactly two subject types with zero distance. In such squares, the proportion of contiguous pairs is constant for all possible pairs.

There is a second feature of the distribution of distances between pairs that can be controlled. In squares balanced for pairwise priority, it is possible to obtain directional symmetry. One can exactly match the distribution of distances for subject types in which Condition  $x$  precedes Condition  $y$  with the distribution obtained when Condition  $x$  follows Condition  $y$ . Thus directional

differences in priority will not be confounded with distance. Latin Squares that control for these two features of distance (directional symmetry and proportion of contiguous pairs) are considered balanced for distance.

#### Construction of Pairwise Balanced Latin Squares

The existence of subsets of Latin Squares incorporating features related to the present concerns has been acknowledged in the past (e.g., "diagram" balanced designs of Wagenaar, 1969, and designs "balanced for the estimation of residual effects", as discussed by Alimena, 1962; Cochran & Cox, 1959; and Williams, 1949). However, these earlier authors have not addressed the special implications of these squares for counterbalancing in psychological research. We have also been able to improve on the procedures presented in this earlier work for identifying acceptable squares (Isaac, McCann & Ostrom, 1983). For example, our procedure for even number designs generates more squares than does Williams' (1949) procedure, and includes squares equivalent to those generated by Alimena (1962) and Wagenaar (1969). Reports of these earlier procedures are absent from many books on statistics (e.g., Winer, 1972; Meyers, 1979) and research design (e.g., Crano & Brewer, 1973; Murphy & Puff, 1982) that appear in the psychological literature.

Since procedures for generating pairwise balanced pairs are available elsewhere, we will not repeat them here. Instead we have prepared tables that summarize squares ranging in size from three to sixteen. Most repeated measures research in psychology

involves designs in that range. We should also note at this point that no single generation procedure exhaustively represents the entire population of pairwise balanced squares for any given  $n$  (see Isaac, McCann, & Ostrom, 1983).

---

Insert Tables 2 and 3 about here

---

The entries in Tables 2 resulted from applying procedures referred to in Isaac et al. (1983). Those in Table 3 were initially developed by Williams (1949). We suspect that even more could be produced through trial and error (see Wagenaar, 1969, and Denes & Keedwell, 1974).

#### How to use Tables 2 and 3

Tables 2 and 3 do not contain the full Latin Squares; rather, they provide only the first line of the one or more pairwise balanced squares given for each size  $n$ . This first line corresponds to "Subject Type I" as described in Table 1. The lines corresponding to the remaining Subject Types are easily produced in the manner described below.

1. Determine the size of Latin Square needed for the research design. The size ( $n$ ) corresponds to the number of treatments in the repeated measures experiment.
2. Select a first line from Table 2 or 3 that corresponds to  $n$ . If more than one is listed in the table, select one randomly.
3. Generate the remaining rows (or Subject Types) of the square. Successive rows are produced by adding one (in modular

arithmetic) to each entry in the previous row. As an illustration, consider the square of  $n = 4$  in Table 2. The second, third, and fourth rows are 1, 2, 0, 3; 2, 3, 1, 0; and 3, 0, 2, 1, respectively.

4. Randomly assign experimental treatments to the  $n$  numbers in the resulting square. Note that this means when an  $n$  by  $n$  square is generated, it forms the basis of  $n!$  squares of experimental treatments.

5. Randomly assign subjects to rows of the square, insuring that an equal number of subjects are assigned to each.

#### Odd size squares

Complications arise when an experiment involves an odd number of experimental conditions. Whereas complete pairwise balance can be achieved with a single square for  $n$  even, this cannot be done in the case of  $n$  odd. For example, it is impossible to achieve pairwise priority for  $n$  odd since the proportion of times Condition  $x$  precedes Condition  $y$  can never be exactly .5. In this case, two squares must be used to achieve design-wide pairwise balance. This can be done by selecting any first row from Table 3 and combining it with a square based on the reverse of the selected first row.

One implication of using two squares for  $n$  odd is that the minimum number of subjects required for full counterbalancing increases from  $n$  to  $2n$ . This suggests that there is a distinct advantage to employing repeated measures designs in which the total number of conditions is even. Thus, if the minimum number of

conditions needed to test the experimental hypothesis is odd, the researcher is urged to consider the benefits of adding one theoretically relevant condition. This would result in increased economy in terms of the minimum number to subjects required.

When the main experimental concern is with trends over a parametric independent variable (e.g., set size or exposure time), adding one more condition will also allow for a test of an additional, higher order orthogonal polynomial. When the repeated measures are the result of a factorial design (e.g., set size by exposure time by familiarity of word type), it is necessary that only one factor have an even number of levels. Conditions under which odd squares should still be used occur when an increase to an even design would result in excessive expense or excessive running time for subjects.

#### Statistical Considerations

Designs reported in this paper are balanced for additive residual or carry-over effects of the immediately preceding treatment. It should be noted that the residual effects may be more complicated; for example, multiplicative effects or those persisting beyond the immediately preceding treatment. If the structure of the residual is of some more complicated sort, these designs or any other Latin Squares may not be appropriate. The investigation of such residual effects is beyond the scope of the present paper.

Latin square designs were originally developed to deal with residual effects that are additive. To this point, the primary concern has been to control for the additive effects of serial



position. Conventional statistical analyses routinely include tests of significance for this factor. (It should be noted that these serial position effects are usually not of interest in psychological research and in practice are rarely even reported.) A unique feature of pairwise balanced Latin Squares is that they make it possible to statistically estimate the contribution of additive residual effects due to pairwise priority. To do this, one must use an analysis proposed by Williams (1949) and illustrated in Cochran and Cox (1957).

Perhaps obviously, a standard statistical analysis of a Latin Square could be used in the present case, assuming pairwise residual effects exist, and that treatments and residual effects are uncorrelated, then there will be a positive bias in the mean square for treatments (i.e., the mean square will be larger than in such residual effects didn't exist), and estimates of treatment effects will be confounded with residual effects. However, since the design is balanced, treatment effects will be confounded with their own residual effects. In contrast, Latin Square designs that are not pairwise balanced will have treatment effects which may be confounded with residual effects of other treatments. Further, there will also be a positive bias in the mean square for treatments, the extent of which will depend on the particular design, but which in general will be greater than that associated with the pairwise balanced designs.

Thus, if it is desirable to estimate the test direct treatment

effects and residual effects separately, the analysis given in Cochran and Cox (1957) is recommended. Note that this analysis applies strictly to pairwise balanced designs, and not to other designs in which carry-over may be suspected. Alternatively, a standard analysis of Latin Square designs, such as given in most textbooks, would be testing, in effect, the significance of an additive combination of treatments and their residual effects when applied to the pairwise balanced designs.

#### Computational Procedures

The computational procedures for pairwise balanced designs were first described by Williams (1949, 1950), and later reported in slightly modified form by Cochran & Cox (1957). Since neither source is commonly available to psychologists, we will present the Cochran & Cox notation, and illustrate its use with an example.

Designs differ in terms of whether more than one square is used and whether more than one subject is assigned to each row of the square. Normally, two or more squares will be used when  $n$  is odd. But also, it will sometimes be advantageous to use several squares in the case of  $n$  even. It will increase error degrees of freedom and can allow for greater control over the

---

Insert Table 4 about here

---

distribution of pairwise distances. We have selected an example employing two  $3 \times 3$  squares with one subject per row.

The analyses presented here assume that row, column, and treatment effects do not interact, and further, that the residual effect simply adds to the effect of the following treatment. Thus, for a row in which B follows A, and C follows B (i.e., treatment order A B C), the period in which C is applied has a total effect attributable to treatments which is  $(t_c + r_b)$ . Similarly, the total observed effect of treatments when B is applied would be  $(t_b + r_a)$ .

To simplify the presentation, let us assume that an experiment involves responses to three attitude statements, A, B, C, all presented to each subject. A given subject responds to a row of an appropriately selected square, and a column corresponds to the position in the order of presentation. Two squares are selected to be pairwise balanced. The squares are given in Table 4. Included in Table 4 is the hypothetical data, with one observation per cell.

The following symbols are used:

n = number of treatments (=3)

m = number of squares (=2)

S = total of sequence (row)

T = treatment total

P = total of position (column) in a given square

R = total of scores in positions immediately following the treatment in question

F = total of sequences (rows) in which this treatment is the last one

Pairwise Balanced Latin Squares

16

$\underline{P}_1$  = total of scores in first position of all sequences

$\underline{G}_j$  = grand total of scores in a given square

$\underline{G}$  = grand total of all scores

The following quantities special to this analysis are needed:

$$\underline{P}_1 - n\underline{G}$$

$$n\underline{P}_1 - (n + 2)\underline{G}$$

Then some of the usual quantities are needed:

Correction factor:  $C = \frac{\underline{G}^2}{\underline{mn}^2}$

Total SS =  $\Sigma X^2 - C$  (df =  $\underline{mn}^2 - 1$ )

Sequences SS =  $\frac{1}{\underline{n}} \Sigma S^2 - C$  (df =  $\underline{mn} - 1$ )

Positions (Order) SS =  $\frac{1}{\underline{n}} \Sigma P^2 - \frac{1}{\underline{n}} \Sigma \underline{G}_j^2$  (df =  $\underline{m}(n - 1)$ )

Note that this is a sum of squares between positions within squares.

Treatments (unadjusted) SS =  $\frac{\Sigma T^2}{\underline{mn}} - C$  (df =  $\underline{n} - 1$ )

In addition, adjusted (for residual or carry-over) treatment effects are computed as follows:

$$\hat{T} = (\underline{n}^2 - \underline{n} - 1) \underline{T} + \underline{nR} + \underline{F} + \underline{P}_1 - \underline{nG}$$

and the adjusted sum of squares for treatments is computed:

$$SS_{\text{trts(adj)}} = \frac{\Sigma \hat{T}^2}{\underline{mn} (\underline{n}^2 - \underline{n} - 1) (\underline{n}^2 - \underline{n} - 2)} \quad (\text{df} = \underline{n} - 1)$$

Similarly, adjusted residual (carry-over) effects are computed:

$$\hat{R} = \underline{nT} + \underline{n}^2 \underline{R} + \underline{nF} + \underline{nP}_1 - (\underline{n} + 2) \underline{G}$$

and

$$SS_{\text{res(adj)}} = \frac{\Sigma \hat{R}^2}{\underline{mn}^3 (\underline{n}^2 - \underline{n} - 2)}$$

The total sum of squares for treatment effects (i.e., direct plus residual) is given by:

$$SS_{\text{trt}} = SS_{\text{trts(unadj)}} + SS_{\text{res(adj)}}$$

$$\text{or} = SS_{\text{trts(adj)}} + SS_{\text{res(unadj)}}$$

Three of these four quantities were computed above; the fourth,  $SS_{\text{res(unadj)}}$  may be computed by subtraction. However, to provide a check on computations,  $SS_{\text{res(unadj)}}$  may be computed directly:

First, for each treatment compute

$$\hat{R}' = \hat{R} + \underline{G} - \underline{nT}$$

Then the sum of squares is given by

$$SS_{\text{res(unadj)}} = \frac{\sum \hat{R}_i^2}{mn^3(n^2 - n - 1)}$$

Finally, SS error is obtained by subtraction from the total sum of squares. Here the sum of squares for Positions within Squares has been removed. If instead the overall sum of squares for Positions is removed (i.e., assuming no differences in Positions effects across squares), the error degrees of freedom becomes  $(n - 1)(mn - 3)$ .

---

Insert Tables 5 and 6 about here

---

Table 5 summarizes the computations using the formulae that are presented above. The analysis of Variance Summary Table is contained in Table 6. Note that the terms in brackets have the same total. Tests on the effects of treatments and residuals should only be made on the adjusted sums of squares. The Error sum of squares is obtained by subtraction of the total of all other non-redundant sums of squares from the Total, i.e.;

$SS_{\text{tot}} = 182.0$ . Thus, only one of the sums in brackets is unsolved in this subtraction, otherwise effects attributable to treatments (direct and residual) would be counted twice. Since  $F_{2,4} = 10.65$  for  $\alpha = .025$ , both direct treatment and residual effects are significant in this example.

This example does not involve a design in which a between Ss treatment effect is included, or in which multiple Ss are assigned to a row of a square. However, the extension of the analysis to such designs is straightforward and follows plans available in standard texts. The adjustment for carry-over (residual) involves adjustment of within subject effects. Further, computation of a Ss x Positions effect (the usual within Ss error) in this case would be unaffected. Between Ss effects would use the usual Between Ss error term.

References

- Alimena, B. S. (1962). A method for determining unbiased distribution in the latin square. Psychometrika, 27, 315-17.
- Campbell, D. T., & Stanley, J. C. (1966). Experimental and quasi-experimental designs for research. Chicago: Rand McNally.
- Carlsmith, J., Ellsworth, P., & Aronson, E. (1976). Methods of research in social psychology. Reading, Mass.: Addison-Wesley.
- Christensen, L. B. (1980). Experimental Methodology (2nd Edition). Boston, Allyn & Bacon.
- Cochran, W. G., & Cox, G. M. (1957). Experimental designs (2nd Edition). New York: Wiley.
- Crano, W. D., Brewer, M. B. (1973). Principles of research in social psychology. New York: McGraw-Hill.
- D'Amato, M. R. (1970). Experimental psychology methodology: Psychophysics and learning. New York: McGraw-Hill.
- Denes, J., & Keedwell, A. D. (1974). Latin squares and their applications. New York: Academic Press.
- Fisher, R. A., & Yates, F. (1953). Statistical tables for biological, agricultural, and medical research (4th Edition). Edinburgh: Oliver & Boyd.
- Greenwald, A. G. (1976). Within-subjects designs: To use or not to use? Psychological Bulletin, 83, 314-320.
- Isaac, P. D., McCann, C. D., & Ostrom, T. M. (1983). Generation of pairwise balanced Latin Squares. Unpublished manuscript,



Ohio State University, Columbus, Ohio.

- Lindman, H. R. (1974). Analysis of variance in complex experimental designs. San Francisco; W. H. Freeman & Co.
- Murphy, M. D., & Puff, C. R. (1982). Free recall: Basic methodology and analyses. In C. R. Puff (Ed.), Handbook of research methods in human memory and cognition. New York: Academic Press.
- Myers, J. S. (1979). Fundamentals of experimental design (3rd Edition). Boston: Allyn & Bacon.
- Poulton, E. C. (1973). Unwanted range effects from using within-subject experimental designs. Psychological Bulletin, 80, 113-21.
- Poulton, E. C. (1974). Range effects are characteristic of a person serving in a within-subjects experiment design -- a reply to Rothstein. Psychological Bulletin, 81, 201-2.
- Poulton, E. C. (1982). Influential companions: Effects of one strategy on another in the within-subjects designs of cognitive psychology. Psychological Bulletin, 91, 673-690.
- Rothstein, L. D. (1974). Reply to Poulton. Psychological Bulletin, 81, 199-200.
- Wagenaar, W. A. (1969). Note on the construction of diagram-balanced latin squares. Psychological Bulletin, 72, 384-6.
- Williams, E. J. (1949). Experimental designs balanced for the estimation of residual effects of treatments. Australian Journal of Scientific Research -- Series A, 2, 149-168.

Williams, E. J. (1950). Experimental designs balanced for pairs of residual effects. Australian Journal of Scientific Research -- Series A, 3, 351-63.

Winer, B. J. (1972). Statistical principles in experimental design. New York: McGraw-Hill.

Table 1  
Latin Square Selection Based on Traditional Criteria

Subject Type	Standard Square				Recommended Square			
	Treatment Order				Treatment Order			
	First	Second	Third	Fourth	First	Second	Third	Fourth
I.	0	1	2	3	3	0	2	1
II.	1	0	3	2	2	1	3	0
III.	2	3	1	0	1	3	0	2
IV.	3	2	0	1	0	2	1	3

Note - Based on Winer (1972, p. 689).

Pairwise Balanced Latin Squares

Table 2

First Rows of Pairwise Balanced Squares for n Even

---

$n=4$	1	3	2													
$n=6$	1	5	2	4	3											
$n=8$	1	7	2	6	3	5	4									
$n=10$	1	9	2	8	3	7	4	6	5							
$n=12$	1	11	2	10	3	9	4	8	5	7	6					
$n=14$	1	13	2	12	3	11	4	10	5	9	6	8	7			
$n=16$	1	15	2	14	3	13	4	12	5	11	6	10	7	9	8	

---

Note. - n = number of experimental treatments

Table 3

First Rows of Pairwise Balanced Squares for n Odd

---

$\frac{n}{0} = 3$	1	2												
$\frac{n}{0} = 5$	1	4	2	3										
0	1	3	4	2										
0	2	1	4	3										
$\frac{n}{0} = 7$	1	6	2	5	3	4								
0	1	3	6	2	4	5								
0	1	4	2	6	5	3								
0	1	4	3	2	5	6								
$\frac{n}{0} = 9$	1	8	2	7	3	6	4	5						
$\frac{n}{0} = 11$	1	10	2	9	3	8	4	7	5	6				
0	1	3	6	10	4	9	2	5	7	8				
$\frac{n}{0} = 13$	1	12	2	11	3	10	4	9	5	8	6	7		
$\frac{n}{0} = 15$	1	14	2	13	3	12	4	11	5	10	6	9	7	8

---

Note. - n = Number of experimental treatments.

Pairwise Balanced Latin Squares

26

Table 4

Pairwise Balanced 3x3 Latin Squares and Simulated Data

Latin Squares

<u>Square 1</u>				<u>Square 2</u>			
Treatment Position				Treatment Position			
<u>Subject Type</u>	<u>First</u>	<u>Second</u>	<u>Third</u>	<u>Subject Type</u>	<u>First</u>	<u>Second</u>	<u>Third</u>
I	0	1	2	IV	2	1	0
II	1	2	0	V	0	2	1
III	2	0	1	VI	1	0	2

Simulated Data

<u>Square 1</u>				<u>Square 2</u>				
				$\Sigma$				
-4	-6	1	-9	6	7	-2	11	
1	1	0	2	-4	-2	2	-4	
3	2	-1	4	0	1	1	2	
$\Sigma$	0	-3	0	-3	$\Sigma$ 2	6	1	9

Table 5  
Summary of Computations

Treatment Number	<u>T</u>	<u>R</u>	<u>F</u>	<u>T̂</u>	<u>t̂=T̂/24</u>	<u>R̂</u>	<u>r̂=R̂/24</u>	<u>k̂'</u>
0	-7	-8	13	-62	-2.58	178	-3.25	-51
1	3	1	0	2	.08	-6	-.25	-9
2	10	11	-7	60	2.50	84	3.5	60

$$\underline{P}_1 - \underline{nG} = 2 - 3(6) = -16$$

$$\underline{nP}_1 - (\underline{n} + 2)G = 3(2) - (3 + 2)6 = -24$$

$$\hat{T} = 5\underline{T} + 3\underline{R} + \underline{F} - 16$$

$$\hat{R}' = 3\underline{T} + 9\underline{R} + 3\underline{F} - 24$$

$$\underline{C} = \frac{G^2}{2 \cdot 3^2} = \frac{36}{18} = 2.0$$

$$SS_{\text{tot}} = 2X^2 - \underline{C} = 184 - 2 = 182$$

$$SS_{\text{seq}} = \frac{1}{3} \{ \underline{\Sigma S}^2 \} - \underline{C} = \frac{1}{3} \{ (-9)^2 + (2)^2 + (2)^2 \} - 2 = 78.67$$

$$SS_{\text{pos w/seq}} = \frac{1}{3} \{ \underline{\Sigma P}^2 \} - \frac{1}{3} 2 \{ \underline{\Sigma G}^2 \}$$

$$= \frac{1}{3} \{ 0^2 + (-3)^2 + \dots + (1)^2 \} - \frac{1}{9} \{ (-3)^2 + (9)^2 \} = 6.67$$

$$SS_{\text{trt(unadj)}} = \frac{\underline{\Sigma T}^2}{2(3)} - \underline{C} = \frac{1}{6} \{ (-7)^2 + (3)^2 + (10)^2 \} - 2 = 24.33$$

Pairwise Balanced Latin Squares

28

$$SS_{\text{trt(adj)}} = \frac{\hat{\Sigma T}^2}{6(5)(4)} = \frac{(-62)^2 + (2)^2 + (60)^2}{120} = 62.07$$

$$SS_{\text{res(adj)}} = \frac{\hat{\Sigma R}^2}{2 \cdot 3^3 (3^2 - 3 - 2)} = \frac{(-78)^2 + (-6)^2 + (84)^2}{216} = 61.00$$

$$SS_{\text{res(unadj)}} = \frac{\hat{\Sigma R}'^2}{2 \cdot 3^3 (3^2 - 3 - 1)} = \frac{(-51)^2 + (-9)^2 + (60)^2}{270} = 23.27$$



Pairwise Balanced Latin Squares

Table 6

Analysis of Variance Summary Table

Source	df	SS	MS	F
Sequences	$\underline{mn} - 1 = 5$	78.67	15.73	
Positions within squares (cols)	$\underline{m}(n - 1) = 4$	6.67	1.67	
Direct Treatment and Residual				
Treatment effects (unadj)	$\underline{n} - 1 = 2$	24.33	12.17	
Residual effects (adj)	$\underline{n} - 1 = 2$	61.00	30.50	10.78
Residual effects (unadj)	$\underline{n} - 1 = 2$	23.27	11.64	
Treatment effects (adj)	$\underline{n} - 1 = 2$	62.07	31.04	10.97
Error	$(\underline{n} - 1)(\underline{mn} - \underline{m} - 2) = 4$	11.32	2.83	
Total	$\underline{mn}^2 - 1 = 17$	182.0		

4420E DISTRIBUTION LIST

LIST 1  
MANDATORY

Defense Technical Information Center (12 copies)  
ATTN: DTIC DDA-2  
Selection and Preliminary Cataloging Section  
Cameron Station  
Alexandria, VA 22314

Library of Congress  
Science and Technology Division  
Washington, D.C. 20540

Office of Naval Research (3 copies)  
Code 4420E  
800 N. Quincy Street  
Arlington, VA 22217

Naval Research Laboratory (6 copies)  
Code 2627  
Washington, D.C. 20375

Office of Naval Research  
Director, Technology Programs  
Code 200  
800 N. Quincy Street  
Arlington, VA 22217

LIST 3  
OPNAV

Deputy Chief of Naval Operations  
(Manpower, Personnel, and Training)  
Head, Research, Development, and  
Studies Branch (Op-115)  
1812 Arlington Annex  
Washington, DC 20350

Director  
Civilian Personnel Division (OP-14)  
Department of the Navy  
1803 Arlington Annex  
Washington, DC 20350

Deputy Chief of Naval Operations  
(Manpower, Personnel, and Training)  
Director, Human Resource Management  
Plans and Policy Branch (Op-150)  
Department of the Navy  
Washington, DC 20350

Chief of Naval Operations  
Head, Manpower, Personnel, Training  
and Reserves Team (Op-964D)  
The Pentagon, 4A478  
Washington, DC 20350

Chief of Naval Operations  
Assistant, Personnel Logistics  
Planning (Op-987H)  
The Pentagon, 5D772  
Washington, DC 20350

LIST 4  
NAVMAT & NPRDC

NAVMAT

Program Administrator for Manpower,  
Personnel, and Training  
MAT-0722  
800 N. Quincy Street  
Arlington, VA 22217

Naval Material Command  
Management Training Center  
NAVMAT O9M32  
Jefferson Plaza, Bldg #2, Rm 150  
1421 Jefferson Davis Highway  
Arlington, VA 20360

Naval Material Command  
Director, Productivity Management Office  
MAT-00K  
Crystal Plaza #5  
Room 632  
Washington, DC 20360

Naval Material Command  
Deputy Chief of Naval Material, MAT-03  
Crystal Plaza #5  
Room 236  
Washington, DC 20360

Naval Personnel R&D Center  
Technical Director  
Director, Manpower & Personnel  
Laboratory, Code 06  
Director, System Laboratory, Code 07  
Director, Future Technology, Code 41  
San Diego, CA 92152

(4 copies)

Navy Personnel R&D Center  
Washington Liaison Office  
Ballston Tower #3, Room 93  
Arlington, VA 22217

4420E  
Aug 83

LIST 5  
BUMED

Commanding Officer  
Naval Health Research Center  
San Diego, CA 92152

Psychology Department  
Naval Regional Medical Center  
San Diego, CA 92134

Commanding Officer  
Naval Submarine Medical  
Research Laboratory  
Naval Submarine Base  
New London, Box 900  
Groton, CT 06349

Director, Medical Service Corps  
Bureau of Medicine and Surgery  
Code 23  
Department of the Navy  
Washington, DC 20372

Commanding Officer  
Naval Aerospace Medical  
Research Lab  
Naval Air Station  
Pensacola, FL 32508

Program Manager for Human  
Performance (Code 44)  
Naval Medical R&D Command  
National Naval Medical Center  
Bethesda, MD 20014

Navy Health Research Center  
Technical Director  
P.O. Box 85122  
San Diego, CA 92138

4420E  
Aug 83

LIST 6  
NAVAL ACADEMY AND NAVAL POSTGRADUATE SCHOOL

Naval Postgraduate School (3 copies)  
ATTN: Chairman, Dept. of  
Administrative Science  
Department of Administrative Sciences  
Monterey, CA 93940

Superintendent  
Naval Postgraduate School  
Code 1424  
Monterey, CA 93940

U.S. Naval Academy  
ATTN: Chairman, Department  
of Leadership and Law  
Stop 7-B  
Annapolis, MD 21402

Superintendent  
ATTN: Director of Research  
Naval Academy, U.S.  
Annapolis, MD 21402

LIST 7  
HRM

Officer in Charge  
Human Resource Management Detachment  
Naval Air Station  
Alameda, CA 94591

Officer in Charge  
Human Resource Management Detachment  
Naval Submarine Base New London  
P.O. Box 81  
Groton, CT 06340

Officer in Charge  
Human Resource Management Division  
Naval Air Station  
Mayport, FL 32228

Commanding Officer  
Human Resource Management Center  
Pearl Harbor, HI 96860

Commander in Chief  
Human Resource Management Division  
U.S. Pacific Fleet  
Pearl Harbor, HI 96860

Officer in Charge  
Human Resource Management Detachment  
Naval Base  
Charleston, SC 29408

Commanding Officer  
Human Resource Management School  
Naval Air Station Memphis  
Millington, TN 38054

Human Resource Management School  
Naval Air Station Memphis (96)  
Millington, TN 38054



List 7 (Continued)

Commanding Officer  
Human Resource Management Center  
1300 Wilson Boulevard  
Arlington, VA 22209

Commanding Officer  
Human Resource Management Center  
5621-23 Tidewater Drive  
Norfolk, VA 23511

Commander in Chief  
Human Resource Management Division  
U.S. Atlantic Fleet  
Norfolk, VA 23511

Officer in Charge  
Human Resource Management Detachment  
Naval Air Station Whidbey Island  
Oak Harbor, WA 98278

Commanding Officer  
Human Resource Management Center  
Box 23  
FPO New York 09510

Commander in Chief  
Human Resource Management Division  
U.S. Naval Force Europe  
FPO New York 09510

Officer in Charge  
Human Resource Management Detachment  
Box 60  
FPO San Francisco 96651

Officer in Charge  
Human Resource Management Detachment  
COMNAVFORJAPAN  
FPO Seattle 98762

4420E  
Aug 83

LIST 8  
NAVY MISCELLANEOUS

Naval Military Personnel Command (2 copies)  
HRM Department (NMPC-6)  
Washington, DC 20350

Naval Training Analysis  
and Evaluation Group  
Orlando, FL 32813

Commanding Officer  
ATTN: TIC, Bldg. 2068  
Naval Training Equipment Center  
Orlando, FL 32813

Chief of Naval Education  
and Training (N-5)  
Director, Research Development,  
Test and Evaluation  
Naval Air Station  
Pensacola, FL 32508

Chief of Naval Technical Training  
ATTN: Code D17  
NAS Memphis (75)  
Millington, TN 38D54

Navy Recruiting Command  
Head, Research and Analysis Branch  
Code 434, Room 8001  
801 North Randolph Street  
Arlington, VA 22203

Navy Recruiting Command  
Director, Recruiting Advertising Dept.  
Code 40  
801 North Randolph Street  
Arlington, VA 22203

Naval Weapons Center  
Code 094  
China Lake, CA 93555

Jesse Orlansky  
Institute for Defense Analyses  
1801 North Beauregard Street  
Alexandria, VA 22311

4420E  
Aug 83

LIST 9  
USMC

Headquarters, U.S. Marine Corps  
Code MPI-20  
Washington, DC 20380

Headquarters, U.S. Marine Corps  
ATTN: Scientific Adviser,  
Code RD-1  
Washington, DC 20380

Education Advisor  
Education Center (E031)  
MCDEC  
Quantico, VA 22134

Commanding Officer  
Education Center (E031)  
MCDEC  
Quantico, VA 22134

Commanding Officer  
U.S. Marine Corps  
Command and Staff College  
Quantico, VA 22134

LIST 10  
OTHER FEDERAL GOVERNMENT

Defense Advanced Research  
Projects Agency  
Director, Cybernetics  
Technology Office  
1400 Wilson Blvd, Rm 625  
Arlington, VA 22209

Dr. Douglas Hunter  
Defense Intelligence School  
Washington, DC 20374

Dr. Brian Usilaner  
GAO  
Washington, DC 20548

National Institute of Education  
EOLC/SMO  
1200 19th Street, N.W.  
Washington, DC 20208

National Institute of Mental Health  
Division of Extramural Research Programs  
5600 Fishers Lane  
Rockville, MD 20852

National Institute of Mental Health  
Minority Group Mental Health Programs  
Room 7 - 102  
5600 Fishers Lane  
Rockville, MD 20852

Office of Personnel Management  
Office of Planning and Evaluation  
Research Management Division  
1900 E Street, N.W.  
Washington, DC 20415

Chief, Psychological Research Branch  
U.S. Coast Guard (G-P-1/2/TP42)  
Washington, D.C. 20593

Social and Developmental Psychology  
Program  
National Science Foundation  
Washington, D.C. 20550

Dr. Earl Potter  
U.S. Coast Guard Academy  
New London, CT 06320

4420E  
Aug 83

LIST 10 CONT'D

OTHER FEDERAL GOVERNMENT

Division of Industrial Science  
& Technological Innovation  
Productivity Improvement Research  
National Science Foundation  
Washington, D.C. 20550

Douglas B. Blackburn, Director  
National Defense University  
Mobilization Concepts Development  
Center  
Washington, D.C. 20319

4420E  
Aug 83

LIST 11  
ARMY

Headquarters, FORSCOM  
ATTN: AFPR-HR  
Ft. McPherson, GA 30330

Army Research Institute  
Field Unit - Leavenworth  
P.O. Box 3122  
Fort Leavenworth, KS 66027

Technical Director  
Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

(3 copies)

Head, Department of Behavior  
Science and Leadership  
U.S. Military Academy, New York 10996

Walter Reid Army Medical Center  
Attn: Dr. Mary Lozano  
W. R. Army Institute of Research  
Division of Neuropsychiatry  
Forest Glen  
Washington, D.C. 20012

4420E  
Aug 83

LIST 12  
AIR FORCE

Air University Library  
LSE 76-443  
Maxwell AFB, AL 36112

Head, Department of Behavioral  
Science and Leadership  
U.S. Air Force Academy, CO 80840

MAJ Robert Gregory  
USAFA/DFBL  
U.S. Air Force Academy, CO 80840

AFOSR/NL  
Building 410  
Bolling AFB  
Washington, DC 20332

Department of the Air Force  
HQUSAF/MPXHL  
Pentagon  
Washington, DC 20330

Technical Director  
AFHRL/MO(T)  
Brooks AFB  
San Antonio, TX 78235

AFMPC/MPCYPR  
Randolph AFB, TX 78150

4420E  
Aug 83

LIST 13  
MISCELLANEOUS

Australian Embassy  
Office of the Air Attache (S3B)  
1601 Massachusetts Avenue, N.W.  
Washington, D.C. 20036

British Embassy  
Scientific Information Officer  
Room 509  
3100 Massachusetts Avenue, N.W.  
Washington, DC 20008

Canadian Defense Liaison Staff,  
Washington  
ATTN: CDRD  
2450 Massachusetts Avenue, N.W.  
Washington, DC 20008

Commandant, Royal Military  
College of Canada  
ATTN: Department of Military  
Leadership and Management  
Kingston, Ontario K7L 2W3

National Defence Headquarters  
DPAR  
Ottawa, Ontario K1A 0K2

Mr. Luigi Petrullo  
2431 North Edgewood Street  
Arlington, VA 22207



Sequential by Principal Investigator

LIST 14  
CURRENT CONTRACTORS

Dr. Clayton P. Alderfer  
Yale University  
School of Organization and Management  
New Haven, Connecticut 06520

Dr. Janet L. Barnes-Farrell  
Department of Psychology  
University of Hawaii  
2430 Campus Road  
Honolulu, HI 96822

Dr. Gary Bowen  
SRA Corporation  
800 18th Street, N.W.  
Washington, D.C. 20006

Dr. Jomills Braddock  
John Hopkins University  
Center for the Social Organization  
of Schools  
3505 N. Charles Street  
Baltimore, MD 21218

Jeanne M. Brett  
Northwestern University  
Graduate School of Management  
2001 Sheridan Road  
Evanston, IL 60201

Dr. Terry Connolly  
Georgia Institute of Technology  
School of Industrial & Systems  
Engineering  
Atlanta, GA 30332

Dr. Richard Daft  
Texas A&M University  
Department of Management  
College Station, TX 77843

Dr. Randy Dunham  
University of Wisconsin  
Graduate School of Business  
Madison, WI 53706

List 14 (continued)

Dr. Henry Emurian  
The Johns Hopkins University  
School of Medicine  
Department of Psychiatry and  
Behavioral Science  
Baltimore, MD 21205

Dr. Arthur Gerstenfeld  
University Faculty Associates  
710 Commonwealth Avenue  
Newton, MA 02159

Dr. J. Richard Hackman  
School of Organization  
and Management  
Box 1A, Yale University  
New Haven, CT 06520

Dr. Wayne Holder  
American Humane Association  
P.O. Box 1266  
Denver, CO 80201

Dr. Daniel Ilgen  
Department of Psychology  
Michigan State University  
East Lansing, MI 48824

Dr. Lawrence R. James  
School of Psychology  
Georgia Institute of  
Technology  
Atlanta, GA 30332

Dr. David Johnson  
Professor, Educational Psychology  
178 Pillsbury Drive, S.E.  
University of Minnesota  
Minneapolis, MN 55455

Dr. F. Craig Johnson  
Department of Educational  
Research  
Florida State University  
Tallahassee, FL 32306

List 14 (continued)

Dr. Dan Landis  
Department of Psychology  
Purdue University  
Indianapolis, IN 46205

Dr. Frank J. Landy  
The Pennsylvania State University  
Department of Psychology  
417 Bruce V. Moore Building  
University Park, PA 16802

Dr. Bibb Latane  
The University of North Carolina  
at Chapel Hill  
Manning Hall 026A  
Chapel Hill, NC 27514

Dr. Edward E. Lawler  
University of Southern California  
Graduate School of Business  
Administration  
Los Angeles, CA 90007

Dr. William H. Mobley  
College of Business Administration  
Texas A&M University  
College Station, TX 77843

Dr. Lynn Oppenheim  
Wharton Applied Research Center  
University of Pennsylvania  
Philadelphia, PA 19104

Dr. Thomas M. Ostrom  
The Ohio State University  
Department of Psychology  
116E Stadium  
404C West 17th Avenue  
Columbus, OH 43210

Dr. William G. Ouchi  
University of California,  
Los Angeles  
Graduate School of Management  
Los Angeles, CA 90024

List 14 (continued)

Dr. Robert Rice  
State University of New York at Buffalo  
Department of Psychology  
Buffalo, NY 14226

Dr. Irwin G. Sarason  
University of Washington  
Department of Psychology, NI-25  
Seattle, WA 98195

Dr. Benjamin Schneider  
Department of Psychology  
University of Maryland  
College Park, MD 20742

Dr. Edgar H. Schein  
Massachusetts Institute of  
Technology  
Sloan School of Management  
Cambridge, MA 02139

Dr. H. Wallace Sinaiko  
Program Director, Manpower Research  
and Advisory Services  
Smithsonian Institution  
801 N. Pitt Street, Suite 120  
Alexandria, VA 22314

Dr. Richard M. Steers  
Graduate School of Management  
University of Oregon  
Eugene, OR 97403

Dr. Siegfried Streufert  
The Pennsylvania State University  
Department of Behavioral Science  
Milton S. Hershey Medical Center  
Hershey, PA 17033

Dr. Barbara Saboda  
Public Applied Systems Division  
Westinghouse Electric Corporation  
P.O. Box 866  
Columbia, MD 21044

Dr. Harry C. Triandis  
Department of Psychology  
University of Illinois  
Champaign, IL 61820

List 14 (continued)

Dr. Anne S. Tsui  
Duke University  
The Fuqua School of Business  
Durham, NC 27706

Andrew H. Van de Ven  
University of Minnesota  
Office of Research Administration  
1919 University Avenue  
St. Paul, MN 55104

Dr. Philip Wexler  
University of Rochester  
Graduate School of Education &  
Human Development  
Rochester, NY 14627

Sabra Woolley  
SRA Corporation  
901 South Highland Street  
Arlington, VA 22204

END

FILMED

02-84

DTIC