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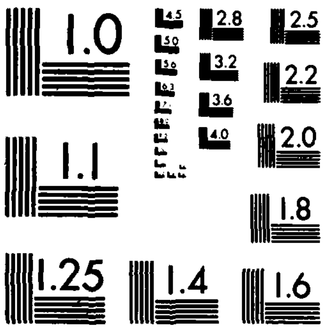

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An Interpolation and Compaction Technique for Gridded Data

by

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An Interpolation and Compaction Technique for Gridded Data

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ABSTRACT

An interpolation technique is implemented which is applicable to terrain data defined on a rectangular grid. The technique also allows for data compaction, i.e., effectively representing the given data using less space than required by the raw data. The technique involves finding bicubic polynomials which represent the terrain surface over small subgrids. These surfaces are then pieced together to form a global surface which is both continuous and smooth over the entire region.

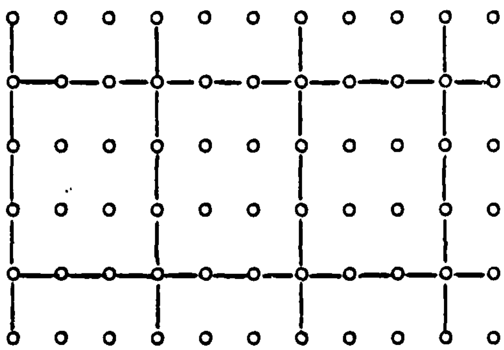
I. INTRODUCTION

The Air Force is engaged in a project to develop techniques for out-the-window scene generation utilizing the digitized terrain elevation data supplied by the Defense Mapping Agency. The data is to be used to generate a terrain surface appropriate for simulation of an out-the-window view of actual terrain as seen from a low flying aircraft. The terrain data is defined on a rectangular grid with a grid point separation of three seconds. A method of data compaction is sought which 1) will allow rapid access to the data, 2) will allow immediate display of the terrain and 3) will represent actual terrain as accurately as possible.

A technique for surface generation from DMA data has already been considered by James Jancaitis[7] for the U.S. Army Engineer Topographic Laboratories. Jancaitis replaces the original data with local least-square polynomial approximations which overlap. These local approximations are then combined using weighting functions to produce a smooth polynomial approximation to the initial data. The polynomial approximation may then be used to visually display the surface. Robert Jablinske[5,6] has applied this method to actual DMA data, and the results have been compared with results obtained by using conventional matrix storage techniques. The results of this comparison indicate that the Jancaitis method introduces too much error into the terrain model for certain types of terrain and that conventional matrix storage techniques work just as well in terms of compaction and error.

II. DESCRIPTION OF INTERPOLATION TECHNIQUE

An alternative method of data compaction and interpolation has been implemented and tested using actual DMA terrain data. This method is described below. The DMA terrain data consists of integer z values defined on a rectangular grid with a grid spacing of 3 seconds. The data is organized into manuscripts each of which covers a one degree by one degree area. Thus the data is organized as shown below in Figure 1. Each "o" indicates the location of a z-value.



NFF = 4

FIGURE 1 - ORGANIZATION OF DMA DATA

The DMA data in Figure 1 has been organized so that 4x4 = 16 terrain heights are considered as a unit for interpolation purposes. The number of data points on a vertical or horizontal boundary of a square subgrid is referred to as the NFF value for that subgrid. In Figure 1, the NFF value for each subgrid is 4. Interpolation is performed for NFF values of 5,6,7,8 and 9.

For a particular NFF value and a particular subgrid of size NFF x NFF a bicubic polynomial of the form

$$f(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^{i} y^{j} \qquad (1)$$

is obtained where all sixteen a_{ij} coefficients are integers. These bicubic surfaces, one for each subgrid of size NFF x NFF are pieced together to form a continuous and smooth global surface. For the derivation of this bicubic polynomial, the gridded data in each subgrid is scaled as indicated in Figure 2.

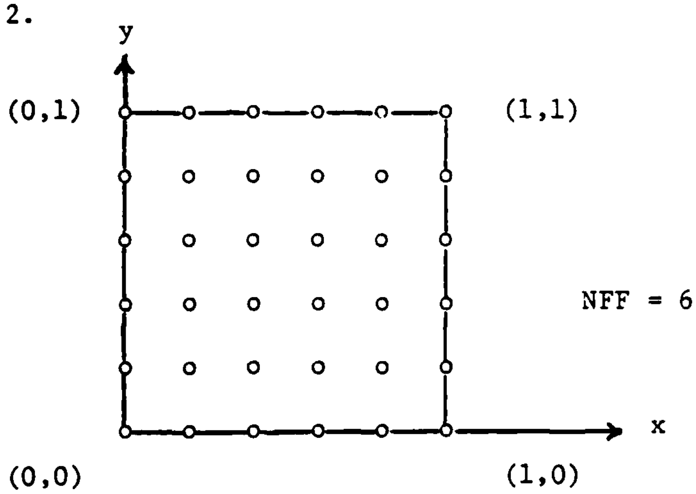


FIGURE 2 - SCALING OF DATA

The bicubic polynomial of Equation 1 is obtained by using the values z, z_x, z_y, z_{xy} at the four corner points of the subregion in Figure 2. These sixteen values completely determine the f(x,y) of Equation 1. The derivation of f(x,y) proceeds as follows:

Let
$$H = XAY^T \tag{2}$$

where
$$H = \begin{bmatrix} H_{00} & H_{10} \\ H_{01} & H_{11} \end{bmatrix} \tag{3}$$

with
$$H_{km} = \begin{bmatrix} z & z_x \\ z_y & z_{xy} \end{bmatrix} \text{ at corner point } (k,m),\ k = 0,1 \text{ and } m = 0,1 \ . \tag{4}$$

Also,

$$X = Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \qquad (5)$$

and

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix}. \qquad (6)$$

Solving Equation 2 for A, we obtain

$$A = X^{-1} H (Y^{-1})^T \qquad (7)$$

where

$$X^{-1} = Y^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & -2 & 3 & -1 \\ 2 & 1 & -2 & 1 \end{bmatrix}. \qquad (8)$$

Thus the bicubic $f(x,y)$ of Equation 1 is easily determined once the values z, z_x, z_y and z_{xy} are known at the four corner points of the subregion.

The bicubic polynomial obtained above interpolates the four corner points of the subgrid. Now consider an adjacent subgrid and the corresponding bicubic surface over this region. It is easily shown[1] that these two surfaces over the common boundary line meet each other in a continuous manner and that they meet in a smooth manner in the sense that first partial derivatives are equal on the boundary line.

Hence a global surface is obtained which is continuous and smooth by piecing together the bicubic polynomials defined over each subgrid.

In order to implement the above process, the following values must be obtained:

(1) values for z_x, z_y and z_{xy} at the corner points of each subgrid.

(2) a value for NFF. For larger NFF values, more data compaction occurs since the NFF x NFF z-values of each subgrid will be represented using the 16 integer coefficients of Equation 1. Also as the NFF value gets larger, the bicubic polynomial of Equation 1 will not represent the actual z-values as well as for smaller NFF values. Thus the NFF value should be chosen to be the largest value for which the corresponding error is acceptable.

The z_x, z_y and z_{xy} values are obtained in two different ways:

(1) Formula method: Both three-point and five-point formulas are available for approximating z_x and z_y[3]. The following five-point formulas are applied to the DMA data:

$$z_x(0,0) = \frac{1}{12h}\left[z(-2h,0) - 8z(-h,0) + 8z(h,0) - z(2h,0) \right]$$
$$z_y(0,0) = \frac{1}{12h}\left[z(0,-2h) - 8z(0,-h) + 8z(0,h) - z(0,2h) \right] \quad (9)$$

The h-value in the above formulas represents the scaled distance between grid points on the x-axis. The z_{xy} value is obtained using the following formula[2]:

$$z_{xy}(i,j) = \frac{z_{i+j,j+1} - z_{i+1,j-1} + z_{i-1,j-1} - z_{i-1,j+1}}{(x_{i+1} - x_{i-1})(y_{j+1} - y_{j-1})} \quad (10)$$

(2) Least squares method: A bicubic polynomial of the form

$$g(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} b_{ij} x^i y^j \tag{11}$$

is obtained using the method of least squares. The b_{ij} values in Equation 11 are real numbers. The bicubic obtained is used to determine z_x, z_y and z_{xy}. To apply the method of least squares, the data on each subgrid is scaled as shown in Figure 3. The NFF value of 5 is fixed for this stage.

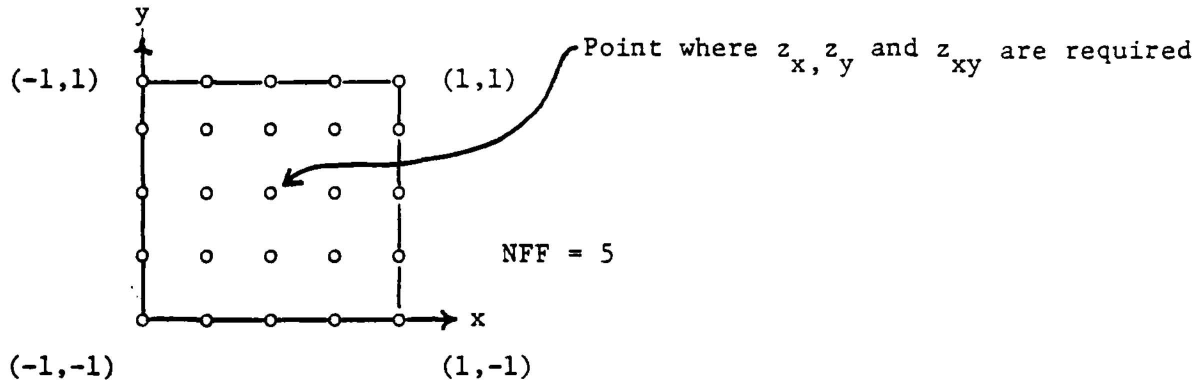


FIGURE 3 - SCALING OF DATA

The method of least squares involves finding a bicubic polynomial as in Equation 11 which is defined over the entire subgrid and which best approximates the given terrain data (z-values) in the following sense: Let

$$S^2 = \sum_{\substack{\text{all } (x,y) \\ \text{in} \\ \text{subgrid}}} (g(x,y) - z(x,y))^2 \tag{12}$$

where $z(x,y)$ is the actual terrain height at the point (x,y). The $g(x,y)$ bicubic polynomial obtained is the bicubic which minimizes the S^2 value in Equation 12.

The above minimization requirement leads to a set of simultaneous normal equations which must be solved to obtain g(x,y). This system theoretically has a solution, but the solution is difficult to obtain accurately because of round-off error. Also considerable computing time is required in solving such a system of equations. Hence an alternative method is used to improve our approximation of the z-values and to reduce the amount of CPU time needed to calculate g(x,y). The alternative method uses orthogonal polynomials and the are obtained as follows [4]:

Polynomials P_i, $i = 0,1,2,3$, are obtained where P_i is a polynomia of a single variable of degree i. The P_i's all have a leading coefficient of 1. Set

$$P_o(x) = 1 \quad . \tag{13}$$

Let

$$S_o = \sum_{n=1}^{NFF} \left[P_o(x_n)\right]^2 \tag{14}$$

and

$$B_o = \left[\sum_{n=1}^{NFF} x_n\right] \Bigg/ S_o \quad . \tag{15}$$

Then

$$P_1(x) = (x - B_o)\cdot P_o(x) \quad . \tag{16}$$

With P_o, P_1 already constructed, P_2 is obtained by letting

$$S_1 = \sum_{n=1}^{NFF} \left[P_1(x_n)\right]^2 \tag{17}$$

$$B_1 = \left[\sum_{n=1}^{NFF} x_n \left[P_1(x_n)\right]^2\right] \Bigg/ S_1 \tag{18}$$

and

$$C_1 = S_1/S_o \quad . \tag{19}$$

Then

$$P_2(x) = (x-B_1)P_1(x) - C_1 \cdot P_o(x) \quad . \tag{20}$$

The polynomial $P_3(x)$ is obtained in a similar manner.

The set of polynomials

$$\left\{ P_i(x) \cdot P_j(y) \,\middle|\, i = 0,1,2,3;\ j = 0,1,2,3 \right\}$$

then form a set of othogonal polynomials for the (x,y)-grid of Figure 3. A bicubic g(x,y) of the form

$$g(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} b_{ij} P_i(x) \cdot P_j(y) \tag{21}$$

is obtained by letting

$$b_{ij} = \frac{\sum_{r=1}^{5} \sum_{s=1}^{5} z(x_r, y_s) \cdot P_i(x_r) \cdot P_j(y_s)}{\sum_{r=1}^{5} P_i^2(x_r) \cdot \sum_{r=1}^{5} P_j^2(y_r)} \tag{22}$$

where

$$x(r) = -1.0 + (r - 1.0)/2, \qquad y(r) = -1.0 + (r - 1.0)/2 \tag{23}$$

Thus the z_x, z_y and z_{xy} are obtained in two ways: (1) Formula method and (2) Least squares method. These two methods are compared to determine which yields a better approximation to the terrain heights.

The method of determining the optimal NFF value to use for a particular manuscript is now described. As mentioned earlier, a large NFF value is desired for compaction purposes; however the error introduced in representing the terrain surface increases as the NFF value increases. Thus to determine an appropriate NFF value, an error analysis is performed on each manuscript for NFF values of 5,6,7,8 and 9. For each of these NFF values, the following error terms are calculated over an entire manuscript:

$$\text{Error}(x,y) = z(x,y) - \text{approximated } z(x,y) \tag{24}$$

$$\text{Relative Error} = \left| \frac{z(x,y) - \text{approximated } z(x,y)}{z(x,y)} \times 100 \right| \tag{25}$$

$$\text{Average Error} = \sum_{\substack{\text{All } (x,y)\\ \text{in}\\ \text{Manuscript}}} \text{Error}(x,y) \Big/ \text{number of terms} \tag{26}$$

$$\text{Average absolute Error} = \sum_{\substack{\text{All } (x,y)\\ \text{in}\\ \text{manuscript}}} \left| \text{Error}(x,y) \right| \Big/ \text{number of terms} \tag{27}$$

$$\text{Standard Deviation of Error} = \sqrt{\frac{\Sigma\,\text{Error}^2(x,y)}{\#\text{of terms}} - (\text{Ave. Error})^2} \tag{28}$$

Also a histogram of error terms is generated for each NFF VALUE. Thus for NFF values from 5 to 9, an error analysis table is generated for the manuscript. Comparing the error analysis tables, one chooses the largest NFF value for which the error values are acceptable. This process associates a single NFF value with each manuscript. To compare the two methods of obtaining the z_x, z_y and z_{xy} values, error analysis tables are generated for each method.

III. INTERPOLATION TECHNIQUE APPLIED TO ACTUAL DMA DATA

The results of applying the above techniques to actual DMA terrain data are described below. The manuscript used for this analysis has a southwest corner point which resides at 47^{o}north, 124^{o} west. The two methods (Formula method and Least Squares method) for generating the z_x, z_y and z_{xy} values are both used. The data values which fall in the shaded region of Figure 4 are not approximated by the bicubic surface. These values however are used in approximating derivative values for interior points.

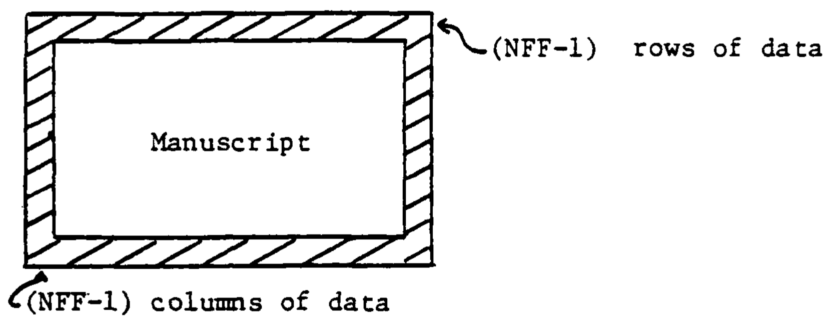


FIGURE 4 - VALUES NOT APPROXIMATED BY BICUBIC POLYNOMIAL

The error analysis tables for this manuscript are found in the Appendix. Table 1 gives the error analysis for a bicubic surface obtained by the method of Least Squares with NFF = 5. This surface represents the bicubic surface that best fits the data in the sense of Equation 12. This local surface however does not yield a global surface which is continuous. Table 1 shows that this surface fits the terrain data with minimal error. Table 2 gives the error analysis for NFF = 5 where the Formula technique is used to generate z_x, z_y and z_{xy}. Table 3 contains the same error analysis except that the Least Squares method is used to obtain z_x, y_z and z_{xy}.

As shown by Tables 1-3, more error is introduced into the bicubic surface in order to obtain global continuity and smoothness. Also, comparing Tables 2 and 3, it seems as though the Formula method of generating z_x, z_y and z_{xy} yields a bicubic surface which better represents the actual terrain data than does the more complex Least Squares method.

Tables 2 and 3 show that the global bicubic surface fits the terrain data reasonably well for NFF=5. In this case 76.1% of the error terms fall in the interval (-10,10) for the Formula method and the corresponding value for the Least Squares method is 60.7%. Also only .2% of the absolute value of the error terms are larger than 100 using the Formula method.

Tables 4-6 present a similar error analysis for NFF = 6. Similar patterns as those for NFF = 5 also appear here. The Formula method again gives less overall error than does the Least Squares technique. However the maximum absolute error using the Formula method (555.87) is significantly larger than the corresponding value for the Least Squares method (356.17). Tables 5 and 6 show that a reasonably good functional approximation to the terrain data is obtained for NFF = 6. For this NFF value, the percent of terms which falls in the interval (-10,10) is 65.0% or 51.9% depending upon whether the Formula method or the Least Squares method is used.

Tables 7-15 present the same error analysis for NFF values of 7-9. In all cases the Formula method gives better results than does the Least Squares method even though the maximum absolute error tends to be larger for the Formula technique. The contents of Tables 1-15 are summarized in Table 16.

For compaction purposes the NFF x NFF z-values on each subgrid are replaced with the 16 integer coefficients of the bicubic polynomial of Equation 1. If

$$\text{Compaction Ratio} = \frac{\text{number of z-values}}{\text{number of coefficients}} \qquad (29)$$

then for NFF = 6, the compaction ratio is 1.6 and for NFF = 7, this value is 2.3. Thus for NFF = 6, the DMA terrain height data set is 1.6 times as large as the data set consisting of coefficients of bicubic polynomials. The compaction ratios for NFF = 5-9 are listed in Table 16.

IV. ADDITIONAL RESEARCH QUESTIONS

(1) For the Least Squares method of finding z, z_x and z_{xy} at a corner point of a subgrid, only the points marked by "x" below in Figure 5 are used in this derivation.

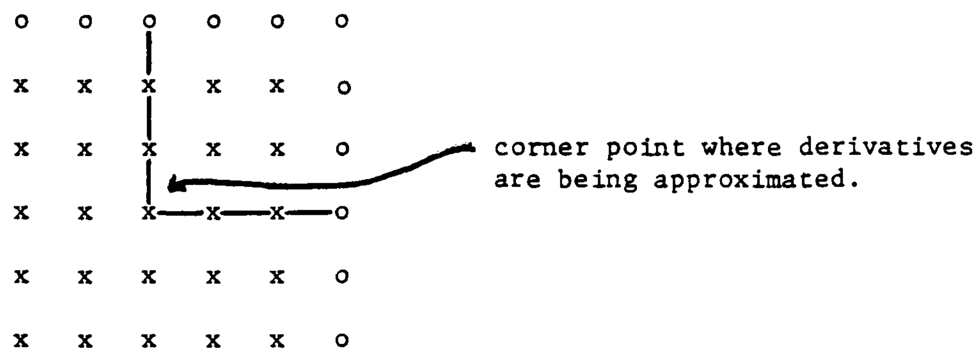


FIGURE 5 - POINTS USED IN METHOD OF LEAST SQUARES

Should the number of points vary as NFF changes?

(2) What effect would a weighted least squares technique for finding z_x, z_y and z_{xy} have on the error analysis?

(3) Can the technique be extended to include the points in the shaded region of Figure 4?

(4) A data set consisting of all z, z_x, z_y and z_{xy} values at corner points of subgrids may be used to represent the terrain surface. With this set much more compaction takes place. However some compaction is lost because z_x, z_y and z_{xy} are real values instead of integers. Can this problem be overcome to increase the amount of compaction obtained without increasing the amount of error.

(5) Does the Formula method produce better results than the Least Squares method over a wide range of terrain types?

(6) Can the maximum absolute error using the Formula method be reduced?

(7) What other formulas might be used to obtain z_x, z_y and z_{xy}?

Note: All programs developed for this grant are written in Pascal and run on a PDP-11/44 computer system which uses the UNIX operating systems.

APPENDIX

TABLE 1

ERROR ANALYSIS USING LOCAL BICUBIC
OBTAINED BY METHOD OF LEAST SQUARES
GLOBAL SURFACE NOT CONTINUOUS

NFF = 5

AVERAGE ERROR	=	-0.00
AVERAGE ABSOLUTE ERROR	=	1.04
MAXIMUM ABSOLUTE ERROR	=	99.51
STANDARD DEVIATION OF ERROR	=	2.19

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	0
# OF ERROR VALUES BETWEEN -100 AND -90	=	1
# OF ERROR VALUES BETWEEN -90 AND -80	=	0
# OF ERROR VALUES BETWEEN -80 AND -70	=	0
# OF ERROR VALUES BETWEEN -70 AND -60	=	8
# OF ERROR VALUES BETWEEN -60 AND -50	=	14
# OF ERROR VALUES BETWEEN -50 AND -40	=	22
# OF ERROR VALUES BETWEEN -40 AND -30	=	138
# OF ERROR VALUES BETWEEN -30 AND -20	=	650
# OF ERROR VALUES BETWEEN -20 AND -10	=	7390
# OF ERROR VALUES BETWEEN -10 AND 0	=	1228381
# OF ERROR VALUES BETWEEN 0 AND 10	=	990264
# OF ERROR VALUES BETWEEN 10 AND 20	=	7287
# OF ERROR VALUES BETWEEN 20 AND 30	=	668
# OF ERROR VALUES BETWEEN 30 AND 40	=	141
# OF ERROR VALUES BETWEEN 40 AND 50	=	51
# OF ERROR VALUES BETWEEN 50 AND 60	=	8
# OF ERROR VALUES BETWEEN 60 AND 70	=	0
# OF ERROR VALUES BETWEEN 70 AND 80	=	1
# OF ERROR VALUES BETWEEN 80 AND 90	=	0
# OF ERROR VALUES BETWEEN 90 AND 100	=	1
# OF ERROR VALUES ABOVE 100	=	0

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	2204606
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	2885
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	750
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	314
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	172
# OF REL. ERROR VALUES ABOVE 25	=	667

TABLE 2

ERROR ANALYSIS USING FORMULA METHOD

NFF = 5

AVERAGE ERROR	=	0.00
AVERAGE ABSOLUTE ERROR	=	8.34
MAXIMUM ABSOLUTE ERROR	=	298.87
STANDARD DEVIATION OF ERROR	=	16.54

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	2097
# OF ERROR VALUES BETWEEN -100 AND -90	=	1229
# OF ERROR VALUES BETWEEN -90 AND -80	=	1887
# OF ERROR VALUES BETWEEN -80 AND -70	=	2837
# OF ERROR VALUES BETWEEN -70 AND -60	=	4354
# OF ERROR VALUES BETWEEN -60 AND -50	=	6913
# OF ERROR VALUES BETWEEN -50 AND -40	=	11565
# OF ERROR VALUES BETWEEN -40 AND -30	=	19915
# OF ERROR VALUES BETWEEN -30 AND -20	=	37724
# OF ERROR VALUES BETWEEN -20 AND -10	=	82563
# OF ERROR VALUES BETWEEN -10 AND 0	=	601500
# OF ERROR VALUES BETWEEN 0 AND 10	=	461563
# OF ERROR VALUES BETWEEN 10 AND 20	=	90907
# OF ERROR VALUES BETWEEN 20 AND 30	=	40499
# OF ERROR VALUES BETWEEN 30 AND 40	=	20854
# OF ERROR VALUES BETWEEN 40 AND 50	=	11516
# OF ERROR VALUES BETWEEN 50 AND 60	=	6394
# OF ERROR VALUES BETWEEN 60 AND 70	=	3807
# OF ERROR VALUES BETWEEN 70 AND 80	=	2300
# OF ERROR VALUES BETWEEN 80 AND 90	=	1359
# OF ERROR VALUES BETWEEN 90 AND 100	=	773
# OF ERROR VALUES ABOVE 100	=	1165

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	1288901
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	70068
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	20072
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	7908
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	3737
# OF REL. ERROR VALUES ABOVE 25	=	7025

TABLE 3

ERROR ANALYSIS USING LEAST SQUARES WITH ORTHOGONAL POLYNOMIALS

NFF = 5

AVERAGE ERROR	=	0.16
AVERAGE ABSOLUTE ERROR	=	13.27
MAXIMUM ABSOLUTE ERROR	=	302.39
STANDARD DEVIATION OF ERROR	=	22.21

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	1619
# OF ERROR VALUES BETWEEN -100 AND -90	=	1320
# OF ERROR VALUES BETWEEN -90 AND -80	=	2509
# OF ERROR VALUES BETWEEN -80 AND -70	=	4758
# OF ERROR VALUES BETWEEN -70 AND -60	=	8650
# OF ERROR VALUES BETWEEN -60 AND -50	=	14800
# OF ERROR VALUES BETWEEN -50 AND -40	=	25458
# OF ERROR VALUES BETWEEN -40 AND -30	=	42002
# OF ERROR VALUES BETWEEN -30 AND -20	=	67609
# OF ERROR VALUES BETWEEN -20 AND -10	=	112539
# OF ERROR VALUES BETWEEN -10 AND 0	=	491296
# OF ERROR VALUES BETWEEN 0 AND 10	=	350668
# OF ERROR VALUES BETWEEN 10 AND 20	=	108196
# OF ERROR VALUES BETWEEN 20 AND 30	=	66921
# OF ERROR VALUES BETWEEN 30 AND 40	=	42574
# OF ERROR VALUES BETWEEN 40 AND 50	=	26301
# OF ERROR VALUES BETWEEN 50 AND 60	=	15697
# OF ERROR VALUES BETWEEN 60 AND 70	=	9215
# OF ERROR VALUES BETWEEN 70 AND 80	=	5263
# OF ERROR VALUES BETWEEN 80 AND 90	=	2834
# OF ERROR VALUES BETWEEN 90 AND 100	=	1630
# OF ERROR VALUES ABOVE 100	=	2350

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	1197347
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	128576
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	33751
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	12528
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	5753
# OF REL. ERROR VALUES ABOVE 25	=	10244

TABLE 4

ERROR ANALYSIS USING LOCAL BICUBIC
OBTAINED BY METHOD OF LEAST SQUARES
GLOBAL SURFACE IS NOT CONTINUOUS

NFF = 6

AVERAGE ERROR	=	-0.00
AVERAGE ABSOLUTE ERROR	=	1.66
MAXIMUM ABSOLUTE ERROR	=	93.76
STANDARD DEVIATION OF ERROR	=	3.20

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	0
# OF ERROR VALUES BETWEEN -100 AND -90	=	5
# OF ERROR VALUES BETWEEN -90 AND -80	=	1
# OF ERROR VALUES BETWEEN -80 AND -70	=	4
# OF ERROR VALUES BETWEEN -70 AND -60	=	8
# OF ERROR VALUES BETWEEN -60 AND -50	=	25
# OF ERROR VALUES BETWEEN -50 AND -40	=	96
# OF ERROR VALUES BETWEEN -40 AND -30	=	331
# OF ERROR VALUES BETWEEN -30 AND -20	=	1698
# OF ERROR VALUES BETWEEN -20 AND -10	=	18323
# OF ERROR VALUES BETWEEN -10 AND 0	=	1033608
# OF ERROR VALUES BETWEEN 0 AND 10	=	981851
# OF ERROR VALUES BETWEEN 10 AND 20	=	18146
# OF ERROR VALUES BETWEEN 20 AND 30	=	1761
# OF ERROR VALUES BETWEEN 30 AND 40	=	344
# OF ERROR VALUES BETWEEN 40 AND 50	=	95
# OF ERROR VALUES BETWEEN 50 AND 60	=	35
# OF ERROR VALUES BETWEEN 60 AND 70	=	10
# OF ERROR VALUES BETWEEN 70 AND 80	=	12
# OF ERROR VALUES BETWEEN 80 AND 90	=	3
# OF ERROR VALUES BETWEEN 90 AND 100	=	0
# OF ERROR VALUES ABOVE 100	=	0

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	2023993
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	5864
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	1267
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	480
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	246
# OF REL. ERROR VALUES ABOVE 25	=	1025

TABLE 5

ERROR ANALYSIS USING FORMULA METHOD

NFF = 6

AVERAGE ERROR	=	0.00
AVERAGE ABSOLUTE ERROR	=	14.63
MAXIMUM ABSOLUTE ERROR	=	555.87
STANDARD DEVIATION OF ERROR	=	28.94

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	13864
# OF ERROR VALUES BETWEEN -100 AND -90	=	4124
# OF ERROR VALUES BETWEEN -90 AND -80	=	5495
# OF ERROR VALUES BETWEEN -80 AND -70	=	7373
# OF ERROR VALUES BETWEEN -70 AND -60	=	10068
# OF ERROR VALUES BETWEEN -60 AND -50	=	14130
# OF ERROR VALUES BETWEEN -50 AND -40	=	20503
# OF ERROR VALUES BETWEEN -40 AND -30	=	30779
# OF ERROR VALUES BETWEEN -30 AND -20	=	48359
# OF ERROR VALUES BETWEEN -20 AND -10	=	88290
# OF ERROR VALUES BETWEEN -10 AND 0	=	509062
# OF ERROR VALUES BETWEEN 0 AND 10	=	395108
# OF ERROR VALUES BETWEEN 10 AND 20	=	98633
# OF ERROR VALUES BETWEEN 20 AND 30	=	53499
# OF ERROR VALUES BETWEEN 30 AND 40	=	33348
# OF ERROR VALUES BETWEEN 40 AND 50	=	21739
# OF ERROR VALUES BETWEEN 50 AND 60	=	14767
# OF ERROR VALUES BETWEEN 60 AND 70	=	10423
# OF ERROR VALUES BETWEEN 70 AND 80	=	7301
# OF ERROR VALUES BETWEEN 80 AND 90	=	5269
# OF ERROR VALUES BETWEEN 90 AND 100	=	3734
# OF ERROR VALUES ABOVE 100	=	10728

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	1173694
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	120716
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	44632
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	20263
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	10946
# OF REL. ERROR VALUES ABOVE 25	=	20575

TABLE 6

ERROR ANALYSIS USING LEAST SQUARES
WITH ORTHOGONAL POLYNOMIALS

NFF = 6

AVERAGE ERROR	=	0.34
AVERAGE ABSOLUTE ERROR	=	20.38
MAXIMUM ABSOLUTE ERROR	=	356.17
STANDARD DEVIATION OF ERROR	=	33.54

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	12154
# OF ERROR VALUES BETWEEN -100 AND -90	=	6554
# OF ERROR VALUES BETWEEN -90 AND -80	=	9619
# OF ERROR VALUES BETWEEN -80 AND -70	=	13670
# OF ERROR VALUES BETWEEN -70 AND -60	=	19197
# OF ERROR VALUES BETWEEN -60 AND -50	=	26212
# OF ERROR VALUES BETWEEN -50 AND -40	=	35894
# OF ERROR VALUES BETWEEN -40 AND -30	=	48707
# OF ERROR VALUES BETWEEN -30 AND -20	=	66736
# OF ERROR VALUES BETWEEN -20 AND -10	=	99444
# OF ERROR VALUES BETWEEN -10 AND 0	=	409952
# OF ERROR VALUES BETWEEN 0 AND 10	=	302879
# OF ERROR VALUES BETWEEN 10 AND 20	=	94794
# OF ERROR VALUES BETWEEN 20 AND 30	=	64926
# OF ERROR VALUES BETWEEN 30 AND 40	=	48636
# OF ERROR VALUES BETWEEN 40 AND 50	=	36416
# OF ERROR VALUES BETWEEN 50 AND 60	=	27179
# OF ERROR VALUES BETWEEN 60 AND 70	=	19905
# OF ERROR VALUES BETWEEN 70 AND 80	=	14525
# OF ERROR VALUES BETWEEN 80 AND 90	=	10119
# OF ERROR VALUES BETWEEN 90 AND 100	=	6942
# OF ERROR VALUES ABOVE 100	=	14396

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	1051168
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	194723
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	67117
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	26833
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	12397
# OF REL. ERROR VALUES ABOVE 25	=	21484

TABLE 7

ERROR ANALYSIS USING LOCAL BICUBIC
OBTAINED BY METHOD OF LEAST SQUARES
GLOBAL SURFACE NOT CONTINUOUS

NFF = 7

AVERAGE ERROR	=	-0.00
AVERAGE ABSOLUTE ERROR	=	2.23
MAXIMUM ABSOLUTE ERROR	=	104.71
STANDARD DEVIATION OF ERROR	=	4.12

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	1
# OF ERROR VALUES BETWEEN -100 AND -90	=	2
# OF ERROR VALUES BETWEEN -90 AND -80	=	7
# OF ERROR VALUES BETWEEN -80 AND -70	=	7
# OF ERROR VALUES BETWEEN -70 AND -60	=	27
# OF ERROR VALUES BETWEEN -60 AND -50	=	58
# OF ERROR VALUES BETWEEN -50 AND -40	=	200
# OF ERROR VALUES BETWEEN -40 AND -30	=	718
# OF ERROR VALUES BETWEEN -30 AND -20	=	3498
# OF ERROR VALUES BETWEEN -20 AND -10	=	31181
# OF ERROR VALUES BETWEEN -10 AND 0	=	949126
# OF ERROR VALUES BETWEEN 0 AND 10	=	920072
# OF ERROR VALUES BETWEEN 10 AND 20	=	30740
# OF ERROR VALUES BETWEEN 20 AND 30	=	3701
# OF ERROR VALUES BETWEEN 30 AND 40	=	778
# OF ERROR VALUES BETWEEN 40 AND 50	=	215
# OF ERROR VALUES BETWEEN 50 AND 60	=	73
# OF ERROR VALUES BETWEEN 60 AND 70	=	29
# OF ERROR VALUES BETWEEN 70 AND 80	=	11
# OF ERROR VALUES BETWEEN 80 AND 90	=	4
# OF ERROR VALUES BETWEEN 90 AND 100	=	0
# OF ERROR VALUES ABOVE 100	=	1

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	1905147
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	9003
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	2020
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	648
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	366
# OF REL. ERROR VALUES ABOVE 25	=	1223

TABLE 8

ERROR ANALYSIS USING FORMULA METHOD

NFF = 7

AVERAGE ERROR	=	-0.15
AVERAGE ABSOLUTE ERROR	=	22.38
MAXIMUM ABSOLUTE ERROR	=	968.37
STANDARD DEVIATION OF ERROR	=	43.94

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	35484
# OF ERROR VALUES BETWEEN -100 AND -90	=	7509
# OF ERROR VALUES BETWEEN -90 AND -80	=	9251
# OF ERROR VALUES BETWEEN -80 AND -70	=	11674
# OF ERROR VALUES BETWEEN -70 AND -60	=	14560
# OF ERROR VALUES BETWEEN -60 AND -50	=	19290
# OF ERROR VALUES BETWEEN -50 AND -40	=	25497
# OF ERROR VALUES BETWEEN -40 AND -30	=	35673
# OF ERROR VALUES BETWEEN -30 AND -20	=	51749
# OF ERROR VALUES BETWEEN -20 AND -10	=	85903
# OF ERROR VALUES BETWEEN -10 AND 0	=	440994
# OF ERROR VALUES BETWEEN 0 AND 10	=	343866
# OF ERROR VALUES BETWEEN 10 AND 20	=	96796
# OF ERROR VALUES BETWEEN 20 AND 30	=	58142
# OF ERROR VALUES BETWEEN 30 AND 40	=	39655
# OF ERROR VALUES BETWEEN 40 AND 50	=	28134
# OF ERROR VALUES BETWEEN 50 AND 60	=	20683
# OF ERROR VALUES BETWEEN 60 AND 70	=	15500
# OF ERROR VALUES BETWEEN 70 AND 80	=	11923
# OF ERROR VALUES BETWEEN 80 AND 90	=	9335
# OF ERROR VALUES BETWEEN 90 AND 100	=	7438
# OF ERROR VALUES ABOVE 100	=	30433

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	1055030
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	158582
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	69310
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	35812
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	20613
# OF REL. ERROR VALUES ABOVE 25	=	44622

TABLE 9

ERROR ANALYSIS USING LEAST SQUARES
WITH ORTHOGONAL POLYNOMIALS

NFF = 7

AVERAGE ERROR	=	0.48
AVERAGE ABSOLUTE ERROR	=	27.76
MAXIMUM ABSOLUTE ERROR	=	462.65
STANDARD DEVIATION OF ERROR	=	45.07

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	36582
# OF ERROR VALUES BETWEEN -100 AND -90	=	12580
# OF ERROR VALUES BETWEEN -90 AND -80	=	15954
# OF ERROR VALUES BETWEEN -80 AND -70	=	20119
# OF ERROR VALUES BETWEEN -70 AND -60	=	24657
# OF ERROR VALUES BETWEEN -60 AND -50	=	30527
# OF ERROR VALUES BETWEEN -50 AND -40	=	37452
# OF ERROR VALUES BETWEEN -40 AND -30	=	48589
# OF ERROR VALUES BETWEEN -30 AND -20	=	61941
# OF ERROR VALUES BETWEEN -20 AND -10	=	88691
# OF ERROR VALUES BETWEEN -10 AND 0	=	367953
# OF ERROR VALUES BETWEEN 0 AND 10	=	273742
# OF ERROR VALUES BETWEEN 10 AND 20	=	86212
# OF ERROR VALUES BETWEEN 20 AND 30	=	61159
# OF ERROR VALUES BETWEEN 30 AND 40	=	48406
# OF ERROR VALUES BETWEEN 40 AND 50	=	39029
# OF ERROR VALUES BETWEEN 50 AND 60	=	31000
# OF ERROR VALUES BETWEEN 60 AND 70	=	25060
# OF ERROR VALUES BETWEEN 70 AND 80	=	20377
# OF ERROR VALUES BETWEEN 80 AND 90	=	16643
# OF ERROR VALUES BETWEEN 90 AND 100	=	13168
# OF ERROR VALUES ABOVE 100	=	39648

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	938772
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	238454
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	100794
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	46287
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	22596
# OF REL. ERROR VALUES ABOVE 25	=	37066

TABLE 10

ERROR ANALYSIS USING LOCAL BICUBIC
OBTAINED BY METHOD OF LEAST SQUARES
GLOBAL SURFACE NOT CONTINUOUS

NFF = 8

AVERAGE ERROR	=	-0.00
AVERAGE ABSOLUTE ERROR	=	2.84
MAXIMUM ABSOLUTE ERROR	=	111.35
STANDARD DEVIATION OF ERROR	=	5.06

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	3
# OF ERROR VALUES BETWEEN -100 AND -90	=	4
# OF ERROR VALUES BETWEEN -90 AND -80	=	8
# OF ERROR VALUES BETWEEN -80 AND -70	=	18
# OF ERROR VALUES BETWEEN -70 AND -60	=	48
# OF ERROR VALUES BETWEEN -60 AND -50	=	115
# OF ERROR VALUES BETWEEN -50 AND -40	=	358
# OF ERROR VALUES BETWEEN -40 AND -30	=	1227
# OF ERROR VALUES BETWEEN -30 AND -20	=	6364
# OF ERROR VALUES BETWEEN -20 AND -10	=	47003
# OF ERROR VALUES BETWEEN -10 AND 0	=	887467
# OF ERROR VALUES BETWEEN 0 AND 10	=	852907
# OF ERROR VALUES BETWEEN 10 AND 20	=	45655
# OF ERROR VALUES BETWEEN 20 AND 30	=	6426
# OF ERROR VALUES BETWEEN 30 AND 40	=	1412
# OF ERROR VALUES BETWEEN 40 AND 50	=	381
# OF ERROR VALUES BETWEEN 50 AND 60	=	115
# OF ERROR VALUES BETWEEN 60 AND 70	=	54
# OF ERROR VALUES BETWEEN 70 AND 80	=	22
# OF ERROR VALUES BETWEEN 80 AND 90	=	6
# OF ERROR VALUES BETWEEN 90 AND 100	=	4
# OF ERROR VALUES ABOVE 100	=	3

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	1809789
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	13813
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	2845
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	898
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	395
# OF REL. ERROR VALUES ABOVE 25	=	1397

TABLE 11

ERROR ANALYSIS USING FORMULA METTHOD

NFF = 8

AVERAGE ERROR	=	-0.09
AVERAGE ABSOLUTE ERROR	=	31.85
MAXIMUM ABSOLUTE ERROR	=	1403.64
STANDARD DEVIATION OF ERROR	=	62.05

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	62649
# OF ERROR VALUES BETWEEN -100 AND -90	=	9877
# OF ERROR VALUES BETWEEN -90 AND -80	=	11791
# OF ERROR VALUES BETWEEN -80 AND -70	=	14278
# OF ERROR VALUES BETWEEN -70 AND -60	=	17522
# OF ERROR VALUES BETWEEN -60 AND -50	=	21974
# OF ERROR VALUES BETWEEN -50 AND -40	=	27998
# OF ERROR VALUES BETWEEN -40 AND -30	=	37081
# OF ERROR VALUES BETWEEN -30 AND -20	=	50928
# OF ERROR VALUES BETWEEN -20 AND -10	=	79672
# OF ERROR VALUES BETWEEN -10 AND 0	=	387648
# OF ERROR VALUES BETWEEN 0 AND 10	=	302871
# OF ERROR VALUES BETWEEN 10 AND 20	=	90140
# OF ERROR VALUES BETWEEN 20 AND 30	=	57793
# OF ERROR VALUES BETWEEN 30 AND 40	=	41552
# OF ERROR VALUES BETWEEN 40 AND 50	=	31494
# OF ERROR VALUES BETWEEN 50 AND 60	=	24061
# OF ERROR VALUES BETWEEN 60 AND 70	=	18963
# OF ERROR VALUES BETWEEN 70 AND 80	=	15650
# OF ERROR VALUES BETWEEN 80 AND 90	=	12429
# OF ERROR VALUES BETWEEN 90 AND 100	=	10452
# OF ERROR VALUES ABOVE 100	=	58506

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	940332
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	182346
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	88962
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	50177
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	30783
# OF REL. ERROR VALUES ABOVE 25	=	77848

TABLE 12

ERROR ANALYSIS USING LEAST SQUARES
WITH ORTHOGONAL POLYNOMIALS

NFF = 8

AVERAGE ERROR	=	0.86
AVERAGE ABSOLUTE ERROR	=	35.40
MAXIMUM ABSOLUTE ERROR	=	519.23
STANDARD DEVIATION OF ERROR	=	57.14

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	65695
# OF ERROR VALUES BETWEEN -100 AND -90	=	15302
# OF ERROR VALUES BETWEEN -90 AND -80	=	18116
# OF ERROR VALUES BETWEEN -80 AND -70	=	21057
# OF ERROR VALUES BETWEEN -70 AND -60	=	24713
# OF ERROR VALUES BETWEEN -60 AND -50	=	30281
# OF ERROR VALUES BETWEEN -50 AND -40	=	37606
# OF ERROR VALUES BETWEEN -40 AND -30	=	44844
# OF ERROR VALUES BETWEEN -30 AND -20	=	55881
# OF ERROR VALUES BETWEEN -20 AND -10	=	77872
# OF ERROR VALUES BETWEEN -10 AND 0	=	326163
# OF ERROR VALUES BETWEEN 0 AND 10	=	244741
# OF ERROR VALUES BETWEEN 10 AND 20	=	77213
# OF ERROR VALUES BETWEEN 20 AND 30	=	55300
# OF ERROR VALUES BETWEEN 30 AND 40	=	43946
# OF ERROR VALUES BETWEEN 40 AND 50	=	37083
# OF ERROR VALUES BETWEEN 50 AND 60	=	30604
# OF ERROR VALUES BETWEEN 60 AND 70	=	25952
# OF ERROR VALUES BETWEEN 70 AND 80	=	22067
# OF ERROR VALUES BETWEEN 80 AND 90	=	19064
# OF ERROR VALUES BETWEEN 90 AND 100	=	16323
# OF ERROR VALUES ABOVE 100	=	70887

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	816639
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	249770
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	123759
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	63158
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	34435
# OF REL. ERROR VALUES ABOVE 25	=	58847

TABLE 13

ERROR ANALYSIS USING LOCAL BICUBIC
OBTAINED BY METHOD OF LEAST SQUARES
GLOBAL SURFACE NOT CONTINUOUS

NFF = 9

AVERAGE ERROR	=	-0.00
AVERAGE ABSOLUTE ERROR	=	3.44
MAXIMUM ABSOLUTE ERROR	=	137.58
STANDARD DEVIATION OF ERROR	=	5.98

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	6
# OF ERROR VALUES BETWEEN -100 AND -90	=	7
# OF ERROR VALUES BETWEEN -90 AND -80	=	7
# OF ERROR VALUES BETWEEN -80 AND -70	=	38
# OF ERROR VALUES BETWEEN -70 AND -60	=	71
# OF ERROR VALUES BETWEEN -60 AND -50	=	171
# OF ERROR VALUES BETWEEN -50 AND -40	=	633
# OF ERROR VALUES BETWEEN -40 AND -30	=	1997
# OF ERROR VALUES BETWEEN -30 AND -20	=	9933
# OF ERROR VALUES BETWEEN -20 AND -10	=	64715
# OF ERROR VALUES BETWEEN -10 AND 0	=	821266
# OF ERROR VALUES BETWEEN 0 AND 10	=	823352
# OF ERROR VALUES BETWEEN 10 AND 20	=	62653
# OF ERROR VALUES BETWEEN 20 AND 30	=	10251
# OF ERROR VALUES BETWEEN 30 AND 40	=	2174
# OF ERROR VALUES BETWEEN 40 AND 50	=	631
# OF ERROR VALUES BETWEEN 50 AND 60	=	232
# OF ERROR VALUES BETWEEN 60 AND 70	=	81
# OF ERROR VALUES BETWEEN 70 AND 80	=	34
# OF ERROR VALUES BETWEEN 80 AND 90	=	17
# OF ERROR VALUES BETWEEN 90 AND 100	=	10
# OF ERROR VALUES ABOVE 100	=	2

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	1751709
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	18797
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	3952
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	1222
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	624
# OF REL. ERROR VALUES ABOVE 25	=	1720

TABLE 14

ERROR ANALYSIS USING FORMULA METHOD

NFF = 9

AVERAGE ERROR	=	0.46
AVERAGE ABSOLUTE ERROR	=	42.08
MAXIMUM ABSOLUTE ERROR	=	1812.03
STANDARD DEVIATION OF ERROR	=	81.55

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	89005
# OF ERROR VALUES BETWEEN -100 AND -90	=	11724
# OF ERROR VALUES BETWEEN -90 AND -80	=	13821
# OF ERROR VALUES BETWEEN -80 AND -70	=	16068
# OF ERROR VALUES BETWEEN -70 AND -60	=	19022
# OF ERROR VALUES BETWEEN -60 AND -50	=	23117
# OF ERROR VALUES BETWEEN -50 AND -40	=	29408
# OF ERROR VALUES BETWEEN -40 AND -30	=	37385
# OF ERROR VALUES BETWEEN -30 AND -20	=	49849
# OF ERROR VALUES BETWEEN -20 AND -10	=	74327
# OF ERROR VALUES BETWEEN -10 AND 0	=	348065
# OF ERROR VALUES BETWEEN 0 AND 10	=	274773
# OF ERROR VALUES BETWEEN 10 AND 20	=	85557
# OF ERROR VALUES BETWEEN 20 AND 30	=	56690
# OF ERROR VALUES BETWEEN 30 AND 40	=	42299
# OF ERROR VALUES BETWEEN 40 AND 50	=	32836
# OF ERROR VALUES BETWEEN 50 AND 60	=	25747
# OF ERROR VALUES BETWEEN 60 AND 70	=	21113
# OF ERROR VALUES BETWEEN 70 AND 80	=	17329
# OF ERROR VALUES BETWEEN 80 AND 90	=	15065
# OF ERROR VALUES BETWEEN 90 AND 100	=	12739
# OF ERROR VALUES ABOVE 100	=	89390

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	849803
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	195173
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	104007
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	63196
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	40232
# OF REL. ERROR VALUES ABOVE 25	=	117912

TABLE 15

ERROR ANALYSIS USING LEAST SQUARES
WITH ORTHOGONAL POLYNOMIALS

NFF = 9

AVERAGE ERROR	=	45.95
AVERAGE ABSOLUTE ERROR	=	255.80
MAXIMUM ABSOLUTE ERROR	=	2010.73
STANDARD DEVIATION OF ERROR	=	382.27

HISTOGRAM DATA FOR ACTUAL ERRORS

# OF ERROR VALUES LESS THAN -100	=	328404
# OF ERROR VALUES BETWEEN -100 AND -90	=	11246
# OF ERROR VALUES BETWEEN -90 AND -80	=	11661
# OF ERROR VALUES BETWEEN -80 AND -70	=	14320
# OF ERROR VALUES BETWEEN -70 AND -60	=	31482
# OF ERROR VALUES BETWEEN -60 AND -50	=	30084
# OF ERROR VALUES BETWEEN -50 AND -40	=	24537
# OF ERROR VALUES BETWEEN -40 AND -30	=	23997
# OF ERROR VALUES BETWEEN -30 AND -20	=	29065
# OF ERROR VALUES BETWEEN -20 AND -10	=	33016
# OF ERROR VALUES BETWEEN -10 AND 0	=	92908
# OF ERROR VALUES BETWEEN 0 AND 10	=	63762
# OF ERROR VALUES BETWEEN 10 AND 20	=	32829
# OF ERROR VALUES BETWEEN 20 AND 30	=	28908
# OF ERROR VALUES BETWEEN 30 AND 40	=	28027
# OF ERROR VALUES BETWEEN 40 AND 50	=	27967
# OF ERROR VALUES BETWEEN 50 AND 60	=	40328
# OF ERROR VALUES BETWEEN 60 AND 70	=	38665
# OF ERROR VALUES BETWEEN 70 AND 80	=	14587
# OF ERROR VALUES BETWEEN 80 AND 90	=	12781
# OF ERROR VALUES BETWEEN 90 AND 100	=	12233
# OF ERROR VALUES ABOVE 100	=	454522

HISTOGRAM DATA FOR RELATIVE ERRORS

# OF REL. ERROR VALUES BETWEEN 0 AND 5	=	180116
# OF REL. ERROR VALUES BETWEEN 5 AND 10	=	93845
# OF REL. ERROR VALUES BETWEEN 10 AND 15	=	87635
# OF REL. ERROR VALUES BETWEEN 15 AND 20	=	83596
# OF REL. ERROR VALUES BETWEEN 20 AND 25	=	81203
# OF REL. ERROR VALUES ABOVE 25	=	843928

TABLE 16

NFF	technique used to obtain z ,z and z	% of terms where absolute value of error is $<$ 10	% of terms where absolute value of error is $>$100	Compaction ratio
5	Formula	76.1%	.2%	1.002
5	Least Squares	60.7%	.3%	1.002
6	Formula	65.0%	1.8%	1.565
6	Least Squares	51.9%	1.9%	1.565
7	Formula	56.7%	4.8%	2.254
7	Least Squares	46.4%	5.5%	2.254
8	Formula	50.4%	8.8%	3.083
8	Least Squares	42.4%	10.1%	3.083
9	Formula	45.5%	13.0%	4.007
9	Least Squares	11.4%	57.1%	4.007

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