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REPORT DOCUMENTATION PAGE — READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR-TR- 83-1347
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER: 2

4. TITLE (and Subtitle): KINETIC THEORY OF GASES, MAGNETO-FLUID DYNAMICS AND THEIR APPLICATION
5. TYPE OF REPORT & PERIOD COVERED: INTERIM, 1 DEC 81-30 NOV 82
6. PERFORMING ORG. REPORT NUMBER:

7. AUTHOR(s): Harold Grad
8. CONTRACT OR GRANT NUMBER(s): AFOSR-81-0020

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Courant Institute for Mathematical Sciences, New York University, 251 Mercer Street, New York NY 10012
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: PE61102F; 2304/A4

11. CONTROLLING OFFICE NAME AND ADDRESS: Mathematical & Information Sciences Directorate, Air Force Office of Scientific Research/NM, Bolling AFB DC 20332
12. REPORT DATE: JAN 83
13. NUMBER OF PAGES: 23

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.

DTIC ELECTE JAN 2 0 1984 A

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):

18. SUPPLEMENTARY NOTES:

DTIC COPY INSPECTED

19. KEY WORDS (Continue on reverse side if necessary and identify by block number):

A1

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper describes research results and cites progress resulting from work performed during this period of the grant. The areas covered in this report are (1) mathematical theory of turbulent fluctuations of a plasma near thermal equilibrium, (2) the theory of non-linear thermal and diffusive waves in finite mass and reacting media, (3) the development of algorithms for the Helmholtz equation, (4) progress in the development of theory for Queer Differential Equations, and (5) spectral theory of non-elliptic operators.

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AFOSR-TR- 83-1347

AFOSR Interim Technical Report
Contract # AFOSR 81-0020
January 31, 1983

"Kinetic Theory of Gases, Magneto-fluid Dynamics and Their Application"

The following summary is a progress report for the research currently carried out under AFOSR Contract #AFOSR 81-0020. The areas covered in this report are 1) mathematical theory of turbulent fluctuations of a plasma near thermal equilibrium, 2) the theory of non-linear thermal and diffusive waves in finite mass and reacting media, 3) the development of algorithms for the Helmholtz equation, 4) progress in the development of theory for Queer Differential Equations, 5) spectral theory of non-elliptic operators.

1. a) The Mathematical Theory of Turbulent Fluctuations of a Plasma Near Thermal Equilibrium

This work is being carried out by Eli Hameiri and K. Riedel and is a continuation of the research initiated during the previous contract year under AFOSR Contract #AFOSR 81-0020.

Fluctuations in plasmas are the rule rather than the exception. Strong evidence has been found in space and solar plasma studies. A well known problem involving MHD-fluid turbulence is found in the radar discriminations of the trailing wake of a re-entry vehicle. A brief description of our previous work is given followed by a discussion of the latest results.

We have constructed a model of fluctuating MHD fluid compressible plasma near thermal equilibrium. Moreover, the thermal equilibrium state is a force-free state. A fluid model already exists due to Landau and is based on adding small "noise" terms to the usual equations which are then linearized. In contrast, our approach has not involved extra terms and is fully non-linear. The two models generally agree in the limit of small fluctuations.

Our treatment is based on discretization of the MHD equations, replacing them by their spatial finite-difference approximation over an equally spaced grid, where for simplicity the plasma domain is taken to be a periodic cubic box. Thus we have a system of evolution equations for grid-valued functions. Different representations of the plasma variables (e.g., momentum rather than velocity) do not yield equivalent discrete systems. We choose the variables which are used when writing the MHD equations in conservation law form. We find that the discrete system conserves a discrete form of the helicity $\int \underset{\sim}{A} \cdot \underset{\sim}{B}$ ($\underset{\sim}{B} = \mathrm{curl}\underset{\sim}{A}$). Moreover, A Liouville theorem holds for the flow in the phase space of the grid values of the plasma variables. This property is used to construct a Gibbs distribution function for the variables, based on the various constants of the motion, in analogy with statistical mechanics. The distribution function enables us to calculate equal-time correlation of plasma variables at different locations. We get agreement with results for magnetic field fluctuations which were known for incompressible plasmas, as well as with known results for classical fluids.

More recently we have constructed the two-time correlations for the model. For this it was necessary to add dissipation and random driving forces to the original equations of motion. We postulate a fluctuation-dissipation theorem for the random forces. As a first step we have treated the linearized evolution equations with small fluctuations. In this case the fluctuation distribution become Gaussian. Our finite difference scheme transforms the evolution equations into a constant coefficient stochastic differential equation. The Fourier modes decouple and yield simple forms for second moments or correlations. We are currently examining Hilbert space settings for the distributions to settle convergence properties.

b) A parallel effort in the study of <u>Stochastic Fluctuations and Their Effect on MHD Waves</u>, began during 1981 has been concluded and we mention here the significant results. This work was carried out by W. Grossmann.

We have investigated the effects of stochastic fluctuations on the propagation of waves in an MHD or plasma medium. For the particular choice of fluctuation background depending only on time but spatially independent ideal MHD waves propagating through an infinite uniform medium have been studied. The stochastic fluctuations are assumed to satisfy the time dependent equilibrium equations of ideal MHD with the further assumption that the response of the equilibrium due to the fluctuations takes place on a time scale much longer than characteristic times associated with MHD waves. Linearization of the governing equations leads to stochastic differential equations for the scattered waves. We have extended a technique due to Keller and Papanicolaou to solve for the coherent part of the wave resulting in stochastic corrections to the dispersion relation for the Alfven and magnetoacoustic signal speeds. Depending on the statistics of the fluctuation background which are assumed known through the power spectrum the waves may become destabilized by the noise. Thermal fluctuations with a white noise spectrum lead to stable wave motion with the wave speed decreased due to scattering. This work which was begun under AFOSR Contract #AFOSR 81-0020 will be continued in collaboration with G. Papanicolaou and H. Grad under separate funding.

2. Considerable progress has been made in the mathematical understanding of non-linear diffusion in inhomogeneous, finite-mass and reacting systems, these problems were begun during 1979-80, reported on in our AFOSR renewal proposal, June 24, 1981 and May 1982. We present below results from significant parts of this research. This work was carried out by P. Rosenau.

a) *Non-linear Diffusion in a Finite Mass Medium*

Here we are concerned with the solutions of the heat equation in an inhomogeneous medium:

$$\rho(\chi)\frac{\partial u}{\partial t} = \frac{\partial^2 A(u)}{\partial \chi^2}, \tag{1a}$$

$$u(\chi,0) = u_o(\chi), \tag{1b}$$

in $R^1 \times [0,\infty)$ subject to the constraints that the total mass m of the medium and the initial energy E are finite, i.e.,

$$m = \int_{-\infty}^{\infty} \rho(\chi)\,d\chi<\infty, \tag{2}$$

$$E = \int_{-\infty}^{\infty} u_o(\chi)\rho(\chi)\,d\chi<\infty. \tag{3}$$

It is assumed that $\rho(\chi)$ is a positive smooth function and A(u) satisfies

(4) $A(0) = 0,\ A'(0) \geqq 0$ and $A'(u) > 0$, if $u > 0$

which, in particular includes the case: $A(u) = u^n$, $n \geqq 1$.

With u being the temperature and $\rho(\chi)$ the particle density, we are thus concerned with thermal phenomena in an unbounded medium <u>but of a finite mass</u>. We show it is exactly this property that causes a remarkable change in the nature of the thermal diffusion as compared with diffusion in a medium of infinite mass.

Thermal phenomena in inhomogeneous ambience are of interest in a variety of situations, e.g., extraterrestrial situations where the response to an impulsively initiated blast takes initially the form of a supersonic wave while the medium is quiescent. It is only in a later stage when the wave slows down to the sonic range that the gas is set into motion.

Previously, we have considered the Cauchy problem in an inhomogeneous medium wherein $A(u) = u^n$ and $\rho(\chi) \sim |\chi|^{-l}$, $0 \leqq l < 1$, as $|\chi| \to \infty$ (thus (2) does not hold!). It was then possible to prove that certain solutions, i.e., the similarity solutions that describe a propagating thermal wave, are asymptotic to the general Cauchy problem provided that the two share the same initial energy and the asymptotic distribution of the density is the same (i.e., the same l). Though, as l is varied, $0 \leqq l < 1$, the asymptotic solution varies as well, the change is continuous and the solutions remain similar to each other. Thus the problem of propagation in a homogeneous medium, $\ell=0$, may be considered as a particular case. However, for $\ell>1$ the similarity solution explodes and thus it cannot be used to represent in any sense the general Cauchy problem. This in turn signals that whenever $\ell>1$, or, as we shall find, more generally--whenever the condition (2) holds, thermal diffusion will be different. The analysis of this case is carried out in the present work.

Simply stated, the main result ensures the isothermalization of the medium to a positive average temperature.

Such a result would be natural in a finite domain with homogeneous Neumann condition. Here it is derived for a Cauchy problem. It is of course drastically different from diffusion in a homogeneous medium or any infinite inhomogeneous mass medium, where the average temperature is zero. In a standard diffusion problem the first non-vanishing term describes the decay to "average" zero of the thermal pulse. On the other hand, in our case the calculation of how this average is approached constitutes the second term in an appropriate asymptotic expansion. We plan to report on this in the near future.

One should, however, distinguish between the approach to the average temperature $\bar{u}$ at a given point and the behavior at infinity. Whether the isothermalization of the whole space takes a finite or infinite time still remains to be answered. Note that the possibility that arbitrarily far particles have a finite temperature is physically plausible. Because there are so few of them all the energy contained in the tail is negligible.

There is another difference between the diffusion in the present case and in infinite mass medium that is worth mentioning. In a finite mass medium, inasmuch as the initial energy and the mass is finite, not only the local density distribution but also the actual form of A(u) hardly matters for the validity of our results. On the other hand, diffusion in an infinite mass medium depends not only on the explicit form of A(u), but the details of the density distribution at infinity must be known and they affect the propagation pattern. Moreover, unless the density field was of monomial form, nothing could be said about the asymptotic pattern of propagation.

b) Nonlinear Thermal Wave in a Reacting Medium

Here, we consider a model problem for the expansion of an instantaneous heat source in a medium in which there is volumetric heating and absorption. Thermal conductivity, thermal sinks and sources are all assumed to be temperature dependent. Such conditions occur in many physical situations. Our interest stems from thermal waves in a heated plasma with losses caused by bremsstrahlung radiation. The model equation (1) may be used to describe the motion of a polytropic gas in a porous medium with appropriate interpretation given to the source and sink terms.

Our model equation in slab symmetry reads

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial \chi} u^p \frac{\partial u}{\partial \chi} + hu^\alpha - \gamma u^\alpha, \qquad (1a)$$

where $h(t) \geq 0$ and $\gamma(t) \geq 0$ are known functions of time.

In the bulk of our work, the family of solutions (p,α,q) will be narrowed to a special one-parameter family $(p,1,1-p)$ i.e.,

$$q=1-p, \quad \alpha=1. \qquad (1b)$$

that has the merit of being explicitly solvable.

The simple linear source term is analytically tractable. Note that by reversing the sign of h and γ, one obtains a linear absorption and a non-linear heating source.

We consider a Cauchy problem in $R^1 \times (0,\infty)$ with

$$u(\chi,0) = E_o \delta(\chi), \qquad (2)$$

where E_o is the initial energy and δ is the Dirac measure. If $\gamma=h=0$, one obtains a thermal wave that propagates with a finite speed. The addition of absorption may completely change the pattern of the thermal evolution. Thus, if $p<q+1$ the thermal wave becomes localized and penetrates only to a finite depth. If, in addition, $q<1$, the pulse has a finite life time $t_*<\infty$ and is thereafter extinguished in the context of the problem considered wherein $q=1-p$, using the results due to Kershner. Martinson has solved problem (1) and (2) under the assumption that $h=0$ and $\gamma=\text{const.}>0$. In addition to the aforementioned features it is found that the wave reverses its motion and thereafter shrinks to zero. In our work, this problem is generalized by adding a linear heating source and allowing γ and h, the coefficients of heating and absorption, to be time dependent.

The addition of even a simple heating source like the one considered here may have both qualitative and quantitative influence on the thermal evolution. We study two cases with γ and h being constant. γ and h are assumed to be time dependent. In particular, it is shown that if γ oscillates, so does the wave front. Under certain conditions these oscillations may cause the extinction of the thermal wave.

A very different class of thermal waves is finally considered. Here instead of constraints (1b) we require $q<\min(\alpha,p+1)$ and construct a stationary thermal wave that is self confined in a finite domain.

c) Non-Linear Thermal Evolution in an Inhomogeneous Medium

Various simple transport models of electron temperature in inhomogeneous plasmas are reducible to the quasilinear equation $\rho(\chi)u_t = [c(\chi)u^n_\chi]_\chi + A(\chi)u^s$, $-1<\chi<1, u(\pm 1)=0$. u is the temperature, $\rho(\chi)$ the density, and $c=g[\rho(\chi)]$ the density-dependent part of the thermal diffusion. $\rho(\chi)$ and $c(\chi)$ may vanish at the plasma edge, rendering the problem singular. The temporal behavior depends critically on the boundedness of $R = \int_{-1}^{+1} c^{-1}(\chi)d\chi$. If $R<\infty$ then in the absence of heat sources, $A\equiv 0$, every initially given state $u(\chi,0)$ evolves toward an algebraically decaying, universal space-time separable solution. Its existence and uniqueness is proved. The method developed in this work may be used to show the equilibration of the solution in the presence of a heat source of the form $A(\chi)u^s, s<n, \rho(\chi)>0$. On the other hand, if $R = \infty$ and $A = 0$ then the system becomes isothermalized: $u \to \bar{u} = \int_{-1}^{+1} u(\chi,0)\rho(\chi)d\chi > 0$. In such a case a[illegible] - tion of heat sources will cause a thermal explosion.

3. The Development of algorithms for the Helmholtz equation is one important area of our general interest in the developme[illegible] of computational methods and algorithms for PDE's of important physical problems. We report below on two projects, begun earlier, where significant results have recently been obtained. This work was carried out by A. Bayliss and E. Turkel.

a) The Helmholtz Equation for Nozzles and Underwater Acoustics

This work concerns radiation boundary conditions for Helmholtz type equations in duct-like geometries. A typical problem would be to solve the equation.

(1.1) a) $\Delta u + k^2 u = u_{\chi\chi} + u_{yy} + k^2 u = 0$

for a function $u(\chi,y)$ in the region $\chi \geq 0$, $0 \leq y \leq H$. Typical boundary conditions would be

(1.1) b) $u(\chi,0) = 0$

c) $u_y(\chi,H) = 0$

Problems of the form (1.1) arise in many physical applications. For example, the problem of acoustic waves propagating in a nozzle is covered by (1.1) in cylindrical coordinates r,z and θ. A typical domain for axially symmetric waves in a straight pipe would be $0 \leq r \leq H$, $z \geq 0$ (Here z is the axial coordinate and r is the distance from the axis). In place of (1.1b and c) some impedence conditions might be imposed on the duct walls.

If the roles of r and z are switched so that the domain becomes $0 \leqslant z \leqslant H$, $r \geqslant r_0$ then problem (1.1) describes the propagation of acoustic waves underwater. In this case z describes the horizontal distance from some incident field (i.e., a sonar beam). A dependence on the azimuthal angle θ is also possible, although for simplicity we neglect this. An important feature of this problem is that the sound speed will in general depend on z, i.e., the k^2u term in (1.1a) will be replaced by a term of the form $k^2n(z)u$ for some specified function n(z).

Other applications of problem (1.1) include electromagnetic radiation in a waveguide and the scattering of waves by a cylindrically confined obstacle. These problems are all connected by the fact that they are posed in a domain which is infinite in only one dimension. In order to numerically compute such problems one must somehow impose that the solution represents radiation propagating outwards or equivalently that there are no sources located at + ∞ radiating into the region of interest.

One way to do this is to impose an artificial boundary at for example $\chi=\chi_1$. The problem is then reduced to one in the bounded domain $0 \leqslant \chi \leqslant \chi_1$, $0 \leqslant y \leqslant H$. However at the artificial boundary it is necessary to impose some boundary condition. This boundary condition must simulate outgoing radiation.

The problem is similar to one we previously studied. There the problem of simulating outgoing radiation (i.e., the Sommerfeld radiation condition) for the fully exterior Helmholtz equation was considered. A family of local boundary conditions, i.e., depending only on the derivatives of the solution was developed. These boundary conditions were based on deriving differential operators which matched the solution to a Laurent expansion in the distance from a fixed origin. It was shown that the higher order members of this family provided very accurate approximations at artificial boundaries close to the region of interest.

In duct-like geometries the situation is very different. This is because there are a discrete number of different waves (depending on k) which propagate outwards. These waves all have different wave numbers. There is thus no single radiation condition even at infinity.

A global boundary condition for problem (1.1) was developed by Fix. This boundary condition coupled all of the boundary points using an integral operator. We have introduced a family of local boundary conditions. In the remainder of our work these boundary conditions are generalized and their properties studied.

b) An Iterative Method for the Helmholtz Equation

We have developed an iterative method to solve the Helmholtz equation.

(1.1) $$\Delta u + k^2(x,y)u = 0,$$

in several geometries and in both two and three dimensions.

An important application of (1.1) is the scattering of acoustic waves by an obstacle. In this case we consider the following boundary value problem in the region Ω exterior to the surface S of the body

(1.2a) $$\Delta u + k^2 u = 0 \qquad \text{in } \Omega,$$

(1.2b) $$\frac{\partial u}{\partial n} = f \qquad \text{on } S,$$

(1.2c) $$\lim_{r \to \infty} r\left(\frac{\partial u}{\partial r} - iku\right) = 0.$$

Condition (1.2b) is for a hard scatterer. For a soft scatterer (1.2b) is replaced by a Dirichlet condition. Condition (1.2c) is the Sommerfeld radiation condition in three dimensions. A similar condition is valid in two dimensions.

Problem (1.2) can be solved by integral equation methods. In this approach (1.2) is replaced by a Fredholm integral equation (typically of the second kind) over the surface S. As k increases, however, the solution becomes more oscillatory and this method requires the inversion of a large full matrix. Asymptotic methods can be developed for large values of k. In practice, many of the features predicted by the asymptotic methods can be at least qualitatively observed at moderate frequencies.

Integral equation methods, in addition to requiring the inversion of large full matrices, are restricted to constant values of k. In this work we will consider the more general approach of introducing an artificial surface Γ, for example the sphere $r = r_1$, which surrounds the surface S. On Γ it is necessary to impose an approximation to the radiation condition (1.2c). The radiation condition can be either global or local.

The continuous problem (1.2) is then replaced by a boundary value problem in a bounded domain. It can therefore be approximated by some standard discretization method such as finite differences or finite elements.

The result of any such discretization is a large, linear system of equations

(1.3) $$A\chi = b,$$

where χ approximates the solution to (1.2) and b is determined by the boundary data. The large, sparse matrix A is difficult to invert by standard iterative methods since the Hermitian part of A will often be indefinite.

The method proposed here is to solve (1.3) by a preconditioned conjugate gradient iteration method. Since the conjugate gradient (CG) method is not directly applicable to indefinite, non-selfadjoint problems we shall consider the normal equations

(1.6) $$A^* A \chi = A^* b,$$

where A^* is the adjoint of A. The matrix A^*A is positive definite and therefore the conjugate gradient iterations will converge.

The matrix A*A is highly ill-conditioned and thus the resulting iterations will converge very slowly. In order to improve the conditioning of the iteration matrix A*A we will precondition A by a partial inverse of the discrete approximation to the Laplacian. Thus, instead of solving (1.6) we will solve the equivalent system

$$A'^{*}A'\chi' = A'^{*}b', \tag{1.7}$$

where $A' = Q^{-1}AQ^{-T}$, $\chi' = Q^{T}\chi$, $b' = Q^{-1}b$ and $M^{-1} = Q^{-T}Q^{-1}$. The matrix M^{-1} is a partial inverse of the discrete Laplacian A_0. This preconditioner will be obtained from the splitting $A_0=M-R$ corresponding to point symmetric successive overrelaxation. Thus the matrix Q^{-1} corresponds to SOR. We have shown that the use of a preconditioner based on the structure of the equation (i.e., Laplacian plus lower order terms) will dramatically accelerate the convergence of the normal equations. The resulting algorithm permits solutions to be computed for practical grid sizes using a relatively small amount of computer time.

4. The Theory of Queer Differential Equations (QDE's)

Since 1974, the use of QDE's in the theory of adiabatic and diffusing plasma media has proved very useful. Numerical algorithms have been developed which accelerate large diffusion codes and the convergence properties of the numerical schemes depend, to a large extent, on as yet incomplete understanding of the theory of QDE's. Recently, research in the mathematical nature of QDE's has been pursued by H. Grad, P. Laurence and E. Stredulinsky. We present here a progress report of this work.

Queer differential equations are a pseudonym introduced by Harold Grad for a new class of functional differential equations which model the slow adiabatic diffusion of a plasma through a magnetic field. The prototype for such an equation is

$$\Delta\psi = F(\chi, V, \psi, \psi', \psi'') \tag{1}$$

where ' denotes differentiation with respect to V the volume enclosed with level sets of $\psi(\chi)$.

Little is known regarding existence, uniqueness and regularity for such equations except for certain special choices of F. For instance for the equation,

$$\Delta\psi = G(\chi, V), \tag{2}$$

under various boundary conditions, existence and regularity of solutions has been demonstrated by Mossino and Temam. Further, a linearized version of (1) has been analyzed by Vigfusson. We consider the model equation:

$$\Delta\psi = -\psi''(V), \tag{3}$$

with ψ specified at the boundary and at an interior point.

Our investigations concern existence and regularity for the above equation in a two dimensional simply connected domain with the unusual "boundary conditions", $\psi(\chi)$ given on $\delta\Omega$, $\psi^*(0) = 0$. Here $\psi^*(V)$ is the increasing rearrangement of $\psi(\chi)$ and the second condition is equivalent to specifying the value of $\psi(\chi)$ at its absolute minimum, an interior point of Ω. An auxiliary problem which we analyze is the existence and regularity of solutions to the same equation on an annulus with boundaries Γ_1(inner) and Γ_2(outer) with $\psi(\chi)=0$ on Γ_1, $\psi(\chi)=1$ on Γ_2.

The only known result for this equation is existence of solutions to a variational problem formulated by Temam which however does not incorporate the boundary condition $\psi^*(0)=0$, but rather replaces this with the condition $\psi^{*\prime}(0)=0$ which turns out at least formally to be a natural boundary condition.

We have introduced two modified variational principles, one in the punctured disk Ω and one in the annulus A for which we have proven existence of solutions. As in Temam's case the passage from a solution to the variational problem to a solution of the actual problem has not been demonstrated. Similarly to his case and unlike the situation for weak or variational formulations of other nonlinear problems, this passage requires more than proving the regularity of the solution. Information is needed about the structure of the closure of the set where the minimum and maximum of the solution is attained. One step in the direction of uncovering the nature of this set has been taken by establishing a weak maximum principle for the solution of the variational problem both in the punctured disk and in the annulus. That is we have shown that the solution $\psi(\chi)$ satisfies $0\leq\psi(\chi)\leq 1$ in $\overline{\Omega}$.

Furthermore, it is possible to introduce a generalized notion of capacity which has the property that the minimizer of the variational problem minimizes this capacity over all families of level sets partitioning Ω. We hope to use this notion of capacity to study further the structure of the level sets of the minimizer. In particular we seek to explore a connection with a free boundary value problem solved by Caffarelli in which he finds the subset of a set Ω with given volume of minimum newtonian capacity and shows the free boundary is C^{∞}.

The generalized notion of capacity also enters Grad's iteration schemes for solving QDE's so that understanding its properties should be useful in proving analytically the convergence of such schemes. We should note that the problem is extremely rich in that there is a large body of literature in a variety of branches of analysis including isoperimetric inequalities and properties of the rearrangement function which can be carried over to generate useful information regarding it.

A partial result towards proving analytically the convergence of such schemes has been obtained through the establishing of an L^2 gradient estimate for the equation in the case of an annulus whose minimum outer radius is larger than one. This gradient estimate can be used to show existence of weak solutions to the problem,

$$\Delta\psi = -\psi^{*''}(\overline{V}),$$

where $\overline{V}(\chi)$ is an independently prescribed volume function whose level sets are not the same as those of $\psi(\chi)$. It uses the fact that scaling such equations onto domains of outer radius one, introduces an $\epsilon<1$ in front of the second order operator on the right hand side of (3).

Furthermore and most recently a norm bound on $\psi^{*\prime}(V)$ for a solution to the variational problems (4) and (5) has been achieved.

5. Spectral Theory of Non-Elliptic Operators

This work, carried out by E. Hameiri and P. Laurence, deals with the spectrum of the linearized ideal magnetohydrodynamics MHD equations, as a non-trivial representative of non-elliptic operators. Most spectral investigations consider elliptic operators, like the Schrölinger operator, where the spectrum is typically discrete unless the potential is singular or the domain is infinite. This is not true in the non-elliptic case. In view of the important ramifications of the existence of a continuous spectrum as the manifestation of many "modes" which may interact to produce non-reversible absorption of energy or generate turbulence, it is of considerable interest to understand the origins, and identify in detail, this part of the spectrum.

We have found that in the non-elliptic case a continuous spectrum may arise from the presence of waves propagating along rays which remian confined and never intersect the boundary. In ideal MHD these rays are simply the magnetic field lines and the waves propagating along them are the Alfven and slow magnetosonic waves. Because the ray does not impinge on the boundary, it is possible to restrict attention to the ray's trajectory and to the quantity transported along it and to view the phenomenon as a one-dimensional wave propagation problem, similar to a string, with the global boundary conditions playing no role.

The continuous spectrum was investigated in great detail in the MHD system. For an axisymmetric configuration the spectrum is the union of the spectra σ_m of modes with fixed azimuthal wave number m. Each σ_m consists of a continuous part, arising from Alfven and slow waves with a particular polarization, and a discrete part. However, when all m are considered, the discrete points become dense and generate additional pieces of continuous spectrum. These pieces can be found directly without going through the accumulation process just described, by considering Alfven and slow waves with various polarizations which are not axisymmetric.

A rather remarkable phenomenon associated with having a continuous spectrum composed of dense eigenvalues is that different spectra may be seen from different rotating frames. A person rotating with frequency Ω in the θ-direction measures the azimuthal angle $\theta'=\theta-\Omega t$, so that a wave $\exp i(m\theta-\omega t)$ is seen as $\exp i\,(m\theta'-\omega' t)$, with the spectrum of frequencies ω transformed into $\omega'=\omega-m\Omega$. If $\omega(m)$

accumulate as $m \to \infty$ then $\omega(m) \to \infty$, thus different accumulation points will be observed in different frames. We found that the most interesting continuous spectrum is generally seen from a moving frame different from the laboratory frame.

APOSR - Interim Technical Report
January 31, 1983

List of Publications

1) A. Bayliss, L. Maestrello, "On the Interaction of a Sound Pulse with the Shear Layer of an Axisymmetric Jet II. Heated Jets," to appear in Journal of Sound and Vibration.

2) A. Bayliss, L. Maestrello, "Flow and Far Field Acoustic Amplification Properties of Heated and Unheated Jets," to appear in revised form in AIAA Journal.

3) A. Bayliss, E. Turkel, "Far Field Boundary Conditions for Compressible Flows, to appear in J. Comp. Phys.

4) A. Bayliss, E. Turkel, "Outflow Boundary Conditions for Fluid Dynamics," SIAM J. Scientific Statistical Computation, Vol. 3, No. 2, June 1982.

5) A. Bayliss, E. Turkel & C. Goldstein, "An Iterative Method for the Helmholtz Equation," to appear in J. Computational Physics.

6) H. Grad, "Correlations, Fluctuations and Turbulence in a Rarefied Gas," in Long-Time Prediction in Dynamics, eds. C.W. Horton, Jr., L.E. Reichl and A.G. Szebehely, John Wiley & Sons, Inc., 1983.

7) W. Grossmann, J. Teichmann, "The Effect of Stochastic Fluctuations on MHD Waves," in preparation for publication in Phys. Fluids.

8) E. Hameiri and J.H. Hammer, "Turbulent Relaxation of Compressible Plasmas," Phys. Fluids 25 (10), October 1982.

9) E. Hameiri, "Variational Principles and Adiabatic Compression of Rotating Plasmas," in preparation.

10) E. Hameiri and H.A. Rose, "Magnetohydrodynamic Fluctuations Near Thermal Equilibrium," Phys. Fluids 25 (12), 2271-2277, December 1982.

11) E. Hameiri, "Adiabatic Compression of Rotating Plasmas," to appear in Phys. Rev. A.

<u>List of Publications (cont'd.)</u>

12. K. Imre and H. Weitzner, "Relativistic Broadening Near Cyclotron Resonance," submitted to Phys. Fluids.

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