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More Optimal Searches

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LEXINGTON, MASSACHUSETTS

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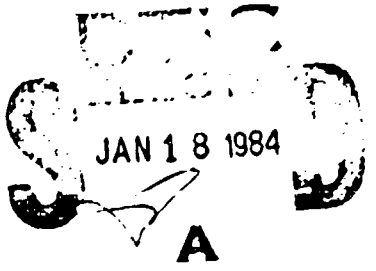

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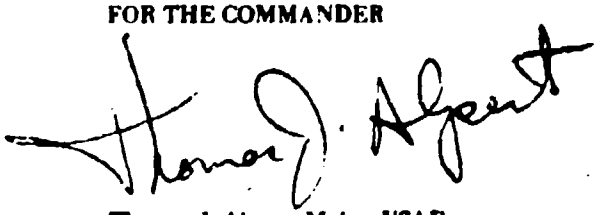

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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MORE OPTIMAL SEARCHES

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Group 94

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Abstract

This Report expands upon our previous work in this area. Specific application is made to searches for near-stationary artificial satellites and to the classical along orbit search. These two reflect different limiting cases for the a priori target distribution (uniform for the near-stationary case) and the conditional detection probability (uniform for the along orbit case). Our treatment of the near-stationary case is as realistic as is currently possible. Atmospheric extinction, eclipses, and phase effects are all included. Similarly we have explored a variety of scenarios for the along orbit search. We conclude with explicit search plan construction and illustrative examples. Finally we mention other work in this area currently underway.

CONTENTS

I. REVIEW

In reference 1 one of us (LGT) introduced the concepts of optimal search theory to the artificial satellite search problem. The best summary of the literature (reference 2) is herein extended in detail to two classical search problems for deep space artificial satellites--near-stationary satellites and along orbit searches. Clearly the latter type is frequently utilized in the low altitude regime and by radars as well as by passive optical sensors. Hence the formalism that allows one to solve such problems should be of interest to a wide audience.

A. Formulation

One looks for artificial satellites on the celestial sphere. In the largest sense this forms the two dimensional search space of the problem. In practice we delineate a limited area of the celestial sphere (say above altitude 30°) that we shall actually search in. Denote this search space by J.

One searches using a telescope with a finite field of view. In practice we always examine an entire field, never a fraction of a field nor more than one field at a time. Hence the search space J consists of a discrete set of fields of view. Number these by the index $j = 1, 2, \ldots$ In particular, since the celestial sphere encompasses 4π steradians, $\max(j) < \infty$.

Before the satellite is found one assigns an a priori target distribution on the search space J, $p: J \to [0,1]$ (the notation means that p is a function defined over the set J which maps elements of J into the domain zero to unity inclusive). The target distribution is the a priori probability of finding an artificial satellite in field of view $j \in J$ before one starts the search. For near-stationary satellites a reasonable model for p

is p is uniform over all geocentric right ascensions and over the geocentric declination range < 10° (or 5° or 20°). For other satellites, both because of parallax effects and the inherent spread over orbital element space (particularly in inclination), a reasonable model for p is that p is uniform over the topocentric celestial sphere. In any case

$$\sum_{j \in \mathcal{J}} p(j) \leq 1$$

When one examines a field of view for an artificial satellite one expends a certain amount of effort trying to detect the satellite. One may look at the same field of view several times. The cost of performing k inspections in the j'th field of view is measured by a cost function $c(j,k)$: $J \times \{0,1,2,\ldots\} \to [0,\infty]$. Clearly $c(j,0) = 0 \; \forall \; j \in J$ (no effort implies no cost). One could measure cost by the time spent examining a field of view plus the time spent in moving to the next field of view (this makes c non-local and is not desirable). Operationally we always spend the same time in each field of view (more or less). Also, because $[\text{area } (J)]^{1/2}/\text{slew speed} \ll$ time spent examining a field of view, the non-local element of c is both unimportant and varies little. Thus we shall measure cost by time and specialize to the case when the incremental cost of the k'th examination in field of view number j, viz.

$$\gamma(j,k) = c(j,k) - c(j,k-1)$$

is a constant independent of both j (i.e., the telescope is fast and all fields of view are treated equally) and k (e.g., the same field of view is equally well inspected each time).

When one does examine a field of view of the search space looking for an artificial satellite then there is a conditional probability of detecting it on or before the k'th inspection of that field of view (given that it is there). This function, for field of view number j and examination k, is denoted by b(j,k): $J \times \{0,1,2,\ldots\} \to [0,1]$. Naturally $b(j,0) = 0 \ \forall \ j \in J$ (you can't find it if you don't look for it). From the detection function b one can construct the probability of failing to detect the satellite on the first k-1 scrutinizations of field of view number j and then succeeding on the k'th one (given that the satellite is in field of view number j); viz.

$$\beta(j,k) = b(j,k) - b(j,k-1)$$

There is a lot of physics and mathematics subsumed in the detection function. Clearly it depends on the satellite's apparent magnitude, the background star density, the night sky background brightness, the resolution element size of the detector(s), the false alarm probability one is willing to accept, how tired one is, etc. Since the celestial sphere is unchanging, atmospheric extinction can be computed, the Moon's position is known, etc. this is a computable function. Operationally, for a fixed set of external parameters, our detection probability has the shape shown in Fig. 1 where m_L is our quoted limiting magnitude (e.g., where the probability of detection is 50%). The form shown in the diagram will be used to compute the optimal search plans given below.

Finally we need to define a search plan. A discrete plan is a sequence $\xi = (\xi_1,\xi_2,\xi_3,\ldots)$ which tells the searcher to first look in cell ξ_1; if the satellite is found there then terminate the search but if it isn't found then

look next in field of view ξ_2, etc. A global way to describe this is by a function which specifies the allocation of effort devoted to each field of view j. To this end define $f: J \to [0,\infty)$; f(j) is the number of examinations in field of view number j.

Above we referred to searches for a fixed target. Clearly the satellites we are trying to find are moving. We've made the assumption that these objects are fixed when compared to our search rate. The mathematical formulation of this approximation is [area (J)/search rate]·satellite angular speed << field of view.

B. Optimal Searches

Given the cost of searching field of view number j a total of k times, c(j,k), the total cost of performing the search plan ξ with allocation f is

$$C[f] = \sum_{j\in J} c(j,f(j))$$

The total number of examinations over all fields of view is $\sum_{j\in J} f(j)$. Similarly the total probability of satellite detection with this allocation of effort is P[f],

$$P[f] = \sum_{j\in J} p(j)b(j,f(j))$$

where b(j,k) is the conditional probability of finding the satellite in field of view number j after k examinations of that field of view given that it's in that field of view.

There are four types of searches one might define as optimal. One might be interested in maximizing the total probability of detection when constrained to a given number of inspections (say K). If the incremental cost function $\gamma(j,k) = c(j,k) - c(j,k-1)$ is a constant, then (after a suitable renormalization) one is demanding that P[f] be a maximum for $C[f] \leq K$. Such a search is termed totally optimal. If one demanded optimality for all K = 1, 2, 3, . . . then the search is called uniformly optimal. A third type of search plan that one might consider is the search plan that maximizes the probability of detection with respect to the incremental cost and does so at every step of the search. Mathematically one finds the value of j which maximizes $p(j)\beta(j,k)/\gamma(j,k)$ at each k. These searches are called locally optimal. Lastly one might entertain a search plan that minimized the total expected cost (i.e., was the fastest) to find the target.

The essential assumptions necessary to cast the artificial satellite search into the simplest form of the mathematical superstructure that Stone[2] outlines are

(1) That the satellite is fixed (i.e., search rate high compared to the satellite's angular speed),

(2) That the search space is discrete (i.e., a fixed field of view),

(3) That the allocation of effort is discrete (i.e., no favored fields of view), and

(4) That γ is bounded away from zero and $p(j)b(j,k)/\gamma(j,k)$ is a decreasing function of j (i.e., no free examinations of a field of view and the larger the search space the more difficult to detect.)

I do not believe that the physics or astronomy is strained by these strictures. In fact (5) γ = constant is not unreasonable (i.e., the telescope moves smartly). The important point is that under these five limitations the totally optimal search plan, the uniformly optimal search plan, the locally optimal search plan, and the fastest searches are all identical. Not only that, it can be explicitly exhibited. See Stone's text for the rigorous mathematical statements of the relevant theorems and their proofs.

C. The Search Plan

We need just a bit more mathematics before we can exhibit the solution to the optimal search problem. The search plan $\xi = (\xi_1, \xi_2, \xi_3, \ldots)$ is a sequence of values $\xi_i \in J$ for $i = 1,2,3, \ldots$ These specify that the i'th examination be in field of view ξ_i if the previous i-1 inspections failed to detect the satellite in fields of view $\xi_1, \xi_2, \ldots, \xi_{i-1}$. Let the set of all such search plans be denoted by Ξ. Introduce the probability $P[n, \xi]$ (and the cost $C[n, \xi]$) of detecting the satellite on or before the n'th examination while performing search plan $\xi \in \Xi$ (of the first n inspections). Finally, let $r(j,n,\xi)$ be the number of scrutinizations out of the first n that are placed in j'th field of view while following search plan ξ. A uniformly optimal search plan [for $\gamma(j,k) = 1$; this is an unimportant normalization] $\xi^* \in \Xi$ is one such that

$$P[n,\xi^*] = \max \{P[n,\xi] : \xi \in \Xi\}, \quad n = 1, 2, \ldots, K$$

A locally optimal search plan ξ^* is one such that ξ_1 is determined by [$\gamma \neq 0$ necessarily]

$$\frac{p(\xi_1)\beta(\xi_1,1)}{\gamma(\xi_1,1)} = \max_{j\epsilon J} \frac{p(j)\beta(j,1)}{\gamma(j,1)}$$

and having determined the field of view for the first n-1 examinations $(\xi_1, \xi_2, \ldots \xi_{n-1})$ the field of view for the n'th one is determined from

$$\frac{p(i)\beta(i,r(i,n-i,\xi)+1)}{\gamma(i,r(i,n-1,\xi)+1)} = \max_{j\epsilon J} \frac{p(j)\beta(j,r(j,n-1,\xi)+1)}{\gamma(j,r(j,n-1,\xi)+1)}$$

with $\xi_n^* = i$. Now define $k_n = r(\xi_n, n, \xi)$. The notation means that the n'th examination of the search plan ξ is placed in field of view ξ_n and that it is the k_n'th time that this field of view has been searched. The average cost to find the satellite can be expressed in a variety of ways if the limit as $n \to \infty$ of $P[n,\xi]$ is unity;

$$\mu(\xi) = \sum_{n=1}^{\infty} C[n,\xi]\,(P[n,\xi] - P[n-1,\xi])$$

$$= \sum_{n=1}^{\infty} \sum_{m=1}^{n} \gamma(\xi_m,k_n)\; p(\xi_n)\; \beta(\xi_n,k_n)$$

$$= \sum_{m=1}^{\infty} \sum_{n=m}^{\infty} \gamma(\xi_m,k_n)\; p(\xi_n)\beta(\xi_n,k_n)$$

$$= \sum_{m=1}^{\infty} \gamma(\xi_m,k_m)\;(1-P[m\ ,\varsigma])$$

since $P[0,\xi] = 0$. If $\gamma(j,k) = 1$ then this reduces to

$$\mu(\xi) = \sum_{n=0}^{\infty} (1-P[n,\xi])$$

Now we can exhibit the solution explicitly. Under the assumptions outlined above if q_j is the probability of detecting the satellite after a single examination of field of view number j (given that it is in field of view number j) then, as each inspection is an independent event, the incremental conditional probability of detection $\beta(j,k) = b(j,k) - b(j,k-1)$ is given by

$$\beta(j,k) = q_j\,(1-q_j)^{k-1} \text{ for } j \in J\ ,\ k = 1, 2, \ldots$$

Normalize such that $\gamma(j,k) = 1\ \forall\ j \in J$, $k = 1, 2, \ldots$ and suppose that an allocation f(j) has total cost (i.e., number of inspections) K,

$$\sum_{j\in J} f(j) = K$$

The total probability of detection for this allocation of effort will be

$$P[f] = \sum_{j\in J} p(j)b(j,f(j)) = \sum_{j\in J} p(j)[1-(1-q_j)^{f(j)}]$$

Consider the search plan defined by: one makes the n'th inspection in field of view number $i \in J$ such that

$$p(i)q_i\,(1-q_i)^{r(i,\,n,\,\xi)} = \max_{j\in J}\ p(j)q_j\,(1-q_j)^{r(j,\,n-1,\,\xi)}$$

Then $\xi = \xi^*$ and is optimal (in all senses). This result is due to Chew[3]. Since J is finite the existence of an i satisfying the above is guaranteed. If one exploits the uniformity of the target distribution p over the search space J, then the result is even simpler,

$$q_i(1-q_i)^{r(i,\,n,\,\xi)} = \max_{j\in J}\ q_j(1-q_j)^{r(j,\,n-1,\,\xi)} \qquad (1)$$

II. NEAR-STATIONARY SATELLITE SEARCHES

We have already argued that the a priori target distribution p(j) can be approximated by a defective uniform distribution over the search space. We have also argued that the incremental cost function is homogeneous over J and independent of the number of looks, $\gamma(j,k) = 1$ (in appropriate units). The probability of detection of the satellite in field of view number J (given that it's there) is q_j. This depends principally on the apparent magnitude of the satellite and the night sky background. Three effects tend to make satellites fainter; atmospheric extinction, loss of brightness due to increasing phase angle, and increasing distance (heliocentric or geocentric).

The extinction is modeled as usual,

$$\varepsilon = \varepsilon_z \sec z$$

where z is the topocentric zenith distance and ε_z is the extinction per unit air mass. We've used a value of 0.13 mag/air mass for ε_z. For the phase function in magnitudes I've used the results in reference 4

$$B(1,0) = B(1,\theta) + 0.538 - 0.134\,|\theta|^{0.714} - 7\ell \quad \text{for } |\theta| < 7°$$

$$B(1,0) = B(1,\theta) - \theta\ell \quad \text{for } |\theta| \geq 7°$$

where B(1,0) is the absolute B magnitude and B(1,θ) is the apparent magnitude corrected for phase angle θ. The parameter of the linear part of the phase function in magnitudes $\ell = 0.039$ mag/deg.

Eclipses can play an important role in searches for near-stationary satellites. Obviously it makes no sense to search that part of the sky currently undergoing eclipse (in the visible wavelength bands). Moreover, it would be a mistake to preferentially find an artificial satellite just prior to its eclipse (eg. westward of the shadow cone). Hence not looking within the penumbra represents one level of search planning refinement and only looking eastward of the penumbra represents yet another step up in sophistication.

The geometrical discussion of artificial satellite eclipses is more complicated than that for the Moon because the satellites are much closer. The results were worked out by one of us (LGT) in 1981. For a near-stationary satellite the half-angle of the umbral cone is 8°.427 while that of the penumbral cone is 8°.960. Since it only takes ∿2 minutes to traverse this ∿0°.5 and the brightness variation during the transition through penumbral eclipse is exceedingly difficult to model, we've chosen to ignore the distinction. The half-angle of the penumbral eclipse cone was then increased by 2% to allow for refraction effects in the Earth's atmosphere. Ellipticity of the Earth, its atmosphere, and its heliocentric orbit have either been ignored (the first two) or averaged over (the latter).

Since the eclipse is centered at the solar opposition point we merely test for the angular distance from the center of a field of view to the opposition point. If this distance is less than 1.02 times the half-angle of the penumbral shadow cone then the apparent magnitude is set equal to ∞ which results in a probability of detection of zero (cf. Fig. 1). Otherwise the apparent magnitude is calculated as indicated above. For the trailing shadow cone searches, the right ascension of the center of the field of view is less than (mod 24)

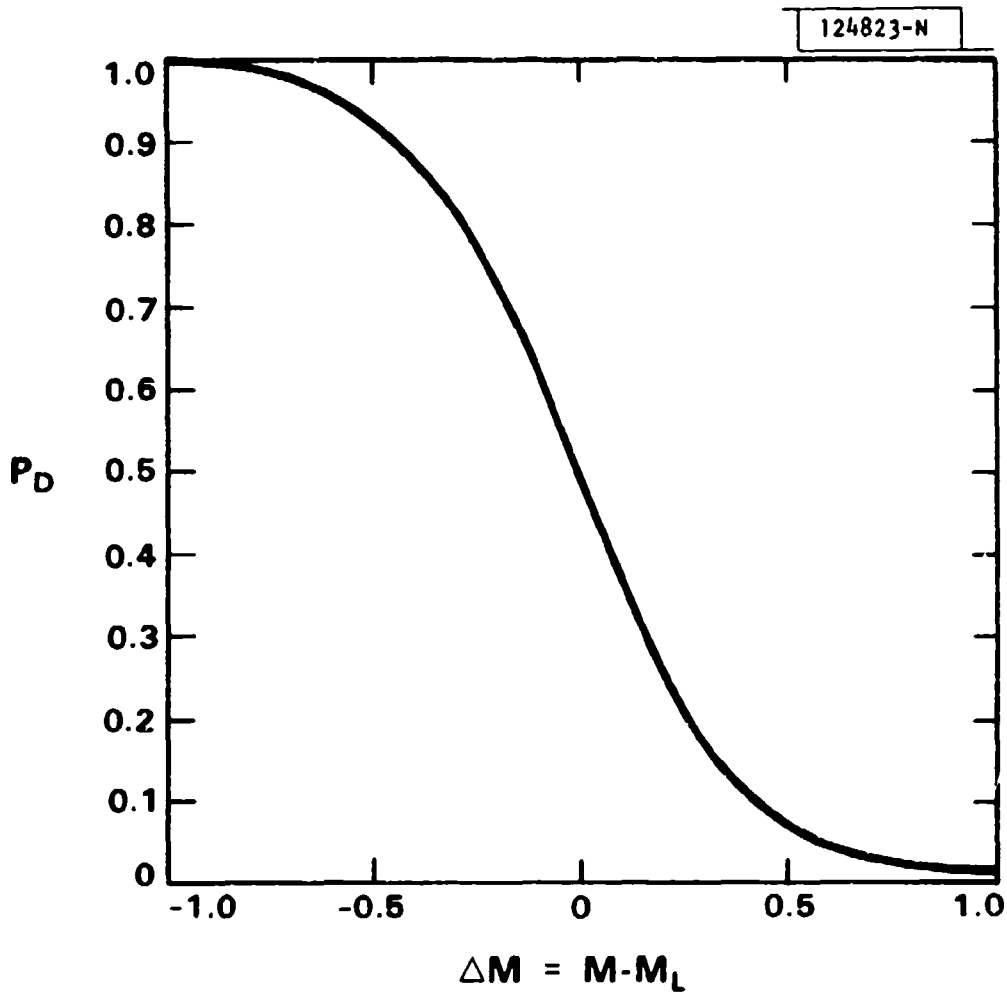

Fig. 1. Probability of detection (P_D) as a function of magnitude difference from the limiting magnitude (m_L).

that of the opposition point, then the probability of detection = 0. Otherwise it is computed as just described. Sample search plans are shown in Figs. 2-8.

All of the search plans have been terminated under the same (artificial) criteria: Either each field of view of the search space ($30° \times 3^h$) in declination x right ascension) is examined once or a field of view of the search space is about to be examined for the seventh time (a combination storage/reasonableness criteria of futility). The fields of view are each 2° x 2° (uncorrected for the cosδ foreshortening). Furthermore all of the sample search plans are illustrated in the same format: Declination increasing up (North) and right ascension to the right (East). The numbers in the individual fields of view are the examination number, following the optimal search plan, of that field of view. No entry means that the field of view wasn't examined before termination. Thus, if you look at Fig. 2, the most northeasterly field of view was never examined while the field of view just to its west was inspected twice--on the 1276 look of the optimal search plan and on the 1372 look. Similarly the highest probability field of view was searched on the first examination of the optimal search plan and on the 562 look. The effort is distributed in elliptical waves, about the opposition point, in Fig. 2. The major axis of the ellipse is vertical--joining the opposition point and the zenith. Actually the "ellipses" are not North-South symmetrical, they bulge more in the North (eg. note the number of empty fields of view in the southwestern and southeastern corners as opposed to the corresponding northern ones). The reason for this is that the satellites are brighter nearer the zenith (because of reduced atmospheric extinction) than

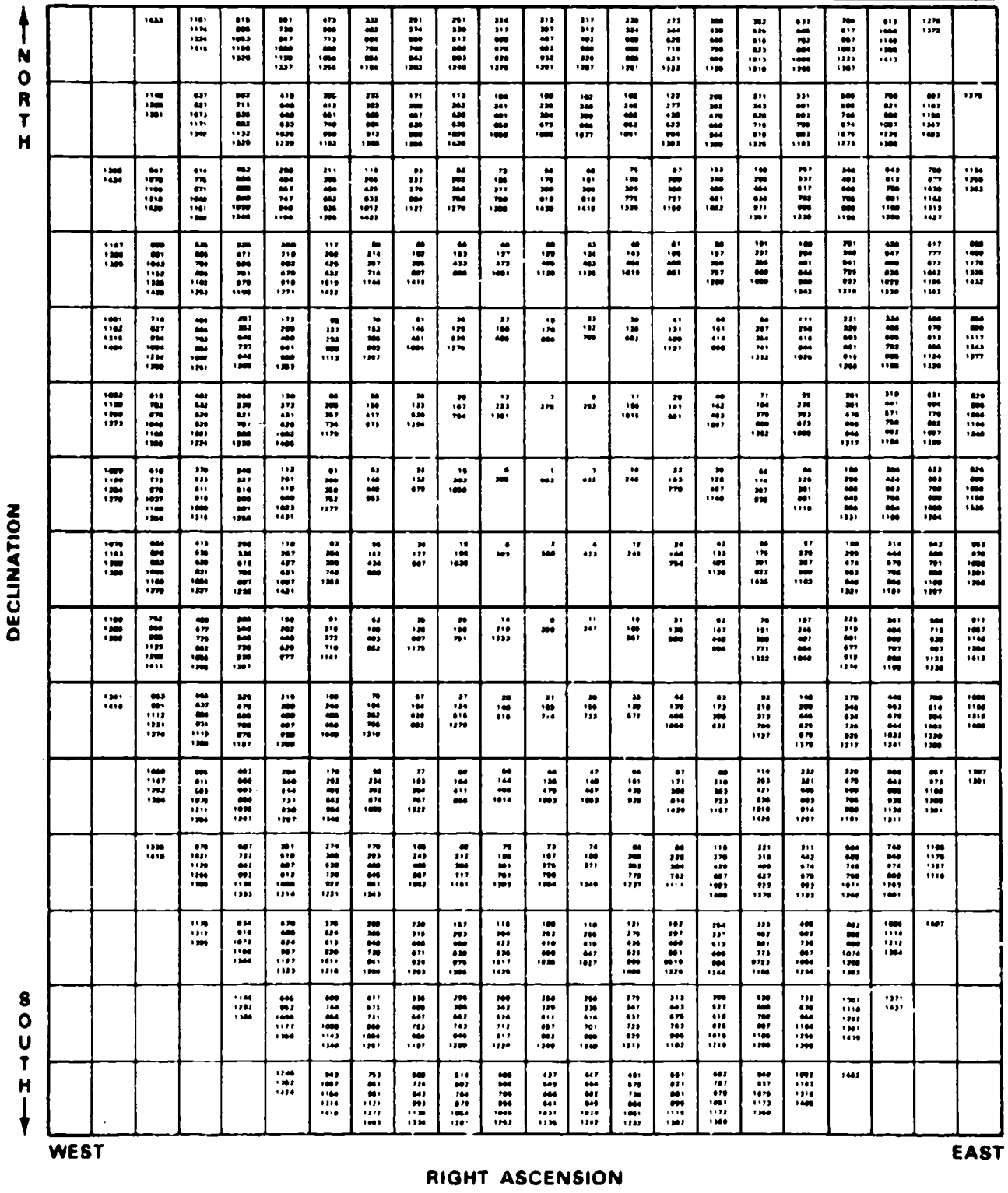

Fig. 2. Optimal search plan for a near-stationary artificial satellite on midnight local time of a winter solstice night. Each rectangular field of view is 2° X 2°. See the text for a fuller description.

they are closer to the nadir. The cumulative probability of detection at the termination of the search was 60.7% assuming that the artificial satellite was in the search space initially. After examining J fields of view the optimal plan was 23% more efficient than the uniform (eg. existing) search plan. This increase in efficiency had dropped to 4.2% after 4J cells had been examined (that is the cumulative probability of detection of the optimal search plan after the firt 4J examinations was 4.2% larger than the cumulative probability of detection of 4 repetitions of the plan that searches each of the J fields of view of the search space once [= the uniform plan]). This loss of effectiveness is easy to understand by actually looking at Fig. 2. As the search wears on the allocation of effort approaches uniformity, hence the relative advantage should approach unity.

The next figure (Fig. 3) shows the same scenario but at 3 A.M. local time instead of at midnight. There is an obvious northeasterly shift of effort (because of extinction). There are further small scale differences but more importantly after J field of view examinations the optimal search is 34.4% more efficient than a uniform one would be (with the same total cost of course). Figure 4 is identical to Fig. 2 but for a satellite a half magnitude brighter. Now each cell of the search is examined because it makes sense to do so. The final cumulative probability of detection is 83.1%.

Figure 5 repeats the scenario of Fig. 2 but now eclipses are included. The hole in the center represents the penumbral shadow cone and this search plan is markedly different from the one in Fig. 2. This time the optimal search plan is 65.7% more efficient than the uniform one after J looks, 42.7% after 2J looks and 32.0% after 3 looks. The cumulative probability of detection at termination is 35.9%.

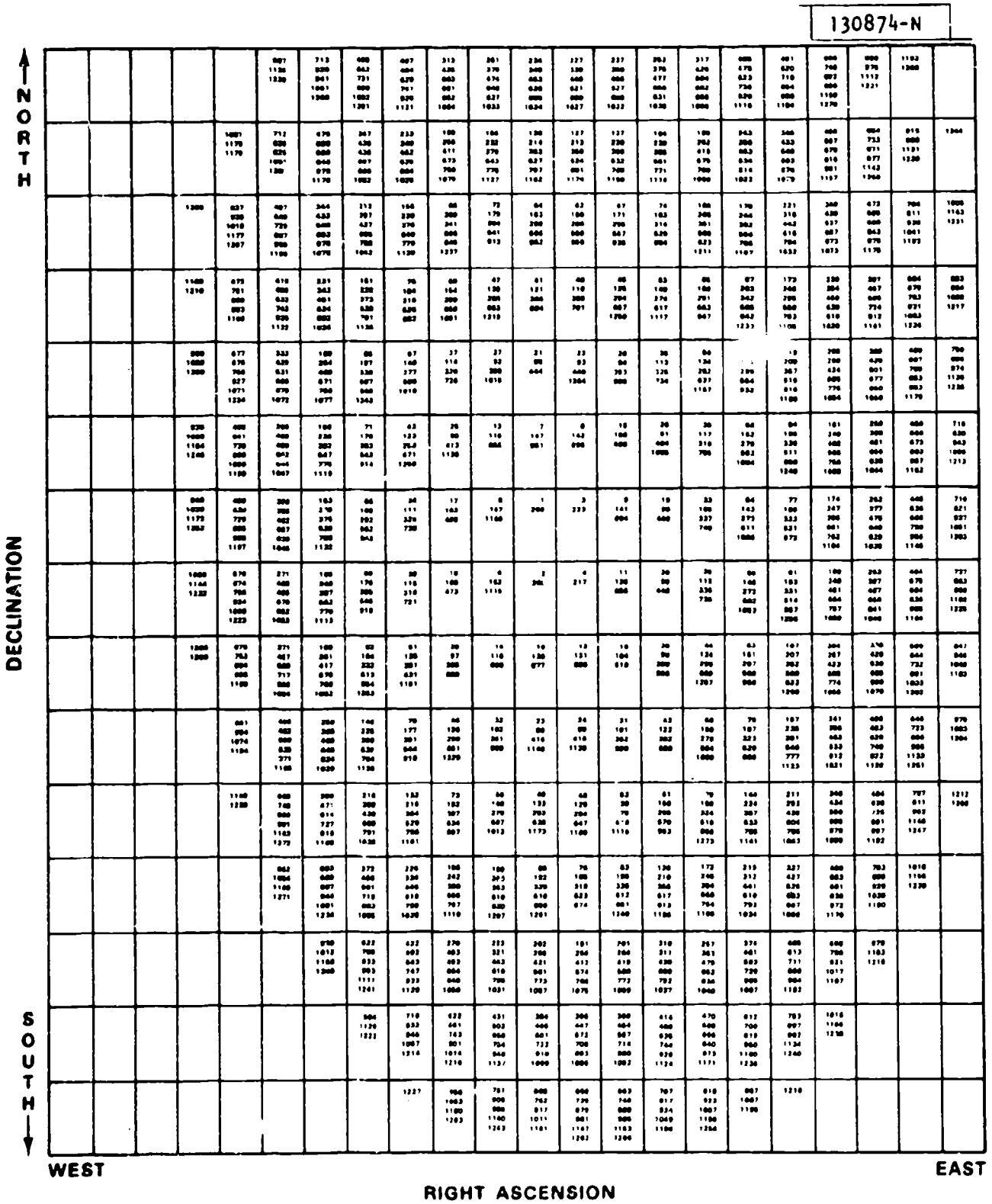

Fig. 3. Same as Fig. 2 but for 3 A.M. local time.

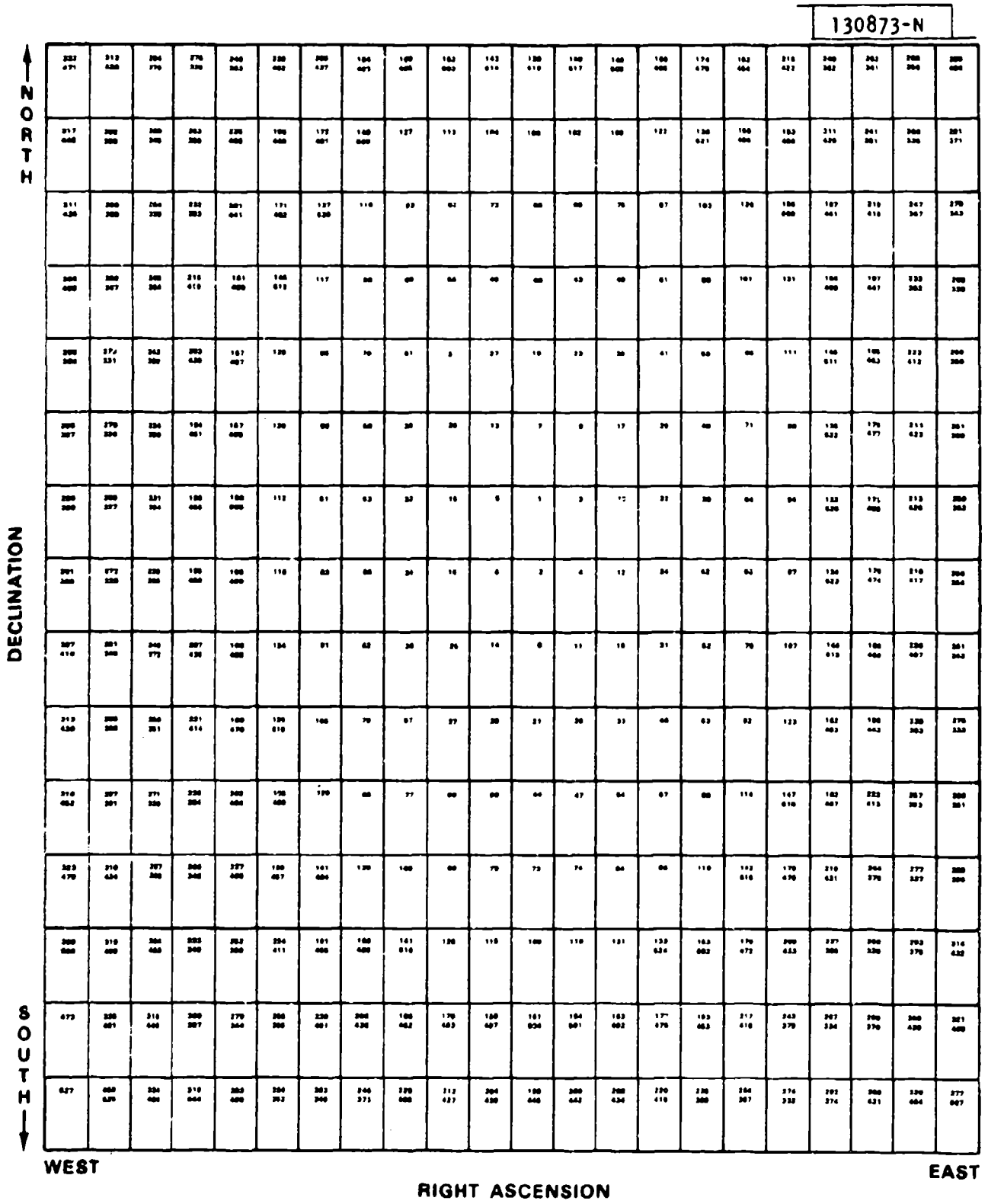

Fig. 4. Same as Fig. 2 but for a satellite of $p^{m}.5$ brighter.

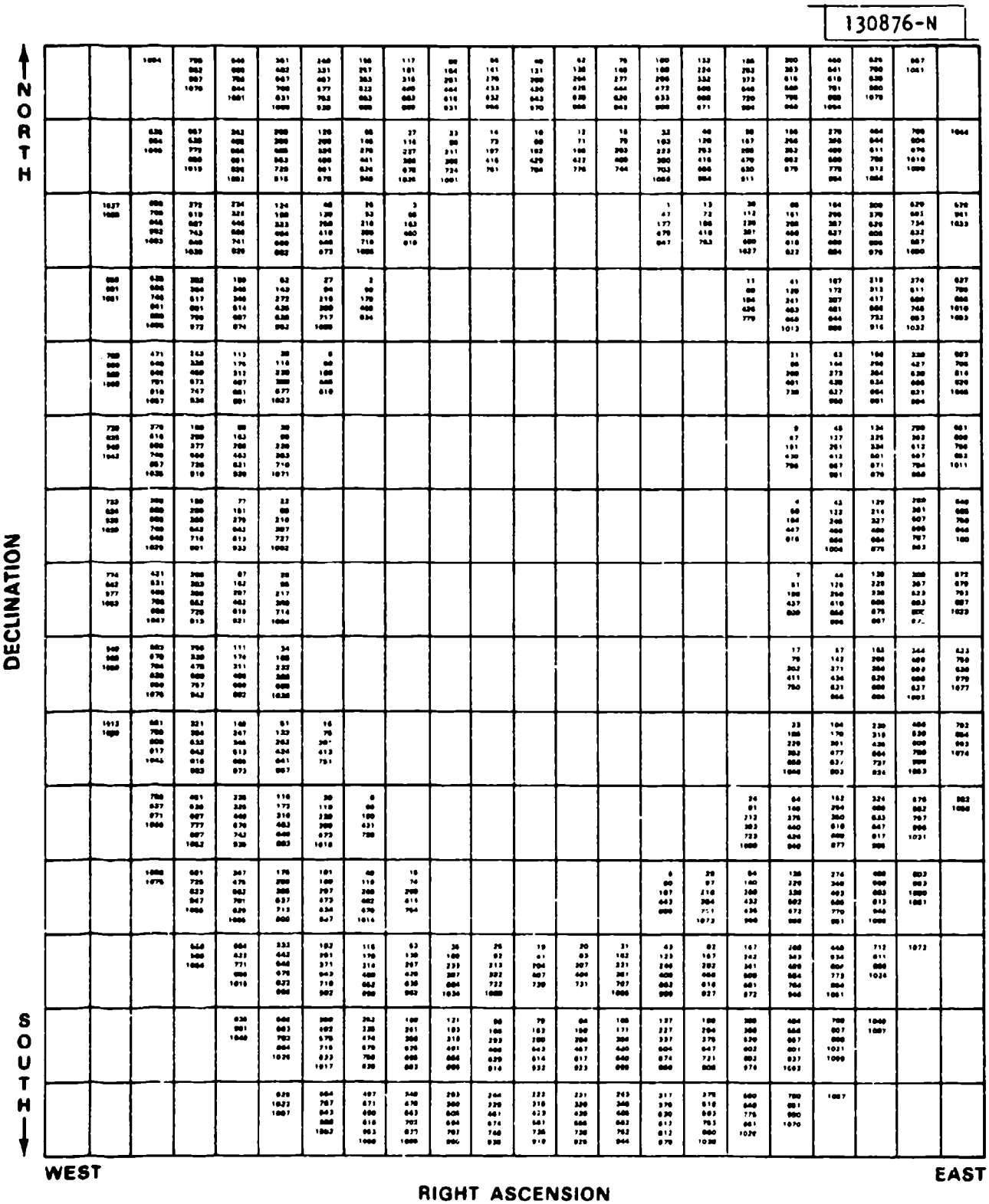

Fig. 5. Same as Fig. 2 but including eclipses.

In Fig. 6 we've changed from the winter solstice to the vernal equinox. Otherwise it's a repetition of the Fig. 3 scenario but at 9 P.M. local time. The North-South asymmetry is especially evident because opposition is lower in the sky ($\delta = 0°$) than it was at the winter solstice ($\delta = \varepsilon$ = obliquity of the ecliptic $\approx 23°.5$). After an expenditure of effort of J scrutinizations the optimal search plan is 60.3% more efficient than is the uniform one. Figure 7 is also on March 21 but at midnight; compare with Fig. 5. Finally, Fig. 8 shows vernal equinox, midnight, trailing edge shadow cone search plan. The first field of view is displaced northeast (extinction) and there's a marked tendency to look in the middle declination fields of view (phase loss).

Hopefully these examples will convince you that optimal search planning is non-trivial, non-intuitive, and important.

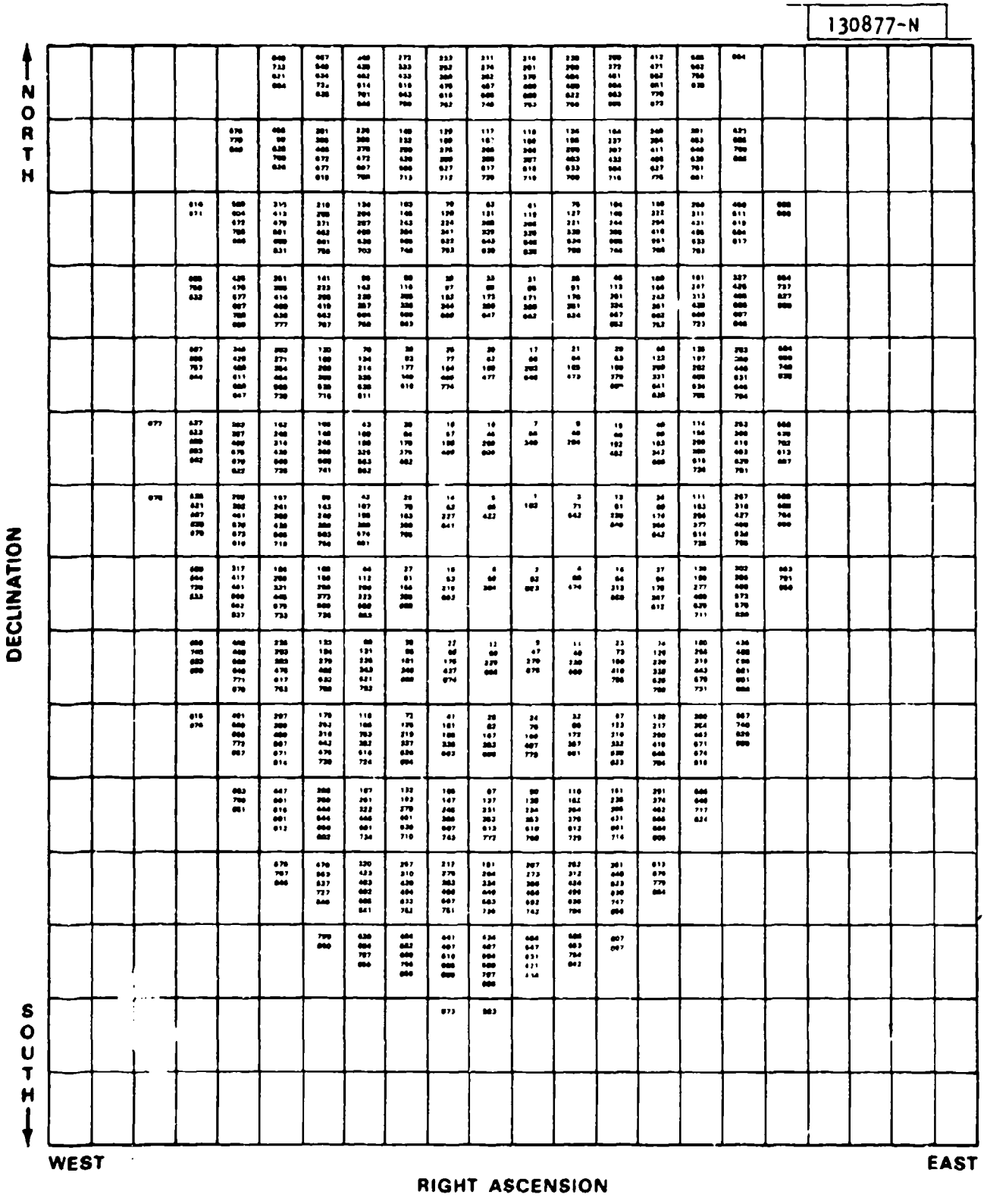

Fig. 6. Same as Fig. 2 but at 9 P.M. local time on the vernal equinox.

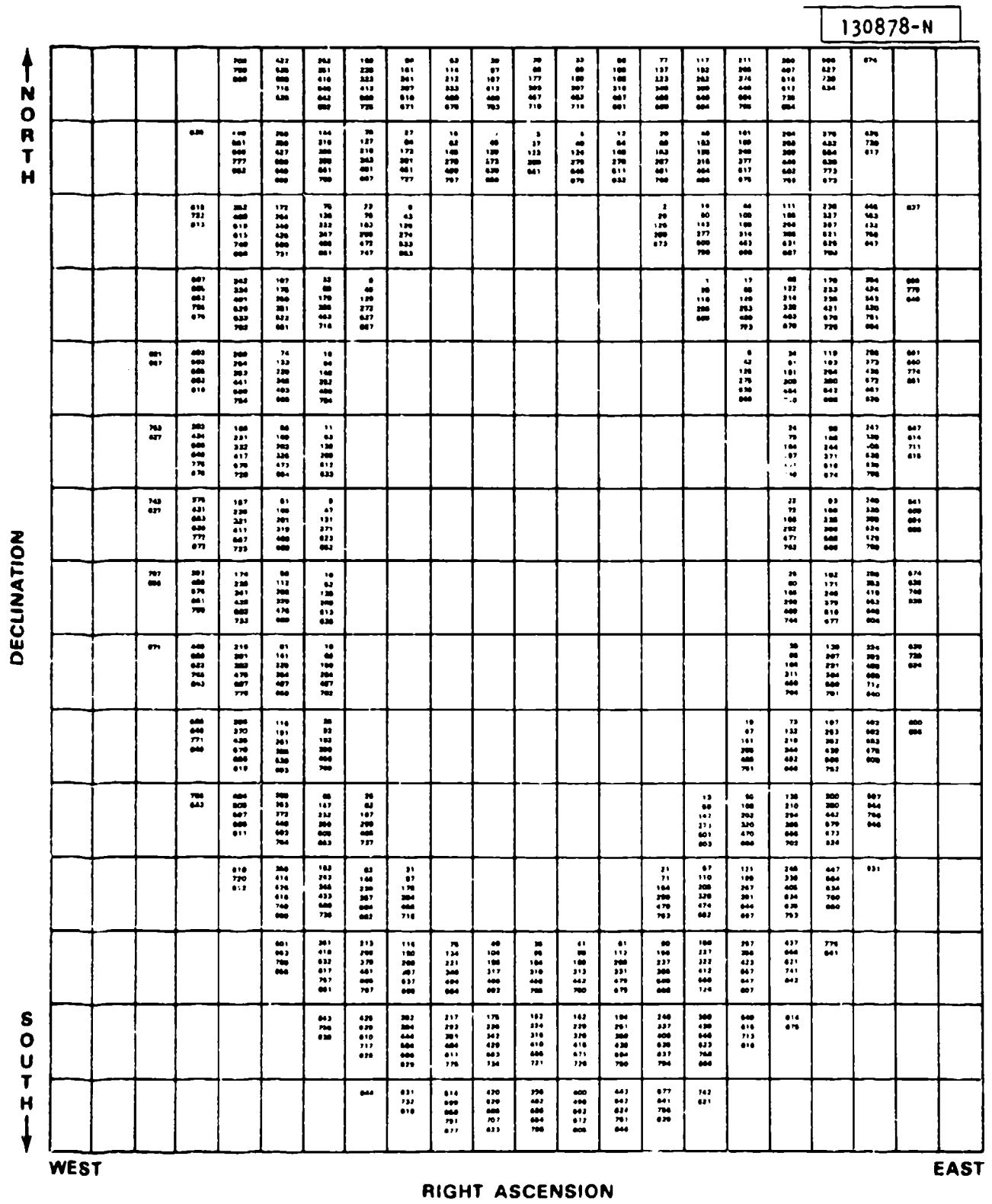

Fig. 7. Same as Fig. 5 but on the vernal equinox.

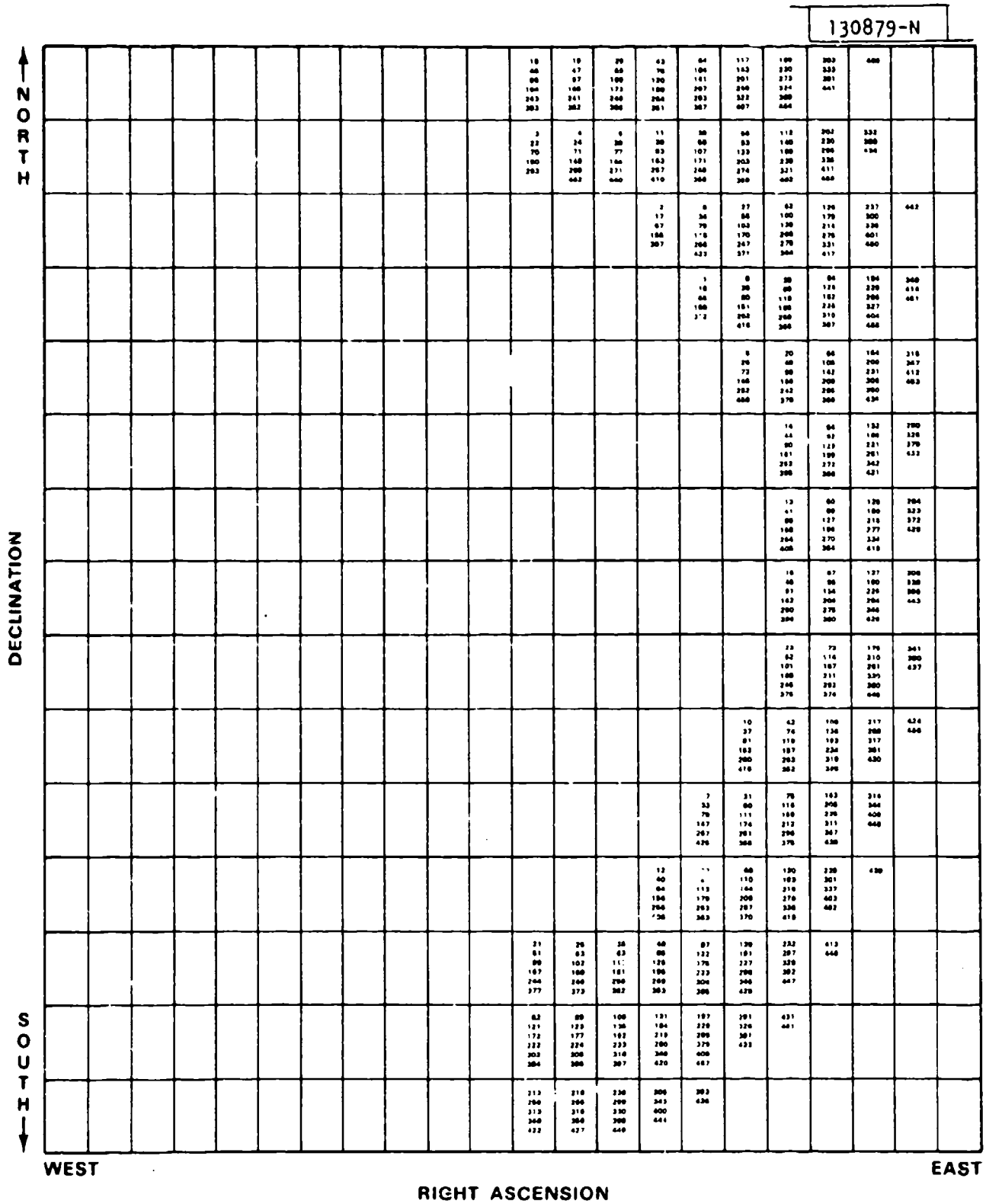

Fig. 8. Midnight local time, vernal equinox optimal search plan including eclipses but only trailing shadow cone edge.

III. ALONG ORBIT SEARCHES

One usually initiates an along orbit search when an examination of the nominal position for the artificial satellite doesn't yield the object of interest. In addition one has assumed no change in the satellite's orbital plane, merely that the satellite is slightly late or slightly early. Hence an along orbit search is one wherein a series of fields of view (or beam widths), along the topocentric path of the assumed orbital plane and travelling with the satellite's presumed mean motion, are examined under the hypothesis that the mean anomaly at epoch is slightly larger or smaller than its nominal value. Such a procedure renders the moving target fixed in the moving coordinate system of the search space. As the usual reason for the non-appearance of a satellite is a perturbation in the mean motion (atmospheric drag) or a poor element set (due to a mean motion/eccentricity swap), the search space is not strictly comoving with the satellite. Therefore to remain within the fixed target scenario, the strong inequality

$$\frac{\text{area search space}}{\text{search rate}} \ll \frac{\text{field of view}}{\text{relative angular speed}}$$

must hold (relative angular speed = |true mean motion-nominal mean motion|). I shall assume this to be the case.

With this point in mind the search space J for the along orbit search is a linear series of field of view (see Fig. 9). I shall label them by $j = -N, -N + 1, \ldots, 0, \ldots, N$. The a priori target distribution p will be assumed to be symmetrical [$p(n) = p(-n)$ $n = 0, 1, 2, \ldots, N$] and unimodal [$1 \geq p(0) \geq p(1) \geq \ldots \geq p(N)$]. We intend to acquire data this Fall at the ETS to provide an empirical estimate for p as a function of orbital type and age. For the

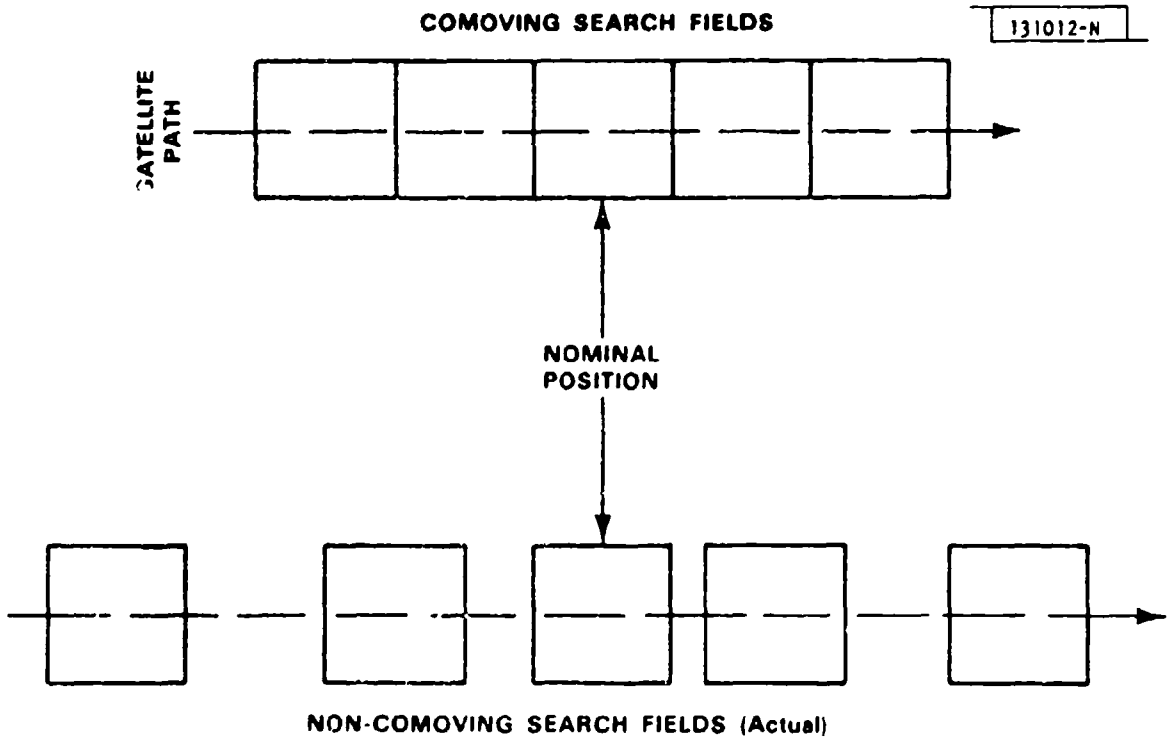

Fig. 9. Comoving and actual fields of view for an along orbit search.

explicit numerical computations listed below we shall further assume that the probability of being in either of the next outer fields of view is equal to that of being in the adjacent inner one, viz.

$$p(n + 1) + p(-n - 1) = p(n) = p(-n)$$

and since p is symmetrical this implies that

$$p(n + 1) = p(n)/2 \qquad n = 1,2, \ldots, N$$

The cost function is most realistically defined in terms of time--the time spent examining a field of view plus the time required to move to the next one. In the case of the GEODSS network, the telescopes accelerate and decelerate so fast that the non-local nature of the cost function for the small areal extent {$\leq(2N + 1)\theta$ where θ is the field of view [N.B. This is not strictly true because J is comoving with the satellite. The real maximum angular extent is a complicated function of N,θ, the satellite's mean motion, and the time spent inspecting each field of view]} involved here shall be neglected. Since the time spent examining each field of view is a constant,

$$c(j,k) = k(t_{look} + t_{move}), \ \forall\, j \in J, \ k = 1,2,\ldots$$

where t_{look} is the time spent looking in a field of view for the artificial satellite and t_{move} is an average duration of a movement from one field of view to another. In the appropriate set of units the incremental cost function $b(j,k) = c(j,k) - c(j,k - 1) = 1$.

The last quantity needed to specify the problem is the conditional detection probability b(j,k). As the portion of the celestial sphere covered

by J is small, the natural background and foreground sources of noise vary smoothly, and the time to complete an along orbit search is short, the differential extinction, phase angle, night sky background, etc., effects are all small. Therefore, the essential approximation for the along orbit search is that b(j,k) is homogeneous,

$$b(i,k) = b(j,k) \quad \forall\, i,j \in J;\ k = 1,2,\ldots$$

If b is the single glimpse probability (i.e., the conditional probability of detection in any field of view given that the satellite is in that field of view) then the incremental probability of detection on the k'th examination is

$$\beta(j,k) = b(1 - b)^{k-1} \qquad k = 1,2,\ldots$$

Note that $\partial b/\partial j = 0$ and $\partial \beta/\partial j = 0$ because of the assumed homogeneity of b(j,k). The algorithm for planning the search is, cf. Eq. (1),

$$p(i)b(1-b)^{r(i,n,\xi)} = \max_{j \in J} p(j)b(1-b)^{r(j,n-1,\xi)}$$

and if the n'th inspection is in field of view $\xi_n = i$ then search plan $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ is optimal.

Let's consider the first few inspections of three different along orbit searches. In each case we'll take J=5 (N=2) and p(-2) + p(-1) + p(0) + p(1) + p(2) = 0.8 so p(-2) = p(2) = 0.08, p(-1) = p(1) = 0.16, and p(0) = 0.32. First let the single glimpse probability of detection be high, b = 95%. Then the optimal search plan for the first 20 examinations is the sequence

$\xi^* = (0,1,-1,2,-2,0,1,-1,2,-2,0,1,-1,2,-2)$

The probabilities of detection with this allocation of effort are 76%, 79.80%, and 79.99% after the first 5, 10, and 15 looks. Note that $P[f] \leq 0.8$ in this case. Since the first five looks of the optimal search plan are in each field of view of the search space, the existing search plans yield the same probability of detection after completion as does the optimal one. This should not be at all surprising since it's been assumed that you'll find the satellite if you look at it (i.e., $b = 0.95$). The difference between the usual search plan and an optimal one becomes more apparent as the probability of detection decreases or the maximum amount of effort (K) increases.

To see this more clearly let us reduce b to 55%. The optimal search plan for the first 21 examinations is (both this plan and the one above appear to be periodic)

$\xi^* = (0,1,-1,0,2,-2,1,-1,0,2,-2,1,-1,0,2,-2,1,-1,0,2,-2)$

The cumulative probabilities of detection following the optimal plan's allocation of effort are 47.52%, 65.38%, 73.42%, and 77.44% after 5, 10, 15, and 20 looks. If we just look in each cell of the search space the corresponding cumulative probabilities of detection would be (for the same <u>total</u> effort) 44%, 63.8%, 72.71%, and 76.72% respectively. Once again, not much difference between the optimal plan and the usual ones. Of course it's been assumed that we have a pretty good chance of detection.

To tip the balance towards the optimal plan consider the case of a faint object, say $b = 0.15$. The first twenty-two stages in the optimal plan are

$\xi^* = (0,0,0,0,0,0,1,-1,0,1,-1,0,1,-1,0,1,-1,0,1,-1,2,-2)$

Note the heavy concentration on that field of view of the search space where the a priori target distribution is highest. The optimal search plan is telling us that when our chance of detection is minimal (b = 15%) then we'd best not look where it's not likely to be. The cumulative probabilities of detection following the allocation of effort of the optimal plan are 17.80%, 28.58%, 36.94%, and 43.50% after 5, 10, 15, and 20 looks. The once in each field of view search plan has only a 12% cumulative probability of detection upon completion. Thus the optimal plan is 48% more efficient than the usual one after the customary expenditure of effort. Repeating the usual plan once or twice yields cumulative probabilities of detection upon completion of 22.2% and 30.87%. The optimal plans with equal expenditure of effort are 29% and 20% more effective in these cases.

IV. ADVANCES

There are two areas of especial interest that we have not dwelled upon. One concerns optimal searches for moving targets. The rigorous theory that supports optimal searches for fixed targets has not been developed for this case. Hence, while an experienced optimal searcher may pursue his (or her) intuition in such matters, certainty is lacking. The second domain concerns optimal searches for targets by multiple searchers--either moving or fixed targets and colocated or separated searchers. Space based surveillance systems will yield yet another order of complexity when non-colocated and moving searchers look for moving targets in an asynchronous fashion. Progress on these topics is being made by others and in particular, the optimal multiple search for constant brightness, near-stationary artificial satellites has been solved (reference 5, see reference 6 too).

References

1. L. G. Taff, "Optimal Search Theory and Optical Artificial Satellite Searches," Project Report ETS-64, Lincoln Laboratory, M.I.T. (5 May 1982), DTIC AD-A115617/3.

2. L. D. Stone, Theory of Optimal Search (Academic Press, N.Y., 1975).

3. M. C. Chew, Ann. Math. Stat. 38, 494, 1967.

4. T. Gehrels and F. F. Tedesco, Astr. J. 84, 1079, 1979.

5. W. Stronquist, private communication, 1983.

6. S. S. Brown, Oper. Res. 28, 1275, 1980.

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This Report expands upon our previous work in this area. Specific application is made to searches for near-stationary artificial satellites and to the classical along orbit search. These two reflect different limiting cases for the a priori target distribution (uniform for the near-stationary case) and the conditional detection probability (uniform for the along orbit case). Our treatment of the near-stationary case is as realistic as is currently possible. Atmospheric extinction, eclipses, and phase effects are all included. Similarly we have explored a variety of scenarios for the along orbit search. We conclude with explicit search plan construction and illustrative examples. Finally we mention other work in this area currently underway.

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