NONLINEAR DYNAMIC POLARIZATION FORCE ON A RELATIVISTIC TEST PARTICLE IN A NONEQUILIBRIUM BEAM-PLASMA SYSTEM (U) HARRY DIAMOND LABS ADELPHI MD H E BRANDT SEP 83 UNCLASSIFIED HDL-TR-1994
Nonlinear Dynamic Polarization Force on a Relativistic Test Particle In a Nonequilibrium Beam-Plasma System

by Howard E. Brandt
The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Citation of manufacturers' or trade names does not constitute an official endorsement or approval of the use thereof.

Destroy this report when it is no longer needed. Do not return it to the originator.

A preliminary version of this report was issued as HDL-PRL-82-6 in May 1982.
The effective charge of a test particle in a nonequilibrium beam-plasma system is the sum of the actual charge and the dynamic polarization charge surrounding it. The contribution of its polarization charge to the total force on a relativistic test particle, namely the dynamic polarization force, is calculated to fourth order in the total electromagnetic field for a slowly varying, nearly spatially independent background distribution with no external fields. This relation is needed in calculations of collective radiation processes and the conditions for the occurrence of radiative instability in relativistic beam-plasma systems.
CONTENTS

1. INTRODUCTION ................................................................. 5
2. THE DYNAMIC POLARIZATION CHARGE DENSITY .......................... 6
3. THE DYNAMIC POLARIZATION CURRENT DENSITY ....................... 12
4. THE NONLINEAR DYNAMIC POLARIZATION FORCE ......................... 17
5. CONCLUSION .................................................................. 21
LITERATURE CITED .................................................................. 22
DISTRIBUTION .................................................................. 23
1. INTRODUCTION

In a nonequilibrium beam-plasma system, the time-average nonlinear force \( \langle F_a \rangle \) on a relativistic test particle due to the total electromagnetic field consists of a part \( \langle F_{oa} \rangle \), acting on the actual charge of species \( a \), and a part \( \langle F_{dp} \rangle \), acting on the dynamic polarization charge surrounding the particle. Thus,

\[
\langle F_a \rangle = \langle F_{oa} \rangle + \langle F_{dp} \rangle .
\] (1)

The polarization charge arises from the redistribution of other particles due to the induced fields produced in the neighborhood of the particle as it moves through the beam-plasma. For a relativistic test particle the time-average force on its actual charge (bare charge) under conditions of the Born approximation for plasma is given by\(^*\)

\[
\langle F_{oa} \rangle = F^{(1)}_a + F^{(2)}_a .
\] (2)

where

\[
F^{(1)}_a = \lim_{t \to \infty} \frac{2\pi}{t} e_a \int \frac{dk}{\omega + i\delta} \delta(\omega - k \cdot v_a) \nabla \cdot \mathbf{E}_k
\] (3)

and

\[
F^{(2)}_a = \lim_{t \to \infty} \frac{m_i}{t} e_a \int \frac{dk}{\omega + i\delta} \int \frac{dk_1}{\omega_1 + i\delta} \frac{\delta(\omega + \omega_1 - (k + k_1) \cdot v_a)}{(\omega + i\delta)(\omega_1 + i\delta)}
\]

\[
\times \mathbf{E}_{k1} \mathbf{E}_{k1j} \left[ \kappa_{A_{ij}}(k_1,k) + \kappa_{A_{ij}}^*(k_1,k) \right]
\] (4)

and

Here $t$ is the time, $e_a$ is the actual charge of the particle of species $a$, $m_a$ is its mass, $k = (\mathbf{k}, \omega)$ is a wave four-vector, $\delta$ is a small imaginary part of the frequency, $\delta(x)$ is the Dirac delta function, $E_{ki}^{(i)}$ is the Fourier transform of the $i$th component of the total electric field, $v_a$ is the particle velocity, $\gamma_a = (1 - v_a^2/c^2)^{-1/2}$, and $c$ is the speed of light. The condition that equation (2) hold, namely, the Born approximation, is that

$$\frac{e_a|\mathbf{p}_a|}{\omega_{pe}|\mathbf{p}_a|} \ll 1,$$

where $\mathbf{p}_a$ is the particle momentum and $\omega_{pe}$ is the electron plasma frequency.

In the present work, the time-average of the dynamic polarization force $\langle \mathbf{F}_{dp} \rangle$ is derived to fourth order in the total field for a slowly varying nearly spatially independent background with no external fields. It is shown to be dependent on the linear, second- and third-order nonlinear conductivity tensors. The result of this calculation agrees with that of Akopyan and Tsytovich.\(^1\) It is important in calculations of collective radiation processes and the conditions for the occurrence of radiative instability in nonequilibrium beam-plasma systems.

2. THE DYNAMIC POLARIZATION CHARGE DENSITY

The dynamic polarization charge which surrounds the test particle results from the background distribution function being disturbed in the neighborhood of the particle. Thus, the associated current density is given by

$$j_{dp} = \sum_s e_s \int \frac{d^3\mathbf{p}_s}{(2\pi)^3} v_s (f(s) - f^R(s)) \mathbf{v}_s,$$

where the sum is over all species and $f(s)$ and $f^R(s)$ are the perturbed and regular background distribution functions, respectively, for species $s$. It is assumed that there are no external fields and the perturbation in the distribution is due to the field associated with the interaction between the test

---

particle and the other particles in the system. The dynamic polarization charge density is related to the current in equation (7) by the equation of continuity, namely,

\[ \frac{\partial \rho_{dp}}{\partial t} + \nabla \cdot j_{dp} = 0. \tag{8} \]

Equation (8) assumes local conservation of the dynamic polarization current. The Fourier decompositions of the polarization charge and current densities are given by

\[ \rho_{dp} = \int dk \, \rho_{dp}(k, t) e^{i(k \cdot \mathbf{r} - \omega t)} \tag{9} \]

and

\[ j_{dp} = \int dk \, j_{dp}(k, t) e^{i(k \cdot \mathbf{r} - \omega t)} \tag{10} \]

where

\[ dk = d^3k \, dw. \tag{11} \]

Using equations (9) and (10) in equation (8), then

\[ \rho_{dp} = \frac{k \cdot j_{dp}}{\omega + i \delta}. \tag{12} \]

Taking the Fourier transform of equation (7) one has also

\[ j_{dp} = \sum_s \epsilon_s \int \frac{d^3p_s}{(2\pi)^3} \left( f_k(s) - f_R(k) \right) \epsilon_s. \tag{13} \]

Substituting equation (13) in equation (12) one obtains the following expression for the Fourier transform of the dynamic polarization charge density:

\[ \rho_{dp} = \sum_s \epsilon_s \int \frac{d^3p_s}{(2\pi)^3} \frac{k \cdot \epsilon_s}{\omega + i \delta} \left( f_k(s) - f_R(k) \right) \epsilon_s. \tag{14} \]

The particle distribution functions \( f(s) \) are determined by the relativistic Vlasov equation, namely,

\[ \frac{\partial f(s)}{\partial t} + \nabla_s \cdot \mathbf{v}_s f(s) + \mathbf{v}_s \cdot \mathbf{v}_s f(s) = 0. \tag{15} \]
where the relativistic relation between velocity and momentum is given by

$$\gamma v_s = \left[1 + \left(\frac{p_s}{m_s c}\right)^2\right]^{-1/2} \frac{p_s}{m_s}$$  \hspace{1cm} (16)

and the force $$\mathbf{f}_s$$ is given by

$$\mathbf{f}_s = e_s (\mathbf{E} + v_s \times \mathbf{B})$$ . \hspace{1cm} (17)

Here $$\mathbf{E}$$ and $$\mathbf{B}$$ are the total electric and magnetic fields. The distribution function and the fields can be expressed in terms of their Fourier transforms. Thus, for example,

$$f(s) = \frac{1}{2\pi} \int dk f_k(s) e^{i(k \cdot \mathbf{r}_s - \omega t)} .$$ \hspace{1cm} (18)

In terms of the Fourier transforms, equation (15) becomes

$$f_k(s) = \frac{1}{i(\omega - k \cdot \mathbf{v}_s + i\delta)} \int dk_1 dk_2 \delta(k - k_1 - k_2) f_{sk_1} \mathbf{v}_{sk_2} f_k(s) .$$ \hspace{1cm} (19)

Expressing the distribution functions as power series in the total field, one has

$$f_k(s) = f_k^R(s) + \sum_{n=1}^{\infty} f_k^{(n)}(s) ,$$ \hspace{1cm} (20)

where $$f_k^R(s)$$ describes the background for the nonequilibrium beam-plasma. The $$f_k^{(n)}$$ describe the perturbation in the background due to the total electromagnetic field. Assuming a slowly varying, nearly spatially independent background which in zeroth approximation is space and time independent and denoted by $$f^R_{ps}(0)$$, then

$$f_k^R(s) = (2\pi)^{-4} \int d^3\mathbf{r} dt f^R_{ps}(0) e^{-i(\mathbf{k} \cdot \mathbf{r} - \omega t)} .$$ \hspace{1cm} (21)

Performing the integration in equation (21), then

$$f_k^R(s) = f^R_{ps}(0) \delta(k) .$$ \hspace{1cm} (22)
where $\delta(k) \equiv \delta^3(\hat{r})\delta(\omega)$ is the four-dimensional Dirac delta function. For stationary turbulence, equation (22) is exact. It follows from equations (19) and (20) by iteration that

$$f_k^{(s)(1)} = \frac{1}{i(\omega - \hat{k} \cdot \hat{v}_s + i\delta)} \int dk_1 dk_2 \delta(k - k_1 - k_2) \hat{F}_{sk_1} \cdot \hat{F}_{ps} \hat{f}_{k_2}^{R(s)},$$

$$f_k^{(s)(2)} = \frac{1}{i(\omega - \hat{k} \cdot \hat{v}_s + i\delta)} \int dk_1 dk_2 dk_4 dk_5 \delta(k - k_1 - k_2) \delta(k_2 - k_3 - k_4)
\times \hat{F}_{sk_1} \cdot \hat{F}_{ps} \frac{1}{i(\omega_2 - \hat{k}_2 \cdot \hat{v}_s + i\delta)} \hat{F}_{sk_3} \cdot \hat{F}_{ps} \hat{f}_{k_4}^{R(s)},$$

and

$$f_k^{(s)(3)} = \frac{1}{i(\omega - \hat{k} \cdot \hat{v}_s + i\delta)} \int dk_1 dk_2 dk_3 dk_4 dk_5 dk_6 \delta(k - k_1 - k_2)
\times \delta(k_2 - k_3 - k_4) \delta(k_4 - k_5 - k_6) \hat{F}_{sk_1} \cdot \hat{F}_{ps} \frac{1}{i(\omega_2 - \hat{k}_2 \cdot \hat{v}_s + i\delta)} \hat{F}_{sk_3} \cdot \hat{F}_{ps} \hat{f}_{k_4}^{R(s)}.$$

Substituting equation (22) in equations (23,24,25) and using the properties of the delta function to do some of the integrations, then

$$f_k^{(s)(1)} = \frac{1}{i(\omega - \hat{k} \cdot \hat{v}_s + i\delta)} \hat{F}_{sk} \cdot \hat{F}_{ps} \hat{f}^{R(0)}(s),$$

$$f_k^{(s)(2)} = \frac{1}{i(\omega - \hat{k} \cdot \hat{v}_s + i\delta)} \int dk_1 dk_2 \delta(k - k_1 - k_2) \hat{F}_{sk_1} \cdot \hat{F}_{ps}
\times \frac{1}{i(\omega_2 - \hat{k}_2 \cdot \hat{v}_s + i\delta)} \hat{F}_{sk_2} \cdot \hat{F}_{ps} \hat{f}^{R(0)}(s),$$

and
\[ \mathbf{F}_{\mathbf{sk}}(s) = \frac{1}{i(\omega - \mathbf{k} \cdot \mathbf{v}_{s} + i\delta)} \int dk_{1}dk_{2}dk_{3}dk_{4} \delta(k - k_{1} - k_{2})\delta(k_{2} - k_{3} - k_{4}) \]

\[ \times \mathbf{F}_{\mathbf{sk}} \mathbf{v}_{p_{s}} \mathbf{p}_{s} \mathbf{p}_{s} \mathbf{p}_{s} \]

The Fourier transform \( \mathbf{F}_{\mathbf{sk}} \) of the Lorentz force \( \mathbf{F}_{s} \) is given by

\[ \mathbf{F}_{\mathbf{sk}} = e_{s}(\mathbf{E}_{k} + \mathbf{v}_{s} \times \mathbf{B}_{k}) . \] (29)

However, from Maxwell's equation,

\[ \partial_{t} \mathbf{B} = -\mathbf{V} \times \mathbf{E} , \] (30)

it follows that

\[ \mathbf{E}_{k} = \frac{1}{\omega + i\delta} \mathbf{k} \times \mathbf{E}_{k} , \] (31)

and substituting equation (31) in equation (29), then

\[ \mathbf{F}_{\mathbf{sk}} = e_{s} \left[ \mathbf{E}_{k} + \frac{\mathbf{v}_{s} \times (\mathbf{k} \times \mathbf{E}_{k})}{\omega + i\delta} \right] . \] (32)

Then using the vector identity, one has

\[ \mathbf{v}_{s} \times (\mathbf{k} \times \mathbf{E}_{k}) = (\mathbf{v}_{s} \cdot \mathbf{E}_{k}) \mathbf{k} - (\mathbf{v}_{s} \cdot \mathbf{k}) \mathbf{E}_{k} , \] (33)

and equation (32) becomes

\[ \mathbf{F}_{\mathbf{sk}} = e_{s} \left[ \mathbf{E}_{k} \left( 1 - \frac{\mathbf{v}_{s} \cdot \mathbf{k}}{\omega + i\delta} \right) + k \frac{\mathbf{v}_{s} \cdot \mathbf{E}_{k}}{\omega + i\delta} \right] . \] (34)

Next substituting equation (20) in equation (14) one obtains for the Fourier transform of the dynamic polarization charge density

\[ \rho_{d_{pk}} = \sum_{s} e_{s} \sum_{n=1}^{\infty} \int \frac{\partial^{3} F_{g}}{(2\pi)^{3}} \omega + i\delta \mathbf{F}_{k} . \] (35)
Similarly, the Fourier transform of the dynamic polarization current density equation (13) becomes

$$
\mathbf{j}_{dpk} = \sum_{s} e_s \sum_{n=1}^{\infty} \int \frac{d^3p_s}{(2\pi)^3} \mathbf{v}^+_s (s)(n) \cdot (s)(n).
$$

Equation (35) may be written as

$$
\rho_{dpk} = \sum_{s} \rho_{dpk},
$$

where

$$
\rho_{dpk} = \sum_{n=1}^{\infty} \rho_{dpk}.
$$

Equation (36) may be written

$$
\mathbf{j}_{dpk} = \sum_{s} \mathbf{j}_{dpk},
$$

where

$$
\mathbf{j}_{dpk} = \sum_{n=1}^{\infty} \mathbf{j}_{dpk}.
$$

Equations (37) and (40) express the Fourier transform of the dynamic polarization charge density and current density in terms of a sum over the contributions of each species. Equations (38) and (41) represent the latter as expansions in the total electric field, the $n$th order terms of which are given by equations (39) and (42). The first three orders can be obtained by substituting equations (26) to (28) and (34) in equations (39) and (42).
3. THE DYNAMIC POLARIZATION CURRENT DENSITY

The currents \( j_{dp}^s(1) \), \( j_{dp}^s(2) \), and \( j_{dp}^s(3) \) are the linear, second-order nonlinear, and third-order nonlinear dynamic polarization current densities, respectively. The linear and nonlinear electric field dependence is given by equations (42), (26) to (28), and (34).

Proceeding to reduce the linear current one has, using equations (42) and (26),

\[
\begin{align*}
\hat{j}_{dpk}^s(1) &= \int \frac{d^3p_s^+}{(2\pi)^3} \frac{eS_s^+}{i(\omega - k \cdot \nabla S_s^+ + i\delta)} \hat{f}_{sk}^s \hat{v}_S^+ R(0). 
\end{align*}
\]

Substituting equation (34) in equation (43) then, equation (43) becomes

\[
\begin{align*}
\hat{j}_{dpk}^s(1) &= \int \frac{d^3p_s^+}{(2\pi)^3} \frac{eS_s^+}{i(\omega - k \cdot \nabla S_s^+ + i\delta)} \left[ \hat{f}_{sk}^s \left( 1 - \frac{k \cdot \nabla S_s^+}{\omega + i\delta} \right) 
\right. \\
&\left. + k \left( \frac{\hat{v}_S^+ \cdot \nabla \xi}{\omega + i\delta} \right) \hat{v}_{ps}^+ R(0) \right].
\end{align*}
\]

Using the Einstein convention with implicit summation over repeated indices, equation (44) may be rewritten as

\[
\begin{align*}
\hat{j}_{dpk}^s(1) &= \delta_{ik} \int \frac{d^3p_s^+}{(2\pi)^3} \frac{eS_s^+}{i(\omega - k \cdot \nabla S_s^+ + i\delta)} \left[ \delta \nabla \left( 1 - \frac{k \cdot \nabla S_s^+}{\omega + i\delta} \right) 
\right. \\
&\left. + \frac{k_m \nabla S_s^+}{\omega + i\delta} \right] \frac{\delta R(0)}{\delta p_{sm}}.
\end{align*}
\]

Equivalently, then, the linear dynamic polarization current density is given by

\[
\begin{align*}
\hat{j}_{dpk}^s(1) &= \sigma^{s}_{il} \hat{E}_{kl},
\end{align*}
\]

where the linear conductivity tensor \( \sigma^{s}_{il} \) is given by \( ^2 \)

\[
\begin{align*}
\sigma^{s}_{il} \hat{E}_{kl} &= \int \frac{d^3p_s^+}{(2\pi)^3} \frac{eS_s^+}{i(\omega - k \cdot \nabla S_s^+ + i\delta)} \left[ \delta \nabla \left( 1 - \frac{k \cdot \nabla S_s^+}{\omega + i\delta} \right) + \frac{k_m \nabla S_s^+}{\omega + i\delta} \right] \frac{\delta R(0)}{\delta p_{sm}}.
\end{align*}
\]

Proceeding to reduce the second-order dynamic polarization current using equations (42) and (27), then

\[ j_{dpk} = \int \frac{d^3p_s}{(2\pi)^3} \frac{e v_s}{s} \frac{dk_1 dk_2}{i(\omega - k_1 v_s + i\delta)} e_S k_1 k_2 \delta(k - k_1 - k_2) \]

Substituting equation (34) in equation (48), then

\[ j_{dpk} = \int \frac{d^3p_s}{(2\pi)^3} \frac{e v_s}{s} \frac{dk_1 dk_2}{i(\omega - k_1 v_s + i\delta)} e_S k_1 k_2 \delta(k - k_1 - k_2) \]

Substituting equation (34) in equation (48), then

\[ j_{dpk} = -e_s \int \frac{dk_1 dk_2}{(\omega_1 + i\delta)(\omega_2 + i\delta)} \frac{S_{ij}(k, k_1, k_2) E_{k_1} E_{k_2}}{S_{ij}(k, k_1, k_2)} \]

where the second-order nonlinear conductivity tensor \( S_{ij}(k, k_1, k_2) \) for species \( s \) is given by\(^1,3-8\)


\[ S_{ijl}(k_1, k_2) = e^2 \int \frac{d^3 p}{(2\pi)^3} \frac{v_{sl}}{(\omega - k \cdot \nu_s + i\delta)} \left[ (\omega_1 - k_1 \cdot \nu_s) \frac{\partial}{\partial p_{sj}} \right. \]
\[ \left. + v_{sj}k_{1m} \frac{\partial}{\partial p_{sm}} \right] \left[ \frac{\partial}{\partial p_{sl}} + \frac{v_{sl}}{\omega_2 - k_2 \cdot \nu_s + i\delta} k_{2n} \frac{\partial}{\partial p_{sn}} \right] \mathbf{R}(0) \quad (51) \]

Equations (50) and (51) are in complete accord with Tsytovich. He has absorbed a factor of \(-e_s/\omega_1 \omega_2\) in equation (50) into his definition of the second-order conductivity. It is to be noted, however, that the Fourier transform and normalization conventions used by Tsytovich are identical to those used here, whereas those used by Akopyan and Tsytovich are not. Note also that the tensor defined by their equations (18) and (20) differs from equation (51) above in that the first complex denominator \(\omega - k \cdot \nu_s + i\delta\) in equation (51) here is implicitly \(\omega - k \cdot \nu_s - i\delta\) there.

Recently, some new exact symmetries of the second-order nonlinear conductivity tensor equation (51) have been discovered and related to long-established approximate symmetries related to the Manley-Rowe relations, crossing symmetry, and the nondissipative nature of the nonlinear current. Also, a useful new polynomial representation for the tensor was obtained in which all derivatives are removed and the pole structure is clearly exhibited. The symmetries are useful in the reduction of collective radiation probabilities. Specifically the new exact symmetries are given by

\[ S_{ijl}(k_1 + k_2, k_1, k_2) + S_{ijl}(k_1 + k_2, k_2, k_1) \]
\[ = S_{ijl}(k_1, k_1 + k_2, k_2) - S_{ijl}(k_1, k_2, k_1 + k_2) \quad (52) \]
and \( s_{ij\ell}^{(g)}(-k_1 - k_2, k_1, k_2) + s_{ij\ell}^{(s)}(-k_1 - k_2, k_2, k_1) \)

\[
= s_{jii}^{(s)}(k_1, k_1, k_2) + s_{jii}^{(s)}(k_1, k_2, k_1) .
\]  

(53)

The approximate symmetries follow from equations (52) and (53) when resonant wave-particle interactions are negligible. For example, under this condition the well-known approximate symmetry--equation (2.83) of Tsytovich\(^2\)--may be obtained from either equation (52) or equation (53),\(^3\)\(^-\)\(^6\)\(^,\)\(^8\)

Proceeding to reduce the third-order polarization current using equations (42) and (28) and integrating over one of the delta functions, then

\[
\begin{align*}
\frac{1}{3} \frac{d^3 P_s}{dpk} & = \int d^3 P_s \frac{e^\phi}{(2\pi)^3} \frac{1}{i(\omega - k \cdot v_s + i\delta)} dk_1 dk_3 dk_4 \delta(k - k_1 - k_3 - k_4) \\
& \times \left( \frac{1}{i(\omega - \omega_1 - (k - k_1) \cdot v_s + i\delta)} \right)^{\frac{1}{2}} P_s \left( \frac{1}{i(\omega - k_4 \cdot v_s + i\delta)} \right)^{\frac{1}{2}} \\
& \times \frac{1}{i(\omega - k_4 \cdot v_s + i\delta)} R(0) .
\end{align*}
\]

(54)

Substituting equation (34) in equation (54) and renaming wave vector variables of integration, then


\(3\) H. E. Brandt, Symmetries of the Nonlinear Conductivity for a Relativistic Turbulent Plasma, Harry Diamond Laboratories, HDL-TR-1927 (March 1981).


\[
\begin{align*}
\int_{d\mathbf{p} \mathbf{k}} \frac{d^3 \mathbf{p}_s \mathbf{k}_s}{(2\pi)^3 i(\omega - \mathbf{k}_s \cdot \mathbf{v}_s + i\delta)} & \, dk_1 dk_2 dk_3 \, \delta(k - k_1 - k_2 - k_3) \\
\times \left[ \varepsilon_{k_1} \left( 1 - \varepsilon_{v_s k_1} \right) + \kappa(\varepsilon_{v_s k_1}) \right] \varepsilon_{v_s k_1} & \, \varepsilon_{v_s k_2} \frac{1}{i[(\omega - \omega_1) - (k - k_3) \cdot \mathbf{v}_s + i\delta]} \\
\times \left[ \varepsilon_{k_2} \left( 1 - \varepsilon_{v_s k_2} \right) + \kappa(\varepsilon_{v_s k_2}) \right] & \, \varepsilon_{v_s k_2} \frac{1}{i(\omega_3 - k_3 \cdot \mathbf{v}_s + i\delta)} \quad \text{(55)} \\
\times \left[ \varepsilon_{k_3} \left( 1 - \varepsilon_{v_s k_3} \right) + \kappa(\varepsilon_{v_s k_3}) \right] & \, \varepsilon_{v_s k_3} \frac{R(0)}{\varepsilon_{v_s k_3}} \, f_{v_s k_3} 
\end{align*}
\]

Equivalently then, the third-order nonlinear dynamic polarization current density--equation (55)--may be written as

\[
\int_{d\mathbf{p} \mathbf{k}_i} \frac{d^3 \mathbf{p}_s \mathbf{k}_s}{(2\pi)^3} \, \frac{\varepsilon_{v_s i} k_i}{\omega - k \cdot \mathbf{v}_s + i\delta} \, \frac{1}{(\omega_1 + i\delta)(\omega_2 + i\delta)(\omega_3 + i\delta)} \\
\times \Sigma_{ij \ell \mu}(k \cdot k_1, k_2, k_3) E_{k_1} E_{k_2} E_{k_3} 
\]

where the third-order nonlinear conductivity tensor for species \( s \) is given by

\[
\Sigma_{ij \ell \mu}(k \cdot k_1, k_2, k_3) = -i e^3 \int \frac{d^3 \mathbf{p}_s \mathbf{k}_s}{(2\pi)^3} \frac{\varepsilon_{v_s i}}{\omega - k \cdot \mathbf{v}_s + i\delta} \\
\times \left[ \delta_{in} (\omega_1 - k_1 \cdot \mathbf{v}_s) + \kappa \, \varepsilon_{v_s j} \right] \frac{\partial}{\partial \mathbf{p}_n} \left( \omega - \omega_1 - (k - k_1) \cdot \mathbf{v}_s + i\delta \right) \\
\times \left[ \delta_{k_2}(\omega_2 - k_2 \cdot \mathbf{v}_s) + \kappa \varepsilon_{v_s} \right] \frac{\partial}{\partial \mathbf{p}_u} \left( \omega_3 - k_3 \cdot \mathbf{v}_s + i\delta \right) \\
\times \left[ \delta_{k_3}(\omega_3 - k_3 \cdot \mathbf{v}_s) + \kappa \varepsilon_{v_s} \right] \frac{R(0)}{\varepsilon_{v_s k_3}} \, f_{v_s k_3} 
\]

Equations (56) and (57) are also in complete accord with Tsytovich.\(^9\) He has absorbed a factor of \((-e_s/\omega_1 \omega_2 \omega_3)\) into the third-order nonlinear conductivity.

4. THE NONLINEAR DYNAMIC POLARIZATION FORCE

The dynamic polarization force \( F_{dp} \) on a test particle is given by the Lorentz force acting on the polarization charge accompanying the particle. Thus,

\[
\hat{F}_{dp} = \sum_s \int d^3\tau [\rho_{dp}(s)(\vec{r}, t)\vec{E}(\vec{r}, t) + j_{dp}(s)(\vec{r}, t)\vec{B}(\vec{r}, t)] .
\]  

(58)

The fields \( \vec{E}(\vec{r}, t) \) and \( \vec{B}(\vec{r}, t) \) are the electric and magnetic fields induced by the test particle interaction with the other particles in the system. It is assumed that there are no external fields. In terms of the Fourier decompositions, equation (58) becomes the following:

\[
\hat{F}_{dp} = \sum_s \int d^3\tau \text{d}k_1 \text{d}k_2 e^{i(k_1 \cdot \vec{r} - \omega_1 t)} e^{i(k_2 \cdot \vec{r} - \omega_2 t)}
\]

\[
\times [\rho_{dpk_1}^s \hat{\vec{E}}_{k_2} + j_{dpk_1}^s \hat{\vec{B}}_{k_2}] ,
\]

(59)

or performing the integral over space, then

\[
\hat{F}_{dp} = \sum_s \int \text{d}k_1 \text{d}k_2 e^{-i(\omega_1 + \omega_2)t} e^{i(\omega_1 + \omega_2)t} \delta(\vec{k}_1 + \vec{k}_2)
\]

\[
\times [\rho_{dpk_1}^s \hat{\vec{E}}_{k_2} + j_{dpk_1}^s \hat{\vec{B}}_{k_2}] .
\]

(60)

The time-average dynamic polarization force is given by

\[
\langle \hat{F}_{dp} \rangle = \lim_{t \to \infty} \frac{1}{2t} \int_{-t/2}^{t/2} \hat{F}_{dp}(t') dt' .
\]

(61)

Substituting equation (60) in equation (61) and replacing the time limits of integration by the infinite limit, then

\[
\langle \hat{F}_{dp} \rangle = \lim_{t \to \infty} \frac{1}{2t} \int_{-\infty}^{\infty} dt' \sum_s \int \text{d}k_1 \text{d}k_2 e^{-i(\omega_1 + \omega_2)t'} e^{i(\omega_1 + \omega_2)t'} \delta(\vec{k}_1 + \vec{k}_2)
\]

\[
\times [\rho_{dpk_1}^s \hat{\vec{E}}_{k_2} + j_{dpk_1}^s \hat{\vec{B}}_{k_2}] .
\]

(62)

Performing the time integration, then equation (62) becomes

\[
\langle \hat{F}_{dp} \rangle = \lim_{t \to \infty} \frac{1}{2t} \sum_s \int \text{d}k_1 \text{d}k_2 \delta(\vec{k}_1 + \vec{k}_2)
\]

\[
\times [\rho_{dpk_1}^s \hat{\vec{E}}_{k_2} + j_{dpk_1}^s \hat{\vec{B}}_{k_2}] ,
\]

(63)
where

\[
\delta(k) \equiv \delta^3(\xi) \delta(\omega) .
\]  

(64)

Using the property of the delta function to integrate over \( k_1 \), and renaming the remaining integration variable \( k_2 \) to be \( k \), then equation (63) becomes

\[
\langle \mathbf{F}_{dp} \rangle = \lim_{t \to \infty} \left( \frac{2\pi}{t} \right)^4 \int_s \left[ \rho_{dp-k}^{(s)} \mathbf{E}_k + j_{dp-k}^{(s)} \times \mathbf{B}_k \right] .
\]

(65)

Using equations (12) and (31), one has

\[
\rho_{dp-k}^{(s)} \mathbf{E}_k + j_{dp-k}^{(s)} \times \mathbf{B}_k = \frac{k^* \rho_{dp-k}^{(s)} \mathbf{E}_k + j_{dp-k}^{(s)} \times (k \times \mathbf{E}_k)}{\omega - i\delta} + \frac{j_{dp-k}^{(s)} \times (k \times \mathbf{E}_k)}{\omega + i\delta} .
\]

(66)

Next using the vector identity

\[
j_{dp-k}^{(s)} \times (k \times \mathbf{E}_k) = (j_{dp-k}^{(s)} \mathbf{E}_k) - (j_{dp-k}^{(s)} \times \mathbf{E}_k) ,
\]

(67)

together with equation (12) and the fact that \( \omega \delta(\omega) = 0 \), then equation (66) becomes

\[
\rho_{dp-k}^{(s)} \mathbf{E}_k + j_{dp-k}^{(s)} \times \mathbf{B}_k = \frac{k^* \rho_{dp-k}^{(s)} \mathbf{E}_k + j_{dp-k}^{(s)} \times (k \times \mathbf{E}_k)}{\omega + i\delta} .
\]

(68)

Therefore, substituting equation (68) in equation (65) one has

\[
\langle \mathbf{F}_{dp} \rangle = \lim_{t \to \infty} \left( \frac{2\pi}{t} \right)^4 \int_s \frac{j_{dp-k}^{(s)} \mathbf{E}_k}{\omega + i\delta} .
\]

(69)

Substituting equation (41) in equation (69), then

\[
\langle \mathbf{F}_{dp} \rangle = \sum_{n=0}^{\infty} \mathbf{F}_{dp}^{(n)} ,
\]

(70)

where

\[
\mathbf{F}_{dp}^{(n)} = \lim_{t \to \infty} \left( \frac{2\pi}{t} \right)^4 \int_s \frac{j_{dp-k}^{(s)} \times (k \times \mathbf{E}_k)}{\omega + i\delta} .
\]

(71)
Next, substituting equation (46) in equation (71), then

$$\mathcal{F}(0) = \lim_{t \to \infty} \frac{(2\pi)^n}{t} \sum_s \int \frac{dk}{\omega + i\delta} \mathbf{E}_k \sigma^{(s)}(\mathbf{k}) \mathbf{E}_k \mathbf{E}^*_{-\mathbf{k}}.$$

Replacing the variable of integration $k$ by $-\mathbf{k}$ in equation (72), and realizing that because the fields are real

$$\mathbf{E}_{-\mathbf{k}} = \mathbf{E}_\mathbf{k},$$

then equation (72) becomes

$$\mathcal{F}(0) = \lim_{t \to \infty} \frac{(2\pi)^n}{t} \sum_s \int \frac{dk}{\omega + i\delta} \mathbf{E}_k \sigma^{(s)}(\mathbf{k}) \mathbf{E}_k \mathbf{E}^*_{-\mathbf{k}}.$$

An alternative form is obtained by introducing an integrated delta function in equation (72) to obtain an equivalent expression, namely,

$$\mathcal{F}(0) = \lim_{t \to \infty} \frac{(2\pi)^n}{t} \sum_s \int \frac{dkdk_1}{\omega + i\delta} \delta(\mathbf{k} + \mathbf{k}_1) \mathbf{E}_k \sigma^{(s)}(\mathbf{k}_1) \mathbf{E}_{\mathbf{k}_1} \mathbf{E}_{-\mathbf{k}_1} \mathbf{E}^*_1.$$

Next substituting equation (50) in equation (71), and noting that the delta function is even, one obtains

$$\mathcal{F}(1) = \lim_{t \to \infty} \frac{(2\pi)^n}{t} \sum_s \int \frac{dkdk_1dk_2}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_2 + i\delta)} \delta(\mathbf{k} + \mathbf{k}_1 + \mathbf{k}_2) \times \mathbf{E}_{\mathbf{k}_1} \mathbf{E}_{\mathbf{k}_2} \mathbf{E}^*_{\mathbf{k}_1} \mathbf{E}^*_{\mathbf{k}_2} S^{(s)}_{ijl}.$$ 

Equation (76) is in complete agreement with equation (18) of Akopyan and Tsytovich since from equation (51) as noted earlier it follows that

$$S^{(s)}_{ijl} = -S^{(s)}_{ijl}(-\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2),$$

where $S^{(s)}_{ijl}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)$ designates the conductivity tensor appearing in Akopyan and Tsytovich in which evidently the first complex denominator is implicitly $\omega - \mathbf{k} \cdot \mathbf{v}_s - i\delta$. This may be seen from equation (18) there where, because of the delta function, one has effectively $S^{(s)}_{ijl}(-\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)$. Assuming that

---

equation (50) holds there also, except for the differing Fourier transform convention, then because of the delta function only $S_{ij}^{(s)}(k_1 + k_2, k, k_2)$ can enter. Equation (18) of Akopyan and Tsytovich has an additional factor of $(2\pi)^{-3}$ which apparently arises from different Fourier transform and background normalization conventions. For example, in their equation (5) the Fourier transform convention employed has a factor of $(2\pi)^{-3}$ in the inverse Fourier transform in the integration over the three-dimensional wave vector space, and a factor of 1 for the integration over frequency, giving a total factor of $(2\pi)^{-3}$, whereas here a total factor of 1 is used as in equation (9), for example, also, the Fourier transform itself has a factor of $(2\pi)^{-1}$ there and $(2\pi)^{-4}$ here as in equation (21), for example. There is also another additional factor of $(2\pi)^{-3}$ in their equation (18) which is apparently due to differing background normalization. Because of the different Fourier transform convention alone, the counterpart of their equation (18) would have an additional factor of $(2\pi)^3$. Apparently it has been absorbed into the normalization of $R^{(0)}$ there. In short, the $R^{(0)}$ there must be $(2\pi)^3$ times that here. Alternatively if the normalization is in fact the same as that here, then there is an erroneous factor of $(2\pi)^{-3}$ appearing there. Also the factor of $1/6$ in Akopyan and Tsytovich arises from the explicit symmetrization chosen there.

Next substituting equation (56) in equation (71) and noting that the delta function is even, one obtains

$$\mathcal{F}^{(2)} = \lim_{t \to \infty} \frac{(2\pi)^4}{t} \sum_{s} e^{i s} \int \frac{dk_1 dk_2 dk_3 \delta(k + k_1 + k_2 + k_3)}{(\omega + i\delta)(\omega_1 + i\delta)(\omega_2 + i\delta)(\omega_3 + i\delta)}$$

$$\times E_{k_1 k_2 k_3} E_{k_1 kj k_3} E_{k_2 k_3 m} \mathcal{F}^{(s)}_{ij lm} (-k, k_1, k_2, k_3).$$

(78)

It is to be noted that equations (78) and (57) are in apparent agreement with equations (19) and (21) of Akopyan and Tsytovich. Evidently in the conductivity tensor $T_{ij lm}(k, k_1, k_2, k_3)$ given by equation (21) there, the first two complex denominators are implicitly $\omega - k_2^2 - i\delta$ and $\omega + \omega_1 - (k + k_1)^2 + i\delta$, respectively, and also there is no overall factor of $i$ as there is in equation (57) here. Therefore

$$\Sigma_{ij lm}(-k, k_1, k_2, k_3) = -iT^{(s)}_{ij lm}(k, k_1, k_2, k_3).$$

(79)

---

Thus, comparing equations (78) and (79) with equation (19) of Akopyan and Tsytovich,\(^1\) one finds that they are in complete agreement. Again the additional factors of \((2\pi)^{-12}\) and \((2\pi)^{-3}\) and \(1/24\) in their equation (19)\(^1\) apparently arise from differing Fourier transform and background normalization conventions and explicit symmetrization, respectively.

In summary, then, using equation (70), the time-average dynamic polarization force on a relativistic test particle to fourth order, in the field in a nonequilibrium beam-plasma system for a slowly varying, nearly spatially independent background with no external fields, is given by

\[
\langle F_{dp} \rangle = F_{dp}^{(0)} + F_{dp}^{(1)} + F_{dp}^{(2)} \tag{80}
\]

The linear, second-order nonlinear, and third-order nonlinear dynamic polarization forces \(F_{dp}^{(0)}, F_{dp}^{(1)},\) and \(F_{dp}^{(2)}\) are given by equations (75), (76), and (78), respectively.

5. CONCLUSION

An expression—equations (80), (75), (76), and (78)—has been obtained for the time-averaged dynamic polarization force on a relativistic test particle to fourth order in the total field in a nonequilibrium beam-plasma system for a slowly varying, nearly spatially independent background with no external fields. This result has been used in the work of Akopyan and Tsytovich in the theory of collective bremsstrahlung in nonequilibrium plasmas.

The present work together with related work by the author\(^10,*,t,E\) is important for ongoing work in calculating collective radiation processes and conditions for the occurrence of radiative instability in relativistic beam-plasma systems.

\(^*\)Other related work, prepared in preprint form, will be published later and is available from the author.
LITERATURE CITED


<table>
<thead>
<tr>
<th>DISTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADMINISTRATOR</td>
</tr>
<tr>
<td>DEFENSE TECHNICAL INFORMATION CENTER</td>
</tr>
<tr>
<td>ATTN DTIC-DDA (12 COPIES)</td>
</tr>
<tr>
<td>CAMERON STATION, BUILDING 5</td>
</tr>
<tr>
<td>ALEXANDRIA, VA 22314</td>
</tr>
<tr>
<td>COMMANDER</td>
</tr>
<tr>
<td>US ARMY MATERIEL DEVELOPMENT &amp; READINESS COMMAND</td>
</tr>
<tr>
<td>5001 EISENHOWER AVENUE</td>
</tr>
<tr>
<td>ALEXANDRIA, VA 22333</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>DEFENSE ADVANCED RESEARCH PROJECTS AGENCY</td>
</tr>
<tr>
<td>ATTN R. GULLICKSON</td>
</tr>
<tr>
<td>ATTN L. C. MARQUET</td>
</tr>
<tr>
<td>ATTN R. SEPUNA</td>
</tr>
<tr>
<td>CPO WILSON BLVD</td>
</tr>
<tr>
<td>ARLINGTON, VA 22209</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>DEFENSE COMMUNICATIONS AGENCY</td>
</tr>
<tr>
<td>WASHINGTON, DC 20305</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>DEFENSE INTELLIGENCE AGENCY</td>
</tr>
<tr>
<td>ATTN D. SPORN</td>
</tr>
<tr>
<td>ATTN DT-1, NUCLEAR &amp; APPLIED SCIENCES DIV</td>
</tr>
<tr>
<td>ATTN ELECTRONIC WARFARE BRANCH</td>
</tr>
<tr>
<td>WASHINGTON, DC 20301</td>
</tr>
<tr>
<td>UNDER SECRETARY OF DEFENSE</td>
</tr>
<tr>
<td>FOR RESEARCH &amp; ENGINEERING</td>
</tr>
<tr>
<td>ATTN DEPUTY UNDER SECRETARY (RESEARCH &amp; ADVANCED TECH)</td>
</tr>
<tr>
<td>ATTN D. L. LAMBERSON</td>
</tr>
<tr>
<td>WASHINGTON, DC 20301</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>OFFICE OF THE UNDER SECRETARY OF DEFENSE FOR RESEARCH &amp; ENGINEERING</td>
</tr>
<tr>
<td>DIR ENERGY TECHNOLOGY OFFICE</td>
</tr>
<tr>
<td>THE PENTAGON</td>
</tr>
<tr>
<td>WASHINGTON, DC 20301</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>DEFENSE NUCLEAR AGENCY</td>
</tr>
<tr>
<td>ATTN DEPUTY DIRECTOR</td>
</tr>
<tr>
<td>SCIENTIFIC TECHNOLOGY</td>
</tr>
<tr>
<td>ATTN G. BAKER</td>
</tr>
<tr>
<td>ATTN RAEV, ELECTRONIC VULNERABILITY</td>
</tr>
<tr>
<td>ATTN J. Z. FARBER</td>
</tr>
<tr>
<td>ATTN G. K. SOPER</td>
</tr>
<tr>
<td>ATTN V. VAN LINT</td>
</tr>
<tr>
<td>WASHINGTON, DC 20305</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>NATIONAL SECURITY AGENCY</td>
</tr>
<tr>
<td>ATTN TECHNICAL LIBRARY</td>
</tr>
<tr>
<td>ATTN F. REDARD</td>
</tr>
<tr>
<td>FT MEADE, MD 20755</td>
</tr>
<tr>
<td>ASSISTANT SECRETARY OF THE ARMY (RDA)</td>
</tr>
<tr>
<td>ATTN DEP FOR SCI &amp; TECH</td>
</tr>
<tr>
<td>WASHINGTON, DC 20301</td>
</tr>
<tr>
<td>OFFICE, DEPUTY CHIEF OF STAFF</td>
</tr>
<tr>
<td>FOR OPERATIONS &amp; PLANS</td>
</tr>
<tr>
<td>DEPT OF THE ARMY</td>
</tr>
<tr>
<td>ATTN DAMO-SSN, NUCLEAR DIV</td>
</tr>
<tr>
<td>WASHINGTON, DC 20310</td>
</tr>
<tr>
<td>OFFICE OF THE DEPUTY CHIEF OF STAFF</td>
</tr>
<tr>
<td>FOR RESEARCH, DEVELOPMENT, &amp; ACQUISITION</td>
</tr>
<tr>
<td>DEPARTMENT OF THE ARMY</td>
</tr>
<tr>
<td>ATTN DIRECTOR OF ARMY RESEARCH, M. E. LASSER</td>
</tr>
<tr>
<td>ATTN DAMC-CSS-N, NUCLEAR TEAM</td>
</tr>
<tr>
<td>ATTN DAMC-ARZ-O, F. D. VERDERAME</td>
</tr>
<tr>
<td>WASHINGTON, DC 20310</td>
</tr>
<tr>
<td>COMMANDER</td>
</tr>
<tr>
<td>US ARMY ABERDEEN PROVING GROUND</td>
</tr>
<tr>
<td>ATTN STEAP-TL, TECH LIB</td>
</tr>
<tr>
<td>ABERDEEN PROVING GROUND, MD 21005</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>BALLISTIC MISSILE DEFENSE</td>
</tr>
<tr>
<td>ADVANCED TECHNOLOGY CENTER</td>
</tr>
<tr>
<td>ATTN D. SCHENK</td>
</tr>
<tr>
<td>PO BOX 1500</td>
</tr>
<tr>
<td>HUNTSVILLE, AL 35807</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>US ARMY BALLISTIC RESEARCH LABORATORY</td>
</tr>
<tr>
<td>ATTN DAMAR-TSB-S (STINFO)</td>
</tr>
<tr>
<td>ATTN D. SCOLLESHELL</td>
</tr>
<tr>
<td>ATTN C. HOLLANDSWORTH</td>
</tr>
<tr>
<td>ATTN R. KREMENS</td>
</tr>
<tr>
<td>ABERDEEN PROVING GROUND, MD 21005</td>
</tr>
<tr>
<td>DIRECTOR</td>
</tr>
<tr>
<td>US ARMY ELECTRONICS TECHNOLOGY &amp; DEVICES LABORATORY</td>
</tr>
<tr>
<td>ATTN DELET-DD</td>
</tr>
<tr>
<td>ATTN W. WILSON</td>
</tr>
<tr>
<td>FT MONMOUTH, NJ 07703</td>
</tr>
<tr>
<td>US ARMY ERADCOM</td>
</tr>
<tr>
<td>ATTN C. M. DESANTIS</td>
</tr>
<tr>
<td>FT MONMOUTH, NJ 07703</td>
</tr>
</tbody>
</table>
DISTRIBUTION (Cont'd)

COMMANDER
US ARMY FOREIGN SCIENCE & TECHNOLOGY CENTER
FEDERAL OFFICE BLDG
ATTN DRXST-SD, SCIENCES DIV
ATTN T. CALDOWELL
220 7TH STREET, NE
CHARLOTTESVILLE, VA 22901

DIRECTOR
US ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
ATTN DRXSY-MP
ABERDEEN PROVING GROUND, MD 21005

COMMANDER
US ARMY MATERIALS & MECHANICS RESEARCH CENTER
ATTN DRXMR-H, BALLISTIC MISSILE DEF MTLS PROG OPC
WATERTOWN, MA 02172

COMMANDER
US ARMY MISSILE COMMAND
ATTN DRUMI-TR, PHYSICAL SCIENCES DIR
REDSTONE ARSENAL, AL 35809

COMMANDER
US ARMY MISSILE & MUNITIONS CENTER & SCHOOL
ATTN ATSK-CTD-F
REDSTONE ARSENAL, AL 35809

ARMY RESEARCH OFFICE (DURHAM)
PO BOX 12211
ATTN H. ROBL
ATTN R. LONTZ
ATTN B. D. GUENTHER
ATTN TECH LIBRARY
RESEARCH TRIANGLE PARK, NC 27709

COMMANDER
US ARMY RSCH & STD GP (EUR)
ATTN CHIEF, PHYSICS & MATH BRANCH
FPO NEW YORK 09510

NAVAL AIR SYSTEMS COMMAND
ATTN LCDR G. BATES, PHS-405
WASHINGTON, DC 20361

NAVAL INTELLIGENCE SUPPORT CENTER
ATTN M. KOONTZ
4301 SUITLAND RD
SUITLAND, MD 20390

NAVAL MATERIAL COMMAND
ATTN T. HOWARTH
2211 JEFFERSON DAVIS HWY
WASHINGTON, DC 20301

OFFICE OF NAVAL RESEARCH
ATTN C. ROBERSON
ATTN W. J. CONDELL
800 N. QUINCY ST
ARLINGTON, VA 22217

SUPERINTENDENT
NAVAL POSTGRADUATE SCHOOL
ATTN LIBRARY, CODE 2124
MONTEREY, CA 93940

DIRECTOR
NAVAL RESEARCH LABORATORY
ATTN 2600, TECHNICAL INFO DIV
ATTN 5540, LASER PHYSICS
ATTN 6000, MTL & RADIATION SCI & TE
ATTN B. RIFIN
ATTN L. A. COSBY
ATTN E. E. KEMPE
ATTN J. T. SCHRIEMPF
ATTN R. F. WENZEL
ATTN R. HETTCHER
ATTN J. GOLDEN
ATTN V. L. GRANATSTEIN
ATTN M. A. REED
ATTN R. K. PARKER
ATTN P. A. SPRANGLER
ATTN S. GOLD
ATTN C. A. KAPETANAKOS
ATTN S. AHN
ATTN T. F. COFFEY
ATTN R. JACKSON
ATTN I. M. VITKOVITSKY
ATTN M. FRIEDMAN
ATTN J. PASOUR
ATTN G. COOPERSTEIN
WASHINGTON, DC 20375

COMMANDER
NAVAL SURFACE WEAPONS CENTER
ATTN V. FUGLIELLI
ATTN DX-21, LIBRARY DIV
DAHLGREN, VA 22448

COMMANDER
NAVAL SURFACE WEAPONS CENTER
ATTN J. Y. CHOE
ATTN F. SAZAMA
ATTN H. UHM
ATTN V. KENYON
ATTN E. MOLTING
ATTN WA-13, HIGH-ENERGY LASER BR
ATTN WA-50, NUCLEAR WEAPONS EFFECTS DIV
ATTN WR, RESEARCH & TECHNOLOGY DEPT
ATTN WR-40, RADIATION DIV
ATTN E-43, TECHNICAL LIB
WHITE OAK, MD 20910
DISTRIBUTION (Cont'd)

<table>
<thead>
<tr>
<th>Command</th>
<th>NASA Headquarters</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAVAL WEAPONS CENTER</td>
<td>WASHINGTON, DC 20546</td>
</tr>
<tr>
<td>HQ USAF/SAMI</td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, DC 20330</td>
<td></td>
</tr>
<tr>
<td>ASSISTANT SECRETARY OF THE AIR FORCE</td>
<td></td>
</tr>
<tr>
<td>(RESEARCH &amp; DEVELOPMENT)</td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, DC 20330</td>
<td></td>
</tr>
<tr>
<td>DIRECTOR</td>
<td></td>
</tr>
<tr>
<td>AF OFFICE OF SCIENTIFIC RESEARCH</td>
<td></td>
</tr>
<tr>
<td>BOLLING AFB</td>
<td></td>
</tr>
<tr>
<td>ATTN NP, DIR OF PHYSICS</td>
<td></td>
</tr>
<tr>
<td>ATTN M. A. STROSCIO</td>
<td></td>
</tr>
<tr>
<td>ATTN R. BARKER</td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, DC 20332</td>
<td></td>
</tr>
<tr>
<td>COMMANDER</td>
<td></td>
</tr>
<tr>
<td>AF WEAPONS LAB, AFSC</td>
<td></td>
</tr>
<tr>
<td>ATTN J. GENEROSA</td>
<td></td>
</tr>
<tr>
<td>ATTN A. H. GUENTHER</td>
<td></td>
</tr>
<tr>
<td>ATTN W. E. PAGE</td>
<td></td>
</tr>
<tr>
<td>ATTN LR, LASER DEV DIV</td>
<td></td>
</tr>
<tr>
<td>KIRTLAND AFB, NM 87117</td>
<td></td>
</tr>
<tr>
<td>WRIGHT-PATTERSON AFB</td>
<td></td>
</tr>
<tr>
<td>FOREIGN TECHNOLOGY DIVISION/ETD</td>
<td></td>
</tr>
<tr>
<td>ATTN J. BUTLER</td>
<td></td>
</tr>
<tr>
<td>WRIGHT-PATTERSON AFB, OH 45433</td>
<td></td>
</tr>
<tr>
<td>CENTRAL INTELLIGENCE AGENCY</td>
<td></td>
</tr>
<tr>
<td>ATTN R. PETTIS</td>
<td></td>
</tr>
<tr>
<td>ATTN D. B. NEWMAN</td>
<td></td>
</tr>
<tr>
<td>PO BOX 1925</td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, DC 20013</td>
<td></td>
</tr>
<tr>
<td>DEPARTMENT OF COMMERCE</td>
<td></td>
</tr>
<tr>
<td>NATIONAL BUREAU OF STANDARDS</td>
<td></td>
</tr>
<tr>
<td>ATTN CENTER FOR RADIATION RESEARCH</td>
<td></td>
</tr>
<tr>
<td>ATTN C. TEAGUE</td>
<td></td>
</tr>
<tr>
<td>ATTN E. MARX</td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, DC 20234</td>
<td></td>
</tr>
<tr>
<td>DEPARTMENT OF ENERGY</td>
<td></td>
</tr>
<tr>
<td>ATTN T. P. GODLOVE</td>
<td></td>
</tr>
<tr>
<td>ATTN M. MURPHY</td>
<td></td>
</tr>
<tr>
<td>ATTN A. COLE</td>
<td></td>
</tr>
<tr>
<td>ATTN G. P. MANLEY</td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, DC 20585</td>
<td></td>
</tr>
<tr>
<td>NATIONAL AERONAUTICS &amp; SPACE ADMINISTRATION</td>
<td></td>
</tr>
<tr>
<td>ATTN R. RAMATY</td>
<td></td>
</tr>
<tr>
<td>GOODFORD SPACE FLIGHT CENTER</td>
<td></td>
</tr>
<tr>
<td>GREENBELT, MD 20771</td>
<td></td>
</tr>
<tr>
<td>University</td>
<td>Address</td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>----------------------------------------------</td>
</tr>
<tr>
<td>UNIVERSITY OF CALIFORNIA</td>
<td></td>
</tr>
<tr>
<td>attn r. lovelace</td>
<td></td>
</tr>
<tr>
<td>attn r. n. sudan</td>
<td></td>
</tr>
<tr>
<td>attn j. nation</td>
<td></td>
</tr>
<tr>
<td>attn d. hammer</td>
<td></td>
</tr>
<tr>
<td>attn h. fleishmann</td>
<td></td>
</tr>
<tr>
<td>ITHACA, NY 14853</td>
<td></td>
</tr>
<tr>
<td>DARTMOUTH COLLEGE</td>
<td></td>
</tr>
<tr>
<td>attn j. e. walsh</td>
<td></td>
</tr>
<tr>
<td>HANOVER, NH 03755</td>
<td></td>
</tr>
<tr>
<td>UNIVERSITY OF THE DISTRICT</td>
<td></td>
</tr>
<tr>
<td>OF COLUMBIA</td>
<td></td>
</tr>
<tr>
<td>VAN NESS CAMPUS</td>
<td></td>
</tr>
<tr>
<td>DEPT OF PHYSICS</td>
<td></td>
</tr>
<tr>
<td>ATTN K. H. BROWN</td>
<td></td>
</tr>
<tr>
<td>4200 CONNECTICUT AVE, NW</td>
<td></td>
</tr>
<tr>
<td>WASHINGTON, DC 20008</td>
<td></td>
</tr>
<tr>
<td>ENGINEERING SOCIETIES LIBRARY</td>
<td></td>
</tr>
<tr>
<td>345 EAST 47TH STREET</td>
<td></td>
</tr>
<tr>
<td>ATTN ACQUISITIONS DEPARTMENT</td>
<td></td>
</tr>
<tr>
<td>NEW YORK, NY 10017</td>
<td></td>
</tr>
<tr>
<td>GENERAL DYNAMICS</td>
<td></td>
</tr>
<tr>
<td>ATTN K. H. BROWN</td>
<td></td>
</tr>
<tr>
<td>PO BOX 2507 ME 44-21</td>
<td></td>
</tr>
<tr>
<td>POMONA, CA 91769</td>
<td></td>
</tr>
<tr>
<td>UNIVERSITY OF ILLINOIS AT</td>
<td></td>
</tr>
<tr>
<td>URBANA--CHAMPAIGN</td>
<td></td>
</tr>
<tr>
<td>DEPT OF PHYSICS</td>
<td></td>
</tr>
<tr>
<td>ATTN N. INAMOTO</td>
<td></td>
</tr>
<tr>
<td>URBANA, IL 61801</td>
<td></td>
</tr>
<tr>
<td>ISTITUTO DI FISICA DELL' UNIVERSITA</td>
<td></td>
</tr>
<tr>
<td>VIA CELORIA 16</td>
<td></td>
</tr>
<tr>
<td>ATTN P. CALDIROLA</td>
<td></td>
</tr>
<tr>
<td>ATTN C. PAIZIS</td>
<td></td>
</tr>
<tr>
<td>20133 MILANO, ITALY</td>
<td></td>
</tr>
<tr>
<td>ISTITUTO DI FISICA DELL' UNIVERSITA</td>
<td></td>
</tr>
<tr>
<td>ATTN A. CAVALIERE</td>
<td></td>
</tr>
<tr>
<td>ATTN R. RUFFINI</td>
<td></td>
</tr>
<tr>
<td>ROME, ITALY</td>
<td></td>
</tr>
<tr>
<td>ISTITUTO DI FISICA GENERALE DELL' UNIVERSITA</td>
<td></td>
</tr>
<tr>
<td>CORSO M. D'AZEGLIO</td>
<td></td>
</tr>
<tr>
<td>ATTN A. FERRARI</td>
<td></td>
</tr>
<tr>
<td>46 TORINO, ITALY</td>
<td></td>
</tr>
<tr>
<td>JAYCOR</td>
<td></td>
</tr>
<tr>
<td>ATTN E. CONRAD</td>
<td></td>
</tr>
<tr>
<td>205 S. WHITING ST</td>
<td></td>
</tr>
<tr>
<td>ALEXANDRIA, VA 22304</td>
<td></td>
</tr>
</tbody>
</table>
DISTRIBUTION (Cont'd)

HARRY DIAMOND LABORATORIES (Cont'd)
ATTN DAVIS, D., 22900
ATTN GRAYBILL, S., 22900
ATTN HUTTLIN, G. A., 22900
ATTN KEHS, A., 22900
ATTN KERRIS, K., 22900
ATTN LAMB, R., 22900
ATTN LINDSAY, D., 22900
ATTN LITZ, M., 22900
ATTN RUTH, B., 22900
ATTN STEWART, A., 22900
ATTN SOLN, J., 22900
ATTN WHITTAKER, D., 22900
ATTN ELBAUM, S., 97100
ATTN BRANST, H. E., 22300
(60 copies)