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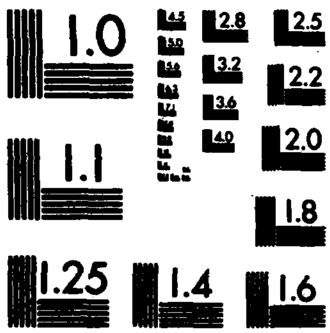

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An expression is obtained for the total electric field associated with collective bremsstrahlung in a nonequilibrium relativistic beam-plasma system in the case of a slowly varying, nearly spatially independent background distribution with no external fields. The field is assumed to consist of an unperturbed regular part corresponding to the nonradiative part of the field of the particles and a stochastic part corresponding to the bremsstrahlung radiation field. The relations derived here are needed in calculations of collective bremsstrahlung processes and the conditions for the occurrence of bremsstrahlung radiative instability in relativistic beam-plasma systems.

CONTENTS

1. INTRODUCTION

In a nonequilibrium relativistic beam-plasma system with a slowly varying, nearly spatially independent background distribution without external fields, the total field, excited in the beam-plasma and associated with a collective bremsstrahlung process involving a test particle, has two parts. First, it has a regular part with a relatively slowly varying phase associated with the nonradiative field of the test particle and its induced dynamic polarization, and it also has a stochastic part with an irregular rapidly varying phase associated with the bremsstrahlung radiation field.[1] Thus, the Fourier transform of the total field involved in the bremsstrahlung process is given by

$$\vec{E}_k = \vec{E}_k^R + \vec{E}_k^{st} \quad , \tag{1}$$

where $\vec{E}_k^R$ and $\vec{E}_k^{st}$ are the regular nonradiative component and the stochastic bremsstrahlung component, respectively. The regular part, $\vec{E}_k^R$, of this field is assumed to consist of four parts in sufficient approximation; namely,

$$\vec{E}_k^R = \vec{E}_k^{(1)} + \vec{E}_k^{(2)} + \vec{E}_{dpk}^{(1)} + \vec{E}_{dpk}^{(2)} \quad . \tag{2}$$

Here $\vec{E}_k^{(1)}$ is the self-field associated with the unperturbed motion of the relativistic bare test particle, ignoring its induced dynamic polarization. The field $\vec{E}_k^{(2)}$ is the field associated with the perturbation in the bare particle motion. The fields $E_{dpk}^{(1)}$ and $\vec{E}_{dpk}^{(2)}$ are fields produced by the dynamic polarization current induced by the test particle to second and third order in the total field, respectively.

In sections 2 through 5, the dispersive characteristics and stochastic properties of the bremsstrahlung field are reviewed, the field due to the perturbed bare test particle trajectory is calculated from the associated current, and the dynamic polarization current is used to calculate the field which it induces. In section 6 an expression for the total field is obtained, and in section 7 the results are summarized. Expressions for these fields are needed in calculations of the total nonlinear force on a relativistic test particle involved in collective bremsstrahlung in a nonequilibrium beam-plasma

[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

system.[1,*,†] The latter is needed in calculations of collective bremsstrahlung processes and the conditions for the occurrence of a bremsstrahlung radiative instability.[1-4,*†‡]

2. DISPERSION RELATION FOR THE BREMSSTRAHLUNG FIELD

In zeroth approximation the dispersive properties of the bremsstrahlung radiation field $\vec{E}_k^{st}$ are governed by the poles of the linear photon Green's function $G_{mn}(k)$ for the beam-plasma. The latter determines the mode dispersion relations. Thus,

$$\vec{E}_k^{st} = \vec{E}_k^{\sigma(0)} \quad , \tag{3}$$

where the electromagnetic waves $\vec{E}_k^{\sigma(0)}$ of mode σ satisfy the following dispersion relation:

$$\left| G_{mn}^{-1}(k) \right| = 0 \quad . \tag{4}$$

The linear photon Green's function $G_{mn}(k)$ follows from the linear Maxwell's equations in the beam-plasma system, namely,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad , \tag{5}$$

and

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

[2] *A. V. Akopyan and V. N. Tsytovich, Transition Bremsstrahlung of Relativistic Particles, Zh. Eksp. Teor. Fiz., 71 (1976), 166 [Sov. Phys. JETP, 44 (1976), 87].*

[3] *A. V. Akopyan and V. N. Tsytovich, Bremsstralung Instability of Relativistic Electrons in a Plasma, Astrofizika, 13 (1977), 717 [Astrophysics, 13 (1977), 423].*

[4] *V. N. Tsytovich, Collective Effects in Bremsstrahlung of Fast Particles in Plasmas, Comments, Plasma Phys. Conf. Fusion, 4 (1978), 73.*

[*] *H. E. Brandt, Nonlinear Force on an Unpolarized Relativistic Test Particle to Second Order in the Total Field in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-82-7 (May 1982) (to be published as HDL-TR-1995, 1983).*

[†] *H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-82-6 (May 1982) (to be published as HDL-TR-1994, 1983).*

[‡] *H. E. Brandt, Theoretical Methods in the Calculation of the Bremsstrahlung Recoil Force in a Nonequilibrium Relativistic Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-83-6 (April 1983) (to be published as HDL-TR-2009, 1983).*

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad . \tag{6}$$

The Fourier decomposition of the fields is given by

$$\vec{E} = \int dk \; \vec{E}_k e^{i(\vec{k}\cdot\vec{r}-\omega t)} \tag{7}$$

and

$$\vec{B} = \int dk \; \vec{B}_k e^{i(\vec{k}\cdot\vec{r}-\omega t)} \quad , \tag{8}$$

where

$$dk = d^3\vec{k} \; d\omega \quad . \tag{9}$$

The assumed forms of the constitutive relations for the beam-plasma system are

$$\vec{H}_k = \frac{\vec{B}_k}{\mu_o} \quad , \tag{10}$$

where μ_o is the permeability of free space, and

$$D_{ki} = \varepsilon_{ij}(k) E_{kj} \quad , \tag{11}$$

where $\varepsilon_{ij}(k)$ is the dielectric tensor for the beam-plasma system, and summation over repeated indices is understood. From equations (5), (10), and (11) it follows that

$$\frac{i}{\mu_o} (\vec{k}\times\vec{B}_k)_i + i\omega\varepsilon_{ij}(k)E_{kj} = j_{ki} \quad , \tag{12}$$

where $\vec{j}_k$ is the Fourier transform of the nonlinear and external current density, and from equation (6)

$$\vec{B}_k = \frac{\vec{k}}{\omega + i\delta} \times \vec{E}_k \quad . \tag{13}$$

Here δ is a small imaginary part of the frequency. Next, substituting equation (13) in equation (12), then

$$\frac{i}{\mu_o(\omega + i\delta)} [\vec{k}\times(\vec{k}\times\vec{E}_k)]_i + i\omega\varepsilon_{ij}E_{kj} = j_{ki} \quad . \tag{14}$$

But one has the following vector identity,

$$\vec{k}\times(\vec{k}\times\vec{E}_k) = (\vec{k}\cdot\vec{E}_k)\vec{k} - k^2\vec{E}_k \quad . \tag{15}$$

Substituting equation (15) in equation (14), then

$$\frac{i}{\mu_o(\omega + i\delta)} \left(\vec{k}\cdot\vec{E}_k k_i - k^2 E_{ki}\right) + i\omega\varepsilon_{ij}E_{kj} = j_{ki} \quad . \tag{16}$$

Equation (16) may be rewritten as

$$G_{ij}^{-1}E_{kj} = -\frac{i}{\omega + i\delta}\, j_{ki} \quad , \tag{17}$$

where

$$G_{ij}^{-1} = \frac{1}{\mu_o(\omega + i\delta)^2}\left(k_i k_j - k^2\delta_{ij}\right) + \varepsilon_{ij} \quad , \tag{18}$$

where δ_{ij} is the Kronecker delta. For a spatially isotropic system, the dielectric permittivity tensor is given by[5,6,7]

$$\varepsilon_{ij}(\vec{k},\omega) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\varepsilon_t(\vec{k},\omega) + \frac{k_i k_j}{k^2}\,\varepsilon_\ell(\vec{k},\omega) \quad , \tag{19}$$

where ε_t and ε_ℓ are the transverse and longitudinal permittivity, respectively. Substituting equation (19) in equation (18) and combining terms, then

$$G_{ij}^{-1} = \left(\varepsilon_t - \frac{k^2}{\mu_o(\omega + i\delta)^2}\right)\delta_{ij} - \left(\varepsilon_t - \varepsilon_\ell - \frac{k^2}{\mu_o(\omega + i\delta)^2}\right)\frac{k_i k_j}{k^2} \quad . \tag{20}$$

Treating equation (17) as a matrix equation, then

$$E_{ki} = -\frac{i}{\omega + i\delta}\, G_{ij} j_{kj} \quad . \tag{21}$$

The matrix G_{ij} inverse to G_{ij}^{-1} is easily obtained from equation (20); namely,

$$G_{ij} = \frac{k_i k_j}{k^2\varepsilon_\ell} + \frac{\omega^2}{\varepsilon_t(\omega + i\delta)^2 - \frac{k^2}{\mu_o}}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \quad . \tag{22}$$

Checking that equation (22) is in fact the inverse, one notes, using equations (20) and (22), that

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[5] A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, *Plasma Electrodynamics*, 1, *Linear Theory*, Pergamon Press (1975), 206.

[6] V. N. Tsytovich, *Nonlinear Effects in Plasma*, Plenum Press, New York (1970), 31.

[7] V. N. Tsytovich, *Theory of Turbulent Plasma* (Consultants Bureau), Plenum Press, New York (1977), 63-65.

$$G_{mi}^{-1}\,G_{ij} = \left[\left(\epsilon_t - \frac{k^2}{\mu_o(\omega + i\delta)^2}\right)\delta_{mi} - \left(\epsilon_t - \epsilon_\ell - \frac{k^2}{\mu_o(\omega + i\delta)^2}\right)\frac{k_m k_i}{k^2}\right]$$

$$\times\left[\frac{k_i k_j}{k^2\epsilon_\ell} + \frac{\omega^2}{\epsilon_t(\omega + i\delta)^2 - \frac{k^2}{\mu_o}}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\right] = \delta_{mj} \tag{23}$$

as it must. Equation (21) can be equivalently written in the following form:

$$E_{kn} = -\frac{i}{\omega + i\delta}\,G_{mn}j_{km} \quad , \tag{24}$$

since the linear photon Green's function equation (22), for the isotropic plasma is a symmetric tensor. According to equation (4), the poles of this function determine the zeroth order dispersion relation for the bremsstrahlung in the isotropic case.

More generally the dielectric tensor is related to the linear conductivity tensor by

$$\epsilon_{ij} = \epsilon_o\delta_{ij} + \frac{i\sigma_{ij}(k)}{\omega + i\delta} \quad . \tag{25}$$

Equation (25) follows through a comparison of equation (12) with the vacuum form of Maxwell's equations; namely,

$$\frac{i}{\mu_o}\left(\vec{k} \times \vec{B}_k\right)_i + i\omega\epsilon_o\delta_{ij}E_{kj} = \sigma_{ij}(k)E_{kj} + j_{ki} \quad , \tag{26}$$

where the explicit linear current in terms of the linear conductivity has been introduced on the right,* j_{ki} designates the nonlinear and external contributions to the current, and the vacuum dielectric tensor has been introduced. Thus, moving the first term on the right of equation (26) to the left and comparing with equation (12), then equation (25) follows.

By equation (47) of HDL-TR-1994,* the linear conductivity tensor σ_{ij} in the beam plasma system is given by

$$\sigma_{ij} = \sum_s e_s^2 \int \frac{d^3\vec{p}_s}{(2\pi)^3}\,\frac{v_{si}\left[\delta_{jm}\left(1 - \frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta}\right) + \frac{k_m v_{sj}}{\omega + i\delta}\right]}{i\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)}\,\frac{\partial f_{p_s}^{R(0)}}{\partial p_{sm}} \quad . \tag{27}$$

**H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-82-6 (May 1982) (to be published as HDL-TR-1994, 1983).*

Here, s is a species label. Substituting equation (27) in equation (25), then the linear dielectric tensor for the beam-plasma system is given by

$$\varepsilon_{ij} = \varepsilon_o \delta_{ij} + \frac{1}{\omega + i\delta} \sum_s e_s^2 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}\left[\delta_{jm}\left(1 - \frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta}\right) + \frac{k_m v_{sj}}{\omega + i\delta}\right]}{\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)} \frac{\partial f_{p_s}^{R(0)}}{\partial p_{sm}} \quad , \tag{28}$$

in which case equation (18) becomes

$$G_{ij}^{-1} = \frac{1}{\mu_o(\omega + i\delta)^2}\left(k_i k_j - k^2\delta_{ij}\right) + \varepsilon_o \delta_{ij}$$
$$+ \frac{1}{\omega + i\delta} \sum_s e_s^2 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}\left[\delta_{jm}\left(1 - \frac{\vec{k}\cdot\vec{v}_s}{\omega + i\delta}\right) + \frac{k_m v_{sj}}{\omega + i\delta}\right]}{\omega - \vec{k}\cdot\vec{v}_s + i\delta} \frac{\partial f_{p_s}^{R(0)}}{\partial p_{sm}} \quad . \tag{29}$$

It should be noted that equations (27), (28), and (29) are in fact symmetric in i and j which follows after a simple integration by parts. The zeros of the determinant of the matrix equation (29) must give the dispersive properties of the bremsstrahlung field.

From equation (19) one has that

$$\varepsilon_\ell = \frac{k_i k_j}{k^2} \varepsilon_{ij} \quad , \tag{30}$$

and

$$\varepsilon_t = \frac{1}{2}\left(\varepsilon_{ii} - \varepsilon_\ell\right) \quad . \tag{31}$$

Substituting equation (28) in equations (30) and (31), then in the spatially isotropic case, the longitudinal and transverse dielectric permittivity are given by

$$\varepsilon_\ell = \varepsilon_o + \frac{1}{\omega + i\delta} \sum_s \frac{e_s^2}{k^2} \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{\vec{k}\cdot\vec{v}_s}{\omega - \vec{k}\cdot\vec{v}_s + i\delta} \vec{k}\cdot\vec{\nabla}_{p_s} f_{p_s}^{R(0)} \tag{32}$$

and

$$\varepsilon_t = \varepsilon_o + \frac{1}{2\left(\omega + i\delta\right)^2} \sum_s \frac{e_s^2}{k^2} \int \frac{d^3\vec{p}_s}{(2\pi)^3} \left[k^2 v_{sm} + \frac{k^2 v_s^2 - \omega\vec{k}\cdot\vec{v}_s}{\omega - \vec{k}\cdot\vec{v}_s + i\delta} k_m\right] \frac{\partial f_{p_s}^{R(0)}}{\partial p_{sm}} \quad . \tag{33}$$

If the background distribution functions $f_{p_s}^{R(0)}$ are also isotropic in momentum space, then they depend only on particle energy; namely,

$$f_{p_s}^{R(0)} = f_{p_s}^{R(0)}\left(\varepsilon_s\right) \quad , \tag{34}$$

where

$$\varepsilon_s = \left(p_s^2c^2 + m_s^2c^4\right)^{1/2} \quad . \tag{35}$$

Then

$$\frac{\partial f_{p_s}^{R(0)}}{\partial p_{sm}} = \frac{\partial f_{p_s}^{R(0)}}{\partial \varepsilon_s}\frac{\partial \varepsilon_s}{\partial p_{sm}} = \frac{\partial f_{p_s}^{R(0)}}{\partial \varepsilon_s}\frac{p_{sm}c^2}{\varepsilon_s} = v_{sm}\frac{\partial f_{p_s}^{R(0)}}{\partial \varepsilon_s} \quad . \tag{36}$$

Substituting equation (36) in equation (28) and simplifying, then for the case of isotropy in momentum space, one has

$$\varepsilon_{ij} = \varepsilon_0\delta_{ij} + \frac{1}{\omega + i\delta}\sum_s e_s^2\int\frac{d^3\vec{p}_s}{(2\pi)^3}\frac{v_{si}v_{sj}}{\left(\omega - \vec{k}\cdot\vec{v}_s + i\delta\right)}\frac{\partial f_{p_s}^{R(0)}}{\partial \varepsilon_s} \quad . \tag{37}$$

Substituting equation (37) in equations (30) and (31), then in the case of isotropy in momentum space, the longitudinal and transverse dielectric permittivity are given by

$$\varepsilon_\ell = \varepsilon_0 + \frac{1}{\omega + i\delta}\sum_s\frac{e_s^2}{k^2}\int\frac{d^3\vec{p}_s}{(2\pi)^3}\frac{(\vec{k}\cdot\vec{v}_s)^2}{\omega - \vec{k}\cdot\vec{v}_s + i\delta}\frac{\partial f_{p_s}^{R(0)}}{\partial \varepsilon_s} \tag{38}$$

and

$$\varepsilon_t = \varepsilon_0 + \frac{1}{2}\frac{1}{\omega + i\delta}\sum_s e_s^2\int\frac{d^3\vec{p}_s}{(2\pi)^3}\frac{v_s^2 - (\vec{k}\cdot\vec{v}_s)^2/k^2}{\omega - \vec{k}\cdot\vec{v}_s + i\delta}\frac{\partial f_{p_s}^{R(0)}}{\partial \varepsilon_s} \quad , \tag{39}$$

and the zeros of the determinant of equation (20) along with equations (38) and (39) then give the dispersion relation for the bremsstrahlung in this case.

3. THE STOCHASTIC PROPERTIES OF THE BREMSSTRAHLUNG FIELD

As stated earlier, it is to be assumed that the bremsstrahlung field resulting from the particle scattering in the system is stochastic, having an irregular rapidly varying phase.[1] To the needed order, the phase of the stochastic bremsstrahlung field is assumed to change randomly during a typical interaction period, and the time average is assumed to be the same as the average over the stochastic ensemble. In particular it is assumed that for this stochastic field the following averages over the statistical phase distribution apply approximately; namely,

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[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

$$\langle E^{st}_{ki} \rangle = 0 \quad , \tag{40}$$

$$\langle E^{st}_{ki} E^{st}_{k_1 j} \rangle = \left| E^{st}_k \right|^2 \delta\left(k + k_1\right) e_{ki} e^*_{kj} \quad , \tag{41}$$

where $\vec{e}_k$ is the unit polarization vector for the stochastic field.

Equations (40) and (41) are deduced as follows. Representing the stochastic electric field in terms of its modulus, A_k, and phase, ϕ_k, one has

$$E^{st}_{ki} = e_{ki} A_k e^{i\phi_k} = e_{ki} A_k \left(\cos \phi_k + i \sin \phi_k\right) \quad . \tag{42}$$

Therefore, the average over the statistical phase distribution $\langle E^{st}_{ki} \rangle$ is given by

$$\langle E^{st}_{ki} \rangle = e_{ki} A_k \left(\langle \cos \phi_k + i \sin \phi_k \rangle\right) \quad . \tag{43}$$

But assuming random phase to the needed order, then clearly

$$\langle \cos \phi_k \rangle = \langle \sin \phi_k \rangle = 0 \quad , \tag{44}$$

and equation (43) becomes equation (40). Similarly,

$$\langle E^{st}_{ki} E^{st*}_{k'j} \rangle = e_{ki} e^*_{k'j} A_k A_{k'} \langle e^{i\phi_k} e^{-i\phi_{k'}} \rangle \quad . \tag{45}$$

But

$$\langle e^{i\phi_k} e^{-i\phi_{k'}} \rangle = \langle e^{i\left(\phi_k - \phi_{k'}\right)} \rangle = \delta_{k'k} \quad , \tag{46}$$

since it is vanishing unless $k = k'$ and unity when $k = k'$. Substituting equation (46) in equation (45), using the property of the Kronecker delta, denoting k' by $-k_1$, and using the reality property of the field $\left(\vec{E}^*_{-k} = \vec{E}_k\right)$, then

$$\langle E^{st}_{ki} E^{st}_{k_1 j} \rangle = e_{ki} e^*_{kj} A^2_k \delta_{-k_1, k} \quad . \tag{47}$$

Next, one may define the quantity $\left| E^{st}_k \right|$ in terms of the modulus A_k by

$$\left| E^{st}_k \right|^2 \delta\left(k + k_1\right) = A^2_k \delta_{-k_1, k} \quad . \tag{48}$$

Then substituting equation (48) in equation (47), equation (41) follows.

Furthermore, using equation (3) in equations (40) and (41) one has for the stochastic properties of the bremsstrahlung radiation field the following:

$$\langle E^{\sigma(0)}_{ki} \rangle = 0 \quad , \tag{49}$$

$$\langle E^{\sigma(0)}_{ki} E^{\sigma(0)}_{k_1 j} \rangle = e^{\sigma}_{ki} e^{\sigma *}_{kj} \left| E^{\sigma(0)}_k \right|^2 \delta\left(k + k_1\right) \quad , \tag{50}$$

where $\vec{e}^{\,\sigma}_k$ is the unit polarization vector for mode σ.

4. THE FIELD DUE TO THE PERTURBED TEST PARTICLE TRAJECTORY

The trajectory of a relativistic test particle of species α moving through the beam-plasma system is given by

$$\vec{r}_\alpha = \vec{v}_\alpha t + \Delta\vec{r}_\alpha \quad , \tag{51}$$

where $\vec{v}_\alpha$ is the unperturbed velocity and $\Delta\vec{r}_\alpha$ and $\Delta\vec{v}_\alpha$ describe the perturbation in the trajectory and the velocity, respectively. Provided the conditions for the plasma Born approximation are satisfied--namely, that the relativistic particle momentum be much greater than the electromagnetic impulse received by the particle in a time interval given by the inverse electron plasma frequency--then the perturbed motion is given by equation (36) of HDL-TR-1995:*

$$\begin{aligned}\Delta\vec{r}_\alpha = &- \frac{e_\alpha}{\gamma_\alpha m_\alpha} \int \frac{d^3\vec{k}\, d\omega}{(\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta)^2} \left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_\alpha}{\omega + i\delta}\right) + \vec{k}\, \frac{\vec{v}_\alpha\cdot\vec{E}_k}{\omega + i\delta}\right] e^{-i(\omega - \vec{k}\cdot\vec{v}_\alpha)t} \\ &+ \frac{e_\alpha \vec{v}_\alpha}{\gamma_\alpha m_\alpha c^2} \int \frac{d^3\vec{k}\, d\omega}{(\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta)^2}\, \vec{v}_\alpha\cdot\vec{E}_k\, e^{-i(\omega - \vec{k}\cdot\vec{v}_\alpha)t} \quad .\end{aligned} \tag{52}$$

Here e_α, m_α, and $\vec{v}_\alpha$ are the charge, mass, and unperturbed velocity of the test particle, respectively. (For notational convenience, the naught in HDL-TR-1995 is dropped here.) The unperturbed relativistic factor γ_α is given by

$$\gamma_\alpha = \left[1 - \left(\frac{v_\alpha}{c}\right)^2\right]^{-1/2} \quad . \tag{53}$$

The perturbation in the velocity is given by equation (35) of HDL-TR-1995:*

$$\begin{aligned}\Delta\vec{v}_\alpha = &\frac{ie_\alpha}{\gamma_\alpha m_\alpha} \int d^3\vec{k}\, d\omega \left[\vec{E}_k\left(1 - \frac{\vec{k}\cdot\vec{v}_\alpha}{\omega + i\delta}\right) + \vec{k}\left(\frac{\vec{v}_\alpha\cdot\vec{E}_k}{\omega + i\delta}\right)\right] \frac{e^{-i(\omega - \vec{k}\cdot\vec{v}_\alpha)t}}{\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta} \\ &- \frac{ie_\alpha \vec{v}_\alpha}{\gamma_\alpha m_\alpha c^2} \int d^3\vec{k}\, d\omega\, \frac{\vec{v}_\alpha\cdot\vec{E}_k e^{-i(\omega - \vec{k}\cdot\vec{v}_\alpha)t}}{\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta} \quad .\end{aligned} \tag{54}$$

The i^{th} component of $\Delta\vec{v}_\alpha$, equation (54), can be equivalently rewritten as follows:

*H. E. Brandt, *Nonlinear Force on an Unpolarized Relativistic Test Particle to Second Order in the Total Field in a Nonequilibrium Beam-Plasma System*, Harry Diamond Laboratories, HDL-PRL-82-7 (May 1982) (to be published as HDL-TR-1995, 1983).

$$\Delta v_{\alpha i} = \frac{-ie_\alpha}{\gamma_\alpha m_\alpha} \int d^3\vec{k}\, d\omega \left(\frac{v_{\alpha i}\, v_{\alpha j}}{c^2} - \delta_{ij}\right) \left[E_{kj}\left(1 - \frac{\vec{k}\cdot\vec{v}_\alpha}{\omega + i\delta}\right)\right. \tag{55}$$

$$\left. + k_j \frac{\vec{v}_\alpha \cdot \vec{E}_k}{\omega + i\delta}\right] \frac{e^{-i(\omega - \vec{k}\cdot\vec{v}_\alpha)t}}{\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta} \ .$$

The current density due to the actual charge of the test particle is then given by

$$\vec{j}_\alpha = e_\alpha(\vec{v}_\alpha + \Delta\vec{v}_\alpha)\delta^3(\vec{r} - \vec{v}_\alpha t - \Delta\vec{r}_\alpha) \ . \tag{56}$$

Expanding the delta function in equation (56) for small $\Delta\vec{r}_\alpha$, then to lowest order in $\Delta\vec{r}_\alpha$,

$$\vec{j}_\alpha = e_\alpha(\vec{v}_\alpha + \Delta\vec{v}_\alpha)[\delta^3(\vec{r} - \vec{v}_\alpha t) - \Delta\vec{r}_\alpha \cdot \vec{\nabla}\delta^3(\vec{r} - \vec{v}_\alpha t)] \ . \tag{57}$$

Thus, to lowest order in $\Delta\vec{r}_\alpha$ and $\Delta\vec{v}_\alpha$, one has

$$\vec{j}_\alpha = \vec{j}_\alpha^{(1)} + \vec{j}_\alpha^{(2)} \ , \tag{58}$$

where

$$\vec{j}_\alpha^{(1)} = e_\alpha \vec{v}_\alpha \delta^3(\vec{r} - \vec{v}_\alpha t) \tag{59}$$

and

$$\vec{j}_\alpha^{(2)} = e_\alpha \Delta\vec{v}_\alpha \delta^3(\vec{r} - \vec{v}_\alpha t) - e_\alpha \vec{v}_\alpha \Delta\vec{r}_\alpha \cdot \vec{\nabla}\delta^3(\vec{r} - \vec{v}_\alpha t) \ . \tag{60}$$

The Fourier components of these currents at the position of the test particle are given by

$$j_{\alpha ki}^{(1)} = \int \frac{d^3\vec{r}\, dt}{(2\pi)^4} j_{\alpha i}^{(1)}(\vec{r},t) e^{-i(\vec{k}\cdot\vec{r} - \omega t)} \tag{61}$$

$$j_{\alpha ki}^{(2)} = \int \frac{d^3\vec{r}\, dt}{(2\pi)^4} j_{\alpha i}^{(2)}(\vec{r},t) e^{-i(\vec{k}\cdot\vec{r} - \omega t)} \ . \tag{62}$$

Substituting equation (59) in equation (61) and integrating over the delta function, then

$$j_{\alpha ki}^{(1)} = \frac{e_\alpha v_{\alpha i}}{(2\pi)^3} \int \frac{dt}{2\pi} e^{-i(\vec{k}\cdot\vec{v}_\alpha - \omega)t} \ . \tag{63}$$

Next, recognizing the integral form of the delta function appearing in equation (63), then

$$j_{\alpha ki}^{(1)} = \frac{e_\alpha v_{\alpha i}}{(2\pi)^3} \delta(\omega - \vec{k}\cdot\vec{v}_\alpha) \ . \tag{64}$$

Next, substituting equation (60) in equation (62) one has

$$j^{(2)}_{\alpha k i} = \int \frac{d^3\vec{r}\,dt}{(2\pi)^4}\, e_\alpha \Delta v_{\alpha i}\, \delta^3(\vec{r} - \vec{v}_\alpha t) e^{-i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$- \int \frac{d^3\vec{r}\,dt}{(2\pi)^4}\, e_\alpha v_{\alpha i}\, \Delta\vec{r}_\alpha \cdot [\vec{\nabla}\delta^3(\vec{r} - \vec{v}_\alpha t)] e^{-i(\vec{k}\cdot\vec{r}-\omega t)} \quad . \tag{65}$$

The second integral in equation (65) can be simplified by integration over space by parts and dropping surface terms. Equation (65) becomes

$$j^{(2)}_{\alpha k i} = \int \frac{d^3\vec{r}\,dt}{(2\pi)^4}\, e_\alpha \Delta v_{\alpha i}\, \delta^3(\vec{r} - \vec{v}_\alpha t) e^{-i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$- \int \frac{d^3\vec{r}\,dt}{(2\pi)^4}\, e_\alpha v_{\alpha i}\, \delta^3(\vec{r} - \vec{v}_\alpha t) i\vec{k}\cdot\Delta\vec{r}_\alpha e^{-i(\vec{k}\cdot\vec{r}-\omega t)} \quad . \tag{66}$$

Integrating over the delta functions, then

$$j^{(2)}_{\alpha k i} = \int \frac{dt}{(2\pi)^4}\, e_\alpha \Delta v_{\alpha i} e^{-i(\vec{k}\cdot\vec{v}_\alpha-\omega)t}$$

$$- i \int \frac{dt}{(2\pi)^4}\, e_\alpha v_{\alpha i} e^{-i(\vec{k}\cdot\vec{v}_\alpha-\omega)t}\, \vec{k}\cdot\Delta\vec{r}_\alpha \quad . \tag{67}$$

Next, substituting equations (52) and (55) in equation (67) and integrating over the time, then

$$j^{(2)}_{\alpha k i} = \frac{-ie_\alpha^2}{\gamma_\alpha m_\alpha}\left(\frac{v_{\alpha i} v_{\alpha j}}{c^2} - \delta_{ij}\right) \int \frac{d^3\vec{k}_1 d\omega_1}{(2\pi)^3} \left[E_{k_1 j}\left(1 - \frac{\vec{k}_1\cdot\vec{v}_\alpha}{\omega_1 + i\delta}\right)\right.$$

$$\left. + k_{1j}\frac{\vec{v}_\alpha\cdot\vec{E}_{k_1}}{\omega_1 + i\delta}\right] \frac{\delta(\omega - \omega_1 - (\vec{k} - \vec{k}_1)\cdot\vec{v}_\alpha)}{\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta}$$

$$+ \frac{ie_\alpha^2}{\gamma_\alpha m_\alpha} v_{\alpha i} \int \frac{d^3\vec{k}_1\, d\omega_1}{(2\pi)^3} \frac{\left[\vec{k}\cdot\vec{E}_{k_1}\left(1 - \frac{\vec{k}_1\cdot\vec{v}_\alpha}{\omega_1 + i\delta}\right) + \frac{\vec{k}\cdot\vec{k}_1 \vec{v}_\alpha\cdot\vec{E}_{k_1}}{\omega_1 + i\delta}\right]}{(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta)^2} \tag{68}$$

$$\times\, \delta(\omega - \omega_1 - (\vec{k} - \vec{k}_1)\cdot\vec{v}_\alpha)$$

$$- \frac{ie_\alpha^2}{\gamma_\alpha m_\alpha c^2} v_{\alpha i} \int \frac{d^3\vec{k}_1 d\omega_1}{(2\pi)^3} \frac{\vec{k}\cdot\vec{v}_\alpha\ \vec{v}_\alpha\cdot\vec{E}_{k_1}}{(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta)^2}\, \delta(\omega - \omega_1 - (\vec{k} - \vec{k}_1)\cdot\vec{v}_\alpha) \quad .$$

Simplifying equation (68), then

$$j_{\alpha ki}^{(2)} = \frac{ie_\alpha^2}{\gamma_\alpha m_\alpha}\int \frac{d^3k_1 d\omega_1}{(2\pi)^3}\,\frac{E_{k_1 j}}{\omega_1 + i\delta}\,\delta\left(\omega - \omega_1 - \vec{k}\cdot\vec{v}_\alpha + \vec{k}_1\cdot\vec{v}_\alpha\right)$$

$$\times\left\{\delta_{ij} - \frac{v_{\alpha i}v_{\alpha j}}{c^2} - \frac{\vec{k}_1\cdot\vec{v}_\alpha\, v_{\alpha i}v_{\alpha j}}{\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta\right)c^2} + \frac{k_{1i}v_{\alpha j}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta}\right. \tag{69}$$

$$\left. + \frac{k_j v_{\alpha i}}{\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta} + \frac{v_{\alpha i}v_{\alpha j}\vec{k}\cdot\vec{k}_1}{\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta\right)^2} - \frac{\omega_1\vec{k}\cdot\vec{v}_\alpha\, v_{\alpha i}v_{\alpha j}}{c^2\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta\right)^2}\right\} .$$

Using the property of the delta function to replace $\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha\right)$ by $\left(\omega - \vec{k}\cdot\vec{v}_\alpha\right)$ in the fourth, fifth, and sixth terms of equation (69), then

$$j_{\alpha ki}^{(2)} = \frac{ie_\alpha^2}{\gamma_\alpha m_\alpha}\int \frac{d^3k_1 d\omega_1}{(2\pi)^3}\,\frac{E_{k_1 j}}{\omega_1 + i\delta}\,\delta\left(\omega - \omega_1 - \vec{k}\cdot\vec{v}_\alpha + \vec{k}_1\cdot\vec{v}_\alpha\right)$$

$$\times\left\{\delta_{ij} + \frac{k_j\cdot v_{\alpha i} + k_{1i}v_{\alpha j}}{\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta} + \frac{\vec{k}\cdot\vec{k}_1 v_{\alpha i}v_{\alpha j}}{\left(\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta\right)^2} - \frac{v_{\alpha i}v_{\alpha j}}{c^2}\right. \tag{70}$$

$$\left. - \frac{\vec{k}_1\cdot\vec{v}_\alpha v_{\alpha i}v_{\alpha j}}{\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta\right)c^2} - \frac{\omega_1}{c^2}\,\frac{\vec{k}\cdot\vec{v}_\alpha v_{\alpha i}v_{\alpha j}}{\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha + i\delta\right)^2}\right\} .$$

Changing the integration variable $k_1 = \left(\omega_1, \vec{k}_1\right)$ to $-k_1$, equation (70) becomes

$$j_{\alpha ki}^{(2)} = \frac{ie_\alpha^2}{\gamma_\alpha m_\alpha}\int \frac{d^3k_1\, d\omega_1}{(2\pi)^3}\,\frac{E_{-k_1 j}}{-\omega_1 + i\delta}\,\delta\left(\omega + \omega_1 - \left(\vec{k} + \vec{k}_1\right)\cdot\vec{v}_\alpha\right)$$

$$\times\left\{\delta_{ij} + \frac{k_j v_{\alpha i} - k_{1i}v_{\alpha j}}{\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta} - \frac{\vec{k}\cdot\vec{k}_1 v_{\alpha i}v_{\alpha j}}{\left(\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta\right)^2} - \frac{v_{\alpha i}v_{\alpha j}}{c^2}\right. \tag{71}$$

$$\left. - \frac{\vec{k}_1\cdot\vec{v}_\alpha v_{\alpha i}v_{\alpha j}}{c^2\left(\bar{\omega}_1 - \vec{k}_1\cdot\vec{v}_\alpha - i\delta\right)} + \frac{\omega_1}{c^2}\,\frac{\vec{k}\cdot\vec{v}_\alpha v_{\alpha i}v_{\alpha j}}{\left(\bar{\omega}_1 - \vec{k}_1\cdot\vec{v}_\alpha - i\delta\right)^2}\right\} .$$

Combining terms and using the property of the delta function, one notes that

$$\delta\left(\omega + \omega_1 - \vec{k}\cdot\vec{v}_\alpha - \vec{k}_1\cdot\vec{v}_\alpha\right)\left[-\frac{v_{\alpha i}v_{\alpha j}}{c^2} - \frac{\vec{k}_1\cdot\vec{v}_\alpha v_{\alpha i}v_{\alpha j}}{\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha - i\delta\right)c^2}\right. \tag{72}$$

$$\left. + \frac{\omega_1}{c^2}\,\frac{\vec{k}\cdot\vec{v}_\alpha v_{\alpha i}v_{\alpha j}}{\left(\omega_1 - \vec{k}_1\cdot\vec{v}_\alpha - i\delta\right)^2}\right] = \delta\left(\omega + \omega_1 - \vec{k}\cdot\vec{v}_\alpha - \vec{k}_1\cdot\vec{v}_\alpha\right)\frac{v_{\alpha i}v_{\alpha j}\,\omega\omega_1}{c^2\left(\omega - \vec{k}\cdot\vec{v}_\alpha + i\delta\right)^2} .$$

Substituting equation (72) in equation (71) and using the reality property of the field $\left(\vec{E}_k = \vec{E}^*_{-k}\right)$, then

$$j^{(2)}_{\alpha ki} = -ie_\alpha \int \frac{dk_1}{(2\pi)^3} \frac{E^*_{k_1 j}}{\omega_1 - i\delta} \delta\left(\omega + \omega_1 - \vec{k}\cdot\vec{v}_\alpha - \vec{k}_1\cdot\vec{v}_\alpha\right)\Lambda^{(\alpha)*}_{ij}(k_1,k) \quad , \tag{73}$$

where

$$\Lambda^{(\alpha)}_{ij}(k_1,k) = \frac{e_\alpha}{\gamma_\alpha m_\alpha}\left\{\delta_{ij} + \frac{v_{\alpha i}k_j - k_{1i}v_{\alpha j}}{\omega - \vec{k}\cdot\vec{v}_\alpha - i\delta} - \frac{v_{\alpha i}v_{\alpha j}\left(\vec{k}\cdot\vec{k}_1 - \frac{\omega\omega_1}{c^2}\right)}{\left(\omega - \vec{k}\cdot\vec{v}_\alpha - i\delta\right)^2}\right\} . \tag{74}$$

The currents $j^{(1)}_{\alpha ki}$ and $j^{(2)}_{\alpha ki}$ given by equations (64) and (73) are the currents associated with the unperturbed motion and the perturbation in the motion, respectively, of the bare particle. By the same arguments leading to equation (21), the components of the field due to the unperturbed and perturbed test particle trajectory are given by

$$E^{(1)}_{kn} = -\frac{i}{\omega + i\delta} G_{nm} j^{(1)}_{km} \tag{75}$$

and

$$E^{(2)}_{kn} = -\frac{i}{\omega + i\delta} G_{nm} j^{(2)}_{km} \quad , \tag{76}$$

respectively. Therefore, substituting equations (64) and (73) in equations (75) and (76), one obtains

$$E^{(1)}_{kn} = -\frac{ie_\alpha}{(2\pi)^3(\omega + i\delta)} v_{\alpha m} G_{nm}(k)\,\delta\left(\omega - \vec{k}\cdot\vec{v}_\alpha\right) \tag{77}$$

and

$$E^{(2)}_{kn} = -\frac{e_\alpha}{(2\pi)^3(\omega + i\delta)} G_{nm}(k) \int \frac{dk_1}{\omega_1 - i\delta} E^*_{k_1 j} \Lambda^{(\alpha)*}_{mj}(k_1,k)\,\delta\left(\omega + \omega_1 - \vec{k}\cdot\vec{v}_\alpha - \vec{k}_1\cdot\vec{v}_\alpha\right). \tag{78}$$

Equations (77) and (78) agree with equations (16) and (17) of Akopyan and Tsytovich, 1975,[1] except for erroneous overall factors of $(2\pi)^{-3}$ and $(2\pi)^{-3}i$ appearing there in equations (16) and (17), respectively. The indices n and m are interchanged there as they are in the defining relation, equation (14) of Akopyan,[1] compared to equation (21) here. However, a spatially isotropic plasma is assumed there, so according to equation (22) the Green's function is symmetric, namely $G_{mn} = G_{nm}$. Also in Akopyan,[1] single-wave particle resonance is ignored, in which case the small imaginary part $i\delta$ in equation (74) is

[1] *A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

ignorable, $\Lambda^{(\alpha)}_{mj}$ becomes effectively real, and the complex conjugate sign in equation (78) may be removed. The additional factors of $(4\pi)(2\pi)^3$ in equation (16) and (4π) in (17) there are due to the Gaussian form of Maxwell's equation used there compared to the MKS form used here, and to the difference in Fourier transform convention. For example, in equation (5) of Akopyan and Tsytovich, 1975,[1] the Fourier transform convention employed has a factor of $(2\pi)^{-3}$ in the inverse Fourier transform in the integration over the three-dimensional wave vector space, and a factor of 1 for the integration over frequency, giving a total factor of $(2\pi)^{-3}$. In this report, a total factor of 1 is used, as in equation (7), for example. Likewise, the Fourier transform itself then has a factor of $(2\pi)^{-1}$ there and $(2\pi)^{-4}$ here, as in equation (61), for example.

5. THE FIELD DUE TO THE DYNAMIC POLARIZATION CURRENT

The Fourier transform of the dynamic polarization current to third-order induced in the beam-plasma is given by[1,*]

$$\vec{j}_{dpk} = \vec{j}^{(1)}_{dpk} + \vec{j}^{(2)}_{dpk} + \vec{j}^{(3)}_{dpk} \quad . \tag{79}$$

The linear polarization current density $\vec{j}^{(1)}_{dpk}$ is given by

$$j^{(1)}_{dpki} = \sum_s \sigma^{(s)}_{i\ell}(k) E_{k\ell} \quad . \tag{80}$$

Its effects are manifest in the linear Green's function for the beam-plasma system as discussed in section 2. The second-order and third-order polarization current densities $j^{(2)}_{dpk}$ and $j^{(3)}_{dpk}$ are given by

$$j^{(2)}_{dpki} = -\sum_s e_s \int \frac{dk_1\, dk_2\, \delta(k - k_1 - k_2)}{(\omega_1 + i\delta)(\omega_2 + i\delta)} S^{(s)}_{ij\ell}(k,k_1,k_2) E_{k_1 j} E_{k_2 \ell} \quad , \tag{81}$$

and

$$j^{(3)}_{dpki} = -\sum_s e_s \int \frac{dk_1\, dk_2\, dk_3\, \delta(k - k_1 - k_2 - k_3)}{(\omega_1 + i\delta)(\omega_2 + i\delta)(\omega_3 + i\delta)} \tag{82}$$

$$\times \Sigma^{(s)}_{ij\ell m}(k,k_1,k_2,k_3) E_{k_1 j} E_{k_2 \ell} E_{k_3 m} \quad ,$$

respectively. The second-order nonlinear and third-order nonlinear conductivities for species s, $S^{(s)}_{ij\ell}$, and $\Sigma^{(s)}_{ij\ell m}$, respectively, are given by

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[1]A. V. Akopyan and V. N. Tsytovich, *Bremsstrahlung in a Nonequilibrium Plasma*, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].

*H. E. Brandt, *Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System*, Harry Diamond Laboratories, HDL-PRL-82-6 (May 1982) (to be published as HDL-TR-1994, 1983).

$$S^{(s)}_{ij\ell}(k,k_1,k_2) = e_s^2 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}}{\omega - \vec{k}\cdot\vec{v}_s + i\delta} \left[\left(\omega_1 - \vec{k}_1\cdot\vec{v}_s\right) \frac{\partial}{\partial p_{sj}} + v_j k_{1m} \frac{\partial}{\partial p_{sm}}\right] \left[\frac{\partial}{\partial p_{s\ell}} + \frac{v_{s\ell}}{\omega_2 - \vec{k}_2\cdot\vec{v}_s + i\delta} k_{2n} \frac{\partial}{\partial p_{sn}}\right] f^{R(0)}_{p_s} , \tag{83}$$

and

$$\Sigma^{(s)}_{ij\ell m}(k,k_1,k_2,k_3) = -ie_s^3 \int \frac{d^3\vec{p}_s}{(2\pi)^3} \frac{v_{si}}{\omega - \vec{k}\cdot\vec{v}_s + i\delta} \left[\delta_{jn}\left(\omega_1 - \vec{k}_1\cdot\vec{v}_s\right) + k_{1n}v_{sj}\right] \times \frac{\partial}{\partial p_{sn}} \frac{1}{\omega - \omega_1 - \left(\vec{k} - \vec{k}_1\right)\cdot\vec{v}_s + i\delta} \left[\delta_{\ell u}\left(\omega_2 - \vec{k}_2\cdot\vec{v}_s\right) + k_{2u}v_{s\ell}\right] \frac{\partial}{\partial p_{su}} \frac{1}{\omega_3 - \vec{k}_3\cdot\vec{v}_s + i\delta} \left[\delta_{mq}\left(\omega_3 - \vec{k}_3\cdot\vec{v}_s\right) + k_{3q}v_{sm}\right] \frac{\partial}{\partial p_{sq}} f^{R(0)}_{p_s} . \tag{84}$$

The function $f^{R(0)}_{p_s}$ is the assumed slowly varying and spatially independent background distribution function for species s, whose Fourier transform is

$$f^{R(s)}_k = f^{R(0)}_{p_s}\delta(k) . \tag{85}$$

To determine the fields defined in equation (2) and associated with the respective parts of the dynamic polarization current, one proceeds as in obtaining equations (75) and (76). Thus, one has for the fields produced by the second- and third-order nonlinear dynamic polarization currents, respectively,

$$E^{(1)}_{dpkn} = -\frac{i}{\omega + i\delta} G_{nm}(k) j^{(2)}_{dpkm} \tag{86}$$

and

$$E^{(2)}_{dpkn} = -\frac{i}{\omega + i\delta} G_{nm}(k) j^{(3)}_{dpkm} . \tag{87}$$

Next, substituting equation (81) in equation (86), one obtains

$$E^{(1)}_{dpkn} = \frac{i}{\omega + i\delta} G_{nm}(k) \sum_s e_s \int \frac{dk_1\, dk_2\, \delta\left(k - k_1 - k_2\right)}{\left(\omega_1 + i\delta\right)\left(\omega_2 + i\delta\right)} \times S^{(s)}_{mj\ell}(k,k_1,k_2) E_{k_1 j} E_{k_2 \ell} . \tag{88}$$

First symmetrizing equation (88) in (j,k_1) and (ℓ,k_2), changing variables of integration from k_1 to $-k_1$ and k_2 to $-k_2$, and using the reality property of the field, then equation (88) becomes

$$E^{(1)}_{dpkn} = \frac{i}{2(\omega + i\delta)} G_{nm}(k) \sum_s e_s \int \frac{dk_1\, dk_2}{(\omega_1 - i\delta)(\omega_2 - i\delta)} \delta(k + k_1 + k_2)$$

$$\times \left[S^{(s)}_{mj\ell}(k,-k_1,-k_2) + S^{(s)}_{m\ell j}(k,-k_2,-k_1)\right] E^*_{k_1 j} E^*_{k_2 \ell} \quad . \tag{89}$$

Comparing equation (89) with equation (22) of Akopyan,[1] a disparity of a factor of $(1/2)(4\pi)(2\pi)^{-6}$ in the latter is apparently due to explicit inclusion there of a self-field factor of (1/2), the use of Gaussian units, and the different Fourier transform and normalization conventions chosen there. The factor of (1/2) is either a typographical error or results from explicit inclusion of a factor of 1/2 associated with self-fields. For the generic expression, equation (89), it is preferable that such a factor be maintained implicit until the formula is applied explicitly and a self-field factor of (1/2) is needed. The factor of (4π) is due to the use of Gaussian units there. One of the factors of $(2\pi)^{-3}$ is due to the differing Fourier transform convention. Because of the different Fourier transform convention, the counterpart of equation (81) would have an additional factor of $(2\pi)^{-3}$. The other factor of $(2\pi)^{-3}$ is apparently due to the different normalization of $f^{R(0)}_{p_s}$ there. In short, the $f^{R(0)}_{p_s}$ there is evidently $(2\pi)^3$ times that here. Alternatively, if the normalization is the same as that here then there is an erroneous factor of $(2\pi)^{-3}$ appearing there. An isotropic plasma is assumed there, in which case by equation (22) the Green's function is symmetric, namely, $G_{mn} = G_{nm}$. However as already noted the indices of G_{ij} are interchanged in Akopyan and Tsytovich[1] in the definition of the Green's function. It appears, however, that there are typographical errors there. The complex conjugate sign on the fields and a factor of e_s are left out. Also, there in the second-order nonlinear conductivity, one notes from equation (83) that*

$$S^{(s)}_{mj\ell}(k,-k_1,-k_2) = S^{(s)*}_{mj\ell}(-k,k_1,k_2) = -\bar{S}^{(s)*}_{mj\ell}(k,k_1,k_2) \quad , \tag{90}$$

where $\bar{S}^{(s)}_{mj\ell}(k,k_1,k_2)$ designates the conductivity tensor appearing in Akopyan and Tsytovich.[1,*] In the last step of equation (90) it is recognized that $\bar{S}^{(s)}_{mj\ell}(k,k_1,k_2)$ differs from $S^{(s)}_{mj\ell}(k,k_1,k_2)$ here in that the first complex denominator $\omega - \vec{k}\cdot\vec{v}_s + i\delta$ in equation (83) here is implicitly $\omega - \vec{k}\cdot\vec{v}_s - i\delta$

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[1]*A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

**H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-82-6 (May 1982) (to be published as HDL-TR-1994, 1983).*

there.[8,9,*] Thus, there is also a disagreement in overall sign between equation (89) here and equation (22) of Akopyan and Tsytovich.[1] In summary then, equation (22) of Akopyan and Tsytovich[1] is in error; it should have an additional factor of $(-e_s)$, the fields and the conductivity tensor on the right-hand side should be complex conjugated, and the explicit additional factor of (1/2) there should be omitted.

Next, substituting equation (82) in equation (87),

$$E^{(2)}_{dpkn} = \frac{i}{\omega + i\delta} G_{nm}(k) \sum_s e_s \int \frac{dk_1\, dk_2\, dk_3\, \delta(k - k_1 - k_2 - k_3)}{(\omega_1 + i\delta)(\omega_2 + i\delta)(\omega_3 + i\delta)}$$

$$\times \Sigma^{(s)}_{mij\ell}(k,k_1,k_2,k_3) E_{k_1 i} E_{k_2 j} E_{k_3 \ell} \quad . \tag{91}$$

Symmetrizing equation (91) in (i,k_1), (j,k_2), (ℓ,k_3), changing variables of integration from k_1 to $-k_1$, k_2 to $-k_2$, and k_3 to $-k_3$, and using the reality property of the field $\left(\vec{E}_{-k} = \vec{E}^*_k\right)$, then equation (91) becomes

$$E^{(2)}_{dpkn} = \frac{-i}{6(\omega + i\delta)} G_{nm}(k) \sum_s e_s \int \frac{dk_1\, dk_2\, dk_3\, \delta(k + k_1 + k_2 + k_3)}{(\omega_1 - i\delta)(\omega_2 - i\delta)(\omega_3 - i\delta)}$$

$$\times \Big[\Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3) + \Sigma^{(s)}_{mi\ell j}(k,-k_1,-k_3,-k_2)$$

$$+ \Sigma^{(s)}_{mji\ell}(k,-k_2,-k_1,-k_3) + \Sigma^{(s)}_{mj\ell i}(k,-k_2,-k_3,-k_1) \tag{92}$$

$$+ \Sigma^{(s)}_{m\ell ij}(k,-k_3,-k_1,-k_2) + \Sigma^{(s)}_{m\ell ji}(k,-k_3,-k_2,-k_1) \Big] E^*_{k_1 i} E^*_{k_2 j} E^*_{k_3 \ell} \quad .$$

Comparing equation (92) with equation (23) of Akopyan and Tsytovich,[1] the disparity of a factor of $(1/2)(4\pi)(2\pi)^{-9}$ in the latter is again due to an explicit self-field factor of (1/2), the use of Gaussian units, and the different Fourier transform and background normalization conventions there.

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[1] A. V. Akopyan and V. N. Tsytovich, *Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

[8] H. E. Brandt, *On the Nonlinear Conductivity Tensor for an Unmagnetized Relativistic Turbulent Plasma, Harry Diamond Laboratories, HDL-TR-1970 (February 1982).*

[9] H. E. Brandt, *Comment on "Exact Symmetry of the Second-Order Nonlinear Conductivity for a Relativistic Turbulent Plasma," Phys. Fluids 25 (1982), 1922.*

*H. E. Brandt, *Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-82-6 (May 1982) (to be published as HDL-TR-1994, 1983).*

Alternatively if the normalization is instead the same as that here, then there is an erroneous factor of $(2\pi)^{-3}$ appearing there. There is apparently an error there. A factor of ie_s has been left out. Note that by equation (84) it follows that

$$\Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3) = -\Sigma^{(s)*}_{mij\ell}(-k,k_1,k_2,k_3) \quad . \tag{93}$$

Also in the conductivity tensor $T^{(s)}_{ij\ell m}(k,k_1,k_2,k_3)$ given by equation (21) of Akopyan and Tsytovich,[1] the first two complex denominators are implicitly $\omega - \vec{k}\cdot\vec{v}_s - i\delta$ and $\omega + \omega_1 - (\vec{k} + \vec{k}_1)\cdot\vec{v}_s - i\delta$, respectively, there.* Also, there is no overall factor of i as there is in equation (84) here. Therefore,

$$\Sigma^{(s)}_{mij\ell}(-k,k_1,k_2,k_3) = -iT^{(s)}_{mij\ell}(k,k_1,k_2,k_3) \quad . \tag{94}$$

Combining equations (93) and (94) then

$$\Sigma^{(s)}_{mij\ell}(k,-k_1,-k_2,-k_3) = -iT^{(s)*}_{mij\ell}(k,k_1,k_2,k_3) \quad . \tag{95}$$

In summary then, equation (23) of Akopyan and Tsytovich[1] is also in error and should also have an additional factor of (ie_s), the complex conjugate of the fourth-order conductivity tensor there must be taken, and the explicit additional factor of (1/2) there should be omitted.

6. THE TOTAL FIELD

Collecting equations (1), (2), and (3) the total field involved in the bremsstrahlung scattering process and acting on the test particle is given by

$$\vec{E}_k = \vec{E}^{\sigma(0)}_k + \vec{E}^{(1)}_k + \vec{E}^{(2)}_k + \vec{E}^{(1)}_{dpk} + \vec{E}^{(2)}_{dpk} \quad . \tag{96}$$

Here $\vec{E}^{\sigma(0)}_k$ is the bremsstrahlung field whose dispersion relations are given by equations (4) and (29) in the spatially isotropic case and equations (4), (20), (38), and (39) in the case of isotropy in both ordinary space and momentum space. The stochastic properties are given by equations (49) and (50). The fields $\vec{E}^{(1)}_k$ and $\vec{E}^{(2)}_k$ given by equations (77) and (78) are the self-fields arising from the relativistic test particle's own motion. The fields $\vec{E}^{(1)}_{dpk}$ and $\vec{E}^{(2)}_{dpk}$ given by equations (89) and (91) are the increasing order fields produced by the nonlinear dynamic polarization current induced by the test particle. The fields $\vec{E}^{(1)}_{dpk}$ and $\vec{E}^{(2)}_{dpk}$ are excited in nonlinear scattering of the field by the induced dynamic polarization current. They involve the three- and four-plasmon vertex, respectively, and play a key role in determining the nonlinear contribution to the collective bremsstrahlung process.

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[1]*A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

**H. E. Brandt, Nonlinear Dynamic Polarization Force on a Relativistic Test Particle in a Nonequilibrium Beam-Plasma System, Harry Diamond Laboratories, HDL-PRL-82-6 (May 1982) (to be published as HDL-TR-1994, 1983).*

7. CONCLUSION

An expression has been obtained, equations (96), (77), (78), (89), and (91), for the total field involved in the collective scattering and bremsstrahlung of a relativistic test particle in a nonequilibrium beam-plasma system. Dispersion relations, equations (4), (29), (20), (38), and (39), and stochastic properties, equations (49) and (50), for the bremsstrahlung field have also been obtained. These results are in agreement with those of Akopyan and Tsytovich[1] with the exception of apparent typographical errors, and have been used by them in their work on nonlinear bremsstrahlung in nonequilibrium plasmas.

The present work, together with related work by the author,[10-14,*†‡§] is important for ongoing work in calculating collective radiation processes and conditions for the occurrence of radiative instability in relativistic nonequilibrium beam-plasma systems.

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[1]*A. V. Akopyan and V. N. Tsytovich, Bremsstrahlung in a Nonequilibrium Plasma, Fiz. Plazmy, 1 (1975), 673 [Sov. J. Plasma Phys., 1 (1975), 371].*

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[§]*Other related work prepared in preprint form will be published later and is available from the author.*

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